## **APPROVAL SHEET**

<b>Title of Thesis:</b> A Fra Agent-Based Models	mework for Predicting and Controlling System-Level Properties	of
Name of Candidate:	Donald P. Miner PhD in Computer Science, 2010	
Thesis and Abstract A	Dr. Marie desJardins Associate Professor Department of Computer Science and Electrical Engineering	
Date Approved:		

#### **Curriculum Vitae**

**Name:** MY-FULL-NAME.

**Permanent Address:** MY-FULL-ADDRESS.

Degree and date to be conferred: DEGREE-NAME, GRADUATION-MONTH

GRADUATION-YEAR.

**Date of Birth:** MY-BIRTHDATE.

Place of Birth: MY-PLACE-OF-BIRTH.

**Secondary Education:** MY-HIGH-SCHOOL, MY-HIGH-SCHOOLS-CITY,

MY-HIGH-SCHOOLS-STATE.

#### **Collegiate institutions attended:**

University of Maryland Baltimore County, DEGREE-NAME MY-MAJOR, GRADUATION-YEAR.

MY-OTHER-DEGREES.

Major: MY-MAJOR.

**Minor:** MY-MINOR.

#### **Professional publications:**

FULL-CITATION-INFORMATION.

FULL-CITATION-INFORMATION.

#### **Professional positions held:**

EMPLOYMENT-INFO. (START-DATE - END-DATE).

EMPLOYMENT-INFO. (START-DATE - END-DATE).

#### **ABSTRACT**

Title of Thesis: A Framework for Predicting and Controlling System-Level Properties of

**Agent-Based Models** 

Donald P. Miner, PhD in Computer Science, 2010

**Thesis directed by:** Dr. Marie desJardins, Associate Professor

Department of Computer Science and

**Electrical Engineering** 

# A Framework for Predicting and Controlling System-Level Properties of Agent-Based Models

by Donald P. Miner

Thesis submitted to the Faculty of the Graduate School of the University of Maryland in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Computer Science 2010

This is my dedication.

#### **ACKNOWLEDGMENTS**

These will be written out later:

- John Way: My excellent high school computer science teacher; he was the first teacher to truly inspire and make me interested in something
- Richard Chang: Unofficial undergrad advisor; Inspired me to work on hard problems with his classes/my undergrad thesis; Indirectly helped me want to pursue graduate school
- Marie: advisor; introducing me to MAS
- Bill Rand: introducing me to NetLogo
- Forrest Stonedahl: working on a similar problem; a conversation which was a turning point in my research focus to ABMs; introduced me to the term 'meta-model'
- Tim Oates: Suggestions dealing with the ML portion of my research
- Undergraduate Researchers (Peter, Kevin, Doug, Nathan?): Helping with researching new domains
- Marc Pickett: Good friend that is always willing to listen to research ideas; Helped me throughout grad school
- Senior grad students who helped me as a young grad student: Adam Anthony, Eric Eaton, Blaz Bulka
- Other supporting CS graduate students: Wes Griffin, Yasaman Haghpanah, Niels Kasch, James MacGlashan, JC Montminy, Sourav M, Patti Ordonez, Soumi Ray, and Brandon Wilson.

## TABLE OF CONTENTS

DEDIC	CATION				
ACKN(	NOWLEDGMENTS iii				
LIST O	OF FIGURES	'n			
LIST O	OF TABLES	i			
Chapte	r 1 INTRODUCTION AND MOTIVATION	1			
1.1	Agent-Based Models	1			
1.2	Wolves, Sheep and Grass	4			
1.3	Overview of the ABM Meta-Modeling Framework	7			
1.4	Summary of Contributions				
1.5	Dissertation Organization				
Chapte	r 2 THE ABM META-MODELING FRAMEWORK 1	0			
2.1	Design Goals of The ABM Meta-Modeling Framework	0			
2.2	Framework Structure				
	2.2.1 Defining System-Level Behavior Properties	4			
	2.2.2 Sampling	4			

