

Ensemble learning

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Further bibliography

- Leo Breiman, Bagging Predictors, Machine Learning , vol. 24, issue 2, pp. 123-140, 1996
- Yoav Freund, Rober E. Schapire, Experiments with a new boosting algorithm, 13th International Conference on Machine Learning, 148–156, Morgan Kaufmann, 1996
- Leo Breiman, Random Forests, Machine Learning 45(1), 5-32, 2001.
- Gautam Kunapuli, Ensemble Methods for Machine Learning, Manning, 2023

Ensemble learning

- Several **base learners** are trained and their **predictions are combined**.
- The way of application of the learners may take place at several levels.
- Classical approaches:
 - Bagging
 - Boosting
 - Random forests

Evolution of Tree Algorithm



Decision
Trees

Bagging

Random
Forest

Boosting

Gradient
Boosting

XG-Boost

Bagging

- Short for **B**ootstrap **A**GGregat**I**NG.
- The training data set of m samples is split into b parts (**b**ags).
 - Every subset has the same number of m elements.
 - The data from each subset are sampled with replacement.
- The base learner is trained on each subset.
- The b constructed models vote the output for new test data:
 - The mean of the outputs for the b models - for regression
 - The class with most appearances in the predictions of the b models - for classification
- Examples not selected (out-of-bag) are used to estimate the generalization error.

Package R **ipred**

- The base learner is a decision tree (**bagged tree**).
 - In this sense package **rpart** is called.
- If the output is seen as factor, classification is considered.
 - If it is continuous, regression is in place.
- Parameter **nbagg** specifies the desired number of bags – default 25.
- m samples are taken in each bag
- Parameter **coob**=true specifies the scope to compute an estimation of the generalization error (out-of-bag estimation)

Code

```
1 library(ipred)
2 library(mlbench)
3 library(e1071)
4
5 data(BreastCancer)
6 dat <- BreastCancer[, -1]
7
8 classColumn <- 10
9 index <- 1:nrow(dat)
10
11 repeats <- 30
12 accuracies <- vector(mode="numeric", length = repeats)
13
14 for (i in 1:repeats){
15   testindex <- sample(index, trunc(length(index) / 4))
16   testset <- dat[testindex, ]
17   trainset <- dat[-testindex, ]
18
19   bg <- bagging(Class ~ ., data = trainset, coob = TRUE)
20
21   bg_pred <- predict(bg, testset[, -classColumn])
22   contab <- table(pred = bg_pred, true = testset[, classColumn])
23   accuracies[i] <- classAgreement(contab)$diag
24 }
25
26 print(bg)
27
28 print(accuracies)
29 print(mean(accuracies))
30 print(sqrt(var(accuracies)))
31 print(summary(accuracies))
```

Result

```
Bagging classification trees with 25 bootstrap replications
```

```
Call: bagging.data.frame(formula = Class ~ ., data = trainset, coob = TRUE)
```

```
Out-of-bag estimate of misclassification error: 0.0391
```

```
[1] 0.9655172 0.9597701 0.9310345 0.9597701 0.9655172 0.9540230 0.9597701  
[8] 0.9482759 0.9540230 0.9770115 0.9540230 0.9195402 0.9367816 0.9425287  
[15] 0.9597701 0.9827586 0.9712644 0.9655172 0.9655172 0.9712644 0.9540230  
[22] 0.9885057 0.9367816 0.9540230 0.9597701 0.9425287 0.9425287 0.9712644  
[29] 0.9540230 0.9310345
```

```
[1] 0.9559387
```

```
[1] 0.01585332
```

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	0.9195	0.9440	0.9569	0.9559	0.9655	0.9885

Python. Data processing WDBC

- For the Wisconsin original data set with 9 attributes and 699 records, we will have to read the .csv file.
- In the collection available from UCI, missing values are denoted by '?'
- Their transformation to 0 will be performed through the use of *applymap* and the anonymous *lambda* function.

```
# change missing values in the form of '?'  
dt = dt.applymap(lambda x: 0 if x == '?' else x)
```

Library BaggingClassifier

- Base estimator is a DecisionTreeClassifier
- n_estimators gives the number of base classifiers in the ensemble
- max_samples denotes the number of records to draw (with replacement) in each bag
- Additionally in this Python library: max_features gives the number of attributes to draw (with replacement) for each classifier
- oob_score specifies whether to use out-of-bag samples to approximate generalization error.

WDBC reading and preprocessing

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sklearn
from sklearn.metrics import accuracy_score
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import train_test_split
from sklearn.ensemble import BaggingClassifier

repeats = 30
accuracies = []

data = pd.read_csv("breast-cancer-wisconsin.csv")
# drop the first column
dt = data.drop(data.columns[0], axis=1)
# change missing values in the form of '?'
dt = dt.applymap(lambda x: 0 if x == '?' else x)

#take first n-1 columns for x and last column for y
brx = dt.iloc[:, :-1]
bry = dt.iloc[:, -1]
```

Results

```
for i in range(0, repeats):  
    brx_train, brx_test, bry_train, bry_test = train_test_split(brx, bry, test_size = 0.25)  
  
