Washington University in St. Louis
JAMES MCKELVEY SCHOOL OF ENGINEERING

Fall 2020 MEMS 4050 Vibrations Laboratory

Pre-Lab 5: Design Your Own Experiment (Beam and Modal Analysis)

Lab Instructor: Dr. Bayly

Group T (Friday 2 PM)

We hereby certify that the lab report herein is our original academic work, completed in accordance to the McKelvey School of Engineering and Undergraduate Student academic integrity policies, and submitted to fulfill the requirements of this assignment:

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1 Question 1

(*2pts*) Describe the physics behind how an impulse force hammer works. How is this different from a quartz accelerometer? Discuss at least two general advantages and disadvantages of each sensor.

An impulse force hammer can be used to apply a force at a specific location as well as measure that force at the same time. Knowing the force appliead and the mass of the beam can then allow us to calculate the acceleration at a point of interest . The impulse force hammer can be applied at different locations.

A quartz accelerometer converts a vibration into a voltage signal and is reliable over a wide range of frequencies[1]. One difference is how the measurement of acceleration is taken. The impulse hammer can measure force and from this, if the mass is known then the acceleration can be calculated at the point of contact. While the quartz accelerometer only measures the force at a specific point.

Some advantages to use the impulse force hammer are that you can measure the applied force at different locations along the beam by moving where the hammer is hit. A second advantage of the impulse hammer is that it accounts for different frequency ranges based on different materials for the tip of the hammer [2]. So, the hammer does not effect the frequency of the system. A disadvantage of the impulse hammer could be that it would be hard to get the exact force wanted when hitting the object. Also, the force hammer would not be able to find the acceleration at other points on the beam. Lastly, it is cheaper than buying a shaker or a system that excited the object.

An advantage of the quartz accelerometer is that you can also measure the acceleration of any point on the beam just by moving the accelerometer to a different spot.[3]. A disadvantage would be that it does not measure angular velocity, only transnational acceleration so as the beam bends it will not measure the full effect [4]. The accelerometer is also less efficient over time. One last advantage of the quartz accelerometer is that it has high sensitivity[4].

2 Question 2

(*4pts*) A cantilever beam extends 50 cm from its clamped base. With no added mass, the lowest natural frequency of the free vibration is 28.75 Hz. When a mass of 0.75 kg is placed on the end of the beam, the natural frequency of its free vibration is 11 Hz. The height of the beam's cross section is 0.35in, and its width is 0.865in. These properties are 0.889cm and 2.197cm respectively. Provide a summary of how you will perform an experiment to estimate these values (see the lab procedure for constraints). The summary should be brief (no more than two pages) and should cover both the Methods and Results & Discussion sections of your lab report. Estimate the Young's modulus (E) and the density (ρ) of the material of the beam. Based on these properties, what is the beam material? Include any references and provide a simple error analysis on your results. Include the schematic(s) of your experiment and use the given values in your sample calculations. Show all work.

2.1 Methods.

2.1.1 Apparatus. A metal beam, with dimensions and mass measured by ruler and scale, is clamped to the table. An impulse force hammer, with a plastic/vinyl tip, strikes the beam (Fig. 1). A force transducer in the hammer [5] measures the impulse and an accelerometer at the striking location on the beam measures the acceleration due to impulse. When appropriate a mass is added to the end of the beam. The signal from the measurement devices is amplified with signal conditioners and then passed to the Quattro. The Quattro analyzes the signals in the frequency and time domains and presents its results using the SignalCalc application. Figure 1 shown below represents the set up for the lab. The length (L), height (H) and Width (W), all in meters, are the dimensions of the beam.

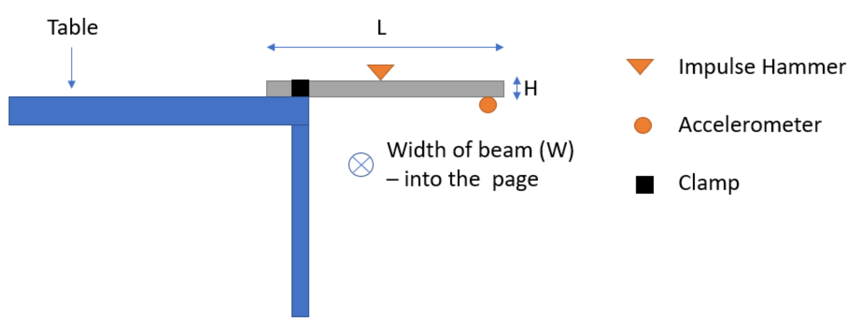

Figure 1 Schematic for the set up of the lab.

