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Effective Short-Term Forecasting for Daily Time Series with Complex Seasonal Patterns

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Abstract

Time series forecasting is a process of estimating future value based on historical data and it plays a crucial role in business decision making in various domains. The selection of a suitable time series forecasting technique depends upon the presence of the following four components: trend, seasonal, cyclical and irregular. Traditional time series techniques like ARIMA, SARIMA, ETS are designed to handle single seasonality in a time series, but with theexistence of multiple seasonality, these techniques fail to perform satisfactorily. Thus, there is a need to use advanced techniques like BATS and TBATS for multiple seasonal data. The main objective of this paper is to develop a successful prediction model after comparing BATS and TBATS models for short-term forecasting of complex time series. The daily time series of natural gas consumption for a manufacturing unit of BHEL, India that exhibits multiple seasonality is used for evaluation purpose. The results of the analysis conclude that TBATS outperforms BATS model for a span of different short-term prediction horizon. The main reason behind is be attributed to thefact that less number of parameters is required to be estimated in TBATS model in comparison to the BATS model.

Keywords: Time Series Analysis; Complex Seasonality; Multiple Seasonality; BATS; TBATS.

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Nomenclature

BATS	Exponential smoothing state space model with Box-Cox transformation, ARMA errors, Trend and Seasonal components
TBATS	Trigonometric Exponential smoothing state space model with Box-Cox transformation, ARMA errors, Trend and Seasonal components
ARMA	Autoregressive moving average model
ARIMA	Autoregressive integrated moving average model
SARIMA	Seasonal Autoregressive integrated moving average model
ETS	Error, Trend, Seasonality model

1. Introduction

Time series modeling is an active research area. Time series modeling aims on study of previous observation on the basis of collected data andestablish an appropriate model that is used to predict future outcomes for the series, i.e.forecast. Act of predicting the future on the basis of previous history can also be defined as time series forecasting. Many disciplines like business, finance, science, engineering etc are using time series modeling to forecast future values and use it for decision making. A time series forecasting model has four components namely seasonal, trend, cyclic and random. The trend component shifts up or down over a long period of time. A cyclical component belongs to longer cycles than the seasonal components and the random component does not comply with any of the above three classes[1]. Seasonality refers to predictable and recurring trends and patterns over a period of time. The pattern repetition may be a day, week, month, quarter etc. These seasonal variations important for future decision making.

A time series can have single or complex seasonal pattern [2]. Complex patterns include non-integer seasonality, multiple seasonal periods and dual-calendar effects. Non-integer seasonality represents that the value of seasonality in time series data doesn't represent the whole integer number. For instance, the monthly seasonal data can't be represented by 30 integer values because the month of February consists of only 28 days while other months are of 30 days or 31 days. This results in the average month length of 30.4 days. Multiple seasonal periods means that data contains more than one seasonality such as Daily time series data may contain weekly and annually seasonality. Dual-calendar effects reflect the role of different calendars in different countries (like India follows both Hindu and Gregorian calendar) and their impact on forecasting.

Traditional times series methods like Naïve method [3], Drift method [4], Simple Exponential Smoothing (SES) [5], Holt method [6], Holt method with drift [7], ETS(Error, trend, seasonal) method [8], and ARIMA [9] are successfully used to model univariatetime series. ETS and SARIMA[10] are the popular time series method to handle a single seasonal pattern but fail to perform satisfactorily when there exist complex seasonal pattern in the time series. Some variants of these approaches can handle seasonality, but seasonality is forced to be periodic.

In today’s world, a time series with thecomplex seasonal pattern is a common phenomenon. For instance, the hourly or daily electricity consumption, natural gas consumption for an organization, call arrival rate at call centers etc don’t have periodic seasonality, rather it contains dynamic seasonality. Considering this fact, the objective of this paper is to develop the most effective short-term univariate forecasting model using BATS and TBATStechniques to predict complex seasonal pattern[11]. The developed model will be tested on the natural gas consumption data provided by CFFP (Central Foundry Forge Plant)[12], a manufacturing unit of BHEL (Bharat Heavy Electricals Limited)[13], India. CFFP is engaged in manufacturing heavy weight castings and forgings. It uses natural gas as one of the fuels.

