Fall 2020 MEMS 4050 Vibrations Laboratory

Lab 2: Basic Vibration Measurements

Lab Instructor: Dr. Bayly

Experiment Dates: Friday, October 9 & 16, 2020

Report Submission Date: Friday, October 23, 2020

Group T (Friday 2PM)

We hereby certify that the lab report herein is our original academic work, completed in accordance to the McKelvey School of Engineering and Undergraduate Student academic integrity policies, and submitted to fulfill the requirements of this assignment:

Aidan Murphy	*Team Leader*
Andrew Brown	*Test Engineer I*
Mitry Anderson	*Data Acquisition Manager*
Matthew Donaldson	*Quality Control Engineer*
Sam Wille	*Test Engineer II*

ABSTRACT: *This experiment aims to make participants familiar with vibration testing tools and methods as well as find the calibration constant of a force transducer and mass of a shaker armature for NYQuist Consulting Labs. The calibration constant is found to be the slope of line of best fit for the graph with force as the independent variable and voltage as the dependent variable. With an assumed DC offset, the constant is found to be 11.862* $\frac{mV}{N}$*, while with no DC offset it 11.65* $\frac{mV}{N}$*. The percent errors from the given theoretical value of 11.4* $\frac{mV}{N}$ *are 4.053% and 2.193% respectively. There is likely error due to electrical resistance, causing the measured acceleration values to be low and causing the measured calibration constant be too high. The mass of the shaker is found by measuring the vibration acceleration of the armature with and without added mass. The armature mass was calculated to be 2.526 kg. When comparing this to the theoretical value of 2.3 kg from the data sheet for the shaker, the percent error is found to be 9.83%. This error is likely due to noise increasing the measured acceleration values. The collected data was rendered and plotted using SignalCalc software and analyzed using MATLAB and Microsoft Excel.*

INTRODUCTION

The purpose of this experiment is to become familiar with vibration testing tools and methods. This is done by measuring the acceleration and force of the horizontal electromagnetic shaker as it accelerates various masses. This information is also used to determine the calibration constants for the force transducer.

In this experiment a displacement is induced on the shaker given by the function:

$$x(t) = x_o Cos(\omega t - \phi) \tag{1}$$

where x_0[m] is the max amplitude of the displacement, ω is the angular velocity [rad/s], t is time in seconds and ϕ is the angular phase shift of the wave in radians. From Eq. 1, the velocity in [$\frac{m}{s}$] and acceleration in [$\frac{m}{s^2}$] of the vibration can be found by taking the derivative and second derivative respectively.

$$\dot{x} = \omega x_0 Cos(\omega t - \phi + \frac{\pi}{2}) \tag{2}$$

$$\ddot{x} = \omega^2 x_0 Cos(\omega t - \phi + \pi) \tag{3}$$

From this it is seen velocity is that the velocity is shifted ahead of the displacement by $\frac{\pi}{2}$ and the acceleration by π.

It is important to understand how the acceleration and force are measured from the shaker. The voltage is generated from a function generator according to the equation:

$$V(t) = V_0 Cos(\omega t - \phi_0). \tag{4}$$

The voltage is then converted to current by the power amplifier resulting in a current of:

$$I(t) = I_0 Cos(\omega t - \phi_1). \tag{5}$$

This, then, causes a force on the armature of the shaker with an equation of

$$F(t) = F_0 Cos(\omega t - \phi_2). \tag{6}$$

This force causes the mass at the end of the armature to accelerate according to Newton's second of motion resulting in the acceleration function:

$$a(t) = a_0 Cos(\omega t - \phi_3). \tag{7}$$

Due to the fact that $\ddot{x} = a(t)$, the maximum displacement can then be solved for by combining equations 3 and 7, given that the maximum displacement amplitude occurs when $\omega t - \phi$ equals $n2\pi$ where n is some integer:

$$x_0 = \frac{a_0}{\omega^2} \tag{8}$$

So, in order to get the displacement an accelerometer was used to measure acceleration at the end of the shaker [1].

These sensors are Piezoelectric, which means that the force transducer and accelerometer each create a small voltage signal when a strain is present. That voltage is proportional to the force and the acceleration. This proportion can be seen below.

$$C = \frac{V_{force}}{F(t)} \tag{9}$$

$$C = \frac{V_{acc}}{a(t)} \tag{10}$$

were C is some calibration constant, V is the voltage produced by the sensors, and F is the force measured by the transducer. The signal output created by the transducer or the accelerometer is then amplified and converted from analog voltage to digital data which then can be put inputted to the data acquisition system [2]. It is important to note here that the force measured by the force transducer is not the force the current is producing on the armature but the force between the armature and the mass. Also, the signal produced by the transducer and accelerometer is weak, so it is passed through a signal conditioner to amplify the signal. That amplification is called a signal gain. The signal is then read into the Quattro for analysis.

In order to estimate the mass of the shaker, measurements of acceleration due to a constant force amplitude were taken with only the mass of the shaker armature (M). Then mass(m) is added, and with the same applied force as before, a new acceleration is measured. Figure 1 below shows a schematic representation of the set up.

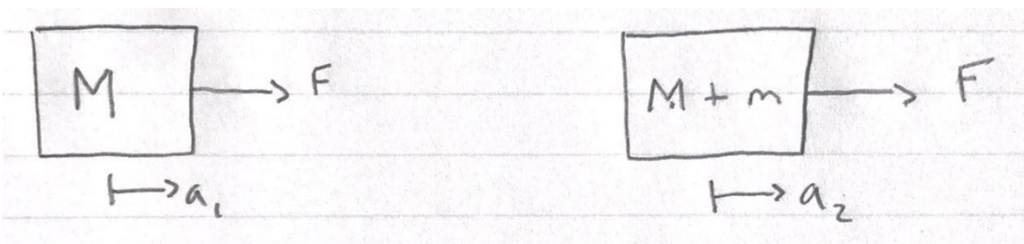

Figure 1 Case 1 (left) correlates the mass of the armature and its corresponding acceleration and case 2 (right) correlates the mass of the armature plus any arbitrary added mass and its corresponding acceleration.