2.1.2 Procedure. To begin, the length, width, height, and mass of the beam will be measured. Also the color and texture of the material will be noted. Then, we will clamp one end of the beam to the table so that the clamped end acts as though it were fully constrained.

Then, using a transfer function test in SignalCalc, the beam will be whacked in multiple known locations. SignalCalc will be set to trigger when the force transducer impacts the beam. The trigger level will be lowered until the hammer triggers a SignlCalc run with a light tap. SignalCalc will record and display the time domain plot of the accelerometer output, the time domain plot of the force transducer output, and the real and imaginary frequency plots of the transfer function of the beam. The accelerometer will always be attached to the bottom of the overhanging edge of the beam. For each run, the beam will be struck three times and the software will output an average of its measurements for each whack.

For the first set of tests, the base natural frequency of each run will be recorded. After each run, mass will be added to the end of the beam. The natural frequency of the beam will be calculated and also measured with SignalCalc. The mass will increased until the calculations agree with the theoretical value for natural frequency. At this point, the system can be treated as a mass-spring system where the spring has negligible mass. With the required mass in place, the beam will be whacked at three different known locations to determine its first three natural frequencies.

Then, to acquire the data necessary to produce a mode shape plot of the beam, the mass will be removed and the beam will be struck at twenty locations,equally spaced from each other.

2.1.3 Analysis. The following equations will be used to find I, E, and ρ for the beam:

$$I = \frac{1}{12}bh^3 \qquad E \approx \frac{m\omega_n^2 L^3}{3I} \qquad \rho \approx \frac{(1.875)^4 EI}{\omega_n^2 A L^4} \tag{1}$$

ω_n will be the base natural frequency of the system with an attached mass large enough such that the mass of the spring can be neglected. The theoretical natural frequency will be given by $\sqrt{(}k/m)$ and compared to the measured ω_n. The added mass is large enough when the theoretical and measured values agree.

The imaginary peak of the transfer function found in Signal Calc from hitting the beam with the force hammer at each location and frequency correspond to the mode shape at that location and natural frequency. Thus, a graph can be generated for the mode shape of the beam at each natural frequency, which can then be compared to the theoretical results from MATLAB and Solidworks.

2.2 Results and Discussion. For the example beam given above, Eq 1 can be used to find the area moment of inertia, young's modulus, and density of the beam. In the real experiment, ω_n will be determined from the accelerometer data.

$$I = \frac{1}{12}(0.02197)(0.00889)^3 = 1.286 * 10^{-9} m^4$$

$$E \approx \frac{m\omega_n^2 L^3}{3I} = \frac{0.75*(11*2*\pi)^2*(0.5)^3}{3*1.286*10^{-9}} = 1.1604 * 10^{11} Pa = 116 GPa \approx E$$

$$\rho \approx \frac{(1.875)^4 EI}{\omega_n^2 A L^4} = \frac{(1.875)^4 1.1604*10^{11}*1.286*10^{-9}}{(28.75*2*\pi)^2((0.00889)(0.02197))(0.5)^4} = 4631.52 kg/m^3$$

With these properties in mind, it seems like the beam is likely made out of Titanium, which has a density of about $4600 kg/m^3$[6] and a young's modulus ranging from 102 to 124 GPa [7]. This method will be used to identify the material of the beam during the lab. The methods described above will also be used to find the mode shapes of the beam from the experimental data in the lab.

3 Question 3

(*2 pts*)) Compute the first three natural frequencies (in Hz) for the beam in Table 1.

a)Tabulate all the natural frequencies generated from your SOLIDWORKS simulation. Identify the mode shapes in the vertical (y) direction.

