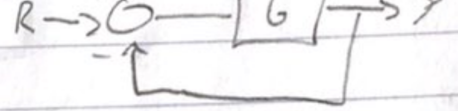
Hw 9. - Ch. 9.

(1)

1. C.	6. C.
2. A.	7. B.
3. B.	8. A.
4. C.	9. A.
5. B.	10. B.

(2) $G(s) = \dfrac{36}{s(s+6)}$



$\zeta = 0.8$, $T_s = 0.5\text{ s}$

$T(s) = \dfrac{G}{1+G} = \dfrac{36}{s^2+6s+36}$

a. $s^2 + 6s + 36 \rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n = 6 \quad 6 = 12\zeta \rightarrow \zeta = 0.5.$

$\boxed{\text{Since } \zeta < 0.8 \text{ we need a Compensator.}}$

b. $\omega_n = \dfrac{4}{\zeta T_s} = \dfrac{4}{(0.8)(0.5)} = 10. \text{ rad/s}$

Desired C.E. $s^2 + 2(10)(.8) + 100$

$s^2 + 16s + 100 = 0$

$s_{1,2} = \dfrac{-16 \pm \sqrt{-144}}{2} = -8 \pm 6i$

$\theta = \cos^{-1}(0.8) = 36.87^\circ$

$\zeta\omega_n = 8.$

$G(s) = \dfrac{36}{s^2+6s} = \dfrac{36}{-\omega^2+6i\omega}$ $\quad \omega^4 + 36\omega^2 - 1296 = 0$

$|G(i\omega)| = 1 = \dfrac{36}{\sqrt{\omega^4+36\omega^2}}$ $\quad \omega^2 = \dfrac{-36 \pm \sqrt{36^2 - 4(-1296)}}{2}$

$\omega_c = 4.72$

$\zeta = 0.8$; $\omega_n = 10$; $\omega_c = 4.72$

$$G_c = \frac{\alpha\left(s + \frac{1}{\alpha t}\right)}{\left(s + \frac{1}{t}\right)} \qquad \phi = 100\zeta = 80^\circ$$

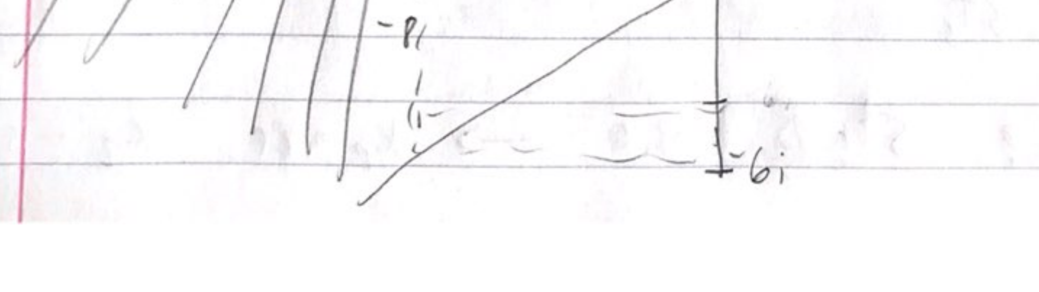
$$M = 10\log\alpha = 21.16 \text{ dB} \approx 22 \text{ dB}. \qquad \alpha = \frac{1+\sin\phi}{1-\sin\phi} = 130.65$$

$$c = 10^{(M/10)} = 158.49$$

$$t = \frac{1}{\omega_c}\sqrt{\frac{c-1}{a^2-c}} = 0.02$$

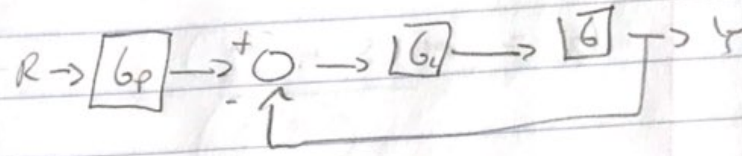
$$\boxed{G_c = 130.65\,\frac{(s + 0.37)}{s + 48.9}}$$

c. $s = -48.9 > -8$



3. $P.O. = 5\%$ $T_s = 1\,s$ $G = 1/s$

$G_c(s) = \frac{1}{s-20}$



$G_c(s) = k_p + k_i(1/s)$.

$$T(s) = \frac{G_p G_c G}{(1 + G_c G)} = G_p\left(\frac{\left(k_p + \frac{k_i}{s}\right)\left(\frac{1}{s-20}\right)}{1 + \left(k_p + \frac{k_i}{s}\right)\left(\frac{1}{s-20}\right)}\right)$$

$$T(s) = G_p\left(\frac{k_p s + k_i}{s(s-20) + k_p s + k_i}\right) = G_p\left(\frac{k_p s + k_i}{s^2 - 20s + k_p s + k_i}\right)$$

$$= G_p\left(\frac{k_p\left(s + \frac{k_i}{k_p}\right)}{s^2 + (k_p - 20)s + k_i}\right)$$ assume $G_p = \left(\frac{\frac{k_i}{k_p}}{s + \frac{k_i}{k_p}}\right)$

$$T(s) = \frac{k_i}{s^2 + (k_p - 20)s + k_i}$$

$\delta = \ln\left(\frac{100}{0.5}\right) = 2.9957$; $\zeta = \frac{\delta}{\sqrt{\delta^2 + \pi^2}} = 0.69$.

$\omega_n = \frac{4}{\zeta T_s} = 5.80$ rad/s

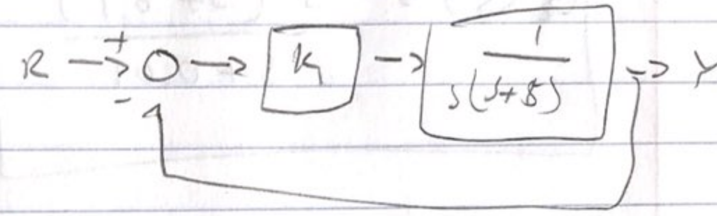
C.E.: $s^2 + 8s + 33.6 \longrightarrow k_p = 28$ $k_i = 33.6$

$$\frac{k_i}{k_p} = 1.2 \approx 2$$

$$\boxed{G_p = \frac{2}{s+2}}$$

$$\boxed{G_c = 28 + 33.6/s}$$

4. $$L(s) = \frac{k_1}{s(s+5)}$$



$K_v = 10$; $\omega_c = 1$

$$K_v = s\left[L(s)\right]_{s=0} = \frac{k_1}{5} \rightarrow k_1 = 5(k_v) = 50.$$

$$L(s) = \frac{k_v}{0.2s^2 + s} = \frac{k_v}{-0.2\omega^2 + i\omega}$$

$$|L| = \frac{k_v}{\sqrt{0.04\omega^4 + \omega^2}}$$

$$dB = 20 \log |L| = 20 \log k_v - 10 \log\left[0.04\omega^4 + \omega^2\right]$$

$\omega = \omega_c = 1$

$$= 20 \log(10) - 10 \log\left[.04 + 1\right]$$

$$= 19.83 = 20 \log \alpha$$

$$\log \alpha = 0.99$$

$$\alpha = 9.81 \approx 10.$$

$z = \frac{\omega_c}{10} = \frac{1}{10}$; $\cancel{\text{}}$ $T = \frac{1}{\frac{1}{10}} = 10.$

$p = \frac{z}{\alpha} = \frac{.1}{10} = \frac{1}{100.} = .01$

$K = \frac{K_1}{\alpha} = 5$

$$\boxed{G_c(s) = \frac{5(s + 0.1)}{s + 0.01}}$$