	2.2.3	The Forward-Mapping Problem	15
	2.2.4	The Reverse-Mapping Problem	16
	2.2.5	Using the Maps	17
	2.2.6	Summary of Configuration Points	17
2.3	Softwar	re Implementation Details	17
2.4	Exampl	e: AMF Applied to Wolf Sheep Predation	17
2.5	Analysi	s of AMF vs. the Design Goals	17
Chapter	3	RELATED WORK	18
Chapter	4	BACKGROUND	19
Chapter	5	DEFINING SYSTEM-LEVEL PROPERTIES	20
Chapter	6	THE FORWARD MAPPING PROBLEM	22
Chapter	7	THE REVERSE MAPPING PROBLEM	23
Chapter	8	USING META-MODELS	24
Chapter	9	RESULTS	25
Chapter	10	CONCLUSIONS AND FUTURE WORK	26
DEEEDI	FNCES		27

## LIST OF FIGURES

1.1	A screenshot of NetLogo's graphical user interface while executing a flock-			
	ing simulation.	2		
1.2	A screen shot from NetLogo's Wolf Sheep Predation model	4		
1.3	The control and monitor interface for the Wolf Sheep Predation model	5		
1.4	Differences in populations based on changes of the sheep-gain-from-food			
	parameter	6		
2.1	An overview of the phases of AMF and how data flows between them	13		

## LIST OF TABLES

## INTRODUCTION AND MOTIVATION

The behavior of individual agents in an agent-based model (ABM) is typically well understood because the agent's program directly controls its local behaviors. What is typically not understood is how changing these programs' agent-level control parameters affect the observed system-level behaviors of the ABM. The aim of this dissertation is to provide researchers and users of ABMs insight into how these agent-level parameters affect system-level properties. In this dissertation, I discuss a learning framework named the ABM Meta-Modeling Framework (AMF) that I have developed that can be used to predict and control system-level behaviors of agent-based models. With this framework, users can interact with ABMs in terms of intuitive system-level concepts, instead of with agent-level controls that only indirectly affect system-level behaviors.

#### 1.1 Agent-Based Models

Agent-based models are used by scientists to analyze system-level behaviors of complex systems by simulating the system bottom-up. At the bottom of these simulations are individual agents that locally interact with other agents and the environment. All the behavior in an ABM, from agent-level local interactions to system-level behaviors, emerge from these local interactions, which are governed by the individual *agent programs*. ABMs can

be used to understand how changes in individuals' agent-level parameters affect system-level properties.



FIG. 1.1. A screenshot of NetLogo's graphical user interface while executing a flocking simulation.

Agent-level control parameters adjust the behaviors of agent-based programs. However, scientists are not typically interested in the local interactions between agents—they are interested in the resulting system-level behaviors that result. For example, researchers that studied agent-based models of lane formations in army ants were interested in the traffic patterns of the lanes, not the individual behaviors of the ants (Couzin & Franks 2003). In other work, researchers that studied locusts were interested in determining at what critical density locusts begin to swarm and destroy crops (Buhl *et al.* 2006). Typically, scientists analyze ABMs by viewing visualizations of the environment or gathering statistical data on the simulation. For instance, NetLogo, an agent-based modeling programming

environment (Tisue & Wilensky 2004), has monitors, plots and visualizations to convey system-level properties to the user. In Figure 1.1, monitors are displaying *density*, *average-heading* and *stddev-heading* statistics for a flocking domain. In addition, a plot of density shows how it has changed over time. These tools are used by a researcher to generate a mental model of how the agent-level control parameters of the flocking domain (the sliders seen in the user interface) affect these system-level properties.

