Ay190 – Worksheet 02
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Note: the formatting is messed up in this but all the work is done!

1 An Unstable Calculation

The sequence defined by the recurrence relation is given as $x_0 = 1, x_1 = \frac{1}{3}, x_{n+1} = \frac{13}{3}x_n - \frac{4}{3}x_{n-1}$ while a direct calculation of any term is given by the alternate expression $x_n = (\frac{1}{3})^n$.

The error arising from the use of single precision floating point numbers as a function of n is shown in Figure 1.

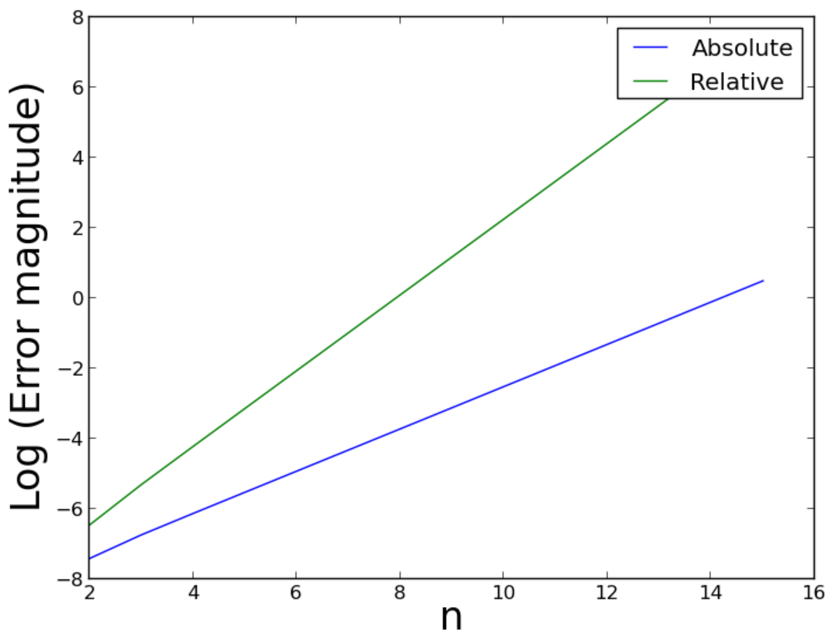

Figure 1: Floating point error in convergence relation.

The absolute error at $n = 15$ is 3.138 while the relative error is $4.503 \times 10^{+07}$.

2 Finite Difference Approximation and Convergence

Figure 2 shows the absolute error in the forward difference derivative (compared to the analytically calculated derivative) of the function $f(x) = x^3 - 5x^2 + x$ for two different step sizes and Figure 3 shows the same thing, but for the central difference derivative.

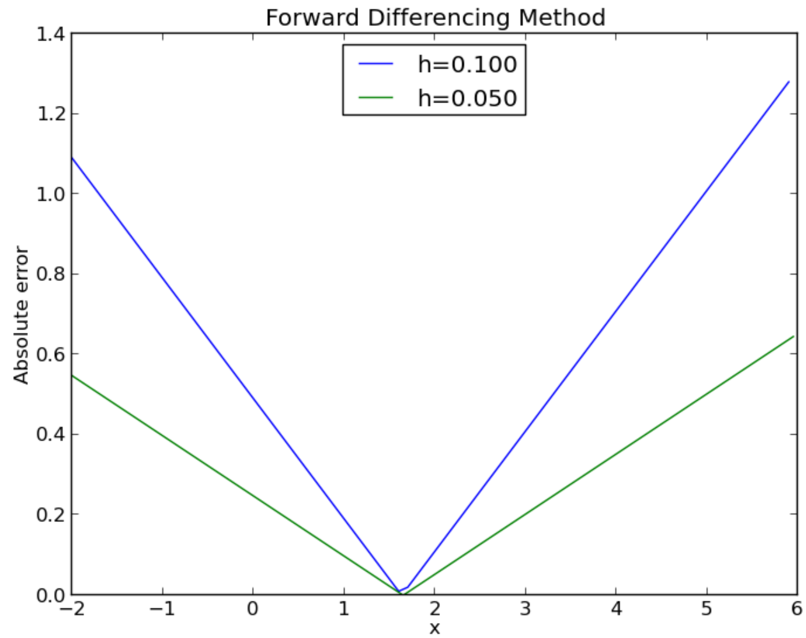

Figure 2: Error in forward differencing for two different step sizes.

