Ay190 – Worksheet 11
Donal O Sullivan
Date: February 17, 2014

1 Advection Equation

1.1 Implementing Moving Gaussian

Running the code $ws11.py$ will demonstrate this working.

1.2 Implementing Upwind Scheme and Testing

1.2.1 Stability

Setting the Courant-Friedrics-Lewy factor to $c_{CFL} = 1.0$ yields $\alpha = 1.0$ which is theoretically stable. Figures 1 and 2 show the results of this.

Setting $c_{CFL} = 1.1$ should theoretically make the solution unstable and sure enough, it does, as seen in Figures 3 and 4

1.2.2 Narrower Gaussian

Setting $\sigma = \frac{1}{5}\sigma_{old}$ yields the results shown in Figures 5 and 6.

The first thing I noticed is that the actual error calculation is going to be dominated by very few points, since the narrower the Gaussian, the less discretized points lie within its FWHM. The second thing I noticed is that at the end, the outflow boundary condition causes an issue in that whent he Gaussian leaves the plotted domain, there is no gradual descent to bring the last point on the curve back down to zero so there remains one point at the boundary still 'floating' above the rest.

1.3 Implementing unstable FTCS scheme

Finally, implementing the FTCS scheme, the instability is shown very clearly in Figures 7 and 8.

1.4 Figures

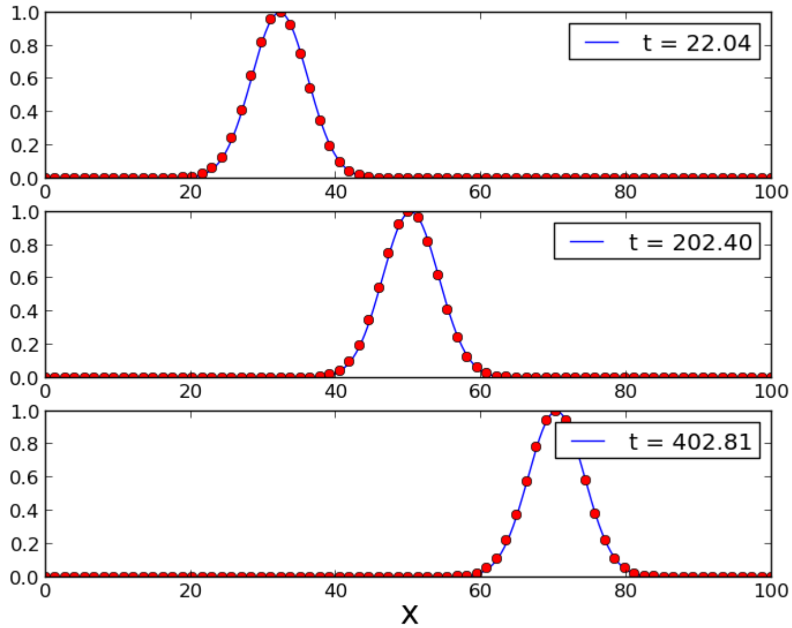

Figure 1: The stable ($c_{CFL} = 1.0$) advected Gaussian function at three different times.