    # base estimator is DecisionTreeClassifier  
    bgm = BaggingClassifier(n_estimators = 25, max_samples = 50, max_features = 2, oob_score = True)  
    bgm.fit(brx_train, bry_train)  
  
    bry_pred = bgm.predict(brx_test)  
    accuracies.append(accuracy_score(bry_test, bry_pred))  
  
print(accuracies)  
print("Oob score", bgm.oob_score_)
```

```
[0.9485714285714286, 0.9714285714285714, 0.9657142857142857, 0.9771428571428571, 0.9828571428571429, 0.9771428571428571,  
Oob score 0.9637404580152672]
```

```
print("Mean accuracy", np.mean(accuracies))  
print("Standard deviation", np.std(accuracies))
```

```
Mean accuracy 0.9649523809523808  
Standard deviation 0.013740715273806808
```

Boosting: Adaboost – **AD**Aptive **BOO**STing

- Aims to progressively increase the performance for a learner.
- The base learner is applied repeatedly.
- Every training data point (of the m) has a weight attached – initially all equal to $1/m$.
- At every step, erroneous points get a higher weight.
- The model resulting at every step receives a vote weighted by its performance
 - The weight is given by the measure of its errors on the training set.

Package R **ada**

- Implements again a decision tree as base learner (**boosted tree**)
 - using the same package **rpart**.
- Parameter **iter** specifies the number of desired iterations.
- The function **plot** offers the possibility of visualization:
 - The error at every iteration of the boosting algorithm
 - The error can be calculated both for the training and the test sets
 - The kappa agreement between the predicted and the real target at each boosting iteration for both sets

Code

```
1 library(ada)
2 library(mlbench)
3 library(e1071)
4
5 data(BreastCancer)
6 dat <- BreastCancer[, -1]
7
8 classColumn <- 10
9 index <- 1:nrow(dat)
10
11 repeats <- 30
12 accuracies <- vector(mode="numeric", length = repeats)
13
14 for (i in 1:repeats){
15   testindex <- sample(index, trunc(length(index) / 4))
16   testset <- dat[testindex, ]
17   trainset <- dat[-testindex, ]
18
19   boost <- ada(Class ~ ., data = trainset, iter = 20)
20
21   boost_pred <- predict(boost, testset[, -classColumn])
22   contab <- table(pred = boost_pred, true = testset[, classColumn])
23   accuracies[i] <- classAgreement(contab)$diag_
24 }
25
26 summary(boost)
```

Rezultate

```
33 print(accuracies)
34 print(mean(accuracies))
35 print(sqrt(var(accuracies)))
36 print(summary(accuracies))
```

```
Loading required package: rpart
Call:
ada(Class ~ ., data = trainset, iter = 20)

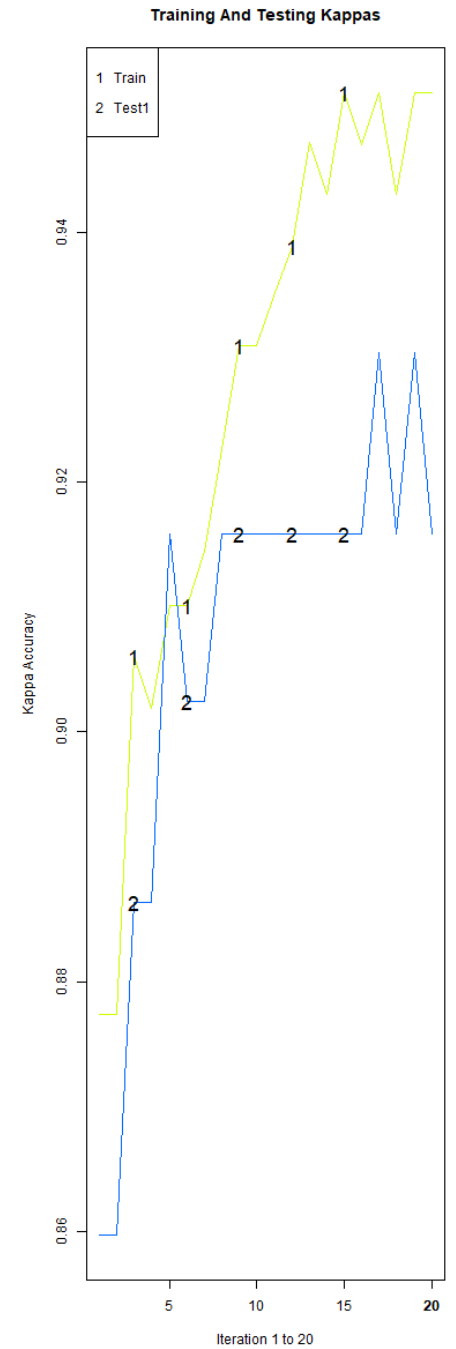
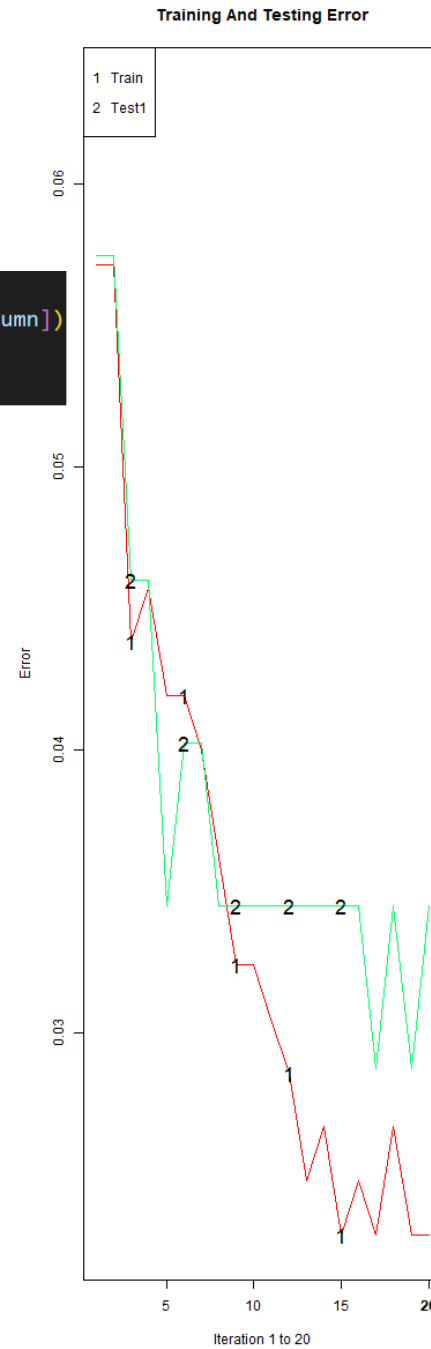
Loss: exponential Method: discrete Iteration: 20

