1. (a) A discrete perceptron network receives inputs $(x_1, x_2)^2 \in \boldsymbol{R}^2$ and has an output $y \in \{0, 1\}$. The decision boundary implemented by the network forms a triangle in the input space as shown in Figure Q1. Draw the network indicating weights and thresholds of the perceptrons.

(12 marks)

Answer

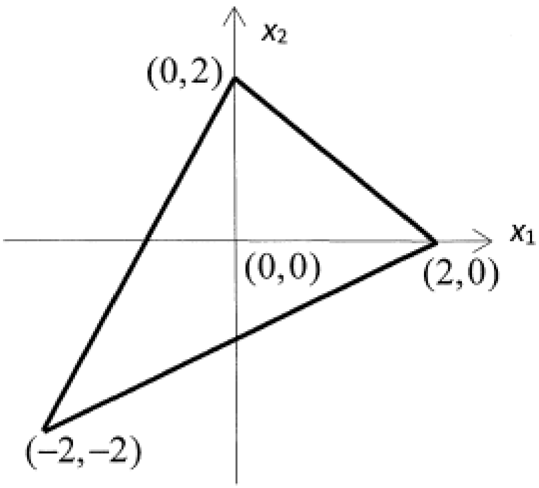


Figure Q1

1. (a) cont

Answer

Line 1 (passing through (0, 2) and (2, 0))
Line 2 (passing through (2, 0) and (-2, -2))
Line 3 (passing through (-2, -2) and (0, 2))

shaded above the line is +1 > 0
shaded below the line is -1 ≤ 0

Line 1 (passing through (0, 2) and (2, 0))

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{2-0} = -1$$

$$y = mx + c$$
$$2 = -1(0) + c$$
$$c = 2$$

$$x_2 = (-1)x_1 + 2$$
$$x_1 + x_2 - 2 = 0$$

$$u_1 = -x_1 - x_2 + 2$$

Line 2 (passing through (2, 0) and (-2, -2))

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

$$y = mx + c$$
$$0 = \frac{1}{2}(2) + c$$
$$c = -1$$

$$x_2 = \frac{1}{2}x_1 - 1$$
$$-\frac{1}{2}x_1 + x_2 + 1 = 0$$

$$u_2 = \frac{1}{2}x_1 - x_2 - 1$$

Line 3 (passing through (-2, -2) and (0, 2))

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{0-(-2)} = \frac{4}{2} = 2$$

