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```
% GM with a fixed step
%
% Least squares: gradient method with fixed step
%
% U. S. Kamilov, CIG, WUSTL, 2021.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

prepare workspace

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear; close all; home;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

load the variables of the optimization problem

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load('dataset.mat');

[m, n] = size(A); % m rows, n cols

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

set up the function and its gradient (* edit this *)

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

evaluateFunc = @(x) (1/2)*norm(A*x-b)^2;
evaluateGrad = @(x) A'*A*x - A'*b;
proj_f = @(x) (x <= 0).*0 + ((0<x)&&(x<1)).*x + (x > 1).* 1 ;
```

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```

parameters of the gradient method

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

xInit = zeros(n, 1); % zero initialization
stepSize = 1/(norm(A,2).^2); % step-size of the gradient method (***
 edit this ***)
tol = 1e-4; % stopping tolerance
maxIter = 200; % maximum number of iterations
tau = 5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

optimize

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% initialize
x = xInit;

% keep track of cost function values
objVals = zeros(maxIter, 1);
infErrs = zeros(maxIter, 1);

% iterate
for iter = 1:maxIter

    % gradient at w
    grad = evaluateGrad(x);

    % CGM
    [M, I] = max(abs(grad));
    e = eye(length(grad));

    s = -tau*sign(grad(I))*e(:,I);
    stepSize = proj_f( ((s-x)'*A'*(b-A*x))/norm(A*(s-x)).^2 );
    xNext = (1-stepSize)*x + stepSize*s;

    % evaluate the objective
    funcNext = evaluateFunc(xNext);

    % store the objective and the classification error
    objVals(iter) = funcNext;
    infErrs(iter) = norm(x(:)-xtrue(:))/norm(xtrue(:));

    fprintf('[%d/%d] [step: %.1e] [objective: %.1e]\n',...
        iter, maxIter, stepSize, objVals(iter));
```

```
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
    % begin visualize data
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

    % plot the evolution
    figure(1);
    set(gcf, 'Color', 'w');
    subplot(2, 2, 1:2);
    stem(1:n, xtrue);
    hold on;
    stem(1:n, x, 'r*');
    hold off;
    xlim([1, n])
    subplot(2, 2, 3);
    semilogy(1:iter, objVals(1:iter), 'b-',...
        iter, objVals(iter), 'b*', 'LineWidth', 2);
    grid on;
    axis tight;
    xlabel('iteration');
    ylabel('objective');
    title(sprintf('cost: %.4e', objVals(iter)));
    xlim([1 maxIter]);
    set(gca, 'FontSize', 16);