Although using ABMs for researching agent-based systems has been proven useful in a number of domains, there is a glaring conceptual disconnect from the user's perspective, between the agent-level controls and the system-level properties. The classical ABM control method of adjusting agent-level properties is unintuitive because they only indirectly affect the system-level properties through emergence. With the current methodology, a simulation has to be executed in order to observe what the values of the system-level properties will be. A time consuming iterative process of guess-and-check is the only way to configure the system to have it exhibit a desired system-level behavior. A determination of what an ABM will do at a system-level, given only the agent-level parameters, is not possible with current software.

The main goal of the ABM Meta-Modeling Framework is to bridge the gap between agent-level parameters and system-level properties. AMF reduces the learning curve of an agent-based model since users are interacting with the system at the system-level, instead of at the agent-level. Qualitative analysis of an ABM's system-level properties will be a more efficient process since researchers deal with an abstraction of the system's controls. In addition, the models learned by AMF can be inspected to gather quantitative data about the correlations between system-level properties and agent-level parameters.



FIG. 1.2. A screen shot from NetLogo's Wolf Sheep Predation model.

#### 1.2 Wolves, Sheep and Grass

Throughout this dissertation, I will use NetLogo's Wolf Sheep Predation model (Wilensky 1997b), which is bundled with NetLogo's standard Model Library, as an example to explain concepts. A snapshot of its NetLogo visualization is shown in Figure 1.2. This multi-agent model simulates a food chain consisting of wolf agents, sheep agents and grass in a two-dimensional space. The model is controlled by seven agent-level control parameters, which directly affect the following agent behaviors:

- The system is initialized with *initial-number-sheep* sheep and *initial-number-wolves* wolves.
- Wolves and sheep move randomly though the space.
- Wolves and sheep die if they run out of energy.

<sup>&</sup>lt;sup>1</sup>http://ccl.northwestern.edu/netlogo/models/WolfSheepPredation

- Wolves eat sheep if they occupy the same space in the environment. Wolves gain wolf-gain-from-food units of energy from eating sheep. The sheep dies.
- Sheep eat grass if they are on a location of the environment that has grass. Sheep gain *sheep-gain-from-food* units of energy from eating grass. The grass dies in that grid location.
- Every time step, each sheep and each wolf has a chance (*sheep-reproduce* and *wolf-reproduce*) to reproduce asexually. Both the parent and the child split the parent's original energy evenly (i.e., parent's energy divided by two).
- Grass regrows after *grass-regrowth-time* number of time steps.



FIG. 1.3. The control and monitor interface for the Wolf Sheep Predation model.

The system-level concepts we are interested in are the number of sheep, the number of wolves and the number of grid locations containing grass. In NetLogo, these properties are displayed with monitors and a plot, as seen in Figure 1.3. The number of each population of agents may change continuously, but the average number of sheep converges. Another interesting feature is some ecosystems fail: either sheep or both sheep and wolves go extinct.



FIG. 1.4. Differences in populations based on changes of the *sheep-gain-from-food* parameter.

After working with this ABM for some time, a user will begin to realize that changes in the control parameters will yield different types of behavior. For example, by setting *sheep-gain-from-food* to 2, 3, and then 4, major differences in system-level behavior are apparent by viewing the graphs in Figure 1.4. When the value of *sheep-gain-from-food* is 4, the system rhythmically exhibits major changes in all three agent populations. When the

value is 2 or 3, the population remains relatively stable, but the average population values are different. When the value is low enough (e.g., 2) the wolves go extinct.

The Wolf Sheep Predation model is a good example of the intuitive disconnect between agent-level parameters and system-level properties. There is no clear *explicit* relationship between the controls presented in the user interface and the resulting system-level properties. An experienced user may have a qualitative understanding of the correlations, but would not be able to predict quantitative concepts, such as the average number of sheep after 2000 time steps. In Chapter 9: Results, I will show that the intuitive disconnect in this domain can easily be solved by AMF.

