Hw 4

1. Error is Given as,

D. All The above

2. To minimize The Disturbance, The controller Should have,

C. Large Gain / Low f.

3. To minimize The noise The Controller Should have,

B. Small gain / ↑ f.

4. Sensitivity of a System T.F. (T) w.r.t. Plant T.F. (G) is

B. $S_G^T = \frac{\partial T}{\partial G} \frac{G}{T}$

5. Sensitivity T(s) w.r.t α in G(s) is,

D. All The above

6. Steady State error is The Value of error function, e(t) evaluated

B. $t = \infty$

7. Gain Setting is defined as,

8. $E(s) = \frac{(s+4)(s+5)}{s(s^2+3s+2)}$ The S.S e is

a. 10.

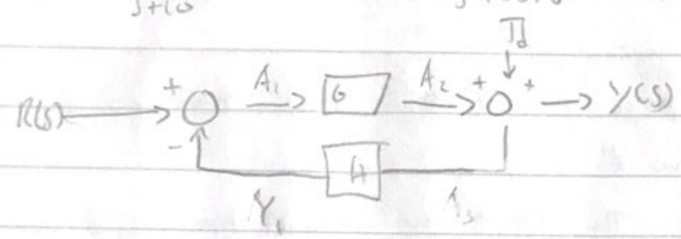
9. $E(s) = \frac{1}{s^2}\left(\frac{s^3+2s^2}{s^3+4s^2+5s+1}\right)$

D. 2.

10. $E(s) = \frac{s^2+3s+4}{s^3+4s^2+(5+2k)s}$ / $e_{ss} = 0.05$ $k = ?$

C. 37.5

2. $G(s) = \frac{K}{s+10}$, $H(s) = \frac{14}{s^2+5s+6}$



a. $A_1 = R - Y_1$, $A_2 = GA_1$, $Y_1 = y(s)H$

$$Y = A_2 + T_d = GA_1 + T_d = G(R - Y_1) + T_d$$
$$= G(R - YH) + T_d = GR - YHG + T_d$$
$$Y + YHG = GR + T_d$$
$$Y(1 + HG) = GR + T_d$$

Transfer Function $\rightarrow$ $\boxed{Y = \left(\frac{GR + T_d}{1 + HG}\right)}$

b. $E = R - Y = R - \frac{GR + T_d}{1 + HG} = R - \frac{GR}{1+HG} - \left(\frac{1}{1+HG}\right)T_d$

$$\boxed{E = R\left(1 - \frac{G}{1+HG}\right) + \left(\frac{1}{1+GH}\right)T_d}$$

$$\lim_{s \to 0} sE = s\left[R\left(1 - \frac{(K/s+10)}{1 + \left(\frac{14}{s^2+5s+6}\right)\left(\frac{K}{s+10}\right)}\right) + \left(\frac{1}{1 + \left(\frac{K}{s+10}\right)\left(\frac{14}{s^2+5s+6}\right)}\right)T_d\right]$$

$$= \frac{s - \left(\frac{sK}{s+10}\right)}{1 + \left(\frac{K}{s+10}\right)\left(\frac{14}{s^2+5s+6}\right)} + \frac{s}{\quad}$$

$$\lim_{s \to 0} sE(s) = 0$$

2. a. $A_1 = R - Y_1$, $A_2 = GA_1$, $Y_1 = g(s)H$

$Y = A_2 + T_d = GA_1 + T_d = G(R - Y_1) + T_d$

$= G(R - YH) + T_d = GR - YHG + T_d$

$Y = \dfrac{GR + T_d}{1 + HG}$

$T(s)$ when $T_d = 0$; $\boxed{T(s) = \dfrac{Y}{R} = \dfrac{G}{1+HG}}$

b. $E = R - Y = R(1 - T(s)) = R\left(1 - \dfrac{G}{1+GH}\right)$

$e_{ss} = \lim_{s \to 0} sE = s\left(\frac{1}{s}\right)\left(1 - \dfrac{(K/s+10)}{1 + \left(\frac{K}{s+10}\right)\left(\frac{14}{s^2+5s+6}\right)}\right)$