From the figure above Newton's Second Law of Motion can be applied.

$$F = Ma_1 \tag{11}$$

$$F = (M + m)a_2 \tag{12}$$

were F is the force on the shaker, M is the mass of the shaker, m is the added mass and a_1 and a_2 are the measured accelerations. Since the force on the mass is kept constant and the mass of the shaker is the only unknown, equations 11 and 12 can be combined to get equation 13 below:

$$M = \frac{ma_2}{a_1 - a_2}. \tag{13}$$

To estimate the the force and displacement amplitudes from the data collected, the force amplitude can first be expressed as

$$F = m\ddot{x} \tag{14}$$

were F is the force amplitude, m is the mass of the object and $\ddot{x}$ is the acceleration amplitude of the object. From equation 14, the displacement amplitude can then be found by solving this differential equation. The displacement function is assumed to be:

$$x(t) = x_0 e^{i\omega t}. \tag{15}$$

Plugging in to equation 14 and solving, the magnitude of the displacement amplitude is then found to be:

$$X_0 = \frac{F_0/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}. \tag{16}$$

If the experiment is done only at high frequencies, damping/ stiffness can be neglected and the resistance to force is mainly due to the mass of the armature. This results in a simplification of equation 16:

$$x_0 = \frac{\ddot{x}}{\omega^2 sin(\omega t)}. \tag{17}$$

From this it is seen that the maximum amplitude is the acceleration over angular velocity squared when $\omega t = \frac{\pi}{2}$.

METHODS

Apparatus. Sine waves produced by a function generator drove an electromagnetic shaker whose osculations were digitized and analyzed. Table 1 contains all of the equipment used, with relevant identification information, to accomplish this. The function generator output its signal, each to the multimeter, power amplifier, and Quattro via BNC cables. The multimeter recorded both the RMS voltage and frequency of the multimeter output. The power amplifier amplified the function generator signal, powering the the horizontally oriented electromagnetic shaker via BNC cable. Both an accelerometer and force transducer were placed on the shaker such that they measured acceleration and force, respectively, along the axis of the shaker's oscillation. As appropriate, various masses were attached to the force transducer. The resulting signals generated by the force transducer and accelerometer were sent to signal conditioners one and two respectively. The conditioners then sent the signal, times some gain, to Quattro ports 2 and 3, respectively. The Quattro then analyzed the signal in the frequency and time domains, presenting its analysis on the PC with the SignalCalc program (Fig 2) [3]. In turn, SignalCalc controlled the Quattro settings.

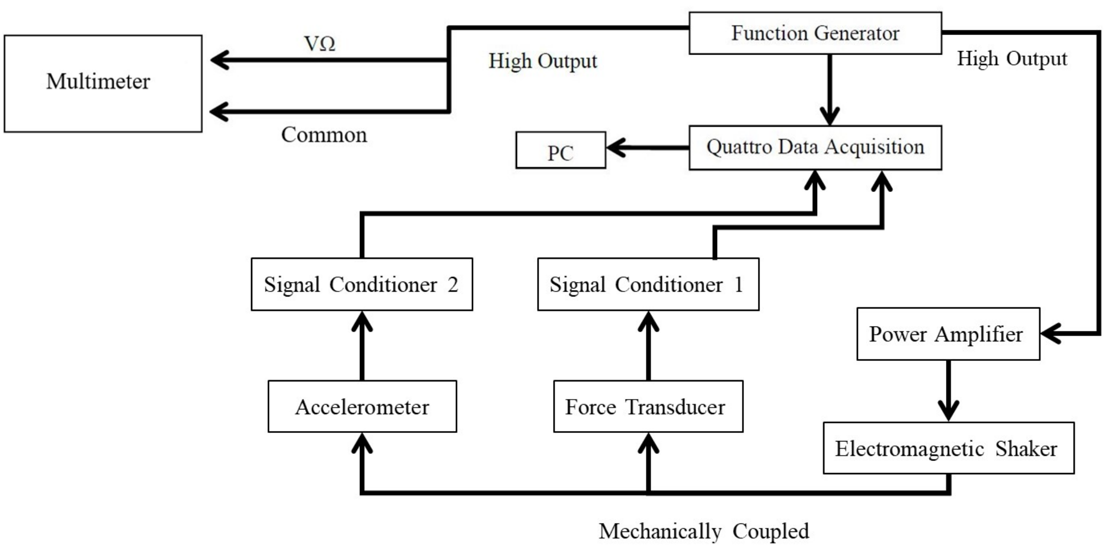

Figure 2 Set up of experimental apparatus [4, 5]

Table 1 Equipment Used to Induce, Measure, and Analyse the Oscillation of Mass with One Degree of Freedom [6].

Equipment	Make	Model	Serial #	Calibration Constant
Laptop	Lenovo	Thinkpad	N/A	N/A
SignalCal 240 Dynamic Signal Analyzer	N/A	N/A	N/A	N/A
Quattro SignalCalc Ace	Data Physics Corporation	N/A	2222	N/A
Multimeter	Fluke	223	N/A	N/A
Function Generator	Beckman	9010	525836	N/A
Accelerometer	PCB	353B33	LW203550	100.4 $\frac{mV}{g}$
Force Transducer	PCB	208B02	13326	Unknown
Signal Conditioner 1	PCB	494A	314	1 mV
Signal Conditioner 2	PCB	494A	816	10 mV
Power Amplifier	APS Dynamics, Inc.	114	339	N/A
Electromagnetic Shaker	APS Dynamics, Inc.	113	685	N/A

Procedure. To determine the unknown mass within the shaker and calculate the calibration constant of the force transducer, in SignalCalc, the Quattro Trigger was set to *Free Run*, the Window to *Hanning*, the Average to *Off*, and auto end to *1 Record*. The accelerometer had a calibration constant of 100.4 $\frac{mV}{g}$ as signal conditioner 2 provided a gain of 10 to the accelerometer output [6]. With this gain, SignalCalc output acceleration values and not voltage values for accelerometer measurements. The force transducer had an arbitrary calibration constant of 1 $\frac{mV}{EU}$ and signal conditioner 1 provided a gain of 1. The Fspan and Lines were initialized to 200 Hz and 400 respectively. However, at user discretion, Fspan and Lines were changed to 400 Hz and 800, respectively. Throughout the experiment, the function generator output approximately 1 V_{RMS}.

