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A Nonhierarchical Formulation of Analytical Target Cascading

Analytical target cascading (ATC) is a method developed originally for translating system-level design targets to design specifications for the components that comprise the system. ATC has been shown to be useful for coordinating decomposition-based optimal system design. The traditional ATC formulation uses hierarchical problem decompositions, in which coordination is performed by communicating target and response values between parents and children. The hierarchical formulation may not be suitable for general multidisciplinary design optimization (MDO) problems. This paper presents a new ATC formulation that allows nonhierarchical target-response coupling between subproblems and introduces system-wide functions that depend on variables of two or more subproblems. Options to parallelize the subproblem optimizations are also provided, including a new bilevel coordination strategy that uses a master problem formulation. The new formulation increases the applicability of the ATC to both decomposition-based optimal system design and MDO. Moreover, it belongs to the class of augmented Lagrangian coordination methods, having thus convergence properties under standard convexity and continuity assumptions. A supersonic business jet design problem is used to demonstrate the flexibility and effectiveness of the presented formulation.
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1 Introduction

The development process of engineering systems often follows a decomposition paradigm. The system is decomposed into components that are developed separately. Accordingly, the optimal system design problem is decomposed into subproblems, each associated with a system component. While these optimal design subproblems are solved autonomously, a systematic coordination method is required to account for interactions and ensure a system-level design that is both consistent and optimal.

Coordination methods for multidisciplinary design optimization (MDO) and decomposition-based optimal system design include collaborative optimization [1–4], bilevel integrated system synthesis [5,6], the quasiseparable decomposition of Ref. [7], the penalty decomposition of Refs. [8,9], and augmented Lagrangian coordination [10–12]. For an overview and classification of coordination methods, the reader is referred to Refs. [13–16]. In this paper, we focus on the analytical target cascading (ATC) method, which, according to the classification of Ref. [16], is an alternating method with closed design constraints and open consistency constraints.

ATC was developed originally for translating system-level design targets to design specifications for the components that comprise the system [17–21]. ATC has been shown to be useful as a coordination method for decomposition-based optimal system design [22,23]. In the ATC paradigm, design targets are cascaded using a hierarchical problem decomposition. Subproblems associated with the components of the system not only determine targets for their children, but also compute responses to targets they receive and communicate these back to their parents. The objective of a subproblem is to minimize the deviations between the target-response pairs while maintaining feasibility with respect to its local design constraints. The process of exchanging targets and responses between subproblems can be shown to converge to optimal system solutions with arbitrarily small deviations between targets and responses under the standard convexity and continuity assumptions [19,23,24].

The traditional ATC formulation uses a hierarchical problem decomposition, in which coordination is performed by communicating target and response values between parents and children. The term hierarchical refers to the functional dependency among system components: Responses of components higher in the hierarchy depend on responses of components lower in the hierarchy, but not vice versa (see Fig. 1(*a*)).

This hierarchical ATC formulation may not be the most suitable for coordinating problems that do not have a hierarchical decomposition structure. For those problems, direct communication between subproblems may be more appropriate. For example, MDO problems are often composed of subproblems governed by analyses among which no hierarchy exists. The first objective of the work presented in this paper was to extend the ATC formulation to include *nonhierarchical target-response* coupling among subproblems so that functional dependencies in nonhierarchical problem decompositions (see Fig. 1(*b*)) can be handled by the ATC process while preserving the unique target-response coupling structure that distinguishes ATC from other coordination methods.

The second objective of the work presented in this paper was to treat coupling of subproblems through *system-wide functions*. There exist decomposed optimal system design problems where functions depend on variables of more than one subproblem (see Fig. 1(*c*)). Such coupling functions often represent a system attribute such as mass, cost, volume, or power. Coordinating system-wide requirements using the hierarchical ATC formulation requires the introduction of at least as many target-response pairs as there are subproblems. The nonhierarchical formulation presented in this paper offers a direct approach to the coordination of system-wide functions that does not require the introduction of a large number of target-response pairs since it operates directly on system-wide functions.

The third contribution of the paper is a strategy for parallelizing the ATC coordination process. This strategy allows the parallel solution of all design optimization subproblems through the introduction of a single coordinating master problem. Hierarchical ATC coordination strategies allow parallel solution of subprob-

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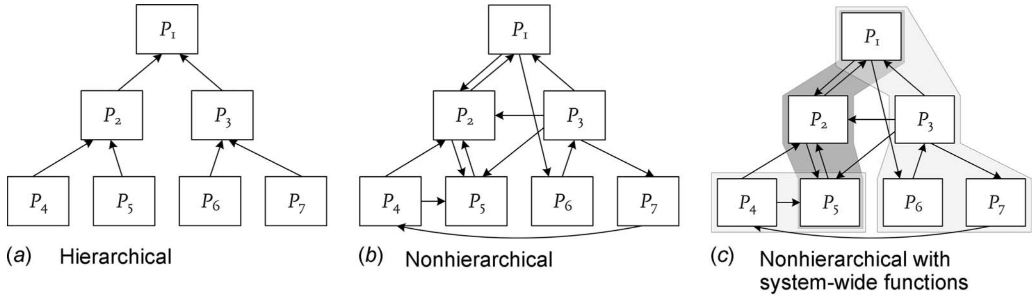

Fig. 1 Functional dependence structure of the original and proposed ATC formulations: The arrows indicate the flow of subproblem responses; the shaded boxes are used to represent the dependence of system-wide functions on subproblem variables

lems only within the same level; subproblems at different levels are solved sequentially. A supersonic business jet design problem [25] is used to illustrate the presented formulation and the parallelization strategy.

It is out of the scope of this paper to investigate which coordination strategy may be best or which optimal system design problems may benefit most from the implementation and application of the nonhierarchical formulation. The objective of the proposed extensions is to increase the flexibility and effectiveness of ATC implementations.

Both the hierarchical and the nonhierarchical ATC formulations belong to the class of augmented Lagrangian coordination (ALC) methods [11]. Therefore, the existing convergence theory for ALC applies to them directly. The feature that distinguishes ATC from other coordination methods, including ALC, is the representation and treatment of variables that couple subproblems using target-response pairings. This target-response coordination paradigm has proven to be useful in optimal system design, and has generated a large body of literature. Currently, ATC is formulated to be applied exclusively to hierarchical problem decompositions. This paper shows how the ATC formulation can be extended to be applied to nonhierarchical problem decompositions (with or without system-wide functions). This development will make ATC applicable to a wider class of optimal system design problems, including MDO problems.

2 ATC Formulation for Nonhierarchical Problems

The hierarchical ATC formulation operates by solving an optimization problem for each component of the decomposed system. Assuming that responses of higher-level components are functions of responses of lower-level components, it aims at minimizing the gap between what higher-level components "want" and what lower-level components "can." The hierarchical augmented Lagrangian ATC subproblem is formulated as [23]

$$\min_{\bar{\mathbf{x}}_{ij}} f_{ij}(\bar{\mathbf{x}}_{ij}) + \phi(\mathbf{t}_{ij} - \mathbf{r}_{ij}) + \sum_{k \in \mathcal{C}_{ij}} \phi(\mathbf{t}_{(i+1)k} - \mathbf{r}_{(i+1)k})$$

$$\text{subject to } \mathbf{g}_{ij}(\bar{\mathbf{x}}_{ij}) \leq \mathbf{0}$$

$$\mathbf{h}_{ij}(\bar{\mathbf{x}}_{ij}) = \mathbf{0}$$

$$\text{with } \mathbf{r}_{ij} = \mathbf{a}_{ij}(\mathbf{x}_{ij}, \mathbf{t}_{(i+1)k_1}, \ldots, \mathbf{t}_{(i+1)k_{c_{ij}}})$$

$$\bar{\mathbf{x}}_{ij} = [\mathbf{x}_{ij}, \mathbf{r}_{ij}, \mathbf{t}_{(i+1)k_1}, \ldots, \mathbf{t}_{(i+1)k_{c_{ij}}}] \tag{1}$$

where $\bar{\mathbf{x}}_{ij}$ are the optimization variables for subproblem j at level i, $\mathbf{x}_{ij}$ are local design variables, and $\mathbf{r}_{ij}$ are response variables related to the targets $\mathbf{t}_{ij}$ computed by the parent of subproblem j. Subproblem j computes targets $\mathbf{t}_{(i+1)k}$ for the set of its children $\mathcal{C}_{ij}$; in turn, the children compute responses $\mathbf{r}_{(i+1)k}$ and return them to the parent. The function f_{ij} is the local objective, and vector functions $\mathbf{g}_{ij}$ and $\mathbf{h}_{ij}$ represent local inequality and equality constraints, respectively (note that a local objective function and local constraints are not required for the ATC process). Functions $\mathbf{a}_{ij}$ represent the analyses required to compute responses $\mathbf{r}_{ij}$. The augmented Lagrangian function ϕ relaxes the consistency equality constraints $\mathbf{c}_{ij}=\mathbf{t}_{ij}-\mathbf{r}_{ij}=\mathbf{0}$ as follows:

$$\phi(\mathbf{t}_{ij} - \mathbf{r}_{ij}) = \mathbf{v}_{ij}^T(\mathbf{t}_{ij} - \mathbf{r}_{ij}) + \|\mathbf{w}_{ij} \circ (\mathbf{t}_{ij} - \mathbf{r}_{ij})\|_2^2 \tag{2}$$

where $\mathbf{v}_{ij}$ and $\mathbf{w}_{ij}$ are penalty parameters selected by an external mechanism as discussed in Sec. 3. The symbol $\circ$ represents the Hadamard product: an entry-wise multiplication of two vectors, such that $\mathbf{a}\circ\mathbf{b}=[a_1, \ldots, a_n]^T\circ[b_1, \ldots, b_n]^T=[a_1b_1, \ldots, a_nb_n]^T$.

2.1 Nonhierarchical Subproblem Coupling. The hierarchical ATC formulation allows communication only between parents and children. In this paper, we extend the ATC formulation so that subproblems can send and receive targets to and from, respectively, *any* other subproblem. In this formulation, subproblems have *neighbors* among which targets and responses are communicated.

