Week 6 Homework

P 1 1 *Suppose the $m \times n$ matrix A has the form $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, where A_1 is a nonsingular matrix of dimension $n \times n$ and A_2 is an arbitrary matrix of dimension $m \quad n) \times n$. Prove that*

$$\|A \ \|_2 \leq \|A_1{}^1\|_2.$$

P 1 2 *Let A be an $m \times n$ matrix. Use the steps below to show that a vector $x \in {}^n$ satis es $Ax = 0$ if and only if $A \ Ax = 0$. This will show that $Null \ A) = Null \ A \ A)$.*
a) Show that if $Ax = 0$, then $A \ Ax = 0$;
b) Suppose $A \ Ax = 0$. Show that $Ax = 0$.

P 1 3 *Show the Vandermonde matrix in (11.4) is nonsingular as long as the points $\{x_i\}$ are distinct.*

P 1 4 *Find a least solution for the following problems:*

a)

$$A = \begin{bmatrix} 1 & 6 \\ 1 & 2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 5 & 1 \\ 1 & 7 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 3 \\ 5 \end{bmatrix}$$

P 1 5 *Find the equation $y = {}_0 + {}_1 x$ of the least-squares line that best ts the data points $2, 1)$, $5, 2)$, $7, 3)$ and $8, 3)$.*