Training Results
Accuracy: 0.977 Kappa: 0.951

[1] 0.9655172 0.9597701 0.9655172 0.9655172 0.9482759 0.9540230 0.9770115
[8] 0.9712644 0.9540230 0.9770115 0.9597701 0.9827586 0.9712644 0.9597701
[15] 0.9540230 0.9425287 0.9712644 0.9597701 0.9770115 0.9712644 0.9712644
[22] 0.9770115 0.9712644 0.9712644 0.9770115 0.9655172 0.9540230 0.9712644
[29] 0.9712644 0.9655172
[1] 0.966092
[1] 0.0097052
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.9425  0.9598   0.9684   0.9661  0.9713   0.9828
```


Plot

```
##add testing data set
boost <- addtest(boost, testset[, -classColumn], testset[, classColumn])
##plot boost
plot(boost, TRUE, TRUE)
```



AdaBoostClassifier in Python

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sklearn
from sklearn.metrics import accuracy_score
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import train_test_split
from sklearn.ensemble import AdaBoostClassifier

repeats = 30
accuracies = []

data = pd.read_csv("breast-cancer-wisconsin.csv")
# drop the first column
dt = data.drop(data.columns[0], axis=1)
# change missing values in the form of '?'
dt = dt.applymap(lambda x: 0 if x == '?' else x)

#take first n-1 columns for x and last column for y
brx = dt.iloc[:, :-1]
bry = dt.iloc[:, -1]

for i in range(0, repeats):
    brx_train, brx_test, bry_train, bry_test = train_test_split(brx, bry, test_size = 0.25)

    # base estimator is DecisionTreeClassifier
    adm = AdaBoostClassifier(n_estimators = 20)
    adm.fit(brx_train, bry_train)

    bry_pred = adm.predict(brx_test)
    accuracies.append(accuracy_score(bry_test, bry_pred))

print(accuracies)
```

Results

```
print("Mean accuracy", np.mean(accuracies))  
print("Standard deviation", np.std(accuracies))
```

✓ 0.0s

Mean accuracy 0.9550476190476191

Standard deviation 0.01366127320155512

Random forests

- It is an ensemble method that combines several decision trees.
- Every tree is generated by random sampling (with replacement) of:
 - Attributes
 - Records
- For every tree there exists automatically
 - A training (in-bag) set
 - An out-of-bag set to measure the generalization error
- The class of a new example is established by vote for classification and by mean for regression.

Pachetul randomForest

- Parameters:
 - *ntree* – number of trees; implicitly equal to 500
 - *mtry* – number of randomly chosen attributes
 - *Gini* – the default split criterion
- Output:
 - An estimate of the generalization error – out-of-bag (OOB) estimate
 - Confusion matrix
 - Attribute importance

Classification Breast Cancer

```
1 library(randomForest)
2 library(mlbench)
3 library(e1071)
4
5 data(BreastCancer)
6 dat <- BreastCancer[, -1]
7
8 classColumn <- 10
9 index <- 1:nrow(dat)
10
11 repeats <- 30
12 accuracies <- vector(mode="numeric", length = repeats)
13
14 for (i in 1:repeats){
15   testindex <- sample(index, trunc(length(index) / 4))
16   testset <- dat[testindex, ]
17   trainset <- dat[-testindex, ]
18
19   rf <- randomForest(class ~ ., data = trainset, importance = TRUE, mtry = 4, na.action = na.omit)
20
21   rf_pred <- predict(rf, testset[, -classColumn])
22   contab <- table(pred = rf_pred, true = testset[, classColumn])
23   accuracies[i] <- classAgreement(contab)$diag
24 }
25
26 print(accuracies)
27 print(mean(accuracies))
28 print(sqrt(var(accuracies)))
29 print(summary(accuracies))
30
31 print(rf)
32
33 print(importance(rf))
34 varImpPlot(rf)
```

Missing values, as type N/A in the Wisconsin original data set from package mlbench, will be omitted.

