

# WORKBOOK EXAMPLES

## CHAPTER 3

### MATH 1100

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# OUTLINE

- 1 §3.2: QUADRATIC EQUATIONS, FUNCTIONS, ZEROS, & MODELS
- 2 §3.3 ANALYZING GRAPHS OF QUADRATIC FUNCTIONS
- 3 §3.4: SOLVING RATIONAL EQUATIONS & RADICAL EQUATIONS

## EXAMPLE

Solve

$$2x^2 - x = 3.$$

SOLUTION:

$$\Rightarrow 2x^2 - x - 3 = 0$$

$$\Rightarrow (2x - 3)(x + 1) = 0$$

$$\Rightarrow 2x - 3 = 0, x + 1 = 0$$

$$\Rightarrow x = \frac{3}{2}, x = -1$$

## EXAMPLE

Solve

$$2x^2 - 10 = 0.$$

SOLUTION:

$$\Rightarrow 2x^2 = 10$$

$$\Rightarrow x^2 = 5$$

$$\Rightarrow x = \pm\sqrt{5}$$

# EXAMPLE

Solve

$$4x^2 = 12.$$

SOLUTION:

$$\Rightarrow x^2 = \frac{12}{4} = 3$$

$$\Rightarrow x = \pm\sqrt{3}.$$

## EXAMPLE

Find the zeros of

$$f(x) = x^2 - 6x - 10$$

by completing the square.

SOLUTION:

Set  $f(x) = 0$ , finding

$$x^2 - 6x - 10 = 0$$

$$\Rightarrow x^2 - 6x = 10$$

$$\Rightarrow x^2 - 6x + 9 = 10 + 9 = 19$$

## EXAMPLE (CONT.)

$$\Rightarrow (x - 3)^2 = 19$$

$$\Rightarrow x - 3 = \pm\sqrt{19}$$

$$\Rightarrow x = 3 \pm \sqrt{19}$$

## EXAMPLE

Find the zeros of

$$f(x) = x^2 - 3x - 3$$

by completing the square.

SOLUTION:

$$\Rightarrow x^2 - 3x - 3 = 0$$

$$\Rightarrow x^2 - 3x = 3$$

$$\Rightarrow x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 3 + \frac{9}{4} = \frac{21}{4}$$



## EXAMPLE (CONT.)

$$\Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{21}{4}$$

$$\Rightarrow x - \frac{3}{2} = \pm \sqrt{\frac{21}{4}}$$

$$\Rightarrow x = \frac{3}{2} \pm \frac{\sqrt{21}}{2}$$

## EXAMPLE

Find the zeros of

$$2x^2 - 3x - 1 = 0$$

by completing the square.

SOLUTION:

$$\Rightarrow 2 \left( x^2 - \frac{3}{2}x \right) = 1$$

$$\Rightarrow 2 \left( x^2 - \frac{3}{2}x + \frac{9}{16} \right) = 1 + \frac{18}{16}$$

$$\Rightarrow 2 \left( x - \frac{3}{4} \right)^2 = \frac{17}{8}$$

## EXAMPLE (CONT.)

$$\Rightarrow \left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$

$$\Rightarrow x - \frac{3}{4} = \pm \sqrt{\frac{17}{16}}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{17}}{4}$$

## EXAMPLE

Solve

$$3x^2 + 2x = 7.$$

SOLUTION:

$$\Rightarrow 3x^2 + 2x - 7 = 0$$

$$\Rightarrow a = 3, b = 2, c = -7$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-7)}}{2(3)}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{22}}{3}$$

## EXAMPLE

Solve

$$2x^2 - 3x - 2 = 0.$$

SOLUTION:

$$\Rightarrow a = 2, b = -3, c = -2$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

$$\Rightarrow x = \frac{3 \pm 5}{4}$$

$$\Rightarrow x = -\frac{1}{2}, 2$$

## EXAMPLE

Solve

$$3x^2 + 8x + 3 = 0.$$

SOLUTION:

$$\Rightarrow x = \frac{-8 \pm \sqrt{64 - 4(3)(3)}}{2(3)}$$

$$\Rightarrow x = \frac{-8 \pm 2\sqrt{7}}{6}$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{7}}{3}$$