The remaining part of the paper is organized as follows. Section 2 contains the discussion on relevant work in the area of forecasting with complex seasonal components followed by section 3 that includes the framework of the forecasting model. Section 4 discusses the efficiency of various models followed by conclusion in section 5.

2. Related Work

Most existing literature available for natural gas consumption forecast includes traditional forecasting methods such as regression, ARIMAand neural network [14]. An individual customerforecasting has been performed with the help of nonlinear regression model [15]. For external process a comparative study for non-linear mixed effects model is provided with anAuto-regressive integrated moving average[16]. A separation model is obtained for specific and common time-varying part by using semi parametric regression model [17]. Iranian natural gas demand prediction is shown by applying adaptive-network-based fuzzy inference system [18].Use of multivariable regression analysis is provided to evaluate the factors affecting gas demands by dividing years into heating and non-heating seasons and use degree day method for theforecast [19]. ARIMA models study is done for natural gas consumption forecasting in Turkey [20]. An autoregressive integrated moving average (ARIMA) model is discussed with historical fuel cost data in thedevelopment of a three-step-ahead fuel cost distribution prediction [21]. A forecasting proposal for shortterm load has been made for natural gas using deep neural network [22]. Study of Holt-Winters exponential smoothing and autoregressive integrated moving average (ARIMA) methods is performed based on time series decomposition [23].Research work addressed the problem of complex seasonality in the natural gas consumption forecasting domain [24].

Seasonality has been explained for time series data [25, 26, 27, 28, 29], Comparing these methods, extended the double seasonal Holt-Winters' method and the DS method in order to accommodate a third seasonal pattern by including a third seasonal component in the model. It is shown that triple seasonal methods outperformed the double seasonal methodsfor short-term predictions using six years of British and French data. Introduction of BATS and TBATSmodels has been provided for time series with complex seasonality [11]. Forecasting for theprice of gold is shown in [30]. Forecasting has also been done for electricity data in [31]. No research is applied to natural gas consumption domain and hence in this paper, these models have been applied to forecast natural gas consumption.

3. Methodology: Forecasting Model with Complex Seasonality

The BATS and TBATS are the two innovations state space modeling framework [10, 11] that can handle complex seasonal time series variations. BATS is a linear homoscedastic model, incorporating non-linearity by integrated Box-Cox transformation [32] and a stationary ARMA [33] representation is utilized in order to capture any autocorrelation in the residuals. Let G_t, $t \in N$ denote an observed natural gas consumption time series. The notation $G_t^{(w)}$represents observations with Box-Cox transformation having parameter ω, where G_t is the observation at time t. After the transformation of time series, decomposition results into a level component l_t, a growth component b_t, an irregular component d_t, and possible seasonal ones $s_t^{(i)}$with seasonal frequencies m_i, for $i = 1,...,M$, where M is overall seasons in the time series. ARMA (p,q) define the irregular component d_t with parameters φ_i for $i = 1,...,p$ and θ_i for $i = 1,...,q$, and $\varepsilon_t$represents an error term which is considered to be Gaussian white noise [34] process with zero mean and constant variance σ. The α, β, γi, represent smoothing parameters, for i = 1,...,M, and the damping parameter ϕ, determine the irregular component effect on the l_k, $b_k$and$s_k^{(i)}$.The notation BATS(w, p, q, ϕ, m1,...mM) is used for these models. w is the Box-Cox parameter and ϕ is the damping parameter; the error is modeled as an ARMA (p,q) process and $m_1,...,m_M$ list the seasonal periods used in the model. BATS modelis not capable of handling non-integer seasonality and has a very large number of parameters that require estimation. The total number of initial seasonal values required for BATS model is m1 + m2 +............+mM.Mathematically the model is described by the equations 1 to 6:

TBATS also uses Box-Cox transformation for non-linearity and an ARMA representation for capturing any autocorrelation in the residuals i.e equation 1, 2, 3, 4 and 6 is same for TBATS model.γ_{1i} and γ_{2i} are identified as smoothing parameters and $\lambda_j^{(i)} = 2\pi\, j/m_i$. The number of harmonics for the ith seasonal component is defined by ki. Separate values of $k_i$are defined for even values of mi, $k_i = m_i/2$, and for odd values of mi. $k_i = (m_i - 1)/2$. The notation TBATS(w, p, q, ϕ, $\{m_1, k_1\}$, $\{m_2, k_2\}$..........$\{m_M, k_M\}$) is used for these models. Where w is the Box-Cox parameter and phi is the damping parameter; the error is modeled as an ARMA (p,q) process, $m_1,...,m_M$ list the seasonal periods and $k_1,...,k_M$are the corresponding number of Fourier terms for each seasonality used in the model. A TBATS model needs initial seasonal values to be estimated as $2(k_1 + k_2 + \cdots + k_T)$, that is smaller as compared to parameters needed by BATS models. The equations 7 to 9 of the TBATS model are:

$$G_t^{(w)} = \begin{cases} \dfrac{G_t^{\,w} - 1}{w} & w \neq 0 \\ \log y_t & w = 0 \end{cases} \tag{1}$$

$$G_t^{(w)} = l_{t-1} + \phi b_{t-1} + \sum_{i=1}^{i=M} s_{t-m_i}^{(i)} + d_t \tag{2}$$

$$l_t = l_{t-1} + \phi b_{t-1} + \alpha d_t \tag{3}$$

$$b_t = \phi b_{t-1} + \beta d_t \tag{4}$$

$$s_t^{(i)} = s_{t-m_i}^{(i)} + \gamma_i d_t \tag{5}$$

$$d_t = \sum_{i=1}^{i=p} \varphi_i d_{t-i} + \sum_{i=1}^{i=q} \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{6}$$

$$s_t^{(i)} = \sum_{j=1}^{j=k_i} s_{j,t}^{(i)} \tag{7}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_{1i} d_t \tag{8}$$

$$s_{jt}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_{2i} d_t \tag{9}$$

Any forecasting technique needs to be evaluated for prediction accuracy. Accuracy measures check the errors in between obtained values and actual values during prediction. Root mean squared error (RMSE) [35] and Mean absolute percentage error (MAPE) [36] measures are commonly used to measure the accuracy of forecast method. To choose the best method to forecast the natural gas data we perform the forecasting with both the BATS and TBATS model. RMSE and MAPE error components have been computed between the test data and obtained forecasted value. The model that provides minimum values for both the error component is chosen for prediction of future value.The complete process of model selection is depicted in figure 1.The error components RMSE and MAPE are denoted by following formulas ify_t denote the t^{th} observation,$\hat{y}_{t|t-1}$ denote its forecast value based on previous values, where t =1,...,T.The formulas for errors are in equation 10 and 11.

$$RMSE = \sqrt{T^{-1} \sum_{t=1}^{t=T} \left(y_t - \hat{y}_{t|t-1} \right)^2} \tag{10}$$

$$MAPE = 100 T^{-1} \sum_{t=1}^{t=T} \frac{\left| y_t - \hat{y}_{t|t-1} \right|}{\left| y_t \right|} \tag{11}$$

