Hw 8

① 1. A
2. B
3. A
4. C
5. B
6. A
7. D
8. C
9. B
10. A

$$|T| = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}} \rightarrow (1-u^2)^2 + (2\zeta u)^2 = \frac{1}{|T|^2}$$

$$u^4 - 2u^2 + 1 + 4\zeta^2 u^2$$

2. $L(s) = \frac{4}{(s+2)^2}$ $M_{dB} = 20 \log |T|$ $\phi = \angle T = \angle N - \angle D = 0 - \tan^{-1}$

$= \tan^{-1}\left(\frac{\omega^4}{8-\omega^2}\right)$

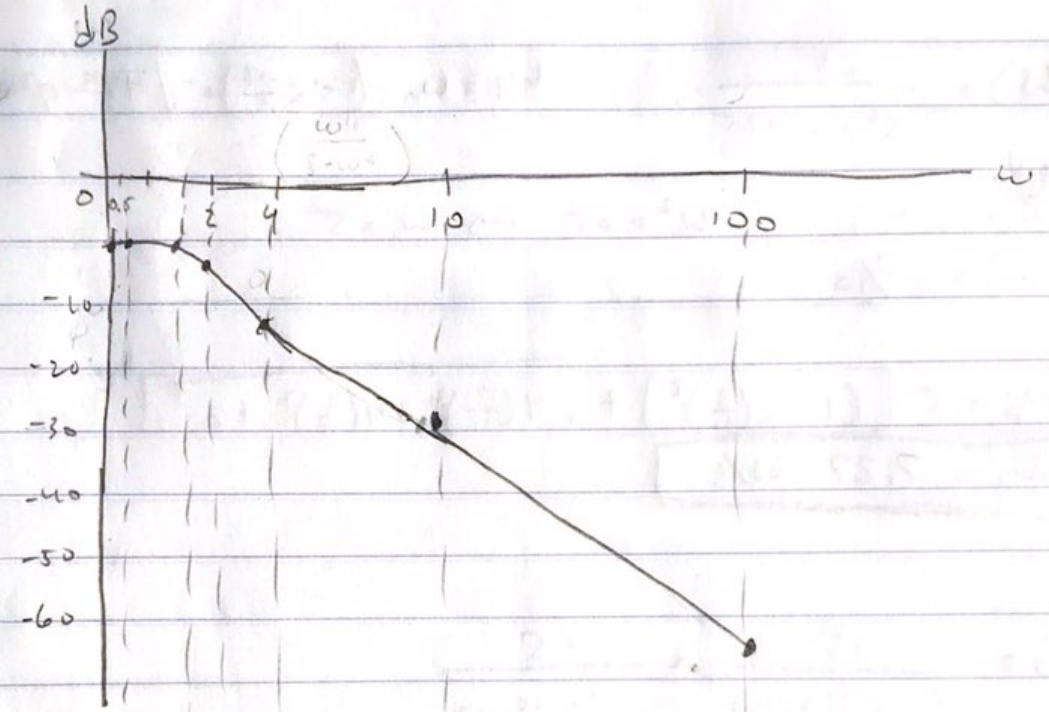
a. $T(s) = \frac{L(s)}{1+L(s)} = \frac{4}{(s+2)^2+4} = \frac{4}{s^2+4s+8}$