# EXAMPLE

Solve

$$x^2 + 1 = x.$$

SOLUTION:

$$\Rightarrow x^2 - x + 1 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{3}i}{2}$$

## EXAMPLE

Solve

$$x^4 - 5x^2 + 4 = 0.$$

SOLUTION: Let  $u = x^2$ .

$$\Rightarrow u^2 - 5u + 4 = 0$$

$$\Rightarrow (u - 4)(u - 1) = 0$$

$$\Rightarrow u = 4, u = 1$$

$$\Rightarrow x^2 = 4, x^2 = 1$$

$$\Rightarrow x = \pm 1, \pm 2$$



## EXAMPLE

Solve

$$x^4 - 3x^2 + 2 = 0.$$

SOLUTION: Let  $u = x^2$ .

$$\Rightarrow u^2 - 3u + 2 = 0$$

$$\Rightarrow (u - 2)(u - 1) = 0$$

$$\Rightarrow u = 2, u = 1$$

$$\Rightarrow x^2 = 2, x^2 = 1$$

$$\Rightarrow x = \pm\sqrt{2}, \pm 1$$

## EXAMPLE

Solve

$$4t^3 - 27t = -12t^2.$$

SOLUTION:

$$\Rightarrow 4t^3 + 12t^2 - 27t = 0$$

$$\Rightarrow t(4t^2 + 12t - 27) = 0$$

$$\Rightarrow t(2t - 3)(2t + 9) = 0$$

$$\Rightarrow t = 0, 2t - 3 = 0, 2t + 9 = 0$$

$$\Rightarrow t = 0, \frac{3}{2}, -\frac{9}{2}$$

## EXAMPLE

Solve

$$18x^3 + 15x^2 + 12x + 10 = 0.$$

SOLUTION:

$$\Rightarrow 3x^2(6x + 5) + 2(6x + 5) = 0$$

$$\Rightarrow (6x + 5)(3x^2 + 2) = 0$$

$$\Rightarrow x = -\frac{5}{6}, \pm\sqrt{\frac{2}{3}}i$$

## EXAMPLE

Solve

$$9x^3 + x^2 - 9x - 1 = 0.$$

SOLUTION:

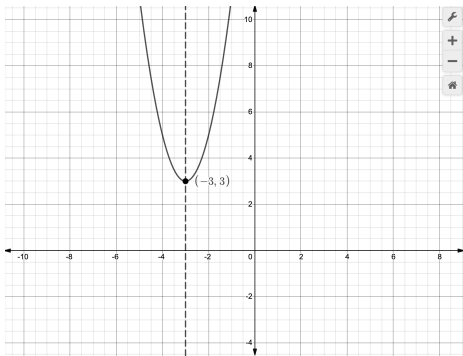
$$\Rightarrow x^2(9x + 1) - (9x + 1) = 0$$

$$\Rightarrow (9x + 1)(x^2 - 1) = 0$$

$$\Rightarrow x = -\frac{1}{9}, \pm 1$$

# EXAMPLE

Given the graph below, a graph of the quadratic function  $f(x) = a(x - h)^2 + k$ , find the vertex, the axis of symmetry, and the minimum or maximum value of the function.



## EXAMPLE (CONT.)

SOLUTION:

The vertex is  $V = (-3, 3)$ .

The axis of symmetry is  $x = -3$ .

The function has a minimum of  $y = 3$ .

## EXAMPLE

Find the vertex, the axis of symmetry, and the maximum or minimum value of

$$f(x) = x^2 + 10x + 28.$$

SOLUTION:

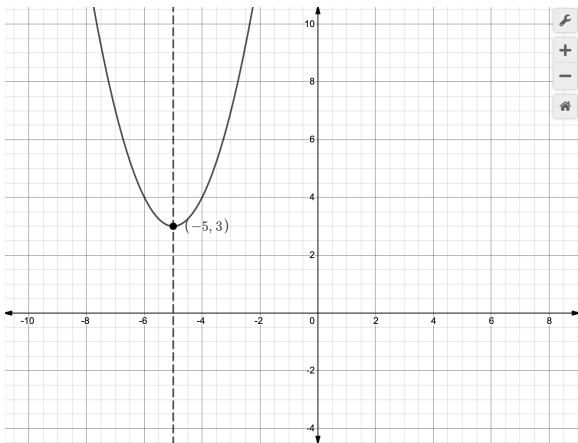
The vertex is  $V = \left(-\frac{10}{2(1)}, f\left(-\frac{10}{2(1)}\right)\right) = (-5, 3)$ .

