# Workbook Examples
# Chapter 3
# Math 1100

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# Outline

# Equations with Absolute Value

For $a > 0$ and an algebraic expression $x$:

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For $a > 0$ and an algebraic expression $x$:

$$|x| = a$$

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For $a > 0$ and an algebraic expression $x$:

$$|x| = a$$

is equivalent to

# Equations with Absolute Value

For $a > 0$ and an algebraic expression $x$:

$$|x| = a$$

is equivalent to

$$x = a \text{ or } x = -a.$$

# Example

Solve

$$|x| = 5.$$

# Example

Solve

$$|x| = 5.$$

Solution:

# Example

Solve

$$|x| = 5.$$

Solution:

$$\Rightarrow x = 5, x = -5.$$

# Example

Solve

$$|5x| = 4.$$

# Example

Solve

$$|5x| = 4.$$

Solution:

# Example

Solve

$$|5x| = 4.$$

Solution:

$$\Rightarrow 5x = 4, 5x = -4$$

# Example

Solve

$$|5x| = 4.$$

Solution:

$$\Rightarrow 5x = 4, 5x = -4$$

$$\Rightarrow x = \frac{4}{5}, x = -\frac{4}{5}.$$

# Example

Solve

$$|x - 3| = 5.$$

# Example

Solve

$$|x - 3| = 5.$$

Solution:

# Example

Solve

$$|x - 3| = 5.$$

Solution:

$$\Rightarrow x - 3 = 5, x - 3 = -5$$

# Example

Solve

$$|x - 3| = 5.$$

Solution:

$$\Rightarrow x - 3 = 5, x - 3 = -5$$

$$\Rightarrow x = 8, x = -2.$$

# Example

Solve

$$|x + 2| - 5 = 9.$$

# Example

Solve

$$|x + 2| - 5 = 9.$$

Solution:

# Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

# Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

$$\Rightarrow x+2=14, x+2=-14$$

# Example

Solve

$$|x+2|-5=9.$$

Solution:

$$\Rightarrow |x+2|=14$$

$$\Rightarrow x+2=14, x+2=-14$$

$$\Rightarrow x=12, x=-16.$$

# Example

Solve

$$|x - 4| + 3 = 9.$$

# Example

Solve

$$|x - 4| + 3 = 9.$$

Solution:

# Example

Solve

$$|x - 4| + 3 = 9.$$

Solution:

$$\Rightarrow |x - 4| = 6$$

# Example

Solve

$$|x-4|+3=9.$$

Solution:

$$\Rightarrow |x-4|=6$$

$$\Rightarrow x-4=6, x-4=-6$$

# Example

Solve

$$|x-4|+3=9.$$

Solution:

$$\Rightarrow |x-4|=6$$

$$\Rightarrow x-4=6, x-4=-6$$

$$\Rightarrow x=10, x=-2.$$

# Example

Solve

$$9 - |x - 2| = 7.$$

# Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

# Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

# Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

# Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2, x - 2 = -2$$

# Example

Solve

$$9 - |x - 2| = 7.$$

Solution:

$$\Rightarrow -|x - 2| = -2$$

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2, x - 2 = -2$$

$$\Rightarrow x = 4, x = 0.$$

# Example

Solve

$$5 - |4x + 3| = 2.$$

# Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

# Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

# Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

# Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

$$\Rightarrow 4x + 3 = 3, 4x + 3 = -3$$

# Example

Solve

$$5 - |4x + 3| = 2.$$

Solution:

$$\Rightarrow -|4x + 3| = -3$$

$$\Rightarrow |4x + 3| = 3$$

$$\Rightarrow 4x + 3 = 3, 4x + 3 = -3$$

$$\Rightarrow x = 0, x = -\frac{3}{2}.$$

# More About Absolute Value Equations

When $a = 0$, $|x| = a$ is equivalent to $x = 0$.

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When $a = 0$, $|x| = a$ is equivalent to $x = 0$.

Note that for $a < 0$, $|x| = a$ has *no* solution,

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Note that for $a < 0, |x| = a$ has *no* solution,

because the absolute value of an expression is never negative.

# More About Absolute Value Equations

When $a = 0, |x| = a$ is equivalent to $x = 0$.