In the hierarchical ATC formulation, the index i denotes the level of the hierarchy at which the subproblem is located. The new formulation is nonhierarchical, so the index i can be dropped. However, we do maintain a double index notation, which now denotes the direction of communication between subproblem j and its neighbor n. The first index denotes the sending subproblem and the second index denotes the receiving subproblem: $\mathbf{t}_{nj}$ are the targets that subproblem j receives from its neighbor n, and $\mathbf{r}_{jn}$ are the responses computed by subproblem j to match the targets from neighbor n. Furthermore, let $\mathcal{T}_j$ be the set of neighbors for which subproblem j sets targets, and let $\mathcal{R}_j=\cup_{n=1}^{M}\{j|j\in\mathcal{T}_n\}$ be the set of neighbors from which subproblem j receives targets (i.e., the set of neighbors for which subproblem j computes responses). Figure 2 illustrates the target-response pairs between subproblem j and both types of neighbors. We have two sets of consistency constraints related to subproblem j:

$$\begin{aligned} \mathbf{c}_{nj} &= \mathbf{t}_{nj} - \mathbf{r}_{jn} = \mathbf{0}, \quad n \in \mathcal{R}_j \\ \mathbf{c}_{jm} &= \mathbf{t}_{jm} - \mathbf{r}_{mj} = \mathbf{0}, \quad m \in \mathcal{T}_j \end{aligned} \tag{3}$$

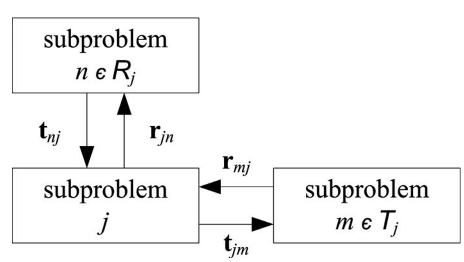

Fig. 2 Nonhierarchical target and response flow between subproblem *j* and its neighbors

The nonhierarchical ATC subproblem formulation is

$$\min_{\bar{\mathbf{x}}_j} f_j(\bar{\mathbf{x}}_j) + \sum_{n \in \mathcal{R}_j} \phi(\mathbf{t}_{nj} - \mathbf{r}_{jn}) + \sum_{m \in \mathcal{T}_j} \phi(\mathbf{t}_{jm} - \mathbf{r}_{mj})$$

$$\text{subject to } \mathbf{g}_j(\bar{\mathbf{x}}_j) \leq \mathbf{0}$$

$$\mathbf{h}_j(\bar{\mathbf{x}}_j) = \mathbf{0}$$

$$\text{with } \mathbf{r}_{jn} = \mathbf{S}_{jn}\mathbf{a}_j(\mathbf{x}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j), \quad n \in \mathcal{R}_j$$

$$\bar{\mathbf{x}}_j = [\mathbf{x}_j, \mathbf{r}_{jn}|n \in \mathcal{R}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j] \tag{4}$$

where $\mathbf{S}_{jn}$ is a binary selection matrix that selects components from $\mathbf{a}_j$ that are sent to subproblem n. These selection matrices reflect the problem decomposition. Common subproblems in sets $\mathcal{T}_j$ and $\mathcal{R}_j$ indicate feedback coupling between subproblem j and the common subproblem.

Formulating consistency constraints for a nonhierarchical problem structure may not be straightforward. If a variable is shared by n subproblems, then $n(n-1)/2$ consistency constraints can be defined to coordinate the n variable copies. However, only $n-1$ of these consistency constraints are strictly necessary to enforce consistency. The selection of consistency constraint sets may affect the efficiency of the coordination process. Although relevant, a discussion of consistency constraint selection is beyond the scope of this paper. The interested reader is referred to Ref. [26] for an approach that addresses this issue. However, the parallelization strategies presented in Sec. 3.3 do not feature this issue since consistency constraints are always uniquely defined with respect to a central system-level copy.

2.2 System-Wide Functions. System-wide functions are objectives or constraints that depend on the variables of more than one subproblem. In the extended ATC formulation proposed here, a system-wide objective $f_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M)$ can be included directly in the objective of a subproblem. System-wide constraints $\mathbf{g}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M)$ and $\mathbf{h}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M)$ are relaxed with augmented Lagrangian penalty functions given by

$$\phi(\mathbf{g}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M), \mathbf{s}) = \mathbf{v}_g^T(\mathbf{g}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M) + \mathbf{s}) + \|\mathbf{w}_g \circ (\mathbf{g}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M) + \mathbf{s})\|_2^2$$

$$\phi(\mathbf{h}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M)) = \mathbf{v}_h^T \mathbf{h}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M) + \|\mathbf{w}_h \circ \mathbf{h}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M)\|_2^2 \tag{5}$$

where $\mathbf{v}_g, \mathbf{w}_g$ and $\mathbf{v}_h, \mathbf{w}_h$ are the penalty parameters for the inequality and equality system-wide constraints, respectively. These penalty parameters have to be determined by an external mechanism, similar to the penalty parameters for the consistency constraints. The positive slack variables $\mathbf{s}$ are used to allow negative (feasible) values for the system-wide inequality constraints $\mathbf{g}_0$.

The ATC subproblem formulation that allows system-wide functions is given by

$$\min_{\bar{\mathbf{x}}_j, \mathbf{s}_j} f_j(\bar{\mathbf{x}}_j) + \sum_{n \in \mathcal{R}_j} \phi(\mathbf{t}_{nj} - \mathbf{r}_{jn}) + \sum_{m \in \mathcal{T}_j} \phi(\mathbf{t}_{jm} - \mathbf{r}_{mj}) + f_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M) + \phi(\mathbf{g}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M) + \mathbf{s}) + \phi(\mathbf{h}_0(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_M))$$

$$\text{subject to } \mathbf{g}_j(\bar{\mathbf{x}}_j) \leq \mathbf{0}$$

$$\mathbf{h}_j(\bar{\mathbf{x}}_j) = \mathbf{0}$$

$$\mathbf{s}_j \geq \mathbf{0}$$

$$\text{with } \mathbf{r}_{jn} = \mathbf{S}_{jn}\mathbf{a}_j(\mathbf{x}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j), \quad n \in \mathcal{R}_j$$

$$\bar{\mathbf{x}}_j = [\mathbf{x}_j, \mathbf{r}_{jn}|n \in \mathcal{R}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j] \tag{6}$$

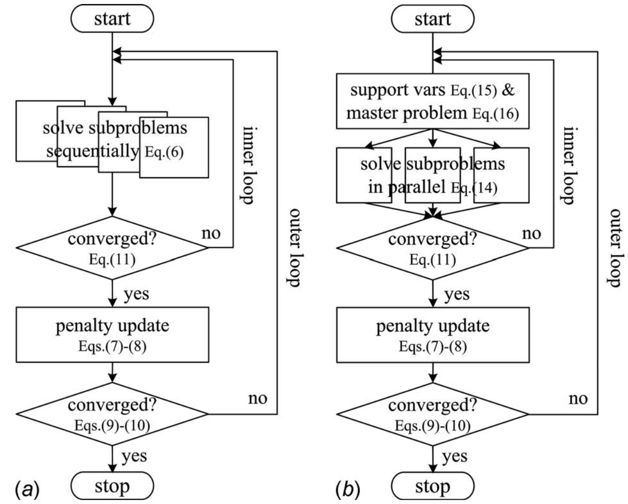

Fig. 3 Illustration of coordination algorithms for the nonhierarchical ATC formulation. Each inner loop may be exact, inexact, or consist of only a single iteration.

The vector $\mathbf{s}_j$ includes the slack variables that are treated as optimization variables in subproblem j. If a system-wide objective does not depend on $\bar{\mathbf{x}}_j$, then it is not included in Eq. (6). Similarly, if a system-wide constraint does not depend on $\bar{\mathbf{x}}_j$, then its penalty is not included in Eq. (6).

The proposed ATC formulation (6) belongs to the class of ALC methods [11]. The feature that distinguishes ATC from the more general ALC formulation is the specific use of target-response pairs to coordinate the interaction between subproblems. This target-response coordination paradigm has proven to be useful in optimal system design, and one objective of this paper is to extend the applicability of the ATC paradigm based on the theory available for ALC.

Several ATC formulations that have been reported in literature are special cases of the proposed formulation (4) without system-wide functions. The hierarchical ATC subproblem formulation of Eq. (1) is obtained when taking $\mathcal{T}_j = \mathcal{C}_{ij}$ and $\mathcal{R}_j = \{p_j\}$, where p_j denotes the parent of j. Furthermore, the ATC formulation for product family design presented in Ref. [27] is obtained if we take $\mathcal{T}_j = \mathcal{C}_{ij}$ and allow a subproblem to have multiple parents at the same level in the hierarchy. The ATC formulation of Ref. [22] that allows feedback targets between parents and children is obtained by taking $\mathcal{T}_j = \mathcal{R}_j = \mathcal{C}_{ij} \cup \{p_j\}$.

3 Coordination Algorithm

Since the proposed formulation belongs to the class of ALC methods [11], the associated algorithm can be applied directly. This section summarizes the ALC algorithm presented in Ref. [11]; a new strategy for parallelizing the coordination process is discussed in Sec. 3.3. Together, these sections present a complete coordination algorithm for the nonhierarchical ATC formulation.

The ALC algorithm for ATC performs two tasks.

1. It selects appropriate penalty parameter values for $\mathbf{v}$ and $\mathbf{w}$ in Eqs. (2) and (5).
2. It manages the coupling of subproblem objectives.

The coordination algorithm operates in inner and outer loops to perform these two tasks. Two coordination algorithm variants are illustrated in Fig. 3. The outer loop takes care of selecting the penalty parameters using the method of multipliers, and the inner loop accounts for the subproblem coupling through an alternating minimization scheme. The outer loop is described in Sec. 3.1; the inner loop is described in Sec. 3.2.

Alternating minimization schemes require the sequential solutions of subproblems that are coupled through target-response pairs and system-wide functions (Fig. 3(*a*)). For certain classes of problems, artificial master problems that allow parallel subproblem solutions can be introduced, as illustrated in Fig. 3(*b*). Two such classes of problems and their parallelization strategies are discussed in Sec. 3.3.

3.1 Outer Loop: Method of Multipliers. In the outer loop, the method of multipliers sets the penalty parameters $\mathbf{v}^{k+1}$ for outer iteration $k+1$ using the update formula [11,28]

$$\mathbf{v}^{k+1} = \mathbf{v}^k + 2\mathbf{w}^k \circ \mathbf{w}^k \circ \mathbf{q}^k \tag{7}$$

where $\mathbf{q}^k$ are the values of the *extended* consistency constraints at termination of the kth inner loop iteration. These are composed of the consistency constraints $\mathbf{c}$ and the system-wide inequality and equality constraints: $\mathbf{q}^T=[\mathbf{c}^T,(\mathbf{g}_0+\mathbf{s})^T,\mathbf{h}_0^T]$.

Large penalty weights can slow down the coordination algorithm and introduce ill-conditioning of the subproblems [23,29]. Therefore, the penalty weights $\mathbf{w}$ are increased by a factor β only when the reduction in the extended consistency constraint value is insufficient. Formally, the penalty weight w_i for the ith extended consistency constraint q_i is updated using the formula [11,28]

$$w_i^{k+1} = \begin{cases} w_i^k & \text{if } |q_i^k| \le \gamma |q_i^{k-1}| \\ \beta w_i^k & \text{if } |q_i^k| > \gamma |q_i^{k-1}| \end{cases} \tag{8}$$

where $\beta>1$ and $0<\gamma<1$. Coordination efficiency depends on the choice of these parameters, especially β. In our experience, the values of $\beta=2.2$ and $\gamma=0.4$ perform well, but may not be optimal for every problem.