The axis of symmetry is  $x = -5$ .

The function has a minimum of  $y = -5$ .

## EXAMPLE (CONT.)

The graph of the function is





## EXAMPLE

Find the vertex, the axis of symmetry, and the maximum or minimum value of

$$f(x) = -2x^2 - 2x + 3.$$

SOLUTION:

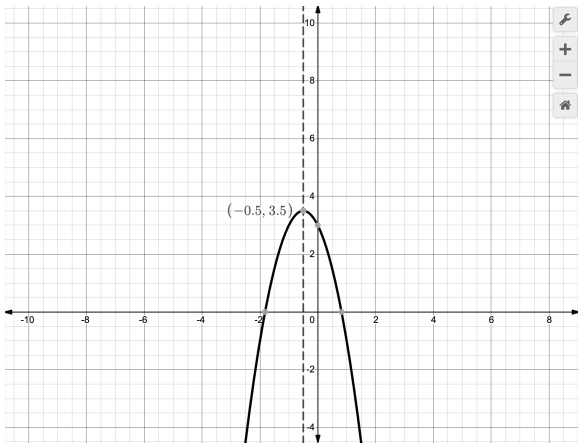
The vertex is  $V = \left(-\frac{-2}{2(-2)}, f\left(-\frac{-2}{2(-2)}\right)\right) = \left(-\frac{1}{2}, \frac{7}{2}\right)$ .

The axis of symmetry is  $x = -\frac{1}{2}$ .

The function has a maximum of  $y = \frac{7}{2}$ .

## EXAMPLE (CONT.)

The graph of the function is



## EXAMPLE

Find the vertex, the axis of symmetry, and the maximum or minimum value of

$$g(x) = \frac{x^2}{2} - 4x + 8.$$

SOLUTION:

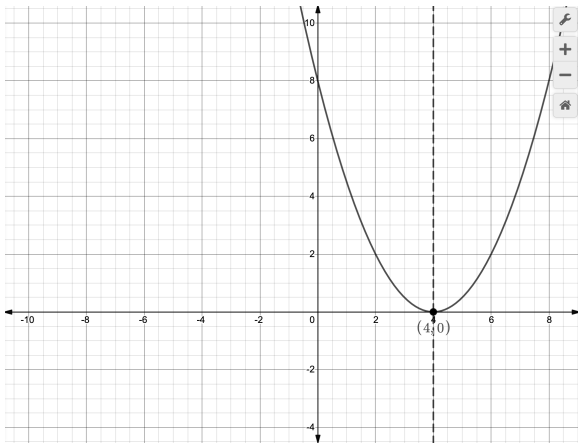
The vertex is  $V = \left(-\frac{-4}{2(1/2)}, f\left(-\frac{-4}{2(1/2)}\right)\right) = (4, 0)$ .

The axis of symmetry is  $x = 4$ .

The function has a minimum of  $y = 0$ .

## EXAMPLE (CONT.)

The graph of the function is



## EXAMPLE

Find the vertex, the axis of symmetry, and the maximum or minimum value of

$$f(x) = x^2 - 9x + 18.$$

SOLUTION:

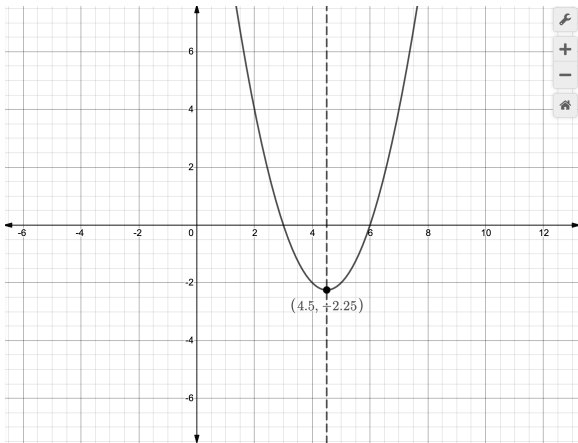
The vertex is  $V = \left(-\frac{-9}{2(1)}, f\left(-\frac{-9}{2(1)}\right)\right) = \left(\frac{9}{2}, -\frac{9}{4}\right)$ .

