m1-peer-reviewed

January 29, 2022

1 Module 1 - Peer reviewed

1.0.1 Outline:

In this homework assignment, there are four objectives.

1. To assess your knowledge of ANOVA/ANCOVA models
2. To apply your understanding of these models to a real-world datasets

General tips:

1. Read the questions carefully to understand what is being asked.
2. This work will be reviewed by another human, so make sure that you are clear and concise in what you are attempting to explain or answer.

[58]:
```
# Load Required Packages
library(tidyverse)
library(ggplot2)
library(plyr)
library(dplyr)
```

1.0.2 Problem #1: Simulate ANCOVA Interactions

In this problem, we will work up to analyzing the following model to show how interaction terms work in an ANCOVA model.

$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon_i$$

This question is designed to enrich understanding of interactions in ANCOVA models. There is no additional coding required for this question, however we recommend messing around with the coefficents and plot as you see fit. Ultimately, this problem is graded based on written responses to questions asked in part **(a)** and **(b)**.

To demonstrate how interaction terms work in an ANCOVA model, let's generate some data. First, we consider the model

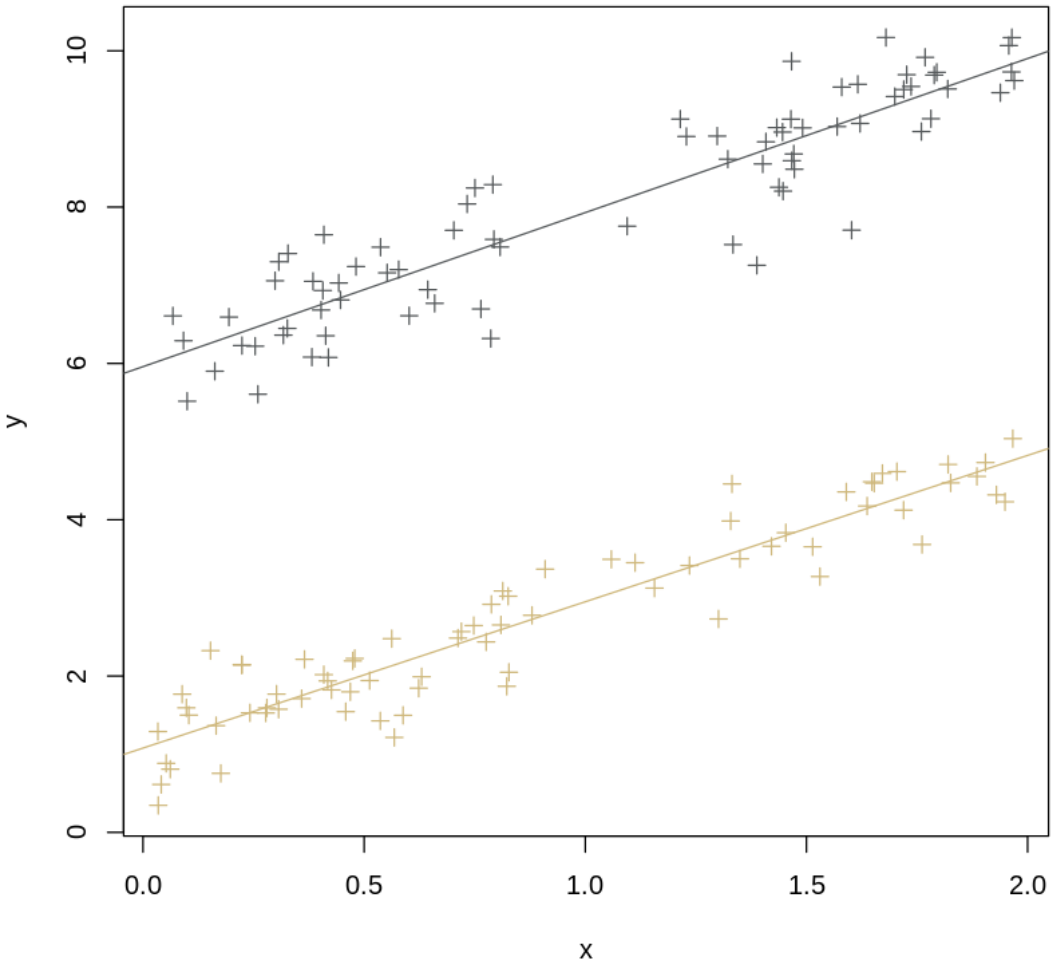
$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon_i$$

where X is a continuous covariate, Z is a dummy variable coding the levels of a two level factor, and $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$. We choose values for the parameters below (b0,…,b2).