The outer loop is terminated when two conditions are satisfied. The first condition requires that the change in the maximal extended consistency constraint value after two outer loop iterations be smaller than some user-defined termination tolerance $\varepsilon>0$ as follows:

$$\|\mathbf{q}^k - \mathbf{q}^{k-1}\|_\infty < \varepsilon \tag{9}$$

The second condition requires that the maximal extended consistency constraint violation be also smaller than the termination tolerance ε as follows:

$$\|\mathbf{q}^k\|_\infty < \varepsilon \tag{10}$$

The first condition ensures convergence of the subproblem solution sequence, while the second condition prevents premature termination.

3.2 Inner Loop: Alternating Optimization. In the inner loop depicted in Fig. 3(*a*) subproblems are solved sequentially for fixed penalty parameters $\mathbf{v}$ and $\mathbf{w}$ using an alternating optimization approach[1] [11]. This procedure is terminated when the relative change in the system objective function value given by

$$F(\overline{\mathbf{x}}_1, \ldots, \overline{\mathbf{x}}_M, \mathbf{s}) = \sum_{j=1}^{M} f_j(\overline{\mathbf{x}}_j) + f_0(\overline{\mathbf{x}}_1, \ldots, \overline{\mathbf{x}}_M) + \phi(\mathbf{q}(\overline{\mathbf{x}}_1, \ldots, \overline{\mathbf{x}}_M, \mathbf{s}))$$

for two consecutive inner loop iterations is smaller than some user-defined termination tolerance $\varepsilon_{\text{inner}}>0$ as follows:

$$\frac{|F^\xi - F^{\xi-1}|}{1 + |F^\xi|} < \varepsilon_{\text{inner}} \tag{11}$$

where ξ denotes the inner loop iteration number. The termination tolerance $\varepsilon_{\text{inner}}$ should be smaller than the outer loop termination tolerance ε to ensure sufficient accuracy of the inner loop solution; we use $\varepsilon_{\text{inner}}=\varepsilon/100$.

An alternative inner loop termination strategy is to exit the inner loop before convergence during the first few iterations by using looser tolerances. Such an inexact approach uses a different tolerance $\varepsilon_{\text{inner}}^k$ for each outer loop iteration. The idea behind such a strategy is that costly inner loop iterations are avoided when the penalty parameters are still far from their optimal values. Convergence for the outer loop in case of an inexact inner loop can still be guaranteed as long as the sequence of termination tolerances $\{\varepsilon_{\text{inner}}^k\}$ is nonincreasing and approaches zero.

An extreme case of the inexact approach is to terminate the inner loop after a single iteration. Such an approach is known as the alternating direction method of multipliers, and typically applies to problems that are coupled only through target-response pairs or linear system-wide functions [30]. Numerical experiments have shown that the alternating direction approach can be more efficient than iterative inner loops [10,12,23,31].

The solutions obtained using ALC algorithms with iterative inner loops have been shown to be Karush–Kuhn–Tucker points of the original nondecomposed optimization problem when the objective and system-wide functions are smooth and subproblem constraint sets are convex [11]. The convergence proof associated with the alternating direction method of multipliers assumes that objective and constraint functions are convex.

3.3 Parallelization of Subproblem Solutions. The nonhierarchical formulation provides flexibility in specifying the ATC process to match existing organizational or computational structures. The price paid for this flexibility is loss of parallelism, since the convergence analysis of the inner loop assumes that subproblems whose objectives are coupled are solved sequentially. Nonhierarchical problems with system-wide functions can be densely coupled, and opportunities for efficiency improvements through parallel solution of subproblems are limited. For certain classes of problems, approaches that allow the parallel solution of subproblems can be derived from augmented Lagrangian coordination methods, as presented in Ref. [12]. In particular, parallelization approaches can be derived for problems that have block-dependent system-wide functions of the form $f_0(\overline{\mathbf{x}}_1,\ldots,\overline{\mathbf{x}}_M)=f_0(\mathbf{f}_1(\overline{\mathbf{x}}_1),\ldots,\mathbf{f}_M(\overline{\mathbf{x}}_M))$, $\mathbf{g}_0(\overline{\mathbf{x}}_1,\ldots,\overline{\mathbf{x}}_M)=\mathbf{g}_0(\mathbf{f}_1(\overline{\mathbf{x}}_1),\ldots,\mathbf{f}_M(\overline{\mathbf{x}}_M))$, and $\mathbf{h}_0(\overline{\mathbf{x}}_1,\ldots,\overline{\mathbf{x}}_M)=\mathbf{h}_0(\mathbf{f}_1(\overline{\mathbf{x}}_1),\ldots,\mathbf{f}_M(\overline{\mathbf{x}}_M))$.

The target-response pairs $(\mathbf{t}_{jn},\mathbf{r}_{nj})$ and system-wide functions f_0, $\mathbf{g}_0$, and $\mathbf{h}_0$ couple the ATC subproblems, therefore preventing parallelization of subproblem solutions. The parallelization approach we propose here decouples the subproblems by introducing support variables and an artificial master problem that determines the optimal values of these support variables. The modified subproblems are coupled only to this master problem, thereby allowing their solution to be performed in parallel, as illustrated in Fig. 3(*b*).

The first set of support variables $\mathbf{F}^T=[\mathbf{F}_1^T,\ldots,\mathbf{F}_M^T]$ is introduced to remove coupling through system-wide functions. Each support variable $\mathbf{F}_j$ assumes the role of the term $\mathbf{f}_j(\overline{\mathbf{x}}_j)$ in the system-wide functions such that $f_0=f_0(\mathbf{F}_1,\ldots,\mathbf{F}_M)$, $\mathbf{g}_0=\mathbf{g}_0(\mathbf{F}_1,\ldots,\mathbf{F}_M)$, and $\mathbf{h}_0=\mathbf{h}_0(\mathbf{F}_1,\ldots,\mathbf{F}_M)$. To enforce consistency between the support variables $\mathbf{F}_j$ and the original terms $\mathbf{f}_j(\overline{\mathbf{x}}_j)$, consistency constraints $\mathbf{d}_j=\mathbf{F}_j-\mathbf{f}_j(\overline{\mathbf{x}}_j)=\mathbf{0}$ are introduced. Each of these constraints is relaxed using an augmented Lagrangian penalty

$$\theta_j(\mathbf{F}_j - \mathbf{f}_j(\overline{\mathbf{x}}_j)) = \mathbf{v}_j^T(\mathbf{F}_j - \mathbf{f}_j(\overline{\mathbf{x}}_j)) + \|\mathbf{w}_j \circ (\mathbf{F}_j - \mathbf{f}_j(\overline{\mathbf{x}}_j))\|_2^2 \tag{12}$$

The penalty terms θ replace the system-wide objective f_0 and penalties (5) on the system-wide constraints $\mathbf{g}_0,\mathbf{h}_0$ of subproblem (4). Since the penalty term θ_j does not depend on the variables of the remaining subproblems, subproblem j is no longer coupled to other subproblems through system-wide functions.

The coupling through target-response pairs is removed by introducing an intermediate response $\hat{\mathbf{r}}$, as illustrated in Fig. 4. For each target-response pair $(\mathbf{t}_{nj},\mathbf{r}_{jn})$ we introduce an intermediate

[1]The alternating optimization approach is also known as "nonlinear Gauss–Seidel" or "block-coordinate descent" [28,30].

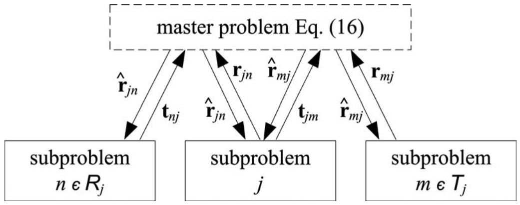

Fig. 4 Target and response flow with intermediate responses $\hat{\mathbf{r}}$

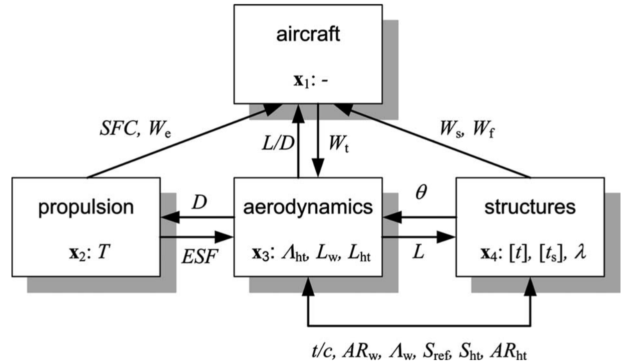

Fig. 5 Functional dependencies of the business jet problem: single arrows indicate flow of responses; double arrows indicate shared variables

response $\hat{r}_{jn}$. Instead of exchanging responses and targets among subproblems directly, consistency constraints (3) for subproblem j are redefined using the intermediate variables as

$$\hat{\mathbf{c}}^r_{nj} = \hat{\mathbf{r}}_{jn} - \mathbf{r}_{jn} = \mathbf{0}, \quad n \in \mathcal{R}_j$$

$$\hat{\mathbf{c}}^t_{jm} = \mathbf{t}_{jm} - \hat{\mathbf{r}}_{mj} = \mathbf{0}, \quad m \in \mathcal{T}_j \tag{13}$$

The superscript t or r is necessary to differentiate between the two consistency constraints associated with each target-response pair: one between the target and the intermediate response and one between the intermediate response and the actual response.

The general ATC subproblem formulation suitable for parallel processing is given by

$$\min_{\overline{\mathbf{x}}_j} f_j(\overline{\mathbf{x}}_j) + \sum_{n\in\mathcal{R}_j} \phi(\hat{\mathbf{r}}_{jn} - \mathbf{r}_{jn}) + \sum_{m\in\mathcal{T}_j} \phi(\mathbf{t}_{jm} - \hat{\mathbf{r}}_{mj}) + \theta_j(\mathbf{F}_j - \mathbf{f}_j(\overline{\mathbf{x}}_j))$$

$$\text{subject to } \mathbf{g}_j(\overline{\mathbf{x}}_j) \le \mathbf{0}$$

$$\mathbf{h}_j(\overline{\mathbf{x}}_j) = \mathbf{0}$$

$$\text{with } \mathbf{r}_{jn} = \mathbf{S}_{jn}\mathbf{a}_j(\mathbf{x}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j), \quad n \in \mathcal{R}_j$$

$$\overline{\mathbf{x}}_j = [\mathbf{x}_j, \mathbf{r}_{jn}|n \in \mathcal{R}_j, \mathbf{t}_{jm}|m \in \mathcal{T}_j] \tag{14}$$

The subproblems are no longer coupled to each other, and can thus be solved in parallel, as illustrated in Fig. 3(*b*).

