ANCOVA with interactions in R

In this lesson, we will learn how to implement and interpret ANCOVA with interaction terms on real data in R.

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Consider the data from a recent study on teacher training. The study was "designed to investigate the effect of a Facebook-based instructional approach on preservice teacher's learning achievement and engagement."

(C. Saini and J. Abraham (2019). "Implementing Facebook-Based Instructional Approach in Pre-Service Teacher Education: An Empirical Investigation," Computers & Education, Vol. 128, pp. 243-255.)

The variables are:

1. `pre_lrn` : Pre-Treatment learning achievement score
2. `Trt` : 1=Facebook Group, 0=Control
3. `post_lrn` : Post-Treatment learning achievement score

In [1]:

```
library(dplyr)
fb = read.table("http://users.stat.ufl.edu/~winner/data/facebook_teach.csv", sep
#https://www.sciencedirect.com/science/article/pii/S0360131518302707#sec5

fb = fb %>%
    mutate(Trt = as.factor(Trt))

levels(fb$Trt) = c("control", "facebook")

head(fb);

fb %>%
    group_by(Trt) %>%
    summarise(n_pre = n(), mean_pre = mean(pre_lrn), n_post = n(), mean_post = m
```

```
Attaching package: ‘dplyr’

The following objects are masked from ‘package:stats’:

    filter, lag

The following objects are masked from ‘package:base’:

    intersect, setdiff, setequal, union
```

pre_lrn	Trt	post_lrn
28.973	facebook	28.502

pre_lrn	Trt	post_lrn
17.533	facebook	21.033
28.995	facebook	20.734
31.416	facebook	28.960
15.897	facebook	47.417
37.776	facebook	51.461

Trt	n_pre	mean_pre	n_post	mean_post
control	37	23.96995	37	34.26997
facebook	31	24.80997	31	41.42006

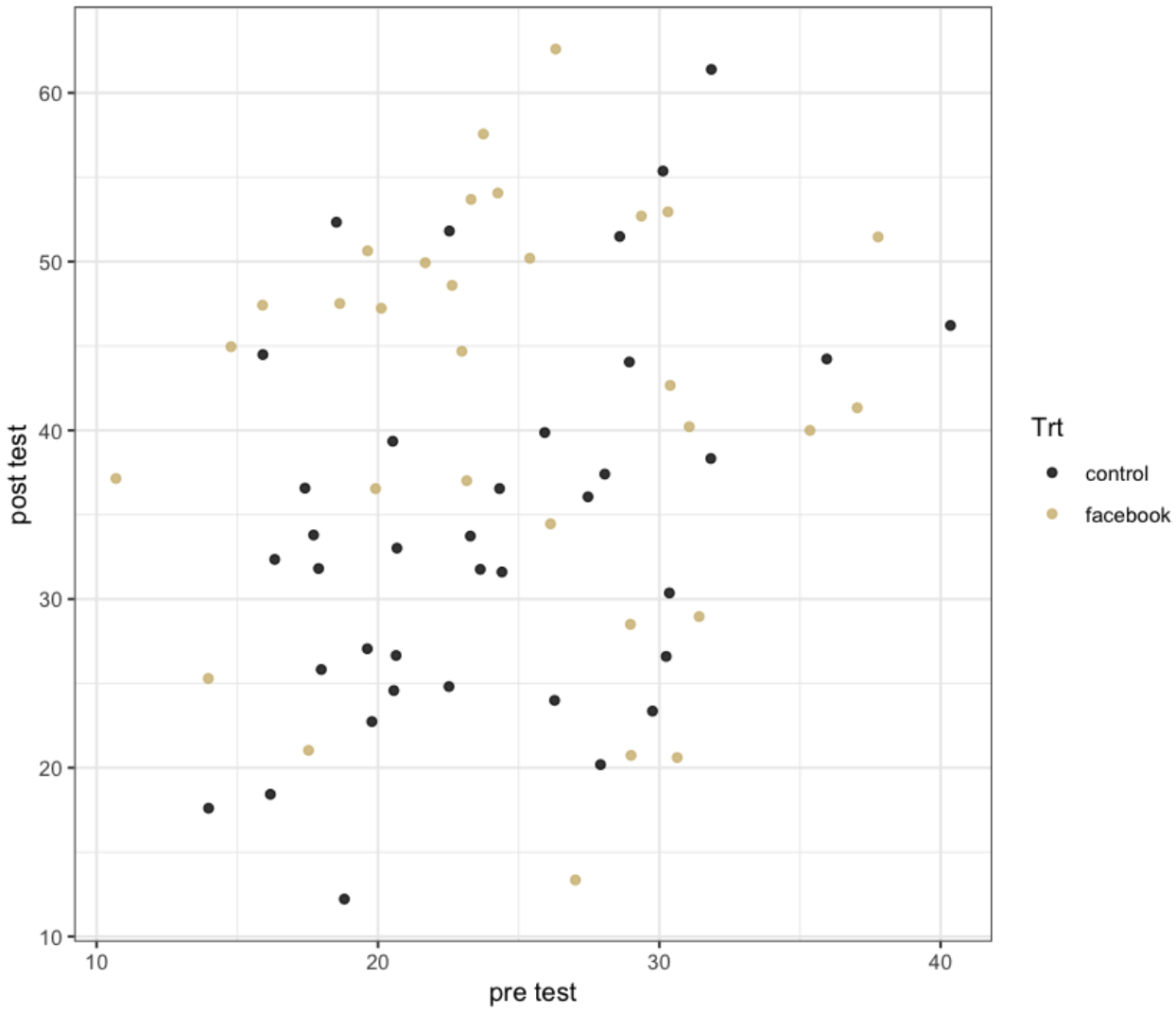
In [2]:

```
library(ggplot2)

p2 = ggplot(data = fb, aes(x = pre_lrn, y = post_lrn, color = Trt))
p2 = p2 + geom_point(alpha = 0.8)
p2 = p2 + scale_color_manual(values=c('black','#CFB87C'))
p2 = p2 + xlab("pre test") + ylab("post test") + theme_bw() + coord_fixed(ratio
#p2 = p2 + ggsave(filename = file.path("~/CU Google Drive/fig1.pdf"))

p2
```

```
Registered S3 methods overwritten by 'ggplot2':
  method         from
  [.quosures     rlang
  c.quosures     rlang
  print.quosures rlang
```



