Washington University in St. Louis

James McKelvey School of Engineering

**Fall 2030 MEMS 505 Engineering and Science Laboratory**

Pre-Lab 1: Periodic Signals, Data Acquisition, and Fourier Analysis

Lab Instructor: Dr. Bayly

**Group T (Friday 11 AM)**

We hereby certify that the lab report herein is our original academic work, completed in accordance to the McKelvey School of Engineering and Undergraduate Student academic integrity policies, and submitted to fulfill the requirements of this assignment:


| | | |
|---|---|---|
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| Andrew Brown | *Quality Control Engineer* | Andrew Brown |
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| Mitry Anderson | *Test Engineer II* | Mitry Anderson |

# 1 Question 1

(*2pts*) Review the SignalCalc - *Summary Guide* document on the main Box Folder.

**a.** Discuss the relationship between the FSPan and Lines parameter.

FSpan represents the upper limit of the frequency range, while the Lines parameter represents the number of samples taken in the frequency domain. So a high FSpan value will take samples over a large range of frequencies, and a high Lines value will mean that within that range, many samples are taken.

**b.** From the Lab 1 procedure, create the required syn file. Include a screenshot of your SignalCalc window.

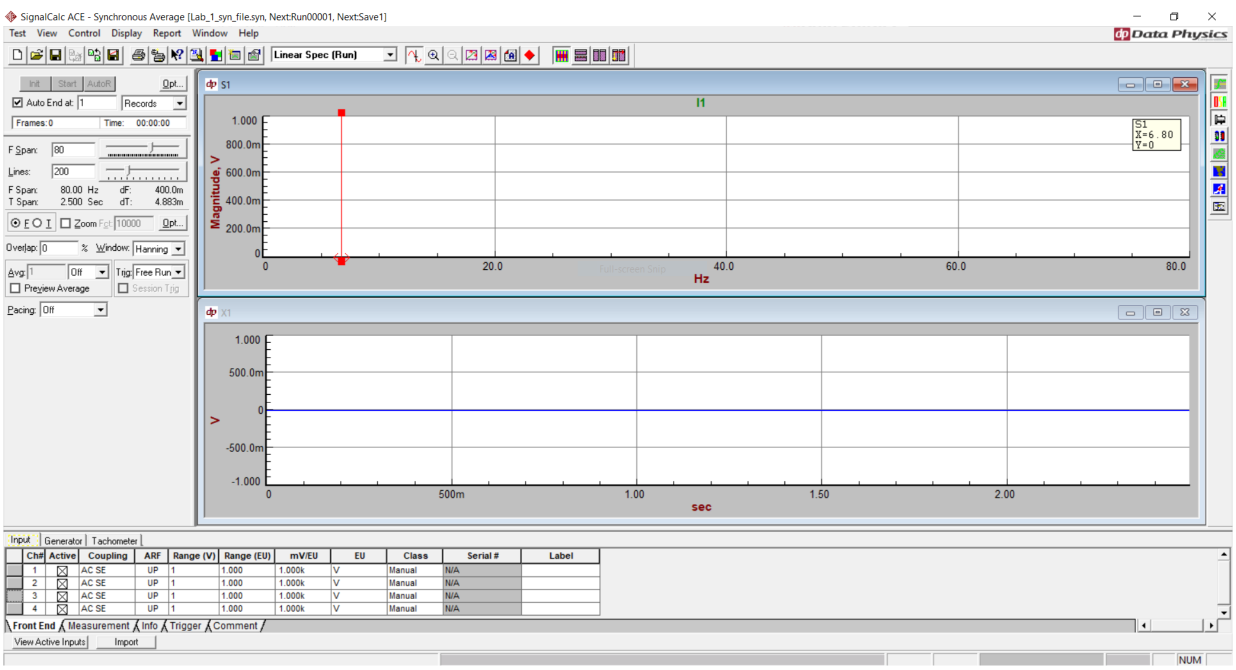



**Figure 1 SignalCalc window with settings based on the Lab 1 procedure.**

# 2 Question 2

(*2pts*) Briefly review the code and run the MATLAB script below. Summarize the code in a few sentences. Include a copy of the generated images( with proper captions). Observe the effects of the sampling parameters in the different figures. Describe your observations of the signal's time series and frequency spectrum plots.

a) aliasing_SamplingFrequency

The MATLAB code generates a sine wave, then samples from that wave at various frequencies. It then plots the results in a series of charts, overlaid with the original signal. It also performs a Fast Fourier Transform (FFT) on the sampled data and plots the result for each sampling frequency. Notably, at low sampling frequencies, the time domain plots are clearly wrong. There is even one plot where the wave happens to be sampled only when its amplitude is zero, such that the result looks like a straight line rather than a wave. As the sampling rate increases, the resulting graphs approximate the original signal with less and less error. In the frequency domain, the same is true, because as the sample rate increases, the FFT results get closer and closer to predicting the proper frequency.