First, the function generator was set to output a 10 Hz sine signal. During the test the amplifier gain was gradually increased until the peak to peak displacement of the shaker armature was between 3 mm and 6 mm. This displacement was estimated with a ruler. The multimeter frequency and RMS voltage readings as well as the SignalCalc analysis were recorded. Then, leaving all other settings constant, the function generator frequency was increased by approximately 10 Hz eight times. Finally, the frequency was increased by 9 Hz to an ultimate frequency of 99 Hz. After each frequency increase the multimeter frequency and RMS voltage readings as well as the SignalCalc analysis were recorded.

Second, the multimeter was set to produce a 15 Hz sine signal. Also, a base mass weighing 26 g was screwed into the force transducer. The gain was then increased until the peak to peak displacement of the shaker armature was between 3 mm and 6 mm. The regular measurements - multimeter frequency and RMS voltage readings and SignalCalc readings - were recorded. Leaving all other settings constant, the function generator was set for frequency output of 114 Hz. The regular measurements were recorded. Then, the mass was removed from the force transducer and the measurements were recorded. Leaving the function generator set to produce a 114 Hz signal, this process was repeated five more times with total masses of 44 g, 203 g, 74 g, 375 g, and 647 g, respectively. Masses were attached magnetically to the base mass to create the total masses listed. For each new mass, a corresponding run was performed without any mass. For each run the regular measurements were recorded.

Next, the 26 g mass was added to the force transducer and the function generator was set to output a sine signal of approximately 15 Hz. Ten runs were performed with these settings, however, on every other run Fspan and lines were doubled from the initial settings to 400 Hz and 800 Hz respectively. The following run, FSPan and lines were returned to their initial settings. After every two runs, the amplifier gain was increased enough to effect our measurement but not enough to cause large displacements for the shaker armature. The regular measurements were recorded after each run. This process was repeated three more times with a mass of 44 g, 74 g, and 203 g, respectively, for a total of 40 runs.

Finally, the force transducer and accelerometer were disconnected from the signal conditioners and connected directly to their corresponding ports on the Quattro. The sinewave measurements described in second paragraph of this section were repeated with this new apparatus setup.

It is also important to note that the multimeter was always powered off when adding or removing a mass. Additionally, whenever the power amplifier was turned off, the amplifier knob was turned to its lowest setting to protect the next amplifier users.

Analysis. SignalCalc was used to preform a synchronous average analysis in the frequency and time domains of the RMS signal outputs for each the function generator, accelerometer, and force transducer. For each run, SignalCalc produced plots of the signal in both the frequency domain and and time domain for each device connected to the Quattro for a total of six plots per run. A MATLAB code was developed to average the signal peaks in the time domain for each run. The code also scanned the frequency domain plot from each run and stored the fundamental frequency. All MATLAB code is in Appendix [D].

The displacement amplitude of the shaker without any added mass was calculated for a range of frequencies between 10 and 100 Hz. Displacement amplitude was given by Eq. 8 where a_0 was the average acceleration as determined by MATLAB through its analysis of the time domain of the acceleration plot. Sample calculations are available in Appendix [B]. x_0 was then plotted as a function of the corresponding fundamental frequency as determined by the MATLAB analysis of the corresponding frequency plot (Fig. 3).

The mass of the electromagnetic shaker armature was determined using Eq. 13 and sample calculations can be seen in Appendix [B]. For various frequencies, the average acceleration of the armature with and without an added mass was determined by the MATLAB analysis of the accelerometer time domain plots. This meant that for each frequency analyzed, an estimate of the armature mass was obtained. These estimates were then averaged together to produce a final mass estimate. Each armature mass estimate as well as the average estimate is seen in Table 3. To better understand the quality of our data set, we took the standard deviation of our estimates. We also determined the percent error from the published mass of the shaker armature.

The force transducer constant was determined by plotting the average maximum force transducer voltage against the force applied to the force transducer. Each value of force transducer voltage was given by the MATLAB analysis of the force transducer time domain plots. The corresponding force was determined using Newton's second law, where acceleration was given by the MATLAB analysis of the time domain plot from the accelerometer data of runs that corresponded to the force transducer data. The mass was the mass attached to the force transducer for corresponding runs. The MATLAB function, fitlm, was used to perform a linear regression on the plotted points and overlay a line of best fit onto the graph of these points [7]. The first fit assumed that the force transducer had no DC offset, forcing the line of best fit to pass through the origin (Fig 4). The second fit allowed for a DC offset and did not force the line of best fit through the origin (Fig 5). In each case the estimated calibration constant for the force transducer was given as the slope of the line of best fit as shown in Eq. 9. We estimated the accuracy of each fit using the coefficient of determination which was calculated with the fitlm function.

Sources of Error. When powering on the function generator, there is a transient response in the system. It is possible that measurements were recorded before the oscillations of the shaker armature reached steady state. Additionally, the Quattro is performing discrete analysis on continuous signals, which mean all of its measured points are really average values over some period of time (dt). The smaller this dt and the longer the Quattro measures, the more accurate it is, but the Quattro output will always be an approximation, never an exact value [3].

RESULTS & DISCUSSION

The vibration data from each setup is summarized below, such that the relationships between equipment changes and the output data can be easily observed.

The first part of this experiment involved finding the acceleration amplitude of an unladen shaker at frequencies between 10 and 100 Hz. From these acceleration amplitudes, the position amplitude at a given frequency was found using Eq. 8. A plot of the calculated displacement amplitude versus the vibration frequency is shown in Fig. 3 below, for trials done using a signal conditioner.

The data fit in the figure below appears to be an exponential decay, which is consistent with the theoretical expectations based on the low fundamental frequency of the armature vibration (Fig. 8). As the frequency gets further from the fundamental, the amplitude of vibration decreases. The effects of the signal conditioner were explored in the fourth section of the lab. The displacement amplitudes were negligibly small, meaning that without the conditioner, meaningful data for the vibration is overshadowed by noise.

