State data agency Statistics Lithuania

Small area estimation using incomplete auxiliary information

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Motivating examples and the problem

To better estimate important population parameters in surveys with probability samples, we may want to exploit

- closely related
- but incomplete auxiliary data (non-probability samples).

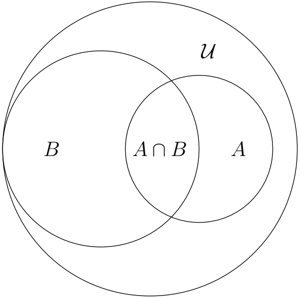
Examples:

1. *Administrative* VAT turnover data for monthly Short-term statistics surveys on the turnover of business enterprises.
2. Investment in tangible assets data from a *cut-off sample* of the Structural Business Statistics for the annual investment survey.
3. *Scraped* online job advertisement data for job vacancies measured in the quarterly statistical survey on earnings.

Problem: to properly integrate samples of non-probability origin for estimation in population domains (areas).

Basic setup

Data structure:
$\mathcal{U} = \{1, \dots, N\}$ is a finite population,
$A \subset \mathcal{U}$ is a probability sample of size n,
$B \subset \mathcal{U}$ is a non-probability sample of size N_B, and the set $A \cap B$ is abundant.



Data model:
The values y_i, $i \in A$, and contaminated values y_i^*, $i \in B$, are observed. Auxiliary vector values $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})'$, $p \geq 1$, $i \in \mathcal{U}$, are available.

A model for a similarity of y to y^*:

$$y_i = f(y_i^*, \mathbf{x}_i) + \epsilon_i, \quad i \in B. \qquad (\mathcal{M})$$

Choices of measurement error model $(\mathcal{M})$ can be:

- linear regression;
- non-linear parametric model;
- non-parametric model.

Direct estimation in population domains

- Let $\mathcal{U} = \mathcal{U}_1 \cup \cdots \cup \mathcal{U}_M$ be the partition of the population into M non-overlapping domains, where $\mathcal{U}_m$ is of size N_m.
- We aim to estimate the domain totals

$$t_m = \sum_{i \in \mathcal{U}_m} y_i, \quad m = 1, \ldots, M.$$

- The probability samples $A_m = A \cap \mathcal{U}_m$ are of sizes $n_m \leq N_m$.
- If the sizes N_m are known, one can apply the direct estimators

$$\hat{t}_m^{\mathrm{H}} = \frac{N_m}{\widehat{N}_m} \sum_{i \in A_m} d_i y_i \quad \text{with} \quad \widehat{N}_m = \sum_{i \in A_m} d_i, \quad m = 1, \ldots, M,$$

  of the totals t_m, where $d_i = 1/\pi_i$ are design weights and π_i are the inclusion probabilities by the sampling design $\mathrm{p}(\cdot)$.
- The variances $\psi_m^{\mathrm{H}} = \mathrm{var}_{\mathrm{p}}(\hat{t}_m^{\mathrm{H}})$ may be too large for small n_m.

Elements of our methodology (main literature)

1. A stratification of $\mathcal{U}$ into B and $\mathcal{U}\backslash B$ by Kim & Tam (2021) suggests treating B as complete auxiliary information.
2. We extend the calibration estimation of Kim & Tam (2021) to the model-calibration approach of Wu & Sitter (2001).
3. The model-calibrated domain estimates are further modeled using, for example, the Fay & Herriot (1979) model.

Kim, J.-K., Tam, S.-M. (2021). Data integration by combining big data and survey sample data for finite population inference. *Int. Stat. Rev.* 89:382–401.

Wu, C., Sitter, R.R. (2001). A model-calibration approach to using complete auxiliary information from survey data. *JASA* 96:185–193.

Fay, R.E., Herriot, R.A. (1979). Estimates of income for small places: an application of James-Stein procedures to census data. *JASA* 74:269–277.

Model-calibration approach

The model $(\mathcal{M})$ is fitted using the data $(y_i, y_i^*, \mathbf{x}_i)$, $i \in A \cap B$. Let $\hat{y}_i$, $i \in B$, be the predictions of y_i obtained from the fitted model.