A master problem is formulated to compute the values of the support and intermediate variables. The master problem is separable in each set of support variables: The first part is a nonlinear programming problem that includes the system-wide functions and the penalty terms θ_j as follows:

$$\min_{\mathbf{F}} f_0(\mathbf{F}_1, \ldots, \mathbf{F}_M) + \sum_{j=1}^{M} \theta_j(\mathbf{F}_j - \mathbf{f}_j(\overline{\mathbf{x}}_j))$$

$$\text{subject to } \mathbf{g}_0(\mathbf{F}_1, \ldots, \mathbf{F}_M) \le \mathbf{0}$$

$$\mathbf{h}_0(\mathbf{F}_1, \ldots, \mathbf{F}_M) = \mathbf{0} \tag{15}$$

When the system-wide functions are block-separable, the master problem becomes a quadratic programming problem that can be solved efficiently [12].

The second part includes only the penalty terms ϕ as follows:

$$\min_{\hat{\mathbf{r}}} \sum_{j=1}^{M} \left[\sum_{n\in\mathcal{R}_j} \phi(\hat{\mathbf{r}}_{jn} - \mathbf{r}_{jn}) + \sum_{m\in\mathcal{T}_j} \phi(\mathbf{t}_{jm} - \hat{\mathbf{r}}_{mj}) \right]$$

$$= \min_{\hat{\mathbf{r}}} \sum_{j=1}^{M} \sum_{n\in\mathcal{R}_j} [\phi(\mathbf{t}_{nj} - \hat{\mathbf{r}}_{jn}) + \phi(\hat{\mathbf{r}}_{jn} - \mathbf{r}_{jn})] \tag{16}$$

Since the augmented Lagrangian penalty is a convex and separable quadratic function in $\hat{\mathbf{r}}_{jn}$, the optimal values for the intermediate responses of master problem (16) can be found analytically as

$$\hat{\mathbf{r}}^*_{jn} = (\hat{W}^t_{jn} + \hat{W}^r_{jn})^{-1}\left(\mathbf{t}^T_{nj}\hat{W}^t_{jn} + \mathbf{r}^T_{jn}\hat{W}^r_{jn} + \tfrac{1}{2}(\hat{\mathbf{v}}^t_{jn} - \hat{\mathbf{v}}^r_{jn})\right) \tag{17}$$

where $\hat{W}^t_{jn}=\text{diag}(\hat{\mathbf{w}}^t_{jn}\circ\hat{\mathbf{w}}^t_{jn})$ and $\hat{W}^r_{jn}=\text{diag}(\hat{\mathbf{w}}^r_{jn}\circ\hat{\mathbf{w}}^r_{jn})$. Furthermore, $\hat{\mathbf{v}}^t_{jn}$ and $\hat{\mathbf{w}}^t_{jn}$ are the penalty parameters associated with the consistency constraints $\hat{\mathbf{c}}^t_{nj}=\mathbf{t}_{nj}-\hat{\mathbf{r}}_{jn}=\mathbf{0}$, while $\hat{\mathbf{v}}^r_{jn}$ and $\hat{\mathbf{w}}^r_{jn}$ are associated with $\hat{\mathbf{c}}^r_{nj}=\hat{\mathbf{r}}_{jn}-\mathbf{r}_{jn}=\mathbf{0}$.

The elimination of coupling through block-dependent system-wide functions and target-response pairs comes at the expense of additional consistency constraints. The coordination of these additional consistency constraints can, and often will, increase the effort required to solve the decomposed problem. Whether this additional effort is compensated by the benefits of parallelization depends on the achieved speed-up of the time-per-iteration in the inner loop. Speed-up values are problem- and decomposition-dependent, and require further investigation.

The obtained parallelized coordination structure resembles a bilevel hierarchical ATC formulation. The difference with the original ATC formulation is that the master problem is an artifact of parallelization, whereas the top-level problem in the hierarchical ATC formulation is associated with the design problem of a physical system. Note that the parallelization approach presented here can also be used to parallelize hierarchical ATC formulations.

In Ref. [31], an alternative parallelization strategy that does not introduce additional variables or a master problem is proposed for hierarchical ATC problems. The approach relies on building separable approximations of the terms that couple subproblems (6). After the parallel solution of subproblems, the approximations are updated and the subproblems are solved again. Since this approach relies on approximations, external mechanisms such as move limits or trust regions are advised to control the quality of the approximations. Our approach does not rely on approximations and therefore does not require such an external mechanism.

4 Supersonic Business Jet Design Example

A supersonic business jet design problem is used to demonstrate the nonhierarchical ATC formulation. The example was reported in Ref. [25]; modified versions have been used previously to demonstrate the use of other coordination algorithms [6,32].

The optimization problem is formulated to minimize the weight of the aircraft while satisfying requirements from the subproblems "structures," "aerodynamics," "propulsion," and "aircraft." The cruise altitude is fixed at $h=55,000$ ft, and the cruise velocity is assumed to be Mach 1.4. The four subproblems and their functional dependencies are displayed in Fig. 5. Table 1 gives a brief description of the variables and lists their reference values. The six shared variables $\mathbf{z}$ are design variables in both the aerodynamics and the structures subproblems, and are depicted in Fig. 5 as double arrows. The ten coupling variables $\mathbf{y}$ are computed as outputs of one subproblem and are used as inputs to other subprob-

Table 1 Design variables for the supersonic business jet problem; MAC=mean aerodynamic chord. Optimal values are listed for the design obtained using MDF.

		Lower $\leq$ reference $\leq$ upper	Units	Optimal
Shared variables $\mathbf{z}$				
t/c	Thickness/chord	$0.01 \leq 0.05 \leq 0.10$		0.0641
AR_w	Wing aspect ratio	$2.5 \leq 3.0 \leq 8.0$		2.5
Λ_w	Wing sweep angle	$40 \leq 60 \leq 70$	deg	70
S_{ref}	Wing surface area	$200 \leq 500 \leq 800$	ft^2	667
S_{ht}	Tail surface area	$50.0 \leq 100.0 \leq 148.9$	ft^2	99.7
AR_{ht}	Tail aspect ratio	$2.5 \leq 5.5 \leq 8.5$		2.5
Local variables $\mathbf{x}$				
T	Thrust	$0.1 \leq 0.6 \leq 1.0$		0.196
Λ_{ht}	Tail sweep	$40 \leq 45 \leq 70$	deg	70.0
L_w	Wing distance[a]	$0.01 \leq 0.15 \leq 0.2$	%MAC	0.01
L_{ht}	Tail distance[a]	$1.0 \leq 1.5 \leq 3.5$	%MAC	3.5
$\mathbf{t}$	Nine thicknesses	$0.1 \leq 3.0 \leq 4.0$	in.	[0.97 0.52 0.21 4.00 3.66 0.91 0.97 0.52 0.21]
$\mathbf{t}_s$	Nine thicknesses	$0.1 \leq 6.0 \leq 9.0$	in.	[2.17 1.48 0.76 4.40 4.02 1.01 2.17 1.48 0.76]
λ	Taper ratio	$0.1 \leq 0.3 \leq 0.4$		0.10
Coupling variables $\mathbf{y}$				
L	Total lift	$5 \leq 25 \leq 100$	10^3 lb	34.3
W_e	Engine weight	$0.1 \leq 15 \leq 30$	10^3 lb	6.49
W_t	Total weight	$5 \leq 25 \leq 100$	10^3 lb	34.3
θ	Wing twist	$0.2 \leq 10.0 \leq 50.0$	deg	19.1
ESF	Engine scaling factor	$0.5 \leq 1.0 \leq 1.5$		0.75
D	Total drag	$1 \leq 40 \leq 70$	10^3 lb	4.80
W_f	Fuel weight	$5 \leq 25 \leq 100$	10^3 lb	10.3
L/D	Lift/drag ratio	$0.1 \leq 5.0 \leq 10.0$		7.03
SFC	Specific fuel consumption	$1.0 \leq 2.0 \leq 4.0$		1.0
W_s	Structural weight	$5 \leq 25 \leq 100$	10^3 lb	17.5

[a]Distance between the aerodynamic center of the wing/horizontal tail and the center of gravity of the aircraft.

lems. Local subproblem variables are denoted by $\mathbf{x}$. The problem has a total of 39 design variables and 46 design constraints. The reader is referred to Ref. [25] for a detailed description of the problem and analysis models.

4.1 All-in-One Problem. The all-in-one design optimization problem for the business jet design optimization is

$$\min_{\mathbf{z},\mathbf{x},\mathbf{y}} W_t$$

$$\text{subject to } g_{\text{aircraft}}(\mathbf{y}) \leq \mathbf{0}$$

$$\mathbf{g}_{\text{prop}}(\mathbf{y},\mathbf{x}_2) \leq \mathbf{0}$$

$$\mathbf{g}_{\text{aero}}(\mathbf{z},\mathbf{y},\mathbf{x}_3) \leq \mathbf{0}$$

$$\mathbf{g}_{\text{struc}}(\mathbf{z},\mathbf{y},\mathbf{x}_4) \leq \mathbf{0}$$

$$\text{where } \mathbf{z} = [t/c, AR_w, \Lambda_w, S_{ref}, S_{ht}, AR_{ht}]$$

$$\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4]$$

$$\mathbf{x}_1 = [\ ], \quad \mathbf{x}_2 = [T], \quad \mathbf{x}_3 = [\Lambda_{ht}, L_w, L_{ht}], \quad \mathbf{x}_4 = [\mathbf{t}, \mathbf{t}_s, \lambda]$$

$$\mathbf{y} = [L, W_e, W_t, \theta, \text{ESF}, D, W_f, L/D, \text{SFC}, W_s] \tag{18}$$

To illustrate the proposed nonhierarchical ATC formulation, our implementation of the problem differs slightly from the original version of Ref. [25]. Besides fixing the cruise altitude and cruise velocity, we minimize the total weight instead of maximizing the range. A constraint g_{aircraft} that requires the range to be at least 2000 nautical miles is added to the problem. Moreover, the structural weight W_s is a newly introduced variable that includes the weight of the total aircraft except the engine weight W_e and fuel weight W_f. Subproblem aircraft computes the total weight W_t of the aircraft. The analysis models, reference design, and variable bounds are taken from Ref. [25].