The axis of symmetry is  $x = \frac{9}{2}$ .

The function has a minimum of  $y = -\frac{9}{4}$ .

## EXAMPLE (CONT.)

The graph of the function is



## EXAMPLE

Find the vertex, the axis of symmetry, and the maximum or minimum value of

$$f(x) = -x^2 - 4x - 5.$$

SOLUTION:

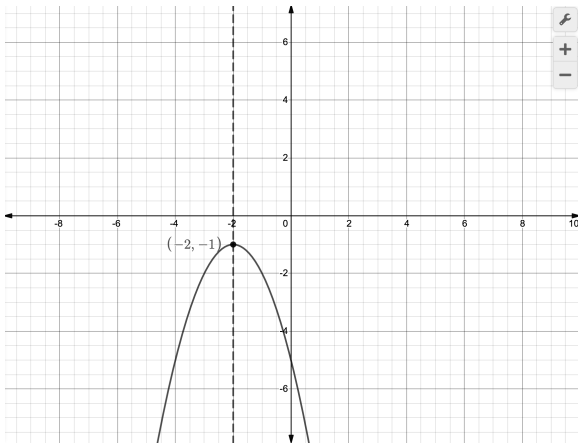
The vertex is  $V = \left(-\frac{-4}{2(-1)}, f\left(-\frac{-4}{2(-1)}\right)\right) = (-2, -1)$ .

The axis of symmetry is  $x = -2$ .

The function has a maximum of  $y = -1$ .

## EXAMPLE (CONT.)

The graph of the function is





## EXAMPLE

For the function

$$f(x) = -x^2 + 14x - 47;$$

(a) find the vertex:

SOLUTION:

$$\Rightarrow V = \left( -\frac{14}{2(-1)}, f\left( -\frac{14}{2(-1)} \right) \right)$$

$$\Rightarrow V = (7, 2)$$

## EXAMPLE (CONT.)

(b) determine whether there is a maximum or minimum value, and find that value:

SOLUTION:

Given that  $a = -1 < 0$ , there is a maximum value for  $f(x)$ .

The value is the  $y$ -value of the vertex, namely

$$y = 2.$$

## EXAMPLE (CONT.)

(c) on what intervals is the function increasing and/or decreasing:

SOLUTION:

$f(x)$  is increasing on  $(-\infty, 7)$ ;

$f(x)$  is decreasing on  $(7, \infty)$ .

## EXAMPLE

Graph the function

$$f(x) = -(x - 3)^2$$

SOLUTION:

We note that the vertex  $V$  is

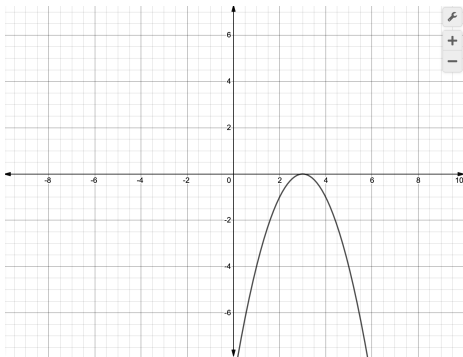
$$V = (3, 0),$$

with  $a = -1 < 0$ , which implies that the graph opens downwards.

## EXAMPLE (CONT.)

The only graph which meets the above criteria is

FIGURE: D.



## EXAMPLE

Graph the function

$$f(x) = 4(x + 9)^2 + 1$$

SOLUTION:

We note that the vertex  $V$  is

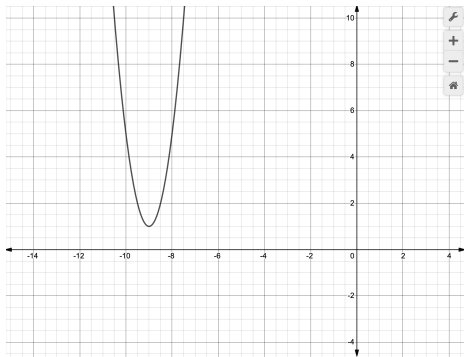
$$V = (-9, 1),$$

with  $a = 4 > 0$ , which implies that the graph opens upwards.

