Washington University in St. Louis

James McKelvey School of Engineering

Fall 2020 MEMS 4050 Vibrations Laboratory

Pre-Lab 3: Response of a Single Degree of Freedom System

Lab Instructor: Dr. Bayly

Group T (Friday 2 PM)

We hereby certify that the lab report herein is our original academic work, completed in accordance to the McKelvey School of Engineering and Undergraduate Student academic integrity policies, and submitted to fulfill the requirements of this assignment:

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1 Question 1

(*3pts*) The time series of the free response of a system that has been viscously damped is given below. Recall the log-dec method where $\zeta_i = \frac{1}{2\pi} ln(\frac{x_i}{x_{i+1}})$ is the i^{th} estimate of the damping ratio at the local maximums.

Using the logarithmic decrement method with at least four peaks, calculate the average damped frequency, ω_d and natural frequency, ω_n.

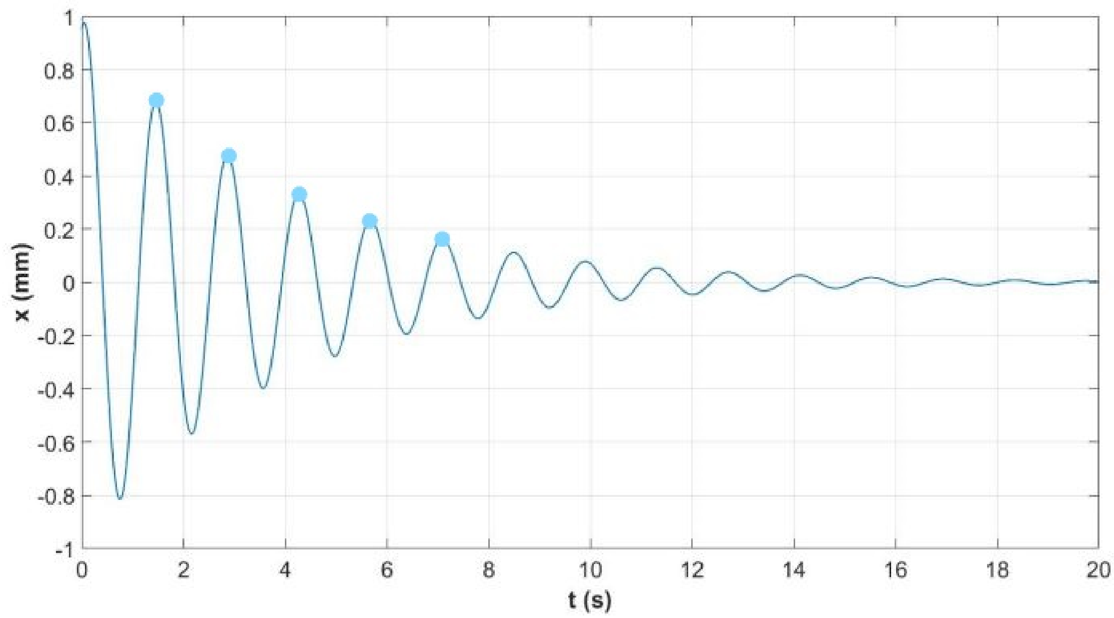


Figure 1 Time series of a viscously damped waveform free response system [1].

The period was estimated to be 1.5 seconds, and the peaks were estimated to be x_1 = 0.99, x_2 = 0.69, x_3 = 0.48, x_4 = 0.34, and x_5 = 0.23. These peaks are shown with blue dots on the graph above. Using the given formula for zeta, $\zeta_i = \frac{1}{2\pi} ln(\frac{x_i}{x_{i+1}})$, four approximate values for zeta were found. These values were then averaged to find an average damping constant for the system. Then, the formula $\omega_n = \frac{2\pi}{\sqrt{1-\zeta^2}}$ was used to calculate the natural frequency, using the four estimates for zeta. These four approximate natural frequencies were averaged to find an average natural frequency. Finally, the formula $\omega_d = \omega_n\sqrt{1-\zeta^2} = \frac{2\pi}{T}$ was used to calculated four approximate values for the damped natural frequency, and these values were again averaged to find an average damped natural frequency. See Table 1 below for a summary of the results.