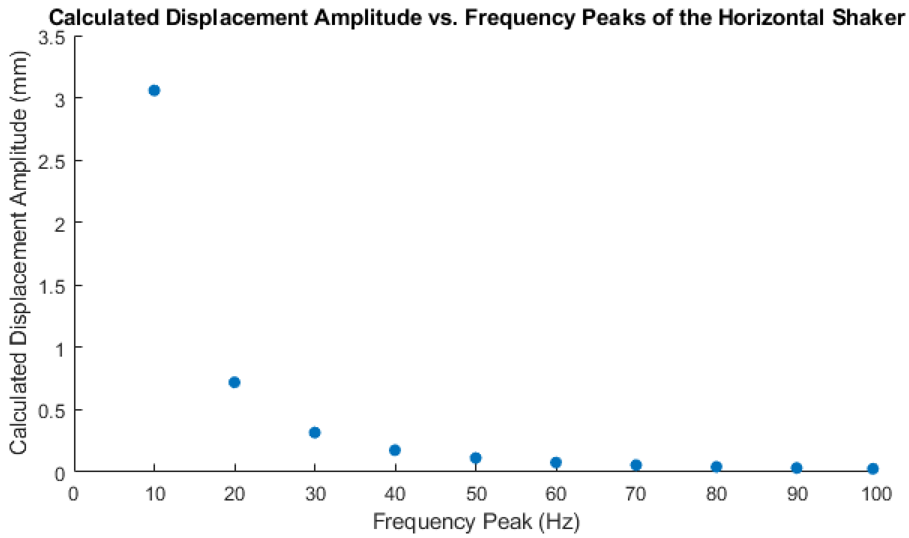

Figure 3 Measured displacement magnitude as a function of frequency for the shaker with signal conditioning.

Next, the force transducer calibration constant is determined. The calculated force magnitude for a given trial is graphed against the input voltage for that trial in order to calculate the calibration constant (the slope of the graph). Figure 4 shows the results of this analysis when a DC offset is assumed.

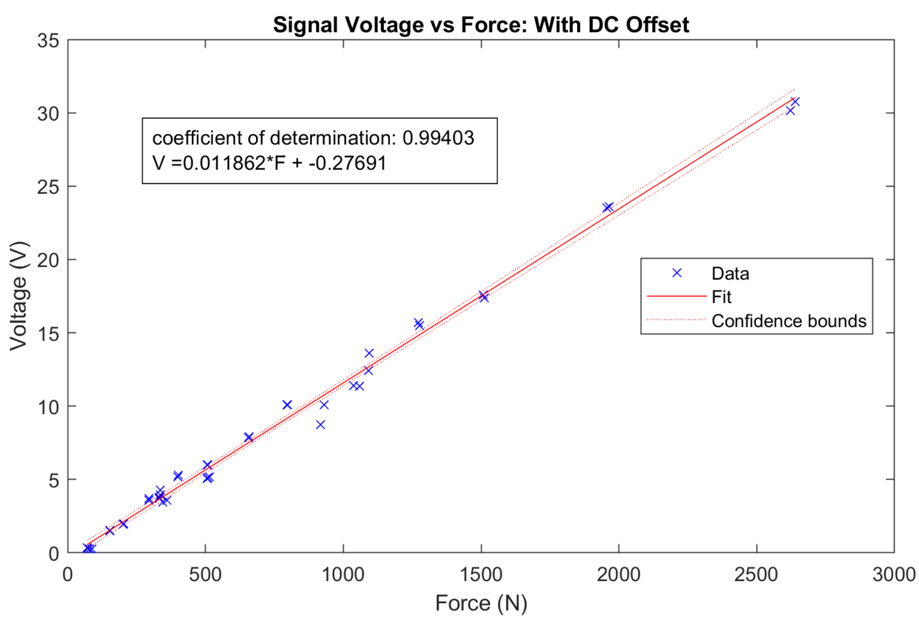

Figure 4 Voltage as a function of force with an assumed DC offset for the derivation of the force transducer calibration constant.

The best fit equation for this linear regression is $V = 0.011862F - 0.27691$ with an R^2 value of 0.99403. This gives an experimental calibration constant of 11.862 $\frac{\mathrm{mV}}{\mathrm{N}}$. These results show a very strong correlation due to the R^2 value being so close to 1. This calibration constant has an experimental error of 4.053% from the theoretical expectation of the calibration constant at 11.4 $\frac{mV}{N}$ [6].

Figure 5 shows the results of the same analysis when the DC offset is assumed to be zero.

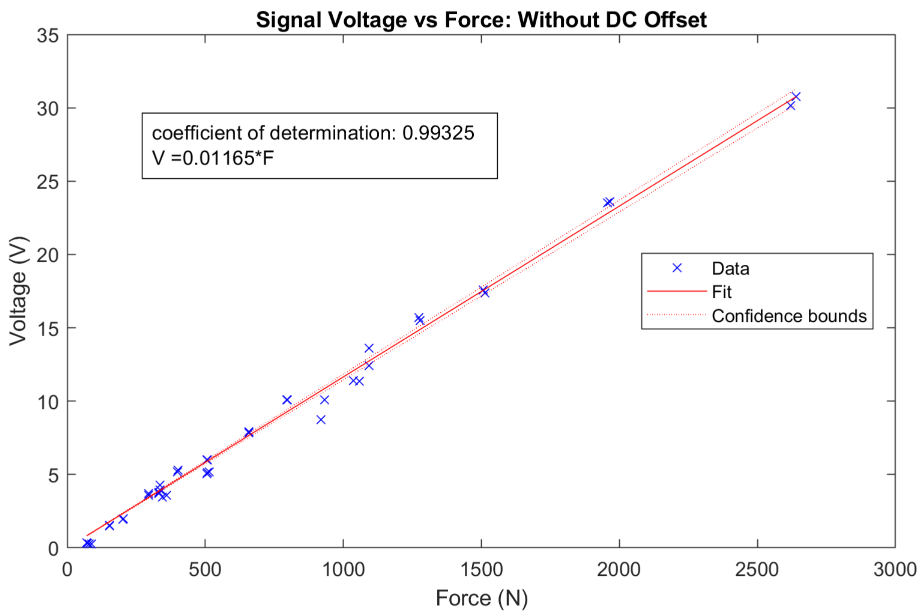

Figure 5 Voltage as a function of force with no assumed DC offset for the derivation of the force transducer calibration constant.