Table 1

Beam 1		
Mode Shape	Frequency (Hz)	Mode Shape Direction
1	38.569	Vertical (y-direction)
2	143.08	Horizontal (x-direction)
3	241.43	Vertical (y-direction)
4	674.93	Vertical (y-direction)
5	882.83	Horizontal (x-direction)

Table 2

Beam 2		
Mode Shape	Frequency (Hz)	Mode Shape Direction
1	19.672	Vertical (y-direction)
2	29.41	Horizontal (x-direction)
3	121.83	Vertical (y-direction)
4	179.6	Horizontal (x-direction)
5	315.4	Rotation/twisting
6	335.01	Horizontal (x-direction)
7	484.01	Vertical (y-direction)

b) How do your theoretical MATLAB frequencies compare to the (vertical) frequencies of your SOLIDWORKS model? Include a table of percent differences. Briefly discuss your observations.

Table 3 Beam 1 Natural Frequencies found in SOLIDWORKS and MATLAB with Percent Errors.

Vertical Mode Shape	**SOLIDWORKS Frequency (Hz)**	**MATLAB Frequency (Hz)**	**Percent Error (%)**
1	38.4114	38.4114	0.4103
2	241.43	240.7374	0.2877
3	674.93	6741.396	0.1172

Table 4 Beam 2 natural frequencies found in SOLIDWORKS and MATLAB with Percent Errors.

Vertical Mode Shape	**SOLIDWORKS Frequency (Hz)**	**MATLAB Frequency (Hz)**	**Percent Error (%)**
1	29.41	29.4389	0.0982
2	179.6	184.5040	2.6579
3	484.01	516.6684	6.3210

c) Provide well-labelled mode shapes of your SOLIDWORKS simulation against their theoretical MATLAB model. Use one graph for each mode shape e.g. Figure 1 should only include the first mode shape for Beam #1 from SOLIDWORKS and MATLAB. Include your completed code. (MATLAB).

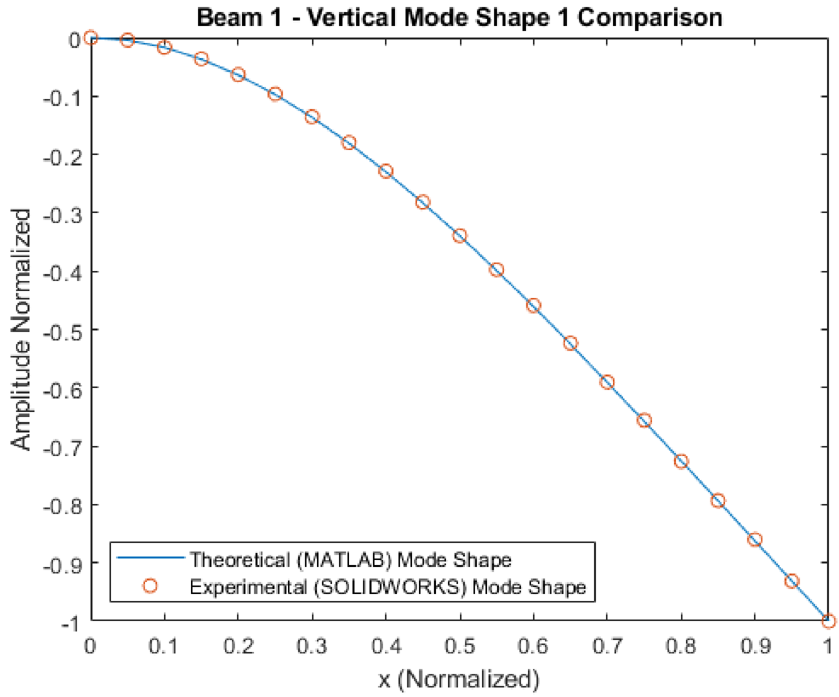

Figure 2 A comparison of vertical mode 1 between the SOLIDWORKS beam and the theoretical model for beam 1.

