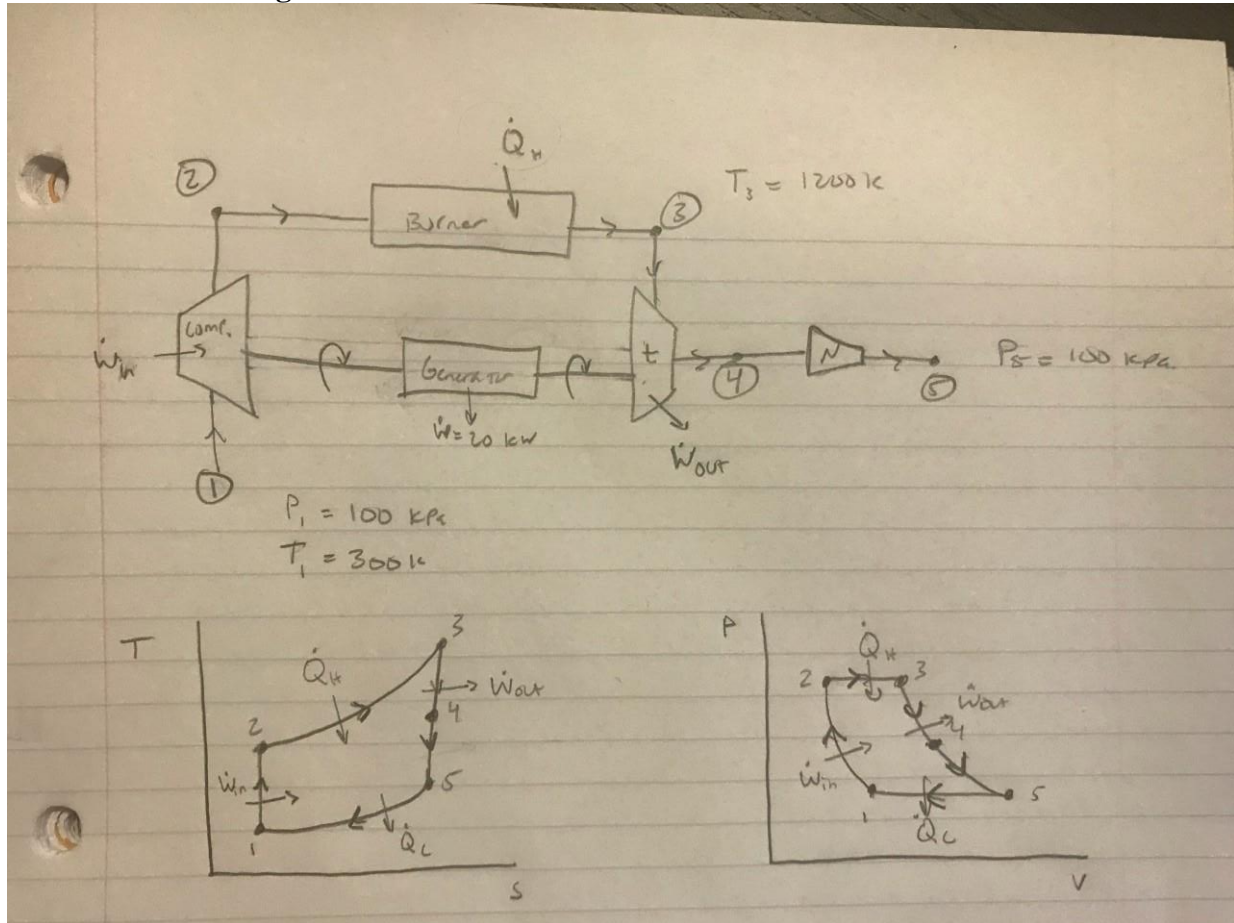# DHW 2 Jet-Engine

Schematics and Diagrams

Assumptions and Boundary Conditions

- Air is an IG. This allows me to use the $C_p * \Delta T = \Delta H$. I honestly assumed this to allow the calculations to be easier, I don't think the temperature change is small enough to assume that Cp is actually constant [4].

- T3 = 1900 K is the max temp in a jet engine if we assume that there are advanced methods of cooling such as a ceramic coating and cooling system in the turbine blades. The reference I used states that fuel can burn in the burner at up to 2273 K with advanced cooling, so I chose a smaller than that than that [1].