Table 1 Summary of calculated values for ζ, ω_n, and ω_d at four peaks.

Iteration	ζ	ω_n [rad/s]	ω_d [rad/s]
1	0.0575	4.1957	4.1888
2	0.0578	4.1958	4.1888
3	0.0549	4.1951	4.1888
4	0.0622	4.1969	4.1888
Average	0.0581	4.1959	4.1888

2 Question 2

(*3pts*) Consider a simple 1DOF system with an effective spring stiffness of 1550 N/m. A time series of the free response of system is shown below.

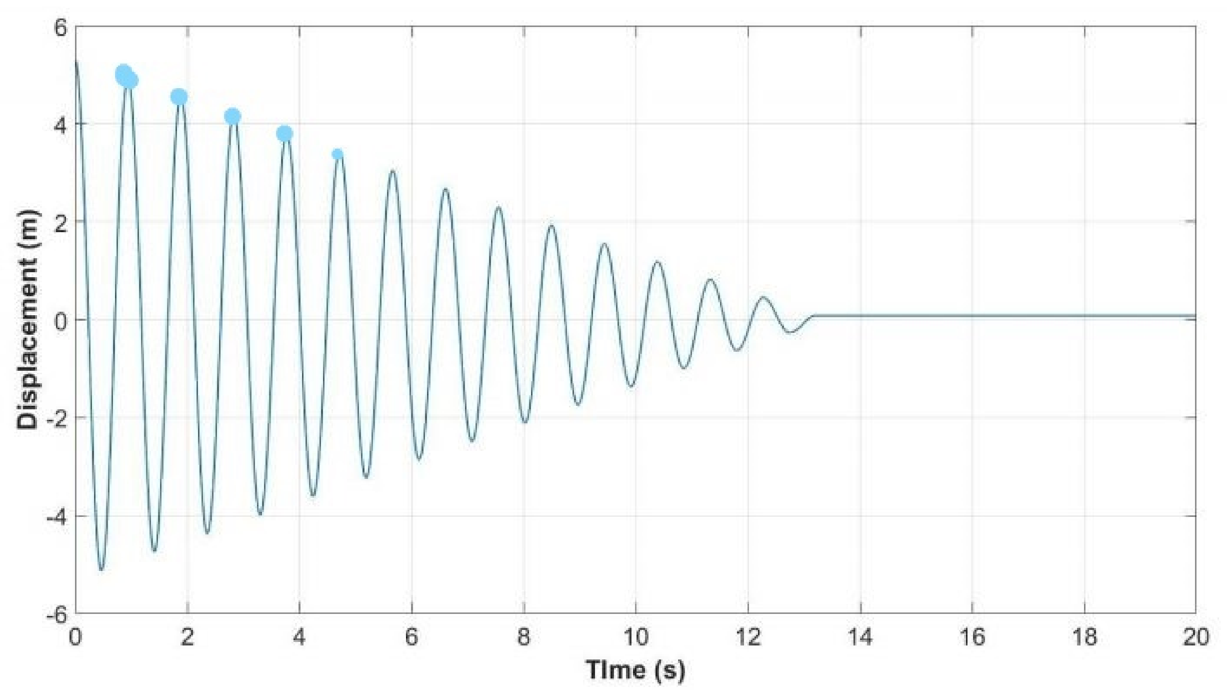


Figure 2 The dots in the graph represent the peak values that we are estimating.From left to right we have (X1,t1), (X2,t2), (X3,t3), (X4,t4), (X5,5).

a) What observation(s) can be made from the signal decay to indicate that Coulomb damping is dominant in the system?

Coulomb damping is dominant in the system because the damping of the wave decreases linearly. You can clearly see this if you draw a line from the first peak (around (0,5)) to the last peak (around (12.5,0.5)), that the line will go through all the consecutive peaks in between.

b) Estimate the Coulomb damping force of the system.