Note that for $a < 0, |x| = a$ has *no* solution,

because the absolute value of an expression is never negative.

The solution set is the *empty set*, denoted $\emptyset$.

# Example

Solve

$$|x - 4| + 3 = 0.$$

# Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

# Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

# Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

but this equation has no solution,

# Example

Solve

$$|x - 4| + 3 = 0.$$

Solution:

$$\Rightarrow |x - 4| = -3,$$

but this equation has no solution,

i.e. the solution set is $\emptyset$.

# Inequalities with Absolute Value

Inequalities sometimes contain absolute–value notation.

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The following properties are used to solve them.

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For $a > 0$ and an algebraic expression $x$:

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Inequalities sometimes contain absolute–value notation.

The following properties are used to solve them.

For $a > 0$ and an algebraic expression $x$:

$$|x| < a \text{ is equivalent to } -a < x < a,$$
$$|x| > a \text{ is equivalent to } x < -a \text{ or } x > a.$$

# Inequalities with Absolute Value

Inequalities sometimes contain absolute–value notation.

The following properties are used to solve them.

For $a > 0$ and an algebraic expression $x$:

$$|x| < a \text{ is equivalent to } -a < x < a,$$
$$|x| > a \text{ is equivalent to } x < -a \text{ or } x > a.$$

Similar statements hold for $|x| \leq a$ and $|x| \geq a$.

# Inequalities with Absolute Value (cont.)

For example,

# Inequalities with Absolute Value (cont.)

For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

# Inequalities with Absolute Value (cont.)

For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

$|y| \geq 1$ is equivalent to $y \leq -1$ or $y \geq 1$.

# Inequalities with Absolute Value (cont.)

For example,

$|x| < 3$ is equivalent to $-3 < x < 3$.

$|y| \geq 1$ is equivalent to $y \leq -1$ or $y \geq 1$.

$|2x + 3| \leq 4$ is equivalent o $-4 \leq 2x + 3 \leq 4$.

# Example

Solve

$$|x| < 5.$$

# Example

Solve

$$|x| < 5.$$

Solution:

# Example

Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

# Example

Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

$$(-5, 5).$$

# Example

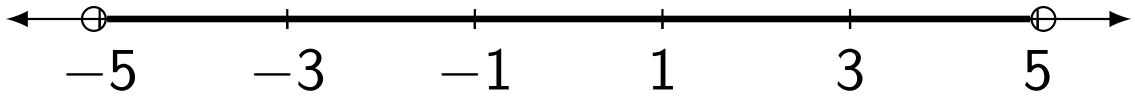
Solve

$$|x| < 5.$$

Solution:

$$\Rightarrow -5 < x < 5,$$

$$(-5, 5).$$



# Example

Solve

$$|x| \geq 6.$$

# Example

Solve

$$|x| \geq 6.$$

Solution:

# Example

Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

# Example

Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

$$(-\infty, -6] \bigcup [6, \infty).$$

# Example

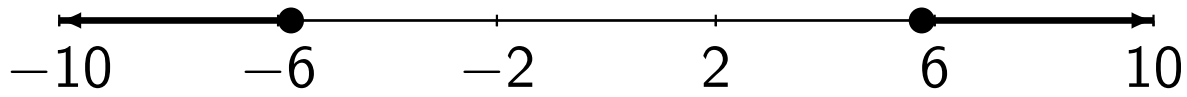
Solve

$$|x| \geq 6.$$

Solution:

$$\Rightarrow x \leq -6 \text{ or } x \geq 6,$$

$$(-\infty, -6] \bigcup [6, \infty).$$



# Example

Solve

$$|x + 6| \leq 10.$$

# Example

Solve

$$|x + 6| \leq 10.$$

Solution:

# Example

Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

# Example

Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

$$\Rightarrow -16 \leq x \leq 4, [-16, 4].$$

# Example

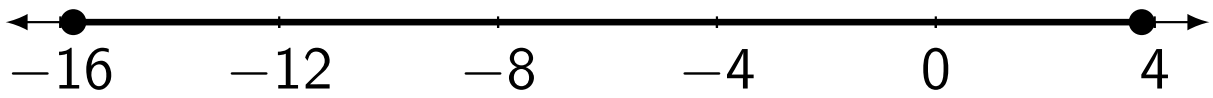
Solve

$$|x + 6| \leq 10.$$

Solution:

$$\Rightarrow -10 \leq x + 6 \leq 10$$

$$\Rightarrow -16 \leq x \leq 4, [-16, 4].$$



# Example

Solve

$$|x + 7| > 10.$$

# Example

Solve

$$|x + 7| > 10.$$

Solution:

# Example

Solve

$$|x + 7| > 10.$$

Solution:

$$\Rightarrow x + 7 < -10 \text{ or } x + 7 > 10$$

# Example

Solve

$$|x + 7| > 10.$$

Solution:

$$\Rightarrow x + 7 < -10 \text{ or } x + 7 > 10$$

$$\Rightarrow x < -17 \text{ or } x > 3, (-\infty, -17) \bigcup (3, \infty).$$

# Example

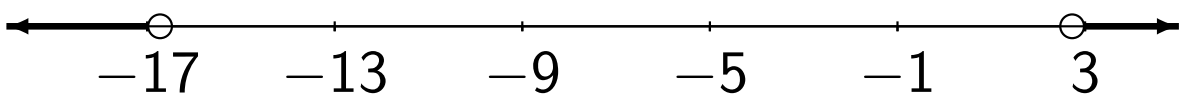
Solve

$$|x+7| > 10.$$

Solution:

$$\Rightarrow x+7 < -10 \text{ or } x+7 > 10$$

$$\Rightarrow x < -17 \text{ or } x > 3, (-\infty, -17) \bigcup (3, \infty).$$



# Example

Solve

$$|3x + 2| < 5.$$

# Example

Solve

$$|3x + 2| < 5.$$

Solution:

# Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

# Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

# Example

Solve

$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

$$\Rightarrow -\frac{7}{3} < x < 1, \left(-\frac{7}{3}, 1\right).$$

# Example

Solve

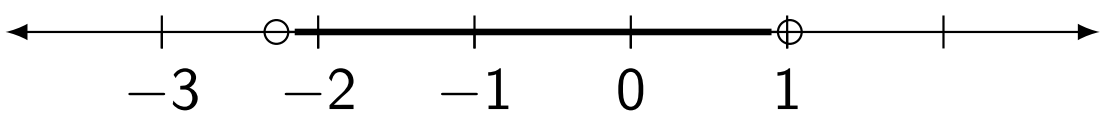
$$|3x + 2| < 5.$$

Solution:

$$-5 < 3x + 2 < 5$$

$$\Rightarrow -7 < 3x < 3$$

$$\Rightarrow -\frac{7}{3} < x < 1, \left(-\frac{7}{3}, 1\right).$$



# Example

Solve

$$|5 - 2x| \geq 1.$$

# Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

# Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

# Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

# Example

Solve

$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 2, (-\infty, 2] \bigcup [3, \infty).$$

# Example

Solve

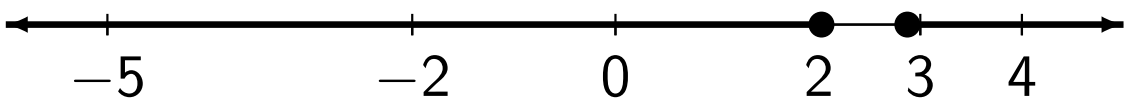
$$|5 - 2x| \geq 1.$$

Solution:

$$\Rightarrow 5 - 2x \leq -1 \text{ or } 5 - 2x \geq 1$$

$$\Rightarrow -2x \leq -6 \text{ or } -2x \geq -4$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 2, (-\infty, 2] \bigcup [3, \infty).$$



# Example

Solve

$$|3x + 4| < 13.$$

# Example

Solve

$$|3x + 4| < 13.$$

Solution:

# Example

Solve

$$|3x + 4| < 13.$$

Solution:

$$\Rightarrow -13 < 3x + 4 < 13$$

# Example

Solve

$$|3x + 4| < 13.$$

Solution:

$$\Rightarrow -13 < 3x + 4 < 13$$

$$\Rightarrow -17 < 3x < 9$$

# Example

Solve

$$|3x + 4| < 13.$$

Solution:

$$\Rightarrow -13 < 3x + 4 < 13$$

$$\Rightarrow -17 < 3x < 9$$

$$\Rightarrow -\frac{17}{3} < x < 3, \left(-\frac{17}{3}, 3\right)$$

# Example

Solve

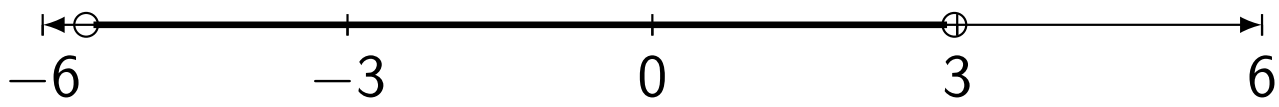
$$|3x + 4| < 13.$$

Solution:

$$\Rightarrow -13 < 3x + 4 < 13$$

$$\Rightarrow -17 < 3x < 9$$

$$\Rightarrow -\frac{17}{3} < x < 3, \left(-\frac{17}{3}, 3\right)$$



# Example

Solve

$$|6 - 4x| \geq 8.$$

# Example

Solve

$$|6 - 4x| \geq 8.$$

Solution:

# Example

Solve

$$|6 - 4x| \geq 8.$$

Solution:

$$\Rightarrow 6 - 4x \leq -8 \text{ or } 6 - 4x \geq 8$$

# Example

Solve

$$|6 - 4x| \geq 8.$$

Solution:

$$\Rightarrow 6 - 4x \leq -8 \text{ or } 6 - 4x \geq 8$$

$$\Rightarrow -4x \leq -14 \text{ or } -4x \geq 2$$

# Example

Solve

$$|6 - 4x| \geq 8.$$

Solution:

$$\Rightarrow 6 - 4x \leq -8 \text{ or } 6 - 4x \geq 8$$

$$\Rightarrow -4x \leq -14 \text{ or } -4x \geq 2$$

$$\Rightarrow x \leq -\frac{1}{2} \text{ or } x \geq \frac{7}{2}, \left(-\infty, -\frac{1}{2}\right] \bigcup \left[\frac{7}{2}, \infty\right)$$

# Example

Solve

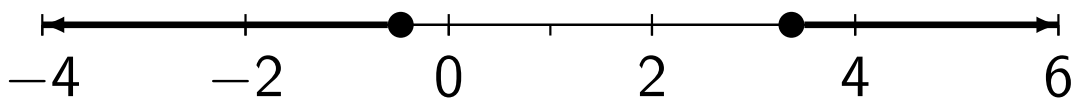
$$|6 - 4x| \geq 8.$$

Solution:

$$\Rightarrow 6 - 4x \leq -8 \text{ or } 6 - 4x \geq 8$$

$$\Rightarrow -4x \leq -14 \text{ or } -4x \geq 2$$

$$\Rightarrow x \leq -\frac{1}{2} \text{ or } x \geq \frac{7}{2}, \left(-\infty, -\frac{1}{2}\right] \bigcup \left[\frac{7}{2}, \infty\right)$$



# Example

Solve

$$|4 + 3x| \leq 13.$$

# Example

Solve

$$|4 + 3x| \leq 13.$$

Solution:

# Example

Solve

$$|4 + 3x| \leq 13.$$

Solution:

$$\Rightarrow -13 \leq 3x + 4 \leq 13$$

# Example

Solve

$$|4 + 3x| \leq 13.$$

Solution:

$$\Rightarrow -13 \leq 3x + 4 \leq 13$$

$$\Rightarrow -17 \leq 3x \leq 9$$

# Example

Solve

$$|4 + 3x| \leq 13.$$

Solution:

$$\Rightarrow -13 \leq 3x + 4 \leq 13$$

$$\Rightarrow -17 \leq 3x \leq 9$$

$$\Rightarrow -\frac{17}{3} \leq x \leq 3, \left[-\frac{17}{3}, 3\right]$$

# Example

Solve

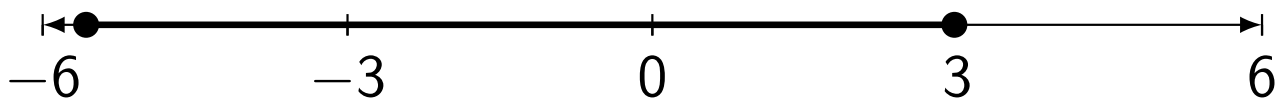
$$|4 + 3x| \leq 13.$$

Solution:

$$\Rightarrow -13 \leq 3x + 4 \leq 13$$

$$\Rightarrow -17 \leq 3x \leq 9$$

$$\Rightarrow -\frac{17}{3} \leq x \leq 3, \left[-\frac{17}{3}, 3\right]$$



# Example

Solve

$$|1 - 6x| < 5.$$

# Example

Solve

$$|1 - 6x| < 5.$$

Solution:

# Example

Solve

$$|1 - 6x| < 5.$$

Solution:

$$\Rightarrow -5 < 1 - 6x < 5$$

# Example

Solve

$$|1 - 6x| < 5.$$

Solution:

$$\Rightarrow -5 < 1 - 6x < 5$$

$$\Rightarrow -6 < -6x < 4$$

# Example

Solve

$$|1 - 6x| < 5.$$

Solution:

$$\Rightarrow -5 < 1 - 6x < 5$$

$$\Rightarrow -6 < -6x < 4$$

$$\Rightarrow -\frac{2}{3} < x < 1, \left(-\frac{2}{3}, 1\right)$$

# Example

Solve

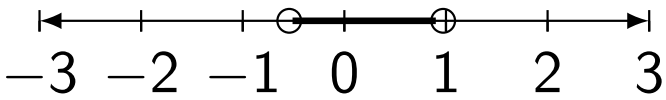
$$|1 - 6x| < 5.$$

Solution:

$$\Rightarrow -5 < 1 - 6x < 5$$

$$\Rightarrow -6 < -6x < 4$$

$$\Rightarrow -\frac{2}{3} < x < 1, \left(-\frac{2}{3}, 1\right)$$



# Example

Solve

$$|7 - x| \geq -4.$$

# Example

Solve

$$|7 - x| \geq -4.$$

Solution:

# Example

Solve

$$|7 - x| \geq -4.$$

Solution:

$$\Rightarrow 7 - x \leq 4 \text{ or } 7 - x \geq -4$$

# Example

Solve

$$|7 - x| \geq -4.$$

Solution:

$$\Rightarrow 7 - x \leq 4 \text{ or } 7 - x \geq -4$$

$$\Rightarrow -x \leq -3 \text{ or } -x \geq -11$$

# Example

Solve

$$|7 - x| \geq -4.$$

Solution:

$$\Rightarrow 7 - x \leq 4 \text{ or } 7 - x \geq -4$$

$$\Rightarrow -x \leq -3 \text{ or } -x \geq -11$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 11, (-\infty, \infty)$$

# Example

Solve

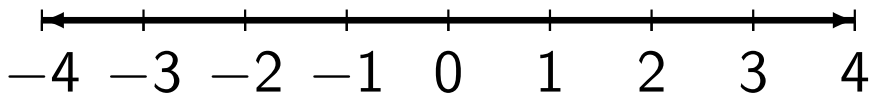
$$|7 - x| \geq -4.$$

Solution:

$$\Rightarrow 7 - x \leq 4 \text{ or } 7 - x \geq -4$$

$$\Rightarrow -x \leq -3 \text{ or } -x \geq -11$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 11, (-\infty, \infty)$$



# Example

Solve

$$|2x + 5| < 0.$$

# Example

Solve

$$|2x + 5| < 0.$$

Solution:

# Example

Solve

$$|2x + 5| < 0.$$

Solution:

$$\Rightarrow 0 < 2x + 5 < 0$$

# Example

Solve

$$|2x + 5| < 0.$$

Solution:

$$\Rightarrow 0 < 2x + 5 < 0$$

$$\Rightarrow -5 < 2x < -5$$

# Example

Solve

$$|2x + 5| < 0.$$

Solution:

$$\Rightarrow 0 < 2x + 5 < 0$$

$$\Rightarrow -5 < 2x < -5$$

$$\Rightarrow -\frac{5}{2} < x < -\frac{5}{2}, \emptyset$$

# Example

Solve

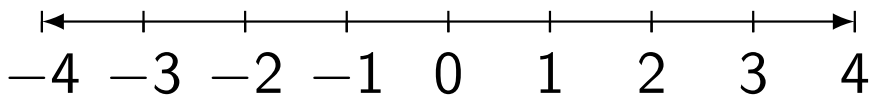
$$|2x + 5| < 0.$$

Solution:

$$\Rightarrow 0 < 2x + 5 < 0$$

$$\Rightarrow -5 < 2x < -5$$

$$\Rightarrow -\frac{5}{2} < x < -\frac{5}{2}, \emptyset$$



# Example

Solve

$$|x - 5| - 8 = -1.$$

# Example

Solve

$$|x - 5| - 8 = -1.$$

Solution:

# Example

Solve

$$|x - 5| - 8 = -1.$$

Solution:

$$\Rightarrow |x - 5| = 7$$

# Example

Solve

$$|x-5|-8=-1.$$

Solution:

$$\Rightarrow |x-5|=7$$

$$\Rightarrow x-5=7, x-5=-7$$

# Example

Solve

$$|x-5|-8=-1.$$

Solution:

$$\Rightarrow |x-5|=7$$

$$\Rightarrow x-5=7, x-5=-7$$

$$\Rightarrow x=12, x=-2 \quad (D.)$$

# Example

Solve

$$|6x + 3| = 7.$$

# Example

Solve

$$|6x + 3| = 7.$$

Solution:

# Example

Solve

$$|6x + 3| = 7.$$

Solution:

$$\Rightarrow 6x + 3 = 7, 6x + 3 = -7$$

# Example

Solve

$$|6x + 3| = 7.$$

Solution:

$$\Rightarrow 6x + 3 = 7, 6x + 3 = -7$$

$$\Rightarrow 6x = 4, 6x = -10$$

# Example

Solve

$$|6x + 3| = 7.$$

Solution:

$$\Rightarrow 6x + 3 = 7, 6x + 3 = -7$$

$$\Rightarrow 6x = 4, 6x = -10$$

$$\Rightarrow x = \frac{2}{3}, x = -\frac{5}{3} \quad (C.)$$

# Example

Solve

$$|5x - 2| \geq 6.$$

# Example

Solve

$$|5x - 2| \geq 6.$$

Solution:

# Example

Solve

$$|5x - 2| \geq 6.$$

Solution:

$$\Rightarrow 5x - 2 \geq 6 \text{ or } 5x - 2 \leq -6$$

# Example

Solve

$$|5x - 2| \geq 6.$$

Solution:

$$\Rightarrow 5x - 2 \geq 6 \text{ or } 5x - 2 \leq -6$$

$$\Rightarrow 5x \geq 8 \text{ or } 5x \leq -4$$

# Example

Solve

$$|5x - 2| \geq 6.$$

Solution:

$$\Rightarrow 5x - 2 \geq 6 \text{ or } 5x - 2 \leq -6$$

$$\Rightarrow 5x \geq 8 \text{ or } 5x \leq -4$$

$$\Rightarrow x \leq -\frac{4}{5} \text{ or } x \geq \frac{8}{5}, \left(-\infty, -4/5\right] \bigcup [8/5, \infty) \quad (D.)$$

# Example

Solve

$$|3x + 2| < 19.$$

# Example

Solve

$$|3x + 2| < 19.$$

Solution:

# Example

Solve

$$|3x + 2| < 19.$$

Solution:

$$\Rightarrow -19 < 3x + 2 < 19$$

# Example

Solve

$$|3x + 2| < 19.$$

Solution:

$$\Rightarrow -19 < 3x + 2 < 19$$

$$\Rightarrow -21 < 3x < 17$$

# Example

Solve

$$|3x + 2| < 19.$$

Solution:

$$\Rightarrow -19 < 3x + 2 < 19$$

$$\Rightarrow -21 < 3x < 17$$

$$\Rightarrow -7 < x < \frac{17}{3}, \left(-7, \frac{17}{3}\right) \quad (D.)$$