The best fit equation for a linear regression is $V = 0.01165F$ with an R^2 value of 0.99325. This gives an experimental calibration constant of 11.65 $\frac{mV}{N}$. This data also has a very high correlation constant (R^2), with an experimental error of 2.193%, when comparing to the theoretical value of 11.4 $\frac{mV}{N}$ [6]. The assumption of a DC offset is shown to increase experimental error, meaning that our final experimental calibration constant is too high. If errors due to resistance lead to low acceleration measurements, then our force values will also be too low, causing the calibration constant to be too high.

Table 3 highlights the preceding data for the force transducer calibration calculations based on each of the voltage vs. force graphs. The percent error from the theoretical value of 11.4 $\frac{mV}{N}$ is also determined for each calculated calibration constant [6].

Table 2 Summary of the calibration constant data with percent error

Fit Type	Calibration constant $\frac{mV}{N}$	R^2	Percent Error (%)
With DC Offset	11.862	0.99403	2.193
Without DC Offset	11.65	0.99325	4.053

Finally, the mass of the shaker armature is estimated to be 2.526 kg based on the data in table 3 below and Eqs. 11, 12 and 13. This finding has reasonable agreement with the armature mass provided in the APS spec sheet of 2.3 kg [8]. The average armature mass has a percent error of 9.83% which could be attributed to noise leading to high measured acceleration values. This error would disproportionately affect the trials with a smaller added mass, meaning that the difference in accelerations $(a_1 - a_2)$ is smaller, causing an overall larger error in the armature mass calculations. The opposite is true for trials that have a larger added mass, the difference in accelerations $(a_1 - a_2)$ is larger, causing an overall smaller error in the armature mass calculations. The following table highlights the armature mass calculations for 6 different frequencies and 6 different added masses, along with simple statistics (Table 3).

Table 3 Calculation of armature mass with simple statistics, including percent error compared to the spec sheet for this specific shaker [8].

Frequency Spectrum Peaks (Hz)	Acceleration Amplitude without Added Mass ($\frac{m}{s^2}$)	Acceleration Amplitude with Added Mass ($\frac{m}{s^2}$)	Added Mass (g)	Estimated Armature Mass (g)	Average Armature Mass (g)	Standard Deviation (g)	Percent Error (%)
100.5	11.99	11.87	26	2623.9	2526.0	127.0	14.1
107	11.95	11.76	44	2593.2			12.7
114	11.83	11.50	74	2611.3			13.5
120	11.71	10.82	203	2477.5			7.7
124.5	11.61	10.00	375	2324.0			1.0

CONCLUSION

This experiment allowed the participants to become familiar with critical vibration measurement equipment, providing foundational knowledge of signal processing and analysis. The experimental data showed stellar agreement with the theoretical relationships governing the expectations for the experiment. The analysis of the shaker vibration showed the exponential decay in the vibration displacement amplitude as frequency increased, a relationship consistent with the low fundamental frequency of the armature vibration.

Additionally, the calibration constant of a force transducer was found for NYQuist Consulting Labs. When a DC offset was assumed, the calibration constant was 11.862 $\frac{mV}{N}$ and when no offset was assumed it was 11.65 $\frac{mV}{N}$. The theoretical calibration constant was given as 11.4 $\frac{mV}{N}$, meaning that the DC offset assumption produced higher error, which did not agree with theoretical expectations. This error was likely due to the calibration constant of the experimental setup being higher than the given value due to electrical resistance causing low measured acceleration values.

Finally, the mass of the shaker armature was evaluated for NYQuist as 2.526 kg . This result was compared to the value given on the APS data sheet of 2.3 kg. The error in this measurement was likely due to noise, which would cause the measured acceleration values to be too high.

These experiments were limited by the experimental error due to noise and resistance. These errors were compounded through the calculation processes as the force and mass were not measured directly. The procedure could be improved by increasing the number of trials required to get more precise results to ensure error is entirely due to resistance and noise. The procedure could also require more data to be taken for the displacement amplitude vs frequency plot in the region of higher expected change (between 0 Hz and 20 Hz).

References

[1] 2020, “Lecture 2:Lab 2,” FL2020.E37.MEMS.4050.01 - Vibrations Lab.

[2] 2020, “LAB 2: Basic Vibration Measurments,” FL2020.E37.MEMS.4050.01 - Vibrations Lab.

[3] n.d., “SignalCalc Summary Guide,” Tech. rep., Data Physics Corporation | Automotive Testing Technology International.

[4] 2020, “LAB 1: PERIODIC SIGNALS, DATA ACQUISITION AND FOURIER ANALYSIS,” FL2020.E37.MEMS.4050.01 - Vibrations Lab.

[5] 2018, “How To Use A Klein Multimeter,” Cool Circuit, accessed Oct. 4, 2020, https://www.coolcircuit.com/how-to-use-a-klein-multimeter/

[6]

[7] “fitlm,” MathWorks, accessed October 22, 2019, https://www.mathworks.com/help/stats/fitlm.html

[8] “APS 113 Electro-Seis: Long Stroke Shaker with Linear Ball Bearings,” Spektra: APS Dynamics, INC., Dresden, Germany, accessed March 2014, file:///C:/Users/matth/Downloads/APS_113_HF_Data_Sheet_en.pdf

[9] 2020, “Introduction to Accelerometers,” Omega, Manchester United Kingdom, https://www.omega.co.uk/prodinfo/accelerometers.html#:~:text=An%20accelerometer%20is%20a%20device,the%20force%20exerted%20upon%20it.

[10] Kleckers, T., 2020, “How does a Force Transducer Actually Work?” HBK, https://www.hbm.com/en/6697/how-does-a-force-transducer-actually-work/

[11] 2020, “What is a signal conditioner?” HBK, https://www.hbm.com/en/7339/what-is-a-signal-conditioner/

[12] Zumbahlen, H., 2008, *Linear Circuit Design Handbook*, Newnes/Elsevier, doi: 10.1055/acprof.oso/97890.0394.000.