- P1 and T1 are STP when on the ground, that is just what pressure and temperature would be.

- P1 and T1 are 23.8 kPa and 222K when about 35,000 ft in the air [2]. This is what I found as a good estimation at that altitude.

- Ideal turbine/compressor so that all the heat created in the burner can be translated to work in the turbine. And also, assuming the compressor is ideal so that I can set the work from the turbine equal to the electrical power and the work from the compressor. Lastly, this also allows me to use isentropic relations across the turbine and compressor since the entropies would be equal [3].
- All the fuel is burned in the burner; this allows me to assume that fuel burns at 45 MJ/kg and that all the fuel will be converted to heat [3].
- Thrust will equal weight of plane. This assumption is not the greatest because you are basically saying that the plane is going straight up but it simplifies the calculations greatly [3].

Temperature Limitation

I set my operating temperature (T3) to 1900 K. The temperature limitation in this jet engine comes from the material used in the blades. Without using any advanced methods of cooling higher temperatures can be obtained, the temperature would melt the material around 1500 K before we could reach the max temperature. By using advanced cooling methods, it allows the operating temperature in the system to go above the melting temperature of the material. One method to increase the maximum temperature is to use "internal cooling passages" to cool the blade by pumping about 1000 K air out of small holes on the blade. Then the air forms a thin layer around the blade allowing it to stay cooler. Another way to increase operating temperature is to coat the blades in a ceramic material. Since ceramics have a low thermal conductivity it causes a greater temperature gradient and keeps the blades cooler.

Calculations

From my assumption about T3 I first wanted to calculate the nozzle velocity. To do this I did an energy balance around the nozzle:

$$(H_4 - H_5) = \frac{1}{2} v^2{}_{nozzle}$$

Using the approximation when C_p is constant gives you,

$$v_{nozzle} = \sqrt{2 * C_p * (T_4 - T_5)}$$

Since I don't know the temperature at either T_4 or T_5 yet, I solved for those using the polytropic relation between step 3 and 5 to solve for T_5 and energy balance between the W_t and W_c.

The polytropic relation is:

$$\frac{T_5}{T_3} = (\frac{P_5}{P_3})^{\frac{k-1}{k}}$$

Which when rearranged for T_5 gives:

$$T_5 = T_3 * (\frac{P_5}{P_3})^{\frac{k-1}{k}}$$

Then solving for T_4 using an energy balance between the work of turbine and the work of the compressor is:

$$\dot{W}_t = \dot{W}_c + P_e$$

The electrical power required is also added because that is the maximum amount of work needed. Continuing to solve gives:

$$H_3 - H_4 = H_2 - H_1 + P_e$$

Then using the approximation with C_p being constant gives.

$$T_4 = T_3 + T_1 - T_2 + \frac{P_e}{C_p}$$

Now you can solve for the velocity in the nozzle. The next step I did was to solve for the total mass flow I the system by balancing the thrust with the weight of the plane. This equation is [5]:

$$\dot{m}_t = W_{plane}/v_{nozzle}$$

Then to find the heat generated first you solve for the work of the turbine by using a energy balance from step 3-4, so equation X is used. This will allow us to plug in $C_p * \Delta T$ for the change in enthalpy across the compressor and the final equation to solve for work of the turbine is:

$$\dot{W}_t = C_p * (T_2 - T_1) + P_e$$

Now since I assumed that all the heat is converted to work, I can solve for the heat rate needed to create the work in the turbine from an energy balance from the turbine and the compressor and then combining the equations to get:

$$\dot{Q} = \dot{m}_t * C_p * (T_4 - T_2) + \dot{W}_t$$

Once the heat rate is found I can find the mass flow of the fuel, knowing the amount of energy a kg of the fuel can burn from:
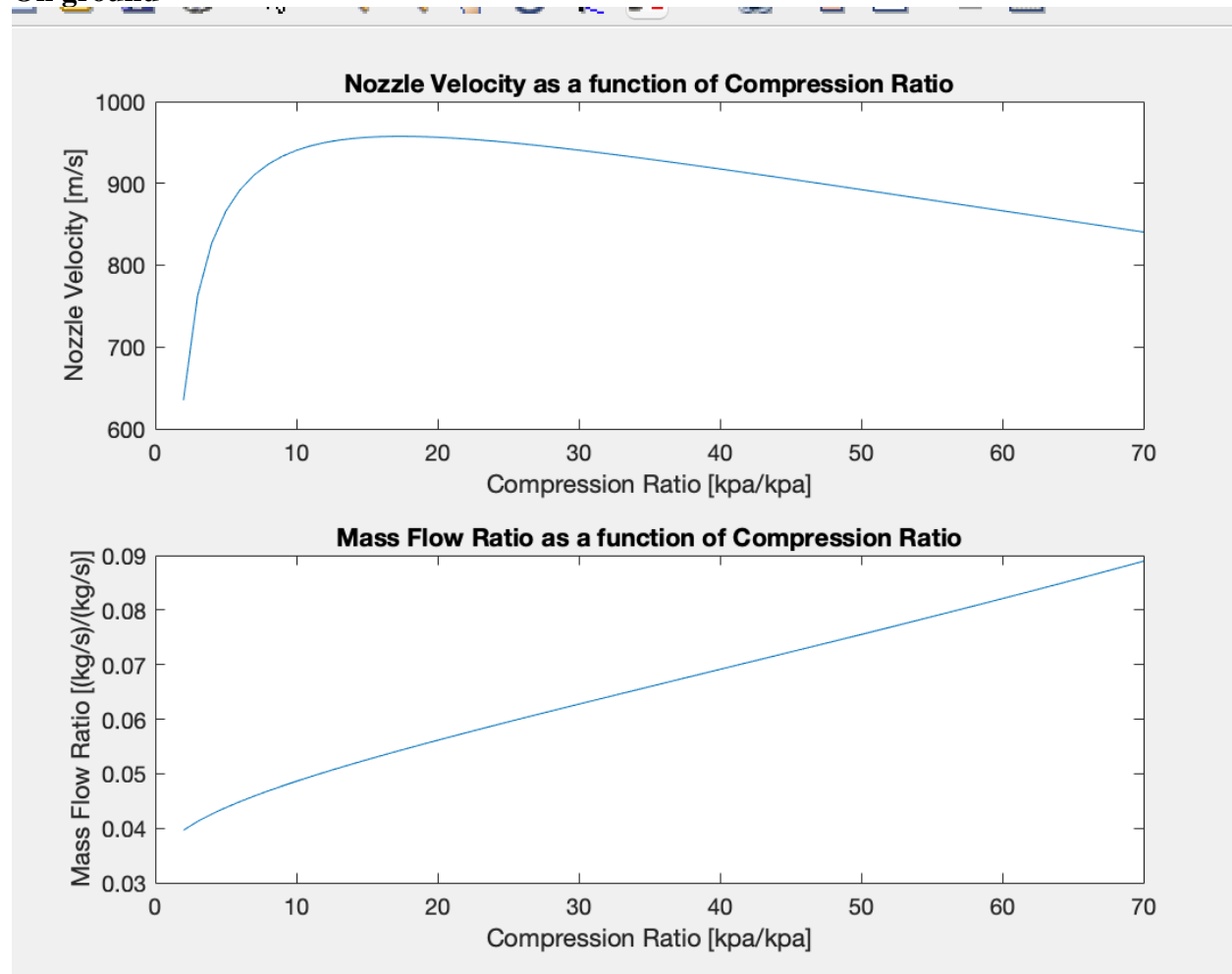
$$\dot{m}_f = \frac{\dot{Q}}{E}$$

Then I can find the mass flow of air by subtracting the mass flow of the fuel from the total mass flow, which then a mass flow ratio can be calculated.