From Fig. 2 the estimated peak values are x = [4.9, 4.5, 4.1, 3.7, 3.3] (m) at a time of t = [1, 1.9, 2.8, 3.7, 4.7] (s). This makes the period between consecutive peaks to be T = [0.9, 0.9, 0.9, 1] (s) using $T = t_2 - t_1$. Equation 1 below can then be rearranged to get equation 2 below:

$$slope = \frac{\Delta X}{T} = \frac{4\mu N}{k_{eq}} \tag{1}$$

$$f_c = \mu N = \frac{\Delta X * k_{eq}}{4T} \tag{2}$$

Where T is the period, ΔX is the difference in peak values for the given period, μN is the damping force, and k_{eq} is the given effective spring stiffness. For Equation 2, f_c is the friction force, which is equivalent to the Coulomb damping force μN. Now we can use equation 2 to get an estimated value for the damping force. Below is a sample calculation for the Coulomb damping force:

$$f_c = \frac{(4.5 - 4.9) * 1550}{4 * (1.9 - 1)} \tag{3}$$

From this equation, the estimated Coulomb damping force is calculated to be -167.6 N.

3 Question

(*4pts*) A cart (mass m = 150g) rolls on a track and is connected by a set of springs (stiffness k = 2750 N/m) to a rigid stop on one end of the track and a rotating motor on the other end. The motor causes an off-center point of attachment to rotate about the fixed motor axis. Energy dissipation is provided by rolling friction between the cart and the track, as well as eddy-current damping provided by a set of magnets on the front of the cart.

- Consider only horizontal motion and forces and neglect the mass of the springs and rotational inertia of the wheels.

- The motor is assumed to rotate about a fixed axis at a specified angular velocity, $\Omega = 10$ rad/sec, which is unaffected by the motion of the cart.
- The spring attachment point is a distance, $\delta = 20$ mm, from the axis of the motor
- The damping force is proportional to velocity: $f_c = c\dot{x}$ (c= 6.55 N-s/m)

a) What is the imposed horizontal displacement of the spring attached to the motor, as a function of motor angular velocity, Ω (rad/sec), and time, t?

The displacement equation for this system is characterized by the angular velocity of the motor. The relationship is characterized by equation 4 below.

$$x(t) = \delta cos(\Omega t) \tag{4}$$

Upon substituting values the result becomes equation 5 below.

