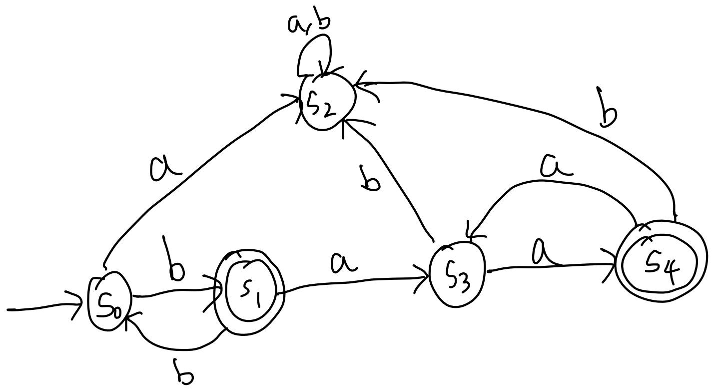
컴퓨터공학과 2021150?? 김동건

자동장치이론 HW 1

1.12 Let $D = \{w |\ w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$. Give a DFA with five states that recognizes D and a regular expression that generates D. (Suggestion: Describe D more simply.)

w doesn't contain ab, so we can simplify D like below.

D = {w | odd number of b followed by even number of a}



regular expression : $b(bb)^*(aa)^*$

1.22 In certain programming languages, comments appear between delimiters such as `/#` and `#/`. Let C be the language of all valid delimited comment strings. A member of C must begin with `/#` and end with `#/` but have no intervening `#/`. For simplicity, assume that the alphabet for C is $\Sigma = \{\mathtt{a}, \mathtt{b}, /, \#\}$.

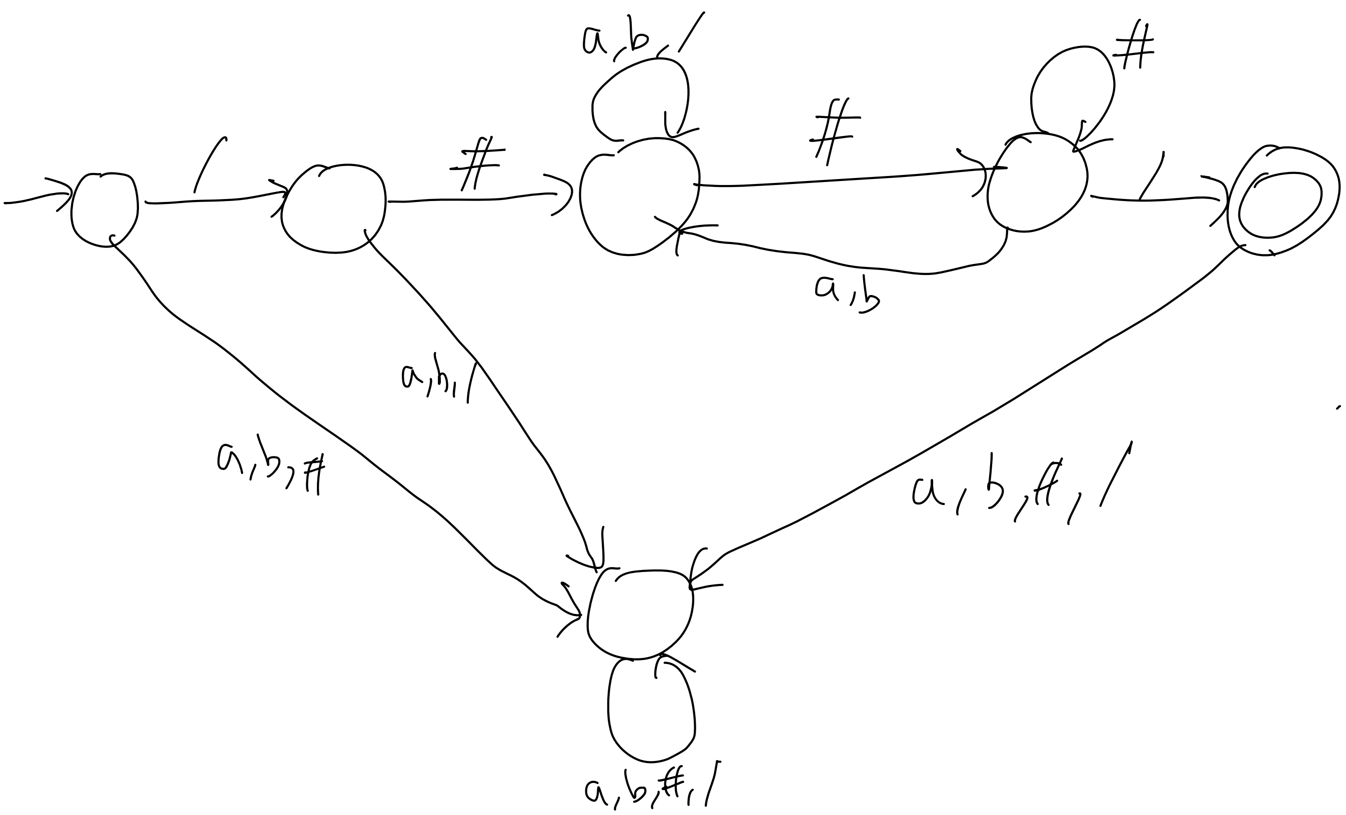
a. Give a DFA that recognizes C.