The optimization problem is solved first using a single-level formulation to obtain a benchmark solution. We used the multidisciplinary feasible (MDF) [33] approach. The system analyzer in MDF computes the values of $\mathbf{y}$ for fixed $\mathbf{z}$ and $\mathbf{x}$ by performing Gauss–Seidel iterations that call the structures, aerodynamics, propulsion, and aircraft analyses sequentially. The problem was solved for 100 different starting points selected randomly within $\pm 10\%$ of the reference design using "fmincon" (MATLAB's sequential quadratic programming solver [34]) with default settings and computing gradients by means of the built-in finite difference routine. The termination tolerances TolX, TolFun, and TolCon were set to 10^{-3}. The objective function W_t is measured in 10^3 lb, and variables are scaled such that the reference values of Table 1 are equal to 1. The obtained results indicate that multiple local minima exist. The optimal weights obtained from the 100 starting points are summarized in Table 2. The optimal design variable values for the most frequently obtained solution with a weight of 34.3×10^3 are listed in Table 1. This solution was obtained from 84 out of the 100 starting points. Another ten solutions are clustered around a design with an optimal weight of 38.1×10^3. The remaining six solutions have optimal weights between 33.6×10^3 and 37.8×10^3. The optimality conditions at these solutions are satisfied within tolerance.

4.2 ATC Formulations. The ATC problem is first formulated hierarchically, with the top-level subproblem aircraft coordinating the propulsion, aerodynamics, and structures subproblems. Figure 6(*a*) depicts the flow of response variables. Coupling between lower-level subproblems is coordinated at the top level by introducing additional target-response pairs. The feedback target for W_t

Table 2 Optimization results for the all-in-one and sequential ATC experiments for the four formulations of Fig. 6 for 100 random starting points within ±10% of the reference design. Optimal weights, maximum constraint violations, and total number of iterations are reported for each formulation. The minimum, average, and maximum for each formulation are determined from the converged runs only.

Formulation			(a)	(b)	(c)	(d)
Coordination		All-in-one	Sequential	Sequential	Sequential	Sequential
Converged runs		100/100	98/100	100/100	89/100	94/100
Optimal weight ($\times 10^3$ lb)	Min	33.6	34.0	33.9	33.6	34.0
	Mean	34.7	34.9	35.0	34.3	34.7
	Max	38.1	39.2	48.5	38.8	46.0
Maximum constraint violation	Min	0.000	0.000	0.000	0.000	0.000
	Mean	0.000	0.001	0.002	0.002	0.003
	Max	0.001	0.009	0.009	0.006	0.006
Iterations (total)	Min	-	181	166	145	76
	Mean	-	244	209	202	85
	Max	-	283	261	242	131

coupling aircraft with aerodynamics can be included in the ATC formulation of Ref. [22], which allows feedback coupling. Subproblem aircraft in the hierarchical ATC formulation has 15 variables, whereas its analysis requires only 5 of them. The other ten variables are introduced solely for coordination. The hierarchical ATC formulation has a total of 26 target-response pairs. The subproblem formulations for this and the following ATC formulations are included in the Appendix.

The nonhierarchical ATC formulation is depicted in Fig. 6(*b*). In this formulation, each subproblem can send targets to the other subproblems directly. The nonhierarchical formulation has 16 target-response pairs, a reduction of 10 with respect to the hierarchical formulation. Note that the lower-level subproblems propulsion and structures cannot be solved in parallel with subproblem aerodynamics due to the direct coupling between them. No obvious target-response relation for the shared variables $\mathbf{z}$ is present, since these variables are not responses of either aerodynamics or structures. The direction of responses for these variables has no influence on the formulation of each subproblem and can therefore be chosen arbitrarily.

The use of a system-wide constraint for g_{aircraft} can eliminate

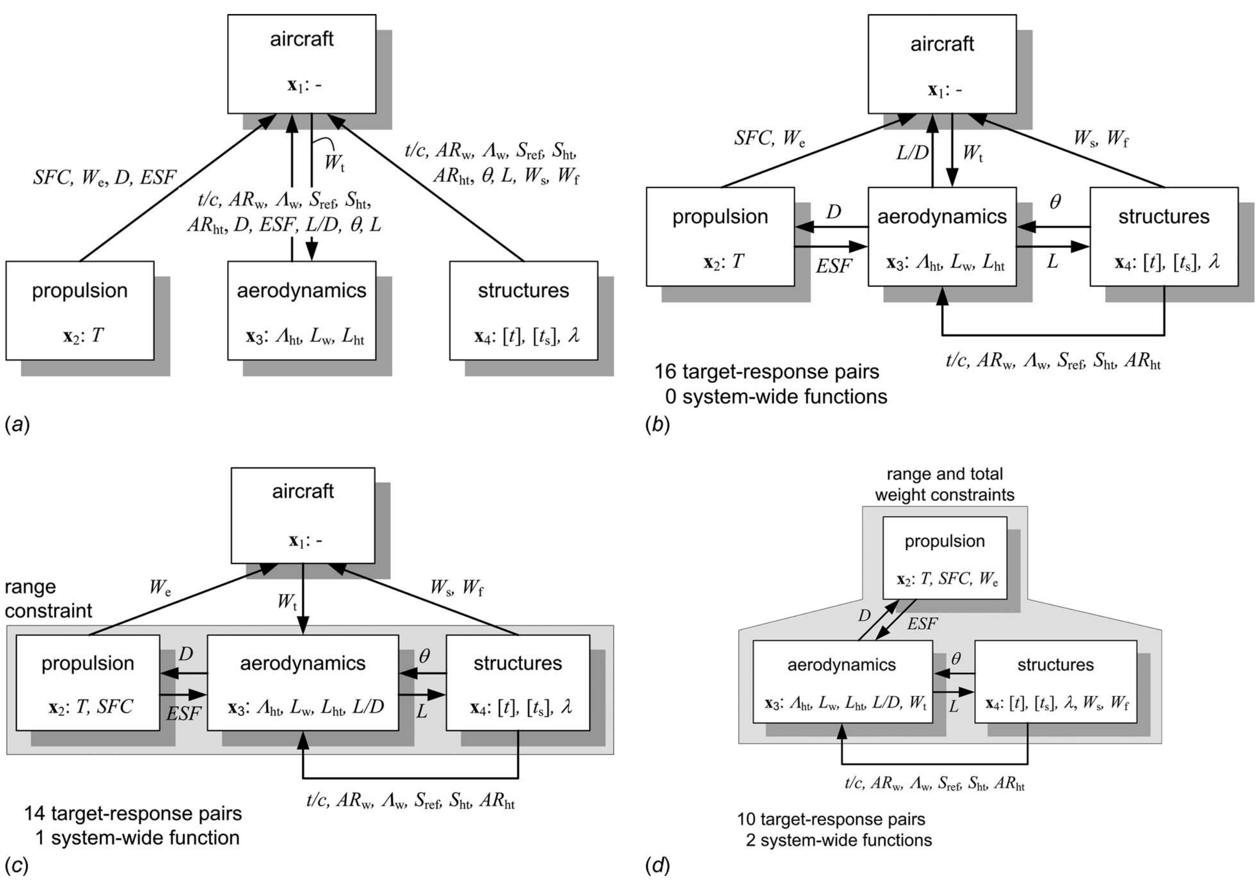

Fig. 6 Four different ATC formulations. Single arrows represent target-response coupling where the direction of the arrow indicates the direction of response flow. The response directions for the shared variables t/c, $\mathrm{AR_w}$, Λ_w, S_{ref}, S_{ht}, and $\mathrm{AR_{ht}}$ for all but the hierarchical ATC formulation are chosen arbitrarily. The dashed boxes annotated by "range constraint" in Fig. 6(*c*) and "range and total weight constraints" in Fig. 6(*d*) represent system-wide constraints.

Table 3 Optimization results for the all-in-one and parallel ATC experiments for the four formulations of Fig. 6 for 100 random starting points within ±10% of the reference design. Optimal weights, maximum constraint violations, and total number of iterations are reported for each formulation. The minimum, average, and maximum for each formulation are determined from the converged runs only. (* =the implementation and thus the results for parallel ATC of formulation (d) are identical to the sequential ATC formulation (a).)

Formulation			(a)	(b)	(c)	(d)
Coordination		All-in-one	Parallel	Parallel	Parallel	Parallel*
Converged runs		100/100	91/100	93/100	97/100	98/100
Optimal weight ($\times 10^3$ lb)	Min	33.6	34.1	34.3	34.1	34.0
	Mean	34.7	35.0	34.6	35.0	34.9
	Max	38.1	38.3	38.0	49.8	39.2
Maximum constraint violation	Min	0.000	0.000	0.000	0.000	0.000
	Mean	0.000	0.001	0.002	0.001	0.001
	Max	0.001	0.008	0.009	0.010	0.009
Iterations (total)	Min	-	206	228	164	181
	Mean	-	343	291	234	244
	Max	-	538	507	277	283

the specific fuel consumption (SFC) and the lift-over-drag ratio L/D as target-response pairs. Since only the range constraint g_{aircraft} depends on these variables, coordinating this constraint as a system-wide constraint eliminates them as target variables from the aircraft subproblem. Instead, SFC becomes a local variable of propulsion and L/D is included in the local variables of aerodynamics. The formulation with the range as a system-wide inequality constraint is depicted in Fig. 6(*c*), where the box annotated "range constraint" represents the system-wide range constraint g_{aircraft}. The total number of target-response pairs for this formulation is 14. Again, the direction of responses for the shared variables $\mathbf{z}$ is chosen arbitrarily.

It is also possible to eliminate the aircraft subproblem completely from the formulation by including the response relation $W_t=W_e+W_f+W_s$ for subproblem aircraft as a system-wide equality constraint $h_{\text{aircraft}}=W_t-W_e-W_f-W_s=0$. The weight variables become local to each subproblem, as illustrated in Fig. 6(*d*), and the total number of target-response pairs is 10.

4.3 Numerical Results. The ATC problem was solved for each formulation using 100 different starting points selected randomly within ±10% of the reference design, which is far from the optimal design. For each formulation, we used an alternating direction method of multiplier algorithm as outlined in Sec. 3 with an initial weight selection as presented in Ref. [11].

A second set of experiments was conducted to investigate the effect of parallelization. Parallel formulations with an artificially introduced master problem were implemented for each formulation using the approach outlined in Sec. 3.3. The master problem is superimposed over the subproblems of the formulations of Fig. 6, and all subproblems are positioned at a lower level (according to Fig. 4). The parallelization is possible since formulations (a) and (b) have only target-response pairs, and since formulations (c) and (d) have block-dependent constraints. We note that the *parallel* formulation (d) is identical to the *sequential* formulation (a); subproblem aircraft of (a) combines the roles of master problems (16) and (15) that would be present in a parallel implementation of (d).

For both sequential and parallel formulations, the initial weight selection strategy is used with an initial objective estimate of $\hat{f}=10$, $\mathbf{w}^0=0.001$, and $\alpha=0.1$. The penalty update parameters are set to $\beta=1.03$ and $\gamma=0.75$, and the termination tolerance is $\varepsilon=10^{-3}$. Disciplinary subproblems are solved using the MATLAB implementation of the sequential quadratic programming algorithm fmincon [34] with default settings.