Plotting the data by group, we do not get a great sense as to whether the least squares line going through the black points should have the same slope as the one going through the gold points. So, we can fit an ANCOVA model (in the regression form) with an interaction term. Here, we use the `lm()` function with the same formula as normal linear regression. Our interaction term enters that formula with `Trt : pre_lrn`.

In [3]:
```
ancova = lm(post_lrn ~  Trt + pre_lrn + Trt:pre_lrn, data = fb)
summary(ancova)
```

```
Call:
lm(formula = post_lrn ~ Trt + pre_lrn + Trt:pre_lrn, data = fb)

Residuals:
     Min       1Q   Median       3Q      Max
-28.0246  -7.4198   0.0964   7.3837  22.1946

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)           16.0895     7.6817   2.095   0.0402 *
Trtfacebook           25.8308    11.0997   2.327   0.0231 *
pre_lrn                0.7585     0.3106   2.442   0.0174 *
Trtfacebook:pre_lrn   -0.7786     0.4403  -1.768   0.0818 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 11.49 on 64 degrees of freedom
Multiple R-squared:  0.1633,    Adjusted R-squared:  0.1241
F-statistic: 4.165 on 3 and 64 DF,  p-value: 0.009307
```

Let's use the default $\alpha = 0.05$. First, our full F-test is signficant, which suggests that we need *some* of the terms in the model. Now, let's decide whether we need the interaction term. The t-test associated with the interaction term is not significant at the 0.05 level. That suggests that we could leave the interaction term out. The `anova()` function would yield the same result.

In this sense, the Pr(>F) test is testing the following.

H_0: The means of the different levels within the variable are the same
H_1: The means of different levels within the variable are different

In [4]:
```
anova(ancova)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Trt	1	862.3384	862.3384	6.527611	0.01300978
pre_lrn	1	375.0380	375.0380	2.838911	0.09687739
Trt:pre_lrn	1	413.1494	413.1494	3.127402	0.08175182
Residuals	64	8454.8020	132.1063	NA	NA

Now let's consider a plot of the lines over the data.

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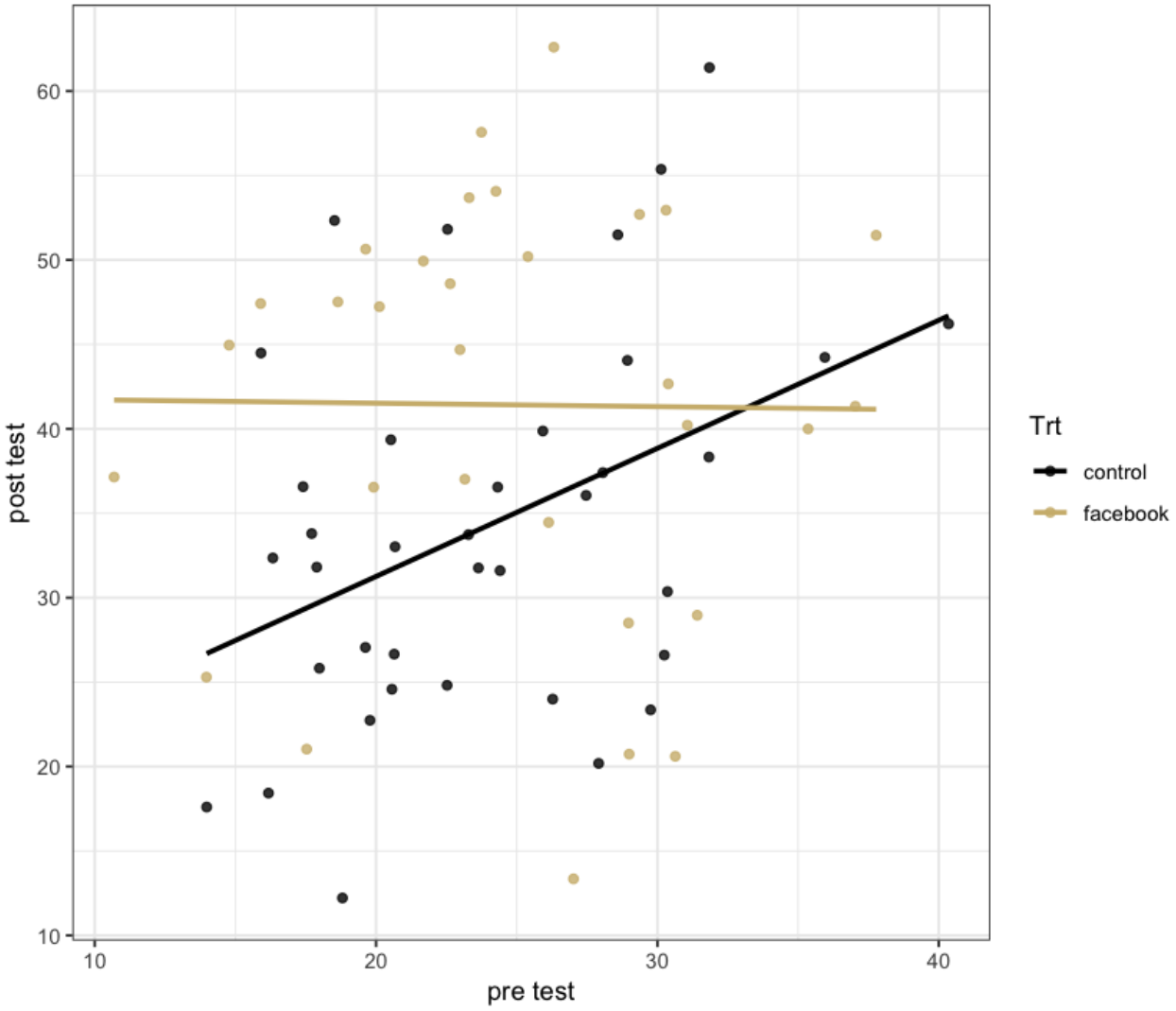
In [5]:
```
p2 = ggplot(data = fb, aes(x = pre_lrn, y = post_lrn, color = Trt))
p2 = p2 + geom_point(alpha = 0.8)
p2 = p2 + scale_color_manual(values=c('black','#CFB87C'))
p2 = p2 + geom_smooth(method = "lm", se = F, alpha = 0.3)
p2 = p2 + xlab("pre test") + ylab("post test") + theme_bw() + coord_fixed(ratio
p2 = p2 + ggsave(filename = file.path("~/CU Google Drive/fig1.pdf"))

p2

#plot separate regression lines without ggplot
#with(fb, plot(pre_lrn,post_lrn, pch = 16, col = c("#CFB87C","#565A5C")[Trt]))
#abline(coef(lm(post_lrn[Trt == "control"] ~ pre_lrn[Trt == "control"], data = f
#abline(coef(lm(post_lrn[Trt == "facebook"] ~ pre_lrn[Trt == "facebook"], data =
```

```
Saving 7 x 7 in image
```



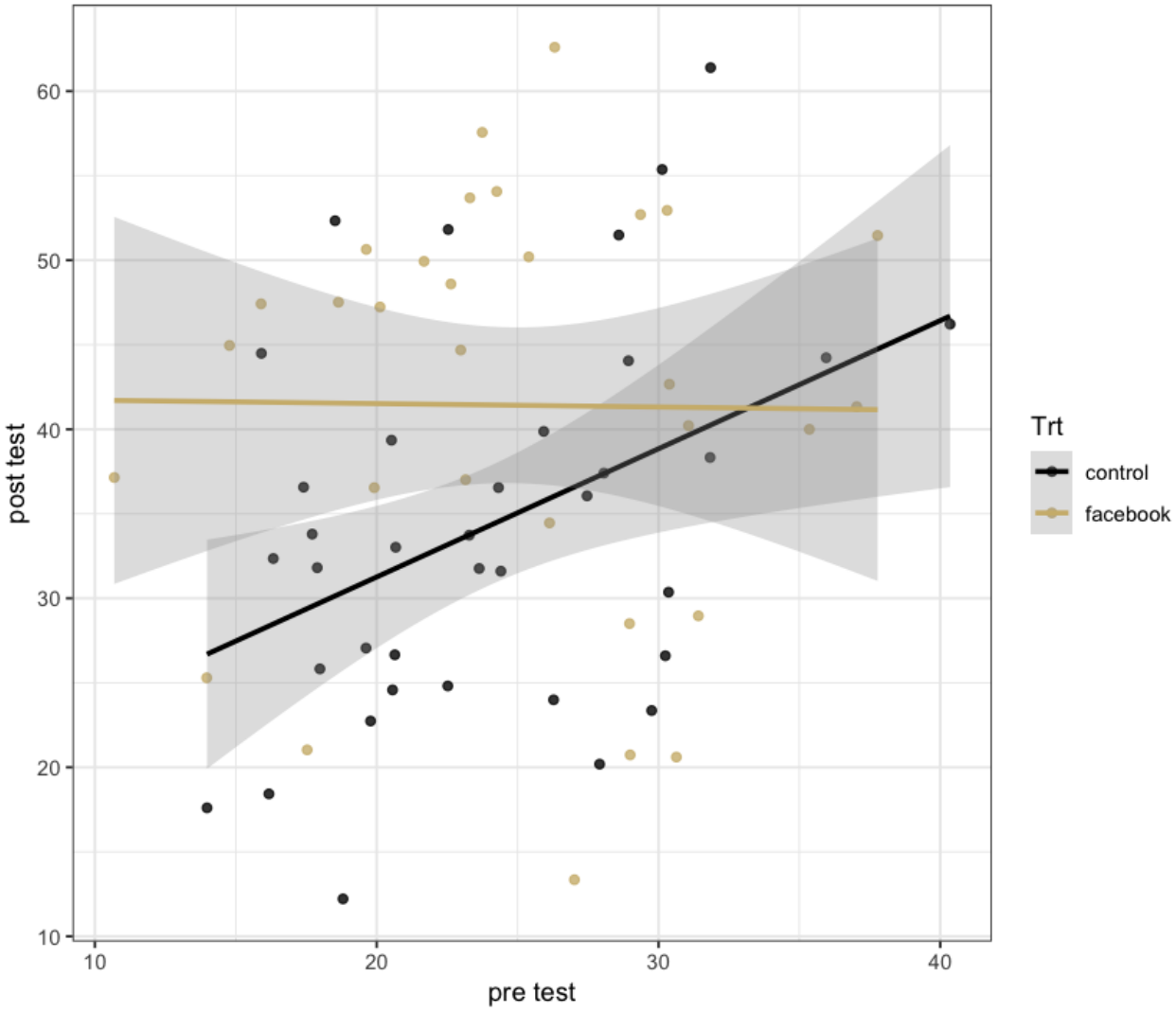
This result might seem inconsistent with the test above! These lines clearly aren't parallel! However, let's plot the lines with the corresponding confidence bands:

In [6]:

```
p2 = ggplot(data = fb, aes(x = pre_lrn, y = post_lrn, color = Trt))
p2 = p2 + geom_point(alpha = 0.8)
p2 = p2 + scale_color_manual(values=c('black','#CFB87C'))
p2 = p2 + geom_smooth(method = "lm", se = T, alpha = 0.3)
p2 = p2 + xlab("pre test") + ylab("post test") + theme_bw() + coord_fixed(ratio
p2 = p2 + ggsave(filename = file.path("~/CU Google Drive/fig1.pdf"))

p2
```

```
Saving 7 x 7 in image
```



Here, we notice that the confidence bands are relatively wide, which reflects the relatively high variability in the data (vertical stretch). Recall the interpretation of these confidence bands: if we resampled the post-test data (at the same values of the pre-test data and same values of the treatment), then we would get a different confidence band. If we did this over and over, then 95% of the bands would cover the true line.

Now, suppose that *this* band covers the true interval (after all, before calcuating it, there was a 0.95 probability...). That suggests that any line that we could draw within the band is "plausible". And, notice that it is possible to redraw the gold line and the black line so that they are parallel! So, while it seems as though the visual and statistical results are inconsistent, in a way, they're not.