    subplot(2, 2, 4);
    semilogy(1:iter, infErrs(1:iter), 'r-',...
        iter, infErrs(iter), 'r*', 'LineWidth', 2);
    grid on;
    axis tight;
    xlabel('iteration');
    ylabel('normalized error');
    title(sprintf('err: %.2e', infErrs(iter)));
    xlim([1 maxIter]);
    set(gca, 'FontSize', 16);
    drawnow;

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
    % end visualize data
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

    % update w
    x = xNext;
end
```

```
[1/200] [step: 2.3e-01] [objective: 4.3e+01]
[2/200] [step: 1.5e-01] [objective: 3.2e+01]
[3/200] [step: 1.3e-01] [objective: 2.5e+01]
[4/200] [step: 8.7e-02] [objective: 2.2e+01]
[5/200] [step: 8.7e-02] [objective: 1.9e+01]
[6/200] [step: 7.1e-02] [objective: 1.7e+01]
```

```
[7/200] [step: 6.8e-02] [objective: 1.5e+01]
[8/200] [step: 7.4e-02] [objective: 1.3e+01]
[9/200] [step: 4.8e-02] [objective: 1.2e+01]
[10/200] [step: 4.9e-02] [objective: 1.1e+01]
[11/200] [step: 6.4e-02] [objective: 9.2e+00]
[12/200] [step: 4.1e-02] [objective: 8.5e+00]
[13/200] [step: 4.3e-02] [objective: 7.8e+00]
[14/200] [step: 3.6e-02] [objective: 7.1e+00]
[15/200] [step: 3.6e-02] [objective: 6.6e+00]
[16/200] [step: 3.6e-02] [objective: 6.1e+00]
[17/200] [step: 2.7e-02] [objective: 5.7e+00]
[18/200] [step: 2.3e-02] [objective: 5.5e+00]
[19/200] [step: 2.0e-02] [objective: 5.3e+00]
[20/200] [step: 2.7e-02] [objective: 5.1e+00]
[21/200] [step: 2.7e-02] [objective: 4.8e+00]
[22/200] [step: 2.7e-02] [objective: 4.5e+00]
[23/200] [step: 2.1e-02] [objective: 4.3e+00]
[24/200] [step: 2.1e-02] [objective: 4.1e+00]
[25/200] [step: 2.1e-02] [objective: 4.0e+00]
[26/200] [step: 2.1e-02] [objective: 3.8e+00]
[27/200] [step: 1.9e-02] [objective: 3.7e+00]
[28/200] [step: 1.8e-02] [objective: 3.5e+00]
[29/200] [step: 1.7e-02] [objective: 3.4e+00]
[30/200] [step: 1.7e-02] [objective: 3.3e+00]
[31/200] [step: 1.7e-02] [objective: 3.2e+00]
[32/200] [step: 1.6e-02] [objective: 3.1e+00]
[33/200] [step: 1.6e-02] [objective: 2.9e+00]
[34/200] [step: 1.5e-02] [objective: 2.9e+00]
[35/200] [step: 1.5e-02] [objective: 2.8e+00]
[36/200] [step: 1.4e-02] [objective: 2.7e+00]
[37/200] [step: 1.4e-02] [objective: 2.6e+00]
[38/200] [step: 1.3e-02] [objective: 2.6e+00]
[39/200] [step: 1.3e-02] [objective: 2.5e+00]
[40/200] [step: 1.3e-02] [objective: 2.4e+00]
[41/200] [step: 1.2e-02] [objective: 2.4e+00]
[42/200] [step: 1.2e-02] [objective: 2.3e+00]
[43/200] [step: 1.2e-02] [objective: 2.2e+00]
[44/200] [step: 1.1e-02] [objective: 2.2e+00]
[45/200] [step: 1.1e-02] [objective: 2.2e+00]
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[47/200] [step: 1.1e-02] [objective: 2.1e+00]