The reader should be aware that it is not our intention to compare the ATC formulations to the all-in-one approach. An all-in-one approach should be used when possible; our contribution is pertinent to problems that either already appear in a decomposed form or need to be decomposed because they are not amenable to the all-in-one approach. We include the all-in-one results solely for the sake of information.

Table 2 summarizes the results for the sequential ATC experiments by reporting the obtained weight, maximum constraint violations, and total number of iterations for each of the four formulations. The minimum, average, and maximum values for these performance measures are determined for each formulation from the converged runs only. Since the alternating direction method performs only a single inner iteration for each outer iteration, an iteration is defined as the solution of all subproblems followed by the update of the penalty parameters. For the parallel implementation case, an iteration also includes the solution of the master problem. The computational effort for coordination is small since the penalty parameter updates and the solution of the master problem are straightforward to perform (analytical expressions in all but the nonparallelized case with system-wide function).

The results show that all ATC formulations with an alternating direction method converge to objective values that are very close to those obtained with the all-in-one implementation, even though the convexity and separability assumptions for the theoretical convergence proof are not satisfied. However, the number of successfully converged runs is smaller than for the all-in-one implementation, which demonstrates empirically that problem decomposition of this example introduces numerical difficulties.

The number of required iterations decreases for each formulation, which demonstrates that computational cost can be reduced by allowing direct communication between subproblems using the proposed nonhierarchical formulation. This cost reduction can be attributed to the reduction in target-response pairs that have to be coordinated. The hierarchical ATC formulation has 26 of these pairs, while the nonhierarchical formulation (b) has only 16 pairs. The results show that reducing this number also reduces the required coordination effort for this example. Coordination of the range constraint as a system-wide constraint removes another two pairs, but does not show a significant reduction in iterations for this example. Although the observed minimum number of iterations is smaller for formulation (c) than for (b), the maximum is increased, and the average number of iterations is similar. Eliminating subproblem aircraft by making the weight response a system-wide equality constraint does reduce computational cost substantially. For this final formulation, four target-response pairs are eliminated, and only ten pairs have to be coordinated between the three remaining subproblems. Note that gains in cost-per-iteration may be gained for formulations (a) and (b) through parallel solution of subproblems that are not coupled (all lower-level subproblems in (a), and propulsion and structures in (b)).

Table 3 lists the results for the parallel formulations of each formulation. The results indicate again that ATC algorithms converge to optimal designs, and that the number of iterations decreases with each formulation, except for formulation (d), which requires slightly more iterations than formulation (c). A comparison between the sequential and parallel solution process shows a substantial increase in the number of iterations due to parallelization. The most likely reason is the increased number of consistency constraints that are introduced to enable parallelization. Whether the benefits of parallelization compensate for this additional number of iterations depends on the achieved speed-up of the time-per-iteration. The average speed-up in terms of required function evaluations per iteration for each of the four formulations are 1.6, 2.7, 3.0, and 1.9, respectively.

5 Concluding Remarks

A new nonhierarchical formulation for ATC has been presented in this paper. The new ATC formulation provides design engineers with more flexibility in specifying the coordination of decomposed optimal system design problems and increases the applicability of ATC to general decomposition-based optimal system design and MDO problems. The formulation extensions maintain the advantageous convergence properties of ATC and augmented Lagrangian coordination under standard convexity and continuity assumptions. A strategy for parallelization that maintains these convergence properties is proposed; it can be applied to the hierarchical as well as the nonhierarchical ATC formulation.

The flexibility of the proposed formulation is demonstrated using a business jet example from literature. Moreover, the flexibility offered by the presented formulation gives model-based decomposition methods as found in, e.g., Refs. [35–38], substantial freedom for identifying computationally inexpensive ATC formulations.

Computational benefits of the new ATC formulation are expected to be greater for systems with a large amount of nonhierarchical coupling and/or with a few system-wide functions that depend on a large number of local variables. Examples of system-wide functions are requirements on distributed quantities such as mass, cost, volume, or power. Most decomposition-based coordination methods available in literature (e.g., bilevel integrated system synthesis or collaborative optimization) focus on problems with lower-order coupling between design groups. The nonhierarchical ATC formulation is applicable to general decomposition-based and MDO problems; its efficiency when solving different classes of problems will hopefully be tested by the community for a wide range of applications.

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Appendix: Subproblem Formulations for Business Jet Design Example

Superscripts denote the subproblem at which variable values are computed: "a" refers to aircraft, "p" to propulsion, "ae" to aerodynamics, and "s" to structures.

1 Formulation (a). Aircraft:

$$\text{Find } \mathrm{SFC}^{\mathrm{a}}, W_{\mathrm{e}}^{\mathrm{a}}, D^{\mathrm{a}}, \mathrm{ESF}^{\mathrm{a}}, L/D^{\mathrm{a}}, t/c^{\mathrm{a}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{a}}, \Lambda_{\mathrm{w}}^{\mathrm{a}}, S_{\mathrm{ref}}^{\mathrm{a}}, S_{\mathrm{ht}}^{\mathrm{a}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{a}}, \theta^{\mathrm{a}}, L^{\mathrm{a}}, W_{\mathrm{f}}^{\mathrm{a}}, W_{\mathrm{s}}^{\mathrm{a}}$$

$$\begin{aligned}
\min\ & W_{\mathrm{t}}^{\mathrm{a}} + \phi(\mathrm{SFC}^{\mathrm{a}} - \mathrm{SFC}^{\mathrm{p}}) + \phi(W_{\mathrm{e}}^{\mathrm{a}} - W_{\mathrm{e}}^{\mathrm{p}}) + \phi(D^{\mathrm{a}} - D^{\mathrm{ae}}) + \phi(D^{\mathrm{a}} - D^{\mathrm{p}}) \\
& + \phi(\mathrm{ESF}^{\mathrm{a}} - \mathrm{ESF}^{\mathrm{ae}}) + \phi(\mathrm{ESF}^{\mathrm{a}} - \mathrm{ESF}^{\mathrm{p}}) + \phi(W_{\mathrm{t}}^{\mathrm{ae}} - W_{\mathrm{t}}^{\mathrm{a}}) + \phi(L/D^{\mathrm{a}} - L/D^{\mathrm{ae}}) \\
& + \phi(t/c^{\mathrm{a}} - t/c^{\mathrm{ae}}) + \phi(t/c^{\mathrm{a}} - t/c^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{w}}^{\mathrm{a}} - \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}) + \phi(\mathrm{AR}_{\mathrm{w}}^{\mathrm{a}} - \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}) \\
& + \phi(\Lambda_{\mathrm{w}}^{\mathrm{a}} - \Lambda_{\mathrm{w}}^{\mathrm{ae}}) + \phi(\Lambda_{\mathrm{w}}^{\mathrm{a}} - \Lambda_{\mathrm{w}}^{\mathrm{s}}) + \phi(S_{\mathrm{ref}}^{\mathrm{a}} - S_{\mathrm{ref}}^{\mathrm{ae}}) + \phi(S_{\mathrm{ref}}^{\mathrm{a}} - S_{\mathrm{ref}}^{\mathrm{s}}) \\
& + \phi(S_{\mathrm{ht}}^{\mathrm{a}} - S_{\mathrm{ht}}^{\mathrm{ae}}) + \phi(S_{\mathrm{ht}}^{\mathrm{a}} - S_{\mathrm{ht}}^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{ht}}^{\mathrm{a}} - \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}) + \phi(\mathrm{AR}_{\mathrm{ht}}^{\mathrm{a}} - \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}) \\
& + \phi(\theta^{\mathrm{a}} - \theta^{\mathrm{ae}}) + \phi(\theta^{\mathrm{a}} - \theta^{\mathrm{s}}) + \phi(L^{\mathrm{a}} - L^{\mathrm{ae}}) + \phi(L^{\mathrm{a}} - L^{\mathrm{s}}) \\
& + \phi(W_{\mathrm{f}}^{\mathrm{a}} - W_{\mathrm{f}}^{\mathrm{s}}) + \phi(W_{\mathrm{s}}^{\mathrm{a}} - W_{\mathrm{s}}^{\mathrm{s}})
\end{aligned}$$

$$\text{subject to } g_{\mathrm{aircraft}}(\mathrm{SFC}^{\mathrm{a}}, W_{\mathrm{e}}^{\mathrm{a}}, L/D^{\mathrm{a}}, W_{\mathrm{f}}^{\mathrm{a}}, W_{\mathrm{s}}^{\mathrm{a}}) \leq 0$$

$$\text{where } W_{\mathrm{t}}^{\mathrm{a}} = W_{\mathrm{t}}(W_{\mathrm{e}}^{\mathrm{a}}, W_{\mathrm{f}}^{\mathrm{a}}, W_{\mathrm{s}}^{\mathrm{a}}) \tag{A1}$$

Propulsion:

$$\text{Find } D^{\mathrm{p}}, T$$

$$\min\ \phi(\mathrm{SFC}^{\mathrm{a}} - \mathrm{SFC}^{\mathrm{p}}) + \phi(W_{\mathrm{e}}^{\mathrm{a}} - W_{\mathrm{e}}^{\mathrm{p}}) + \phi(D^{\mathrm{a}} - D^{\mathrm{p}}) + \phi(\mathrm{ESF}^{\mathrm{a}} - \mathrm{ESF}^{\mathrm{p}})$$

$$\text{subject to } \mathbf{g}_{\mathrm{prop}}(D^{\mathrm{p}}, T) \leq \mathbf{0}$$

$$\begin{aligned}
\text{where } W_{\mathrm{e}}^{\mathrm{p}} &= W_{\mathrm{e}}(D^{\mathrm{p}}, T) \\
\mathrm{SFC}^{\mathrm{p}} &= \mathrm{SFC}(D^{\mathrm{p}}, T) \\
\mathrm{ESF}^{\mathrm{p}} &= \mathrm{ESF}(D^{\mathrm{p}}, T)
\end{aligned} \tag{A2}$$

Aerodynamics:

$$\text{Find } W_t^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}, ESF^{ae}, \theta^{ae}$$

$$\begin{aligned} \min\ & \phi(W_t^{ae} - W_t^{a}) + \phi(t/c^{a} - t/c^{ae}) + \phi(AR_w^{a} - AR_w^{ae}) + \phi(\Lambda_w^{a} - \Lambda_w^{ae}) \\ & + \phi(S_{ref}^{a} - S_{ref}^{ae}) + \phi(S_{ht}^{a} - S_{ht}^{ae}) + \phi(AR_{ht}^{a} - AR_{ht}^{ae}) + \phi(D^{a} - D^{ae}) \\ & + \phi(ESF^{a} - ESF^{ae}) + \phi(L/D^{a} - L/D^{ae}) + \phi(\theta^{a} - \theta^{ae}) + \phi(L^{a} - L^{ae}) \end{aligned}$$

$$\text{subject to } \mathbf{g}_{prop}(W_t^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}, ESF^{ae}, \theta^{ae}) \le \mathbf{0}$$

$$\text{where } D^{ae} = D(W_t^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}, ESF^{ae}, \theta^{ae})$$

$$L/D^{ae} = L/D(W_t^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}, ESF^{ae}, \theta^{ae})$$

$$L^{ae} = L(W_t^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}, ESF^{ae}, \theta^{ae}) \quad \text{(A3)}$$