$= 1 - \dfrac{K}{(s+10)\left(1 + \left(\frac{K}{s+10}\right)\left(\frac{14}{s^2+5s+6}\right)\right)} = 1 - \dfrac{K}{10\left(1 + \left(\frac{K}{10}\right)\left(\frac{14}{6}\right)\right)}$

$e_{ss} = 1 - \dfrac{K}{10\left(\frac{60 + 14K}{60}\right)} = \boxed{1 - \dfrac{3K}{7K+30}}$

c. $T(s)$ when $R(s) = 0$; $T(s) = \dfrac{Y(s)}{T_d(s)} = \dfrac{1}{1+HG}$

$E(s) = -Y = -\dfrac{T_d}{1+GH}$

$e_{ss} = \lim_{s \to 0} sE = -s\left(\frac{1}{s}\right)\left(\dfrac{1}{1 + \left(\frac{K}{s+10}\right)\left(\frac{14}{s^2+5s+6}\right)}\right) = -\dfrac{1}{1 + \left(\frac{K}{10}\right)\left(\frac{14}{6}\right)}$

$e_{ss} = -\dfrac{60}{60 + 14K} = \boxed{-\dfrac{30}{30 + 7K}}$

d. $S_K^T = S_G^T S_K^G = \left(\frac{\partial T}{\partial G}\right)\left(\frac{\partial G}{\partial K}\right)\left(\frac{G}{T}\right)\left(\frac{K}{G}\right) = \left(\frac{\partial T}{\partial G}\right)\left(\frac{\partial G}{\partial K}\right)\left(\frac{K}{T}\right)$

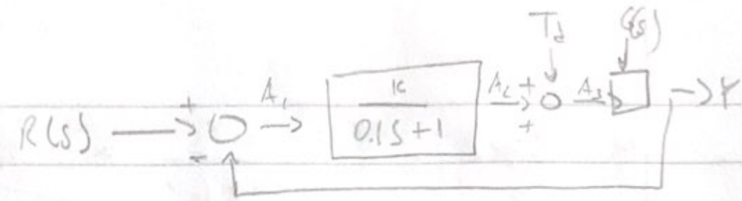
$\frac{\partial}{\partial G}\left(\frac{G}{1+HG}\right) = \frac{(1+HG)(1) - (G)(H)}{(1+GH)^2} = \frac{1+HG-HG}{(1+GH)^2} = \frac{1}{(1+HG)^2}$

$\frac{\partial G}{\partial K}\left(\frac{K}{s+10}\right) = \frac{1}{s+10}$

$S_K^T = \left(\frac{1}{(1+HG)^2}\right)\left(\frac{1}{s+10}\right)\left(\frac{K}{T}\right) = \frac{1}{(1+HG)^2} \quad G\left(\frac{(1+HG)}{G}\right) = \frac{1}{1+HG}$

$\boxed{S_K^T = \frac{1}{1+HG}}$

3. $G(s) = \frac{200}{s^2+25s+200}$



a.

$A_1 = R(s) - Y(s)$; $A_2 = A_1 \frac{K}{0.1s+1}$; $A_3 = A_2 + T_d$

$Y = A_3 G(s) = (A_2 + T_d) G = \left(\left(A_1 \frac{K}{0.1s+1}\right) + T_d\right) G =$

$= \left((R-Y)\left(\frac{K}{0.1s+1}\right) + T_d\right) G = \left(\left(R\frac{K}{0.1s+1}\right) - \left(Y\frac{K}{0.1s+1}\right) + T_d\right) G$

$= \left(GR\frac{K}{0.1s+1}\right) - \left(YG\frac{K}{0.1s+1}\right) + T_d G$

$Y + YG\frac{K}{0.1s+1} = \left(GR\frac{K}{0.1s+1}\right) + T_d G$

$Y = \frac{\left(\frac{GK}{0.1s+1}\right)R + GT_d}{1 + G\frac{K}{0.1s+1}} = \frac{GPR + GT_d}{1+GP}$, $P = \frac{K}{0.1s+1}$

$T(s)$ when $T_d = 0$; $T(s) = \frac{GP}{1+GP}$

$S^T_K = S^T_P S^P_K = \left(\frac{\partial T}{\partial P}\right)\left(\frac{P}{T}\right)\left(\frac{\partial P}{\partial K}\right)\left(\frac{K}{P}\right) = \left(\frac{\partial T}{\partial P}\right)\left(\frac{\partial P}{\partial K}\right)\left(\frac{K}{T}\right)$