[59]:
```
rm(list = ls())
set.seed(99)

#simulate data
n = 150
# choose these betas
b0 = 1; b1 = 2; b2 = 5; eps = rnorm(n, 0, 0.5);
x = runif(n,0,2); z = runif(n,-2,2);
z = ifelse(z > 0,1,0);
# create the model:
y = b0 + b1*x + b2*z + eps
df = data.frame(x = x,z = as.factor(z),y = y)
head(df)

#plot separate regression lines
with(df, plot(x,y, pch = 3, col = c("#CFB87C","#565A5C")[z]))
abline(coef(lm(y[z == 0] ~ x[z == 0], data = df)), col = "#CFB87C")
abline(coef(lm(y[z == 1] ~ x[z == 1], data = df)), col = "#565A5C")
```

A data.frame: 6 × 3

	x <dbl>	z <fct>	y <dbl>
1	0.09159879	1	6.290179
2	1.96439135	1	10.168612
3	0.57805656	1	7.200027
4	0.03370108	0	1.289331
5	1.82614045	0	4.470862
6	0.71220319	0	2.485743



1. (a) What happens with the slope and intercept of each of these lines? In this case, we can think about having two separate regression lines–one for Y against X when the unit is in group $Z = 0$ and another for Y against X when the unit is in group $Z = 1$. What do we notice about the slope of each of these lines?

The slope of the black line and yellow line are the same; however, the two regression lines have different intercepts.

1. (b) Now, let's add the interaction term (let $\beta_3 = 3$). What happens to the slopes of each line now? The model now is of the form:

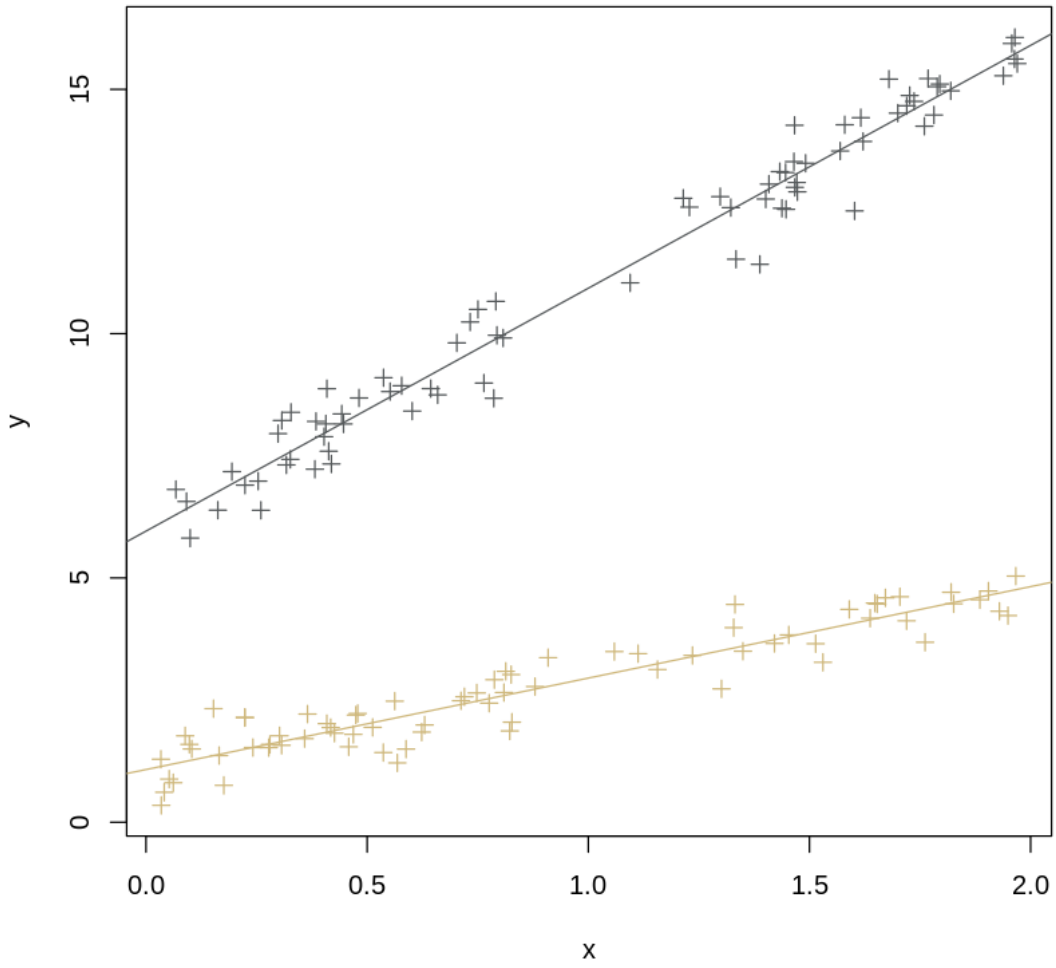
$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon_i$$