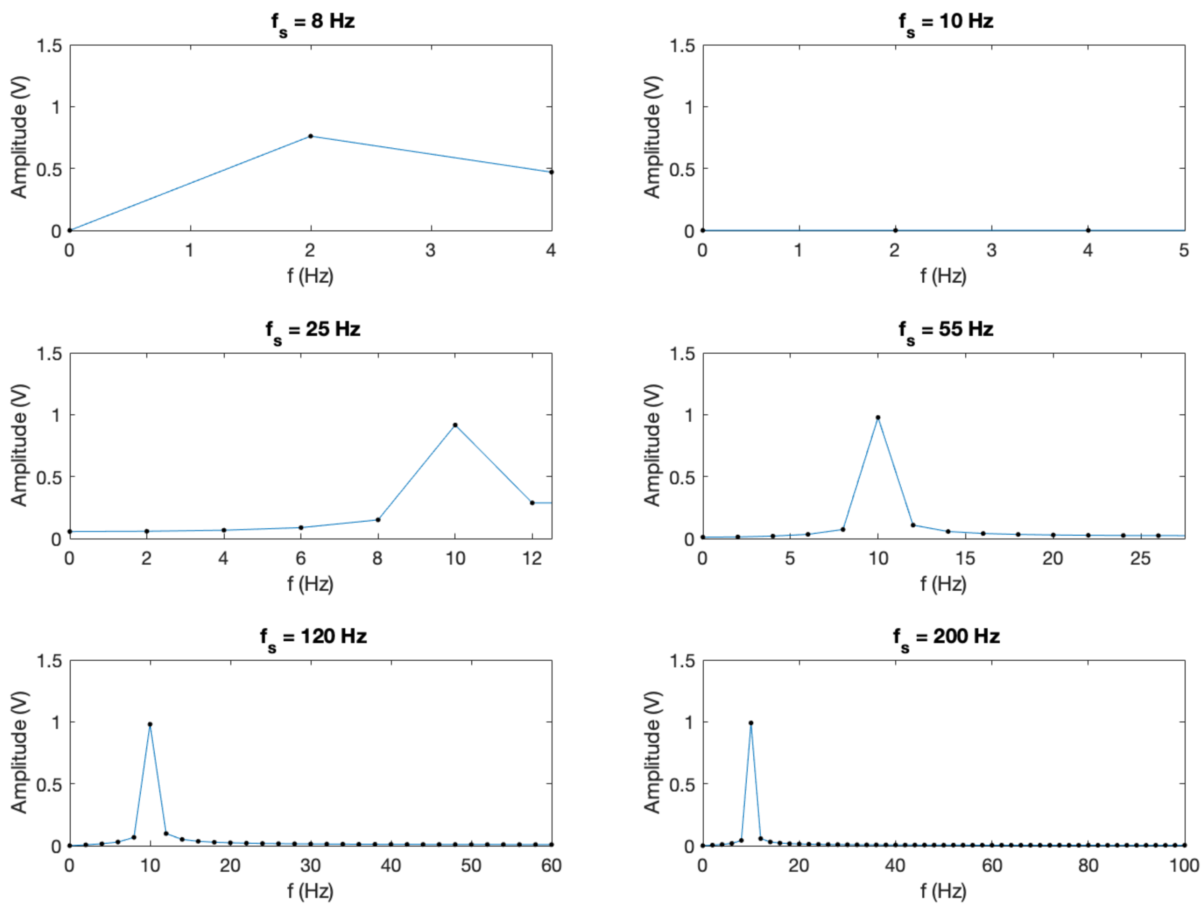



**Figure 2 Shows that an increase in sampling frequency creates a better approximation of the sine wave in the frequency domain.**