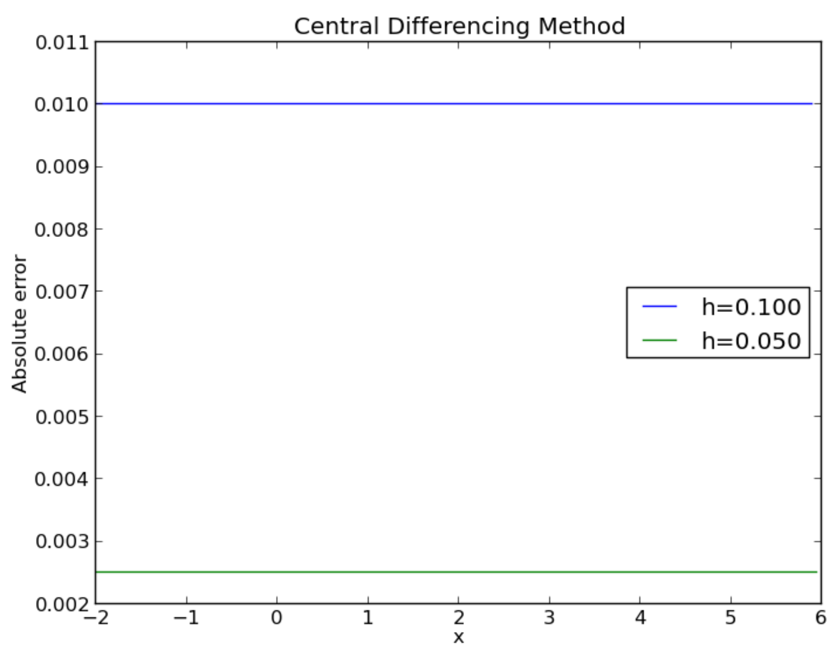

Figure 3: Error in central differencing for two different step sizes.

3 Second Derivative

Taking the first 3 terms of the Taylor expansion:

$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2} f''(x_0) + O(h^3)$

$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2} f''(x_0) + O(h^3)$

$f''(x_0) = \frac{2}{h^2}(f(x_0 + h) - f(x_0)) - \frac{2}{h} f'(x_0) - +O(h^3)$

Using first derivative from forward differencing: $f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} + O(h^2)$

$f''(x_0) = \frac{2}{h^2}(f(x_0 + h) - f(x_0)) - \frac{1}{h^3}(f(x_0 + h) - f(x_0 - h)) + O(h^2)$

4 Interpolation: Cepheid Lightcurve

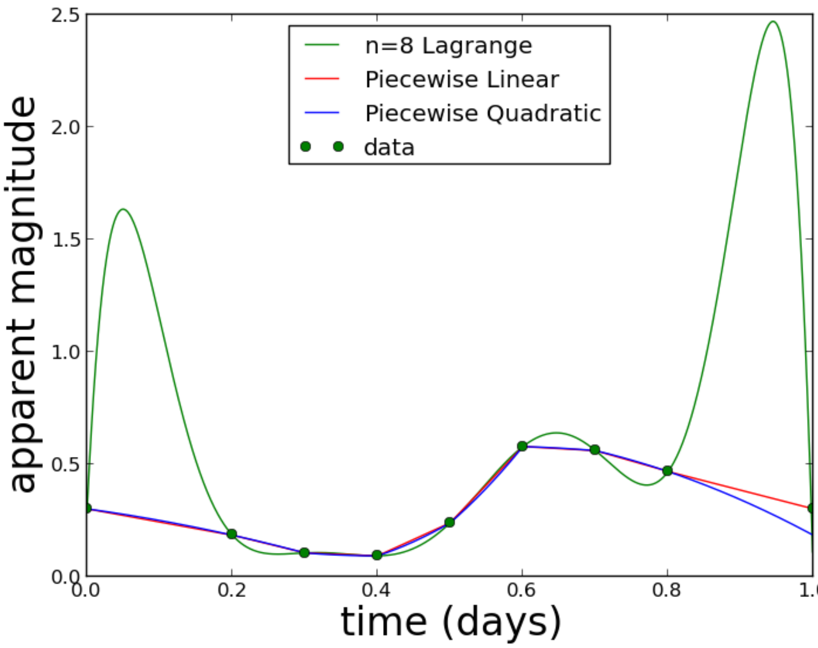

Figure 4: Interpolation of cepheid variable data using order 8 Lagrange interpolation, piecewise linear and piecewise quadratic interpolation.

5 More Cepheid Lightcurve Interpolation

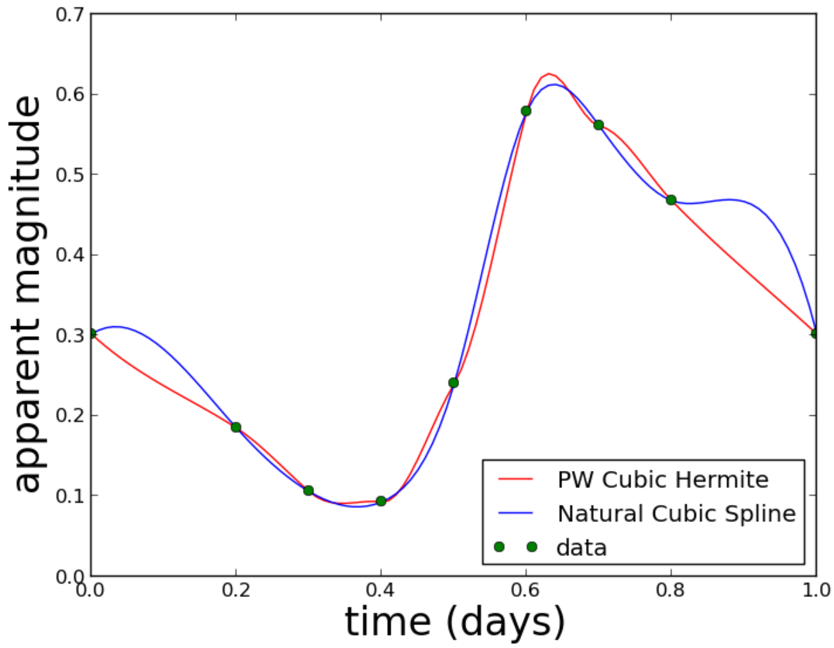

Figure 5: Interpolation of cepheid variable data using piecewise cubic Hermite and natural cubic spline interpolation.