The model-calibration approach by Wu & Sitter (2001) means to find the weights w_i, $i \in A$, in

$$\hat{t}_m^{\mathrm{MC}} = \sum_{i \in A_m} w_i y_i, \quad m = 1, \ldots, M,$$

minimizing the distance measure

$$\Phi_m = \sum_{i \in A_m} d_i \left(\frac{w_i}{d_i} - 1 \right)^2,$$

for each $m = 1, \ldots, M$, subject to certain area-specific *calibration constraints* as in Kim & Tam (2021) but where auxiliary data are used through the fitted values $\hat{y}_i$, $i \in B$.

Calibration constraints for incomplete auxiliary data

Let us introduce the indicator variable

$$\delta_i = \begin{cases} 1 & \text{if } i \in B, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that all intersections of the sets A_m and $B_m = B \cap \mathcal{U}_m$ are neither empty nor too small.

For each $m = 1, \ldots, M$, we find the weights $\{w_i,\ i \in A_m\}$ by minimizing the distance Φ_m subject to the calibration constraints

$$\sum_{i \in A_m} w_i \delta_i = N_{B_m}, \quad \sum_{i \in A_m} w_i \delta_i \hat{y}_i = \sum_{i \in B_m} \hat{y}_i,$$

and

$$\sum_{i \in A_m} w_i (1 - \delta_i) = N_m - N_{B_m},$$

where N_{B_m} is the size of the non-probability sample subset B_m.

Note: the auxiliary variable y^* is already integrated.

The data for the Fay–Herriot (FH) model (Fay & Herriot, 1979):

- The model-calibrated estimators $\hat{t}_m^{\mathrm{MC}}$ are treated as the direct estimators because they are approximately design-unbiased under certain conditions (Wu & Sitter, 2001).
- Estimators $\tilde{\psi}_m^{\mathrm{MC}}$ of the variances $\psi_m^{\mathrm{MC}} = \mathrm{var}(\hat{t}_m^{\mathrm{MC}})$.
- Exactly known area-level covariates $\mathbf{z}_m = (z_{m1}, \ldots, z_{mq})'$, $q \leq p$, selected from aggregates of auxiliary data $\mathbf{x}_i$, $i \in \mathcal{U}_m$.

The standard FH model is the linear mixed model

$$\hat{t}_m^{\mathrm{MC}} = \mathbf{z}_m' \boldsymbol{\beta} + v_m + \varepsilon_m, \quad m = 1, \ldots, M,$$

where $\varepsilon_m \overset{\text{ind}}{\sim} \mathcal{N}(0, \psi_m^{\mathrm{MC}})$ are sampling errors, $v_m \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_v^2)$ are random area effects independent of ε_m, and $\boldsymbol{\beta}$ are fixed effects.

EBLUP based on the FH model

The empirical best linear unbiased predictions (EBLUPs) of the domain totals t_m, $m = 1, \ldots, M$, are expressed as the linear combinations

$$\hat{t}_m^{\mathsf{FH}} = \hat{\gamma}_m \hat{t}_m^{\mathsf{MC}} + (1 - \hat{\gamma}_m)\mathbf{z}_m' \hat{\boldsymbol{\beta}} \quad \text{with} \quad \hat{\gamma}_m = \frac{\hat{\sigma}_v^2}{\tilde{\psi}_m^{\mathsf{MC}} + \hat{\sigma}_v^2},$$

and

$$\hat{\boldsymbol{\beta}} = \left(\sum_{m=1}^{M} \frac{\mathbf{z}_m \mathbf{z}_m'}{\tilde{\psi}_m^{\mathsf{MC}} + \hat{\sigma}_v^2} \right)^{-1} \sum_{m=1}^{M} \frac{\mathbf{z}_m \hat{t}_m^{\mathsf{MC}}}{\tilde{\psi}_m^{\mathsf{MC}} + \hat{\sigma}_v^2},$$

where $\hat{\sigma}_v^2$ is an estimator of the variance σ_v^2 of random area effects.

Note: for skewed data, a variance-stabilizing transformation – such as the logarithm – might be applied if needed.

Monthly turnover example

Aim – to improve direct *GREG* estimates of total turnover in NACE activity groups of service enterprises.

Main variables:

- y_i – monthly turnover;
- y_i^* – corresponding turnover derived from VAT data;
- x_{i1} – previous year's annual turnover.