$$x(t) = .02cos(\Omega t) \tag{5}$$

b) Draw the free body-diagrams of all five different components

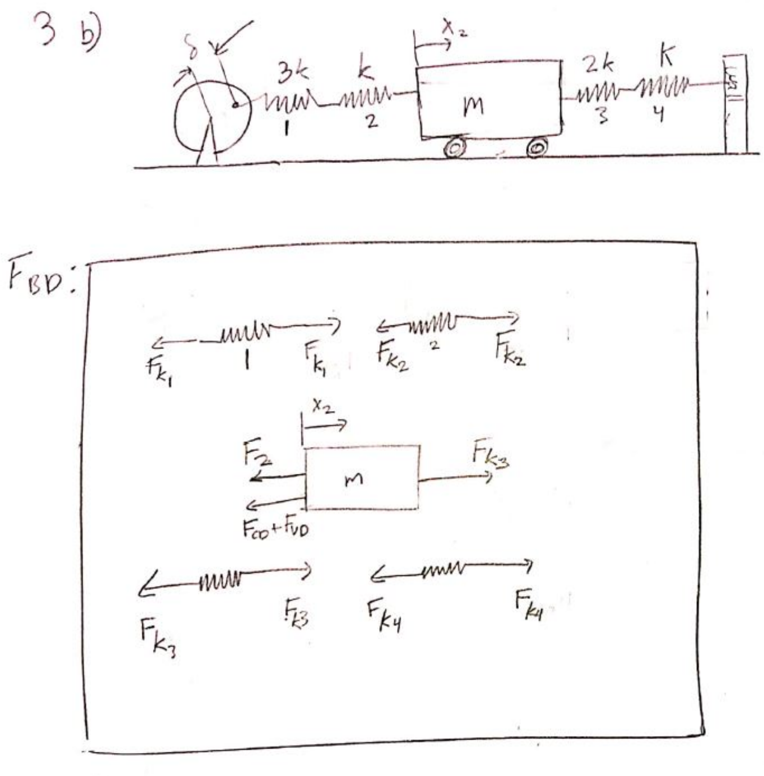


Figure 3 Free body diagrams for all 5 components in the cart and spring system.

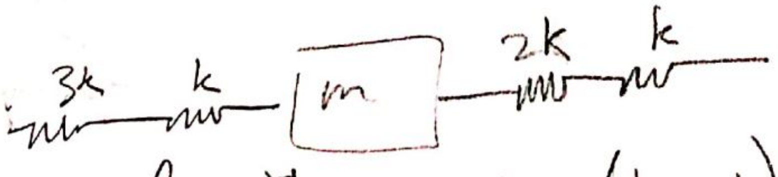
c) What is the equation of motion of the system?

- Generate an equation in terms of variables x2, m, k, c, δ and Ω.
- With known values plugged in, generate a well labelled time series plot of the base system's free response.

The general equation of motion for the system is given by Eq. 6.

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_0 Cos(\Omega t)}{m} \tag{6}$$

Assuming a solution of $X_0 e^{-i\Omega t}$ the real part of the solution comes out to be:



$$k_{eq_1} = \left(\frac{1}{3k} + \frac{1}{k}\right)^{-1} \qquad k_{eq_2} = \left(\frac{1}{2k} + \frac{1}{k}\right)^{-1} \qquad k_{eq\,TOT} = \frac{3k}{4} + \frac{2k}{3} = \frac{9k}{12} + \frac{8k}{12}$$

$$k_{eq_1} = \left(\frac{1}{3k} + \frac{3}{3k}\right)^{-1} = \frac{3k}{4} \qquad k_{eq_2} = \frac{2k}{3} \qquad k_{eq\,TOT} = \frac{17k}{12}$$

$$\ddot{X} + C\dot{X} + k_{eq_T} X = F_F$$

$$\ddot{x}_2 + \frac{C\dot{x}_2}{m} + \frac{17k}{12m} x_2 = \frac{17k}{m\,12} \delta \cos(\Omega t)$$

$$-\Omega X \cancel{e^{i\Omega t}} + \frac{c}{m} i\Omega X \cancel{e^{i\Omega t}} + \frac{17k X e^{i\Omega t}}{12m} = \frac{17k}{12m} \cancel{e^{i\Omega t}}$$

$$X\left(-\Omega + \frac{17k}{12m} + \frac{c\,i}{m}\right) = \frac{17k}{12m}$$

$$X = \frac{\frac{17k}{12m}}{-\Omega + \frac{17}{12m}k + \frac{c}{m}i} = \frac{\frac{17(2750)}{12(.15)}}{\left(-10 + \frac{(17)(2750)}{12(.15)}\right) + \frac{6.55}{.15}i}$$

$$|X| = \frac{\sqrt{25972.22^2}}{\sqrt{25962.22^2 + 43.66^2}} = 1$$

$$\angle X = \tan^{-1}\left(\frac{0}{25972.22}\right) - \tan^{-1}\left(\frac{43.66}{25962.22}\right)$$

$$= 0 - 1.569$$

$$x(t) = 1\,\text{m} \cos\left(10\,\text{rad/s}\; t - 0.00168\right)$$

Figure 4 Work to determine $x_2(t)$

$$x_2(t) = X_0 * Cos(\Omega t - 0.00168) \quad (7)$$

where the absolute value of X_0 is taken to get the real part of the coefficient to be plotted and is defined below.