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[49/200] [step: 1.0e-02] [objective: 2.0e+00]
[50/200] [step: 7.0e-03] [objective: 1.9e+00]
[51/200] [step: 1.1e-02] [objective: 1.9e+00]
[52/200] [step: 1.0e-02] [objective: 1.9e+00]
[53/200] [step: 1.1e-02] [objective: 1.8e+00]
[54/200] [step: 8.2e-03] [objective: 1.8e+00]
[55/200] [step: 9.1e-03] [objective: 1.8e+00]
[56/200] [step: 9.1e-03] [objective: 1.7e+00]
[57/200] [step: 9.4e-03] [objective: 1.7e+00]
[58/200] [step: 8.4e-03] [objective: 1.7e+00]
[59/200] [step: 8.6e-03] [objective: 1.6e+00]
[60/200] [step: 8.2e-03] [objective: 1.6e+00]
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[61/200] [step: 8.5e-03] [objective: 1.6e+00]
[62/200] [step: 8.2e-03] [objective: 1.6e+00]
[63/200] [step: 8.0e-03] [objective: 1.5e+00]
[64/200] [step: 7.8e-03] [objective: 1.5e+00]
[65/200] [step: 7.7e-03] [objective: 1.5e+00]
[66/200] [step: 7.6e-03] [objective: 1.5e+00]
[67/200] [step: 7.5e-03] [objective: 1.4e+00]
[68/200] [step: 7.4e-03] [objective: 1.4e+00]
[69/200] [step: 7.3e-03] [objective: 1.4e+00]
[70/200] [step: 7.2e-03] [objective: 1.4e+00]
[71/200] [step: 7.0e-03] [objective: 1.4e+00]
[72/200] [step: 7.0e-03] [objective: 1.3e+00]
[73/200] [step: 6.9e-03] [objective: 1.3e+00]
[74/200] [step: 6.8e-03] [objective: 1.3e+00]
[75/200] [step: 6.7e-03] [objective: 1.3e+00]
[76/200] [step: 6.6e-03] [objective: 1.3e+00]
[77/200] [step: 6.5e-03] [objective: 1.3e+00]
[78/200] [step: 6.4e-03] [objective: 1.2e+00]
[79/200] [step: 6.4e-03] [objective: 1.2e+00]
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[81/200] [step: 6.2e-03] [objective: 1.2e+00]
[82/200] [step: 6.1e-03] [objective: 1.2e+00]
[83/200] [step: 6.1e-03] [objective: 1.2e+00]
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[85/200] [step: 5.9e-03] [objective: 1.1e+00]
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[114/200] [step: 4.4e-03] [objective: 8.4e-01]
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[142/200] [step: 3.5e-03] [objective: 6.8e-01]
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[162/200] [step: 3.1e-03] [objective: 6.0e-01]
[163/200] [step: 3.1e-03] [objective: 5.9e-01]
[164/200] [step: 3.1e-03] [objective: 5.9e-01]
[165/200] [step: 3.0e-03] [objective: 5.8e-01]
[166/200] [step: 3.0e-03] [objective: 5.8e-01]
[167/200] [step: 3.0e-03] [objective: 5.8e-01]
[168/200] [step: 3.0e-03] [objective: 5.7e-01]
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[169/200] [step: 3.0e-03] [objective: 5.7e-01]
[170/200] [step: 2.9e-03] [objective: 5.7e-01]
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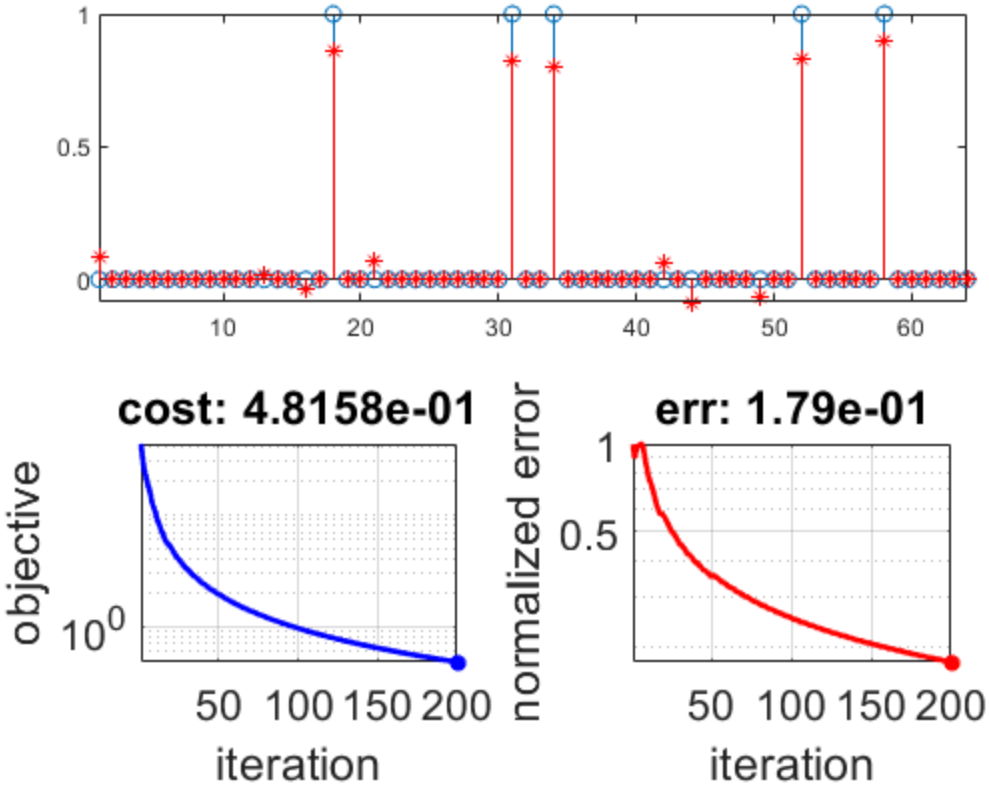

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