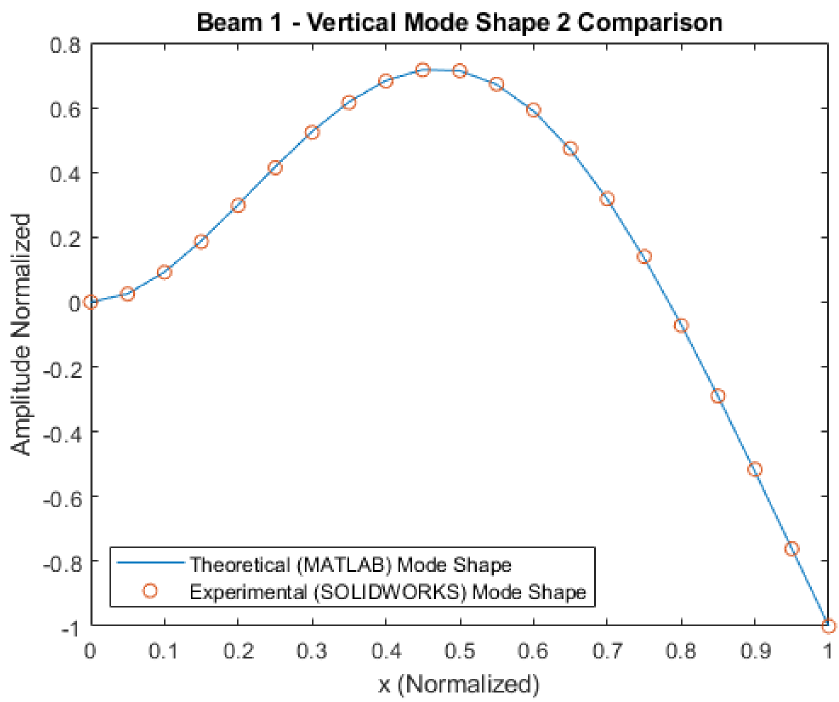

Figure 3 A comparison of vertical mode 2 between the SOLIDWORKS beam and the theoretical model for beam 1.

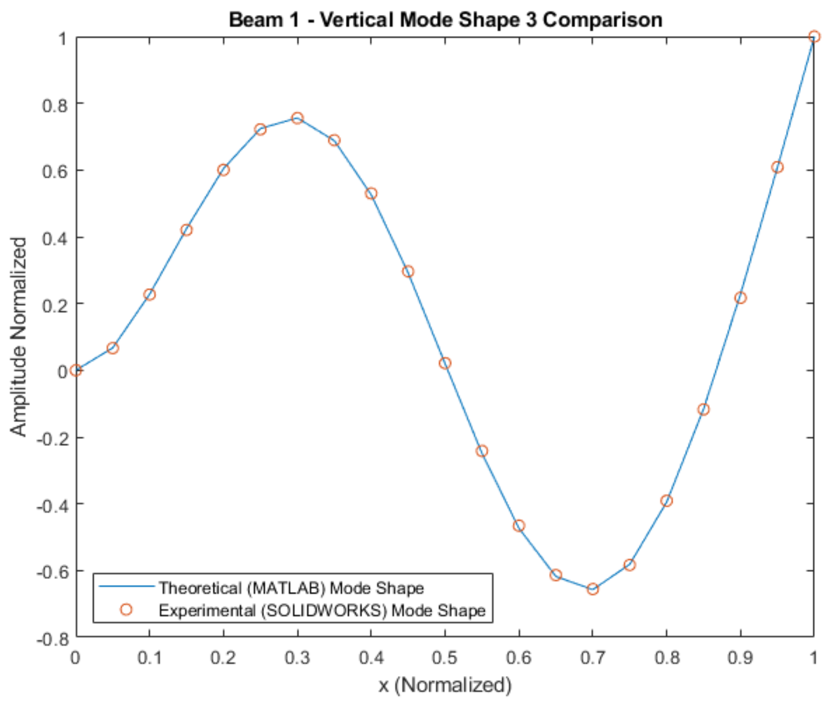

Figure 4 A comparison of vertical mode 3 between the SOLIDWORKS beam and the theoretical model for beam 1.

