HW 7

① 1. A.
2. A.
3. B.
4. D.
5. B.
6. D.
7. C.
8. B
9. D.
10. D.

② $L(s) = kP(s) = \dfrac{k(s+1)}{s^2+4s+5} \rightarrow T(s) = \dfrac{k(s+1)}{s^2+4s+5 + k(s+1)}$

$m = 1$ Zero's : $k(s+1) = 0 \rightarrow s = -1$

$n = 2$ Poles : $s^2 + (4+k)s + (5+k) = 0 \rightarrow s = \dfrac{-(4+k) \pm \sqrt{(4+k)^2 - 4(5+k)}}{2}$

$s = \dfrac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i2$

$s_1 = -2 - i \quad s_2 = -2 + i$

Loci = n = 1 # Loci That Travels $\rightarrow \infty$ = n-m = 1

$$\sigma_A = \frac{\Sigma \text{ poles} - \Sigma \text{ zeros}}{n-m} = \frac{(-2+i + -2-i) - (-1)}{2-1}$$

$$\sigma_A = -3$$

$$\phi_0 = 180$$

No cross over point since Asymptote never hits y-axis.

$$k = -\frac{1}{P(s)} = -\frac{s^2+4s+5}{s+1} \rightarrow \frac{dk}{ds}\left(-\frac{1}{P(s)}\right) = 0$$

$$-\left[\frac{(s+1)(+2s+4) - (s^2+4s+5)(1)}{(s+1)^2}\right] = 0$$

$$\frac{-2s^2 - 6s - 4 + s^2 + 4s + 5}{(s^2+1)^2} = 0 \qquad -s^2 - 2s + 1 = 0$$

$$s^2 + 2s - 1 = 0$$

$$s = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = -1 \pm \frac{\sqrt{8}}{2} = -1 \pm \sqrt{2}$$

$$s = 0.414, \ -2.414$$

Break in points

$$P(s) = \frac{(s+1)}{(s-2-j)(s-2+j)}$$

$$P'(s) = \frac{(s+2-i)(s+1)}{\cancel{(s+2-j)}(s+2+j)} = \left(\frac{s+1}{s+2+i}\right)_{s=-2+i}$$

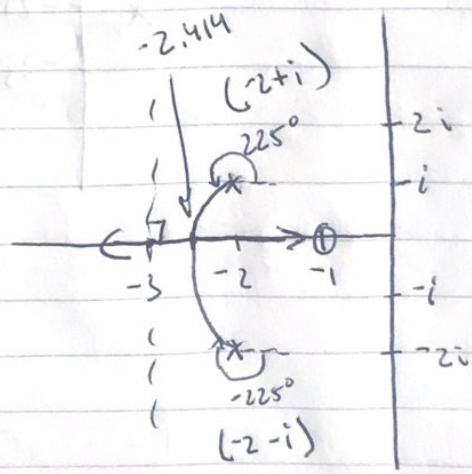
$$p' = \frac{-1+j}{2j}$$

$$\angle P' = \angle N' - \angle D' = \tan^{-1}\left(\frac{1}{-1}\right) - \tan^{-1}\left(\frac{2}{0}\right) = -45-90$$
$$= -135^\circ$$

