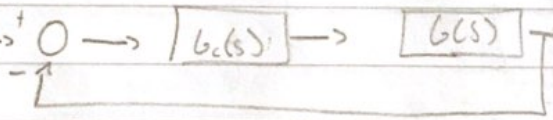
HW 5 Controls

(1)
1. A.
2. D.
3. C.
4. B.
5. C.
6. D.
7. A.
8. B.
9. D.
10. C.

(2)



$L(s) = G_c(s)G(s) = \frac{k}{s(s+\sqrt{2k})}$

a. Step input: $R(s) = 1/s$

$T(s) = \frac{L}{1+L} = \frac{k/s(s+\sqrt{2k})}{1 + k/s(s+\sqrt{2k})} = \frac{k}{s^2 + s\sqrt{2k} + k}$

$Y(s) = \frac{k}{s(s^2 + s\sqrt{2k} + k)}$ → $k = \omega_n^2$ $\quad \sqrt{2k} = 2\zeta\omega_n$

$2\zeta = \sqrt{2} \rightarrow \zeta = \frac{\sqrt{2}}{2}$

$0.02 = e^{-\zeta\omega_n t_s}$

$t_s = \frac{\ln 50}{\zeta\omega_n}$ $\quad \zeta = \frac{\ln(50)}{(\sqrt{k})(\frac{\sqrt{2}}{2})} = \boxed{\frac{5.53}{\sqrt{k}} = t_s}$

$P.O. = 100\, e^{-\pi\zeta/\beta}$ $\quad \beta = \sqrt{1-\zeta^2} = 0.707$

$P.O. = 100\, e^{-\frac{\sqrt{2}}{2}\pi/.707} = \boxed{4.32 = P.O}$

b. $t_s \geq \frac{5.53}{\sqrt{k}} \rightarrow 1 \geq \frac{5.53}{\sqrt{k}} \rightarrow \sqrt{k} \geq 5.53$

$$k \geq 5.53^2 = 30.61$$

3. $R(s) \rightarrow \boxed{G_p(s)} \rightarrow \overset{+}{\underset{-}{\bigcirc}} \rightarrow \boxed{\frac{1}{s(s+2)}} \rightarrow Y(s)$

$$\boxed{\frac{s+3}{s+0.1}}$$

a. $G_p(s) = 1$

$R(s) \rightarrow \boxed{G_p(s)} \rightarrow \boxed{\frac{s+0.1}{s(s+2)(s+0.1)+s+3}} \rightarrow Y(s)$

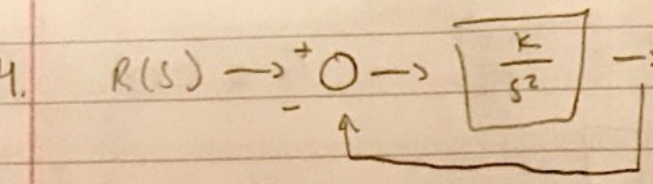
$$T(s) = \frac{s+0.1}{s(s+2)(s+0.1)+s+3}$$

$$E(s) = R(s)(1 - T(s)) = \frac{1}{s}\left(1 - \frac{s+0.1}{s(s+2)(s+0.1)+s+3}\right)$$

$$e_{ss} = sE(s)\Big|_{s=0} = 1 - \frac{s+0.1}{s(s+2)(s+0.1)+s+3}\Big|_{s=0} = \boxed{0.97}$$

b. $0 = 1 - \frac{G_p(s)\ s+0.1}{s(s+2)(s+0.1)+s+3}\Big|_{s=0} \Rightarrow 1 = \frac{G_p(0)\ (0.1)}{3}$

$$\boxed{G_p(s) = 30}$$ ✓

4. 

$R(t) = t^2$

$R(s) = 2/s^3$

$$T(s) = \frac{\frac{K}{s^2}}{1 + \frac{K}{s^2}} = \frac{K}{s^2 + K}$$

$$E(s) = R(s)\left(1 - T(s)\right) = \frac{2}{s^3}\left(1 - \frac{K}{s^2 + K}\right)$$

$$e_{ss} = s\,E(s)\Big|_{s=0} \leq 0.5$$

$$\frac{2}{s^2}\left(1 - \frac{K}{s^2+K}\right)\Big|_{s=0} \leq 0.5 \longrightarrow \frac{2}{s^2} - \frac{2K}{s^2(s^2+K)}\Big|_{s=0} \leq 0.5$$

$$\frac{2\left(s^2(s^2+K)\right) - 2Ks^2}{s^4(s^2+K)}\Big|_{s=0} \leq 0.5$$

$$\frac{2(s^2+K) - 2K}{s^2(s^2+K)}\Big|_{s=0} = \frac{2s^2+2K-2K}{s^4+Ks^2}\Big|_{s=0} \leq 0.5$$

$$\frac{2}{s^2+K}\Big|_{s=0} \leq 0.5 \longrightarrow \boxed{K \geq 4}$$