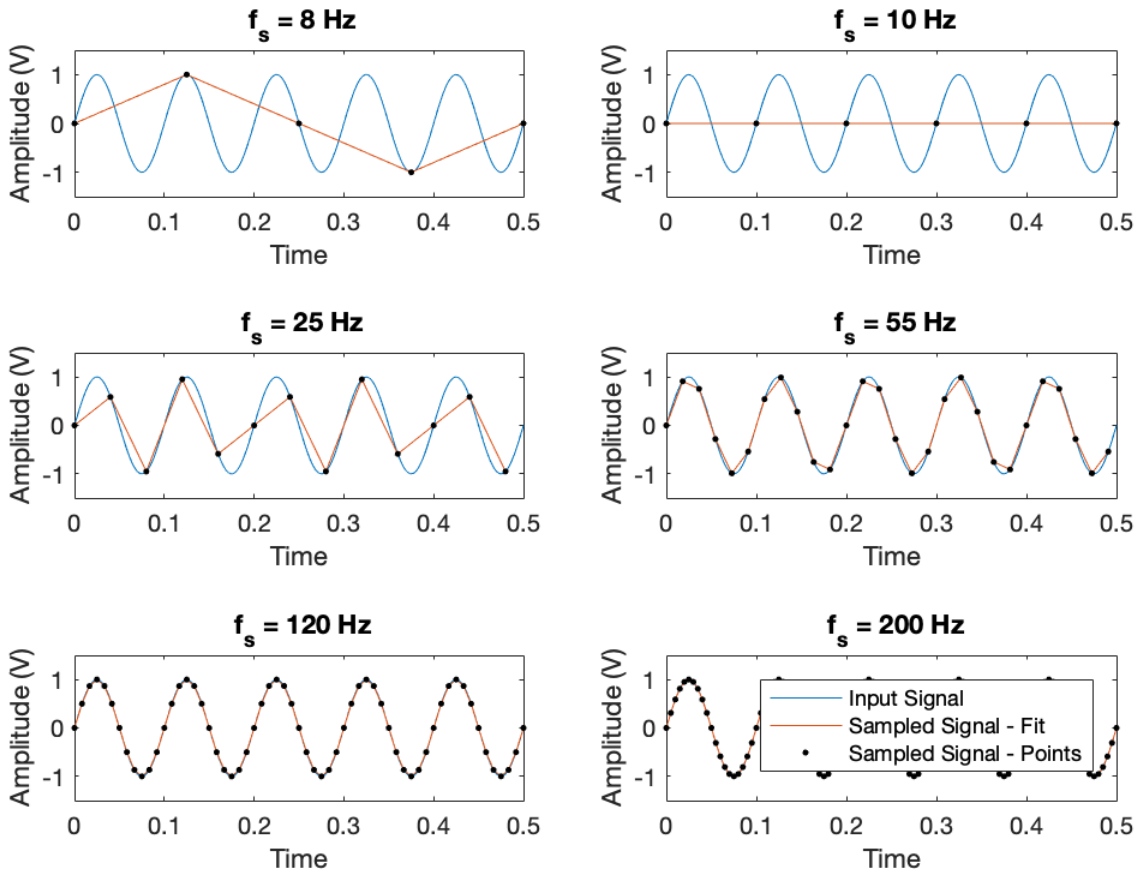



**Figure 3 Shows that an increase in sampling frequency creates a better approximation of the sine wave in the time domain.**

b) aliasing_InputSignals

This MATLAB code keeps the sampling frequency constant, but varies the frequency of the sample. In the time domain, because the bandwidth is 80Hz, when sampling at 80Hz no signal is detected, because it always will sample the same value from the sine wave. At 40Hz and 120Hz, it will detect the peaks, troughs, and zero points of the signal, such that at triangular looking function is produced, despite the true function being sinusoidal. However, in the frequency domain, the correct frequency is predicted in every case except for the 80Hz case using a Fast Fourier Transform. At frequencies close to 80Hz it appears less accurate, but still predicts the proper frequency. Above 80Hz, aliasing occurs, causing the distortion. Because of this, it may be worth it to check the frequency found from FFT and then refine your sampling frequency based off the results to get a clearer picture of the time domain signal.

