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```
% GM with a fixed step
%
% Least squares: gradient method with fixed step
%
% U. S. Kamilov, CIG, WUSTL, 2021.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

prepare workspace

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear; close all; home;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

load the variables of the optimization problem

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load('dataset.mat');

[m, n] = size(A); % m rows, n cols

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

set up the function and its gradient (* edit this *)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

evaluateFunc = @(x) (1/2)*norm(A*x-b)^2;
evaluateGrad = @(x) A'*A*x - A'*b;
proj_4 = @(x) piecewise(x < 0,0 ,x > 0,x);
```

```
proj_5 = @(x) piecewise(x < 0,0 ,x > 0,x); % Cond 1, val 1, cond 2,
 val 2, else val
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

parameters of the gradient method

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%

xInit = zeros(n, 1); % zero initialization
stepSize = 1/(norm(A,2).^2); % step-size of the gradient method (***
 edit this ***)
tol = 1e-4; % stopping tolerance
maxIter = 200; % maximum number of iterations
tau = 5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

optimize

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%

% initialize
x = xInit;

% keep track of cost function values
objVals = zeros(maxIter, 1);
infErrs = zeros(maxIter, 1);

% iterate
for iter = 1:maxIter

    % gradient at w
    grad = evaluateGrad(x);
    % update using GM(*** edit this ***)
    xNext = x - stepSize*grad;

    %%%%%%%%%%%%%%%% update using PGM %%%%%%%%%%%%%%%%%%%
    %xNext = proj_4(x - stepSize*grad);

    %%%%%%%%%%%%%%%% update using CGM %%%%%%%%%%%%%%%%%%%%%%%%%%
    %[M, I] = max(abs(grad));
    %e = grad(I)/norm(grad(I));

    %s = -tau*sign(grad(I));
    %stepSize = proj_5(((s-x)'*A'*(b-A*x))/norm(A*(s-x)).^2);
    %xNext = (1-stepSize)*x + stepSize*s;

    % evaluate the objective
    funcNext = evaluateFunc(xNext);
```

```
    % store the objective and the classification error
    objVals(iter) = funcNext;
    infErrs(iter) = norm(x(:)-xtrue(:))/norm(xtrue(:));

    fprintf('[%d/%d] [step: %.1e] [objective: %.1e]\n',...
        iter, maxIter, stepSize, objVals(iter));

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
    % begin visualize data
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

    % plot the evolution
    figure(1);
    set(gcf, 'Color', 'w');
    subplot(2, 2, 1:2);
    stem(1:n, xtrue);
    hold on;
    stem(1:n, x, 'r*');
    hold off;
    xlim([1, n])
    subplot(2, 2, 3);
    semilogy(1:iter, objVals(1:iter), 'b-',...
        iter, objVals(iter), 'b*', 'LineWidth', 2);
    grid on;
    axis tight;
    xlabel('iteration');
    ylabel('objective');
    title(sprintf('cost: %.4e', objVals(iter)));
    xlim([1 maxIter]);
    set(gca, 'FontSize', 16);
    subplot(2, 2, 4);
    semilogy(1:iter, infErrs(1:iter), 'r-',...
        iter, infErrs(iter), 'r*', 'LineWidth', 2);
    grid on;
    axis tight;
    xlabel('iteration');
    ylabel('normalized error');
    title(sprintf('err: %.2e', infErrs(iter)));
    xlim([1 maxIter]);
    set(gca, 'FontSize', 16);
    drawnow;

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
    % end visualize data
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

    % update w
    x = xNext;
end
```

```
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[2/200] [step: 6.5e-03] [objective: 6.7e+00]
[3/200] [step: 6.5e-03] [objective: 4.4e+00]
[4/200] [step: 6.5e-03] [objective: 3.1e+00]
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[54/200] [step: 6.5e-03] [objective: 1.1e-03]
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[162/200] [step: 6.5e-03] [objective: 1.7e-08]
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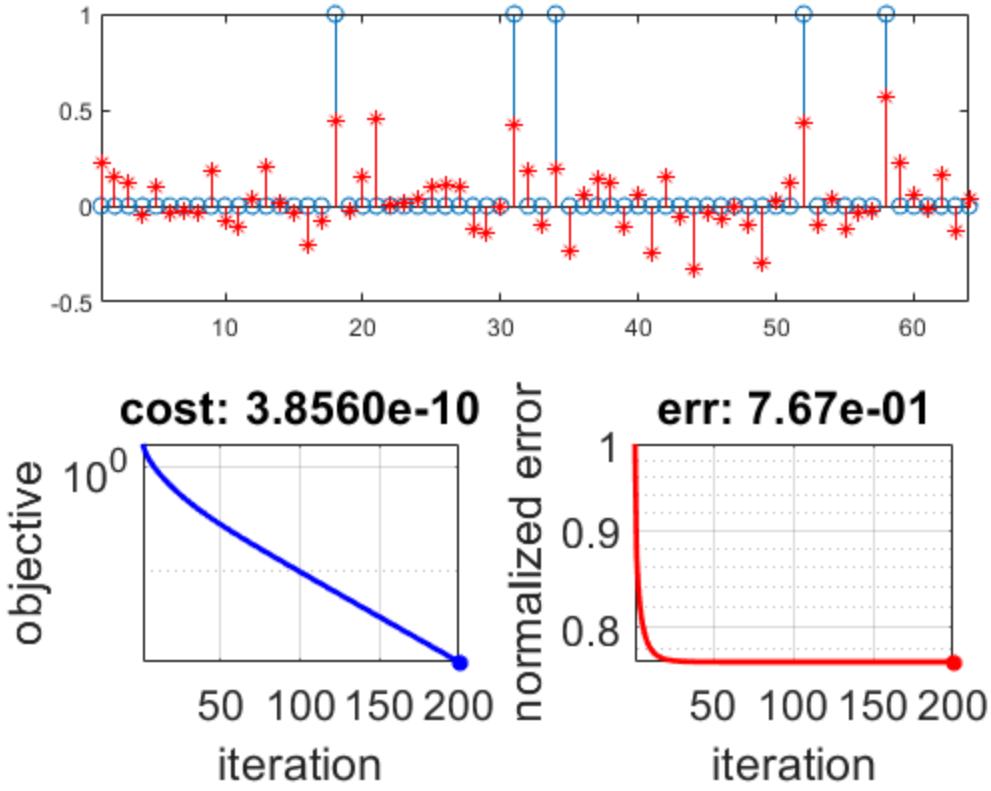

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