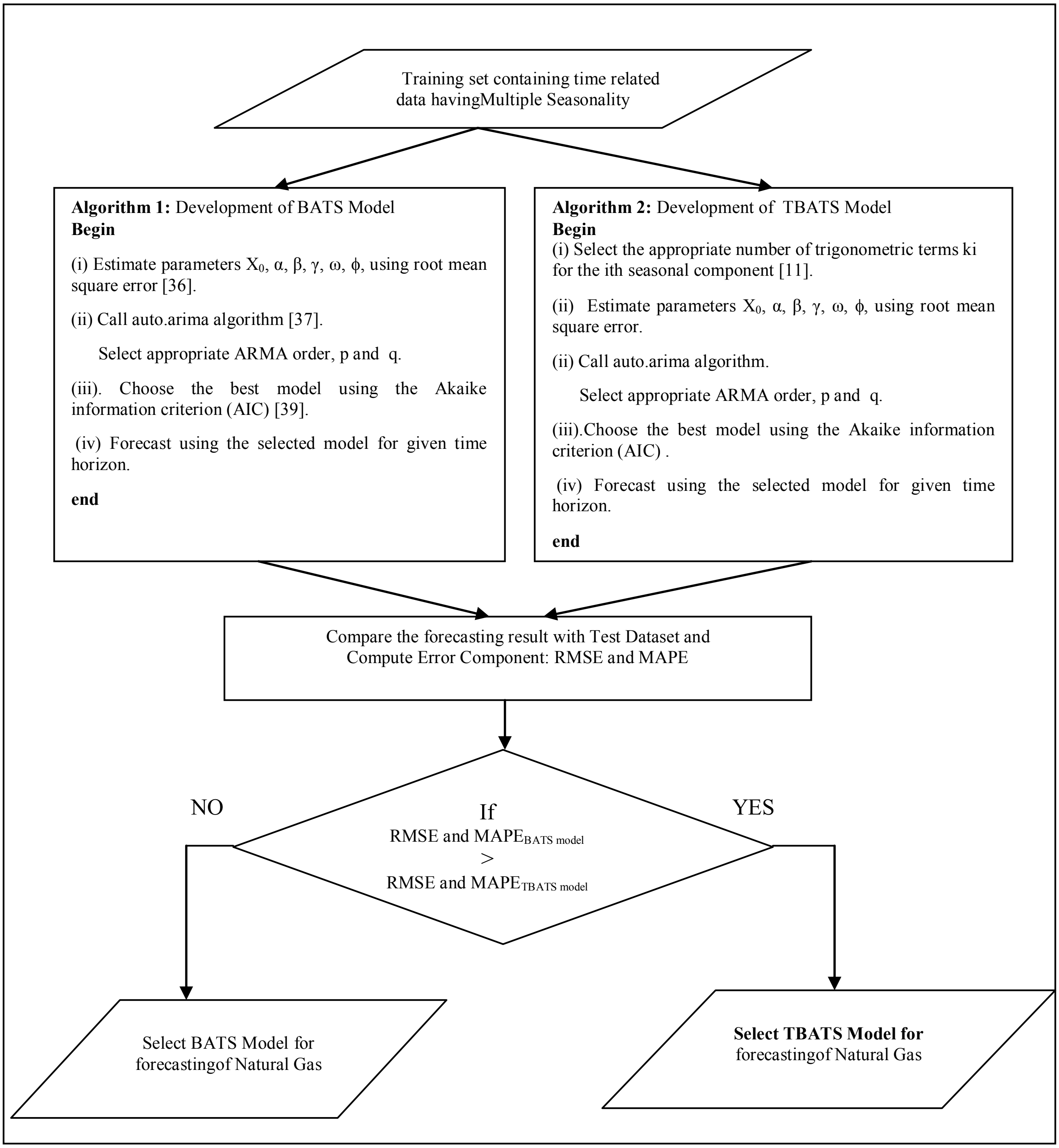

Fig. 1. Framework for Model Selection

4. Experiments and Results

The developed model is tested on the data provided by Central Foundry Forge Plant (CFFP), BHEL, India. A monthly dataset is prepared by collecting natural gas consumption data from CFFP, a manufacturing unit of Bharat

Heavy Electricals Limited, India about their consumption of natural gas for production of Castings products from year April 2014 to January 2017 and the time series is drawn in figure 2.