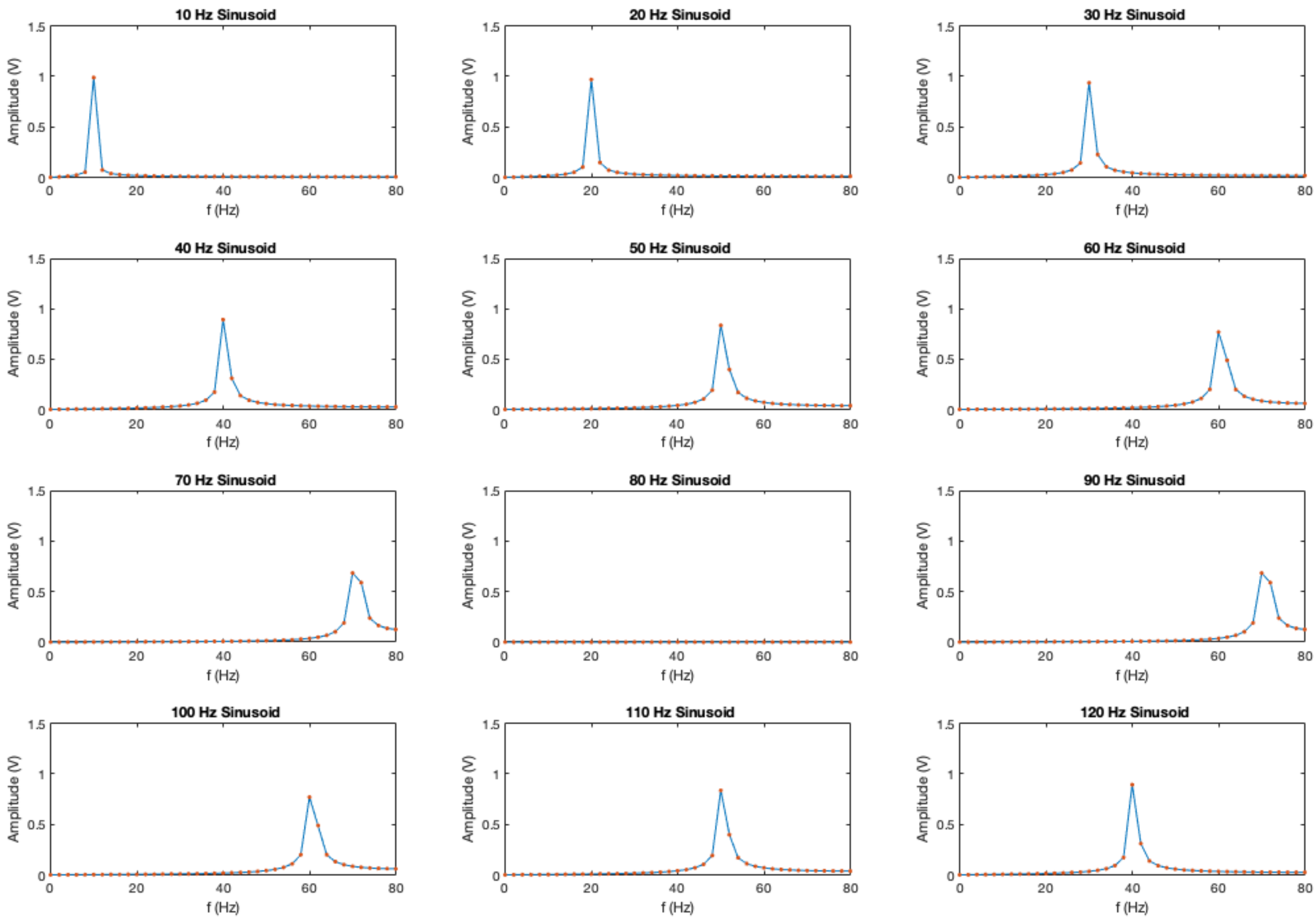



**Figure 4 Shows that when the sampling frequency is constant peaks and troughs are detected for 40Hz and 120 Hz and at 80Hz nothing is detected in the frequency domain.**

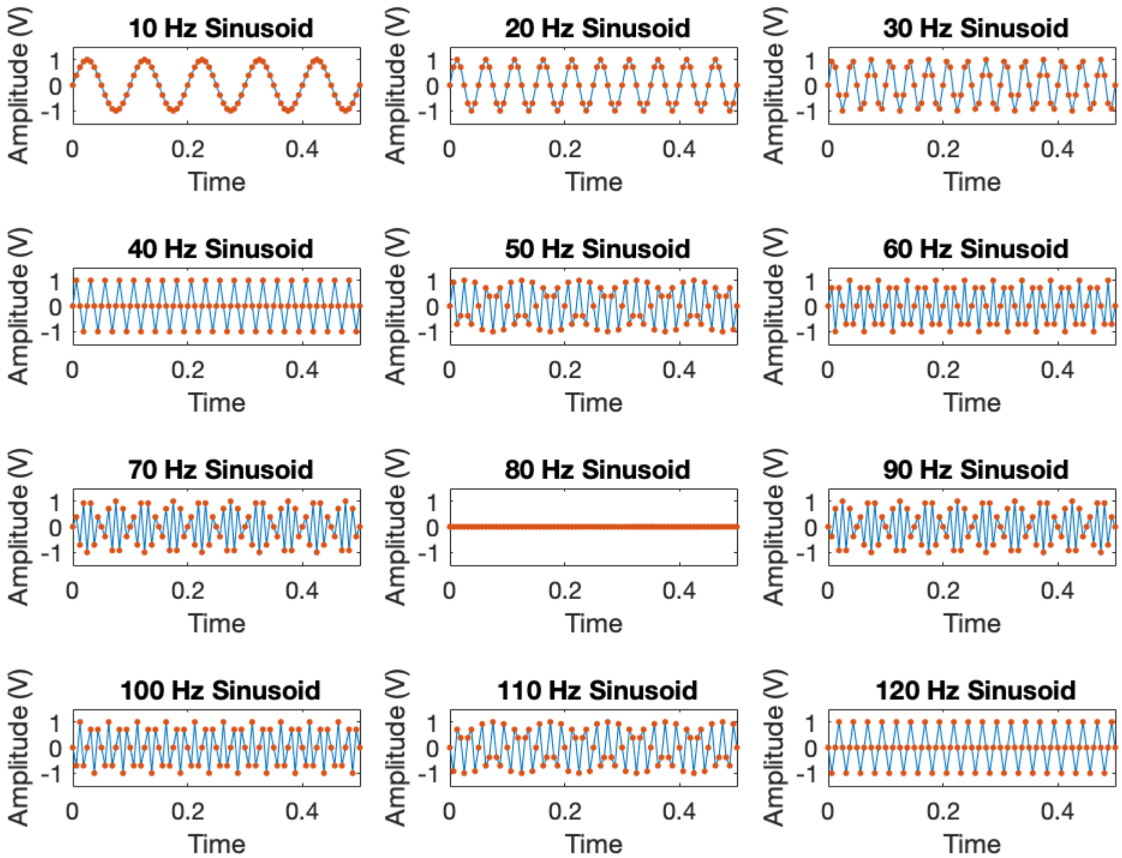



**Figure 5** **Shows that near or at 80Hz the Fourier Transform produces inaccurate and no data respectively. Aliasing occurs above 80 Hz.**

## 3 Question 3

(*2 pts*) Briefly define the following terms below. Include proper ASME citations for each term in a reference list.

- **Anti-aliasing filter:** A filter which analyzes input frequencies, and selectively excludes outlier frequencies above the Nyquist frequency. This maintains the integrity of measurements by excluding frequencies for which the sampling rate is less than twice the input frequency. [1].
- **Nyquist Frequency:** The Nyquist Frequency is half of the sampling rate. This frequency is the maximum input frequency for which a sampling rate can accurately capture a signal. [2].
- **Root Mean Square Voltage:** The Root Mean Square Voltage is the representation of the average

magnitude of a sinusoidal signal. It represents the magnitude of an AC voltage as if it were DC, hence it is often referred to as the DC equivalent voltage. [3].