b. Give a regular expression that generates C.

a.

/# #/

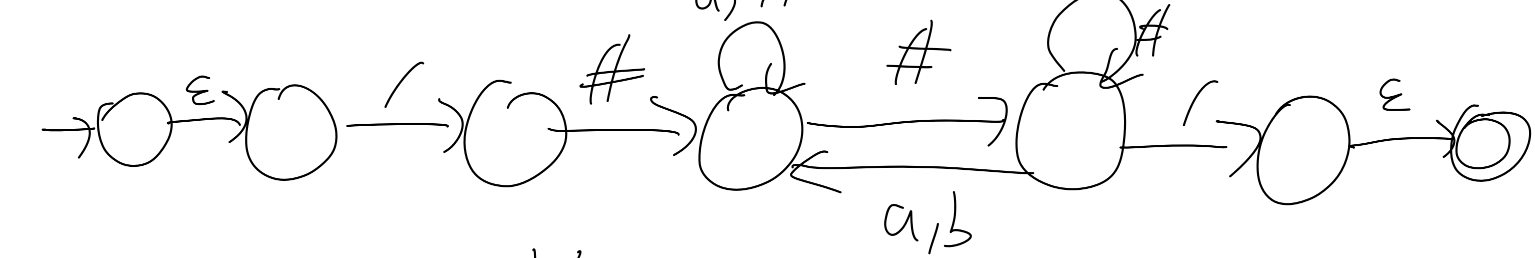
#/ should not appear...

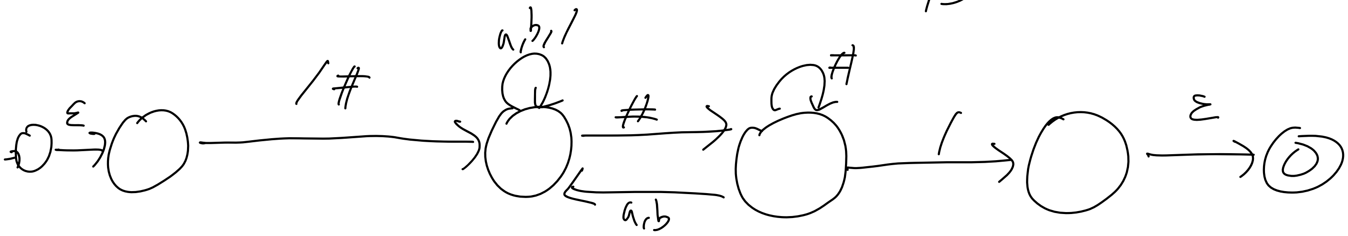


b.