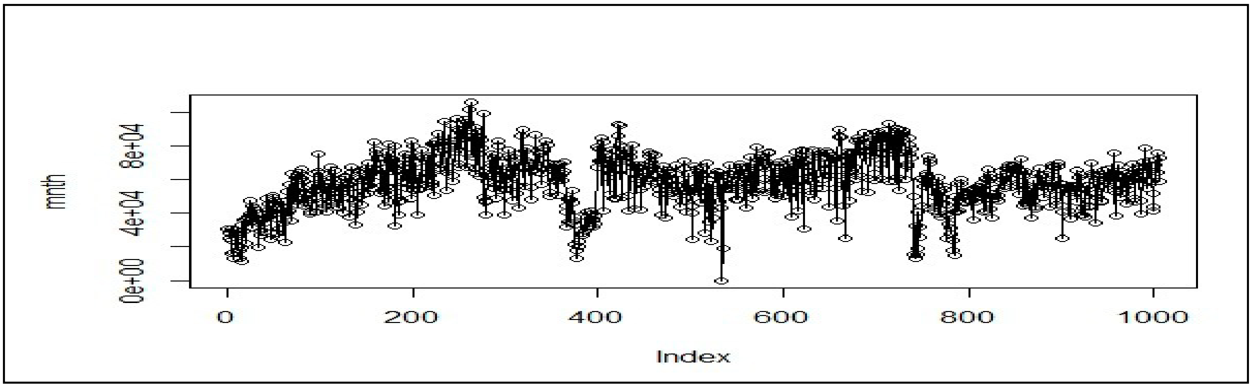

Fig. 2. Monthly natural gas consumption in CFFP, BHEL

For any forecasting method, in sample period and out-of-sample period should be defined. The dataset that is used for identifying the relationship between the data is termed as training set or in the sample period. On the other hand test set or out-sample period is the data set that is used to examine the strength of the predicted relationship. From figure 1, it is obtained that the dataset is the collection of 1006 days observations.

1. Training set: Day1 to Day 800 used for model development.
2. Test set: Day 801 to Month 1006 used for training Purpose.

To further analyze the randomness of the considered data, auto correlation (ACF) plot and partial auto correlation (PACF) plot as shown in figure 3were created. This is used to measure the relationship between lagged values of time series. At consecutive lags the value is significant, representing the strong correlation between successive values.

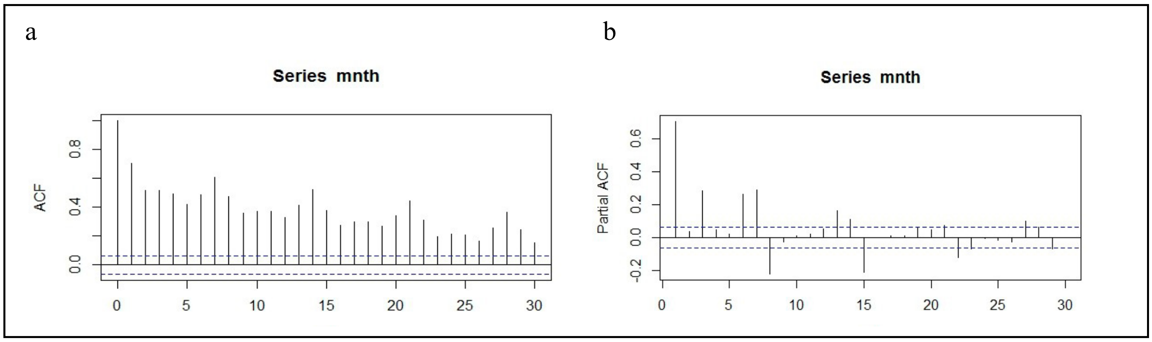

Fig. 3. (a) ACF Plot (b) PACF plot

From ACF and PACF plot, a clear seasonality at 7, 14, 21 and so on is evident. It indicates the seasonality of length 7 or weekly seasonality exists in data. As the natural gas consumption series is a daily data series, it also contains monthly seasonality(average month length of 30.4 days) and annual seasonality (average annual year length of 365.25 days). Therefore the existing frequencies in natural gas consumption series are m1=7, m2= 30.4, m3=365.25. These frequencies represent weekly, monthly and annual seasonality respectively. Now BATS and TBATS models have been applied to this data and perform error analysis. In applying the BATS(w, p, q, ϕ, m_1,...m_J)

and TBATS(p,q,$\{m_1,k_1\},\{m_2,k_2\},...,\{m_M,k_M\}$) model as discussed in section 3, on the natural gas consumption data of CFFP, BHEL, the obtained model structures are represented in table 1.