- **Clipping (in signal processing):** Clipping is a signal distortion process by which upper and lower bounds are implemented on an oscillating signal. Any values above or below those respective bounds are said to be the value of the limit. [4].

- **Gibbs phenomenon:** States that no matter the amount of terms that are included in the Fourier series for a signal, there will always be an error at or near a jump discontinuity in the signal in the form of an overshoot. This overshoot is always around 9% of the size of the jump. [5].

## 4 Question 4

(*4pts*) Calculate the Fourier series representations of the waveforms below. You must begin with the general Fourier representation of a wave to obtain the equation for the wave.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n sin(nw_0 t) \tag{1}$$

You can assume the wave switches to positive at time t = 0.

a) 35Hz square wave with a 12.45V amplitude.

a)

$$f(t) = \frac{a_0}{2} + \sum a_n \cos(n\omega_0 t) + \sum b_n \sin(n\omega_0 t)$$

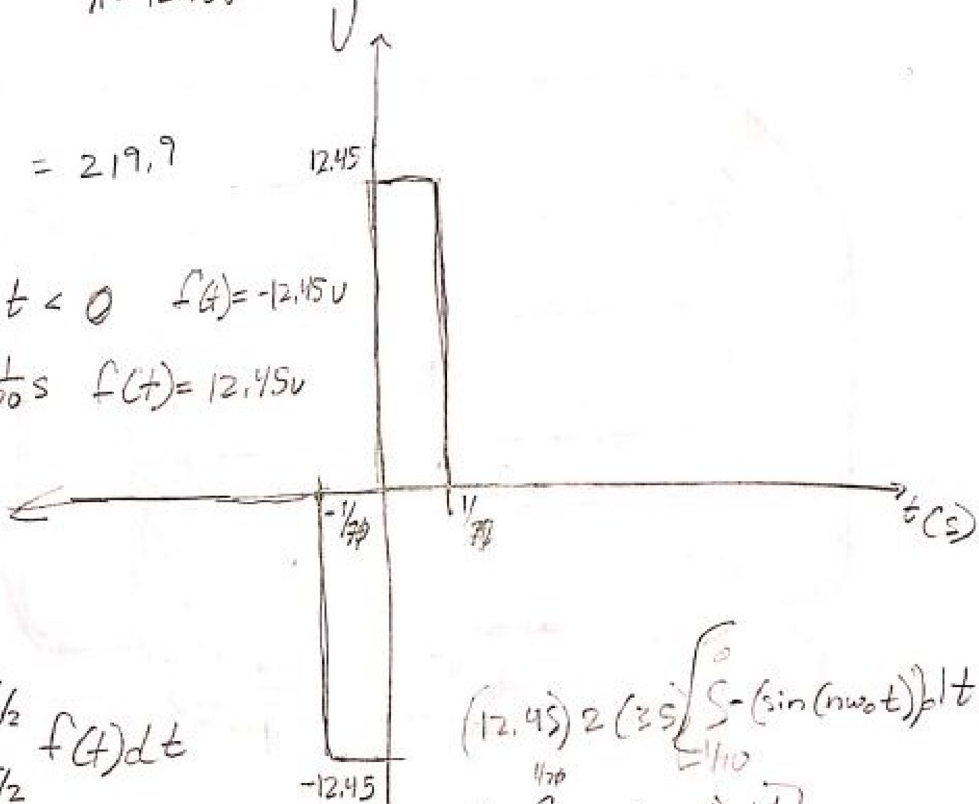
$f = 35\,Hz$ $\quad A = 12.45V$

$T = \frac{1}{35}$

$\omega_0 = \frac{2\pi}{T} = 219.9$

for $-1/70\,s < t < 0$ $\quad f(t) = -12.45\,V$

for $0 < t < \frac{1}{70}\,s$ $\quad f(t) = 12.45\,V$




$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega_0 t)\,dt$$

$$(12.45)\,2(35)\left[\int_{-1/70}^{0} -(\sin(n\omega_0 t))\,dt + \int_{0}^{1/70} \sin(n\omega_0 t)\,dt\right]$$

$$= 871.5\left[\frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_{-1/70}^{0} - \frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_{-1/70}^{0}\right]$$

$f(t)$ is odd function so $a_n = 0$ for $n = 1$ to $\infty$

and $a_0 = 0$

$$b_1 = 2(35)\left[\int_{-1/70}^{0} -12.45\sin(1(219.9)t)\,dt + \int_{0}^{1/70} 12.45\sin(1\,219.9t)\,dt\right]$$

$$= 70\left[\frac{+12.45\cos(219.9t)}{219.9}\Big|_{-1/70}^{0} - \frac{12.45\cos(219.9t)}{219.9}\Big|_{0}^{1/70}\right]$$

a) cont

$$b_1 = \frac{871.5}{219.9}\left[(1+1) - (\cancel{-1} \overset{0}{} - 1)\right]$$

$$= \frac{871.5}{219.9}\left[2+2\right]$$

$=$

$$\boxed{b_1 = 19.85}$$

14.92
10.56

$$b_2 = 2(35)\left[\int_{-1/70}^{0} -12.45\left(\sin\left(2(219.9)t\right)dt + \int_{0}^{1/70} 12.45 \sin\left(2(219.9)t\right)dt\right.\right]$$