where X is a continuous covariate, Z is a dummy variable coding the levels of a two level factor, and $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$. We choose values for the parameters below (b0,…,b3).

[78]:
```
#simulate data
set.seed(99)
n = 150
# pick the betas
b0 = 1; b1 = 2; b2 = 5; b3 = 3; eps = rnorm(n, 0, 0.5);

#create the model
y = b0 + b1*x + b2*z + b3*(x*z) + eps
df = data.frame(x = x,z = as.factor(z),y = y)
head(df)

lmod = lm(y ~ x + z, data = df)
lmodz0 = lm(y[z == 0] ~ x[z == 0], data = df)
lmodz1 = lm(y[z == 1] ~ x[z == 1], data = df)
# summary(lmod)
# summary(lmodz0)
# summary(lmodz1)

# lmodInt = lm(y ~ x + z + x*z, data = df)
# summary(lmodInt)

#plot separate regression lines
with(df, plot(x,y, pch = 3, col = c("#CFB87C","#565A5C")[z]))
abline(coef(lm(y[z == 0] ~ x[z == 0], data = df)), col = "#CFB87C")
abline(coef(lm(y[z == 1] ~ x[z == 1], data = df)), col = "#565A5C")
```

A data.frame: 6 × 3

	x <dbl>	z <fct>	y <dbl>
1	0.09159879	1	6.564975
2	1.96439135	1	16.061786
3	0.57805656	1	8.934197
4	0.03370108	0	1.289331
5	1.82614045	0	4.470862
6	0.71220319	0	2.485743



In this case, we can think about having two separate regression lines–one for Y against X when the unit is in group $Z = 0$ and another for Y against X when the unit is in group $Z = 1$. **What do you notice about the slope of each of these lines?**

The slopes of the lines are different.