$$\theta_d = \pm(-135+360) = \pm 225^\circ$$

$-2-j: \theta_d = -225^\circ$

$-2+j: \theta_d = 225^\circ$



a. $L(s) = KP(s) = \dfrac{K}{s(s^2+2s+5)}$

$T(s) = \dfrac{K}{s(s^2+2s+5)+K}$

a. Zeros: $m = 0$ $\quad s = $ none

Poles: $s(s^2+2s+5)+K = 0$

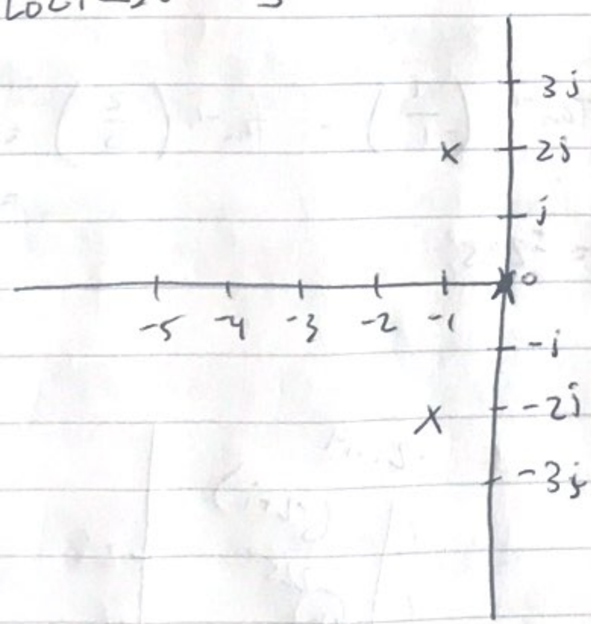
$s^3 + 2s^2 + 5s + K = 0$

at $K=0$ $\quad s=0 \quad s^2+2s+5=0$

$s = \dfrac{-2 \pm \sqrt{4-4(1)(5)}}{2} = -1 \pm 2j$

Loci = 3

Loci $\to \infty = 3$



$\sigma_A = \dfrac{\Sigma p - \Sigma z}{n-m} = \dfrac{0-1+2j-1-}{3}$

$= -2/3$

$\phi = \dfrac{(2k+1)}{n-m}(180) = 60°$

180

300

$\phi_0 = 60°$

$\phi_1 = 180$

$\phi_2 = 300°$

$1 + kP(s) = 0$

$P(s) = \frac{1}{s(s^2+2s+5)} = \frac{1}{s^3+2s^2+5s+k}$ ← C. E.

s^3	1	5	0
s^2	2	k	0
s	b_1	0	0
s^0	c_1	0	0

$c_1 = k.$, $k \geq 0$

$b_1 = \left(-\frac{k}{2}\right)(1) + 5 = -\frac{1}{2}k + 5 = \frac{10-k}{2}$

$k \geq 10$

plug k into auxillary polynomial

$2s^2 + 10 = 0$

$s = \frac{\sqrt{-10}}{\sqrt{2}} = \pm j\sqrt{\frac{10}{2}} =$

$\boxed{\begin{array}{c}\text{Loci cross over points} \\ \downarrow \\ s = j(2.23) \text{ or } -j2.23\end{array}}$

Asymptote cross over points:

$\pm j\left(-\frac{2}{3}\right)\tan(60) = \pm j\,1.155$

∡ of Departure. $s = -1 + 2j$

$$P(s) = \frac{1}{s(s+1+2j)(s+1-2j)}$$

$$P'(s) = \left[(s+1-j(2))\frac{1}{s(s+1+2j)(s+1-2j)}\right]_{s=-1+2j}$$

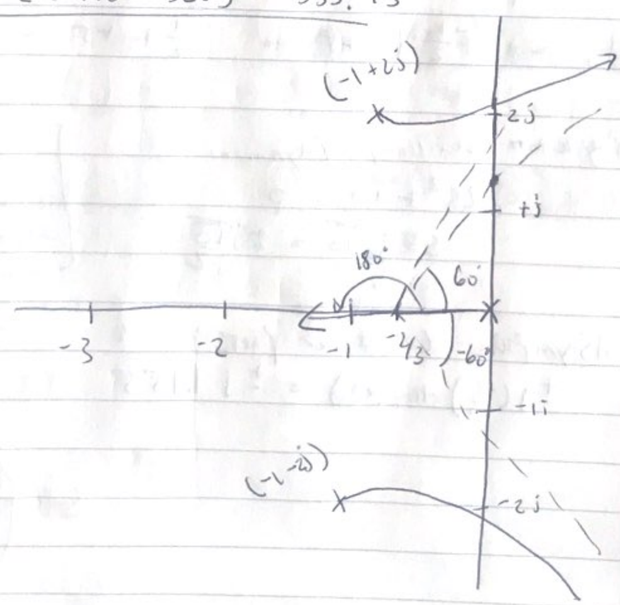
$$P'(s) = \left[\frac{1}{s(s+1+2j)}\right]_{s=-1+2j} = \frac{1}{(-1+2j)(4j)}$$

$$\angle P' = \angle N' - \angle D' = 0 - \left[\tan^{-1}\left(\frac{2j}{-1}\right) + 90^\circ\right]$$

$$= -\left[-63.43 + 180 + 90\right] = -26.56^\circ$$

$$\theta_d = \pm(-26.56 + 360) = 333.43^\circ$$

b.



HW 7.

4. $L(s) = KP(s) = \frac{K}{s(s+2)(s+5)}$

a.

$m=0$ Zeros: none

$n=3$ Poles: $s=0$, $s=-2$, $s=-5$

$s^3 + 7s^2 + 10s$

Loci = 3 # Loci $\to \infty$ = 3

$K = \frac{-1}{P(s)} = -s(s+2)(s+5) = 0$

$\boxed{\begin{array}{l} K = -8.21 \text{ @ } -3.786 \\ K = 4.06 \text{ @ } -.8803 \end{array}}$