Results

```
[1] 0.9595376 0.9647059 0.9642857 0.9580838 0.9825581 0.9473684 0.9768786
[8] 0.9761905 0.9532164 0.9823529 0.9704142 0.9644970 0.9880952 0.9649123
[15] 0.9881657 0.9595376 0.9766082 0.9761905 0.9651163 0.9705882 0.9644970
[22] 0.9882353 0.9647059 0.9649123 0.9532164 0.9705882 0.9763314 0.9709302
[29] 0.9821429 0.9640719
[1] 0.9696312
[1] 0.01074724
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.9474  0.9643  0.9678  0.9696  0.9765  0.9882
```

```
Call:
randomForest(formula = Class ~ ., data = trainset, importance = TRUE,      mtry = 4, na.action = na.omit)
  Type of random forest: classification
    Number of trees: 500
No. of variables tried at each split: 4
```

OOB estimate of error rate: 2.33%

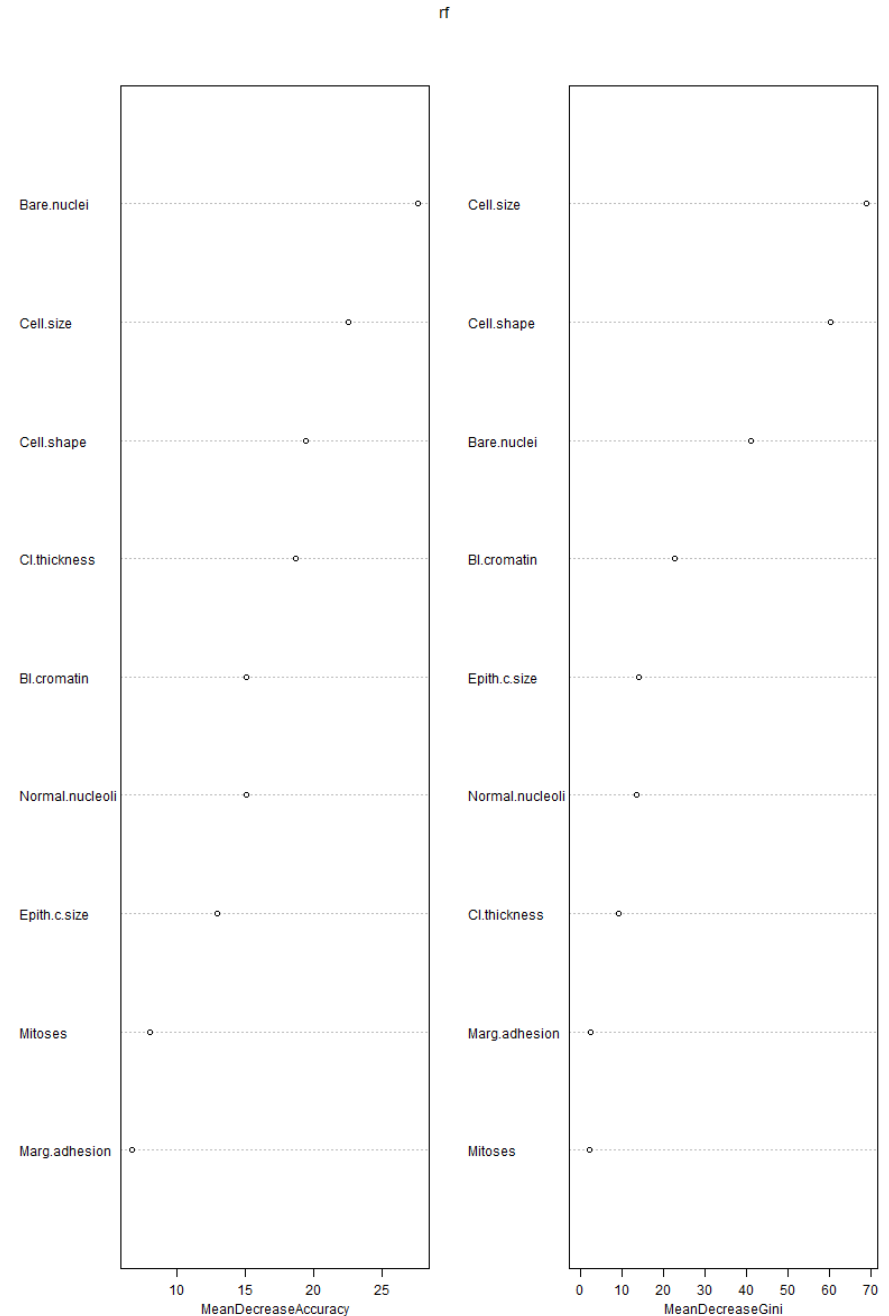
Confusion matrix:

	benign	malignant	class.error
benign	328	8	0.02380952
malignant	4	176	0.02222222

Variable importance

	benign	malignant	MeanDecreaseAccuracy	MeanDecreaseGini
Cl.thickness	15.431359	16.5033867	18.747926	9.028927
Cell.size	16.813055	15.0669505	22.590839	69.008282
Cell.shape	7.572875	17.7918011	19.494297	60.304563
Marg.adhesion	4.424217	5.6358816	6.789697	2.380213
Epith.c.size	9.212465	10.4729200	13.003310	14.059009
Bare.nuclei	18.988259	24.0511769	27.643494	41.090238
Bl.cromatin	10.396394	11.9273734	15.141829	22.531616
Normal.nucleoli	14.213698	6.6185920	15.098342	13.549149
Mitoses	8.409287	0.2111715	8.104082	2.068840

- *Mean decrease in accuracy* on OOB data and shows the amount of loss in accuracy if the variable values are permuted (total and per class)
- *Mean impurity decrease (Gini)* shows the contribution of the variable to the homogeneity of the forest