#### 1.3 Overview of the ABM Meta-Modeling Framework

The foundation of this work is framing the problem of building a meta-model of an ABM as two sub-problems: the *forward-mapping problem* and the *reverse-mapping problem*. In Chapter 6: The Forward-Mapping Problem, I will discuss how AMF maps given values of the agent-level parameters to expected system-level property values with standard regression approaches. In Chapter 7: The Reverse-Mapping Problem, I will discuss how AMF maps a set of desired system-level property values to a set of agent-level parameters that would generate this behavior. My general approach to solving the reverse-mapping problem is to interpolate configurations using the forward mapping to approximate a smooth and continuous surface. This interpolated surface represents the space of configurations that would satisfy the system-level requirements set out by the user. Also, in Chapter 7, I will discuss alternative methods for solving the revers-mapping problem.

AMF is simple and has only a few configuration points. This allows researchers to focus on the analysis of the system, instead of on the details of AMF. The framework consists of three major steps: sampling, solving the forward-mapping problem and solving

the reverse-mapping problem. In these three steps, the only configurations the user must perform are: define how to measure system-level properties of interest, provide the ranges of parameters to be sampled, and plug in a regression algorithm.

I will show in Chapter 9: Results that my framework is able to generate models of system-level behavior. For example, AMF is able to predict the number of sheep and wolves in the Wolf Sheep Predation model, given the configuration parameter values (the forward-mapping problem). Also, AMF is able to make suggestions for the values for the control parameters, given the desired system-level property outcome (the reverse-mapping problem).

A more comprehensive overview of AMF is provided in Chapter 2: The ABM Meta-Modeling Framework.

#### 1.4 Summary of Contributions

My main contribution presented in this dissertation is an in-depth analysis of metamodels of agent-based models. This analysis includes a discussion of methods for using regression to build models of the correlations between agent-level parameters and systemlevel properties. In addition, this dissertation contains a survey of ways that that metamodels can be used to inspect system-level behaviors of agent-based models.

The ABM Meta-Modeling Framework encapsulates my methodology for building meta-models of ABMs. The implementation of AMF as software serves as a proof-of-concept to show that my approach is implementable and applicable to a variety of domains. The software itself is a contribution, since it is available to be used by researchers interested in building meta-models of NetLogo ABMs. The design of the general framework is a contribution as well, since it could be implemented to interact with other agent-based modeling systems similar to NetLogo, or totally independent agent-based models.

#### 1.5 Dissertation Organization

This dissertation is divided into nine chapters, including this one. Chapter 2: The ABM Meta-Modeling Framework explains each framework component in detail, explains how a new user would tailor AMF to a new ABM, discusses implementation details and gives an introduction to the forward- and reverse-mapping problems. Chapter 3: Related Work compares and contrasts approaches similar to AMF in motivation, with AMF. Chapter 4: Background provides information about NetLogo and algorithms used by AMF. Chapter 5: Defining System-Level Properties discusses the numerous different ways that system-level properties of ABM can be defined. Chapters 6 and 7 discuss my solutions to the forward- and reverse-mapping problems. Chapter 8: Using Meta-Models is a survey of different ways the meta-models generated by AMF can be used to analyze system-level properties of ABMs. Chapter 9: Results evaluates AMF on a domain-by-domain basis and provides explicit examples of how AMF has been used. Chapter 10: Conclusions and Future Work summarizes this dissertation, provides additional thoughts I have regarding this work and possible directions for future work.

## THE ABM META-MODELING FRAMEWORK

ABM Meta-Modeling Framework builds meta-models that map the values of agent-level control parameters and system-level properties, and vice versa. Learning both of these mappings are separate problems, which I call the *forward-mapping problem* and the *reverse mapping problem*. To solve these problems, a user of AMF "plugs in" a regression algorithm of their choice. The framework uses this regression algorithm to learn the mappings and provide a user with an interface to query them. Once the mappings are learned, the user can query either for *prediction* or *control*. A prediction query asks the forward mapping to give a value for system-lev Most of the implementation details and inner workings of AMF are abstracted away from the user, who only has to attend to a limited number of configuration points.

