# Chapter 02 - Linear regression

March 19, 2022

## 1 Linear Regression

In this lab we would be going through: - Simple linear regression - Multiple linear regression - Interaction Terms - Non-linear Transformation of the Predictors

We would be working with the Boston data set available in package ISLR2.

[41]:
```r
library(ISLR2)
head(Boston)
```

A data.frame: 6 × 13

| | crim <dbl> | zn <dbl> | indus <dbl> | chas <int> | nox <dbl> | rm <dbl> | age <dbl> | dis <dbl> | rad <int> | tax <db |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00632 | 18 | 2.31 | 0 | 0.538 | 6.575 | 65.2 | 4.0900 | 1 | 296 |
| 2 | 0.02731 | 0 | 7.07 | 0 | 0.469 | 6.421 | 78.9 | 4.9671 | 2 | 242 |
| 3 | 0.02729 | 0 | 7.07 | 0 | 0.469 | 7.185 | 61.1 | 4.9671 | 2 | 242 |
| 4 | 0.03237 | 0 | 2.18 | 0 | 0.458 | 6.998 | 45.8 | 6.0622 | 3 | 222 |
| 5 | 0.06905 | 0 | 2.18 | 0 | 0.458 | 7.147 | 54.2 | 6.0622 | 3 | 222 |
| 6 | 0.02985 | 0 | 2.18 | 0 | 0.458 | 6.430 | 58.7 | 6.0622 | 3 | 222 |

Lets try using `lstat` as the predictor variable to predict the value `medv` using a `lm()` model

[42]:
```r
#To know more about lm( );
#?lm()
```

[43]:
```r
lm.fit <- lm(medv ~ lstat, data = Boston)
lm.fit
```

```
Call:
lm(formula = medv ~ lstat, data = Boston)

Coefficients:
(Intercept)        lstat
      34.55        -0.95
```

[44]:
```r
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ lstat, data = Boston)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-15.168  -3.990  -1.318   2.034  24.500 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 34.55384    0.56263   61.41   <2e-16 ***
lstat       -0.95005    0.03873  -24.53   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.216 on 504 degrees of freedom
Multiple R-squared:  0.5441,Adjusted R-squared:  0.5432 
F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

We can extract the coefficients of the fit by using `lm.fit$coefficient` or also by using the extractor functions like `coef()`

Similarly, to get the confidence interval for the coefficient estimates, we can use the `confint()` function

[45]:
```
coef(lm.fit) #to get the coefficients
confint(lm.fit) #to get the confidence interval
```

**(Intercept)** 34.5538408793831 **lstat** -0.950049353757991

A matrix: 2 × 2 of type dbl

| | 2.5 % | 97.5 % |
|---|---|---|
| (Intercept) | 33.448457 | 35.6592247 |
| lstat | -1.026148 | -0.8739505 |

The `predict()` function can be used to produce confidence intervals and prediction intervals for the prediction of `medv` for a given value of `lstat`.

[46]:
```
predict(lm.fit, data.frame(lstat = (c(5, 10, 15))), interval = "confidence")
predict(lm.fit, data.frame(lstat = (c(5, 10, 15))),interval = "prediction")
```

A matrix: 3 × 3 of type dbl

| | fit | lwr | upr |
|---|---|---|---|
| 1 | 29.80359 | 29.00741 | 30.59978 |
| 2 | 25.05335 | 24.47413 | 25.63256 |
| 3 | 20.30310 | 19.73159 | 20.87461 |

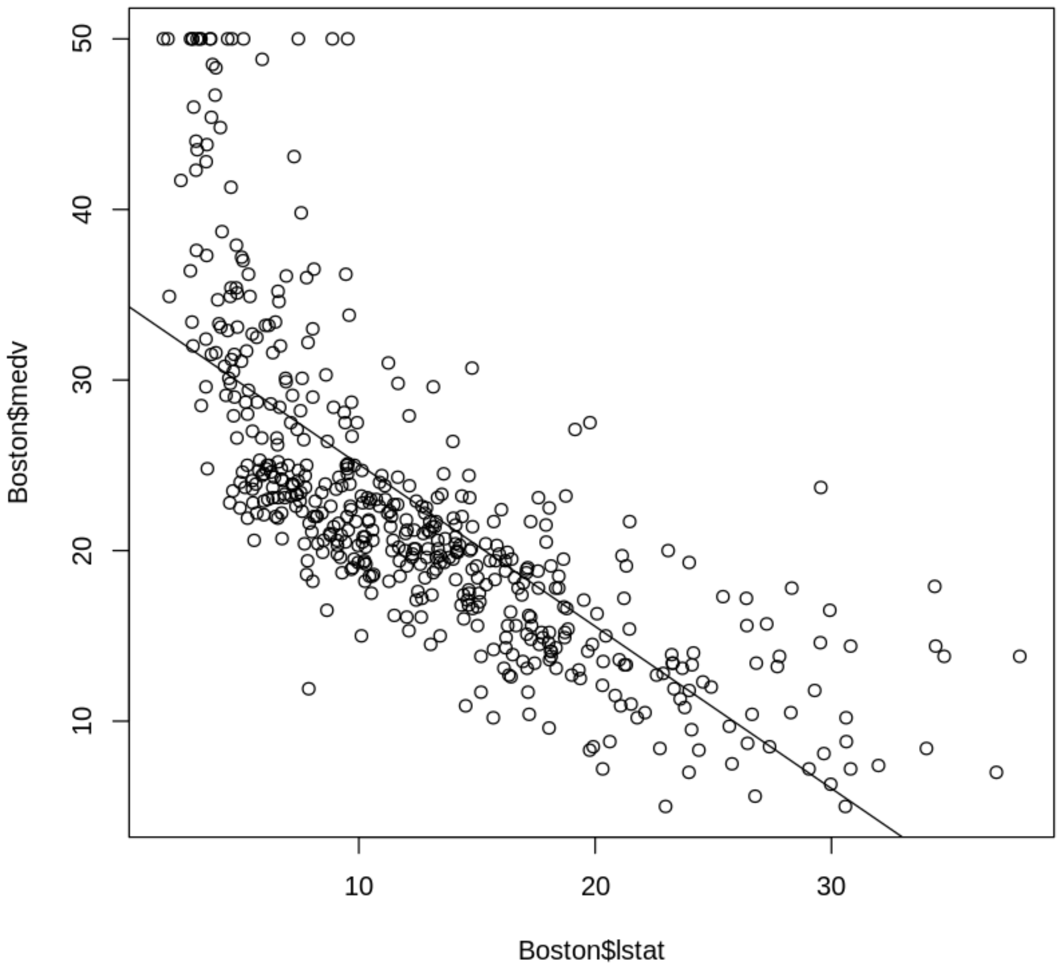
A matrix: 3 × 3 of type dbl

| | fit | lwr | upr |
|---|---|---|---|
| 1 | 29.80359 | 17.565675 | 42.04151 |
| 2 | 25.05335 | 12.827626 | 37.27907 |
| 3 | 20.30310 | 8.077742 | 32.52846 |

These values imply that, the 95 % confidence interval associated with a `lstat` value of 10 is (24.47, 25.63), and the 95 % prediction interval is (12.828, 37.28).

As expected, the confidence and prediction intervals are centered around the same point (a predicted value of 25.05 for `medv` when `lstat` equals 10), but the latter are substantially wider.

[47]:
```
plot(Boston$lstat, Boston$medv)
abline(lm.fit)
```




Plotting the points and an `abline` of the fit, we can observe evidences for non-linearity in the relationship between `lstat` and `medv`.

### 1.1 Multiple Linear Regression

The syntax `lm(y  x1 + x2 + x3)` is used to fit a model with three predictors, `x1`, `x2`, and `x3`.

The `summary()` function now outputs the regression coefficients for all the predictors.

[48]:
```
#fit a model that responds medv with lstat and age as predictors
lm.multiple.fit = function(){
    # your code here
    return(lm(medv ~ lstat + age, data=Boston))
}
```

[49]:
```
# Test residuals of the model
residuals = summary(lm.multiple.fit())$residuals
stopifnot(length(summary(lm.multiple.fit())$residuals) == 506)
stopifnot(round(median(residuals),2)== -1.28) #median of residuals
stopifnot(round(max(residuals),2)== 23.16)    #max of residuals

#Test Coefficients of the models
coefficients = summary(lm.multiple.fit())$coefficients
stopifnot(round(coefficients[2],2) == -1.03) #lstat estimate
stopifnot(round(coefficients[3],2) == 0.03)  #age estimate
```

The `Boston` data set contains 12 variables, and so it would be cumbersome to have to type all of these in order to perform a regression using all of the predictors.

Instead, we can use the following short-hand:

[50]:
```
lm.fit = lm(medv~., data = Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ ., data = Boston)

Residuals:
     Min       1Q   Median       3Q      Max
-15.1304  -2.7673  -0.5814   1.9414  26.2526