Sample B size relative to A: $N_B/n \approx 6.2$.

Measurement error model $(\mathcal{M})$ – *linear regression* since y and y^* are continuous variables.

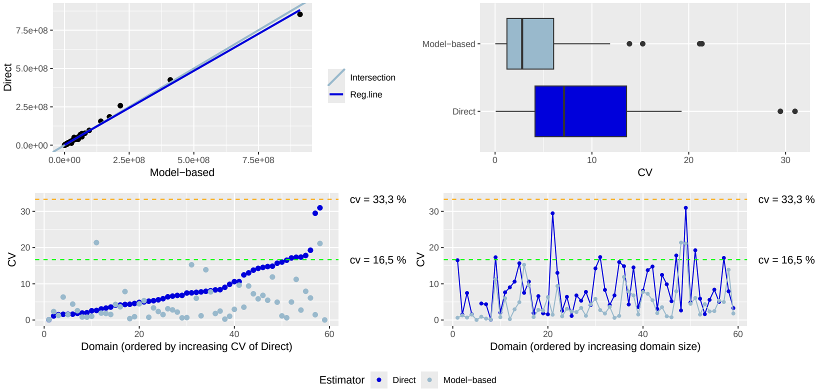


Figure 1(a): Comparison of direct estimates and EBLUPs for 2024 M11.

Good (CV $\leqslant$ 16.5%), sufficient (16.5% $<$ CV $\leqslant$ 33.3%), unreliable (CV $>$ 33.3%).

Monthly turnover example | multiple months

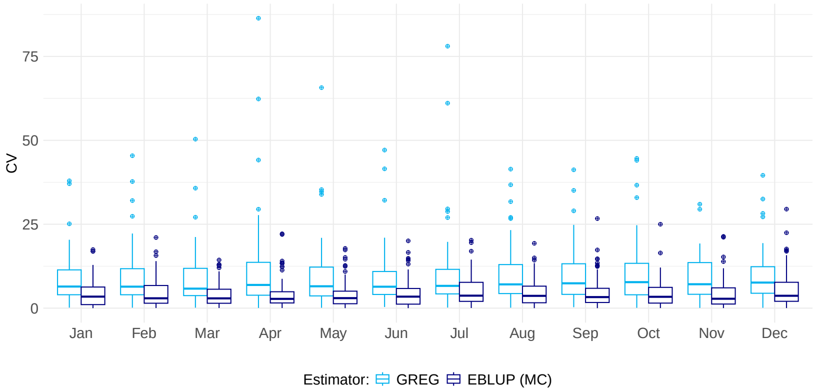


Figure 1(b): Comparison of direct GREG estimates and EBLUPs.

Annual investment example

Aim – to improve direct Horvitz–Thompson estimates of total investment in tangible assets in NACE activity groups.

Main variables:

- y_i – annual investment (sum over quarters);
- y_i^* – corresponding investment from a cut-off sample of the Structural Business Statistics;
- x_{i1} – previous year's annual investment from the Structural Business Statistics.

Sample B size relative to A: $N_B/n \approx 1.1$.

Measurement error model $(\mathcal{M})$ – *two-part approach:*
(a) logistic regression for the probability of zero;
(b) linear regression for the positive part;
since y and y^* are continuous variables with many zeros.

Annual investment example | a single year

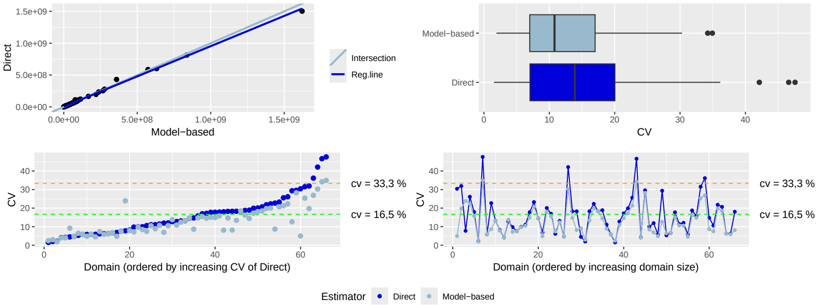


Figure 2(a): Comparison of direct estimates and EBLUPs for 2023.

