Workbook Examples
Chapter 4
Math 1100

Don D. Kim

6 April 2020

Outline

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

Factor polynomial functions, and find the zeros and their multiplicities.

Polynomial Function

A *polynomial function* P is given by

Polynomial Function

A *polynomial function* P is given by

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

Polynomial Function

A *polynomial function* P is given by

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Solution:

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Solution:

The leading term, i.e. the term with the highest degree, is

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$\frac{1}{6}x^3.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$\frac{1}{6}x^3.$$

The degree of $g(x)$ is 3.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -\frac{1}{6}x^3 - 4x + 8.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$\frac{1}{6}x^3.$$

The degree of $g(x)$ is 3.

The polynomial is a cubic polynomial.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Solution:

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Solution:

The leading term, i.e. the term with the highest degree, is

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$2.4x^4.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$2.4x^4.$$

The degree of $g(x)$ is 4.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = 2.4x^4 + 5x^2 - x + \frac{5}{8}.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$2.4x^4.$$

The degree of $g(x)$ is 4.

The polynomial is a quartic polynomial.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Solution:

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$x^3.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$x^3.$$

The degree of $g(x)$ is 3.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -3x + 7x^2 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$x^3.$$

The degree of $g(x)$ is 3.

The polynomial is a cubic polynomial.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Solution:

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$9x^4.$$

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$9x^4.$$

The degree of $g(x)$ is 4.

Example

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as a constant, linear, quadratic, cubic or quartic.

$$g(x) = -4x^2 + 9x^4 + x^3.$$

Solution:

The leading term, i.e. the term with the highest degree, is

$$9x^4.$$

The degree of $g(x)$ is 4.

The polynomial is a quartic polynomial.

Example

Given

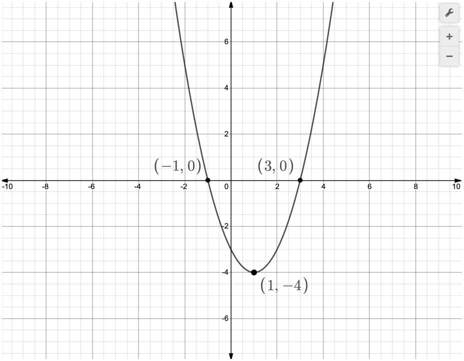
$$f(x) = x^2 - 2x - 3 = (x+1)(x-3),$$

Example

Given

$$f(x) = x^2 - 2x - 3 = (x+1)(x-3),$$

Figure: $f(x) = x^2 - 2x - 3$



Example (cont.)

find the zeros of $f(x)$:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

the domain:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

the range:

Example (cont.)

find the zeros of $f(x)$:

Solution: $x = -1, 3$;

the $x-$intercepts:

Solution: $x = -1, 3$;

the $y-$intercept:

Solution: $y = -3$;

the minimum:

Solution: $y = -4$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

the range:

Solution: $[-4, \infty)$.

Example

Given

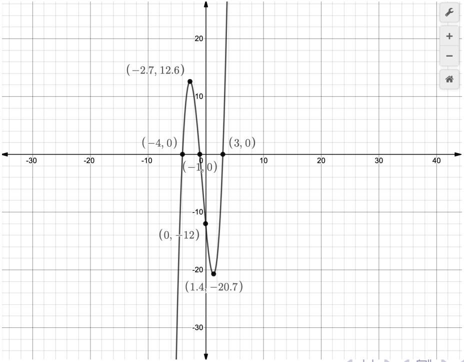
$$g(x) = x^3 + 2x^2 - 11x - 12 = (x+4)(x+1)(x-3),$$

Example

Given

$$g(x) = x^3 + 2x^2 - 11x - 12 = (x+4)(x+1)(x-3),$$

Figure: $g(x) = x^3 + 2x^2 - 11x - 12$



Example (cont.)

find the zeros of $g(x)$:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

EXAMPLE (CONT.)

find the zeros of $g(x)$:

SOLUTION: $x = -4, -1, 3$;

the $x-$intercepts:

SOLUTION: $x = -4, -1, 3$;

the $y-$intercept:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Solution: ∞;

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Solution: ∞;

the domain:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Solution: ∞;

the domain:

Solution: $(-\infty, \infty)$;

the range:

Example (cont.)

find the zeros of $g(x)$:

Solution: $x = -4, -1, 3$;

the $x-$intercepts:

Solution: $x = -4, -1, 3$;

the $y-$intercept:

Solution: $y = -12$;

the minimum:

Solution: $-\infty$;

the maximum:

Solution: ∞;

the domain:

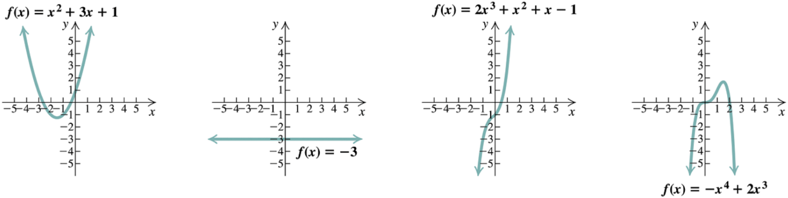
Solution: $(-\infty, \infty)$;

the range:

Solution: $(-\infty, \infty)$.

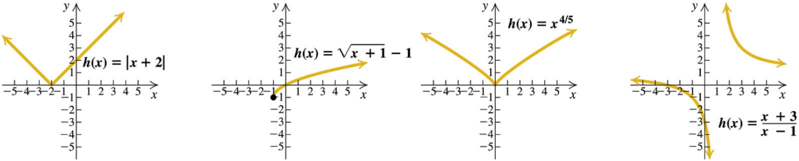
Examples of Polynomial Functions

Figure: Polynomial Functions



Examples of Nonpolynomial Functions

Figure: Nonpolynomial Functions



Polynomial Functions

The graph of a polynomial function is *continuous*; that is, it has no holes or breaks.

Polynomial Functions

The graph of a polynomial function is *continuous*; that is, it has no holes or breaks.

It is also smooth; there are no "sharp" corners.

Polynomial Functions

The graph of a polynomial function is *continuous*; that is, it has no holes or breaks.

It is also smooth; there are no "sharp" corners.

Furthermore, the domain of a polynomial function is the set of all real numbers.

The Leading–Term Test

If $a_n x^n$ is the leading term of a polynomial function,

The Leading–Term Test

If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ or as $x \to -\infty$

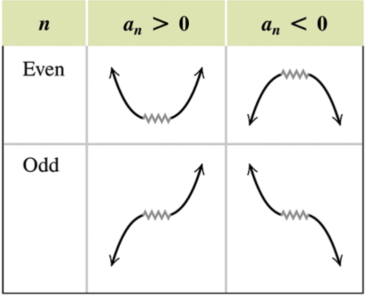
The Leading–Term Test

If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ or as $x \to -\infty$ can be described in one of the four following ways.

The Leading–Term Test

If $a_n x^n$ is the leading term of a polynomial function, then the behavior of the graph as $x \to \infty$ or as $x \to -\infty$ can be described in one of the four following ways.

Figure:



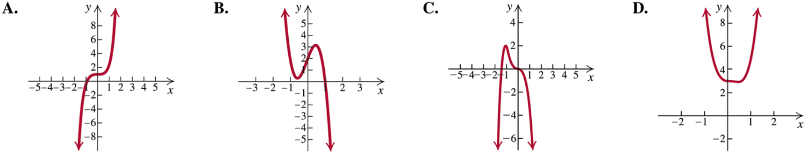
Example

Using the leading term test, match each of the following functions with one of the graphs **A–D** that follow.

Example

Using the leading term test, match each of the following functions with one of the graphs **A–D** that follow.

Figure:



Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3$.

Solution: **D**

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Solution: **B**

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Solution: **B**

c. $f(x) = x^5 + \frac{1}{4}x + 1$

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Solution: **B**

c. $f(x) = x^5 + \frac{1}{4}x + 1$

Solution: **A**

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Solution: **B**

c. $f(x) = x^5 + \frac{1}{4}x + 1$

Solution: **A**

d. $f(x) = -x^6 + x^5 - 4x^3$

Example (cont.)

a. $f(x) = 3x^4 - 2x^3 + 3.$

Solution: **D**

b. $f(x) = -5x^3 - x^2 + 4x + 2$

Solution: **B**

c. $f(x) = x^5 + \frac{1}{4}x + 1$

Solution: **A**

d. $f(x) = -x^6 + x^5 - 4x^3$

Solution: **C**

Example

Describe the end behavior of the graph of the following functions.

Example

Describe the end behavior of the graph of the following functions.