## EXAMPLE (CONT.)

The only graph which meets the above criteria is

FIGURE: D.



## EXAMPLE

Given the function

$$f(x) = x^2 - 10x + 24$$

(a) find the vertex,

SOLUTION:

$$V = \left( -\frac{-10}{2(1)}, f\left(-\frac{-10}{2(1)}\right) \right)$$

$$\Rightarrow V = (5, -1)$$

(b) determine whether there is a maximum or a minimum value,



## EXAMPLE (CONT.)

SOLUTION:

Since  $a = 1 > 0$ , the graph of  $f(x)$  opens upwards, and there is a minimum value, which is the  $y$ -value of the vertex, namely

$$y = -1.$$

(c) find the range,

SOLUTION:

The range comprises the possible  $y$ -values of  $f(x)$ , and so is

$$[-1, \infty).$$

## EXAMPLE (CONT.)

(d) find the intervals on which the function is increasing and the intervals on which the function is decreasing,

SOLUTION:

The function  $f(x)$  is increasing on the interval

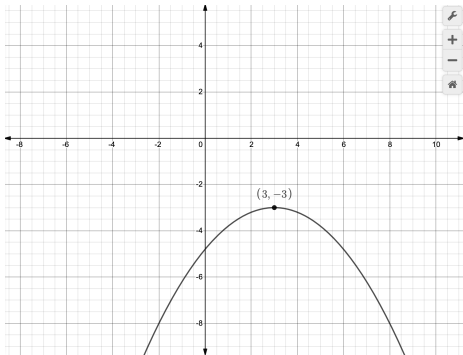
$$(5, \infty),$$

and the function  $f(x)$  is decreasing on the interval

$$(-\infty, 5).$$

# EXAMPLE

Use the graph below to find the vertex, the axis of symmetry, and the maximum or minimum of the function.



## EXAMPLE (CONT.)

SOLUTION:

The vertex  $V$  is

$$V = (3, -3).$$

The axis of symmetry is

$$x = 3.$$

The function has a maximum value of

$$y = -3.$$

## EXAMPLE

Find the vertex of the parabola

$$f(x) = -2x^2 - 20x - 53.$$

SOLUTION:

$$V = \left( -\frac{-20}{2(-2)}, f\left(-\frac{-20}{2(-2)}\right) \right)$$

$$\Rightarrow V = (-5, -3) \quad (B.)$$

## EXAMPLE

Find the vertex of the parabola

$$f(x) = -x^2 + 7x - 5.$$

SOLUTION:

$$V = \left( -\frac{7}{2(-1)}, f\left(-\frac{7}{2(-1)}\right) \right)$$

$$\Rightarrow V = \left( \frac{7}{2}, \frac{29}{4} \right) \quad (C.)$$

## EXAMPLE

Find the axis of symmetry of the given function

$$f(x) = -3x^2 + 6x.$$

SOLUTION:

We need only find

$$-\frac{b}{2a}.$$

$$\Rightarrow x = -\frac{6}{2(-3)} = 1. \quad (B.)$$

## EXAMPLE

Find the axis of symmetry of the given function

$$f(x) = 4x^2 - 16x + 20$$

SOLUTION:

We need only find

$$-\frac{b}{2a}.$$

$$\Rightarrow x = -\frac{-16}{2(4)} = 2. \quad (C.)$$



## EXAMPLE

Determine whether there is a maximum or minimum value for the given function, and find that value:

$$f(x) = x^2 + 4x - 5.$$

SOLUTION:

Since  $a = 1 > 0$ , there is a minimum value of  $f(x)$ .

The minimum value is the  $y$ -value of the vertex. which is

$$f(x) = -9 \quad (D.)$$

## EXAMPLE

Determine whether there is a maximum or minimum value for the given function, and find that value:

$$f(x) = -2x^2 - 4x - 8.$$

SOLUTION:

Since  $a = -2 < 0$ , there is a maximum value of  $f(x)$ .