$$m_{air} = m_{tot} - m_f$$

$$\dot{m}_{ratio} = \frac{\dot{m}_{air}}{\dot{m}_f}$$

Graphs and final answer
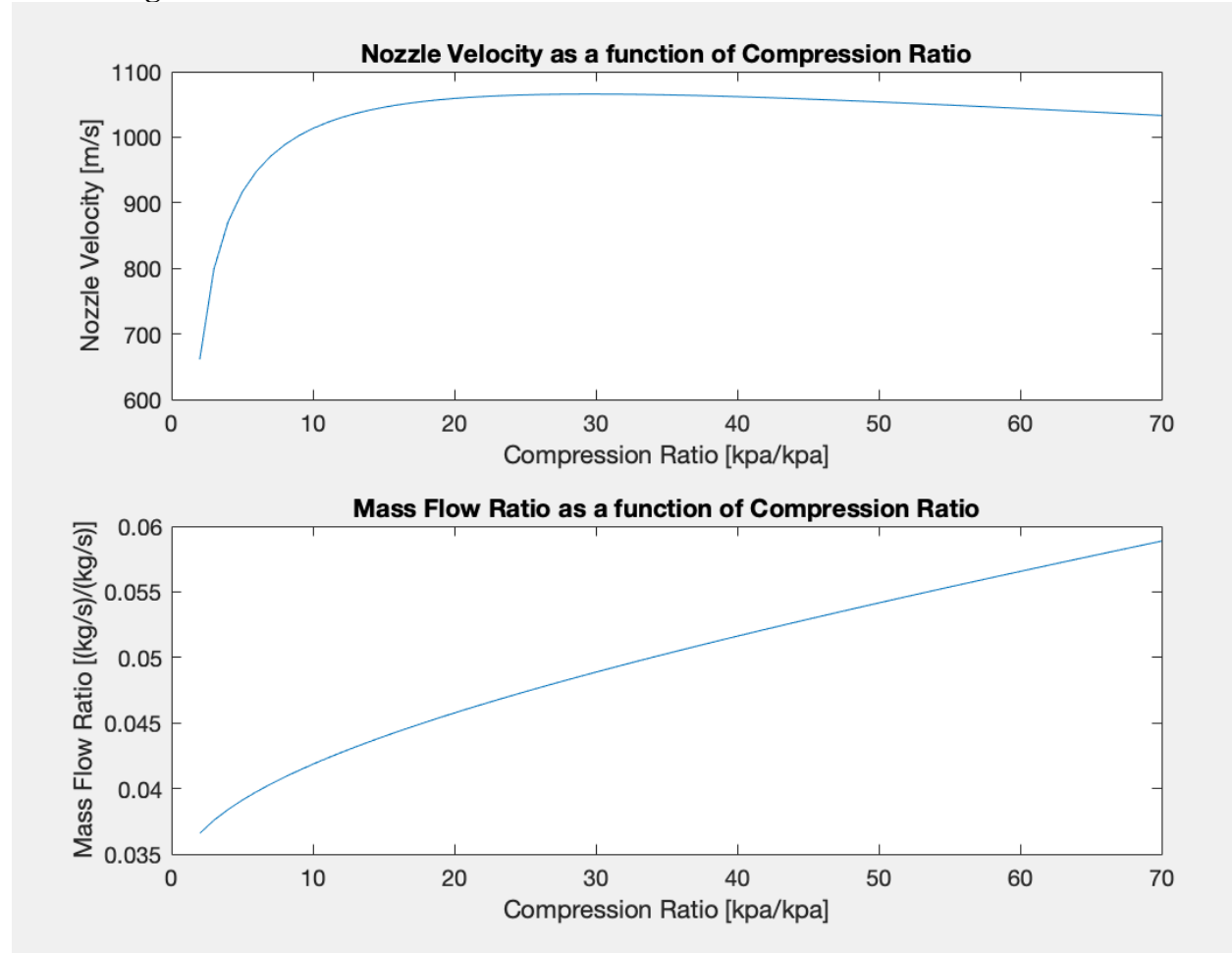
On ground

Rp = 70
Velocity nozzle = 840.522 [m/s]
Mass Flow Ratio = 0.0889504
Mass flow of f = 52484.6 [kg/s]
Mass flow of air = 4668.53 [kg/s]
Total Mass flow = 57153.2 [kg/s]

At cruising altitude

Rp = 70
Velocity nozzle = 1033.16 [m/s]
Mass Flow Ratio = 0.0588719
Mass flow of f = 64513.9 [kg/s]
Mass flow of air = 3798.06 [kg/s]
Total Mass flow = 68312 [kg/s]

