Workbook Examples
Chapter 4
Math 1100

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29 March 2020

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Outline

Objectives

Determine the behavior of the graph of a polynomial function using the leading term test.

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Determine the behavior of the graph of a polynomial function using the leading term test.

Factor polynomial functions, and find the zeros and their multiplicities.

Polynomial Function

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where the coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and the exponents are whole numbers.

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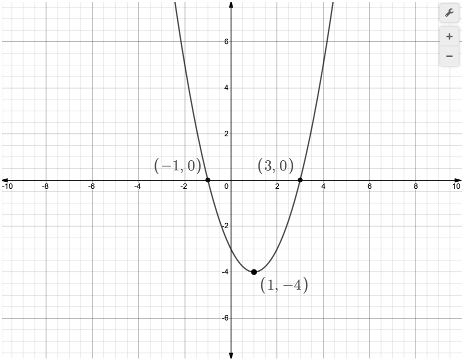
Figure: $f(x) = x^2 - 2x - 3$



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Solution: $(-\infty, \infty)$;

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Solution: $[-4, \infty)$.

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$$g(x) = x^3 + 2x^2 - 11x - 12 = (x+4)(x+1)(x-3),$$

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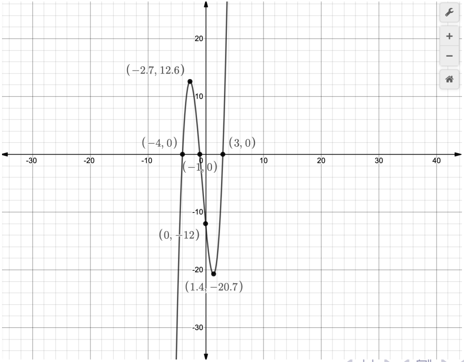

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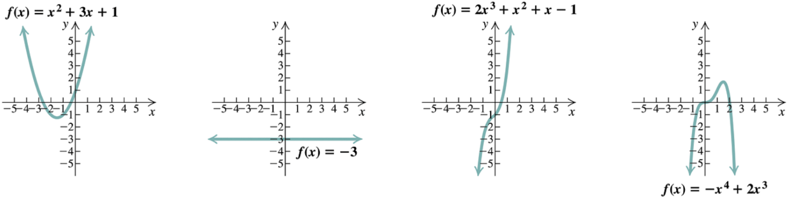
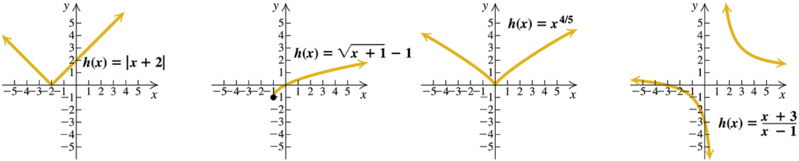
Examples of Polynomial Functions

Figure: Polynomial Functions



Examples of Nonpolynomial Functions

Figure: Nonpolynomial Functions



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Furthermore, the domain of a polynomial function is the set of all real numbers.

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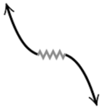
n	$a_n > 0$	$a_n < 0$
Even		
Odd		

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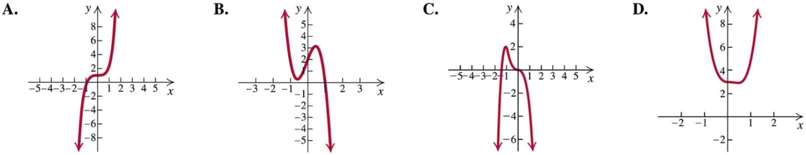

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a. $f(x) = 3x^4 - 2x^3 + 3.$

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Solution: **B**

c. $f(x) = x^5 + \frac{1}{4}x + 1$

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Solution: **A**

d. $f(x) = -x^6 + x^5 - 4x^3$

Example (cont.)

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Solution: **B**

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Solution: **C**

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Solution: x^5 is the leading term. So $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$.

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(4) $f(x) = x^2 - x^3 - 2x + 4$.

Example (cont.)

(3) $f(x) = 2x^4 - x^2 + 1$

Solution: $2x^4$ is the leading term. So $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$.

(4) $f(x) = x^2 - x^3 - 2x + 4$.

Solution: $-x^3$ is the leading term. So $f(x) \to \infty$ as $x \to -\infty$, and $f(x) \to -\infty$ as $x \to \infty$.

Example

Choose the end behavior diagram that best describes the function $f(x) = -3.9x^4 + x^6 + 0.1x^7$.

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Solution: $f(x) \approx x^7 \Rightarrow$ C.

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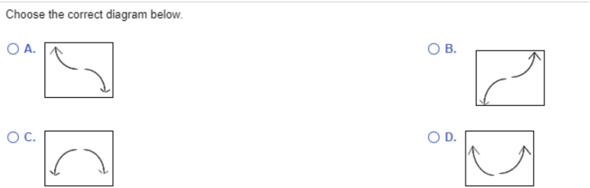

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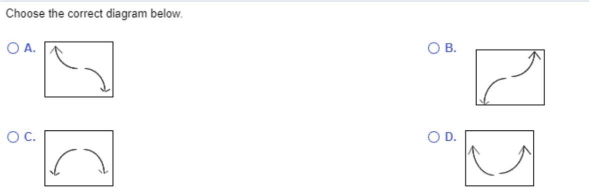

Solution: $f(x) \approx x^4 \Rightarrow$D.

Example

Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

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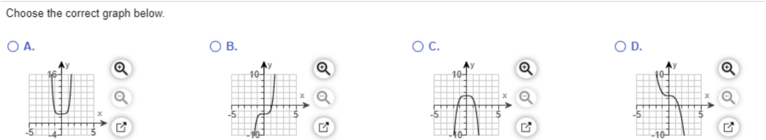
Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

Figure:



Example

Use the leading term test to match the function $f(x) = x^5 + \frac{1}{14}x - 3$ with one of the given graphs.

Figure:

Choose the correct graph below.

Solution: $f(x) \approx x^5 \Rightarrow$B.

Finding Zeros of Polynomial Functions

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If c is a real zero of a function (that is, $f(c) = 0$),
then $(c, 0)$ is an $x-$intercept of the graph of the function.

Even and Odd Multiplicity

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(i) k is odd, then the graph crosses the $x-$axis at $(c, 0)$;

(ii) k is even, then the graph is tangent to the $x-$axis at $(c, 0)$.

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Use substitution to determine whether 3 is a zero of

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$$\Rightarrow f(3) = 27 - 90 + 51 + 12 = 0$$

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Thus 3 is a zero of $f(x)$.

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$$\Rightarrow f(3) = -75$$

Thus 3 is *not* a zero of $f(x)$.

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Determine whether the following statement is true or false.

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The statement is true (Statement B).

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Choose D.