The maximum value is the  $y$ -value of the vertex. which is

$$f(x) = -6 \quad (D.)$$

## EXAMPLE

Solve

$$\frac{x+6}{4} - \frac{x-3}{5} = 3$$

SOLUTION:

$$\text{LCD} = 20 \Rightarrow 20 \left( \frac{x+6}{4} \right) - 20 \left( \frac{x-3}{5} \right) = 20(3)$$

$$\Rightarrow 5(x+6) - 4(x-3) = 60$$

$$\Rightarrow 5x + 30 - 4x + 12 = 60$$

$$\Rightarrow x + 42 = 60$$

$$\Rightarrow x = 18.$$

# EXAMPLE

Solve

$$\frac{x-8}{3} - \frac{x-3}{2} = 0.$$

SOLUTION:

$$\begin{aligned}\text{LCD} = 3(2) = 6 &\Rightarrow 6 \cdot \frac{x-8}{3} - 6 \cdot \frac{x-3}{2} = 6 \cdot 0. \\ &\Rightarrow 2(x-8) - 3(x-3) = 0 \\ &\Rightarrow 2x - 16 - 3x + 9 = 0 \\ &\Rightarrow -x - 7 = 0 \\ &\Rightarrow x = -7.\end{aligned}$$

## EXAMPLE

Solve

$$\frac{2}{x+4} = \frac{4}{x}$$

SOLUTION:

$$\begin{aligned}\text{LCD} &\Rightarrow x(x+4) \left( \frac{2}{x+4} \right) = x(x+4) \left( \frac{4}{x} \right) \\ &\Rightarrow 2x = 4(x+4) \\ &\Rightarrow 2x = 4x + 16 \\ &\Rightarrow -2x = 16 \\ &\Rightarrow x = -8.\end{aligned}$$

## EXAMPLE

Solve

$$\frac{x^2}{x-3} = \frac{9}{x-3}.$$

SOLUTION:

$$\text{LCD} = x - 3 \Rightarrow (x - 3) \left( \frac{x^2}{x - 3} \right) = (x - 3) \left( \frac{9}{x - 3} \right)$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm 3$$

But  $x = 3$  gives us  $\frac{3^2}{0} = \frac{9}{0}$ , which does not exist.

$$\Rightarrow x = -3.$$

## EXAMPLE

Solve

$$\frac{y^2}{y+7} = \frac{49}{y+7}.$$

SOLUTION:

$$\text{LCD} = 7 \Rightarrow y + 7 \left( \frac{y^2}{y+7} \right) = (y+7) \left( \frac{49}{y+7} \right)$$

$$\Rightarrow y^2 = 49$$

$$\Rightarrow y = \pm 7$$

$$\Rightarrow \cancel{y = -7}, y = 7.$$

## EXAMPLE

Solve

$$\frac{x+9}{3} - \frac{x-12}{4} = 3$$

SOLUTION:

$$\begin{aligned}\text{LCD} = 12 &\Rightarrow 12\left(\frac{x+9}{3}\right) - 12\left(\frac{x-12}{4}\right) = 12(3) \\ &\Rightarrow 4(x+9) - 3(x-12) = 36 \\ &\Rightarrow 4x - 36 - 3x + 36 = 36 \\ &\Rightarrow x = -36.\end{aligned}$$



## EXAMPLE

Solve

$$\frac{4}{x+2} = \frac{6}{x}$$

SOLUTION:

$$\begin{aligned}\text{LCD} = x(x+2) &\Rightarrow x(x+2) \left( \frac{4}{x+2} \right) = x(x+2) \left( \frac{6}{x} \right) \\ &\Rightarrow 4x = 6(x+2) \\ &\Rightarrow 4x = 6x + 12 \\ &\Rightarrow -2x = 12 \\ &\Rightarrow x = -6.\end{aligned}$$