Table 1. Structure of BATS and TBATS models

Data	BATS Model	TBATS Model
Model	BATS (1, {5,3}, 0.94, {7, 30, 365})	TABTS (1, {1,2}, -, {<7, 3>, <30.4, 4>, < 365.25, 4>})
Initial Seasonal Values	402	22
Parameter	α: 0.57653	α : 0.02959359
	β: -0.02165474 ϕ: 0.939613 γ: -0.009506386 -0.03321486 -0.6301767 p:0.160883, 0.12109, 0.085695, 0.025865, -0.010232 q: -0.254549, -0.284276, -0.105684	γ1: 0.00105814, -0.003042091, -0.0010745 γ2: 0.0005051707, 0.001803545, -0.0008674448 p: 0.85559 q: -0.31464, -0.178581

The BATS and TBATS model were applied to the training data set repeatedly. The obtained BATS (1, {5,3}, 0.94, {7, 30, 365}) model represents w=1 i.e. no Box-Cox transformation was included in the model, the order of ARMA error are (5, 3) and the damping parameter is 0.93 near to1. The total number of initial seasonal values are (7+ 30 + 365 is equal to 402. The obtained TABTS (1, {1,2}, -, {<7, 3>, <30.4, 4>, < 365.25, 4>}) model represents w=1 i.e. no Box-Cox transformation, the order of ARMA error are (1, 2) , the no damping parameter. The number of harmonics are k1 =3,k2 =23 and k3 =3. The total number of initial seasonal values are 2(3+ 4 + 4) is equal to 22. As it can be seen that estimation of initial seasonal values in TBATS model is very less in comparison to BATS model, TBATS model turns out to be less complex and its execution time will also be less.

A time series is decomposed into its components having latent subseries provides a key role in more realistic, retrospective analysis of the respective time series for several reasons. Firstly, decomposition can be employed as a preliminary stage in choosing an appropriate forecasting model. Secondly, an efficient decomposition method provides graphical insights into the behavior of a time series, thus leading to the identification of possible causes of any variation in the structure of a series. Thirdly, decomposing a time series allows inferences to be drawn about patterns of change over time, leading to physically interpretable latent processes underlying the data. Decomposition of natural gas consumption data from BATS and TBATS model are represented in figure 4.

The decomposition obtained from the BATS (1, {5,3}, 0.94, {7, 30, 365}) model is represented in six parts. The first part shows the observed data. Second and third part displays the level and slope respectively. Rest of the three parts define seasonality in the natural gas consumption data. Season1, season2 and season3 show weekly, monthly and yearly seasonality respectively. The decomposition generated from the TBATS (1, {1,2}, -, {<7, 3>, <30.4, 4>, < 365.25, 4>}) model is partitioned in five parts. The first part shows the observed data. Second displays the trend component of data. The rest of the three parts define seasonality in the natural gas consumption data. Season1, season2 and season3 show weekly, monthly and yearly seasonality respectively.