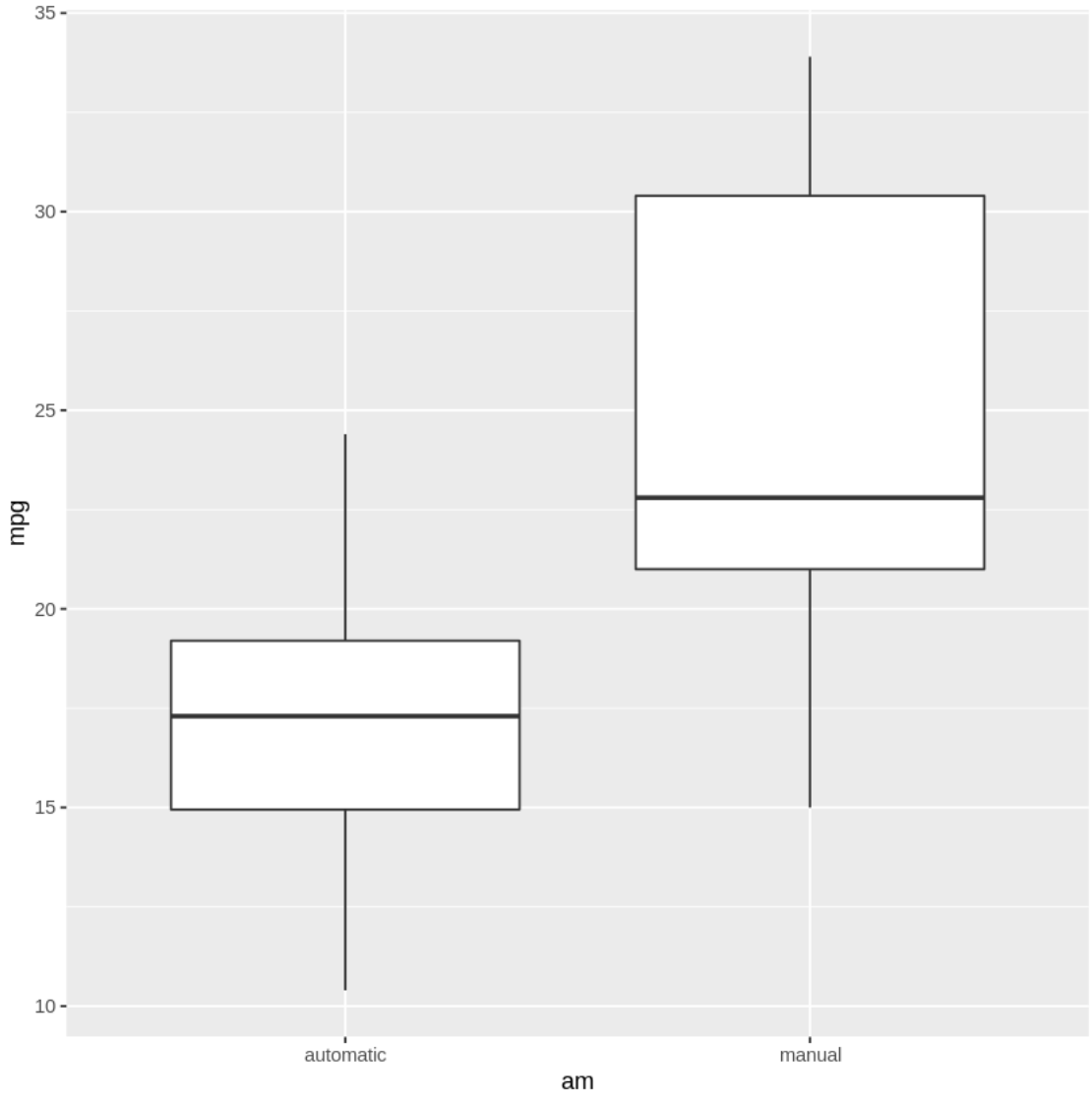
1.1 Problem #2

In this question, we ask you to analyze the `mtcars` dataset. The goal if this question will be to try to explain the variability in miles per gallon (mpg) using transmission type (am), while adjusting for horsepower (hp).

To load the data, use `data(mtcars)`

2. (a) Rename the levels of am from 0 and 1 to "Automatic" and "Manual" (one option for this is to use the revalue() function in the plyr package). Then, create a boxplot (or violin plot) of mpg against am. What do you notice? Comment on the plot

[61]:
```
data(mtcars)

# your code here
new_mtcars = mtcars %>%
    mutate(am = as.factor(am))
new_mtcars$am = revalue(new_mtcars$am, c("0"="automatic", "1"="manual"))

ggplot(new_mtcars, aes(x=am, y=mpg)) +
    geom_boxplot()
```



We can see that automatic cars have a less median of miles per gallon than manual cars. Overall, it seems an automatic vehicle has less mpg than a manual one.

2. (b) Calculate the mean difference in mpg for the Automatic group compared to the Manual group.