In this chapter, I will discuss the design goals of AMF, the framework structure, software implementation details, and how well AMF conforms to the design goals.

#### 2.1 Design Goals of The ABM Meta-Modeling Framework

My specific goal in designing AMF is to make controlling and interacting with agentbased models more intuitive. In addition to this central goal, AMF strives to be:

• Domain independent – the design of AMF should minimize the amount of configu-

ration for each domain,

- Algorithm independent any regression algorithm should be able to be plugged into AMF,
- Accurate AMF should generate accurate predictions and control suggestions,
- Fast for the user interactions with the models generated by AMF should require minimal computational time.

Domain independence is paramount because of the variety in which ABMs come. I have designed AMF such that the same general approach would work for any ABM. Also, I strove to minimize the amount of configuration needed to apply AMF to a new domain. These constraints I have set on the design make AMF broadly applicable to a number of domains, without the need of in-depth domain knowledge. To reinforce this claim, I have tested AMF on a number of diverse domains and used the same general approach for each.

Algorithm independence in a learning framework is important because different algorithms are more effective for modeling different agent-based models. In general, the learning algorithms that will be discussed in this dissertation will satisfy the requirements for modeling most agent-based models. However, an in depth analysis of which types of algorithms should be used for different classes of ABMs is outside the scope of this dissertation research. In addition, algorithm independence allows AMF to scale with new advances in machine learning research, since future state-of the art regression algorithms can be plugged in just as easily as current approaches.

Accuracy and fast user response time appear to be obvious design goals. However, AMF requires that the user spends a significant amount of computational time sampling different configurations of the target ABM. These large training sets can be used to build static meta-models of ABMs that are both accurate and fast to query. In contrast, an ac-

tive learning approach would be able to learn models faster, but would require interaction with the user, increasing the amount of user interaction. Likewise, optimization approaches could be used to generate arbitrarily accurate results, but typically require numerous iterations and would significantly increase the response time for a user's query. In summary, I am making the assumption that users studying ABMs with AMF are more interested in achieving more accurate results for their research, than generating their results faster.

#### 2.2 Framework Structure

The framework is split into several phases: sampling, solving the forward mapping problem, solving the reverse mapping problem, querying for prediction, and querying for control (i.e., suggesting a configuration). This section uses the diagram in Figure 2.1 to explain how the different phases interact with one another. From an exterior perspective, the framework only has a limited about of input and output: AMF takes in observations of an agent-based model and then provides interfaces to query for prediction and control. These queries interact with the user, as well as the agent-based model.

Many configuration points exist, which allow users of AMF to modify the behavior of the framework. However, the individual phases take the same output and provide the same output, no matter the configuration. This is what makes AMF a framework and not a collection of algorithms. For example, even though different regression algorithms can be used to learn the forward mapping, the forward mapping still provides predictions of how an ABM will behave, from the user's perspective.

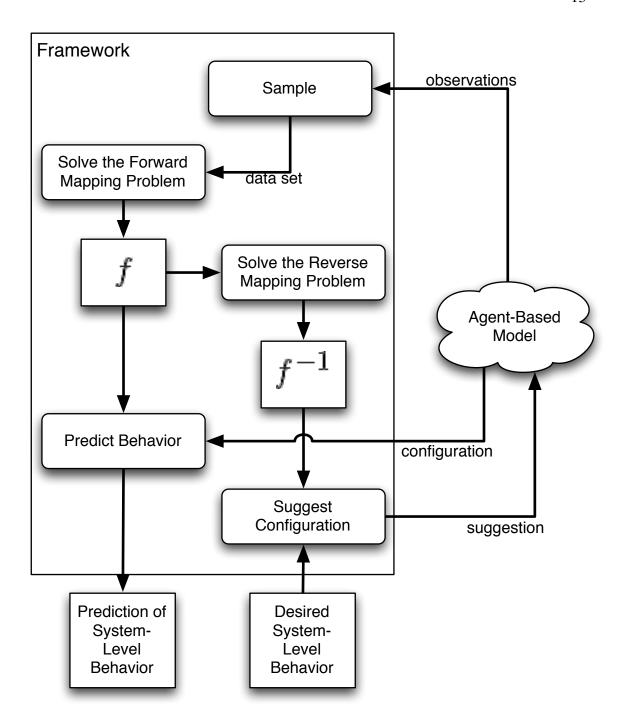


FIG. 2.1. An overview of the phases of AMF and how data flows between them.

