Workbook Examples
Chapter 4
Math 1100

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Outline

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

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Determine the behavior of the graph of a polynomial function using the leading term test.

Factor polynomial functions, and find the zeros and their multiplicities.

Polynomial Function

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where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

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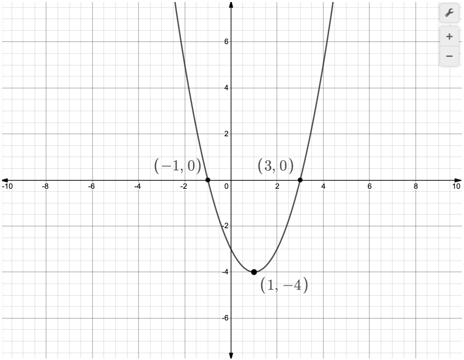
Figure: $f(x) = x^2 - 2x - 3$



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Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

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the minimum:

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Solution: ∞;

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Solution: $(-\infty, \infty)$;

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find the zeros of $f(x)$:

Solution: $x = -1, 3$;

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Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

the range:

Solution: $[-4, \infty)$.

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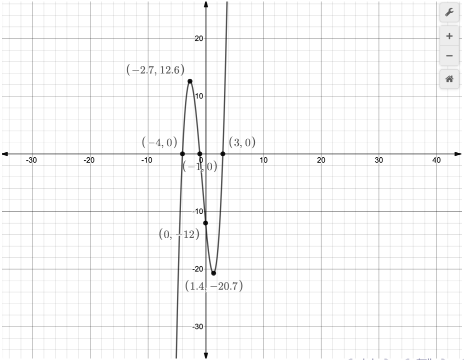
Figure: $g(x) = x^3 + 2x^2 - 11x - 12$



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SOLUTION: $y = -12$;

the minimum:

SOLUTION: $-\infty$;

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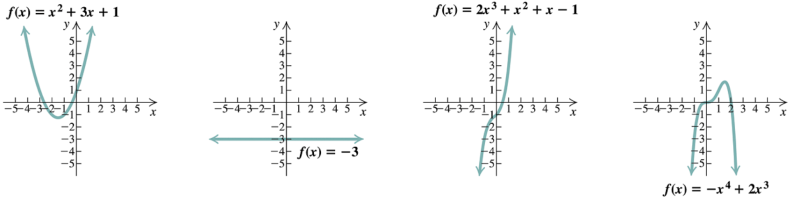
Examples of Polynomial Functions

Figure: Polynomial Functions



Examples of Nonpolynomial Functions

Figure: Nonpolynomial Functions