[13] “DC Offset,” Audacity, https://manual.audacityteam.org/man/dc_offset.html

[14] *Calibrate*.

[15] “Error Analysis (Uncertainty Analysis),” https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-621-experimental-projects-i-spring-2003/lecture-notes/10_errors03.pdf

[16] “Coefficient of Determination (R Squared): Definition, Calculation,” Statics How To, https://www.statisticshowto.com/probability-and-statistics/coefficient-of-determination-r-squared/

A Glossary

- **a. Accelerometer:** An accelerometer consists of a mass surrounded by piezoelectric material. When the mass accelerates, it will exert a force on the piezoelectric material, which will produce an electric charge. The force and charge are proportional, and the mass is constant, so the charge is proportional to the acceleration experienced by the device as a whole. [9]

  A practical application for an accelerometer is to sense when you shake your cellphone. The phone has a tiny accelerometer inside it, and when you shake your phone, the accelerometer registers the shaking and can trigger a software action from it.

- **b. Force transducer:** The force transducer combines a spring with a known Young's Modulus and Poisson's ratio, and one or more strain gauges in order to measure the force applied to the spring. A strain gauge is essentially a piece of wire stuck fast to a film. When the film stretches, the wire gets longer and the resistance increases. This change in resistance can be measured and related to the change in length of the strain gauge. When a strain gauge is connected to a spring (the simplest case would just be a solid metal cylinder), the strain on that spring can be related to the stress with Hooke's Law, and then the area can be divided out to get the force that was applied. If necking occurred, then the material's Poisson ratio can be used to find what the new area is. [10]

  One application for force transducers is in industrial processes where a certain force must be applied to every action (stamping, press brake, etc).

- **c. Signal conditioner/voltage amplifiers:** A signal conditioner is a general purpose device that takes a signal from a sensor and conditions it so that the measurement device can better read the signal. This may include amplifying the signal with a voltage amplifier, as some sensors output very low voltages or voltages within a very tight range such that in order to accurately measure differences in voltage, the voltage must first be amplified. A voltage amplifier uses operational amplifiers and other complex integrated circuits to change the output voltage of a signal by some constant factor called the gain. A signal conditioner may also filter out noise, linearize a reading that has a non-linear relationship, or even convert analog to digital. [11]

A signal conditioner can be used in a variety of laboratory and industrial applications, such as in a large scale automation and machine control operations, or in data acquisition. Any process that involves interfacing with various sensors would benefit from a signal conditioner.

- **Signal gain:** The signal gain is the gain that is applied to an input signal with a feedback loop connected. Signal gain can be inverting or non-inverting. Signal gain is simply an overall general term for the voltage gain of an amplifier (for electronics) but relates to the resistances the signal feels through the amplifier [12]. The signal gain in a non-electronic sense would be the amplification of a vibration or any oscillatory motion.

- **DC offset:** "[It] is the mean amplitude displacement from zero". You can see the DC offset on a graph because there will be a y-intercept for the function. [13]

- **Calibration:** It is having equipment standardized to a specified result. It allows for a precise measurement and accurate data. [14]

- **Uncertainty analysis:** It its a method used to tells you how good the results are. This helps show you if the data agrees with the theory. Error can come from the equipment, human error etc... [15]

- **Coefficient of determination(R^2):** It is used to determine how one variable is affected by a second variable. The usefulness is that it will tell you the probability of how likely one variable is to happen given another event. [16]

B Sample Calculations

Using Eq. 8, the displacement amplitude could be calculated for a given acceleration amplitude and frequency. What follows is the calculation for an acceleration of 1 m/s^2 and an angular frequency of 2 rad/s.

$$x_0 = \frac{1}{2^2} = 0.25m$$

This process was used to find the displacement peak for each acceleration value.

Using Eq. 13, the mass could be calculated from experimental data. What follows is an example from a trial with average acceleration 1 (without added mass) of 11.9911, average acceleration 2 (with added mass) of 11.8735, and an added mass of 26g.

$$M = \frac{26 * 11.8735}{11.9911 - 11.8735} = 2623.907g = 2.62kg$$

This same process was used to process the data from each added mass.

C Raw Data

Table C.1 below contains the hand recorded data from the experiments discussed in this report. They appear in an unconventional order, going section 3, 4, 2, 1, because that is the order in which data was recorded during the experiment to make efficient use of time in the laboratory. Section 3 has data from the trials used to determine the mass of the armature. section 4 has data that was not used for any particular section in the report, but that confirmed the importance of the signal conditioners. Section 2 contains the data used to find the force transducer calibration constant. Section 1 contains the data used to find the displacement amplitude at various frequencies.

Table C.1 Raw Data

Section	Run #	Mass (g)	Frequency (Hz)	V_{rms} (V)	FSpan	Lines
Section 3	1	N/A	N/A	N/A	N/A	N/A
	2	26	15.02	1.046	200	400
	3	26	15.02	1.046	400	800
	4	26	15.02	1.046	200	400
	5	26	15.02	1.046	400	800
	6	26	15.02	1.046	200	400
	7	26	15.02	1.046	400	800
	8	26	15.02	1.046	200	400
	9	26	15.02	1.046	400	800
	10	26	15.02	1.046	200	400
	11	26	15.02	1.046	400	800
	12	44	18	1.056	200	400

Table C.1 Continued.

Section	Run #	Mass (g)	Frequency (Hz)	V_{rms} (V)	FSpan	Lines
Section 3	13	44	18	1.056	400	800
	14	44	18	1.056	200	400
	15	44	18	1.056	400	800
	16	44	18	1.056	200	400
	17	44	18	1.056	400	800
	18	44	18	1.056	200	400
	19	44	18	1.056	400	800
	20	44	18	1.056	200	400
	21	44	18	1.056	400	800
	22	74	12.99	1.049	200	400
	23	74	12.99	1.049	400	800
	24	74	12.99	1.049	200	400
	25	74	12.99	1.049	400	800
	26	74	12.99	1.049	200	400
	27	74	12.99	1.049	400	800
	28	74	12.99	1.049	200	400
	29	74	12.99	1.049	400	800
	30	74	12.99	1.049	200	400
	31	74	12.99	1.049	400	800
	32	203	19.95	1.058	200	400
	33	203	19.95	1.058	500	800
	34	203	19.95	1.058	400	800
	35	203	19.95	1.058	200	400
	36	203	19.95	1.058	400	800
	37	203	19.95	1.058	200	400
	38	203	19.95	1.058	400	800

Table C.1 Continued.