# EXAMPLE

Solve

$$\frac{2}{7} + \frac{1}{4} = \frac{1}{x}.$$

SOLUTION:

$$\text{LCD} = 28x \Rightarrow 28x \left( \frac{2}{7} \right) + 28x \left( \frac{1}{4} \right) = 28x \left( \frac{1}{x} \right)$$

$$\Rightarrow 8x + 7x = 28$$

$$\Rightarrow 15x = 28$$

$$\Rightarrow x = \frac{28}{15}.$$

## EXAMPLE

Solve

$$\frac{x+9}{3} - \frac{x-12}{4} = 3.$$

SOLUTION:

$$\text{LCD} = 3(4) = 12 \Rightarrow 12\left(\frac{x+9}{3}\right) - 12\left(\frac{x-12}{4}\right) = 12(3)$$

$$\Rightarrow 4(x+9) - 3(x-12) = 36$$

$$\Rightarrow 4x + 36 - 3x + 36 = 36$$

$$\Rightarrow x = -36.$$

## EXAMPLE

Solve

$$\frac{7}{p} + p = -8.$$

SOLUTION:

$$\text{LCD} = p \Rightarrow p \left( \frac{7}{p} \right) + p(p) = -8p$$

$$\Rightarrow 7 + p^2 = -8p$$

$$\Rightarrow p^2 + 8p + 7 = 0$$

$$\Rightarrow (p + 7)(p + 1) = 0$$

## EXAMPLE (CONT.

$$\Rightarrow p + 7 = 0, p + 1 = 0$$

$$\Rightarrow p = -7, p = -1.$$

## EXAMPLE

Solve for  $p$  in

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{s}.$$

SOLUTION:

$$\text{LCD} = fps \Rightarrow fps \left( \frac{1}{f} \right) = fps \left( \frac{1}{p} \right) + fps \left( \frac{1}{s} \right)$$

$$\Rightarrow ps = fs + fp$$

$$\Rightarrow ps - fp = fs$$

## EXAMPLE (CONT.)

$$\Rightarrow p(s - f) = fs$$

$$\Rightarrow p = \frac{fs}{s - f}.$$

## EXAMPLE

Solve for  $x$  in

$$\frac{1}{x} = \frac{1}{y} + \frac{1}{m}.$$

SOLUTION:

$$\begin{aligned}\text{LCD} = xym &\Rightarrow xym \left( \frac{1}{x} \right) = xym \left( \frac{1}{y} \right) + xym \left( \frac{1}{m} \right) \\ &\Rightarrow ym = xm + xy \\ &\Rightarrow ym = x(m + y) \\ &\Rightarrow x = \frac{my}{m + y}.\end{aligned}$$



## EXAMPLE

Solve for  $x$ :

$$\frac{2}{x+1} + \frac{1}{x-1} = \frac{20}{x^2-1}.$$

SOLUTION:

$$x^2 - 1 = (x-1)(x+1) \Rightarrow \text{LCD} = (x-1)(x+1)$$

$$\Rightarrow (x+1)(x-1) \left( \frac{2}{x+1} \right) + (x+1)(x-1) \left( \frac{1}{x-1} \right) = (x^2-1) \frac{20}{x^2-1}$$

## EXAMPLE (CONT.)

$$\Rightarrow 2(x - 1) + x + 1 = 20$$

$$\Rightarrow 2x - 2 + x + 1 = 20$$

$$\Rightarrow 3x - 1 = 20$$

$$\Rightarrow 3x = 21$$

$$\Rightarrow x = 7.$$

## EXAMPLE

Solve for  $x$ :

$$\frac{15}{y-15} - \frac{3}{y} = \frac{17y+5}{y^2-25}.$$

SOLUTION:

$$\begin{aligned}\text{LCD} &= y(y^2 - 25)(y - 15) \\ \Rightarrow y(y^2 - 25)(y - 15) \left( \frac{15}{y-15} \right) - y(y^2 - 25)(y - 15) \left( \frac{3}{y} \right) \\ &= y(y^2 - 25)(y - 15) \left( \frac{17y+5}{y^2-25} \right) \\ \Rightarrow 15y(y^2 - 25) - 3(y - 15)(y^2 - 25) &= y(y - 15)(17y - 5)\end{aligned}$$

## EXAMPLE (CONT.)

$$\begin{aligned}\Rightarrow & 15y^3 - 375y - 3(y^3 - 15y^2 - 25y + 375) \\ \Rightarrow & 17y^3 - 250y^2 - 75y \\ \Rightarrow & 5y^3 - 295y^2 + 225y + 1125 = 0 \\ \Rightarrow & y \approx -1.5911, 2.4314, 58.160.\end{aligned}$$