$$|X_0| = (F_0/m)/(-\Omega^2 + \frac{c}{m}\Omega + \frac{k_eq}{m}) \quad (8)$$

Specific variables from Eq. 8 are defined by the following equations.

$$F_0 = F_{eq1}\delta = \frac{3}{4}k\delta \quad (9)$$

$$k_{eq} = k_{eq1} + k_{eq2} = \frac{3k * k}{3k + k} + \frac{2k * k}{2k + k} = \frac{17}{12}k \quad (10)$$

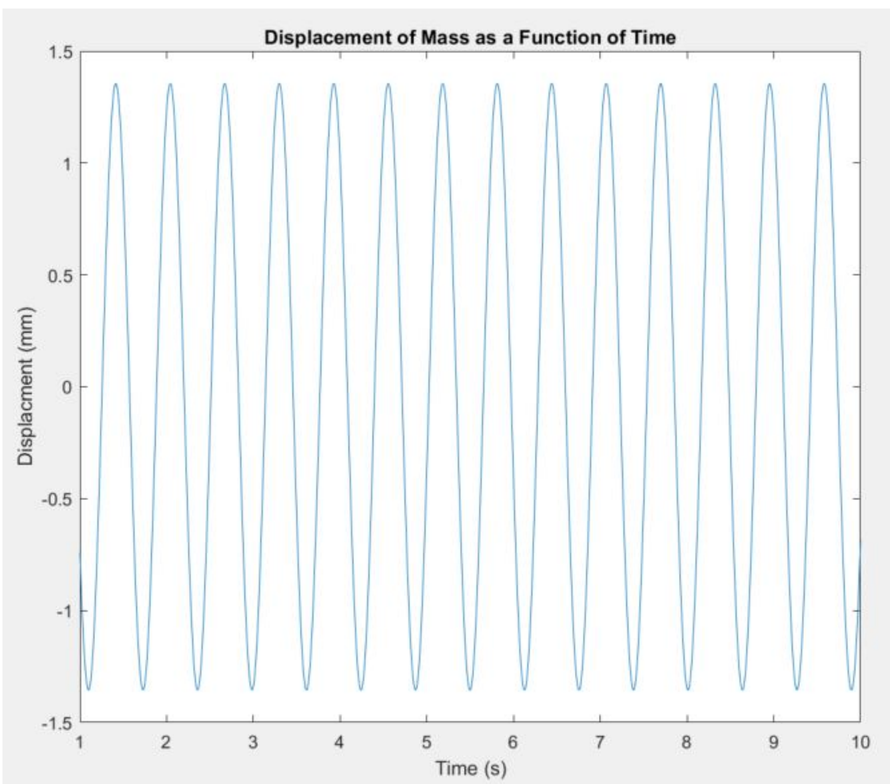


Figure 5 Displacement vs Time Plot for the Free Response of This System

Figure 5, was genereated utilizing the following code.

```
%%
clc;
clear;
clf;

%% Knowns
Δ = 20; %m
c = 6.55;     %Ns/m
omega = 10 ; % rad/s
k = 1550;     %N/m
m = 150;      %g
n = 10;

%% Calculating Variables
k_eq1 = (3/4)*k;
k_eq2 = (2/3)*k;
k_eq = k_eq1 + k_eq2;
f_eq = k_eq1*Δ;
F_0 = f_eq/m;
x_0 = F_0./(sqrt(( ((k_eq/m)^2 - omega.^2).^2 + ((c/m).*omega).^2)));

%% Solving for x2
t = 1:0.01:n;
for i = 1:length(t)
   x2(i) = x_0.*cos(omega.*t(i) - 1.569);
end


plot(t,x2)
title("Displacement of Mass as a Function of Time");
xlabel("Time (s)");
ylabel("Displacment (mm)");
```

References

[1] 2020, "PRE-LAB 3: RESPONSE OF A SINGLE DEGREE-OF-FREEDOM SYSTEM," FL2020.E37.MEMS.4050.01 - Vibrations Lab.