Library RandomForestClassifier

- *n_estimators* to specify number of trees
- *max_features* to indicate number of features randomly chosen as candidates for each split
- *criterion* to give the desired split modality – default Gini

WDBC reading and preprocessing

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sklearn
from sklearn.metrics import accuracy_score
from sklearn.metrics import confusion_matrix
from sklearn.model_selection import train_test_split
from sklearn.ensemble import RandomForestClassifier
from sklearn import datasets
from sklearn.metrics import ConfusionMatrixDisplay

repeats = 30
accuracies = []

data = pd.read_csv("breast-cancer-wisconsin.csv")
# drop the first column
dt = data.drop(data.columns[0], axis=1)
# change missing values in the form of '?'
dt = dt.applymap(lambda x: 0 if x == '?' else x)

# for variable importance plots
# take the names of the features - all columns without class
dtNames = dt.drop(dt.columns[9], axis=1)
features = dtNames.columns

#take first n-1 columns for x and last column for y
brx = dt.iloc[:, :-1]
bry = dt.iloc[:, -1]
```

RF Classifier in Python

```
for i in range(0, repeats):
    brx_train, brx_test, bry_train, bry_test = train_test_split(brx, bry, test_size = 0.25)

    # default for split criterion: gini
    rfm = RandomForestClassifier(n_estimators = 500, max_features = 4)
    rfm.fit(brx_train, bry_train)

    bry_pred = rfm.predict(brx_test)
    accuracies.append(accuracy_score(bry_test, bry_pred))

print(accuracies)
```

```
print("Mean accuracy", np.mean(accuracies))
print("Standard deviation", np.std(accuracies))
cm = confusion_matrix(bry_test, bry_pred)
```

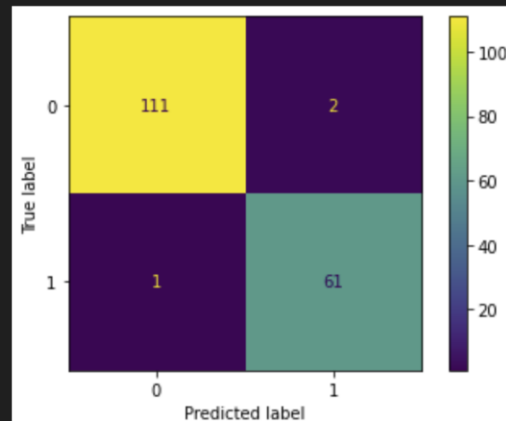
```
ConfusionMatrixDisplay(cm).plot()
```

✓ 0.1s

Mean accuracy 0.9655238095238095

Standard deviation 0.013561316199106709

<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x265057ec970>



Variable importance

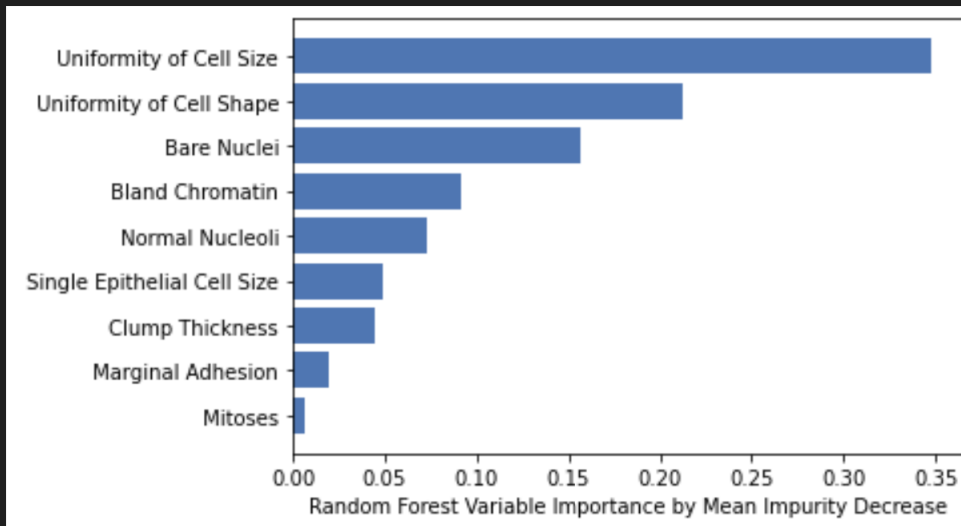
- Impurity-based importance
 - based on training set statistics (Gini)
 - biased towards high cardinality (favors numerical variables)
- Permutation importance
 - computed on a held-out test set
 - measures the change in performance when each variable is shuffled randomly
 - shows the generalization ability of the variable for prediction on the test data

Impurity-based importance