Section	Run #	Mass (g)	Frequency (Hz)	V_{rms} (V)	FSpan	Lines
Section 3	39	203	19.95	1.058	200	400
	40	203	19.95	1.058	400	800
	41	203	19.95	1.058	200	400
	42	203	19.95	1.058	400	800
Section 4	43	N/A	10.05	1.037	200	400
	44	N/A	20.05	1.058	200	400
	45	N/A	29.96	1.063	200	400
	46	N/A	40.07	1.064	200	400
	47	N/A	50.05	1.065	200	400
	48	N/A	59.96	1.065	200	400
	49	N/A	70	1.065	200	400
	50	N/A	79.96	1.065	200	400
	51	N/A	89.94	1.065	200	400
	52	N/A	99.95	1.065	200	400
Section 2	53	26	15.03	1.052	200	400
	54	26	15.03	1.052	400	800
	55	26	100.3	1.065	200	400
	56	26	100.3	1.065	400	800
	57	0	100.3	1.065	200	400
	58	0	100.3	1.065	400	800
	59	44	107.1	1.065	200	400
	60	44	107.1	1.065	400	800
	61	0	107.1	1.065	200	400
	62	0	107.1	1.065	200	400
	63	0	107.1	1.065	400	800
	64	74	114.1	1.064	200	400

Table C.1 Continued.

Section	Run #	Mass (g)	Frequency (Hz)	V_{rms} (V)	FSpan	Lines
Section 2	65	74	114.1	1.064	400	800
	66	0	114.1	1.045	200	400
	67	0	114.1	1.045	400	800
	68	203	119.9	1.064	200	400
	69	203	119.9	1.064	400	800
	70	0	119.9	1.064	200	400
	71	0	119.9	1.064	400	800
	72	375	124.5	1.064	200	400
	73	375	124.5	1.064	400	800
	74	0	124.5	1.064	200	400
	75	0	124.5	1.064	400	800
Section 1	76	N/A	10.05	1.038	200	400
	77	N/A	20.07	1.058	200	400
	78	N/A	30.03	1.063	200	400
	79	N/A	39.97	1.064	200	400
	80	N/A	50.05	1.065	200	400
	81	N/A	60.01	1.064	200	400
	82	N/A	69.95	1.065	200	400
	83	N/A	80	1.065	200	400
	84	N/A	80	1.065	400	800
	85	N/A	89.95	1.065	200	400
	86	N/A	89.95	1.065	400	800
	87	N/A	99.32	1.065	200	400
	88	N/A	99.32	1.065	400	800

D MATLAB Code

The following program was used to find the average acceleration amplitude for each trial from the first section of the lab, in order to calculate the displacement amplitude.

```
%%
clear all; close all; clc;
%% Graphing run function for either file path

%First File path (STRICTLY for local file path of stored files)
% mainFilepath = 'D:\WashU\Classes\Fall 2020\Vibrations LAB\Lab 2\Run ...
    Files\Run 1 Files';
%Second File path (STRICTLY for local file path of stored files)
mainFilepath = 'D:\WashU\Classes\Fall 2020\Vibrations LAB\Lab 2\Run Files\Run ...
    2 Files';

% Runs 76-88 for displacement amplitudes
startRun = 76;
finalRun = 88;

frequencies = zeros(1,finalRun-startRun+1);
mean_acceleration = zeros(1,finalRun-startRun+1);

for currentRun = startRun:finalRun
    localPath = strcat(mainFilepath,'/Run');
    if currentRun < 10
        localPath = strcat(localPath,'0000');
    end
    if currentRun ≥ 10 && currentRun < 100
        localPath = strcat(localPath,'000');
    end
    if currentRun ≥ 100 && currentRun < 1000
        localPath = strcat(localPath,'00');
    end
    localPath = strcat(localPath,int2str(currentRun));
    localPath = strcat(localPath,'/MATLAB/DPsv00000.mat');
    load(localPath);
```

```
    % Section 2 calculations
    [Vpk_Splot,fpk_Splot] = findpeaks(abs(S1(:,2)));
    [voltage_Splot, frequency_Splot]=findPeaksGreaterThan(Vpk_Splot,S1, ...
                                                        fpk_Splot,0.2);
    [Accel_Xplot,time_Xplot] = findpeaks(X3(:,2));
    [acceleration_Xplot, frequency_Xplot]= findPeaksGreaterThan(Accel_Xplot,X3,...
                                                        time_Xplot,0.2);
    % Storing the frequencies and the mean acceleration peaks
    for i = (currentRun-startRun+1)
        frequencies(:,i) = frequency_Splot;
    end
    for i = (currentRun-startRun+1)
        mean_acceleration(:,i) = mean(acceleration_Xplot);
    end
end

%% Section 2 Plotting

% removing repeated frequencies
frequencies = unique(frequencies);

% % removing corresponding mean acceleration values
mean_acceleration(:,13) = [];
mean_acceleration(:,11) = [];
mean_acceleration(:,9) = [];

% displacement amplitude calculation @ sin(x)=1
displacement_amplitude = (mean_acceleration./((frequencies.*2*pi).^2))*1000; %mm

figure(1)
scatter(frequencies,displacement_amplitude,'filled');
xlim([0 100])
xlabel('Frequency Peak (Hz)')
ylabel('Calculated Displacement Amplitude (mm)')
title_1 = {'Calculated Displacement Amplitude vs. Frequency Peaks of the ...
    Horizontal Shaker'};
title(title_1);
```

```
function [peaks, locations]= findPeaksGreaterThan(pks,arr,locs,val)
    currentindex = 1;
    arrSizea = size(pks);
    arrSize = arrSizea(1);
    peaks = zeros(arrSize,1);
    locations = zeros(arrSize,1);
    for i=1:arrSize
        if(pks(i) ≥ val)
            peaks(currentindex) = pks(i);
            locations(currentindex) = arr(locs(i),1);
            currentindex = currentindex + 1;
        end
    end
    peaks = peaks(peaks ≠ 0);
    locations =  locations(locations ≠ 0);
end
```