Good (CV $\leqslant$ 16.5%), sufficient (16.5% $<$ CV $\leqslant$ 33.3%), unreliable (CV $>$ 33.3%).

Annual investment example | a single year

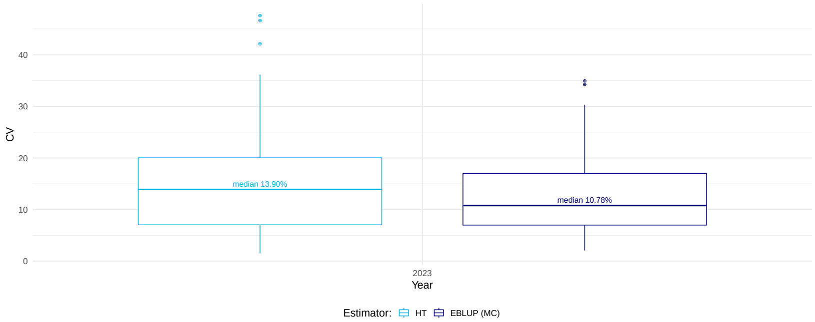


Figure 2(b): Comparison of Horvitz–Thompson estimates and EBLUPs.

Quarterly job vacancies example

Aim – to improve direct Hájek estimates of municipal job vacancies.

Main variables:

- y_i – job vacancies at the end of the quarter;
- y_i^* – corresponding online job advertisement information;
- x_{i1} – last month's number of employees.

Sample B size relative to A: $N_B/n \approx 1.8$.

Measurement error model $(\mathcal{M})$ – *non-parametric k-nearest-neighbors imputation* since y and y^* are count variables with many zeros.

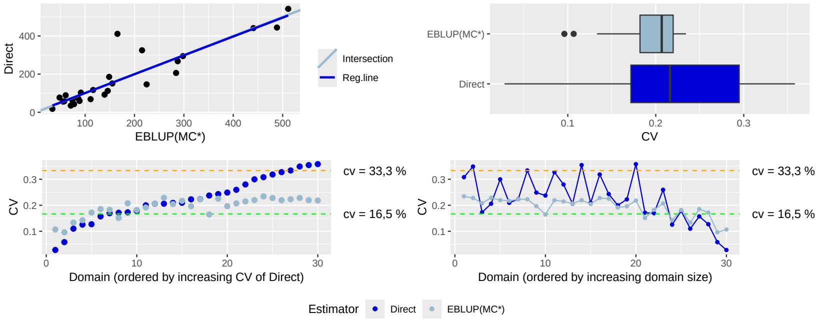


Figure 3(a): Comparison of direct estimates and EBLUPs for 2024 Q2.

Good (CV $\leqslant$ 16.5%), sufficient (16.5% $<$ CV $\leqslant$ 33.3%), unreliable (CV $>$ 33.3%).

Quarterly job vacancies example | multiple quarters

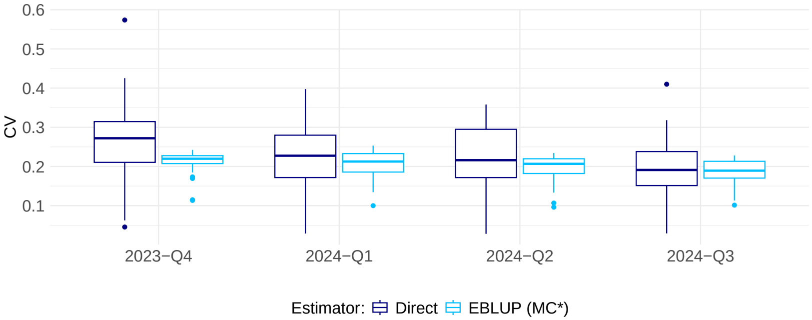


Figure 3(b): Comparison of direct Hájek estimates and EBLUPs.

Conclusions and suggestions

- Important variables for policy decisions often have analogs in other data sources, but these external data cover only a part of the target population. Our methodology provides a way to address this issue.
- The examples show that the improvement depends on how large the non-probability sample is relative to the probability sample.
- As a second step, for key variables, estimation in domains might be considered through small area estimation techniques.

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Thank you!

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