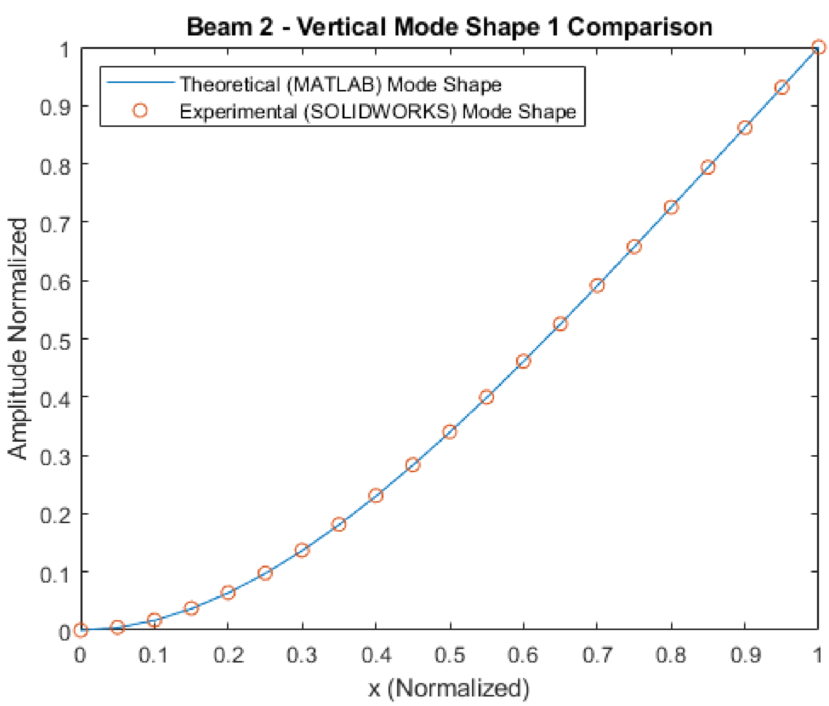

Figure 5 A comparison of vertical mode 1 between the SOLIDWORKS beam and the theoretical model for beam 2.

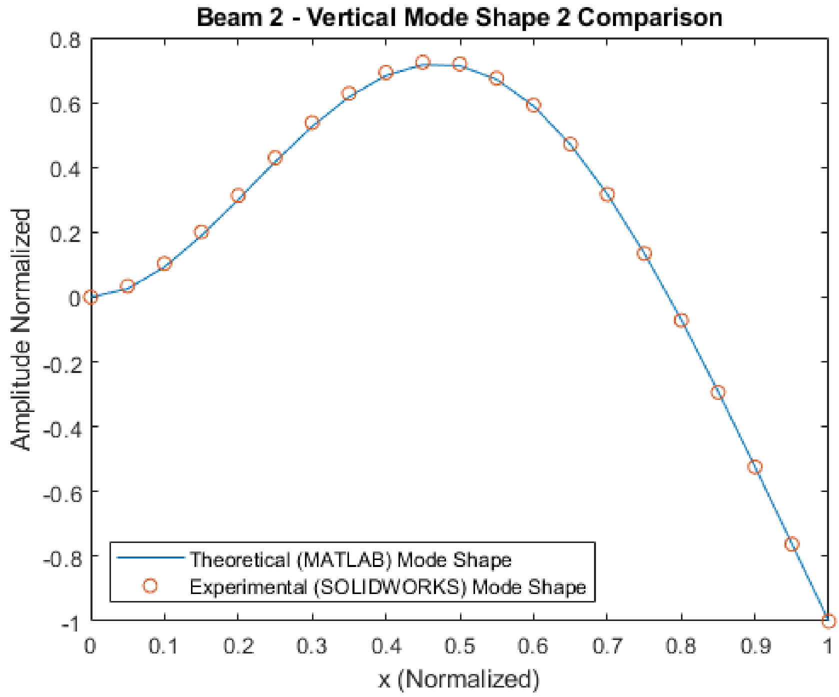

Figure 6 A comparison of vertical mode 2 between the SOLIDWORKS beam and the theoretical model for beam 2.

