

# Derivatives and Integrals Cheatsheet

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## 1 The fundamental derivatives

### Powers of $x$

$$\frac{d}{dx} k = 0$$

$$\frac{d}{dx} x^a = ax^{a-1}, \quad a \in \mathbb{R}$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}, \quad x > 0$$

$$\frac{d}{dx} \sqrt[n]{x} = \frac{1}{n\sqrt[n]{x^{n-1}}}, \quad x > 0, n \in \mathbb{N}$$

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

### Logarithmic and exponential functions

$$\frac{d}{dx} a^x = a^x \ln a, \quad a > 0$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log_a x = \frac{\log_a e}{x}, \quad x > 0$$

$$\frac{d}{dx} \ln x = \frac{1}{x}, \quad x > 0$$

### Trigonometric functions

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$\frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$$

### Inverse trigonometric functions

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \operatorname{arccot} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

### The rules of differentiation

$$\frac{d}{dx} [k \cdot f(x)] = k \cdot f'(x)$$

$$\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx} [f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx} [f(x) \cdot g(x) \cdot z(x)] = f'(x) \cdot g(x) \cdot z(x) + f(x) \cdot g'(x) \cdot z(x) + f(x) \cdot g(x) \cdot z'(x)$$

$$\frac{d}{dx} [f(x)]^a = a[f(x)]^{a-1} \cdot f'(x), \quad a \in \mathbb{R}$$

$$\frac{d}{dx} \left[ \frac{1}{f(x)} \right] = -\frac{f'(x)}{f^2(x)}$$

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$\frac{d}{dx} [f(g(x))] = f'(z) \cdot g'(x), \quad z = g(x)$$

$$\frac{d}{dx} [f(g(z(x)))] = f'(u) \cdot g'(t) \cdot z'(x), \quad t = z(x), \quad u = g(t)$$

$$\frac{d}{dx} [f(x)]^{g(x)} = [f(x)]^{g(x)} \left[ g'(x) \ln f(x) + \frac{g(x) \cdot f'(x)}{f(x)} \right]$$

$$\frac{d}{dx} [f^{-1}(y)] = \frac{1}{f'(x)}, \quad x = f^{-1}(y)$$

## 2 Basic integrals

- **Potenze di  $x$**

$$\int dx = x + c$$

$$\int x^p dx = \frac{x^{p+1}}{p+1} + c \quad \forall p \in \mathbb{R} \setminus \{-1\}$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int \sqrt{x} dx = \frac{2}{3} \sqrt{x^3} + c$$

- **Logarithmic and exponential functions**

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c \quad \forall a \in \mathbb{R} \setminus \{1\}$$

- **Trigonometric functions**

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln |\cos x| + c$$

$$\int \cot x dx = \ln |\sin x| + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

- **Inverse trigonometric functions**

$$\int \frac{1}{1+x^2} = \begin{cases} \arctan(x) + c \\ -\operatorname{arccot}(x) + c \end{cases}$$

$$\int \frac{1}{\sqrt{1-x^2}} = \begin{cases} \arcsin(x) + c \\ -\arccos(x) + c \end{cases}$$

- **Other more complex basic integrals**

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + c$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{x}{\sqrt{x^2+a}} dx = \sqrt{x^2+a} + c$$

$$\int \frac{x}{\sqrt{a-x^2}} dx = -\sqrt{a-x^2} + c$$

$$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}| + c$$

$$\int \frac{1}{\sin x} dx = \ln \left| \tan \frac{x}{2} \right| + c$$

$$\int \frac{1}{\cos x} dx = \ln \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right| + c$$

$$\int \sin^2 x dx = \frac{1}{2} (x - \sin x \cos x) + c$$

$$\int \cos^2 x dx = \frac{1}{2} (x + \sin x \cos x) + c$$

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2} (a^2 \arcsin \frac{x}{a} + x \sqrt{a^2-x^2}) + c$$