(1) $f(x) = -x^6 + \frac{3}{4}x^4$

Example

Describe the end behavior of the graph of the following functions.

(1) $f(x) = -x^6 + \frac{3}{4}x^4$

Solution: $-x^6$ is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$ and as $x \to \infty$.

Example

Describe the end behavior of the graph of the following functions.

(1) $f(x) = -x^6 + \frac{3}{4}x^4$

Solution: $-x^6$ is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$ and as $x \to \infty$.

(2) $f(x) = 3 - \frac{1}{10}x + x^5$

Example

Describe the end behavior of the graph of the following functions.

(1) $f(x) = -x^6 + \frac{3}{4}x^4$

Solution: $-x^6$ is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$ and as $x \to \infty$.

(2) $f(x) = 3 - \frac{1}{10}x + x^5$

Solution: x^5 is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$.

Example (cont.)

(3) $f(x) = 2x^4 - x^2 + 1$

Example (cont.)

(3) $f(x) = 2x^4 - x^2 + 1$

Solution: $2x^4$ is the leading term. So $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$.

Example (cont.)

(3) $f(x) = 2x^4 - x^2 + 1$

Solution: $2x^4$ is the leading term. So $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$.

(4) $f(x) = x^2 - x^3 - 2x + 4$.

Example (cont.)

(3) $f(x) = 2x^4 - x^2 + 1$

Solution: $2x^4$ is the leading term. So $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$.

(4) $f(x) = x^2 - x^3 - 2x + 4$.

Solution: $-x^3$ is the leading term. So $f(x) \to \infty$ as $x \to -\infty$, and $f(x) \to -\infty$ as $x \to \infty$.

Example

Choose the end behavior diagram that best describes the function $f(x) = -3.9x^4 + x^6 + 0.1x^7$.

Example

Choose the end behavior diagram that best describes the function $f(x) = -3.9x^4 + x^6 + 0.1x^7$.

Figure:

Example

Choose the end behavior diagram that best describes the function $f(x) = -3.9x^4 + x^6 + 0.1x^7$.

Figure:

Solution: $f(x) \approx x^7 \Rightarrow$C.

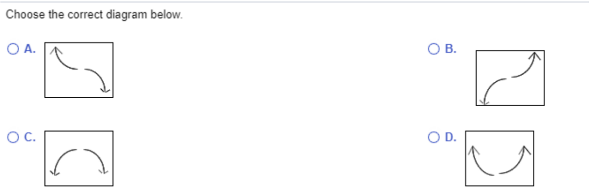
Example

Choose the end behavior diagram that best describes the function $f(x) = 5 + \frac{1}{5}x^4 - \frac{1}{2}x^3$.

Example

Choose the end behavior diagram that best describes the function $f(x) = 5 + \frac{1}{5}x^4 - \frac{1}{2}x^3$.

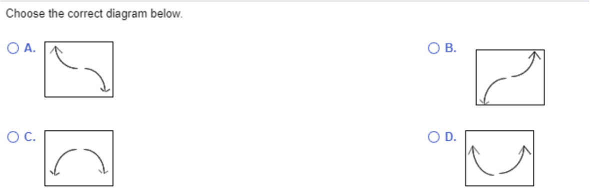
Figure:



Example

Choose the end behavior diagram that best describes the function $f(x) = 5 + \frac{1}{5}x^4 - \frac{1}{2}x^3$.

Figure:



Solution: $f(x) \approx x^4 \Rightarrow$ D.

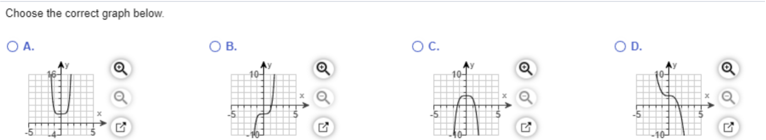
Example

Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

Example

Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

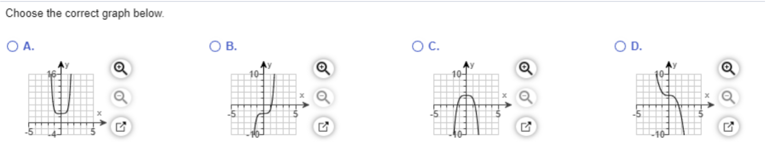
Figure:



Example

Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

Figure:



Solution: $f(x) \approx x^5 \Rightarrow$B.