[71]:
```
# your code here
mean_df = new_mtcars %>%
    group_by(am) %>%
    dplyr::summarise(mean_mpg = mean(mpg))
mean_df
abs(mean_df[1,2] - mean_df[2,2])
```

A tibble: 2 × 2

am	mean_mpg
<fct>	<dbl>
automatic	17.14737
manual	24.39231

A data.frame: 1 × 1

mean_mpg
<dbl>
7.244939

Manual cars, on average, have 7.24 better mpg than automatic vehicles.

2. (c) Construct three models:

1. An ANOVA model that checks for differences in mean mpg across different transmission types.
2. An ANCOVA model that checks for differences in mean mpg across different transmission types, adjusting for horsepower.
3. An ANCOVA model that checks for differences in mean mpg across different transmission types, adjusting for horsepower and for interaction effects between horsepower and transmission type.

Using these three models, determine whether or not the interaction term between transmission type and horsepower is significant.

[72]:
```
# your code here
# mean mpg across different transmission types
lm1 = lm(mpg~am, data=new_mtcars)
summary(lm1)
```

```
Call:
lm(formula = mpg ~ am, data = new_mtcars)

Residuals:
    Min      1Q  Median      3Q     Max
-9.3923 -3.0923 -0.2974  3.2439  9.5077
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   17.147      1.125  15.247 1.13e-15 ***
ammanual       7.245      1.764   4.106 0.000285 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.902 on 30 degrees of freedom
Multiple R-squared:  0.3598,Adjusted R-squared:  0.3385
F-statistic: 16.86 on 1 and 30 DF,  p-value: 0.000285
```

[73]:
```
# mean mpg across differen transmission types, adjusting for horsepower
lm2 = lm(mpg ~ am + hp, data=new_mtcars)
summary(lm2)
```

```
Call:
lm(formula = mpg ~ am + hp, data = new_mtcars)

Residuals:
    Min      1Q  Median      3Q     Max
-4.3843 -2.2642  0.1366  1.6968  5.8657

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 26.584914   1.425094  18.655  < 2e-16 ***
ammanual     5.277085   1.079541   4.888 3.46e-05 ***
hp          -0.058888   0.007857  -7.495 2.92e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.909 on 29 degrees of freedom
Multiple R-squared:  0.782,Adjusted R-squared:  0.767
F-statistic: 52.02 on 2 and 29 DF,  p-value: 2.55e-10
```

[75]:
```
# mean mpg across differen transmission types, adjusting for horsepower
# and transmission type
lm3 = lm(mpg ~ am + hp + am:hp, data=new_mtcars)
summary(lm3)
```

```
Call:
lm(formula = mpg ~ am + hp + am:hp, data = new_mtcars)

Residuals:
```