```
# feature importance computed with  
# mean impurity decrease (Gini)  
# and ordered decreasingly  
sorted_idx = rfm.feature_importances_.argsort()  
  
plt.barh(features[sorted_idx], rfm.feature_importances_[sorted_idx])  
plt.xlabel("Random Forest Variable Importance by Mean Impurity Decrease")
```

✓ 0.2s

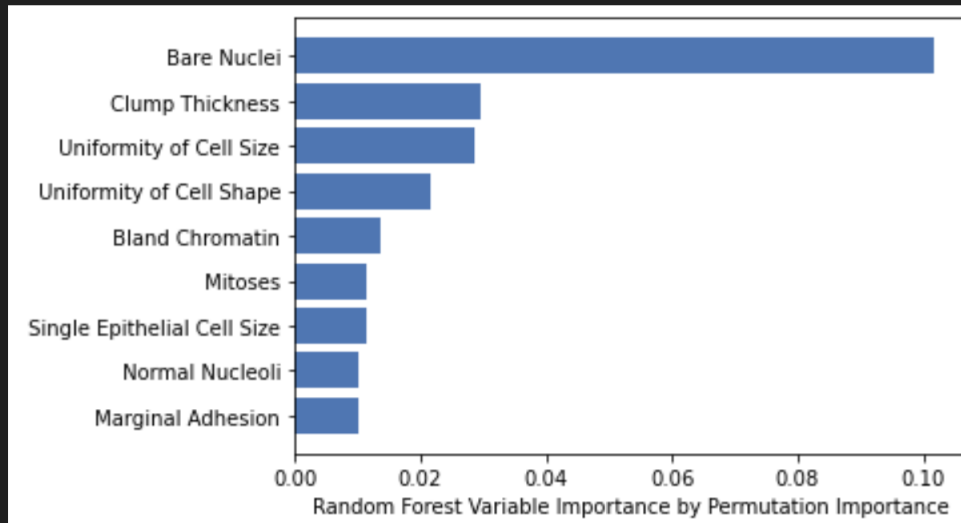
Text(0.5, 0, 'Random Forest Variable Importance by Mean Impurity Decrease')



Permutation importance

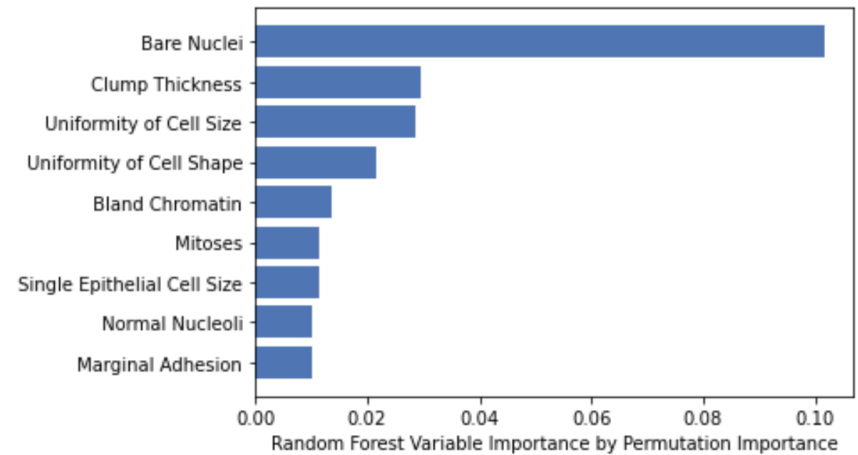
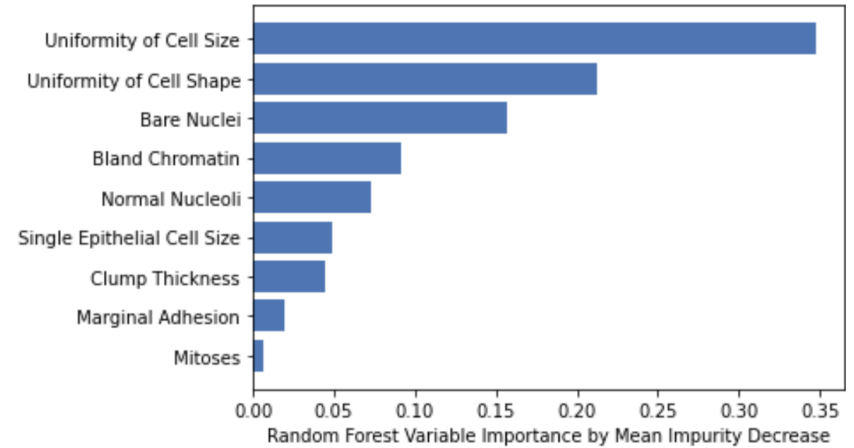
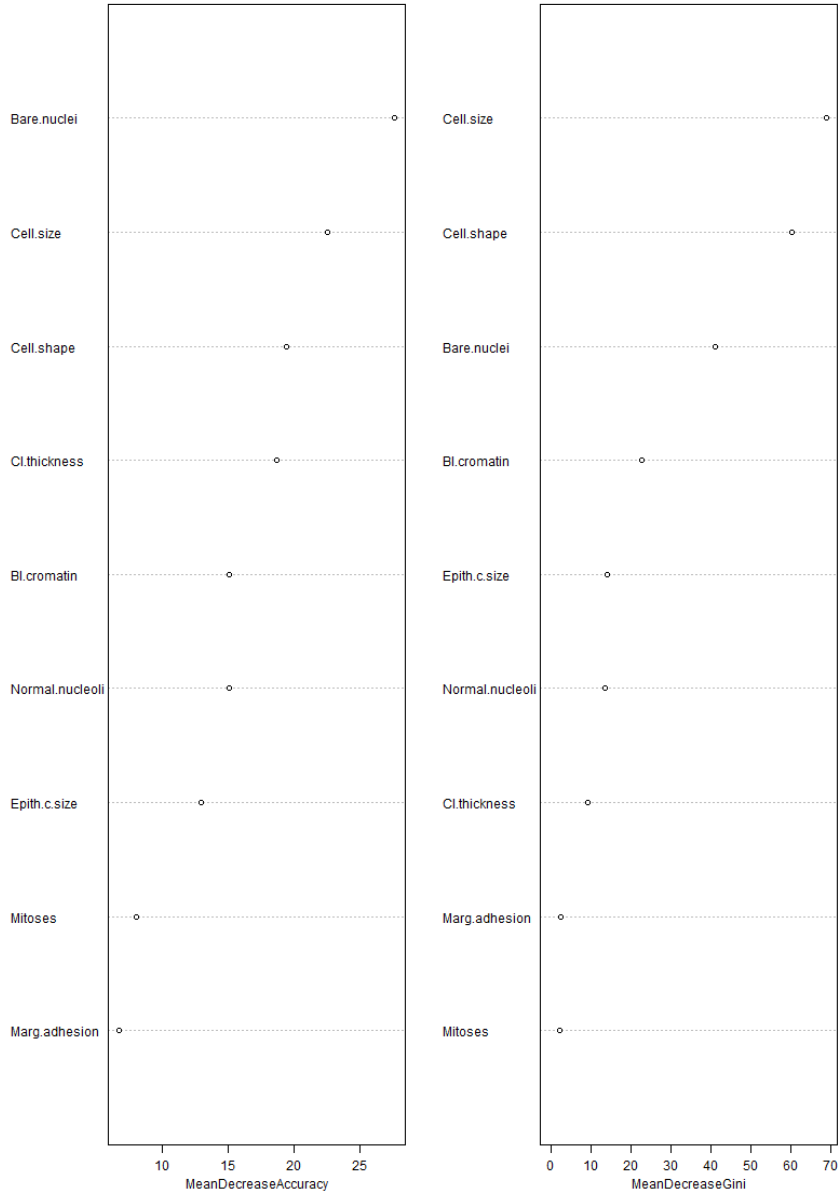
```
# feature importance computed with  
# permutation importance (mean decrease in accuracy)  
  