Finding Zeros of Polynomial Functions

If c is a real zero of a function (that is, $f(c) = 0$),

Finding Zeros of Polynomial Functions

If c is a real zero of a function (that is, $f(c) = 0$),
then $(c, 0)$ is an $x-$intercept of the graph of the function.

Even and Odd Multiplicity

If $(x-c)^k, k \geq 1$, is a factor of a polynomial function $P(x)$ and $(x-c)^{k+1}$ is not a factor and:

Even and Odd Multiplicity

If $(x-c)^k, k \geq 1$, is a factor of a polynomial function $P(x)$ and $(x-c)^{k+1}$ is not a factor and:

(i) k is odd, then the graph crosses the $x-$axis at $(c, 0)$;

Even and Odd Multiplicity

If $(x - c)^k, k \geq 1$, is a factor of a polynomial function $P(x)$ and $(x - c)^{k+1}$ is not a factor and:

(i) k is odd, then the graph crosses the $x-$axis at $(c, 0)$;

(ii) k is even, then the graph is tangent to the $x-$axis at $(c, 0)$.

Example

Use substitution to determine whether 3 is a zero of

Example

Use substitution to determine whether 3 is a zero of

$$(a)\ f(x) = x^3 - 10x^2 + 17x + 12.$$

Example

Use substitution to determine whether 3 is a zero of

$$(a)\ f(x) = x^3 - 10x^2 + 17x + 12.$$

Solution:

Example

Use substitution to determine whether 3 is a zero of

$$(a)\ f(x) = x^3 - 10x^2 + 17x + 12.$$

Solution:

$$f(3) = 3^3 - 10(3^2) + 17(3) + 12$$

Example

Use substitution to determine whether 3 is a zero of

$$(a)\ f(x) = x^3 - 10x^2 + 17x + 12.$$

Solution:

$$f(3) = 3^3 - 10(3^2) + 17(3) + 12$$

$$\Rightarrow f(3) = 27 - 90 + 51 + 12 = 0$$

Example

Use substitution to determine whether 3 is a zero of

$$(a)\ f(x) = x^3 - 10x^2 + 17x + 12.$$

Solution:

$$f(3) = 3^3 - 10(3^2) + 17(3) + 12$$

$$\Rightarrow f(3) = 27 - 90 + 51 + 12 = 0$$

Thus 3 is a zero of $f(x)$.

Example

Use substitution to determine whether 3 is a zero of

Example

Use substitution to determine whether 3 is a zero of

$$(b)\ f(x) = x^4 - 5x^3 - 2x - 15.$$

Example

Use substitution to determine whether 3 is a zero of

$$(b)\ f(x) = x^4 - 5x^3 - 2x - 15.$$

Solution:

Example

Use substitution to determine whether 3 is a zero of

$$(b)\ f(x) = x^4 - 5x^3 - 2x - 15.$$

Solution:

$$f(3) = 3^4 - 5(3^3) - 2(3) - 15$$

Example

Use substitution to determine whether 3 is a zero of

$$(b)\ f(x) = x^4 - 5x^3 - 2x - 15.$$

Solution:

$$f(3) = 3^4 - 5(3^3) - 2(3) - 15$$

$$\Rightarrow f(3) = -75$$

Example

Use substitution to determine whether 3 is a zero of

$$(b)\ f(x) = x^4 - 5x^3 - 2x - 15.$$

Solution:

$$f(3) = 3^4 - 5(3^3) - 2(3) - 15$$

$$\Rightarrow f(3) = -75$$

Thus 3 is *not* a zero of $f(x)$.

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Rewrite $f(x) = 5(x-2)^3(x+1)$.

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Rewrite $f(x) = 5(x-2)^3(x+1)$.

$x = 2$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Rewrite $f(x) = 5(x-2)^3(x+1)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 3.

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Rewrite $f(x) = 5(x-2)^3(x+1)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 3.

$x = -1$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = 5(x-2)(x-2)(x-2)(x+1)$ and determine the multiplicity of each.

Solution:

Rewrite $f(x) = 5(x-2)^3(x+1)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 3.

$x = -1$ is a zero of $f(x)$

with a multiplicity of 1

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Solution:

$x = 1$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Solution:

$x = 1$ is a zero of $f(x)$

with a multiplicity of 2.

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Solution:

$x = 1$ is a zero of $f(x)$

with a multiplicity of 2.