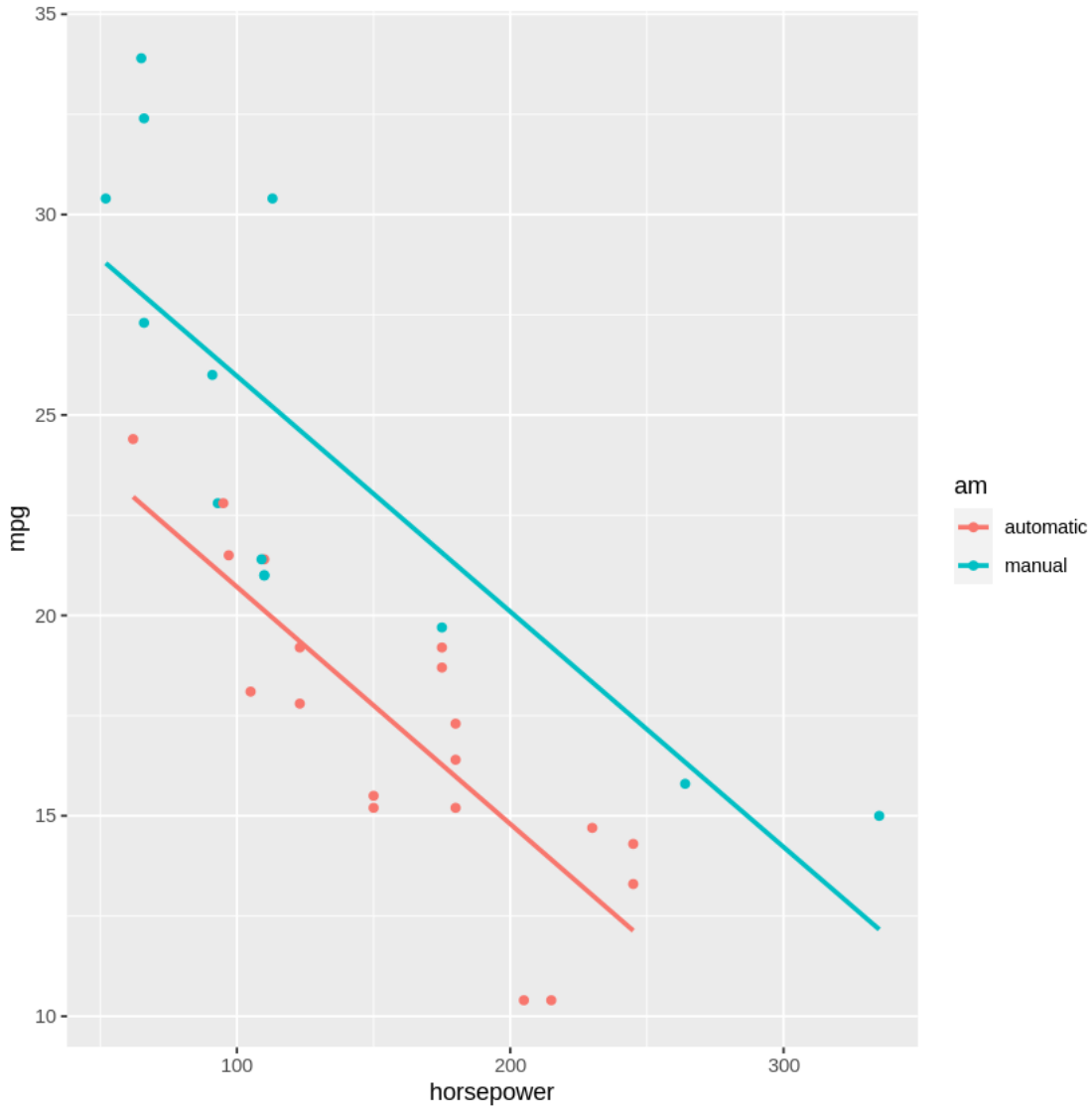
```
    Min      1Q  Median      3Q     Max
-4.3818 -2.2696  0.1344  1.7058  5.8752

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 26.6248479  2.1829432  12.197 1.01e-12 ***
ammanual     5.2176534  2.6650931   1.958   0.0603 .
hp          -0.0591370  0.0129449  -4.568 9.02e-05 ***
ammanual:hp  0.0004029  0.0164602   0.024   0.9806
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.961 on 28 degrees of freedom
Multiple R-squared:  0.782,Adjusted R-squared:  0.7587
F-statistic: 33.49 on 3 and 28 DF,  p-value: 2.112e-09
```

In the first model, it was shown that the “am” variable is significant as both the P-value of the t-test and F-test are very close to 0. Controlling for horsepower in the second model, The “am” factor is still signifcant; also, the “hp” predictor is significant as well. The p-value of the F-test remains close to 0; also, the asjusted R-squared increased significantly. The third model suggests the interaction term is not needed. It is not statistically signifiacnt. This causes the “am” factor to be insignificant. Therefore, our best model is to use the ‘am’ factor while controlling for horsepower.

2. (d) Construct a plot of mpg against horsepower, and color points based in transmission type. Then, overlay the regression lines with the interaction term, and the lines without. How are these lines consistent with your answer in (b) and (c)?

[77]:
```
# your code here
ggplot(data=new_mtcars, aes(x=hp, y=mpg, color=am)) +
    geom_point() +
    xlab("horsepower") + ylab("mpg") +
    geom_smooth(method = "lm", se=F)
```

```
`geom_smooth()` using formula 'y ~ x'
```



This is confirms with what we stated in b and c. We can see the intercept of manual vehicles is about 7 mpg higher than automatic (via the intercept). Also, we can see both regression lines have the same slope; therefore, an interaction would be useless in this model.

[ ]: