Kuratowski's Ordered Pairs

Donato Martinelli

Definition 0.1. Let x and y be entities. The set

$$\{\{x\}, \{x, y\}\}$$

is called the *pair with first coordinate x and second coordinate y*, and is denoted by the symbol

$$(x, y).$$

Note 1. From the definition, it follows in particular that

$$(x, x) = \{\{x\}\}.$$

Lemma 0.1.

$$(x_1, y_1) = (x_2, y_2) \quad \Leftrightarrow \quad x_1 = x_2 \ \wedge \ y_1 = y_2.$$

Proof. Assume $(x_1, y_1) = (x_2, y_2)$.

If $x_1 = y_1$, we have

$$(x_1, y_1) = \{\{x_1\}\},$$

so

$$(x_2, y_2) = \{\{x_2\}, \{x_2, y_2\}\}$$

is also constituted of a single element, and therefore

$$x_1 = x_2 = y_2.$$

Let $x_1 \neq y_1$. From

$$\{\{x_1\}, \{x_1, y_1\}\} = \{\{x_2\}, \{x_2, y_2\}\}.$$

it follows that $x_2 \neq y_2$, so

$$\{x_1\} = \{x_2\} \text{ and } \{x_1, y_1\} = \{x_2, y_2\}.$$

Therefore, we have

$$x_1 = y_1 \text{ and } y_1 = y_2.$$

□