Tutorial 5
Neural Network: Backpropagation

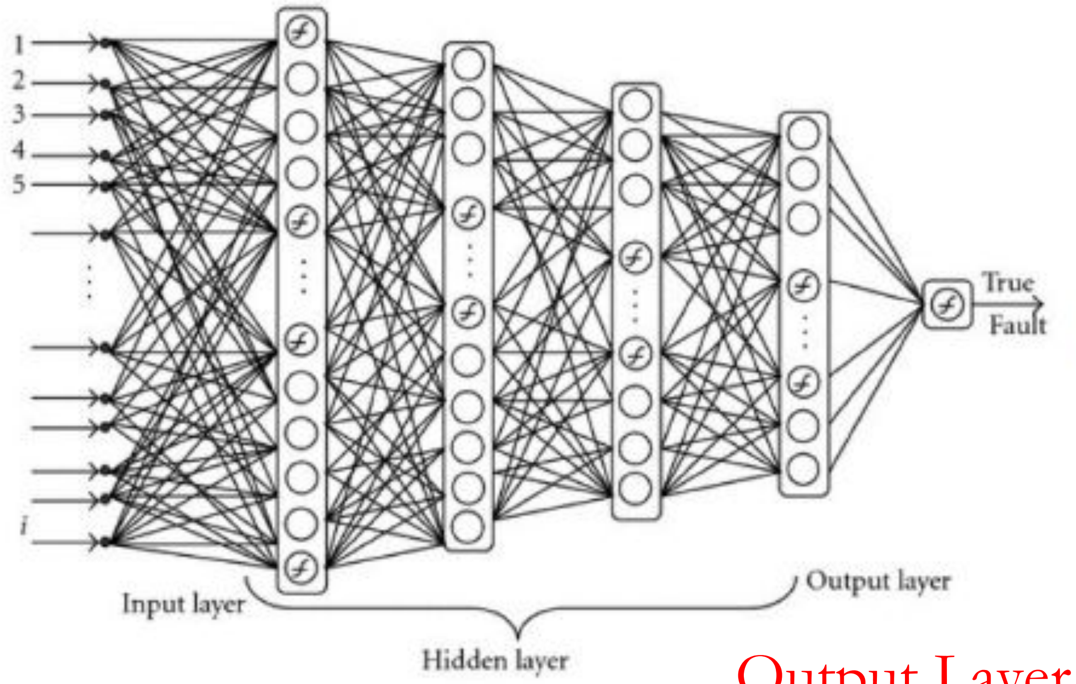
Agenda

- Understand FeedFoward and Backpropagation
- Discussion about Programming Assignment 5
  - Build Neural Network model
  - Multi-class classification and prediction
- Python Implementation
  - Tensorflow and Keras

Neural Network

NN (perceptron) consists of three layers:

Input data



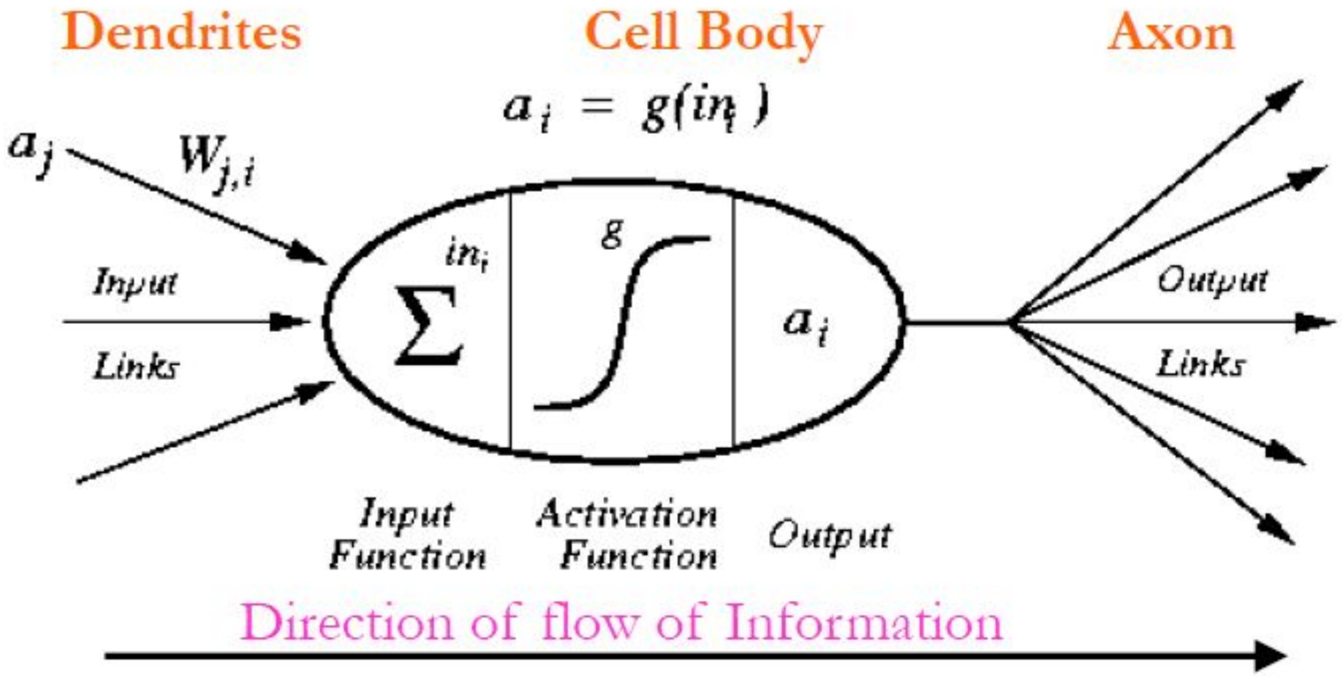
Output data

Input Layer

Hidden Layers

Output Layer

Neuron in Neural Network

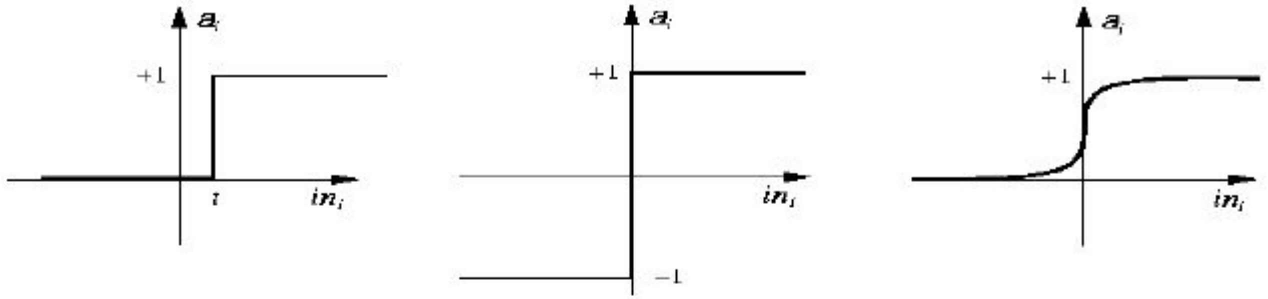


Each unit does a local computation based on inputs from its neighbours & compute a new activation level – sends along each of its output links

- a_j: Activation value of unit j
- $w_{j,i}$: Weight on the link from unit j to unit i
- in_i: Weighted sum of inputs to unit i
- a_i: Activation value of unit i
- g: Activation function.

Hidden Layer: Activation Functions

- Common Choices:



(a) Step function (b) Sign function (c) Sigmoid function

threshold function logistic function

Step(x)	=	1 if x >= 0, else 0
Sign(x)	=	1 if x >= 0, else –1
Sigmoid(x)	=	$1/(1+e^{-x})$

Simply put, activation function calculates a "weighted sum" of its input, and then decides whether it should be "fired" or not

$$Y = \sum (weight * input) + bias$$

Question: Why do we need it?

Activation functions are really important for NN to learn and make sense of something really Complicated and Non-linear complex functional mappings between the inputs and outputs. They introduce non-linear properties to our NN.

https://towardsdatascience.com/activation-functions-and-its-types-which-is-better-a9a5310cc8f

Hidden Layer: Activation Functions

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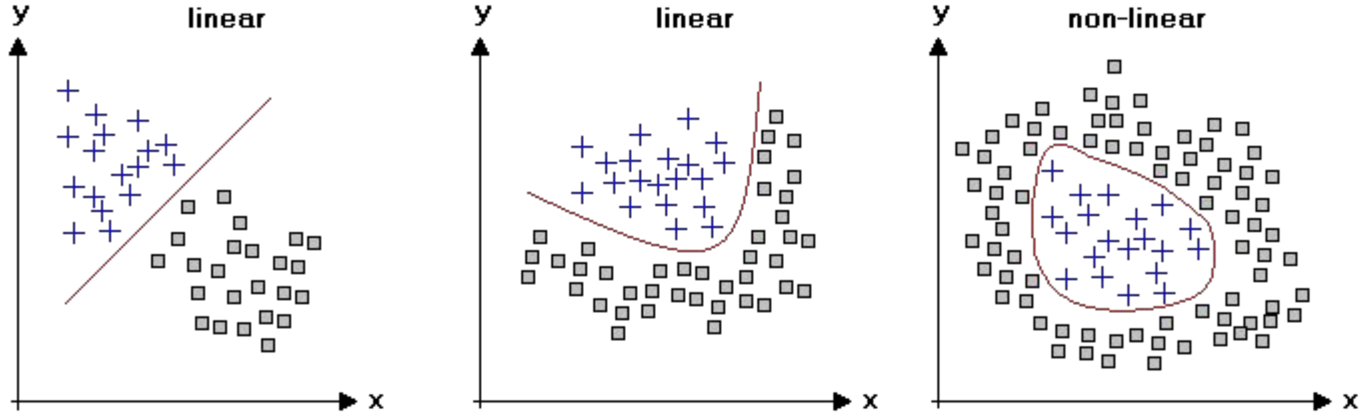