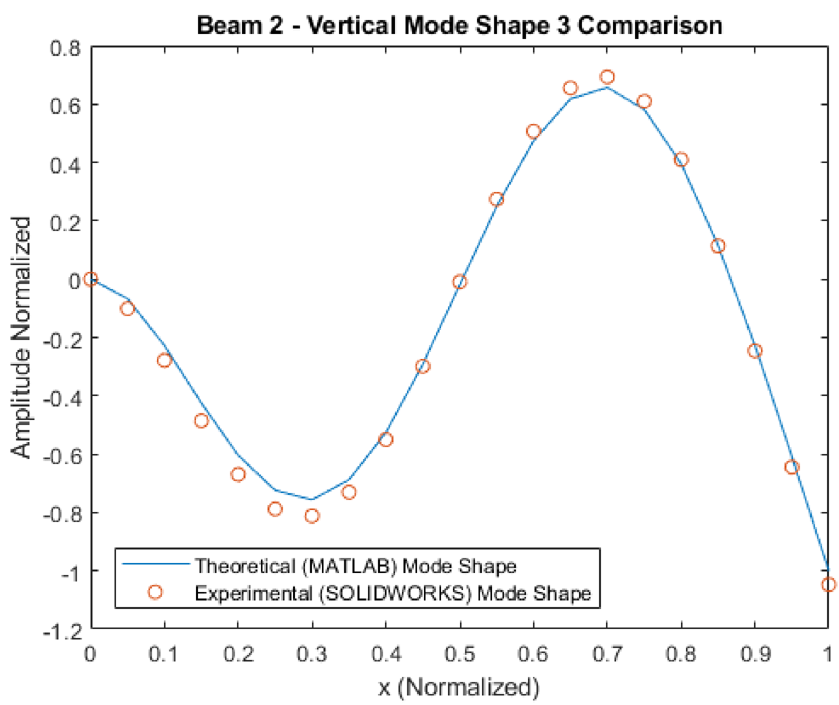

Figure 7 A comparison of vertical mode 3 between the SOLIDWORKS beam and the theoretical model for beam 2.

