Assignment 10 (66 Points)

Work on the following questions, staple them together and hand them out to TA in your next recitation. **No late submission will be accepted**. These questions are from our textbook (third edition). The question numbering may be different if you use the second edition.

Chapter 5

1 (10 points) Let $ALL_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ Show that $ALL_{\text{DFA}} \leq_m E_{\text{DFA}}$ by constructing a reduction f from ALL_{DFA} to E_{DFA} (in a form of a Turing machine) such that

$$\langle A \rangle \in ALL_{\text{DFA}} \leftrightarrow f(\langle A \rangle) \in E_{\text{DFA}}$$

2 (10 points) Let $ALL_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a TM that accepts all strings}\}$. Show that ALL_{TM} is undecidable by showing that if ALL_{TM} is decidable, A_{TM} is decidable.

3 (10 points) From question 2 above, show that $A_{\text{TM}} \leq_m ALL_{\text{TM}}$ by showing that there exists a computable function f in a form of a Turing machine such that

$$\langle M, w \rangle \in A_{\text{TM}} \leftrightarrow f(\langle M, w \rangle) \in ALL_{\text{TM}}$$

4 (10 points) Show that $DECIDER_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a decider}\}$ is undecidable by showing that if $DECIDER_{\text{TM}}$ is decidable, A_{TM} is also decidable.

5 (10 points) From question 4 above, show that $A_{\text{TM}} \leq_m DECIDABLE_{\text{TM}}$ by showing that there exists a computable function f in a form of a Turing machine such that

$$\langle M, w \rangle \in A_{\text{TM}} \leftrightarrow f(\langle M, w \rangle) \in DECIDER_{\text{TM}}$$

Chapter 7

1 (16 points) Answer each part TRUE or FALSE.

a. $2n = \mathcal{O}(n)$.

b. $n^2 = \mathcal{O}(n)$.

c. $3^n = 2^{\mathcal{O}(n)}$.

d. $2^{2^n} = \mathcal{O}(2^{2^n})$.

e. $n = o(2n)$.

f. $2n = o(n^2)$.

g. $n = o(\log n)$.

h. $1 = o(1/n)$.