Ay190 – Worksheet 08
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1 ODE Integration: Simplified Stellar Structure

1.1 Inspecting the Template

The program -ignoring the missing bits- uses the Forward Euler integration method to set up the initial conditions (1-D profiles of pressure, density and mass ($P, \rho and M$) as a function of radius) for a White Dwarf (WD) under the constraints of hydrostatic equilibrium and mass conservation. In general, the code works for any Polytropic equation of state (EOS) but the values given in the code are specifically for WDs.

The program sets a max radius of $2000km$ and uses a resolution of 1000 points to create grids for $P(r), \rho(r)$ and $M(r)$. It then establishes a central pressure and density which are required to create an overall profile. The loop runs over grid points, using each set of grid points P_i, ρ_i and M_i to calculate P_{i+1}, ρ_{i+1} and M_{i+1} using numerical integration of the given ordinary differential equations (ODEs).

The loop checks every iteration whether the pressure is below a certain minimum or not and, when it is, saves that point as the surface and sets all subsequent values of pressure, density and mass to the respective surface values.

1.2 Implementing Forward Euler

Not sure what to write here except that yep, the values I got match the given values quite well. I get $M = 1.45069 M_\odot$ and $R = 1,501.5km$.

1.3 Implementing Runge-Kutta 2nd Order

I wasn't sure what the question meant by a 'rate' of 2 or 1, but I chose a plot that I think clearly demonstrates the different convergence rates of the Forward Euler and RK2 methods.

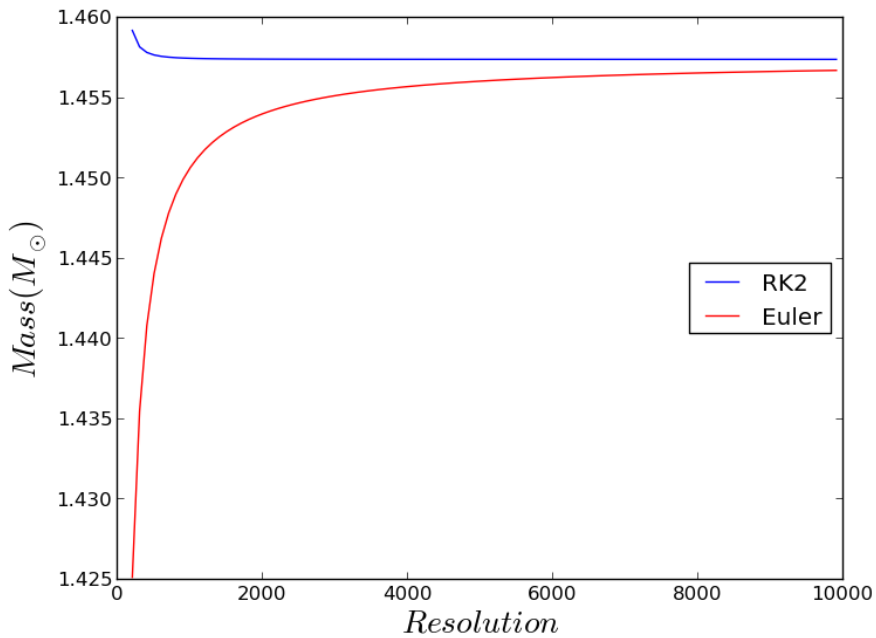

Figure 1: Showing the convergence of the Forward Euler and Runge-Kutta 2nd Order numerical integration methods.

1.4 Plotting Profiles

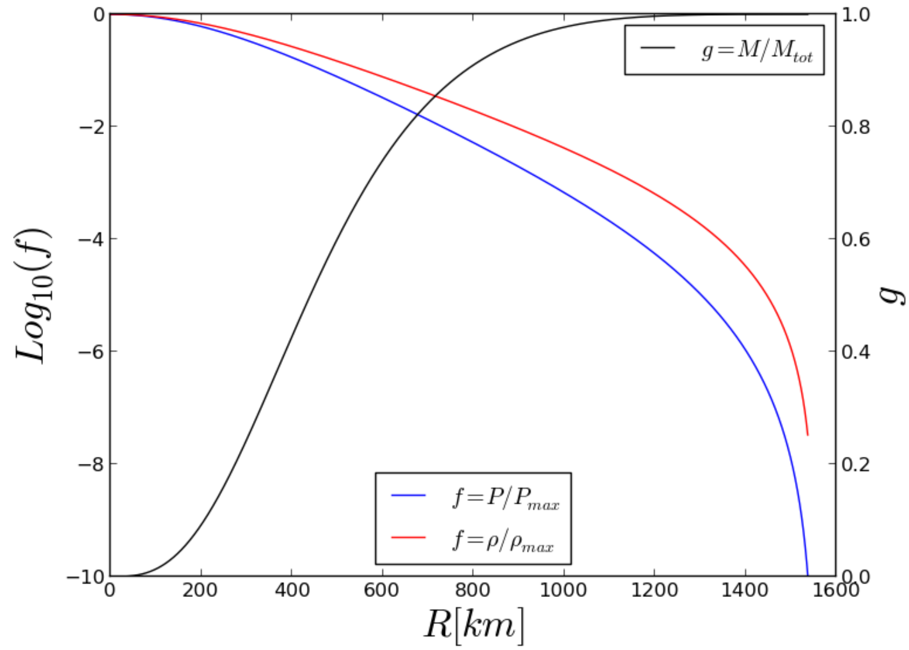

Figure 2: Plot showing pressure P, density ρ and mass M as a function of radius for the White Dwarf generated by the program. The left vertical axis shows the log scale for the two logged properties displayed, while the right axis shows the normalised scale for the mass profile.