#### 2.2.1 Defining System-Level Behavior Properties

System-level properties of an ABM must be defined by the user of AMF. The measurement is some sort of statistical or mathematical calculation based on the state of the ABM over some period of time. The one assumption made by AMF is that the measurement is *stable*, meaning that as the same configuration is sampled again and again, the measurement's value should vary very much. One way to measure stability is to calculate the standard deviation of measured values of several runs of the system. The need for this assumption, ways to conform to it and a more detailed explanation of how to define system-level properties is provided in Chapter 5: Defining System-Level Behaviors.

#### 2.2.2 Sampling

The first phase is *sampling*. In this phase, numerous observations are made on different configurations of an agent-based model. A data set is generated that contains the independent variables (the agent-level control parameter values) and the resultant system-level behaviors that are measured. This process can be significantly time consuming, dependent on several factors:

- Execution time of the model the models will be executed numerous times, so the longer a model takes to execute, the longer the whole set of experiments will take to execute,
- Granularity more detailed samplings will take more time, since more points must be sampled,
- Dimensionality more agent-level control parameters result in a larger search space and naturally more points to sample,

Small samples, such as 120 observation sample of the NetLogo *Fires* model<sup>1</sup> (Wilensky 1997a), required about thirty seconds.<sup>2</sup> Meanwhile, a large sample of 50000 observation of a Reynolds boid flock (Reynolds 1987) took approximately four days.

The user is required to have a minimal amount of domain knowledge to specify to AMF what ranges of values should be sampled. AMF is not able to automatically infer which value ranges are interesting, and thus must be explicitly defined.

One redeeming quality of this process is that it is easily parallelizable. Since each experiment is independent on one another, the set of all experiments can be segmented among a number of systems and processors to significantly reduce the computation time. Once each subset of experiments are done executing, the results can be concatenated into one data set.

For the scope of this dissertation, I limit AMF to use a simple random sampling method (i.e., randomly select points within a specified range) or a systematic sampling method (i.e., given ranges, sample evenly spaced points). I acknowledge that an intelligent sampling strategy can be improve the performance of many machine learning techniques, however, none are used for the sake of focusing on other portions of the framework. In future work, different sampling techniques' effect on accuracy and speed could be measured.

#### 2.2.3 The Forward-Mapping Problem

The forward-mapping problem is to develop a function  $f: \mathbb{R}^n \to \mathbb{R}$  that maps a provided configuration vector  $\mathbf{x} = \{x_1, x_2, ..., x_n\}$  to a single system-level behavior property  $\hat{y}$ :

$$\hat{y} \leftarrow f(\mathbf{x})$$

<sup>&</sup>lt;sup>1</sup>See Chapter 9 for detailed results

<sup>&</sup>lt;sup>2</sup>Most experiments are executed on a 2.2GHz Intel Core 2 Duo running Mac OS X

The set of values  $\vec{x}$  consists of all the agent-level parameters (independent variables) in the data set provided by the sample step. An individual mapping is learned for each system-level behavior measurement provided by the user.

This problem is solved with straightforward regression, such as k-nearest neighbor. In this phase of AMF, the regression algorithm is "initialized," if necessary. For example, linear regression would need to solve the least-squares problem. Meanwhile, an algorithm like k-nearest neighbor would not need to initialize anything. Once this phase is completed, the user is provided with f, an interface to the mapping built by the regression algorithm.