$\frac{dK}{ds} = 0 = 3s^2 + 14s + 10$

$s = \frac{-14 \pm \sqrt{14^2 - 4(3)(10)}}{6} = \frac{-14 \pm 8.72}{6}$

$s = -3.786$, -0.8803

b. $\sigma_A = \frac{\sum P - \sum Z}{n-m} = \frac{(-2-5)-0}{3} = -\frac{7}{3}$

σ_A crosses over ϕ gives

$\phi_1 = \frac{2k+1}{3}(180) = 60°$
$= 180°$
$= 300°$

$\pm j(\frac{-7}{3}) \tan 60 = \pm j4.04$

s^3	1	10	0
s^2	7	K	0
s	b_1	0	0
s^0	c_1	0	0

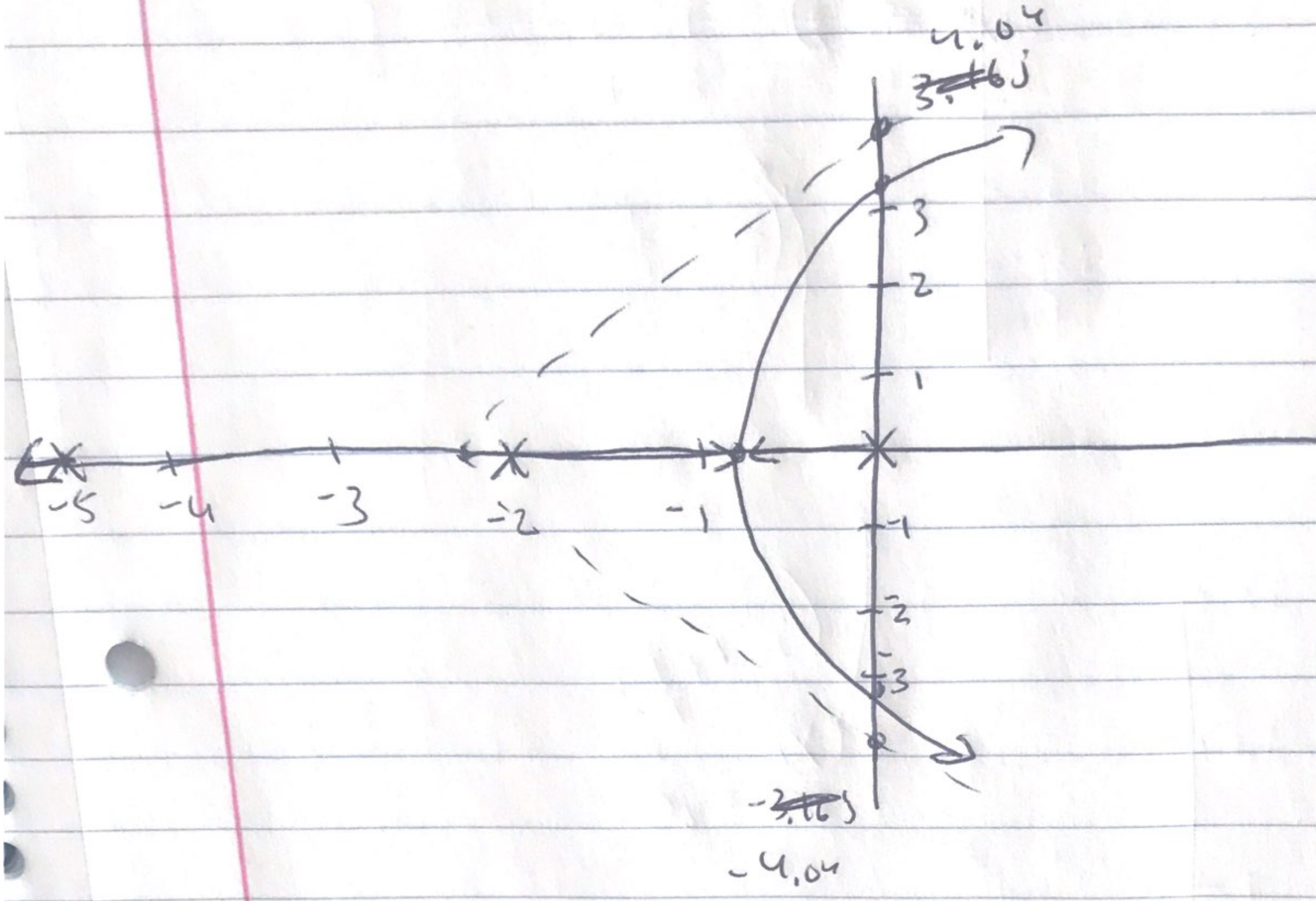
$c_1 = K$ $K \geq 0$

$b_1 = (-\frac{K}{7})1 + 10$

$K \geq 70$

$7s^2 + 70 = 0$

$s = \sqrt{-10} = \pm 3.16j$



4.04
~~3.16~~j
3
2
1
-5
-4
-3
-2
-1
-1
-2
-3
-~~3.16~~j
-4.04