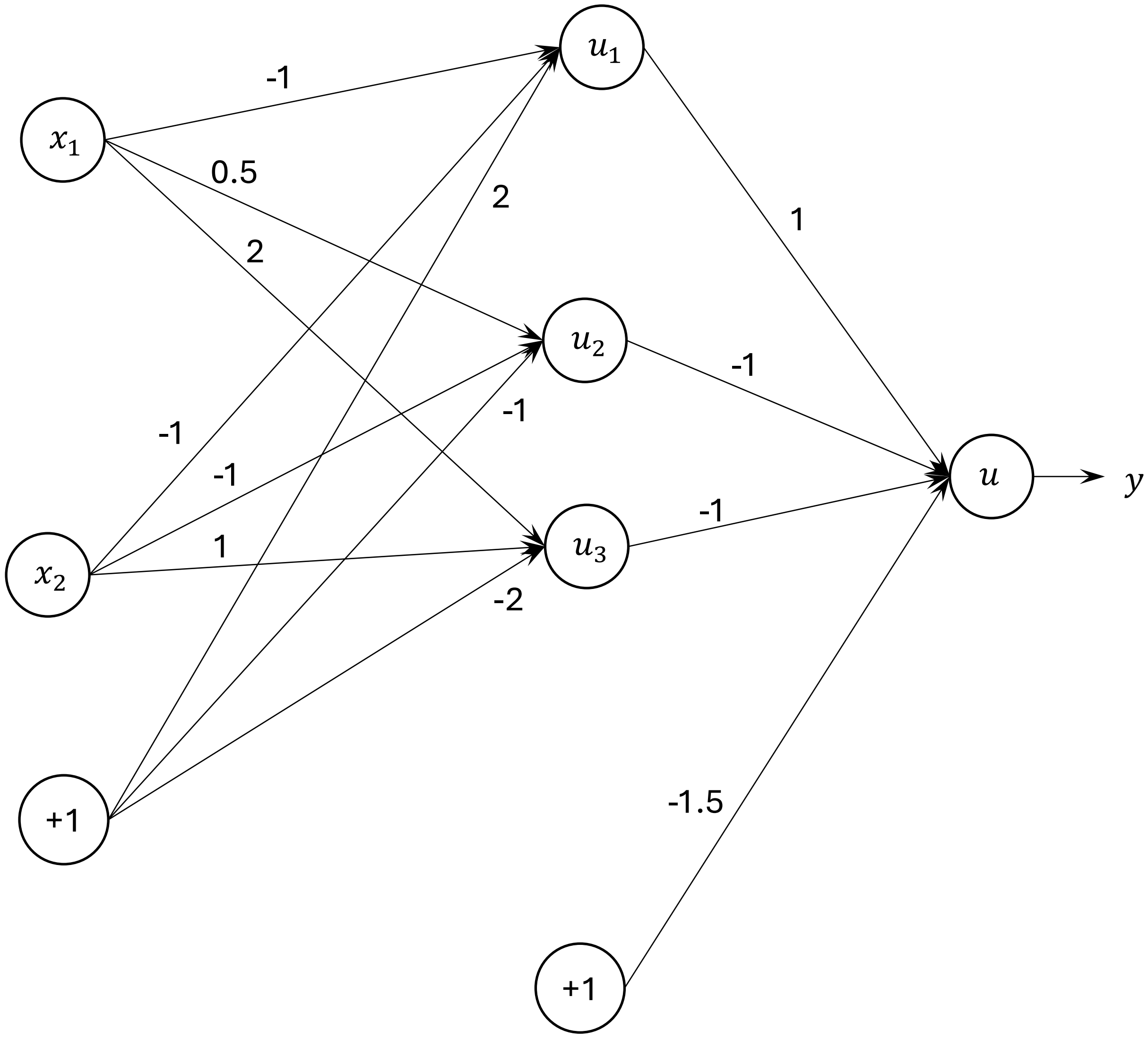
$$y = mx + c$$
$$-2 = 2(-2) + c$$
$$c = 2$$

$$x_2 = (2)x_1 + 2$$
$$2x_1 + x_2 - 2 = 0$$

$$u_3 = 2x_1 + x_2 - 2$$

1. (b) cont

Answer



$$u = y_1 - y_2 - y_3 - 1.5$$

1 (b)

2. A two-layer perceptron network has 2 input nodes, 3 hidden neurons, and 2 output neurons. The activation functions of the hidden layer neurons and output layer neurons are g and f, respectively, where

$$g(z) = \frac{1 - e^{-s}}{1 + e^{-s}} \quad \text{and} \quad f(u) = \frac{1}{1 + e^{-0.5u}}$$

The weight matrices to the hidden layer $\boldsymbol{W}$ and output layer $\boldsymbol{V}$ are initialized as follows:

$$\boldsymbol{W} = \begin{pmatrix} 0.50 & -1.35 \\ 1.25 & 0.25 \\ -0.75 & 0.30 \end{pmatrix} \text{and} \quad \boldsymbol{V} = \begin{pmatrix} 0.40 & -0.20 & 0.40 \\ 0.60 & 0.25 & -0.34 \end{pmatrix}$$

The thresholds of all neurons are initialized to 0.05.