from sklearn.inspection import permutation_importance  
  
perm_importance = permutation_importance(rfm, brx_test, bry_test)  
sorted_idx = perm_importance.importances_mean.argsort()  
plt.barh(features[sorted_idx], perm_importance.importances_mean[sorted_idx])  
plt.xlabel("Random Forest Variable Importance by Permutation Importance")  
  
✓ 2.5s
```

Text(0.5, 0, 'Random Forest Variable Importance by Permutation Importance')



R vs Python

rf



RF Regressor in Python

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sklearn
from sklearn.metrics import mean_squared_error
from sklearn.metrics import r2_score
from sklearn.model_selection import train_test_split
from sklearn.ensemble import RandomForestRegressor
from sklearn import datasets

repeats = 30
accuracies = []

mses = []
r2s = []

bx, by = datasets.load_boston(return_X_y=True)

for i in range(0, repeats):
    #random generation of training and test, 75-25%
    bx_train, bx_test, by_train, by_test = train_test_split(bx, by, test_size = 0.25)

    # Default values for parameters: n_estimators = 100, max_features = 1, criterion = squared_error
    rfm = RandomForestRegressor()
    rfm.fit(bx_train, by_train)

    by_pred = rfm.predict(bx_test)
    mses.append(mean_squared_error(by_test, by_pred))
    r2s.append(r2_score(by_test, by_pred))

print(mses)
print(r2s)
```


Results

```
print("Mean MSE:", np.mean(mses))  
print("Standard deviation:", np.std(mses))  
print("R^2:", np.mean(r2s))
```

```
Mean MSE: 10.292878972427873  
Standard deviation: 2.5748469636950917  
R^2: 0.876793938937226
```