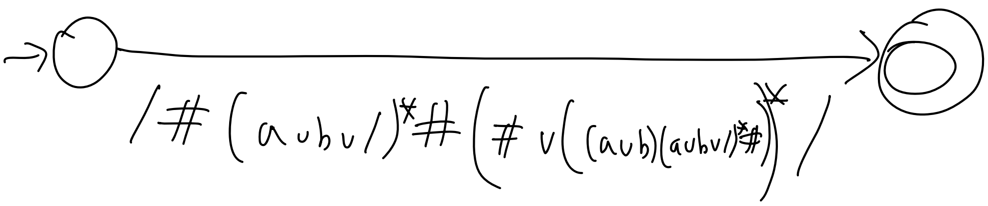
$$/\#(a \cup b \cup /)^*\#\left(\# \cup (a \cup b)(a \cup b \cup /)^*\#\right)^*/$$

a,b,/


ε
/
#
#
#
a,b
/
ε


ε
/#
a,b,/
#
#
a,b
/
ε


ε
/#(a∪b∪/)*#
#∪((a∪b)(a∪b∪/)*#)
/
ε


/#(a∪b∪/)*#(#∪((a∪b)(a∪b∪/)*#))*/

1.30 Describe the error in the following "proof" that 0^*1^* is not a regular language. (An error must exist because 0^*1^* *is* regular.) The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose s to be the string 0^p1^p. You know that s is a member of 0^*1^*, but Example 1.73 shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

error: 0^p1^p cannot be pumped.

Let's pump...

$w = 0^p1^p$,

$w = xyz$, $|xy| \le p$, $|y| \ge 1$

So, y contains at least one 0

let $y = 0$.

① $xy^iz = 0^{i+p-1}1^p$ is in 0^*1^* for any $i \ge 0$

② $|y| \ge 1$

③ $|xy| = 2 \le p$

So, it satiesfy pumpping lemma

1.40 Recall that string x is a ***prefix*** of string y if a string z exists where $xz = y$, and that x is a ***proper prefix*** of y if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

[A]**a.** $NOPREFIX(A) = \{w \in A|$ no proper prefix of w is a member of $A\}$.
b. $NOEXTEND(A) = \{w \in A|\, w$ is not the proper prefix of any string in $A\}$.

Let DFA $M = (Q, \Sigma, \delta, q_0, F)$ recognizes A

a. we can construct DFA M that recognizes NOPREFIX(A).

$M' = (Q', \Sigma, \delta', q_0', F')$

$Q' = Q$, $q_0' = q_0$, $F' = F$, $\delta' = (x, a) = \begin{cases} \delta(x, a) & \text{if } x \notin F \\ \emptyset & \text{else} \end{cases}$

(It means M' cut the progress if string is in A)

we can construct M'. so, NOPREFIX(A) is regular language.

b. we can construct DFA M' that recognizes NOEXTEND(A).

$M' = (Q', \Sigma, \delta', q_0', F')$. $Q' = Q$, $\delta' = \delta$, $q_0' = q_0$

$F' \subseteq F$ because state x ($x \in F$ and there is route x to another final state) should $x \notin F'$.

So, we can construct M'

1.51 Let x and y be strings and let L be any language. We say that x and y are ***distinguishable by L*** if some string z exists whereby exactly one of the strings xz and yz is a member of L; otherwise, for every string z, we have $xz \in L$ whenever $yz \in L$ and we say that x and y are ***indistinguishable by L***. If x and y are indistinguishable by L, we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

A★**1.52** **Myhill–Nerode theorem.** Refer to Problem 1.51. Let L be a language and let X be a set of strings. Say that X is ***pairwise distinguishable by L*** if every two distinct strings in X are distinguishable by L. Define the ***index of L*** to be the maximum number of elements in any set that is pairwise distinguishable by L. The index of L may be finite or infinite.

a. Show that if L is recognized by a DFA with k states, L has index at most k.

b. Show that if the index of L is a finite number k, it is recognized by a DFA with k states.

c. Conclude that L is regular iff it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.

a. Let's assume that index of L is greater than k.

Then, There is set S that size is at least $k+1$.

and pairwise distinguishable by L.