The network is trained to produce a desired output $d = (0.00 \quad 1.00)^T$ for an input pattern $x = (0.75 \quad -0.80)^T$. The learning factor is 0.2. For one iteration of training, compute

2. (a) the synaptic input $\boldsymbol{z}$ and activation $\boldsymbol{h}$ at the hidden layer;

(5 marks)

Answer

$$W = \begin{pmatrix} 0.50 & 1.25 & -0.75 \\ -1.35 & 0.25 & 0.30 \end{pmatrix} \quad b = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix} \quad V = \begin{pmatrix} 0.40 & 0.60 \\ -0.20 & 0.25 \\ 0.40 & -0.35 \end{pmatrix} \qquad c = \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} 0.75 \\ -0.80 \end{pmatrix}$$

Synaptic input to the hidden-layer

$$\begin{aligned}
\boldsymbol{z} = \boldsymbol{W}^T\boldsymbol{x} + \boldsymbol{b} &= \begin{pmatrix} 0.50 & -1.35 \\ 1.25 & 0.25 \\ -0.75 & 0.30 \end{pmatrix} \begin{pmatrix} 0.75 \\ -0.80 \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix} \\
&= \begin{pmatrix} (\ 0.50)(0.75) + (-1.35)(-0.80) \\ (\ 1.25)(0.75) + (\ 0.25)(-0.80) \\ (-0.75)(0.75) + (\ 0.30)(-0.80) \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix} \\
&= \begin{pmatrix} 1.455 \\ 0.738 \\ -0.803 \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix} = \begin{pmatrix} 1.505 \\ 0.788 \\ -0.753 \end{pmatrix}
\end{aligned}$$

Output of the hidden-layer

$$\boldsymbol{h} = g(\boldsymbol{z}) = \frac{1 - e^{-s}}{1 + e^{-s}} = \begin{pmatrix} 0.637 \\ 0.735 \\ -0.360 \end{pmatrix}$$

2. (b) the synaptic input $\boldsymbol{u}$ and activation $\boldsymbol{y}$ at the output layer;

(2 marks)

Answer

$$W = \begin{pmatrix} 0.50 & 1.25 & -0.75 \\ -1.35 & 0.25 & 0.30 \end{pmatrix} \quad b = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix} \quad V = \begin{pmatrix} 0.40 & 0.60 \\ -0.20 & 0.25 \\ 0.40 & -0.35 \end{pmatrix} \qquad c = \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

$$\boldsymbol{h} = \begin{pmatrix} 0.637 \\ 0.735 \\ -0.360 \end{pmatrix}$$

Synaptic input to output-layer

$$\boldsymbol{u} = \boldsymbol{V}^T\boldsymbol{h} + \boldsymbol{c} = \begin{pmatrix} 0.40 & -0.20 & 0.40 \\ 0.60 & 0.25 & -0.35 \end{pmatrix}\begin{pmatrix} 0.637 \\ 0.735 \\ -0.360 \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

$$= \begin{pmatrix} (0.4)(0.637) + (-0.20)(0.735) + (\ \ 0.40)(-0.36) \\ (0.6)(0.637) + (\ \ 0.25)(0.735) + (-0.35)(-0.36) \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

$$= \begin{pmatrix} -0.036 \\ 0.692 \end{pmatrix} + \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix} = \begin{pmatrix} 0.014 \\ 0.742 \end{pmatrix}$$

Output of the output-layer

$$\boldsymbol{y} = f(\boldsymbol{u}) = \frac{1}{1 + e^{-0.5u}} = \begin{pmatrix} 0.502 \\ 0.592 \end{pmatrix}$$

2. (c) the error terms $\nabla_u J$ and $\nabla_V J$, at the output layer and hidden layer, respectively;

(10 marks)

Answer

Derivatives of $f(u)$ $\qquad \boldsymbol{y} = f(\boldsymbol{u}) = \dfrac{1}{1 + e^{-0.5u}} = \begin{pmatrix} 0.502 \\ 0.592 \end{pmatrix} \qquad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$f'(\boldsymbol{u}) = 0.5\boldsymbol{y}\cdot(\boldsymbol{1} - \boldsymbol{y}) = 0.5\begin{pmatrix} 0.502 \\ 0.592 \end{pmatrix}\cdot\left(\begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix} - \begin{pmatrix} 0.502 \\ 0.592 \end{pmatrix}\right) = \begin{pmatrix} 0.251 \\ 0.296 \end{pmatrix}\cdot\begin{pmatrix} 0.498 \\ 0.408 \end{pmatrix} = \begin{pmatrix} 0.125 \\ 0.121 \end{pmatrix}$$