MATLAB Code. What follows below is the MATLAB code used in the analysis for this report, based off the given code [8].

```
clear all;
close all;
clc;

%% Beam 1
E_1 = 193e9;        % young's modulus (Pa)
rho_1 = 6500;      % density (kg/m^3)
L_1 = 35.5/100;         % length (m)
b_1 = 2.05/100;       % width (m)
h_1 = 0.55/100;      % thickness (m)

I_1  =  b_1*h_1^3/12; % area moment of inertia
A_1 = b_1*h_1;         % cross section area

B1L_1 = 1.875;     % eignevalue 1
B2L_1 = 4.694;     % eigenvalue 2
B3L_1 = 7.855;     % eigenvalue 3

% COMPUTE NATURAL FREQUENCIES
BL_1 = [B1L_1;B2L_1;B3L_1];
wn_1 = (BL_1.^2)*sqrt(E_1*I_1/(rho_1*A_1*L_1^4));      % natural frequencies in rad/sec
fn_1 = wn_1/(2*pi);                         % natural frequencies in Hz

% COMPUTE MODE SHAPES
alpha_1 = (sin(BL_1) + sinh(BL_1))./(cos(BL_1) + cosh(BL_1));
dx_1 = L_1/20;
x_1=0:dx_1:L_1;
W1_1 =  (sin(BL_1(1)*x_1/L_1) - sinh(BL_1(1)*x_1/L_1) ...
    -alpha_1(1)*(cos(BL_1(1)*x_1/L_1) - cosh(BL_1(1)*x_1/L_1)));
W2_1 =  (sin(BL_1(2)*x_1/L_1) - sinh(BL_1(2)*x_1/L_1) ...
    -alpha_1(2)*(cos(BL_1(2)*x_1/L_1) - cosh(BL_1(2)*x_1/L_1)));
W3_1 =  (sin(BL_1(3)*x_1/L_1) - sinh(BL_1(3)*x_1/L_1) ...
    -alpha_1(3)*(cos(BL_1(3)*x_1/L_1) - cosh(BL_1(3)*x_1/L_1)));
```

```
% SCALE SO W(L)=1
W1_1  = W1_1/W1_1(end);
W2_1  = W2_1/W2_1(end);
W3_1  = W3_1/W3_1(end);

% MASS NORMALIZATION OF MODE SHAPES
% NOT NEEDED FOR PRE-LAB 5
% m1_1 = sum(rho_1*A_1*W1_1.^2*dx_1);
% W1_1   = W1_1/sqrt(m1_1);
% mx1_1 = 1.2*max(abs(W1_1));
%
% m2_1 = sum(rho_1*A_1*W2_1.^2*dx_1);
% W2_1   = W2_1/sqrt(m2_1);
% mx3_1 = 1.2*max(abs(W3_1));
%
% m3_1 = sum(rho_1*A_1*W3_1.^2*dx_1);
% W3_1   = W3_1/sqrt(m3_1);
% mx2_1 = 1.2*max(abs(W2_1));


% PLOT NORMALIZED MODE SHAPES FROM MATLAB
% Note the different length axis options!

figure(1)
subplot(3,1,1),plot(100*x_1,W1_1); title('Mode 1')
xlabel(sprintf('x (cm) \n'))
ylabel('Amplitude (Normalized)')

subplot(3,1,2),plot(x_1,W2_1); title('Mode 2')
xlabel(sprintf('x (m) \n'))
ylabel('Amplitude (Normalized)')

subplot(3,1,3),plot(x_1/L_1(end),W3_1); title('Mode 3')
xlabel('x (Normalized)')
ylabel('Amplitude (Normalized)')

%% Beam 2
```

```
E_2 = 5.25e9;        % young's modulus (Pa)
rho_2 = 889;      % density (kg/m^3)
L_2 = 1;         % length (m)
b_2 = 5/100;       % width (m)
h_2 = 7.5/100;      % thickness (m)

I_2  =  b_2*h_2^3/12; % area moment of inertia
A_2 = b_2*h_2;         % cross section area

B1L_2 = 1.875;     % eignevalue 1
B2L_2 = 4.694;     % eigenvalue 2
B3L_2 = 7.855;     % eigenvalue 3

% COMPUTE NATURAL FREQUENCIES
BL_2 = [B1L_2;B2L_2;B3L_2];
wn_2 = (BL_2.^2)*sqrt(E_2*I_2/(rho_2*A_2*L_2^4));      % natural frequencies in ...
    rad/sec
fn_2 = wn_2/(2*pi);                          % natural frequencies in Hz

% COMPUTE MODE SHAPES
alpha_2 = (sin(BL_2) + sinh(BL_2))./(cos(BL_2) + cosh(BL_2));
dx_2 = L_2/20;
x_2=0:dx_2:L_2;
W1_2 =  (sin(BL_2(1)*x_2/L_2) - sinh(BL_2(1)*x_2/L_2) ...
    -alpha_2(1)*(cos(BL_2(1)*x_2/L_2) - cosh(BL_2(1)*x_2/L_2)));
W2_2 =  (sin(BL_2(2)*x_2/L_2) - sinh(BL_2(2)*x_2/L_2) ...
    -alpha_2(2)*(cos(BL_2(2)*x_2/L_2) - cosh(BL_2(2)*x_2/L_2)));
W3_2 =  (sin(BL_2(3)*x_2/L_2) - sinh(BL_2(3)*x_2/L_2) ...
    -alpha_2(3)*(cos(BL_2(3)*x_2/L_2) - cosh(BL_2(3)*x_2/L_2)));

% SCALE SO W(L)=1
W1_2  = W1_2/W1_2(end);
W2_2  = W2_2/W2_2(end);
W3_2  = W3_2/W3_2(end);

% MASS NORMALIZATION OF MODE SHAPES
% NOT NEEDED FOR PRE-LAB 5
% m1_2 = sum(rho_2*A_2*W1_2.^2*dx_2);
```

```
% W1_2  = W1_2/sqrt(m1_2);
% mx1_2 = 1.2*max(abs(W1_2));
%
% m2_2 = sum(rho_2*A_2*W2_2.^2*dx_2);
% W2_2  = W2_2/sqrt(m2_2);
% mx3_2 = 1.2*max(abs(W3_2));
%
% m3_2 = sum(rho_2*A_2*W3_2.^2*dx_2);
% W3_2  = W3_2/sqrt(m3_2);
% mx2_2 = 1.2*max(abs(W2_2));


%% PLOT NORMALIZED MODE SHAPES FROM MATLAB
% Note the different length axis options!

figure(2)
subplot(3,1,1),plot(100*x_2,W1_2); title('Mode 1')
xlabel(sprintf('x (cm) \n'))
ylabel('Amplitude (Normalized)')

subplot(3,1,2),plot(x_2,W2_2); title('Mode 2')
xlabel(sprintf('x (m) \n'))
ylabel('Amplitude (Normalized)')

subplot(3,1,3),plot(x_2/L_2(end),W3_2); title('Mode 3')
xlabel('x (Normalized)')
ylabel('Amplitude (Normalized)')


%% PLOT NORMALIZED MODE SHAPES FROM SOLIDWORKS
Beam1_MS1 = xlsread('Beam1_ModeShape1.csv'); % Beam 1 Mode Shape 1
Beam1_MS3 = xlsread('Beam1_ModeShape3.csv'); % Beam 1 Mode Shape 3
Beam1_MS4 = xlsread('Beam1_ModeShape4.csv'); % Beam 1 Mode Shape 4

%% Beam 2
Beam2_MS2 = xlsread('Beam2_ModeShape2.csv'); % Beam 2 Mode Shape 2
Beam2_MS4 = xlsread('Beam2_ModeShape4.csv'); % Beam 2 Mode Shape 4
Beam2_MS7 = xlsread('Beam2_ModeShape7.csv'); % Beam 2 Mode Shape 7
```

```
%% Beam 1 comparison
figure(3)
plot(x_1/L_1(end),W1_1*(-1))
hold on
plot(Beam1_MS1(:,2)./20,Beam1_MS1(:,3),'o'),
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','southwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 1 - Vertical Mode Shape 1 Comparison');

figure(4)
plot(x_1/L_1(end),W2_1*(-1))
hold on
plot(Beam1_MS4(:,2)./20,Beam1_MS4(:,3),'o')
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','southwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 1 - Vertical Mode Shape 2 Comparison');

figure(5)
plot(x_1/L_1(end),W3_1)
hold on
plot(Beam1_MS3(:,2)./20,Beam1_MS3(:,3),'o')
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','southwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 1 - Vertical Mode Shape 3 Comparison');

%% Beam 2 comparison
figure(6)
plot(x_2/L_2(end),W1_2)
```

```
hold on
plot(Beam2_MS2(:,2)./20,Beam2_MS2(:,3),'o')
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','northwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 2 - Vertical Mode Shape 1 Comparison');

figure(7)
plot(x_2/L_2(end),W2_2*(-1))
hold on
plot(Beam2_MS4(:,2)./20,Beam2_MS4(:,3),'o')
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','southwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 2 - Vertical Mode Shape 2 Comparison');

figure(8)
plot(x_2/L_2(end),W3_2*(-1))
hold on
plot(Beam2_MS7(:,2)./20,Beam2_MS7(:,3),'o')
legend('Theoretical (MATLAB) Mode Shape',...
       'Experimental (SOLIDWORKS) Mode Shape',...
       'Location','northwest');
xlabel('x (Normalized)');
ylabel('Amplitude Normalized');
title('Beam 2 - Vertical Mode Shape 3 Comparison');
```