Discussions
I do not believe any of my answers make sense. The only thing I thought was correct was that the velocity went up when at higher altitude because you would need to go faster to maintain the

thrust to weight balance. Starting with the compression ratio, I got 70, but that was just because from my graph the highest value for compression ratio was optimal at whatever the highest value was. And comparing this to a compression ratio of 40 [1], my number is just really far off. Also, looking at a typical speed of air coming out of a jet engine, it is about 580 m/s while I am calculating mine at 840 m/s and more. So again, this is pretty far off. My answer could be off for many reasons. Apart from doing the problem wrong, I assumed you could use to cold air assumption which is probably not true because Cp would vary over the range of temperatures used. Also, none of the parts would be ideal in real life so there would be some loss of energy in the system due to friction and other factors. Lastly, all the fuel my not be burned up fully or produce exactly 45MJ/kg of energy which would also affect the results. But honestly, I think I messed up somewhere in my calculations to get the answers I did.

References

[1] https://cs.stanford.edu/people/eroberts/courses/ww2/projects/jet-airplanes/how.html

[2] https://www.quora.com/What-happens-to-an-airliners-tyre-pressures-at-35000-ft

[3] P. Weissensee, *1st Design Homework – Geothermal Power Plant,* MEMS 412 – Design of thermal systems, Washington University in Saint Louis.

[4] Sonntag, R. E., J., V. W. G., & Borgnakke, C. (2008). *Fundamentals of thermodynamics*. Hoboken, NJ: Wiley.

[5] https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/thrsteq.html

[6] https://science.howstuffworks.com/transport/flight/modern/turbine8.htm

Matlab code:

```matlab
% Change these parameters depending on if on ground or at cruising altitude
% STP if ground, T1 = 222 K, P1 = 23.8 kPa if at cruising speed.
T1 = 222;              % K
P1 = 23.8;               % kPa;


Wp = 400*1000*9.81; % Newtons
cp = 1000;             % kj/K
cv = 718;              % kj/K
k = cp/cv;


Pe = 20;
E = 45000;             % kj/kg of fuel
T3 = 1500;             % K
P4 = P1;               % kPa
P5 = P1;               % kPa


rp = 2:70;             % compression ratio
P2 = P1 .*rp;          % kPa
P3 = P2;
% Use compression ratio and polytropic relations to get T2, T4, T5.
T2 = T1 .* rp.^((k-1)/k);      % K
% E.B. across turbine/ compressor
T4 = T3 + T1 - T2 + Pe/cp;     % K
% Polytropic relation
T5 = T3 * (P5./P3).^((k-1)/k); % K
% E.B. across nozzle
V5 = sqrt(2*cp.*(T4 - T5));    % m/s


% Mass flow from balance b/w weight of plane and thrust equation (NASA)
M_dot_a = Wp ./ V5;
% Work from turbine is the power needed + work from compressor
Wt = M_dot_a .* cp .* (T2-T1) + Pe;
Q_dot = M_dot_a .* cp .*(T4-T2) + Wt;


% you can calc. mass flow of fuel from heat needed in system
M_dot_f = Q_dot/E;
M_ratio = M_dot_a ./ M_dot_f;


fprintf('\nVelocity out = %g [m/s]\n', V5(rp == max(rp)))
fprintf('Total Mass flow = %g [kg/s]\n', M_dot_a(rp == max(rp)) + M_dot_f(rp
== max(rp)))
fprintf('Rp = %g\n', max(rp))
fprintf('Mass Flow Ratio = %g \n', M_ratio(rp == max(rp)))
fprintf('Mass flow of f = %g [kg/s]\n', M_dot_f(rp == max(rp)))
fprintf('Mass flow of air = %g [kg/s]\n', M_dot_a(rp == max(rp)))


subplot(2,1,1)
plot(rp, V5)
title('Nozzle Velocity as a function of Compression Ratio')
xlabel('Compression Ratio [kpa/kpa]')
ylabel('Nozzle Velocity [m/s]')
subplot(2,1,2)
plot(rp, M_ratio)
```

```
title('Mass Flow Ratio as a function of Compression Ratio')
xlabel('Compression Ratio [kpa/kpa]')
ylabel('Mass Flow Ratio [(kg/s)/(kg/s)]')
```