Because L's number of state is k, there is

$x \in S$, $y \in S$, that $\delta(q_0, x) = \delta(q_0, y)$. ($\because$ pigeon hole)

So, there is $\delta(q_0, xz) = \delta(q_0, yz)$, $z \in \Sigma^*$

$\therefore$ x and y are not distinguishable by L.

by contradiction, if L is recognized by DFA with k states, L has index at most k

b. **Show that if the index of L is a finite number k, it is recognized by a DFA with k states.**

let $X = \{s_1, \cdots, s_k\}$ that $s_1, \cdots, s_k$ are pair wise dintinguishable by L.

Let's construct DFA $M = (Q, \Sigma, \delta, q_0, F)$.

M has k states and recognize L.

$\delta(q_i, x) = q_j$ $(s_i \in X, \underline{s_j \equiv_L s_i x})$. (If not, $X \cup s_j x$ have $k+1$ pairwise distinguishable.)

$F = \{q_i \mid s_i \in L\}$.

$\therefore$ M recognize L with k states.

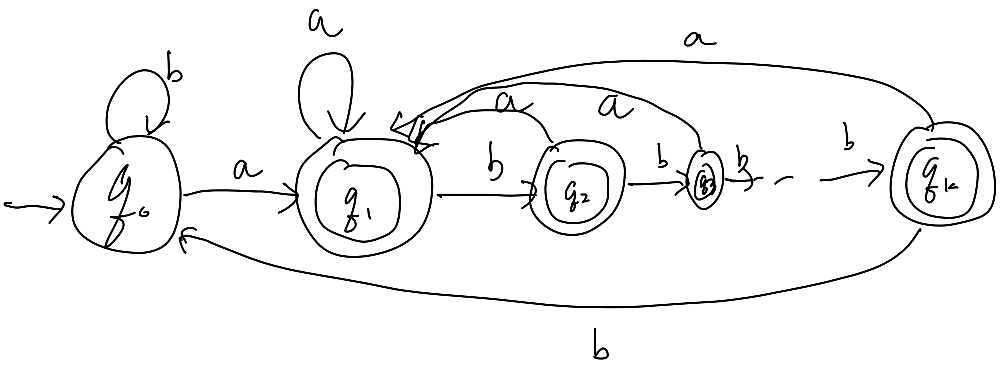
c. **Conclude that L is regular $\overset{\leftarrow}{\underset{\rightarrow}{\text{iff}}}$ it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.**

$\rightarrow$ if L is regular, L has index at most k. ($\because$ a)

$\leftarrow$ if L has index k, DFA with k states recognize L. So it is regular ($\because$ b)

Suppose L's index is k. by (b), there is DFA with k states reccognize L. If DFA has smaller than k states, by (a) it's index is less then k. so by contradiction, index is the size of smallest DFA recognizing it.

1.62 Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. For each $k \geq 1$, let D_k be the language consisting of all strings that have at least one $\mathtt{a}$ among the last k symbols. Thus $D_k = \Sigma^* \mathtt{a}(\Sigma \cup \varepsilon)^{k-1}$. Describe a DFA with at most $k+1$ states that recognizes D_k in terms of both a state diagram and a formal description.



$$M = (Q_k, \Sigma, \delta_k, q_0, F)$$

$$Q_k = \{q_0, \cdots, q_k\}$$

$$\Sigma = \{a, b\}$$

$$\delta_k = q_k(q_i, x) = \begin{cases} q_1 & x = a \\ q_0 & i = 0,\ x = b \\ q_{i+1} & 1 \leq i \leq k-1,\ x = b \\ q_0 & i = k,\ x = b \end{cases}$$

$$q_0 = \{q_0\}$$

$$F = \{q_1, \cdots, q_k\}$$

1.69 Let $\Sigma = \{0,1\}$. Let $WW_k = \{\overset{|2k|}{ww} \mid w \in \Sigma^* \text{ and } w \text{ is of length } k\}$.

a. Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.

b. Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k.

a. DFA have to check string of length $2k$.