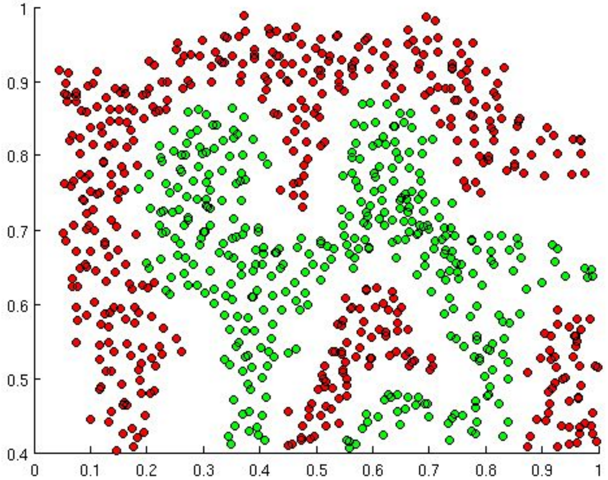

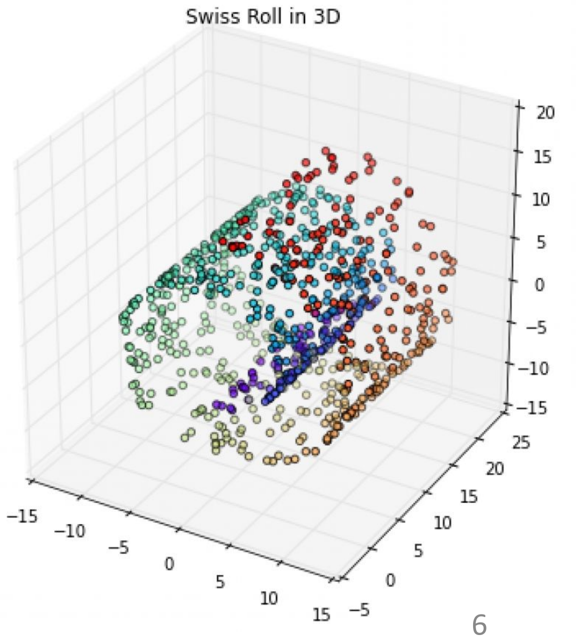

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