Feature importance - SHAP

- Model-agnostic variable ranker
- Based on game theory
- Popular for AI explainability
- *pip install shap*

```
import shap

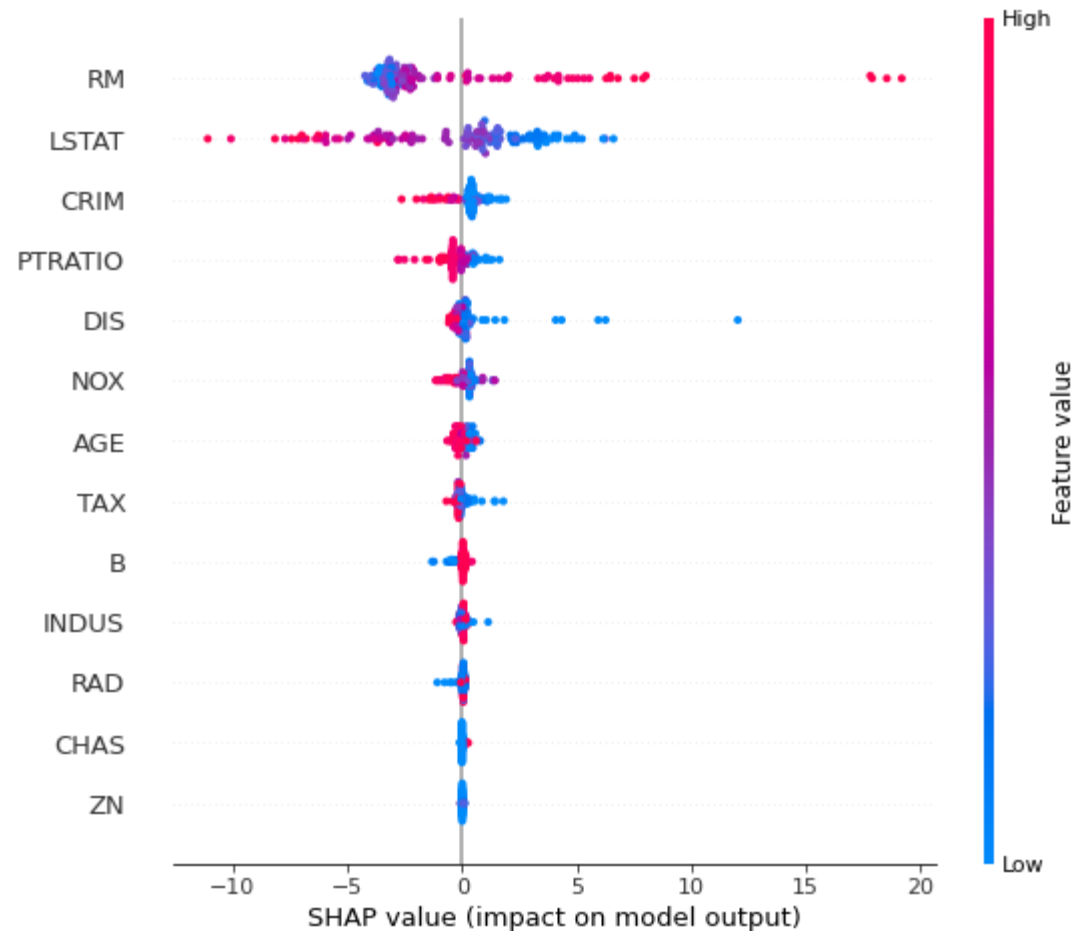
data = datasets.load_boston(return_X_y=False)
features = data.feature_names

explainer = shap.TreeExplainer(rfm)
shap_values = explainer.shap_values(bx_test)

shap.summary_plot(shap_values, bx_test, feature_names = features)
```

Variable importance

- Variable values where resulting $\text{SHAP} \geq 0$ have a positive impact on the prediction
- Idem for negative
- RM with higher values has a positive impact
- LSTAT with higher values has a negative one



Pixel importance in an image

```
from sklearn.ensemble import RandomForestClassifier
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
from sklearn import datasets

mx, my = datasets.load_digits(return_X_y = True) # 8x8 images of digits

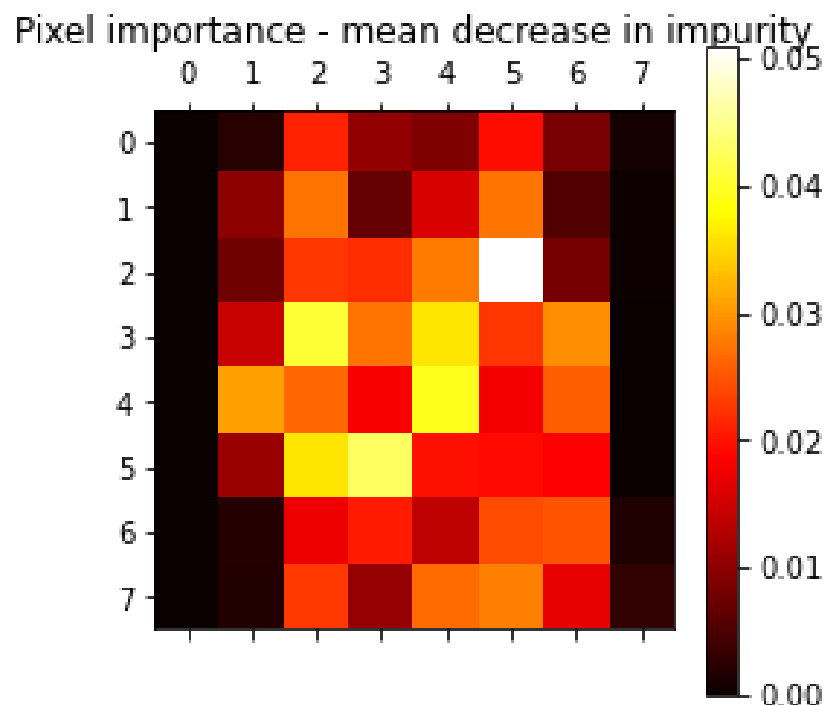
mx_train, mx_test, my_train, my_test = train_test_split(mx, my, test_size = 0.25)

forest = RandomForestClassifier(n_estimators=1000)
forest.fit(mx_train, my_train)

img_shape = (8, 8)
importance = forest.feature_importances_

imp_resaped = importance.reshape(img_shape)
plt.matshow(imp_resaped, cmap=plt.cm.hot)
plt.title("Pixel importance - mean decrease in impurity")
plt.colorbar()
plt.show()
```

MNIST pixel importance

[illegible]

Homework

- Implement in R or Python a decision tree model for
 - either Pima Indians Diabetes
 - or a car price / customer purchasing behaviour.
- Represent the resulting trees through a plot.
- Implement bagging, boosting and random forests in comparison.
- Implement also a neural network as the black-box model.

Extra

- Research XGBoost (gradient boosting), explain its concept of weak learners training on residuals and apply it on your previous data set of choice.



XGBoost

The entire
deep-learning
ecosystem



medium.com