DFA have to determine first k character is equal to last k character. (in order)

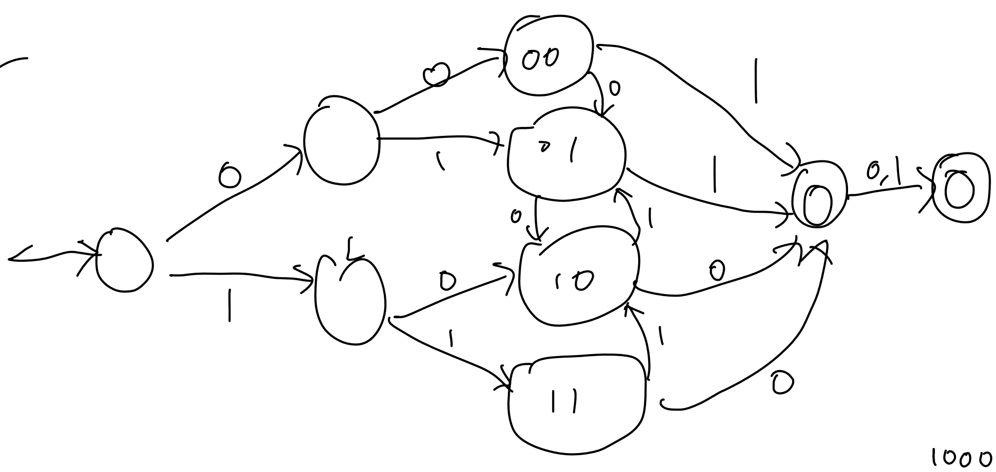
but, after check first k character, DFA can't know information of first k character. So, DFA represent first k character as states.

So, DFA need at least 2^k states.

b. $\overline{WW_k} = \{ uv \mid u \in \Sigma^*, v \in \Sigma^* \text{ and } |u| = |v| = k \text{ and } u \neq v \}$

$k = 2$

$2^k + k$ state



0000 X
0001 O
0010 O
0011 O

0100 O
0101 X
0110 O
0111 O

1000 O
1001 O
1010 X
1011 O

1100 O
1101 O
1110 O
1111 X

1.71 Let $\Sigma = \{0,1\}$.

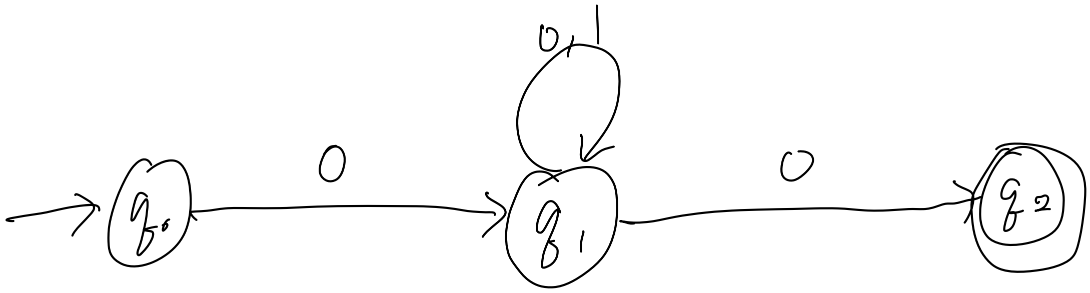
a. Let $A = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that A is regular.

b. Let $B = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that B is not regular.

a. We can draw NFA of A, so A is regular.

String s has at least one 0 at start, and at least one 0 at last

$s \in A$ because we can think another 0 is in u.



it recognizes A. $0\underbrace{0110010}_{u}00$

b. $0^k 1 u 0^k \ldots$

We choose $w = 0^p 1 0^p$.

$w \in L$ and $|w| = 2p+1 \geq p$

$w = xyz$, $|xy| \leq p$, $|y| > 0$. $(1 \leq k \leq p)$ $y = 0^k$

So y has at least one 0, and y is composed only of 0s.

$\therefore$ pump up to $xy^2z = 0^{p+k} 1 0^k \notin L$.

It violates pumpping lemma, so it is not regular.