Workbook Examples
Chapter 4
Math 1100

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Mizzou
University of Missouri

Outline

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

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Determine the behavior of the graph of a polynomial function using the leading term test.

Factor polynomial functions, and find the zeros and their multiplicities.

Polynomial Function

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where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

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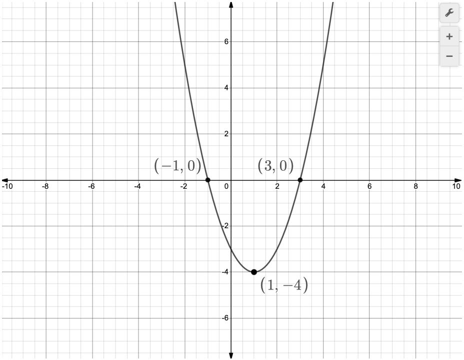
Figure: $f(x) = x^2 - 2x - 3$



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Solution: $(-\infty, \infty)$;

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Example

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$$g(x) = x^3 + 2x^2 - 11x - 12 = (x + 4)(x + 1)(x - 3),$$

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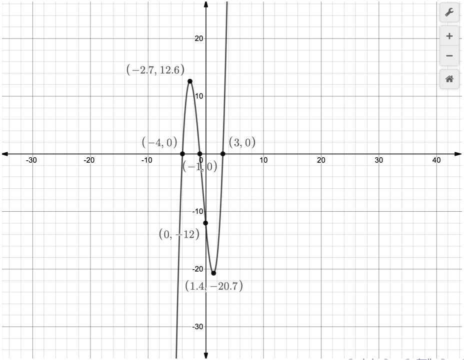

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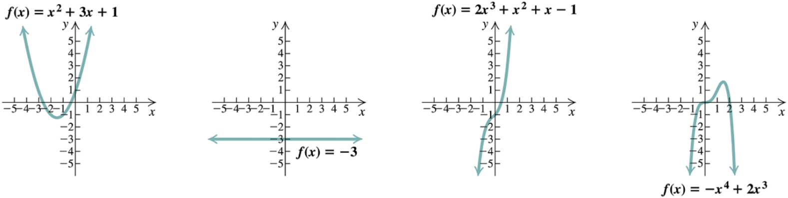
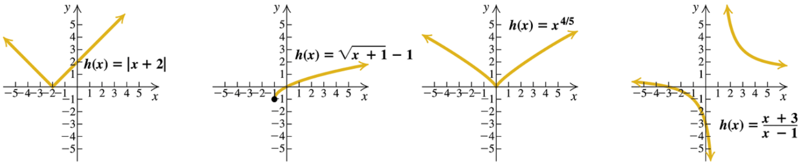
Examples of Polynomial Functions

Figure: Polynomial Functions



Examples of Nonpolynomial Functions

Figure: Nonpolynomial Functions



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Furthermore, the domain of a polynomial function is the set of all real numbers.

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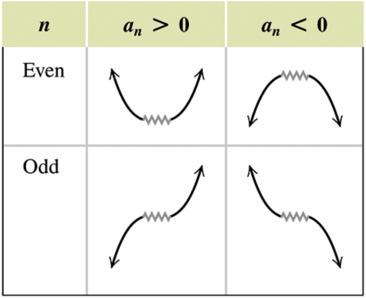

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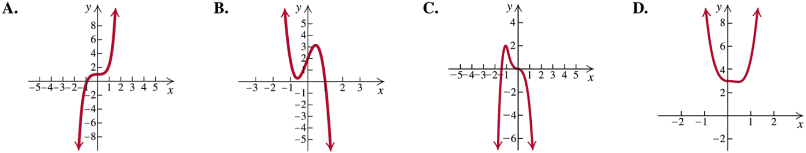

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Solution: **B**

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Solution: x^5 is the leading term.

So $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$.