Workbook Examples
Chapter 3
Math 1100

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Outline

1 §3.5: Solving Equations & Inequalities w/ Abs. Value

Equations with Absolute Value

For $a > 0$ and an algebraic expression x:

Equations with Absolute Value

For $a > 0$ and an algebraic expression x:

$$|x| = a$$

Equations with Absolute Value

For $a > 0$ and an algebraic expression x:

$$|x| = a$$

is equivalent to

Equations with Absolute Value

For $a > 0$ and an algebraic expression x:

$$|x| = a$$

is equivalent to

$$x = a \text{ or } x = -a.$$

Example

Solve

$$|x| = 5.$$

Example

Solve

$$|x| = 5.$$

Solution:

Example

Solve

$$|x| = 5.$$

Solution:

$$\Rightarrow x = 5, x = -5.$$

Example

Solve

$$|5x| = 4.$$

Example

Solve

$$|5x| = 4.$$

Solution:

Example

Solve

$$|5x| = 4.$$

Solution:

$$\Rightarrow 5x = 4, 5x = -4$$

Example

Solve

$$|5x| = 4.$$

Solution:

$$\Rightarrow 5x = 4, 5x = -4$$

$$\Rightarrow x = \frac{4}{5}, x = -\frac{4}{5}.$$

Example

Solve

$$|x - 3| = 5.$$

Example

Solve

$$|x - 3| = 5.$$

Solution:

Example

Solve

$$|x - 3| = 5.$$

Solution:

$$\Rightarrow x - 3 = 5, x - 3 = -5$$

Example

Solve

$$|x - 3| = 5.$$

Solution:

$$\Rightarrow x - 3 = 5, x - 3 = -5$$

$$\Rightarrow x = 8, x = -2.$$

Example

Solve

$$|x + 2| - 5 = 9.$$

Example

Solve

$$|x + 2| - 5 = 9.$$

Solution:

Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

$$\Rightarrow x+2=14, x+2=-14$$

Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

$$\Rightarrow x+2=14, x+2=-14$$

$$\Rightarrow x=12, x=-16.$$

Example

Solve

$$|x - 4| + 3 = 9.$$

Example

Solve

$$|x - 4| + 3 = 9.$$

Solution:

Example

Solve

$$|x - 4| + 3 = 9.$$

Solution:

$$\Rightarrow |x - 4| = 6$$

Example

Solve

$$|x-4|+3=9.$$

Solution:

$$\Rightarrow |x-4|=6$$

$$\Rightarrow x-4=6, x-4=-6$$

Example

Solve

$$|x-4|+3=9.$$

Solution:

$$\Rightarrow |x-4|=6$$

$$\Rightarrow x-4=6, x-4=-6$$

$$\Rightarrow x=10, x=-2.$$

Example

Solve

$$9 - |x - 2| = 7.$$

Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2, x - 2 = -2$$

Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2, x - 2 = -2$$

$$\Rightarrow x = 4, x = 0.$$

Example

Solve

$$5 - |4x + 3| = 2.$$

Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

$$\Rightarrow 4x + 3 = 3, 4x + 3 = -3$$

Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

$$\Rightarrow 4x + 3 = 3, 4x + 3 = -3$$

$$\Rightarrow x = 0, x = -\frac{3}{2}.$$

More About Absolute Value Equations

When $a = 0$, $|x| = a$ is equivalent to $x = 0$.

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When $a = 0, |x| = a$ is equivalent to $x = 0$.

Note that for $a < 0, |x| = a$ has *no* solution,

because the absolute value of an expression is never negative.

The solution set is the *empty set*, denoted $\emptyset$.

Example

Solve

$$|x - 4| + 3 = 0.$$

Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

but this equation has no solution,

Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

but this equation has no solution,

i.e. the solution set is $\emptyset$.

Inequalities with Absolute Value

Inequalities sometimes contain absolute–value notation.

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The following properties are used to solve them.

For $a > 0$ and an algebraic expression x:

$$|x| < a \text{ is equivalent to } -a < x < a,$$
$$|x| > a \text{ is equivalent to } x < -a \text{ or } x > a.$$

Inequalities with Absolute Value

Inequalities sometimes contain absolute–value notation.

The following properties are used to solve them.

For $a > 0$ and an algebraic expression x:

$$|x| < a \text{ is equivalent to } -a < x < a,$$
$$|x| > a \text{ is equivalent to } x < -a \text{ or } x > a.$$

Similar statements hold for $|x| \leq a$ and $|x| \geq a$.

Inequalities with Absolute Value (cont.)

For example,

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For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

Inequalities with Absolute Value (cont.)

For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

$|y| \geq 1$ is equivalent to $y \leq -1$ or $y \geq 1$.

Inequalities with Absolute Value (cont.)

