P_M5_1

October 29, 2021

1 Module 5 Peer Review Assignment

2 Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y.

X	Y						P(X=x)
	1	2	3	4	5	6	
1	$\frac{1}{36}$	0	0	0	0	0	$\frac{1}{36}$
2	$\frac{2}{36}$	$\frac{1}{36}$	0	0	0	0	$\frac{3}{36}$
3	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0	0	0	$\frac{5}{36}$
4	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0	0	$\frac{7}{36}$
5	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0	$\frac{9}{36}$
6	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{11}{36}$
P(Y=y)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{1}{36}$	1

b) What is $P(X \geq 3, Y = 1)$?

$\frac{8}{36} = \frac{2}{9}$

c) What is $P(X \geq Y + 2)$?

$\frac{20}{36} = \frac{5}{9}$

d) Are X and Y independent? Explain.

No they are not. We know independence can be implied when $P(X = x, Y = y) = P(X = x)P(Y = y)$. In our distribution, this doesn't happen when X=1 and Y=1. $P(X = 1, Y = 1) = \frac{1}{36}$. However, $P(X = 1)P(Y = 1) = \frac{1}{36} \cdot \frac{11}{36} = \frac{11}{1296}$. Because $\frac{1}{36} \neq \frac{11}{1296}$, we know X and Y are not independent.

3 Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x,y) = \begin{cases} cxy^2 & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

C=6 as shown below

$$\int_0^1 \int_0^1 cxy^2 dydx = 1 \Rightarrow c \int_0^1 \int_0^1 xy^2 dydx = 1 \Rightarrow \frac{c}{3} \int_0^1 \left[xy^3\right]_0^1 dx = 1 \Rightarrow \frac{c}{3} \int_0^1 xdx = 1 \Rightarrow \frac{c}{3}\left[\frac{x^2}{2}\right]_0^1 = 1 \Rightarrow \frac{c}{6} = 1 \Rightarrow$$

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

For $f_x(x)$:

$$\int_0^1 6xy^2 dy \Rightarrow \left[\frac{6xy^3}{3}\right]_{y=0}^{y=1} \Rightarrow 2x$$

for $f_Y(y)$:

$$\int_0^1 6xy^2 dx \Rightarrow \left[\frac{6x^2y^2}{2}\right]_{x=0}^{x=1} \Rightarrow \frac{6y^2}{2} \Rightarrow 3y^2$$

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

For $E[X]$ we have:

$$\int_0^1 x \cdot 2xdx \Rightarrow \int_0^1 2x^2 dx \Rightarrow \left[\frac{2x^3}{3}\right]_0^1 \Rightarrow \frac{2}{3}$$

For $E[Y]$ we have:

$$\int_0^1 3y^2 \cdot ydy \Rightarrow \int_0^1 3y^3 dy \Rightarrow \left[\frac{3y^4}{4}\right]_0^1 \Rightarrow \frac{3}{4}$$

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

For E[XY] we have:

$$\int_0^1 \int_0^1 6xy^2 \cdot xydydx \Rightarrow \int_0^1 \int_0^1 6x^2y^3 dydx \Rightarrow \int_0^1 \left[\frac{6x^2y^4}{4}\right]_{y=0}^{y=1} \Rightarrow \int_0^1 \frac{3x^2}{2} dx \Rightarrow \left[\frac{3x^3}{6}\right]_0^1 \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$$

Part e)

Are X and Y independent?

Yes they are, we know if the two distributions are independent then $f(x,y) = f_x(x)f_y(y)$ and $E[XY] = E[X]E[Y]$. Evaluating f(x,y) for independence we can show the joint PDFs is the same as the marginal PDFs multiplied by each other:

$$f(x,y) = f_x(x)f_y(y) \Rightarrow 6xy^2 = (2x)(3y^2) \Rightarrow 6xy^2 = 6xy^2$$

Also, evaluating $E[XY] = E[X]E[Y]$:

$$\frac{1}{2} = \frac{2}{3} \cdot \frac{3}{4} \Rightarrow \frac{1}{2} = \frac{6}{12} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

Thus, $f_x(x) and f_y(y)$ are independent