$$= \frac{871.5}{439.8}\left[\cos(439.8t)\Big|_{-1/70}^{0} - \cos(439.2t)\Big|_{0}^{1/70}\right]$$

$$= \frac{871.5}{439.8}\left[(1 - \cancel{.1}\overset{0}{}) - (.\cancel{9979}\overset{0}{} - 1)\right]$$

$$\boxed{b_2 = 0}$$

$$b_3 = 871.5\left[\int_{-1/70}^{0} -\sin\left(3(219.9t)\right)dt + \int_{0}^{1/70} \sin\left(3(219.9)t\right)dt\right]$$

$$= \frac{871.5}{659.7}\left[\cos(659.7t)\Big|_{-1/70}^{0} - \cos(659.7t)\Big|_{0}^{1/70}\right]$$

$$= \frac{871.5}{659.7}\left[(1+1) - (-1-1)\right]$$

$$\boxed{b_3 = 5.28}$$

$$b_4 = 871.5\left[\int_{-1/70}^{0} -\sin\left(4(219.9t)\right)dt + \int_{0}^{1/70} \sin\left(4(219.9)t\right)dt\right]$$

$$= \frac{871.5}{879.6}\left[\cos(879.6t)\Big|_{-1/70}^{0} - \cos(879.6t)\Big|_{0}^{1/70}\right]$$

$$= \frac{871.5}{879.6}\left[(1 - \cancel{.976}\overset{0}{}) - (.\cancel{976}\overset{0}{} - 1)\right]$$

$$\boxed{b_4 = 0}$$

$$b_5 = 871.5\left[\int_{-1/170}^{0}\left(-\sin(5(219.9)t)\right)dt + \int_{0}^{1/170}\sin(5(219.9)t)\,dt\right]$$

$$= \frac{871.5}{1099.5}\left[\cos(1099.5t)\Big|_{-1/170}^{0} - \cos(1099.5t)\Big|_{0}^{1/170}\right]$$

$$= .793\left[(1 - -1) - (-1 - 1)\right]$$

$$= .793[4]$$

$$\boxed{b_5 = 3.17}$$

$$b_6 = 871.5\left[\int_{-1/170}^{0}\left(-\sin(6(219.9)t)\right)dt + \int_{0}^{1/170}\sin[6(219.9)t)\,dt\right]$$

$$= \frac{871.5}{1319.4}\left[\cos(6(219.9)t)\Big|_{-1/170}^{0} - \cos[6(219.9t)]\Big|_{0}^{1/170}\right]$$

$$= .6605\left[(1 - 1) - (1 - 1)\right]$$

$$\boxed{b_6 = 0}$$

$$b_7 = 871.5\left[\int_{-1/170}^{0}\left(-\sin(7(219.9)t)\right)dt + \int_{0}^{1/170}\sin(7(219.9)t)\,dt\right]$$

$$= .56617\left[\cos(7(219.9)t)\Big|_{-1/170}^{0} - \cos(7(219.9)t)\Big|_{0}^{1/170}\right]$$

$$= .56617\left[(1 - -1) - (-1 - 1)\right]$$

$$= .56617[4]$$

$$\boxed{b_7 = 2.26}$$

$$\boxed{f(t) = 15.85\sin(219.9t) + 5.28\sin(439.8t) + 3.17\sin(1099.5t) + 2.26\sin(1539.3t)}$$

**Figure 6** The images above show the work done to arrive at the four term Fourier Series representation of the given signal. Due to symmetry, $a_0$ and $a_n$ are equal to zero. Also, for $b_n$, even terms are also equal to zero.

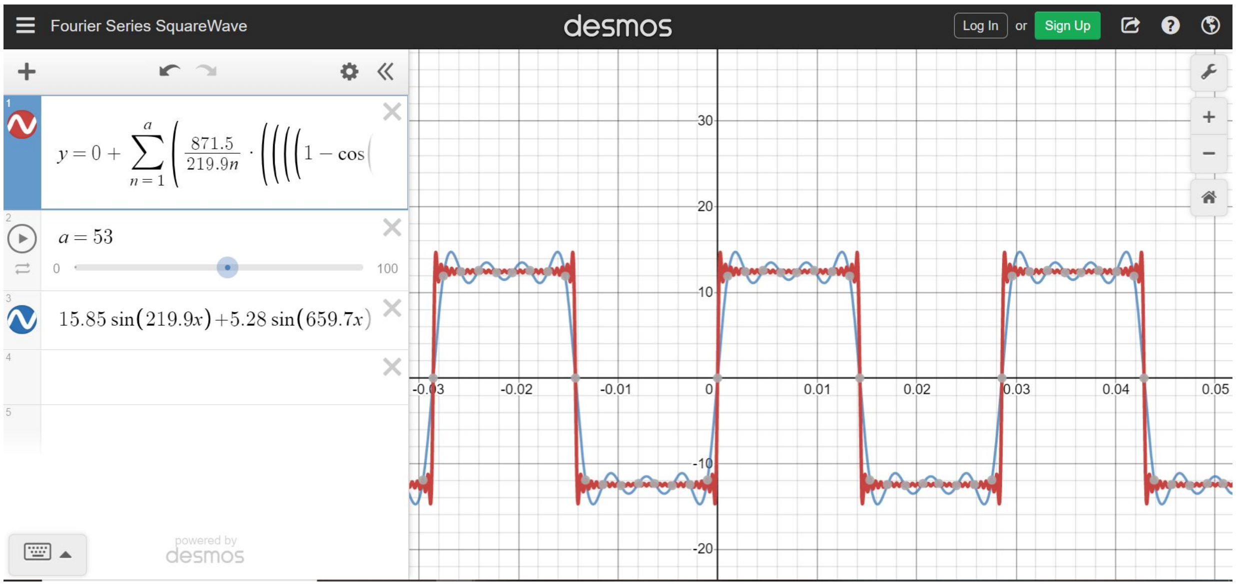
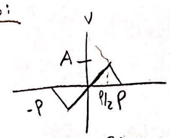