$x = -2$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = -(x-1)^2(x+2)^2$ and determine the multiplicity of each.

Solution:

$x = 1$ is a zero of $f(x)$

with a multiplicity of 2.

$x = -2$ is a zero of $f(x)$

with a multiplicity of 2

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 1.

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 3$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 3$ is a zero of $f(x)$

with a multiplicity of 1

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 3$ is a zero of $f(x)$

with a multiplicity of 1

$x = -3$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = x^3 - 2x^2 - 9x + 18$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-2) - 9(x-2) = (x-2)(x-3)(x+3)$.

$x = 2$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 3$ is a zero of $f(x)$

with a multiplicity of 1

$x = -3$ is a zero of $f(x)$

with a multiplicity of 1.

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

with a multiplicity of 1.

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

with a multiplicity of 1.

$x = \sqrt{7}$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

with a multiplicity of 1.

$x = \sqrt{7}$ is a zero of $f(x)$

with a multiplicity of 1

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x - 1) - 7(2x - 1) = (2x - 1)(x^2 - 7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

with a multiplicity of 1.

$x = \sqrt{7}$ is a zero of $f(x)$

with a multiplicity of 1

$x = -\sqrt{7}$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = 2x^3 - x^2 - 14x + 7$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(2x-1) - 7(2x-1) = (2x-1)(x^2-7)$.

$x = \frac{1}{2}$ is a zero of $f(x)$

with a multiplicity of 1.

$x = \sqrt{7}$ is a zero of $f(x)$

with a multiplicity of 1

$x = -\sqrt{7}$ is a zero of $f(x)$

with a multiplicity of 1.

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-1)$.

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x - 1)$.

$x = 1$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-1)$.

$x = 1$ is a zero of $f(x)$

with a multiplicity of 1.

EXAMPLE

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

SOLUTION:

Factor $f(x) = x^2(x-1)$.

$x = 1$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 0$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = x^3 - x^2$ and determine the multiplicity of each.

Solution:

Factor $f(x) = x^2(x-1)$.

$x = 1$ is a zero of $f(x)$

with a multiplicity of 1.

$x = 0$ is a zero of $f(x)$

with a multiplicity of 2

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Solution:

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Solution:

$x = -1$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Solution:

$x = -1$ is a zero of $f(x)$

with a multiplicity of 4.

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Solution:

$x = -1$ is a zero of $f(x)$

with a multiplicity of 4.

$x = 3$ is a zero of $f(x)$

Example

Find the zeros of $f(x) = (x+1)^4(x-3)$ and determine the multiplicity of each.

Solution:

$x = -1$ is a zero of $f(x)$

with a multiplicity of 4.

$x = 3$ is a zero of $f(x)$

with a multiplicity of 1

Example

Determine whether the following statement is true or false.

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Solution:

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Solution:

$P(x)$ has a zero at $x = -5$.

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Solution:

$P(x)$ has a zero at $x = -5$.

This zero has a multiplicty of 2, an even number.

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Solution:

$P(x)$ has a zero at $x = -5$.

This zero has a multiplicty of 2, an even number.

Thus $P(x)$ is tangent to the $x-$axis at $(-5, 0)$

Example

Determine whether the following statement is true or false.

"If $P(x) = (x-7)^7(x+5)^2$, then the graph of the polynomial function $y = P(x)$ is tangent to the $x-$axis at $(-5, 0)$. "

Solution:

$P(x)$ has a zero at $x = -5$.

This zero has a multiplicty of 2, an even number.

Thus $P(x)$ is tangent to the $x-$axis at $(-5, 0)$

The statement is true (Statement B).

Example

Find the correct end behavior diagram for the given polynomial function.

Example

Find the correct end behavior diagram for the given polynomial function.

$$f(x) = -2.76x^4 - x^3 + x^2 - 2x + 1.$$

Example

Find the correct end behavior diagram for the given polynomial function.

$$f(x) = -2.76x^4 - x^3 + x^2 - 2x + 1.$$

Solution:

Example

Find the correct end behavior diagram for the given polynomial function.

$$f(x) = -2.76x^4 - x^3 + x^2 - 2x + 1.$$

Solution:

$f(x) \sim -x^4$.

Example

Find the correct end behavior diagram for the given polynomial function.

$$f(x) = -2.76x^4 - x^3 + x^2 - 2x + 1.$$

Solution:

$f(x) \sim -x^4$.

Choose D.