### 3 Basic integrals of composite functions

- Composite trigonometric functions

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int \frac{f'(x)}{\cos^2 f(x)} dx = \tan f(x) + c$$

$$\int \frac{f'(x)}{\sin^2 f(x)} dx = -\cot f(x) + c$$

- Composite inverse trigonometric functions

$$\int \frac{f'(x)}{1 + [f(x)]^2} dx = \begin{cases} \arctan f(x) + c \\ -\operatorname{arccot} f(x) + c \end{cases}$$

$$\int \frac{f'(x)}{\sqrt{1 - [f(x)]^2}} dx = \begin{cases} \arcsin f(x) + c \\ -\arccos f(x) + c \end{cases}$$

- Composite powers of  $x$

$$\int f^\alpha(x) f'(x) dx = \frac{f^{\alpha+1}}{\alpha+1} + c \quad \forall \alpha \in \mathbb{R} \setminus \{-1\}$$

- Composite logarithmic and exponential functions

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int e^{f(x)} f'(x) dx = e^{f(x)} + c$$

$$\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\ln a} + c$$

- More complex composite inverse trigonometric functions

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \arcsin \frac{f(x)}{|a|} + c, \quad a \neq 0$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \arctan \frac{f(x)}{a} + c, \quad a \neq 0$$

## 4 Summary of derivatives and integrals of normal and composite trigonometric functions

$$\frac{d}{dx} \sin x = \cos x \quad \Longleftrightarrow \quad \int \cos x \, dx = \sin x + c$$

$$\frac{d}{dx} \cos x = -\sin x \quad \Longleftrightarrow \quad \int \sin x \, dx = -\cos x + c$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} \quad \Longleftrightarrow \quad \int \frac{1}{\cos^2 x} \, dx = \tan x + c$$

$$\frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} \quad \Longleftrightarrow \quad \int \frac{1}{\sin^2 x} \, dx = -\cot x + c$$

$$\left. \begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \end{aligned} \right\} \quad \Longleftrightarrow \quad \int \frac{1}{\sqrt{1-x^2}} = \begin{cases} \arcsin(x) + c \\ -\arccos(x) + c \end{cases}$$

$$\left. \begin{aligned} \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \frac{d}{dx} \operatorname{arccot} x &= -\frac{1}{1+x^2} \end{aligned} \right\} \quad \Longleftrightarrow \quad \int \frac{1}{1+x^2} = \begin{cases} \arctan(x) + c \\ -\operatorname{arccot}(x) + c \end{cases}$$

$$\frac{d}{dx} \sin f(x) = f'(x) \cos f(x) \quad \Longleftrightarrow \quad \int f'(x) \cos f(x) \, dx = \sin f(x) + c$$

$$\frac{d}{dx} \cos f(x) = -f'(x) \sin f(x) \quad \Longleftrightarrow \quad \int f'(x) \sin f(x) \, dx = -\cos f(x) + c$$

$$\frac{d}{dx} \tan f(x) = \frac{f'(x)}{\cos^2 f(x)} \quad \Longleftrightarrow \quad \int \frac{f'(x)}{\cos^2 f(x)} \, dx = \tan f(x) + c$$

$$\frac{d}{dx} \cot f(x) = -\frac{f'(x)}{\sin^2 f(x)} \quad \Longleftrightarrow \quad \int \frac{f'(x)}{\sin^2 f(x)} \, dx = -\cot f(x) + c$$

$$\left. \begin{aligned} \frac{d}{dx} \arcsin f(x) &= \frac{f'(x)}{\sqrt{1-[f(x)]^2}} \\ \frac{d}{dx} \arccos f(x) &= -\frac{f'(x)}{\sqrt{1-[f(x)]^2}} \end{aligned} \right\} \quad \Longleftrightarrow \quad \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} \, dx = \begin{cases} \arcsin f(x) + c \\ -\arccos f(x) + c \end{cases}$$

$$\left. \begin{aligned} \frac{d}{dx} \arctan f(x) &= \frac{f'(x)}{1+[f(x)]^2} \\ \frac{d}{dx} \operatorname{arccot} f(x) &= -\frac{f'(x)}{1+[f(x)]^2} \end{aligned} \right\} \quad \Longleftrightarrow \quad \int \frac{f'(x)}{1+[f(x)]^2} \, dx = \begin{cases} \arctan f(x) + c \\ -\operatorname{arccot} f(x) + c \end{cases}$$