**Figure 7 The image above shows a graph of the first four terms of the series as calculated above in blue on top of a graph of the first 100 terms in red.**

b) 20Hz triangle wave with a 55.55V amplitude.

Vibrations Lab Prelab 1, 4b:




$P = \frac{T}{2}$, $T = \frac{1}{f}$, $f = 20\text{Hz}$

$A = 55.55\text{V}$ $\omega_0 = \frac{2\pi}{T} = 40\pi$

$a_0 = \frac{1}{P}\int_{-P}^{P} f(x)\,dx$

$a_n = \frac{1}{P}\int_{-P}^{P} f(x)\cos\left(\frac{n\pi}{P}x\right)dx \quad n=1,2,3,\ldots$

$b_n = \frac{1}{P}\int_{-P}^{P} f(x)\sin\left(\frac{n\pi}{P}x\right)dx \quad n=1,2,3,\ldots$

$$f(t) = \begin{cases} \frac{2A}{P}t & 0 < t < P/2 \\ 2A - \frac{2A}{P}t & P/2 < t < P \end{cases}$$

$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{P}\int_0^P f(t)\sin\left(\frac{n\pi}{P}t\right)dt = \frac{2}{P}\left[\int_0^{P/2}\left(\frac{2A}{P}\right)t\sin\left(\frac{n\pi}{P}t\right)dt + \int_{P/2}^{P}\left(2A - \frac{2A}{P}t\right)\sin\left(\frac{n\pi}{P}t\right)dt\right]$$

$$= \frac{2}{P}\left[\frac{2A}{P}\frac{\sin\left(\frac{n\pi}{P}t\right) - \frac{n\pi}{P}t\cos\left(\frac{n\pi}{P}t\right)}{\left(\frac{n\pi}{P}\right)^2}\Bigg|_0^{P/2} + \frac{2A\cos\left(\frac{n\pi}{P}t\right)}{\frac{n\pi}{P}}\Bigg|_{P/2}^{P} - \frac{2A}{P}\frac{\sin\left(\frac{n\pi}{P}t\right) - \frac{n\pi}{P}t\cos\left(\frac{n\pi}{P}t\right)}{\left(\frac{n\pi}{P}\right)^2}\Bigg|_{P/2}^{P}\right]$$

$$= \frac{2}{P}\left[\frac{2A}{P}\frac{\sin\left(\frac{n\pi}{P}\frac{P}{2}\right) - \frac{n\pi P}{P\,2}\cos\left(\frac{n\pi}{P}\frac{P}{2}\right)}{\left(\frac{n\pi}{P}\right)^2} - 0 - \frac{2A\left(\cos\left(\frac{n\pi}{P}P\right) - \cos\left(\frac{n\pi}{P}\frac{P}{2}\right)\right)}{\frac{n\pi}{P}}\right.$$

$$\left. - \frac{2A}{P}\frac{\sin\left(\frac{n\pi}{P}P\right) - \frac{n\pi}{P}P\cos\left(\frac{n\pi}{P}P\right)}{\left(\frac{n\pi}{P}\right)^2} + \frac{2A}{P}\frac{\sin\left(\frac{n\pi}{P}\frac{P}{2}\right) - \frac{n\pi}{P}\frac{P}{2}\cos\left(\frac{n\pi}{P}\frac{P}{2}\right)}{\left(\frac{n\pi}{P}\right)^2}\right]$$

$$= \frac{2}{P}\cdot\frac{2A}{P}\cdot\frac{1}{\frac{n^2\pi^2}{P^2}}\left[\sin\left(\frac{n\pi}{2}\right) - \frac{n\pi}{2}\cos\left(\frac{n\pi}{2}\right) - n\pi\left(\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right) - \sin(n\pi) + n\pi\cos(n\pi)\right.$$

$$\left. + \sin\left(\frac{n\pi}{2}\right) - \frac{n\pi}{2}\cos\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{4A}{n^2\pi^2}\left[2\sin\left(\frac{n\pi}{2}\right) \cancel{- 2n\pi\cos\left(\frac{n\pi}{2}\right)} \cancel{- n\pi\cos(n\pi)} + \cos\left(\frac{n\pi}{2}\right) - \sin(n\pi) + \cancel{n\pi\cos(n\pi)}\right]$$

$$= \frac{4A}{n^2\pi^2}\left[2\sin\left(\frac{n\pi}{2}\right) - \sin(n\pi)\right]$$

$$b_n = \frac{4A}{n^2\pi^2}\left(2\sin\left(\frac{n\pi}{2}\right) - \sin(n\pi)\right) = \frac{4A}{n^2\pi^2}\left(2\sin\left(\frac{n\pi}{2}\right)\right) = \frac{8A}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$b_1 = \frac{8(55.55)}{1^2\pi^2}\left(\sin\left(\frac{\pi}{2}\right)\right) = 45.027\text{V}$