References

[1] “Quartz Accelerometer,” PCB Piezoelectric INC., accessed November 15, 2020, http://tactileresearch.org/rcholewi/files/PCB_Accelerometer.pdf

[2] “Impulse Force Accelerometer,” Kistler Instrument Corp., 30280 Hudson Drive 48377 Novi, MI, accessed November 15, 2020, https://www.kistler.com/en/glossary/term/impulse-force-hammer/

[3] 2020, “In Class Lecture:Lab 5,” FL2020.E37.MEMS.4050.01 - Vibrations Lab.

[4] “Advantages of Accelerometers, Disadvantages of Accelerometers,” RF Wireless, accessed November 15, 2020, https://www.rfwireless-world.com/Terminology/Advantages-and-Disadvantages-of-Accelerometer.html

[5] “Impulse Hammer 086C03,” PCB Group Inc., Depew, NY, accessed 2019, https://www.pcb.com/products?m=086C03

[6] “Density of selected solids,” Engineering ToolBox, accessed 2009, https://www.engineeringtoolbox.com/density-solids-d_1265.html

[7] “YOUNG’S MODULUS OF TITANIUM ALLOYS,” ASM International, From Metals Handbook. Properties and Selection: Irons, Steels, and High- Performance Alloys., accessed 1990, https://amesweb.info/Materials/Youngs-Modulus-of-Titanium.aspx

[8] Bayly, P., Woodhams, L., and Asinugo, C., 2020, “Class Material,” FL2020.E37.MEMS.4050.01 - Vibrations Lab.