Structures:

$$\text{Find } t/c^{s}, AR_w^{s}, \Lambda_w^{s}, S_{ref}^{s}, S_{ht}^{s}, AR_{ht}^{s}, L^{s}, [t], [t_s], \lambda$$

$$\begin{aligned} \min\ & \phi(t/c^{a} - t/c^{s}) + \phi(AR_w^{a} - AR_w^{s}) + \phi(\Lambda_w^{a} - \Lambda_w^{s}) + \phi(S_{ref}^{a} - S_{ref}^{s}) \\ & + \phi(S_{ht}^{a} - S_{ht}^{s}) + \phi(AR_{ht}^{a} - AR_{ht}^{s}) + \phi(\theta^{a} - \theta^{s}) + \phi(L^{a} - L^{s}) + \phi(W_s^{a} - W_s^{s}) + \phi(W_f^{a} - W_f^{s}) \end{aligned}$$

$$\text{subject to } \mathbf{g}_{struc}(t/c^{s}, AR_w^{s}, \Lambda_w^{s}, S_{ref}^{s}, S_{ht}^{s}, AR_{ht}^{s}, L^{s}, [t], [t_s], \lambda) \le \mathbf{0}$$

$$\text{where } W_s^{s} = W_s(t/c^{s}, AR_w^{s}, \Lambda_w^{s}, S_{ref}^{s}, S_{ht}^{s}, AR_{ht}^{s}, L^{s}, [t], [t_s], \lambda)$$

$$W_f^{s} = W_f(t/c^{s}, AR_w^{s}, \Lambda_w^{s}, S_{ref}^{s}, S_{ht}^{s}, AR_{ht}^{s}, L^{s}, [t], [t_s], \lambda)$$

$$\theta^{s} = \theta(t/c^{s}, AR_w^{s}, \Lambda_w^{s}, S_{ref}^{s}, S_{ht}^{s}, AR_{ht}^{s}, L^{s}, [t], [t_s], \lambda) \quad \text{(A4)}$$

2 Formulation (b). Aircraft:

$$\text{Find } SFC^{a}, W_e^{a}, L/D^{a}, W_s^{a}, W_f^{a}$$

$$\begin{aligned} \min\ & W_t^{a} + \phi(SFC^{a} - SFC^{p}) + \phi(W_e^{a} - W_e^{p}) \\ & + \phi(L/D^{a} - L/D^{ae}) + \phi(W_t^{ae} - W_t^{a}) + \phi(W_s^{a} - W_s^{s}) + \phi(W_f^{a} - W_f^{s}) \end{aligned}$$

$$\text{subject to } g_{aircraft}(SFC^{a}, W_e^{a}, L/D^{a}, W_f^{a}, W_s^{a}) \le \mathbf{0}$$

$$\text{where } W_t^{a} = W_t(W_e^{a}, W_f^{a}, W_s^{a}) \quad \text{(A5)}$$

Propulsion:

$$\text{Find } D^{p}, T$$

$$\min\ \phi(SFC^{a} - SFC^{p}) + \phi(W_e^{a} - W_e^{p}) + \phi(D^{p} - D^{ae}) + \phi(ESF^{ae} - ESF^{p})$$

$$\text{subject to } \mathbf{g}_{prop}(D^{p}, T) \le \mathbf{0}$$

$$\text{where } W_e^{p} = W_e(D^{p}, T)$$

$$SFC^{p} = SFC(D^{p}, T)$$

$$ESF^{p} = ESF(D^{p}, T) \quad \text{(A6)}$$

Aerodynamics:

$$\text{Find } ESF^{ae}, W_t^{ae}, \theta^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}$$

$$\begin{aligned} \min\ & \phi(ESF^{ae} - ESF^{p}) + \phi(D^{p} - D^{ae}) + \phi(W_t^{ae} - W_t^{a}) + \phi(L/D^{a} - L/D^{ae}) \\ & + \phi(L^{s} - L^{ae}) + \phi(\theta^{ae} - \theta^{s}) + \phi(t/c^{ae} - t/c^{s}) + \phi(AR_w^{ae} - AR_w^{s}) \\ & + \phi(\Lambda_w^{ae} - \Lambda_w^{s}) + \phi(S_{ref}^{ae} - S_{ref}^{s}) + \phi(S_{ht}^{ae} - S_{ht}^{s}) + \phi(AR_{ht}^{ae} - AR_{ht}^{s}) \end{aligned}$$

$$\text{subject to } \mathbf{g}_{prop}(W_t^{ae}, \theta^{ae}, ESF^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht}) \le \mathbf{0}$$

$$\text{where } D^{ae} = D(W_t^{ae}, \theta^{ae}, ESF^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht})$$

$$L/D^{ae} = L/D(W_t^{ae}, \theta^{ae}, ESF^{ae}, t/c^{ae}, AR_w^{ae}, \Lambda_w^{ae}, S_{ref}^{ae}, S_{ht}^{ae}, AR_{ht}^{ae}, \Lambda_{ht}, L_w, L_{ht})$$

$$L^{\mathrm{ae}} = L(W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, \mathrm{ESF}^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}}) \tag{A7}$$

Structures:

$$\begin{aligned}
&\text{Find } L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda \\
&\min\ \phi(W_{\mathrm{f}}^{\mathrm{a}} - W_{\mathrm{f}}^{\mathrm{s}}) + \phi(W_{\mathrm{s}}^{\mathrm{a}} - W_{\mathrm{s}}^{\mathrm{s}}) + \phi(L^{\mathrm{s}} - L^{\mathrm{ae}}) + \phi(\theta^{\mathrm{ae}} - \theta^{\mathrm{s}}) \\
&\quad + \phi(t/c^{\mathrm{ae}} - t/c^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}) \\
&\quad + \phi(\Lambda_{\mathrm{w}}^{\mathrm{ae}} - \Lambda_{\mathrm{w}}^{\mathrm{s}}) + \phi(S_{\mathrm{ref}}^{\mathrm{ae}} - S_{\mathrm{ref}}^{\mathrm{s}}) + \phi(S_{\mathrm{ht}}^{\mathrm{ae}} - S_{\mathrm{ht}}^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}) \\
&\text{subject to } \mathbf{g}_{\mathrm{struc}}(L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda) \le \mathbf{0} \\
&\text{where } W_{\mathrm{s}}^{\mathrm{s}} = W_{\mathrm{s}}(L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda) \\
&W_{\mathrm{f}}^{\mathrm{s}} = W_{\mathrm{f}}(L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda) \\
&\theta^{\mathrm{s}} = \theta(L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda)
\end{aligned} \tag{A8}$$

3 Formulation (c). Aircraft:

$$\begin{aligned}
&\text{Find } W_{\mathrm{e}}^{\mathrm{a}}, W_{\mathrm{s}}^{\mathrm{a}}, W_{\mathrm{f}}^{\mathrm{a}} \\
&\min\ W_{\mathrm{t}}^{\mathrm{a}} + \phi(W_{\mathrm{e}}^{\mathrm{a}} - W_{\mathrm{e}}^{\mathrm{p}}) + \phi(W_{\mathrm{t}}^{\mathrm{ae}} - W_{\mathrm{t}}^{\mathrm{a}}) + \phi(W_{\mathrm{s}}^{\mathrm{a}} - W_{\mathrm{s}}^{\mathrm{s}}) + \phi(W_{\mathrm{f}}^{\mathrm{a}} - W_{\mathrm{f}}^{\mathrm{s}}) \\
&\text{where } W_{\mathrm{t}}^{\mathrm{a}} = W_{\mathrm{t}}(W_{\mathrm{e}}^{\mathrm{a}}, W_{\mathrm{f}}^{\mathrm{a}}, W_{\mathrm{s}}^{\mathrm{a}})
\end{aligned} \tag{A9}$$

Propulsion:

$$\begin{aligned}
&\text{Find } D^{\mathrm{p}}, T, s \\
&\min\ \phi(W_{\mathrm{e}}^{\mathrm{a}} - W_{\mathrm{e}}^{\mathrm{p}}) + \phi(D^{\mathrm{p}} - D^{\mathrm{ae}}) + \phi(\mathrm{ESF}^{\mathrm{ae}} - \mathrm{ESF}^{\mathrm{p}}) \\
&\quad + \phi(g_{\mathrm{aircraft}}(\mathrm{SFC}^{\mathrm{p}}, W_{\mathrm{e}}^{\mathrm{p}}, L/D^{\mathrm{ae}}, W_{\mathrm{s}}^{\mathrm{s}}, W_{\mathrm{f}}^{\mathrm{s}}) + s) \\
&\text{subject to } \mathbf{g}_{\mathrm{prop}}(D^{\mathrm{p}}, T) \le \mathbf{0} \\
&s \ge 0 \\
&\text{where } W_{\mathrm{e}}^{\mathrm{p}} = W_{\mathrm{e}}(D^{\mathrm{p}}, T) \\
&\mathrm{SFC}^{\mathrm{p}} = \mathrm{SFC}(D^{\mathrm{p}}, T) \\
&\mathrm{ESF}^{\mathrm{p}} = \mathrm{ESF}(D^{\mathrm{p}}, T)
\end{aligned} \tag{A10}$$

Aerodynamics:

$$\begin{aligned}
&\text{Find } \mathrm{ESF}^{\mathrm{ae}}, W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}} \\
&\min\ \phi(\mathrm{ESF}^{\mathrm{ae}} - \mathrm{ESF}^{\mathrm{p}}) + \phi(D^{\mathrm{p}} - D^{\mathrm{ae}}) + \phi(W_{\mathrm{t}}^{\mathrm{ae}} - W_{\mathrm{t}}^{\mathrm{a}}) \\
&\quad + \phi(L^{\mathrm{s}} - L^{\mathrm{ae}}) + \phi(\theta^{\mathrm{ae}} - \theta^{\mathrm{s}}) + \phi(t/c^{\mathrm{ae}} - t/c^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}) \\
&\quad + \phi(\Lambda_{\mathrm{w}}^{\mathrm{ae}} - \Lambda_{\mathrm{w}}^{\mathrm{s}}) + \phi(S_{\mathrm{ref}}^{\mathrm{ae}} - S_{\mathrm{ref}}^{\mathrm{s}}) + \phi(S_{\mathrm{ht}}^{\mathrm{ae}} - S_{\mathrm{ht}}^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}) \\
&\quad + \phi(g_{\mathrm{aircraft}}(\mathrm{SFC}^{\mathrm{p}}, W_{\mathrm{e}}^{\mathrm{p}}, L/D^{\mathrm{ae}}, W_{\mathrm{s}}^{\mathrm{s}}, W_{\mathrm{f}}^{\mathrm{s}}) + s) \\
&\text{subject to } \mathbf{g}_{\mathrm{prop}}(W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, \mathrm{ESF}^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}}) \le \mathbf{0} \\
&\text{where } D^{\mathrm{ae}} = D(W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, \mathrm{ESF}^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}}) \\
&L/D^{\mathrm{ae}} = L/D(W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, \mathrm{ESF}^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}}) \\
&L^{\mathrm{ae}} = L(W_{\mathrm{t}}^{\mathrm{ae}}, \theta^{\mathrm{ae}}, \mathrm{ESF}^{\mathrm{ae}}, t/c^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}}, \Lambda_{\mathrm{w}}^{\mathrm{ae}}, S_{\mathrm{ref}}^{\mathrm{ae}}, S_{\mathrm{ht}}^{\mathrm{ae}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}}, \Lambda_{\mathrm{ht}}, L_{\mathrm{w}}, L_{\mathrm{ht}})
\end{aligned} \tag{A11}$$