$$\nabla_{\boldsymbol{u}} J = -(\boldsymbol{d} - \boldsymbol{y})\cdot f'(\boldsymbol{u}) = -\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0.502 \\ 0.592 \end{pmatrix}\right)\cdot\begin{pmatrix} 0.125 \\ 0.121 \end{pmatrix} = -\begin{pmatrix} -0.063 \\ 0.049 \end{pmatrix} = \begin{pmatrix} 0.063 \\ -0.049 \end{pmatrix}$$

$$\boldsymbol{h} = g(\boldsymbol{z}) = \frac{1 - e^{-s}}{1 + e^{-s}} = \begin{pmatrix} 0.637 \\ 0.735 \\ -0.360 \end{pmatrix}$$

$$g'(\boldsymbol{z}) = \boldsymbol{0.5}(\boldsymbol{1} - \boldsymbol{h}^{\boldsymbol{2}}) = 0.5\left(\begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} - \begin{pmatrix} (\ \ 0.637)^2 \\ (\ \ 0.735)^2 \\ (-0.360)^2 \end{pmatrix}\right) = \begin{pmatrix} 0.297 \\ 0.230 \\ 0.435 \end{pmatrix}$$

$$\nabla_{\boldsymbol{z}} J = V(\nabla_{\boldsymbol{u}} J)\cdot g'(\boldsymbol{z}) = \begin{pmatrix} 0.40 & 0.60 \\ -0.20 & 0.25 \\ 0.40 & -0.35 \end{pmatrix}\begin{pmatrix} 0.063 \\ -0.049 \end{pmatrix}\cdot\begin{pmatrix} 0.297 \\ 0.230 \\ 0.435 \end{pmatrix}$$

$$= \begin{pmatrix} (\ \ 0.40)(0.063) + (\ \ 0.60)(-0.049) \\ (-0.20)(0.063) + (\ \ 0.25)(-0.049) \\ (\ \ 0.40)(0.063) + (-0.35)(-0.049) \end{pmatrix}\cdot\begin{pmatrix} 0.297 \\ 0.230 \\ 0.435 \end{pmatrix} =\cdot \begin{pmatrix} -0.004 \\ -0.025 \\ 0.042 \end{pmatrix}\cdot\begin{pmatrix} 0.297 \\ 0.230 \\ 0.435 \end{pmatrix} = \begin{pmatrix} -0.001 \\ -\ 0.006 \\ 0.018 \end{pmatrix}$$

3. (a) unknown

3. (b) A convolution neural network has an input layer of dimensions 3x3. The first hidden layer has a convolution layer consisting of two filters $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$, and a mean pooling layer of pooling dimensions 2x2:

$$\boldsymbol{w}_1 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.6 \end{pmatrix} \text{ and } \boldsymbol{w}_2 = \begin{pmatrix} 0 & 0.4 \\ 0.4 & 0 \end{pmatrix}$$

Input pattern $\boldsymbol{X}$ is presented to the input layer of the network:

$$X = \begin{pmatrix} 0.10 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.40 & 0.60 & 0.20 \end{pmatrix}$$

Determine feature maps at

(i) the first convolution layer, assuming sigmoid activation functions and thresholds of 0.1 for all neurons;

(7 marks)

3. (b) (i) cont

Answer

$$X = \begin{pmatrix} 0.10 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.40 & 0.60 & 0.20 \end{pmatrix} \qquad \boldsymbol{w}_1 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.6 \end{pmatrix} \text{ and } \boldsymbol{w}_2 = \begin{pmatrix} 0 & 0.4 \\ 0.4 & 0 \end{pmatrix}$$

