Workbook Examples
Chapter 4
Math 1100

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22 March 2020

Outline

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

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Determine the behavior of the graph of a polynomial function using the leading term test.

Factor polynomial functions, and find the zeros and their multiplicities.

Polynomial Function

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where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

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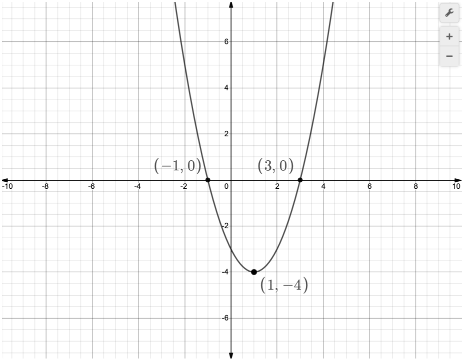
Figure: $f(x) = x^2 - 2x - 3$



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Solution: $(-\infty, \infty)$;

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Solution: $[-4, \infty)$.

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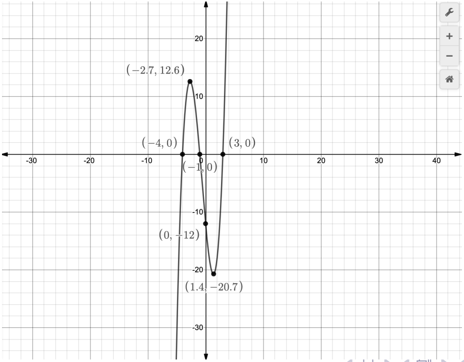

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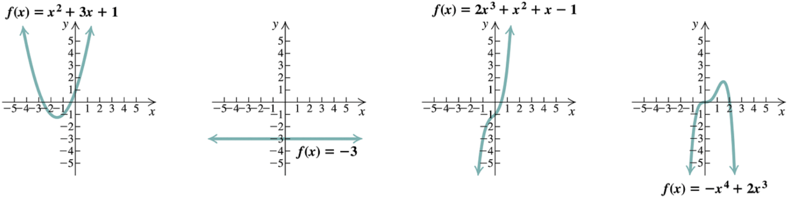
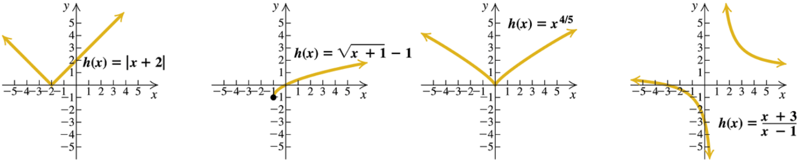
Examples of Polynomial Functions

Figure: Polynomial Functions



Examples of Nonpolynomial Functions

Figure: Nonpolynomial Functions



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Furthermore, the domain of a polynomial function is the set of all real numbers.

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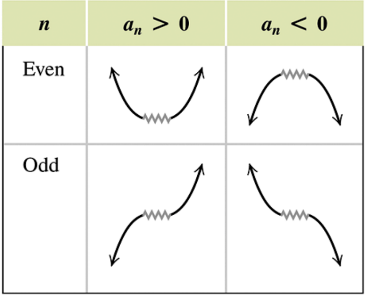

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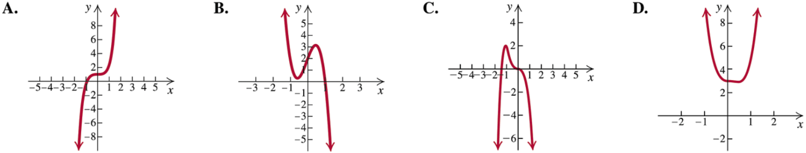

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c. $f(x) = x^5 + \frac{1}{4}x + 1$

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Solution: **A**

d. $f(x) = -x^6 + x^5 - 4x^3$

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Solution: **B**

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Solution: x^5 is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$.

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(4) $f(x) = x^2 - x^3 - 2x + 4$.

Solution: $-x^3$ is the leading term. So $f(x) \to \infty$ as $x \to -\infty$, and $f(x) \to -\infty$ as $x \to \infty$.

Example

Choose the end behavior diagram that best describes the function $f(x) = -3.9x^4 + x^6 + 0.1x^7$.

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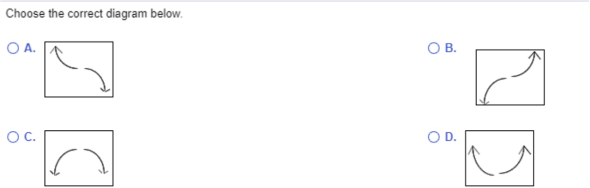

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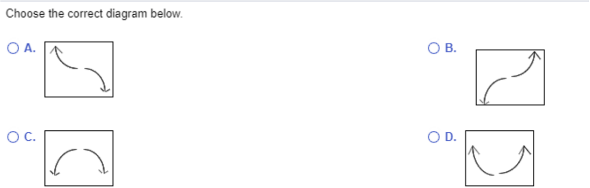

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Choose the correct graph below.

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Finding Zeros of Polynomial Functions

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Finding Zeros of Polynomial Functions

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(ii) k is even, then the graph is tangent to the $x-$axis at $(c, 0)$.