

$$1. \quad C_L = \frac{4 \cos \phi}{\sigma'} \left(\frac{a'}{1+a'} \right) = (1+a') \left(\frac{1-a}{\lambda_r \tan \phi} \right)$$

$$C_L = \frac{4 \cos \phi}{\sigma'} \left(a' \frac{\lambda_r \tan \phi}{1-a} \right) = \frac{4 \sin \phi}{\sigma'} \lambda_r \left(\frac{a'}{1-a} \right)$$

Note: $a = \frac{\sigma' C_L}{4 \sin \phi \tan \phi + \sigma' C_L}$

$$a' = \frac{\sigma' C_L}{4 \cos \phi - \sigma' C_L}$$

$$1 - \sigma' a' = 1 - a = \frac{1 - \frac{\sigma' C_L}{4 \cos \phi - \sigma' C_L}}{\frac{4 \sin \phi \tan \phi + \sigma' C_L}{4 \sin \phi \tan \phi + \sigma' C_L}} = \frac{4 \sin \phi \tan \phi}{4 \sin \phi \tan \phi + \sigma' C_L}$$

$$C_L = \frac{4 \sin \phi}{\sigma'} \lambda_r \left(\frac{\sigma' C_L}{4 \cos \phi - \sigma' C_L} \right) \left(\frac{4 \sin \phi \tan \phi + \sigma' C_L}{4 \sin \phi \tan \phi} \right)$$

$$C_L = \frac{4 \sin \phi}{\sigma'} \lambda_r \frac{\sigma' C_L (4 \sin \phi \tan \phi + \sigma' C_L)}{(4 \cos \phi - \sigma' C_L) (4 \sin \phi \tan \phi)}$$

$$1 = \frac{4 \sin \phi \lambda_r (4 \sin \phi \tan \phi + \sigma' C_L)}{(4 \cos \phi - \sigma' C_L) (4 \sin \phi \tan \phi)}$$

$$(4 \cos \phi - \sigma' C_L) (4 \sin \phi \tan \phi) = 4 \sin \phi (4 \lambda_r \sin \phi \tan \phi + \lambda_r \sigma' C_L)$$

$$16 \cos \phi \sin \phi \tan \phi - 4 \sin \phi \tan \phi \sigma' C_L = 16 \lambda_r \sin^2 \phi \tan \phi + 4 \lambda_r \sin \phi \sigma' C_L$$

$$16 \sin^2 \phi - 16 \lambda_r \sin^2 \phi \tan \phi = 4 \lambda_r \sin \phi \sigma' C_L + 4 \sin \phi \tan \phi \sigma' C_L$$

$$16 \sin^2 \phi (1 - \lambda_r \tan \phi) = 4 \sin \phi \sigma' C_L (\lambda_r + \tan \phi)$$

$$C_L = \frac{4 \sin \phi}{\sigma'} \frac{(1 - \lambda_r \tan \phi)}{(\lambda_r + \tan \phi)}$$

2. 3 blades, $D = 10m$, $\lambda = 7$, $\alpha = 7^\circ$, $B = 3$.

$$P/R = 0.5, C_t = 1$$

$$\phi = \tan^{-1} \left(\frac{2}{3 \lambda r} \right) \quad \theta_p = \phi - \alpha \quad \theta_r = \theta_p - \theta_{p_0}$$

$$\theta_{p_0} = (\theta_p)_{tip} = \phi_{tip} - \alpha$$

$$\phi_{tip} = \tan^{-1} \left(\frac{2}{3 \lambda} \right), \quad C = \frac{8 \pi r \sin \phi}{3 B C_t \lambda r}$$

$$\lambda_r = \lambda \frac{r}{R} = 7(0.5) = 3.5$$

$$\phi = \tan^{-1} \left(\frac{2}{3} (3.5) \right) = 66.8^\circ \quad \boxed{\phi = 10.78}$$

$$\theta_p = \phi - \alpha = 10.78 - 7^\circ \quad \boxed{\theta_p = 3.78}$$

$$\theta_r = \phi - \phi_{tip} \quad \phi_{tip} = \tan^{-1} \left(\frac{2}{3} \lambda \right) = 5.44$$

$$\theta_r = 10.78 - 5.44 = 5.34 \quad \boxed{\theta_r = 5.34}$$

$$C = \frac{8 \pi r \sin \phi}{3 B C_t \lambda r} = \frac{8 \pi (5) \sin (10.78)}{3 (3) (1) (3.5)} = 0.746$$

$$\boxed{C = 0.746}$$

3. note 3 is Done in matlab.

4. note 4 is Done in matlab