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  41.617270   4.936039   8.431 3.79e-16 ***
crim         -0.121389   0.033000  -3.678 0.000261 ***
zn            0.046963   0.013879   3.384 0.000772 ***
indus         0.013468   0.062145   0.217 0.828520
chas          2.839993   0.870007   3.264 0.001173 **
nox         -18.758022   3.851355  -4.870 1.50e-06 ***
rm            3.658119   0.420246   8.705  < 2e-16 ***
age           0.003611   0.013329   0.271 0.786595
dis          -1.490754   0.201623  -7.394 6.17e-13 ***
rad           0.289405   0.066908   4.325 1.84e-05 ***
tax          -0.012682   0.003801  -3.337 0.000912 ***
ptratio      -0.937533   0.132206  -7.091 4.63e-12 ***
lstat        -0.552019   0.050659 -10.897  < 2e-16 ***
---
```

```
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.798 on 493 degrees of freedom
Multiple R-squared:  0.7343,Adjusted R-squared:  0.7278
F-statistic: 113.5 on 12 and 493 DF,  p-value: < 2.2e-16
```

## 1.2 Interaction Terms

It is easy to include interaction terms in a linear model using the `lm()` function. - `lstat:black` (tells `R` to include an interaction term between `lstat` and `black`) - `lstat * age` (`lstat` + `age` + `lstat:age`)

[51]:
```
#Fit a lm model that responds with lstat, age and their interaction as␣
↪predictors
lm.interaction.fit = function(){
    # your code here
    return(lm(medv ~ lstat + age + lstat:age, data=Boston))
}
```

[52]:
```
interaction.summary = summary(lm.interaction.fit())
interaction.summary$coefficients

stopifnot(length(interaction.summary$coefficients) == 16) #intercept, lstat,␣
↪age and lstate:age
```

A matrix: $4 \times 4$ of type dbl

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 36.0885359346 | 1.469835463 | 24.55277263 | 4.907116e-88 |
| lstat | -1.3921168406 | 0.167455532 | -8.31335236 | 8.780730e-16 |
| age | -0.0007208595 | 0.019879171 | -0.03626205 | 9.710878e-01 |
| lstat:age | 0.0041559518 | 0.001851795 | 2.24428275 | 2.524911e-02 |

## 1.3 Non-linear Transformations of the Predictors

The `lm()` function can also accommodate non-linear transformations of the predictors. For instance, given a predictor `X`, we can create a predictor `X^2` using `I(X^2)`.

The function `I()` is needed since the ^ has a special meaningin a formula object; wrapping as we do allows the standard usage in `R`, which is to raise `X` to the power 2.

[53]:
```
lm.transform.fit = lm(medv~lstat + I(lstat^2), data=Boston)
```

[54]:
```
summary(lm.transform.fit)
```

```
Call:
lm(formula = medv ~ lstat + I(lstat^2), data = Boston)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-15.2834  -3.8313  -0.5295   2.3095  25.4148 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 42.862007   0.872084   49.15   <2e-16 ***
lstat       -2.332821   0.123803  -18.84   <2e-16 ***
I(lstat^2)   0.043547   0.003745   11.63   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.524 on 503 degrees of freedom
Multiple R-squared:  0.6407,Adjusted R-squared:  0.6393 
F-statistic: 448.5 on 2 and 503 DF,  p-value: < 2.2e-16
```

In order to create a cubic fit, we can include a predictor of the form `I(X^3)`.

However, this approach can start to get cumbersome for higher- order polynomials. A better approach involves using the `poly()` function to create the polynomial within `lm()`

[55]:
```
# Fit a regression model that would respond medv with an lstat polynomial of
 ↪degree 4 as predictor
#?poly(); to understand more about poly function
lm.poly.fit = function(){
    # your code here
    return (lm.transform.fit = lm(medv~poly(lstat,4), data=Boston))
}
```

[56]:
```
# Test - check the number of coefficients that are considered in the curve
fit = lm.poly.fit()
stopifnot(length(summary(fit)$coefficients) == 20) # 1- intercept; 4 -
 ↪variables against 4 columns
```

We use the `anova()` function to further quantify the extent to which the quadratic fit is superior to the linear fit.

[57]:
```
anova(lm.fit, lm.transform.fit)
```

A anova: 2 × 6

| | Res.Df | RSS | Df | Sum of Sq | F | Pr(>F) |
|---|---|---|---|---|---|---|
| | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> |
| 1 | 493 | 11349.42 | NA | NA | NA | NA |
| 2 | 503 | 15347.24 | -10 | -3997.824 | 17.36589 | 3.822984e-27 |

[58]:
```
#since the p-value is small, we can say the quadratic model is betters
```