## EXAMPLE

Solve

$$\sqrt{3x + 1} = 4.$$

SOLUTION:

$$\Rightarrow 3x + 1 = 16$$

$$\Rightarrow 3x = 15$$

$$\Rightarrow x = 5.$$

## EXAMPLE

Solve

$$5 + \sqrt{x + 7} = x.$$

SOLUTION:

$$\Rightarrow \sqrt{x + 7} = x - 5$$

$$\Rightarrow x + 7 = x^2 - 10x + 25$$

$$\Rightarrow x^2 - 11x + 18 = 0$$

$$\Rightarrow (x - 9)(x - 2) = 0$$

$$\Rightarrow x - 9 = 0, x - 2 = 0$$

## EXAMPLE (CONT.)

$$\Rightarrow x = 9, x = 2$$

But with  $x = 2$  :

$$5 + \sqrt{9} = 5 + 3 = 8 \neq 2.$$

$$\Rightarrow x = 9.$$

## EXAMPLE

Solve

$$\sqrt{x+3} + 3 = 0.$$

SOLUTION:

$$\Rightarrow \sqrt{x+3} = -3$$

$$\Rightarrow x + 3 = 9$$

$$\Rightarrow x = 6.$$

But

$$\sqrt{6+3} + 3 = 6 \neq 0.$$

$$\Rightarrow \emptyset \text{ (no solution).}$$



## EXAMPLE

Solve

$$\sqrt{x + 81} + 9 = x.$$

SOLUTION:

$$\Rightarrow \sqrt{x + 81} = x - 9$$

$$\Rightarrow x + 81 = x^2 - 18x + 81$$

$$\Rightarrow x^2 - 19x = 0$$

$$\Rightarrow x(x - 19) = 0$$

## EXAMPLE (CONT.)

$$\Rightarrow x = 0, x = 19.$$

But with  $x = 0$ :

$$\sqrt{81} + 9 = 18 \neq 0.$$

$$\Rightarrow x = 19.$$

## EXAMPLE

Solve

$$\sqrt{1 - 2x} = 3.$$

SOLUTION:

$$\Rightarrow 1 - 2x = 9$$

$$\Rightarrow 8 = 2x$$

$$\Rightarrow x = 4$$

$$\text{But } \sqrt{1 - 2(4)} = i\sqrt{7} \neq 3$$

$$\Rightarrow \emptyset.$$

## EXAMPLE

Solve

$$\sqrt{5 - x} = 1.$$

SOLUTION:

$$\Rightarrow 5 - x = 1$$

$$\Rightarrow x = 4.$$

## EXAMPLE

Solve

$$\sqrt{3a+3} = a+1.$$

SOLUTION:

$$\Rightarrow 3a+3 = (a+1)^2$$

$$\Rightarrow 3a+3 = a^2 + 2a + 1$$

$$\Rightarrow a^2 - a - 2 = 0$$

$$\Rightarrow (a-2)(a+1) = 0$$

## EXAMPLE (CONT.)

$$\Rightarrow a - 2 = 0, a + 1 = 0$$

$$\Rightarrow a = 2, a = -1.$$

# EXAMPLE

Solve

$$\sqrt{b+3} - 2 = 1.$$

SOLUTION:

$$\Rightarrow \sqrt{b+3} = 3$$

$$\Rightarrow b+3 = 9$$

$$\Rightarrow b = 6.$$

## EXAMPLE

Solve

$$\frac{6}{y+3} - \frac{4}{y-3} = \frac{6}{y^2-9}.$$

SOLUTION:

$$\begin{aligned}\text{LCD} &= y^2 - 9 = (y - 3)(y + 3) \\ \Rightarrow 6(y - 3) - 4(y + 3) &= 6 \\ \Rightarrow 6y - 18 - 4y - 12 &= 6 \\ \Rightarrow 2y - 30 &= 6 \\ \Rightarrow 2y &= 36 \\ \Rightarrow y &= 18 \quad (A).\end{aligned}$$



# EXAMPLE

Solve for  $t$ :

$$\frac{1}{A} = \frac{1}{m} + \frac{1}{t}.$$

SOLUTION:

$$\text{LCD} = Amt$$

$$\Rightarrow mt = At + Am$$

$$\Rightarrow mt - At = Am$$

$$\Rightarrow t(m - A) = Am$$

$$\Rightarrow t = \frac{Am}{m - A} \quad (C).$$