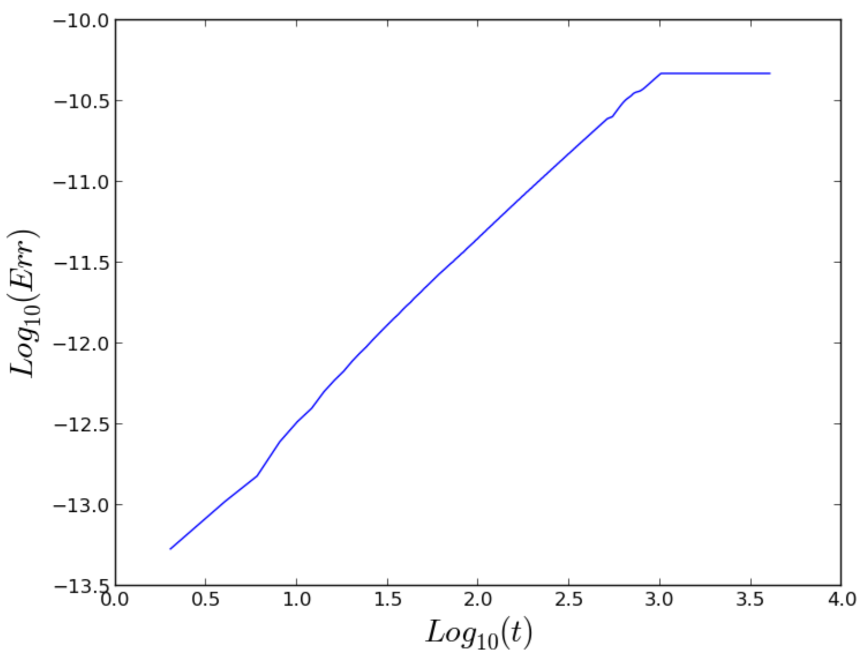

Figure 2: Error as a function of time for the stable ($c_{CFL} = 1.0$) advected Gaussian.

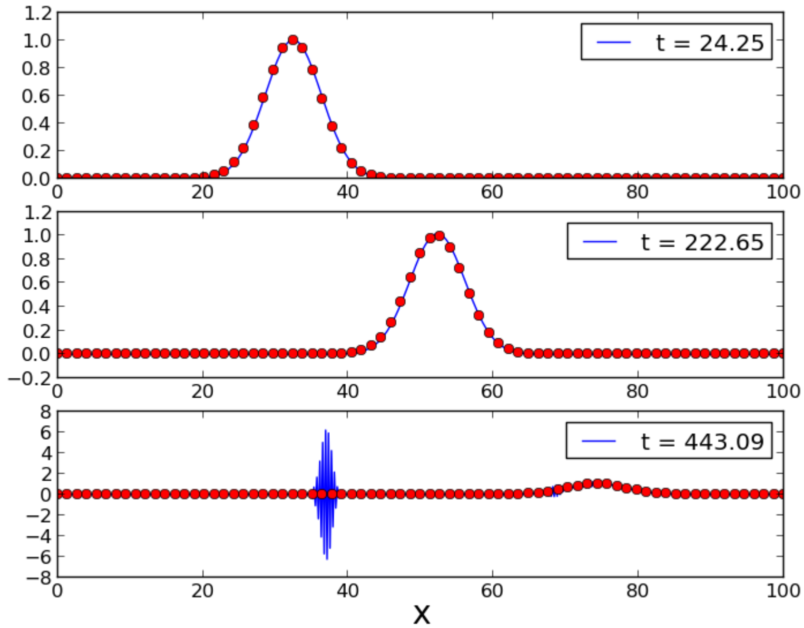

Figure 3: The unstable ($c_{CFL} = 1.1$) advected Gaussian function at three different times.

