

# Trigonometry Cheatsheet

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# 1 Valori trigonometrici per angoli noti

$\alpha^\circ$	$\alpha$	$\cos \alpha$	$\sin \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$0^\circ$	0	1	0	0	$\infty$	1	$\infty$
$30^\circ$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$45^\circ$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60^\circ$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
$90^\circ$	$\frac{\pi}{2}$	0	1	$\infty$	0	$\infty$	1
$120^\circ$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
$135^\circ$	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$150^\circ$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
$180^\circ$	$\pi$	-1	0	0	$\infty$	-1	$\infty$
$210^\circ$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2
$225^\circ$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$240^\circ$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
$270^\circ$	$\frac{3\pi}{2}$	0	-1	$\infty$	0	$\infty$	-1
$300^\circ$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
$315^\circ$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$330^\circ$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2

## 2 Formule per gli archi associati

Angoli complementari

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

Angoli che differiscono di un angolo retto

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot \alpha$$

$$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan \alpha$$

Angoli supplementari

$$\sin(\pi - \alpha) = \sin \alpha$$

$$\cos(\pi - \alpha) = -\cos \alpha$$

$$\tan(\pi - \alpha) = -\tan \alpha$$

$$\cot(\pi - \alpha) = -\cot \alpha$$

Angoli che differiscono di un angolo piatto

$$\sin(\pi + \alpha) = -\sin \alpha$$

$$\cos(\pi + \alpha) = -\cos \alpha$$

$$\tan(\pi + \alpha) = \tan \alpha$$

$$\cot(\pi + \alpha) = \cot \alpha$$

Angoli la cui somma è  $\frac{3\pi}{2}$

$$\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$$

$$\tan\left(\frac{3\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{3\pi}{2} - \alpha\right) = \tan \alpha$$

Angoli che differiscono di  $\frac{3\pi}{2}$

$$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin \alpha$$

$$\tan\left(\frac{3\pi}{2} + \alpha\right) = -\cot \alpha$$

$$\cot\left(\frac{3\pi}{2} + \alpha\right) = -\tan \alpha$$

Angoli opposti

$$\sin(-\alpha) = -\sin \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\tan(-\alpha) = -\tan \alpha$$

$$\cot(-\alpha) = -\cot \alpha$$

### 3 Formule goniometriche

- Addizione e sottrazione

$$\begin{aligned} - \sin(\alpha \pm \beta) &= \sin(\alpha) \cos(\beta) \pm \sin(\beta) \cos(\alpha) \\ - \cos(\alpha \pm \beta) &= \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \\ - \tan(\alpha \pm \beta) &= \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)} \\ - \cot(\alpha \pm \beta) &= \frac{\cot(\alpha) \cot(\beta) \mp 1}{\cot(\beta) \pm \cot(\alpha)} \end{aligned}$$

- Duplicazione

$$\begin{aligned} \sin(2\alpha) &= 2 \sin(\alpha) \cos(\alpha) \\ \cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) \\ \tan(2\alpha) &= \frac{2 \tan(\alpha)}{1 - \tan^2(\alpha)} \\ \cot(2\alpha) &= \frac{\cot^2 \alpha - 1}{2 \cot(\alpha)} \end{aligned}$$

- Bisezione

$$\begin{aligned} \sin\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 - \cos(\alpha)}{2}} \\ \cos\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 + \cos(\alpha)}{2}} \\ \tan\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}} = \frac{\sin(\alpha)}{1 + \cos(\alpha)} = \\ &\quad \frac{1 - \cos(\alpha)}{\sin(\alpha)} \\ \cot\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 + \cos(\alpha)}{1 - \cos(\alpha)}} = \frac{\sin(\alpha)}{1 - \cos(\alpha)} = \\ &\quad \frac{1 + \cos(\alpha)}{\sin(\alpha)} \end{aligned}$$

- Parametriche o razionali  $t = \tan(\frac{\alpha}{2})$ .

$$\begin{aligned} \sin(\alpha) &= \frac{2t}{1 + t^2} \\ \cos(\alpha) &= \frac{1 - t^2}{1 + t^2} \\ \tan(\alpha) &= \frac{2t}{1 - t^2} \\ \cot(\alpha) &= \frac{1 - t^2}{2t} \end{aligned}$$

- Prostaferesi

$$\begin{aligned} \sin(\alpha) + \sin(\beta) &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \sin(\alpha) - \sin(\beta) &= 2 \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right) \\ \cos(\alpha) + \cos(\beta) &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \cos(\alpha) - \cos(\beta) &= -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \end{aligned}$$

- Werner

$$\begin{aligned} \sin(\alpha) \sin(\beta) &= \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \\ \cos(\alpha) \cos(\beta) &= \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2} \\ \sin(\alpha) \cos(\beta) &= \frac{\sin(\alpha - \beta) + \sin(\alpha + \beta)}{2} \\ \sin(\beta) \cos(\alpha) &= \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2} \end{aligned}$$