- P = 0
- S = 1
- bias = 0.3
- filter size = 2 x 2

convolution size:

$$\text{row} = \frac{R - F + 2P}{S} + 1 = \frac{3 - 2 + 2(0)}{1} + 1 = 2$$

$$\text{col} = \frac{C - F + 2P}{S} + 1 = \frac{3 - 2 + 2(0)}{1} + 1 = 2$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$: $u_1 = Conv(\boldsymbol{X}, \boldsymbol{w}_1) + b$

$$u(1,1) = (0.1)(0.6) + (0.2)(0) + (0.1)(0) + (0.8)(0.6) + 0.1$$
$$= 0.06 + 0 + 0 + 0.48 + 0.1 = 0.64$$
$$u(1,2) = (0.2)(0.6) + (0)(0) + (0.8)(0) + (0.1)(0.6) + 0.1$$
$$= 0.12 + 0 + 0 + 0.06 + 0.1 = 0.28$$
$$u(2,1) = (0.1)(0.6) + (0.8)(0) + (0.4)(0) + (0.6)(0.6) + 0.1$$
$$= 0.06 + 0 + 0 + 0.36 + 0.1 = 0.52$$
$$u(2,2) = (0.8)(0.6) + (0.1)(0) + (0.6)(0) + (0.2)(0.6) + 0.1$$
$$= 0.48 + 0 + 0 + 0.12 + 0.1 = 0.7$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_2$:

$$u(1,1) = (0.1)(0) + (0.2)(0.4) + (0.1)(0.4) + (0.8)(0) + 0.1$$
$$= 0 + 0.08 + 0.04 + 0 + 0.1 = 0.22$$
$$u(1,2) = (0.2)(0) + (0)(0.4) + (0.8)(0.4) + (0.1)(0) + 0.1$$
$$= 0 + 0 + 0.32 + 0 + 0.1 = 0.42$$
$$u(2,1) = (0.1)(0) + (0.8)(0.4) + (0.4)(0.4) + (0.6)(0) + 0.1$$
$$= 0 + 0.32 + 0.16 + 0 + 0.1 = 0.58$$
$$u(2,2) = (0.8)(0) + (0.1)(0.4) + (0.6)(0.4) + (0.2)(0) + 0.1$$
$$= 0 + 0.04 + 0.24 + 0 + 0.1 = 0.38$$

3. (b) cont

(ii) the first pooling layer.

(3 marks)

Answer

$$u_1 = \begin{pmatrix} 0.64 & 0.28 \\ 0.52 & 0.70 \end{pmatrix}$$

$$u_2 = \begin{pmatrix} 0.22 & 0.42 \\ 0.58 & 0.38 \end{pmatrix}$$

pooling size:

$$\text{row} = \frac{R-F}{S} + 1 = \frac{2-2}{2} + 1 = 1$$

$$\text{col} = \frac{C-F}{S} + 1 = \frac{2-2}{2} + 1 = 1$$

Feature maps at the convolution layer:

$$y_1 = sigmoid(u_1) = \frac{1}{1+e^{-\boldsymbol{u}_1}} = \begin{pmatrix} 0.65 & 0.57 \\ 0.63 & 0.67 \end{pmatrix}$$

$$y_2 = sigmoid(u_2) = \frac{1}{1+e^{-\boldsymbol{u}_2}} = \begin{pmatrix} 0.55 & 0.60 \\ 0.64 & 0.59 \end{pmatrix}$$

$$\boldsymbol{y} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0.64 & 0.28 \\ 0.52 & 0.70 \end{pmatrix} \\ \begin{pmatrix} 0.22 & 0.42 \\ 0.58 & 0.38 \end{pmatrix} \end{pmatrix}$$

Pooling 2×2

$$P_{ave1} = \frac{0.65 + 0.57 + 0.63 + 0.67}{4} = \frac{2.52}{4} = 0.63$$

$$P_{ave2} = \frac{0.22 + 0.42 + 0.58 + 0.38}{4} = \frac{1.6}{4} = 0.40$$

Feature maps at the pooling layer:
Mean-pooling:

$$P_{ave} = \begin{pmatrix} (0.63) \\ (0.40) \end{pmatrix}$$

4. (a) unknown

4. (b) unknown