Balancing Robot

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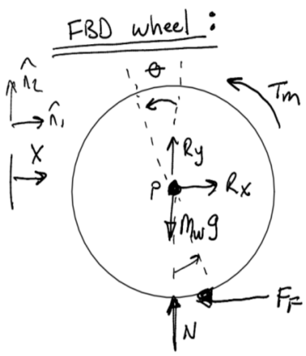
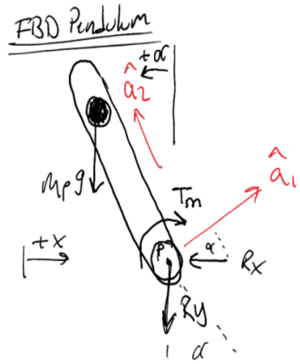
Free body diagrams





Acceleration vectors

$\vec{r}^{m_w} = x\hat{n}_1 + 0\hat{n}_2$
$\vec{v}^{m_w} = \dot{x}\hat{n}_1 + 0\hat{n}_2$
$\vec{a}^{m_w} = \ddot{x}\hat{n}_1 + 0\hat{n}_2$
$\vec{r}_{m_p} = (x - L\sin\alpha)\hat{n}_1 + L\cos\alpha\hat{n}_2$
$\vec{v}^{m_p} = (\dot{x} - \dot{\alpha}L\cos\alpha)\hat{n}_1 - \dot{\alpha}L\sin\alpha\hat{n}_2$
$\vec{a}^{m_p} = (\ddot{x} - \ddot{\alpha}L\cos\alpha + \dot{\alpha}^2L\sin\alpha)\hat{n}_1 - (\ddot{\alpha}L\sin\alpha + \dot{\alpha}^2L\cos\alpha)\hat{n}_2$

Equations of motion

Newton's Law for m_w

$$(R_x - F_F)\hat{n}_1 + (R_y + N - m_w g)\hat{n}_2 = (m_w\ddot{x})\hat{n}_1 + 0\hat{n}_2$$

Newton's Law for m_p

$$(-R_x)\hat{n}_1 + (-R_y - m_p g)\hat{n}_2 = m_p(\ddot{x} - \ddot{\alpha}L\cos\alpha + \dot{\alpha}^2L\sin\alpha)\hat{n}_1 - m_p(\ddot{\alpha}L\sin\alpha + \dot{\alpha}^2L\cos\alpha)\hat{n}_2$$

Sum of moments for m_w

$$T_m - F_F R_w = \frac{1}{2}m_w R_w^2$$

Sum of moments for m_p

$$R_y L\sin\alpha + R_x L\cos\alpha - T_m = 0$$

System of equations

(1) $m_w\ddot{x} + F_F - R_x = 0$
(2) $R_y + N = m_w g$
(3) $m_p\ddot{x} - m_p\ddot{\alpha}L\cos\alpha + R_x = -m_p\dot{\alpha}^2L\sin\alpha$
(4) $m_p\ddot{\alpha}L\sin\alpha - R_y = m_p g - m_p\dot{\alpha}^2L\cos\alpha$
(5) $F_F R_w = T_m - \frac{1}{2}m_w R_w^2$
(6) $R_y L\sin\alpha + R_x L\cos\alpha = T_m$
unknowns: x, α, N, R_x, R_y, F_F (6 total)

Torque from Voltage Input

$$\beta = \theta - \alpha$$
$$\omega = \dot{\beta} = \dot{\theta} - \dot{\alpha} = \frac{-\dot{x}}{R_w} - \dot{\alpha}$$
$$K_b\omega = V - R_m I$$
$$K_b(\frac{-\dot{x}}{R_w} - \dot{\alpha}) = V - R_m I$$
$$I = \frac{V - K_b(\frac{-\dot{x}}{R_w} - \dot{\alpha})}{R_m}$$
$$T = K_t I = K_t \frac{V - K_b(\frac{-\dot{x}}{R_w} - \dot{\alpha})}{R_m}$$

Linear Model

$$A = 1e4 \begin{bmatrix} 0 & 0.0001 & 0 & 0 \\ 0 & -0.1401 & 0.0051 & -0.0030 \\ 0 & 0 & 0 & 0.0001 \\ 0 & -1.3914 & 0.0640 & -0.0301 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ -61.1453 \\ 0 \\ -607.1709 \end{bmatrix}$$