For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

$|y| \geq 1$ is equivalent to $y \leq -1$ or $y \geq 1$.

$|2x + 3| \leq 4$ is equivalent o $-4 \leq 2x + 3 \leq 4$.

Example

Solve

$$|x| < 5.$$

Example

Solve

$$|x| < 5.$$

Solution:

Example

Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

Example

Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

$$(-5, 5).$$

Example

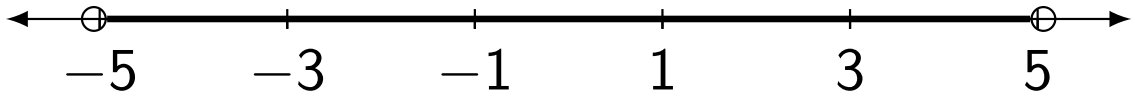
Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

$$(-5, 5).$$



Example

Solve

$$|x| \geq 6.$$

Example

Solve

$$|x| \geq 6.$$

Solution:

Example

Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

Example

Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

$$(-\infty, -6] \bigcup [6, \infty).$$

Example

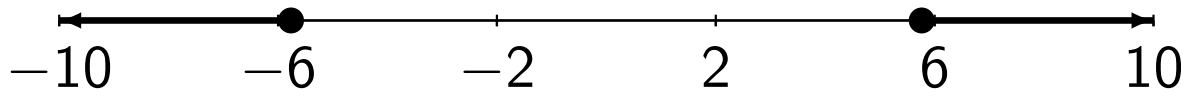
Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

$$(-\infty, -6] \bigcup [6, \infty).$$



Example

Solve

$$|x + 6| \leq 10.$$

Example

Solve

$$|x + 6| \leq 10.$$

Solution:

Example

Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

Example

Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

$$\Rightarrow -16 \leq x \leq 4, [-16, 4].$$

Example

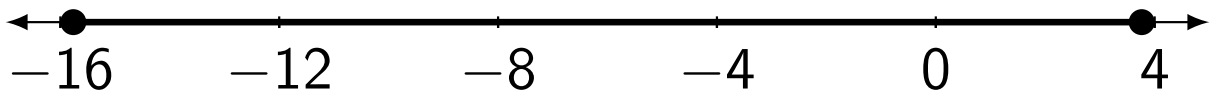
Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

$$\Rightarrow -16 \leq x \leq 4, [-16, 4].$$



Example

Solve

$$|x + 7| > 10.$$

Example

Solve

$$|x + 7| > 10.$$

Solution:

Example

Solve

$$|x + 7| > 10.$$

Solution:

$$\Rightarrow x + 7 < -10 \text{ or } x + 7 > 10$$

Example

Solve

$$|x + 7| > 10.$$

Solution:

$$\Rightarrow x + 7 < -10 \text{ or } x + 7 > 10$$

$$\Rightarrow x < -17 \text{ or } x > 3, (-\infty, -17) \bigcup (3, \infty).$$

Example

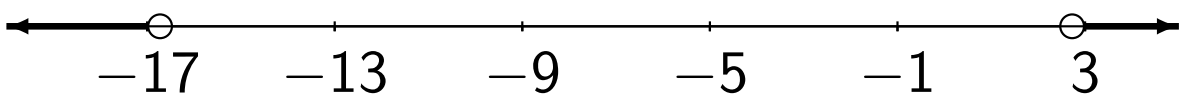
Solve

$$|x+7| > 10.$$

Solution:

$$\Rightarrow x+7 < -10 \text{ or } x+7 > 10$$

$$\Rightarrow x < -17 \text{ or } x > 3, (-\infty, -17) \bigcup (3, \infty).$$



Example

Solve

$$|3x + 2| < 5.$$

Example

Solve

$$|3x + 2| < 5.$$

Solution:

Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

$$\Rightarrow -\frac{7}{3} < x < 1, \left(-\frac{7}{3}, 1\right).$$

Example

Solve

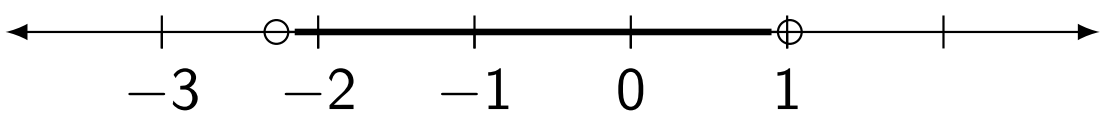
$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

$$\Rightarrow -\frac{7}{3} < x < 1, \left(-\frac{7}{3}, 1\right).$$



Example

Solve

$$|5 - 2x| \geq 1.$$

Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 2, (-\infty, 2] \bigcup [3, \infty).$$

Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 2, (-\infty, 2] \bigcup [3, \infty).$$