HW 5

1. $L = 2$ $m = 1500$ kg $d = .15$ m $E = 210$ GPa $G = 80.8$ GPa

$P = 275$ kW $\omega = 60$ rpm $= \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 2\pi$

b. $t = 4$ min, $\Omega = 2(60 \text{ rpm}) = 120$ rpm $P = \Omega_0 T$, $T = I_m \alpha$

$\alpha = \frac{\Omega - \Omega_0}{t} = \frac{4\pi - 2\pi}{4 \text{ min}}$ $\alpha = 0.026$

$T = \frac{275}{2\pi} = 43.77$ kN

$I_m = \frac{T}{\alpha} = \frac{43.77 \text{ kN}}{0.026} = 1.68 \cdot 10^6$ kg m^2

b. $\delta = \frac{wL^3}{3EI_0}$ $I_0 = \frac{\pi d^4}{64} = \frac{\pi (.15)^4}{64} = 2.49 \cdot 10^{-5}$

$\delta = \frac{(1500)g\,(2)^3}{3(210\text{GPa})(2.49 \cdot 10^{-5})} = 0.0075$ m

$\delta = 7.5$ mm

$\sigma_{max} = \frac{Mc}{I_0} = \frac{wLc}{I_0} = \frac{wLd}{2I_0} = \frac{(1500)(9.81)(2)(.15)}{2(2.49 \cdot 10^{-5})}$

$\sigma_{max} = 88.8$ MPa

$\tau_{max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{w}{A} = \frac{4}{3}\frac{(1500)g}{\pi\left(\frac{d}{2}\right)^2} = \frac{\frac{4}{3}(1500)g}{\pi(.15)^2}$

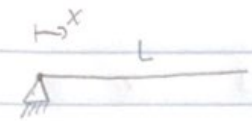
$\tau_{max} = 1.11$ MPa

c. $\phi = \frac{TL}{GJ} = \frac{(P/\Omega_0)L}{G(2I_0)} = \frac{(43.77 \text{ kN})(2 \text{ m})}{(80.9 \text{ GPa})(2 \cdot 2.49 \cdot 10^{-5})}$ $\phi = 0.021$ rad

$\tau = \frac{TL}{J} = \frac{Td}{2J} = \frac{(43.77 \text{ kN})(.15 \text{ m})}{2(2.49 \cdot 10^{-5})}$

$\tau = 65.9$ MPa

2.



$x=0$	$x=L$
$y(0)=0$	$y''(L)=0$
$y''(0)=0$	$y''(L)=0$

$$y = A\sinh\beta x + B\cosh\beta x + C\sin\beta x + D\cos\beta x$$
$$y' = A\beta\cosh\beta x + B\beta\sinh\beta x + C\beta\cos\beta x - D\beta\sin\beta x$$
$$y'' = A\beta^2\sinh\beta x + B\beta^2\cosh\beta x - C\beta^2\sin\beta x - D\beta^2\cos\beta x$$
$$y''' = A\beta^3\cosh\beta x + B\beta^3\sinh\beta x - C\beta^3\cos\beta x + D\beta^3\sin\beta x$$

$$y(0) = 0 = B\cosh 0 + D\cos 0 \rightarrow B = -D$$

$$y''(0) = 0 = B\cosh 0 + D\cos 0 \rightarrow B - D = 0 \quad \text{so } B, D = 0.$$
$$y''(L) = 0 = A\sinh\beta L - C\sin\beta L$$
$$A\sinh\beta L = C\sin\beta L \qquad \frac{A}{C} = \frac{\sin\beta L}{\sinh\beta L}$$

$$y''' = A\cosh\beta L - C\cos\beta L = 0$$
$$\frac{A}{C} = \frac{\cos\beta L}{\cosh\beta L}$$

$$\text{So} \quad \frac{\sin\beta L}{\sinh\beta L} = \frac{\cos\beta L}{\cosh\beta L} \longrightarrow \tan\beta L = \tanh\beta L$$

$\beta L = 3.927, \ 7.069$ ← Roots intersections on Desmos.

$$\beta = \frac{3.927}{L}, \ \frac{7.069}{L}$$

$$\omega_n = \beta_n^2\sqrt{\frac{EI}{\rho A}}$$

$$\omega_1 = \left(\frac{3.927}{L}\right)^2\sqrt{\frac{EI}{\rho A}} \qquad \omega_2 = \left(\frac{7.069}{L}\right)^2\sqrt{\frac{EI}{\rho A}}$$

3. $M = \frac{1}{n}\sum_{1}^{10} \log(n_i) = \frac{1}{10}\sum_{1}^{10} \log n_i = 15.4843$ ran code in matlab

$$\delta = \sqrt{\frac{1}{n-1}\sum_{1}^{10}(\log n_i - m)^2} = \sqrt{\frac{1}{9}\sum_{1}^{10}(\log n_i - m)^2}$$

$\delta = 0.1195$ found in matlab

$\log N = M - Y\delta$

$P^{\log N} = .98$

$$Y = Y_2 - \left(\frac{P_2 - P_1}{P_2 - P_1}\right)(Y_2 - Y_1)$$

$$2.054 = 2.06 - \left(\frac{.9803 - .98}{.9803 - .9798}\right)(2.06 - 2.05)$$

$\log N = M - Y\delta$

$= 5.4843 - (2.054)(0.1195)$

$\log N = 5.2388$

$N = 173319$ cycles

4. See attached work

5. $\sigma_x = -10$ ksi $\quad \sigma_y = 2$ ksi $\quad \tau_{xy} = 5$ ksi

a. $\frac{\sigma_x + \sigma_y}{2} = \frac{-10 \text{ ksi} + 2 \text{ ksi}}{2} = -4$ ksi

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{(-10) - 2}{2}\right)^2 + 5^2} = 7.81$$

$\sigma_1 = -4 + 7.81 = 3.81$ ksi

$\sigma_2 = -4 - 7.81 = -11.81$ ksi

$\sigma_m = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sqrt{3.81^2 - (3.81)(-11.81) + (11.81)^2} = 14.1$

So material 2 is better