A more in depth definition of the forward-mapping problem, specific examples of regression methods used in AMF, and the role of regression is given in Chapter 6: The Forward-Mapping Problem.

#### 2.2.4 The Reverse-Mapping Problem

The reverse-mapping problem is to *invert* the forward mapping. This will produce a mapping  $f^{-1}: \mathbb{R} \to \hat{S}$  from a given system-level behavior property y to a set of configurations  $\hat{S} = \{\hat{\mathbf{x}} | f(\hat{\mathbf{x}}) = y\}$ . That is, S is the set of behaviors that will produce behavior y.

Developing the mapping  $f^{-1}$  is what I call "inverted regression problem," and has several unique challenges. The main challenge is  $f^{-1}$  does not describe a functional mapping. This is because  $f^{-1}$  describes a one-to-many relationship, since f is many-to-one. Therefore, AMF cannot use standard regression techniques to solve this problem. Instead, AMF approximates the inverse of the forward mapping, with a method that I developed. As an aside, this approach has the benefit of using the forward mapping to develop the reverse mapping. No additional input from the user or data set are needed. The intricacies of inverting a functional mapping developed by a regression algorithm are discussed in Chapter 7: The Reverse-Mapping Problem.

- 2.2.5 Using the Maps
- **2.2.6** Summary of Configuration Points
- 2.3 Software Implementation Details
- 2.4 Example: AMF Applied to Wolf Sheep Predation
- 2.5 Analysis of AMF vs. the Design Goals

## **RELATED WORK**

## **BACKGROUND**

## **DEFINING SYSTEM-LEVEL PROPERTIES**

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

For example, in the Wolf Sheep Predation domain, one simple system-level measurement could be the average number of wolves over a thousand time steps. This number can be misleading, because certain configurations will sometimes result in the wolves going extinct (i.e., zero wolves). Other times, the same configurations will have the wolves converge to a stable non-zero population. Therefore, the expected value for the average number of wolves  $\hat{w}$  to be:

$$E(\bar{w}) = P(extinct) * (\bar{w}|extinct) + (1 - P(extinct)) * (\bar{w}|\neg extinct)$$

where extinct is whether the wolves went extinct or not, P(extinct) is the probability wolves go extinct, and  $(\bar{w}|\neg extinct)$  is the average number of wolves, given they did not go extinct.  $\hat{w}$  is not very stable, because sometimes it is zero and sometimes it is  $(\bar{w}|\neg extinct)$ .

Making a prediction based on the expected value of  $\hat{w}$  will always have a specific amount of error associated with it. To remedy this problem

## THE FORWARD MAPPING PROBLEM

## THE REVERSE MAPPING PROBLEM

## **USING META-MODELS**

## **RESULTS**

## **CONCLUSIONS AND FUTURE WORK**

#### **REFERENCES**

- [1] Buhl, J.; Sumpter, D.; Couzin, I.; Hale, J.; Despland, E.; Miller, E.; and Simpson, S. 2006. From Disorder to Order in Marching Locusts. *Science* 312(5778):1402–1406.
- [2] Couzin, I., and Franks, N. 2003. Self-organized lane formation and optimized traffic flow in army ants. In *Proceedings of the Royal Society of London, Series B*, volume 270, 139–146.
- [3] Reynolds, C. W. 1987. Flocks, herds, and schools: A distributed behavioral model. In *Computer Graphics*, 21(4) (SIGGRAPH '87 Conference Proceedings), 25–34.
- [4] Tisue, S., and Wilensky, U. 2004. NetLogo: A simple environment for modeling complexity. In *International Conference on Complex Systems*, 16–21.
- [5] Wilensky, U. 1997a. NetLogo Traffic Basic model. Technical report, Northwestern University.
- [6] Wilensky, U. 1997b. NetLogo Wolf Sheep Predation model. Technical report, Northwestern University.