$\frac{\partial}{\partial P}\left(\frac{GP}{1+GP}\right) = \frac{(1+GP)(G) - (GP)(G)}{(1+GP)^2} = \frac{G}{(1+GP)^2}$

$\frac{\partial}{\partial K}\left(\frac{K}{0.1s+1}\right) = \frac{1}{0.1s+1}$

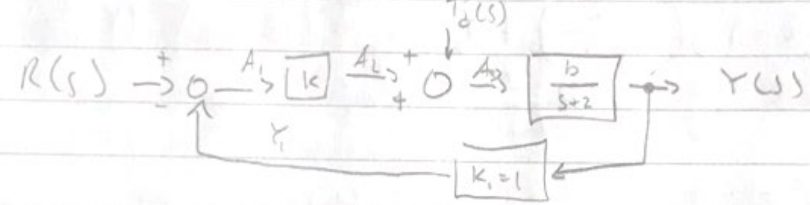
$S^T_K = \left(\frac{G}{(1+GP)^2}\right)\left(\frac{1}{0.1s+1}\right)\left(\frac{K}{T}\right) = \frac{\cancel{G}}{(1+GP)^{\cancel{2}}}(\cancel{P})\left(\frac{\cancel{(1+GP)}}{\cancel{GP}}\right)$

$\boxed{S^T_K = \frac{1}{1+GP}} = \boxed{\frac{1}{1+\left(\frac{GK}{0.1s+1}\right)}}$

b. $R=0 \quad Y = \frac{G T_d}{1+GP}$

$$T(s) = \frac{Y}{T_d} = \boxed{\frac{G}{1+G\left(\frac{k}{0.1s+1}\right)}}$$

4.



a.

$$A_1 = R - Y \; ; \; A_2 = A_1 K \; ; \; A_3 = A_2 + T_d \; ; \; Y = A_3\left(\frac{b}{s+2}\right)$$

$$Y = (A_2 + T_d)\left(\frac{b}{s+2}\right) = (A_1 K + T_d)\left(\frac{b}{s+2}\right) = ((R-Y)K + T_d)\left(\frac{b}{s+2}\right)$$

$$Y = ((RK - YK) + T_d)\frac{b}{s+2} = \frac{bKR}{s+2} - \frac{bYK}{s+2} + \frac{bT_d}{s+2}$$

$$Y\left(1 + \frac{bK}{s+2}\right) = \frac{bKR}{s+2} + \frac{bT_d}{s+2}$$

$$Y = \frac{PKR + PT_d}{1+PK} \; ; \; P = \frac{b}{s+2}$$

$$\boxed{T(s) = \frac{Y}{R} = \frac{PK}{1+PK} \; ; \; P = \frac{b}{s+2}}$$

b. $S_b^T = S_P^T \, S_b^P = \left(\frac{\partial T}{\partial P}\right)\left(\frac{\partial P}{\partial b}\right)\left(\frac{b}{T}\right)$

$$\frac{\partial}{\partial P}\left(\frac{PK}{1+PK}\right) = \frac{(1+PK)(K) - (PK)(K)}{(1+PK)^2} = \frac{K}{(1+PK)^2}$$

$$\frac{\partial}{\partial b}\left(\frac{b}{s+2}\right) = \frac{1}{s+2}$$

$$S_b^T = \left(\frac{k}{(1+Pk)^2}\right)\left(\frac{1}{s+2}\right)\left(\frac{b}{T}\right) = \left(\frac{k}{(1+Pk)^2}\right)\left(P\right)\left(\frac{1+Pk}{Pk}\right)$$

$$S_b^T = \frac{1}{1+Pk} = \boxed{\frac{1}{1+\frac{kb}{s+2}}}$$

C. $$\frac{Y}{T_d} = \frac{P}{1+Pk} = \frac{\frac{b}{s+2}}{1+\frac{bk}{s+2}} = \frac{b}{s+2+bk}$$

To make S_b^T / $\frac{Y}{T_d}$ minimum we want $\boxed{k = 50}$ since k is in the Denominator for both equations.