Ay190 – Worksheet 07
Donal O Sullivan
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1 Measuring π with an MC Experiment

Shown in Fig 1 is the error in the Monte Carlo estimation of π. Although it does decrease as a general trend, it has a considerably high variance.

1.1 Bonus: Better Number Generator

The random.SystemRandom() generator uses a module from the operating system package, os.urandom(n), which returns a string of 'n' random bytes which is suitable for cryptographic use. I'm not sure what exactly is uses this for but I'd guess as a seed for the random number generator, which would make the random numbers more random than using a simple or common seed value.

Though it could be due to a fault in my code, I saw no significant increase in convergence using the random.SystemRandom() generator. The results are also shown in Fig 1.

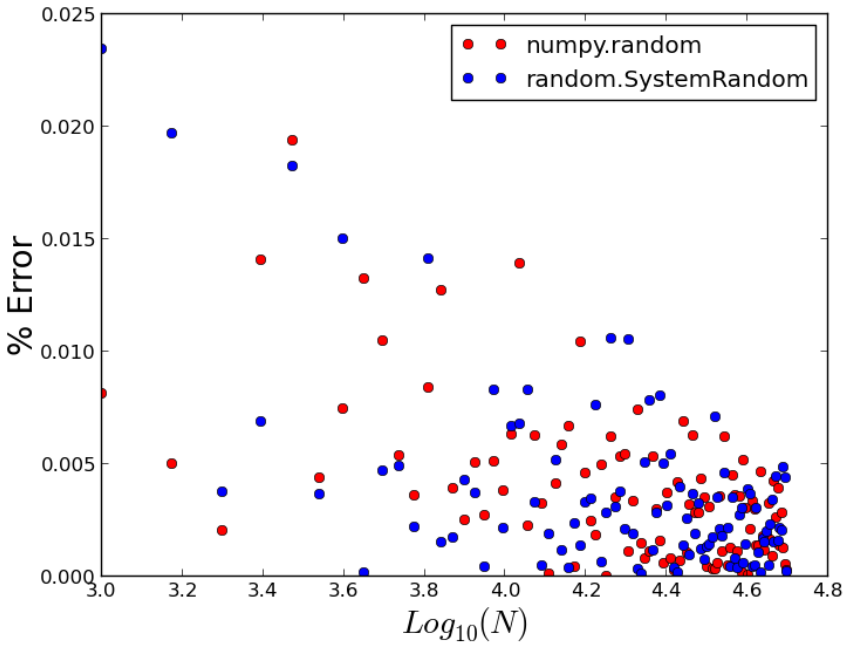

Figure 1: Error convergence on MC estimation of π using two different random number generators.

2 The Birthday Paradox

The analytical calculation of the probability, P, that at least two people within a group of size N will share a birthday is easily calculated as $P = 1 - P'$ where P' is the probability that none of them share the same birthday. $P' = \frac{365!}{(365-N+1)!(365)^N}$ The analytical and Monte Carlo probability are plotted in Fig 2. The Monte Carlo result indicates that at a group size of $N = 23$, the probability of two people having the same birthday is $P = 0.51$.

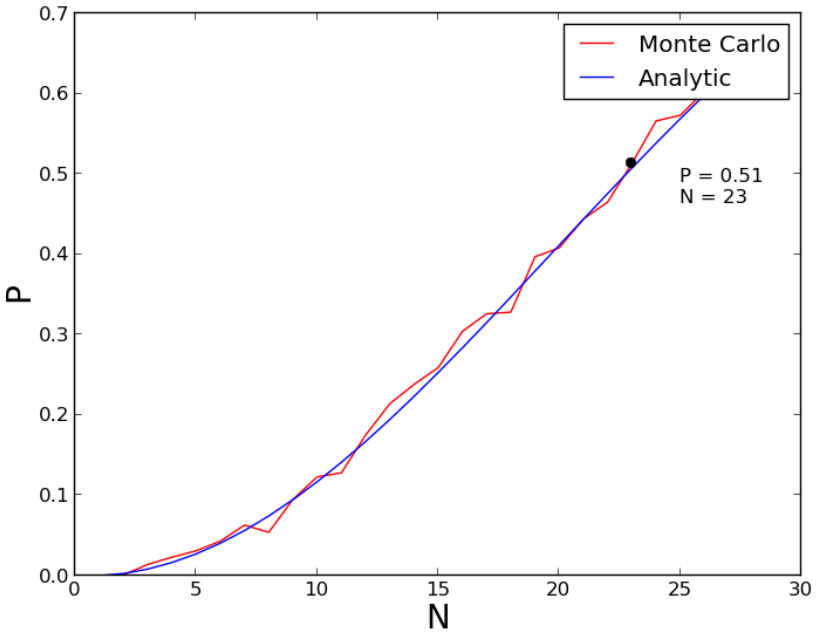

Figure 2: Probability, P, as a function of group size, N, that at least two people within the group will share a birthday, calculated both analytically and using Monte Carlo simulations.

3 BONUS: More MC Integration

Fig 3 shows the results of the Monte Carlo integration of the function $f(x) = x^2 + 1$. The error seems to converge rapidly from around $N = 10$ to $N = 3000$ and then slow down, coming to a roughly horizontal trend just below 1 percent error.

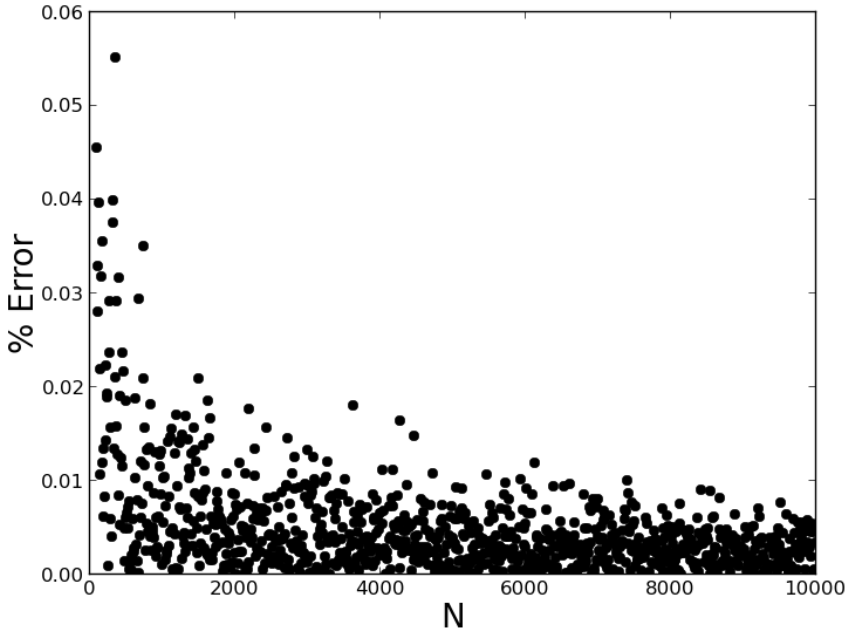

Figure 3: Convergence of Monte Carlo Integration of $f(x) = x^2 + 1$.