Structures:

$$\begin{aligned}
&\text{Find } L^{\mathrm{s}}, t/c^{\mathrm{s}}, \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}, \Lambda_{\mathrm{w}}^{\mathrm{s}}, S_{\mathrm{ref}}^{\mathrm{s}}, S_{\mathrm{ht}}^{\mathrm{s}}, \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}, [t], [t_{\mathrm{s}}], \lambda \\
&\min\ \phi(W_{\mathrm{f}}^{\mathrm{a}} - W_{\mathrm{f}}^{\mathrm{s}}) + \phi(W_{\mathrm{s}}^{\mathrm{a}} - W_{\mathrm{s}}^{\mathrm{s}}) + \phi(L^{\mathrm{s}} - L^{\mathrm{ae}}) + \phi(\theta^{\mathrm{ae}} - \theta^{\mathrm{s}}) \\
&\quad + \phi(t/c^{\mathrm{ae}} - t/c^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{w}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{w}}^{\mathrm{s}}) \\
&\quad + \phi(\Lambda_{\mathrm{w}}^{\mathrm{ae}} - \Lambda_{\mathrm{w}}^{\mathrm{s}}) + \phi(S_{\mathrm{ref}}^{\mathrm{ae}} - S_{\mathrm{ref}}^{\mathrm{s}}) + \phi(S_{\mathrm{ht}}^{\mathrm{ae}} - S_{\mathrm{ht}}^{\mathrm{s}}) + \phi(\mathrm{AR}_{\mathrm{ht}}^{\mathrm{ae}} - \mathrm{AR}_{\mathrm{ht}}^{\mathrm{s}}) \\
&\quad + \phi(g_{\mathrm{aircraft}}(\mathrm{SFC}^{\mathrm{p}}, W_{\mathrm{e}}^{\mathrm{p}}, L/D^{\mathrm{ae}}, W_{\mathrm{s}}^{\mathrm{s}}, W_{\mathrm{f}}^{\mathrm{s}}) + s)
\end{aligned}$$

$$\text{subject to}\ \ \mathbf{g}_{\text{struc}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda) \le \mathbf{0}$$

$$\text{where}\ \ W_{\text{s}}^{\text{s}} = W_{\text{s}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda)$$

$$W_{\text{f}}^{\text{s}} = W_{\text{f}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda)$$

$$\theta^{\text{s}} = \theta(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda) \tag{A12}$$

4 Formulation (d). Propulsion:

$$\text{Find}\ \ D^{\text{p}},T,s$$

$$\begin{aligned}\min\ &\phi(D^{\text{p}}-D^{\text{ae}})+\phi(\text{ESF}^{\text{ae}}-\text{ESF}^{\text{p}})\\ &+\phi(g_{\text{aircraft}}(\text{SFC}^{\text{p}},W_{\text{e}}^{\text{p}},L/D^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}})+s)\\ &+\phi(h_{\text{aircraft}}(W_{\text{e}}^{\text{p}},W_{\text{t}}^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}}))\end{aligned}$$

$$\text{subject to}\ \ \mathbf{g}_{\text{prop}}(D^{\text{p}},T) \le \mathbf{0}$$

$$s \ge 0$$

$$\text{where}\ \ W_{\text{e}}^{\text{p}} = W_{\text{e}}(D^{\text{p}},T)$$

$$\text{SFC}^{\text{p}} = \text{SFC}(D^{\text{p}},T)$$

$$\text{ESF}^{\text{p}} = \text{ESF}(D^{\text{p}},T) \tag{A13}$$

Aerodynamics:

$$\text{Find}\ \ \text{ESF}^{\text{ae}},W_{\text{t}}^{\text{ae}},\theta^{\text{ae}},t/c^{\text{ae}},\text{AR}_{\text{w}}^{\text{ae}},\Lambda_{\text{w}}^{\text{ae}},S_{\text{ref}}^{\text{ae}},S_{\text{ht}}^{\text{ae}},\text{AR}_{\text{ht}}^{\text{ae}},\Lambda_{\text{ht}},L_{\text{w}},L_{\text{ht}}$$

$$\begin{aligned}\min\ &\phi(\text{ESF}^{\text{ae}}-\text{ESF}^{\text{p}})+\phi(D^{\text{p}}-D^{\text{ae}})+\phi(L^{\text{s}}-L^{\text{ae}})\\ &+\phi(\theta^{\text{ae}}-\theta^{\text{s}})+\phi(t/c^{\text{ae}}-t/c^{\text{s}})+\phi(\text{AR}_{\text{w}}^{\text{ae}}-\text{AR}_{\text{w}}^{\text{s}})+\phi(\Lambda_{\text{w}}^{\text{ae}}-\Lambda_{\text{w}}^{\text{s}})\\ &+\phi(S_{\text{ref}}^{\text{ae}}-S_{\text{ref}}^{\text{s}})+\phi(S_{\text{ht}}^{\text{ae}}-S_{\text{ht}}^{\text{s}})+\phi(\text{AR}_{\text{ht}}^{\text{ae}}-\text{AR}_{\text{ht}}^{\text{s}})\\ &+\phi(g_{\text{aircraft}}(\text{SFC}^{\text{p}},W_{\text{e}}^{\text{p}},L/D^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}})+s)\\ &+\phi(h_{\text{aircraft}}(W_{\text{e}}^{\text{p}},W_{\text{t}}^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}}))\end{aligned}$$

$$\text{subject to}\ \ \mathbf{g}_{\text{prop}}(W_{\text{t}}^{\text{ae}},\theta^{\text{ae}},\text{ESF}^{\text{ae}},t/c^{\text{ae}},\text{AR}_{\text{w}}^{\text{ae}},\Lambda_{\text{w}}^{\text{ae}},S_{\text{ref}}^{\text{ae}},S_{\text{ht}}^{\text{ae}},\text{AR}_{\text{ht}}^{\text{ae}},\Lambda_{\text{ht}},L_{\text{w}},L_{\text{ht}}) \le \mathbf{0}$$

$$\text{where}\ \ D^{\text{ae}} = D(W_{\text{t}}^{\text{ae}},\theta^{\text{ae}},\text{ESF}^{\text{ae}},t/c^{\text{ae}},\text{AR}_{\text{w}}^{\text{ae}},\Lambda_{\text{w}}^{\text{ae}},S_{\text{ref}}^{\text{ae}},S_{\text{ht}}^{\text{ae}},\text{AR}_{\text{ht}}^{\text{ae}},\Lambda_{\text{ht}},L_{\text{w}},L_{\text{ht}})$$

$$L/D^{\text{ae}} = L/D(W_{\text{t}}^{\text{ae}},\theta^{\text{ae}},\text{ESF}^{\text{ae}},t/c^{\text{ae}},\text{AR}_{\text{w}}^{\text{ae}},\Lambda_{\text{w}}^{\text{ae}},S_{\text{ref}}^{\text{ae}},S_{\text{ht}}^{\text{ae}},\text{AR}_{\text{ht}}^{\text{ae}},\Lambda_{\text{ht}},L_{\text{w}},L_{\text{ht}})$$

$$L^{\text{ae}} = L(W_{\text{t}}^{\text{ae}},\theta^{\text{ae}},\text{ESF}^{\text{ae}},t/c^{\text{ae}},\text{AR}_{\text{w}}^{\text{ae}},\Lambda_{\text{w}}^{\text{ae}},S_{\text{ref}}^{\text{ae}},S_{\text{ht}}^{\text{ae}},\text{AR}_{\text{ht}}^{\text{ae}},\Lambda_{\text{ht}},L_{\text{w}},L_{\text{ht}}) \tag{A14}$$

Structures:

$$\text{Find}\ \ L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda$$

$$\begin{aligned}\min\ &\phi(L^{\text{s}}-L^{\text{ae}})+\phi(\theta^{\text{ae}}-\theta^{\text{s}})+\phi(t/c^{\text{ae}}-t/c^{\text{s}})+\phi(\text{AR}_{\text{w}}^{\text{ae}}-\text{AR}_{\text{w}}^{\text{s}})\\ &+\phi(\Lambda_{\text{w}}^{\text{ae}}-\Lambda_{\text{w}}^{\text{s}})+\phi(S_{\text{ref}}^{\text{ae}}-S_{\text{ref}}^{\text{s}})+\phi(S_{\text{ht}}^{\text{ae}}-S_{\text{ht}}^{\text{s}})+\phi(\text{AR}_{\text{ht}}^{\text{ae}}-\text{AR}_{\text{ht}}^{\text{s}})\\ &+\phi(g_{\text{aircraft}}(\text{SFC}^{\text{p}},W_{\text{e}}^{\text{p}},L/D^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}})+s)\\ &+\phi(h_{\text{aircraft}}(W_{\text{e}}^{\text{p}},W_{\text{t}}^{\text{ae}},W_{\text{s}}^{\text{s}},W_{\text{f}}^{\text{s}}))\end{aligned}$$

$$\text{subject to}\ \ \mathbf{g}_{\text{struc}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda) \le \mathbf{0}$$

$$\text{where}\ \ W_{\text{s}}^{\text{s}} = W_{\text{s}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda)$$

$$W_{\text{f}}^{\text{s}} = W_{\text{f}}(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda)$$

$$\theta^{\text{s}} = \theta(L^{\text{s}},t/c^{\text{s}},\text{AR}_{\text{w}}^{\text{s}},\Lambda_{\text{w}}^{\text{s}},S_{\text{ref}}^{\text{s}},S_{\text{ht}}^{\text{s}},\text{AR}_{\text{ht}}^{\text{s}},[t],[t_{\text{s}}],\lambda) \tag{A15}$$

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