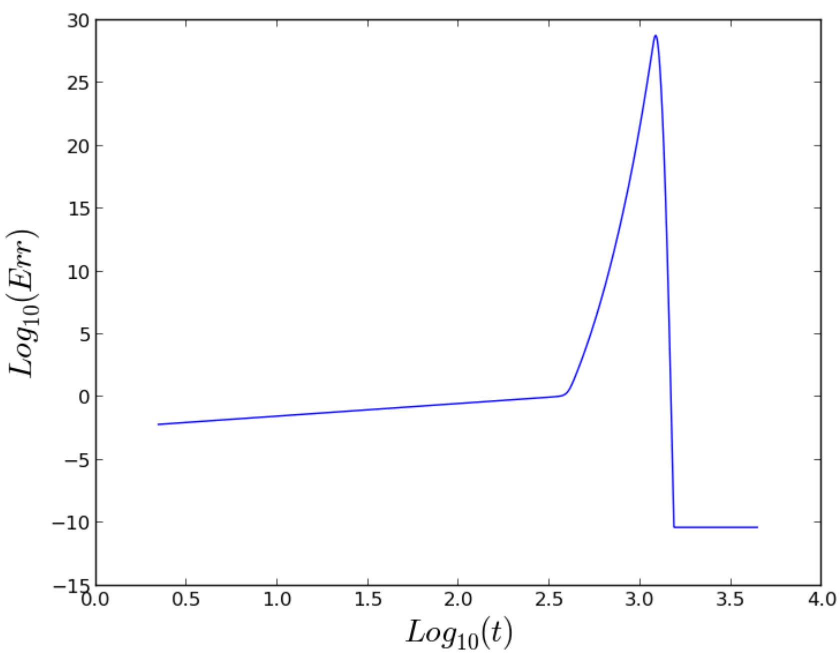

Figure 4: Error as a function of time for the unstable ($c_{CFL} = 1.1$) advected Gaussian.

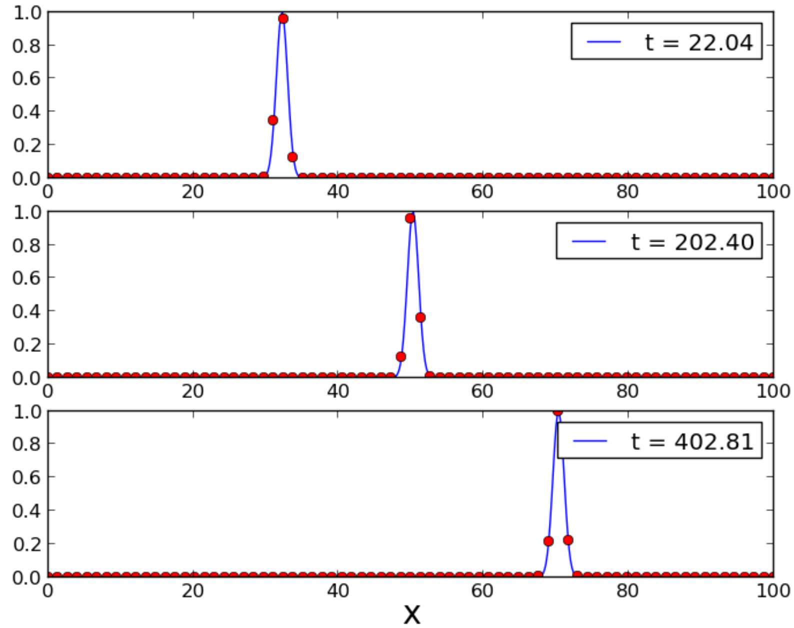

Figure 5: A narrower Gaussian ($\sigma = \frac{1}{5}\sigma_{old}$).

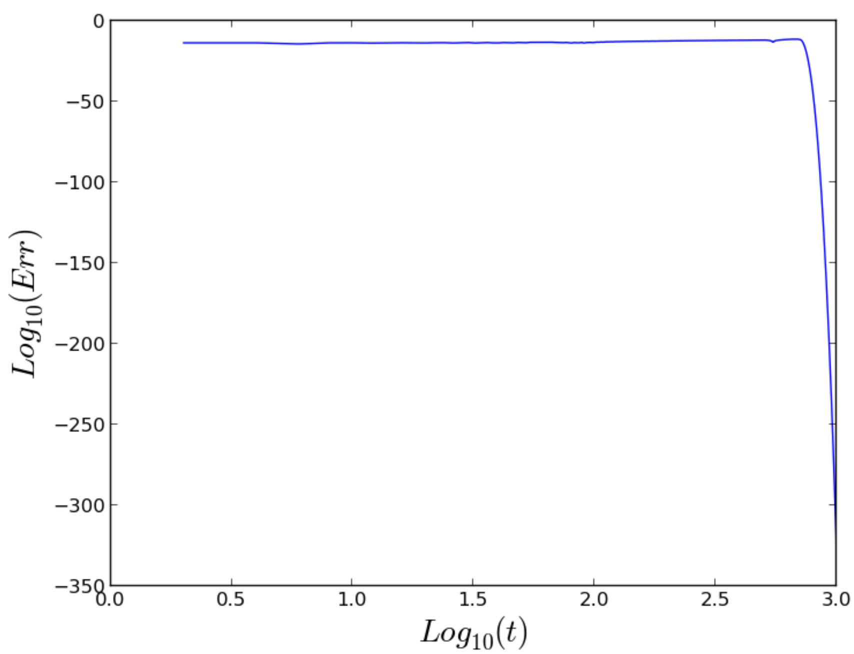

Figure 6: Error as a function of time for the narrower Gaussian.