The following program was used to find the average acceleration and force amplitudes in order to find the calibration constant for the force transducer.

```
%%
clear all;
clc;
close all;

%%
% mainFilepath = 'D:\WashU\Classes\Fall 2020\Vibrations LAB\Lab 1\Run Files';
mainFilepath = 'C:\Users\mitry\Google Drive\Run 2 Files';

% excellFilename = '/first_runs.xlsx';
excellFilename = '/MassEstimation.xlsx';

cd(mainFilepath)
A = {'Run number','S-plot peak (m/s^2)','S-plot frequency(Hz)', ...
     'X-plot avg Acceleration peak (m/s^2)','X-plot avg Force peak (N)'};
```

```
xlswrite(strcat(mainFilepath,excellFilename),A)

firstRun = 53;
lastRun = 75;
% numFolders = 73;
height = 2;

for currentRun = firstRun:lastRun
    localPath = strcat(mainFilepath,'/Run');
    if currentRun < 10
        localPath = strcat(localPath,'0000');
    end
    if currentRun ≥ 10 && currentRun < 100
        localPath = strcat(localPath,'000');
    end
    if currentRun ≥ 100 && currentRun < 1000
        localPath = strcat(localPath,'00');
    end
    localPath = strcat(localPath,int2str(currentRun));
    localPath = strcat(localPath,'/MATLAB/DPsv00000.mat');
    load(localPath);

    figure(1)
    plot(abs(S3(:,1)),abs(S3(:,2)));
    figure(2)
    plot(X3(:,1),X3(:,2));

    [Vpk_Splot,fpk_Splot] = findpeaks(abs(S3(:,2)));
    [voltage_Splot, frequency_Splot]=findPeaksGreaterThan(Vpk_Splot,S3, ...
                                                            fpk_Splot,0.5);

    [Vpk_Xplot,fpk_Xplot] = findpeaks(X3(:,2));
    [voltage_Xplot, frequency_Xplot]= findPeaksGreaterThan(Vpk_Xplot,X3,...
                                                            fpk_Xplot,0.5);

    [Fpk_Xplot,Ffpk_Xplot] = findpeaks(X2(:,2));
    [force_Xplot, ForceFrequency_Xplot]= findPeaksGreaterThan(Fpk_Xplot,X2,...
                                                            Ffpk_Xplot,0.5);
```

```
    size_Splot = size(voltage_Splot);
    testNumberArr_Splot = zeros(size_Splot(1),size_Splot(2));
    mean_voltage_xplot = zeros(size_Splot(1),size_Splot(2));
    mean_force_xplot = zeros(size_Splot(1),size_Splot(2));
    for i = 1:size(voltage_Splot)
        testNumberArr_Splot(i) = currentRun;
        mean_voltage_xplot(i) = mean(voltage_Xplot);
        mean_force_xplot(i) = mean(force_Xplot);
    end

    writematrix([testNumberArr_Splot, voltage_Splot, frequency_Splot, ...
                 mean_voltage_xplot,mean_force_xplot],strcat(mainFilepath, ...
                 excellFilename), 'Range',strcat('A',int2str(height)));

    height = height + size_Splot(1);

end

function [peaks, locations]= findPeaksGreaterThan(pks,arr,locs,val)
    currentindex = 1;
    arrSizea = size(pks);
    arrSize = arrSizea(1);
    peaks = zeros(arrSize,1);
    locations = zeros(arrSize,1);
    for i=1:arrSize
        if(pks(i) ≥ val)
            peaks(currentindex) = pks(i);
            locations(currentindex) = arr(locs(i),1);
            currentindex = currentindex + 1;
        end
    end
    peaks = peaks(peaks ≠ 0);
    locations =  locations(locations ≠ 0);
end
```

The following program made graphs from the output of the previous program, in order to find the calibration constant for the force transducer.

```
clear all;
clc;
close all;

% Load in Data
mainFilepath = 'C:\Users\mitry\Google Drive\Run 2 Files\AccelerationPeaks.xlsx';
T = readtable(mainFilepath)
A = table2array(T);

mass = A(:,6)
signalAmplitude = A(:,5)
frequency = A(:,3)
acceleration = A(:,4)

%omega = 2*pi*f
angularFrequency = frequency.*(2*pi());
%F = m*a
forceAmplitude = mass.*acceleration;
%x0 = a0/omega^2
displacementAmplitude = acceleration./(angularFrequency.^2);

%With Offset
figure(1)
dlm = fitlm(forceAmplitude,signalAmplitude)
plot(dlm);
title('Signal Voltage vs Force: With DC Offset');
xlabel('Force (N)');
ylabel('Voltage (V)');
r2 = dlm.Rsquared.Ordinary;
slopeandintercept = dlm.Coefficients.Estimate;
m = slopeandintercept(2);
b = slopeandintercept(1);
str = {['coefficient of determination: ',num2str(r2)] ['V =', num2str(m),'*F', ...
    ' + ', num2str(b)]};
annotation('textbox',[.2 .5 .3 .3],'String',str,'FitBoxToText','on');

%Without Offset
figure(2);
```

```
dlm = fitlm(forceAmplitude,signalAmplitude,'Intercept',false)
plot(dlm);
title('Signal Voltage vs Force: Without DC Offset');
xlabel('Force (N)');
ylabel('Voltage (V)');
r2 = dlm.Rsquared.Ordinary;
slopeandintercept = dlm.Coefficients.Estimate;
m = slopeandintercept(1);
str = {['coefficient of determination: ',num2str(r2)] ['V =', num2str(m),'*F']};
annotation('textbox',[.2 .5 .3 .3],'String',str,'FitBoxToText','on');
```