

HW 6.

1.  $B = 3$      $d = 38.1 \text{ m}$      $L = 1 \text{ m}$      $\omega = 50 \text{ rpm} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 5.23 \text{ rad/s}$   
 $\omega_{na} = 1.67 \text{ Hz} \cdot 2\pi$      $\omega_p = 1.42 \text{ Hz} \cdot 2\pi$      $m = 89 \text{ kg}$

a.  $e = \frac{2\varepsilon}{2\varepsilon + 3}$      $\varepsilon = \frac{1}{\omega^2} (\omega_a^2 - \omega_{na}^2 - \omega^2)$   
 $= \frac{1}{5.23^2} (1.42^2 - 1.67^2 - 5.23^2)$   
 $= 0.3$

$e = \frac{2(0.3)}{2(0.3) + 3} = 0.16$

b.  $I = \frac{1}{3} M_d R^2 (1 - e)^3 = \frac{1}{3} (89 \text{ kg}) \left( \frac{38.1}{4} \right)^2 (1 - 0.16)^3$

$I_s = 6.4 \cdot 10^4 \text{ kg m}^2$

c.  $k_p = \omega_{na}^2 I_b = (1.67 \cdot 2\pi)^2 (6.4 \cdot 10^4) =$

$k_p = 6.98 \cdot 10^6 \text{ kg m}^2/\text{s}^2 \rightarrow \text{N-m}$

2.  $U = 8 \text{ m/s}$      $\text{Stoke curve} = 2\pi$      $r_0 = 1/3$      $\theta_p = 1.5^\circ$   
 $\Delta = 1.225 \text{ kg/m}^3$

$\beta_1 = \frac{B_1}{A_1}$      $B_1 = \frac{\gamma}{8} \left( \frac{4}{3} L - \theta_p \right)$

$\gamma = \frac{\Delta a C K^4}{I_b} = \frac{(1.225) \left( \frac{4}{3} \right) (1 \text{ m}) \left( \frac{2.94}{2} \right)^4}{6}$

$\gamma = 15.8$

$B_1 = \frac{15.8}{8} \left( \frac{4}{3} \left( \frac{8(1 - 1/3)}{5.23(38.1/2)} \right) - 1.8 \cdot 2\pi \right)$

$B_1 = 4.52^\circ$

$$3 \quad a. \quad \beta_o = \frac{r}{8k} \left[ \frac{4}{3} \frac{V}{\pi R} (1-a_o) - \theta_r \right]$$

$$\frac{\partial \beta_o}{\partial V} = \frac{r}{8k} \left( \frac{4(1-a_o)}{3\pi R} \right)$$

$$b. \quad a_o = 1/3 \quad \frac{\partial \beta_o}{\partial V} = \frac{r}{k} \left( \frac{1}{9\pi R} \right)$$

$$c. \quad \frac{\partial \beta_o}{\partial \theta_r} = -\frac{r}{8k}$$

$$A_1 = 1 + \epsilon + \frac{k_B}{R^2 I_b} = 1 + 0.3 + \frac{6.95 \cdot 10^6}{(5.23)^2 (6.4 \cdot 10^7)} = 5.29$$

$$\beta_o = \frac{4.52}{5.29} = 0.85$$