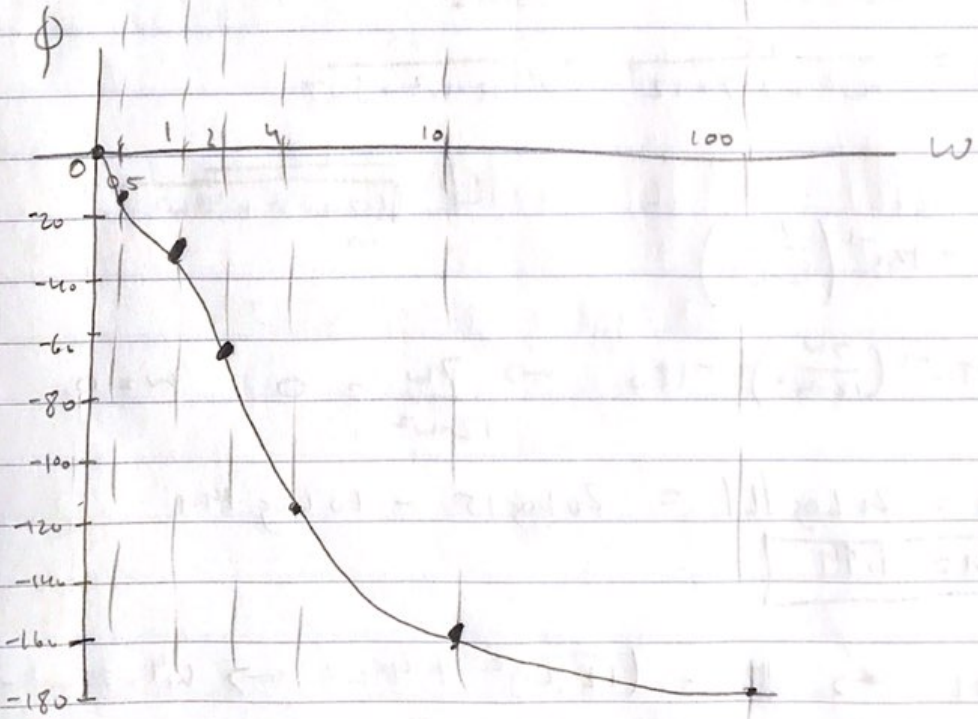
$$T(j\omega) = \frac{4}{(j\omega)^2 + 4(j\omega) + 8} = \frac{4}{(-\omega^2+8) + j\omega 4}$$

$$dB = 20\log(4) - 10\log(\omega^4 + 64)$$

| $\omega$ | 0 | 0.5 | 1 | 2 | 4 | 10 | 100 |
|---|---|---|---|---|---|---|---|
| dB | -6.021 | -6.025 | -6.088 | -6.99 | -13.01 | -27.99 | -67.96 |
| $\phi$ | 0 | -14.47 | -24.74 | -63.43 | -116.6 | -156.5 | -177.61 |

b.


dB
ω
0
0.5
1
2
4
10
100
-10
-20
-30
-40
-50
-60


ϕ
ω
0
0.5
1
2
4
10
100
-20
-40
-60
-80
-100
-120
-140
-160
-180

C. In MATLAB.

3. $T(s) = \dfrac{25}{s^2 + 3s + 25}$ $\qquad \omega_B = \omega_n \left[ (1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2} \right]^{1/2}$

$2\zeta\omega_n = 3 \qquad \omega_n^2 = 25 \rightarrow \omega_n = 5$

$\zeta = 3/10$

$\omega_B = 5 \left[ \left(1 - 2\left(\frac{3}{10}\right)^2\right) + \sqrt{4\left(\frac{3}{10}\right)^4 - 4\left(\frac{3}{10}\right)^2 + 2} \right]^{1/2}$

$\boxed{\omega_B = 7.27 \text{ rad/s}}$

4. $L(s) = \dfrac{15}{(s+3)(s+4)} = \dfrac{15}{s^2 + 7s + 12}$

$L(j\omega) = \dfrac{15}{-\omega^2 + j\omega 7 + 12} = \dfrac{15}{12 - \omega^2 + j\omega 7}$

$|L| \dfrac{15}{\sqrt{(12-\omega^2)^2 + 49\omega^2}}$

$\angle L = -\tan^{-1}\left(\dfrac{7\omega}{12 - \omega^2}\right)$

Set $\quad -\tan^{-1}\left(\dfrac{7\omega}{12-\omega^2}\right) = -180 \rightarrow \dfrac{7\omega}{12-\omega^2} = 0 \qquad \omega = 0.$

$GM = 20 \log |L| = 20 \log 15 - 10 \log 144$

$\boxed{GM = 1.94}$

Set $|L| = 1 \Rightarrow 225 = (12-\omega^2)^2 + 49\omega^2 \Rightarrow \omega^4 + 25\omega^2 - 81 = 0$

Let $x = \omega^2 \rightarrow x^2 + 25x - 81 = 0$

$x = \dfrac{-25 \pm \sqrt{949}}{2} = 2.903 \rightarrow \omega^2 = x \rightarrow \omega = 1.7$

$\phi = -\tan^{-1}\left(\dfrac{7(1.7)}{12 - 1.7^2}\right) \Rightarrow \boxed{\phi = -52.56^\circ}$