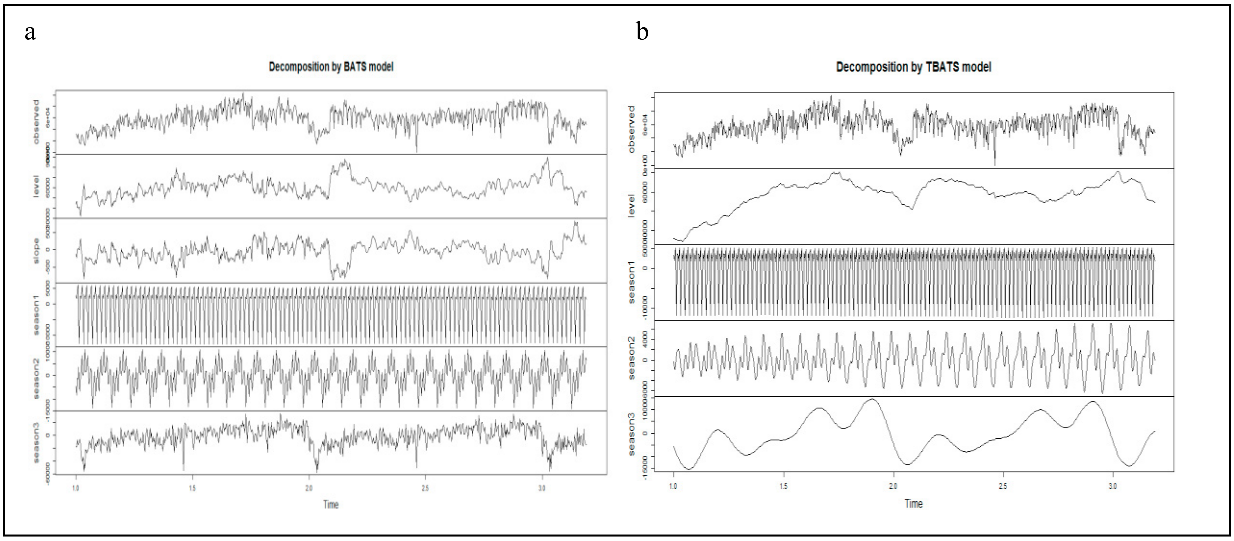

Fig. 4. (a) Decomposition from BATS model; (b) Decomposition from BATS model

The monthly seasonal pattern seems to evolve with time, while the weekly seasonal pattern stays relatively more stable. Large variation can be seen in the components obtained from BATS model (figure 4) in comparison of the components from TBATS model this is due to a large number of the parameter used by BATS model. Now by applying these two models on training data, out of sample forecast values to ten different forecasting horizon as h=1, 24, 48, 72, 96, 120, 144, 168, 192 and 206 have been generated. The generated forecast value at h=24, i.e. 24 days ahead in figure 5 gives a graphical representation of the comparison of forecast values and actual test data values.

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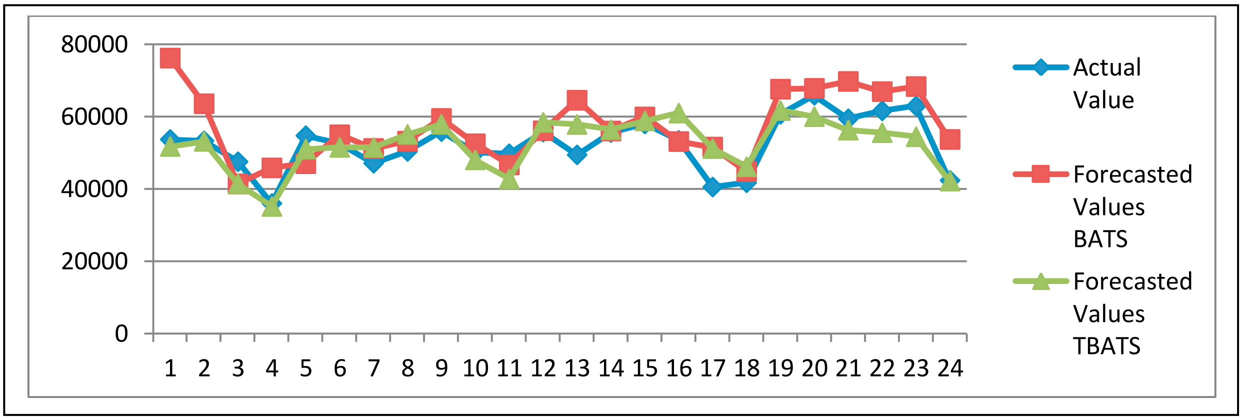

Figure 5: Comparison of 24 days ahead forecast values from BATS and TBATS model

The results show forecasted values obtained from TBATS model is near to the actual test data. The forecasted values obtained from BATS model have a significant difference as compared to actual test data set. From the above figure, a clear comparison can be derived from the actual test data and forecasted values generated from BATS and TBATS model. Thus, TBATS model performs better BATS model when there is the presence of complex seasonality.