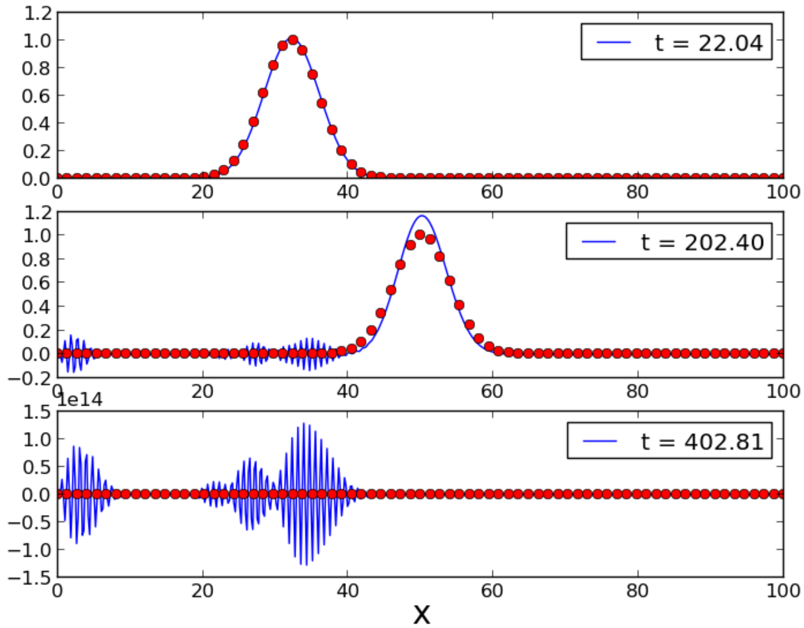

Figure 7: FTCS scheme used to advect a Gaussian.

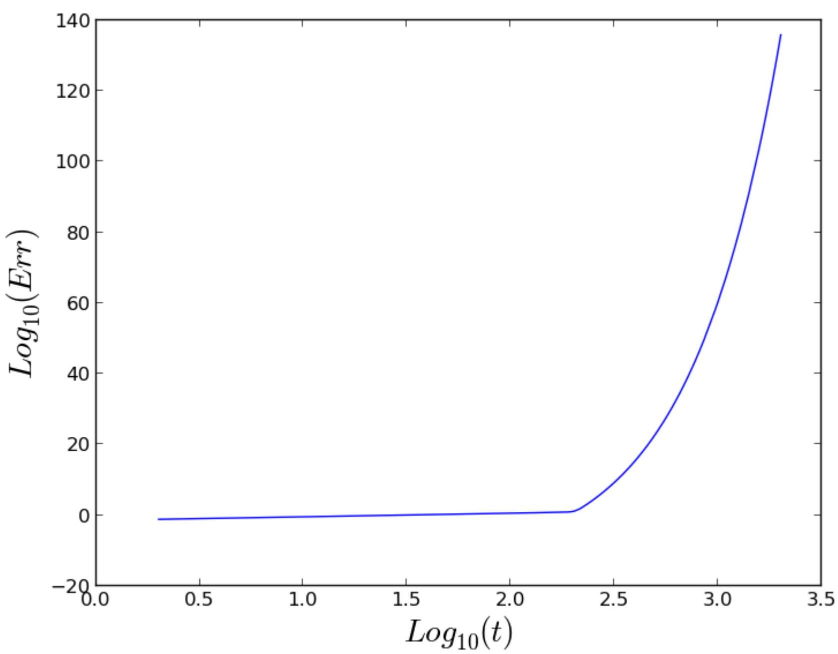

Figure 8: Error as a function of time for the FTCS scheme.