$b_2 = \frac{8(55.55)}{2^2\pi^2}\left(\sin(\pi)\right) = 0\text{V}$

$b_3 = \frac{8(55.55)}{3^2\pi^2}\left(\sin\left(\frac{3\pi}{2}\right)\right) = -5.003\text{V}$

$b_4 = \frac{8(55.55)}{4^2\pi^2}\left(\sin(2\pi)\right) = 0\text{V}$

$$b_n = \begin{cases} \frac{4A}{n^2\pi^2} & n=1,5,9\ldots \\ 0 & n=2,6,10\ldots \\ -\frac{4A}{n^2\pi^2} & n=3,7,11\ldots \\ 0 & n=4,8,12\ldots \end{cases}$$

so $b_5 = 1.801\text{V}$

& $b_7 = -0.9191\text{V}$

$f(t) \approx 45.027\sin(40\pi t) - 5.003\sin(120\pi t) + 1.801\sin(200\pi t) - 0.919\sin(280\pi t)$

$\boxed{f(t) \approx 45.027\sin(125.663t) - 5.003\sin(376.991t) + 1.801\sin(628.32t) - 0.919\sin(879.65t)}$

where $f(t)$ is in volts.

**Figure 8 The figure shows the work done to arrive at the four term Fourier Series representation of the given signal. Due to symmetry, $a_0$ and $a_n$ are equal to zero**

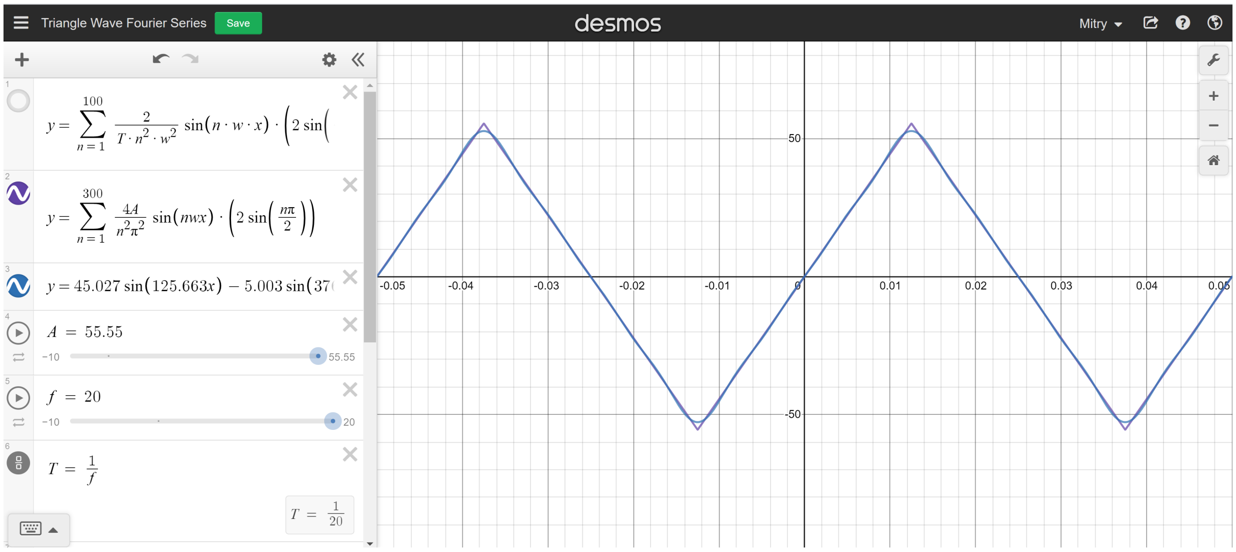



**Figure 9 The image aboves show a graph of the first four terms of the series as calculated above in blue on top of a graph of the first 300 terms in purple.**

## References


[1] "Anti-Aliasing Filters and Their Usage Explained," National Instruments, Austin TX, updated March 14, 2019, accessed September 17, 2020, https://www.ni.com/en-us/innovations/white-papers/18/anti-aliasing-filters-and-their-usage-explained.html

[2] "Nyquist Frequency," Gatan, Pleasanton CA, accessed September 17, 2020, https://www.gatan.com/nyquist-frequency

[3] "RMS Voltage Tutorial," Electronics Tutorials, accessed September 17, 2020, https://www.electronics-tutorials.ws/accircuits/rms-voltage.html

[4] Esqueda, F., Bilbao, S., and Välimäki, V., 2016, "Aliasing Reduction in Clipped Signals," IEEE Transactions on Signal Processing, **64**(20), pp. 5255–5267.

[5] "Gibbs' Phenomenon," MIT OpenCourseWare, accessed Fall 2011, https://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/unit-iii-fourier-series-and-laplace-transform/operations-on-fourier-series/MIT18_03SCF11_s22_7text.pdf