Finally, to further ensure the accuracy of the models, an error analysis has been done to check best performing model between these two models. For this purpose, two error components RMSE and MAPE(explained in section 3) has been computed to check the out-of-sample performances for different forecasting horizon as h=1, 24, 48, 72, 96, 120, 144, 168, 192 and 206. Figure 6 represents the comparison between RMSE and MAPE error components obtained from BATS and TBATS model.

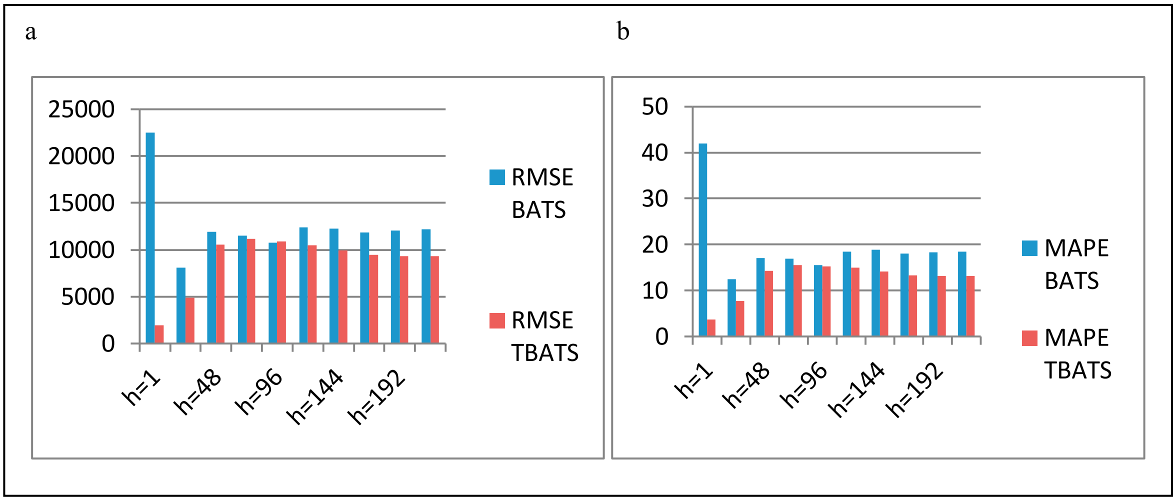

Figure 6: Comparison of average RMSE values from BATS and TBATS model

The minimum value of error components RMSE and MAPE is the significance of better forecasting. It has been observed that value of RMSE and MAPE provided by TBATS model for different forecasting horizon is smaller than the RMSE value provided by BATS model. Figure 6 also represents the comparison between RMSE and MAPE error values from both the models. Considering different forecasting horizon as h=1, 24, 48, 72, 96, 120, 144, 168, 192 and 206, the result can be explained by the ability of the TBATS model containing trigonometric terms to accommodate non-integer seasonality as well as a lesser number of initial parameter estimation. Thus, it is concluded that TBATS is good techniques to forecast with complex seasonality for the available training and test data set.

5. Conclusion

In Today's era, the complex seasonality is frequently appearing in long time series such as daily, hourly etc. Due to the limitations of traditional forecasting methods to handle complex seasonality, new forecasting methods like BATS and TBATS become more important. In this paper, both the models have been applied to daily natural gas consumption data of CFFP, BHEL. The analysis revealed the existence of weekly, monthly and annual multiple seasonality in the data. Forecast values were produced by these two models for different forecasting horizons. Two error components RMSE and MAPE were studied at different forecasting horizon to compare the forecasting accuracies of these two models. TBATS model with a lesser number of parameters is found to be a more suitable technique for forecasting of this particular data set with minimum error. The future scope of the work can include the effect of Indian holidays on daily time series data and the effect of temperature on natural gas consumption can also be considered for further analysis at themicro level.

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