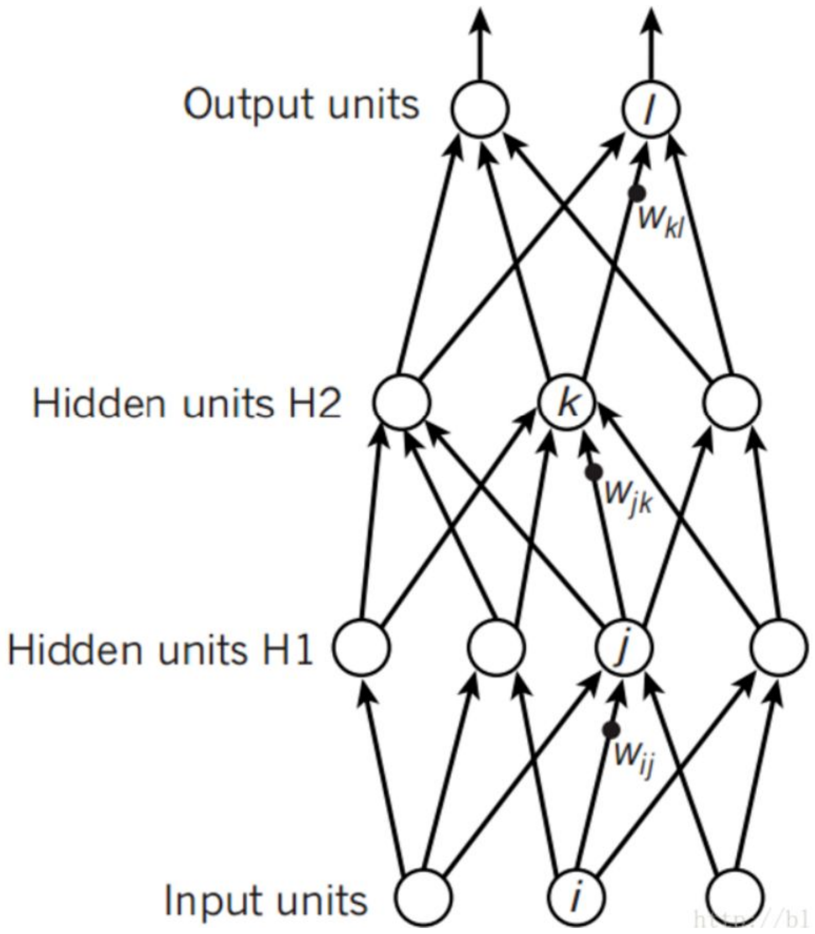
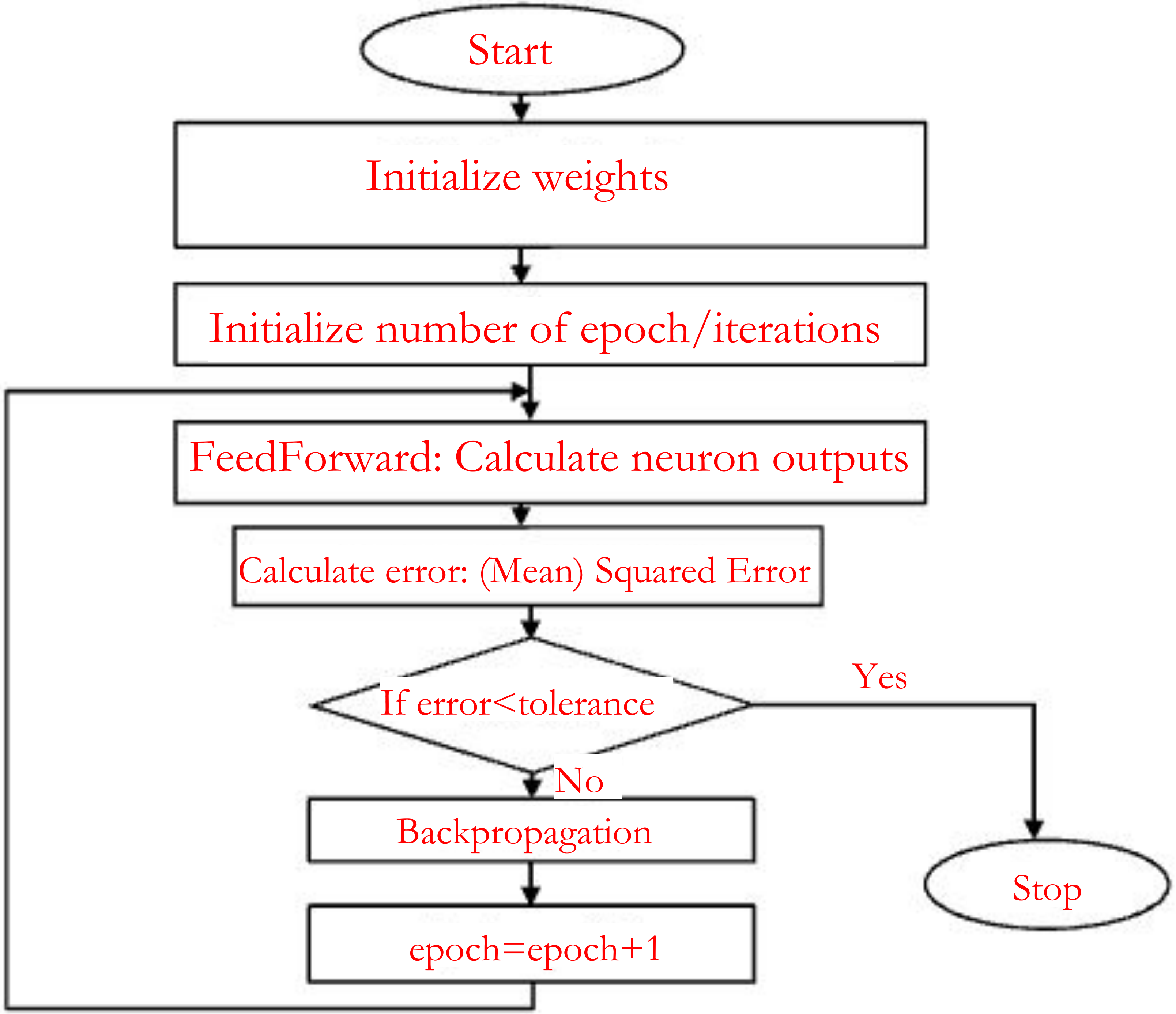
Neural Network: Feed Forward and Backpropagation





Feed Forward

Input Layer

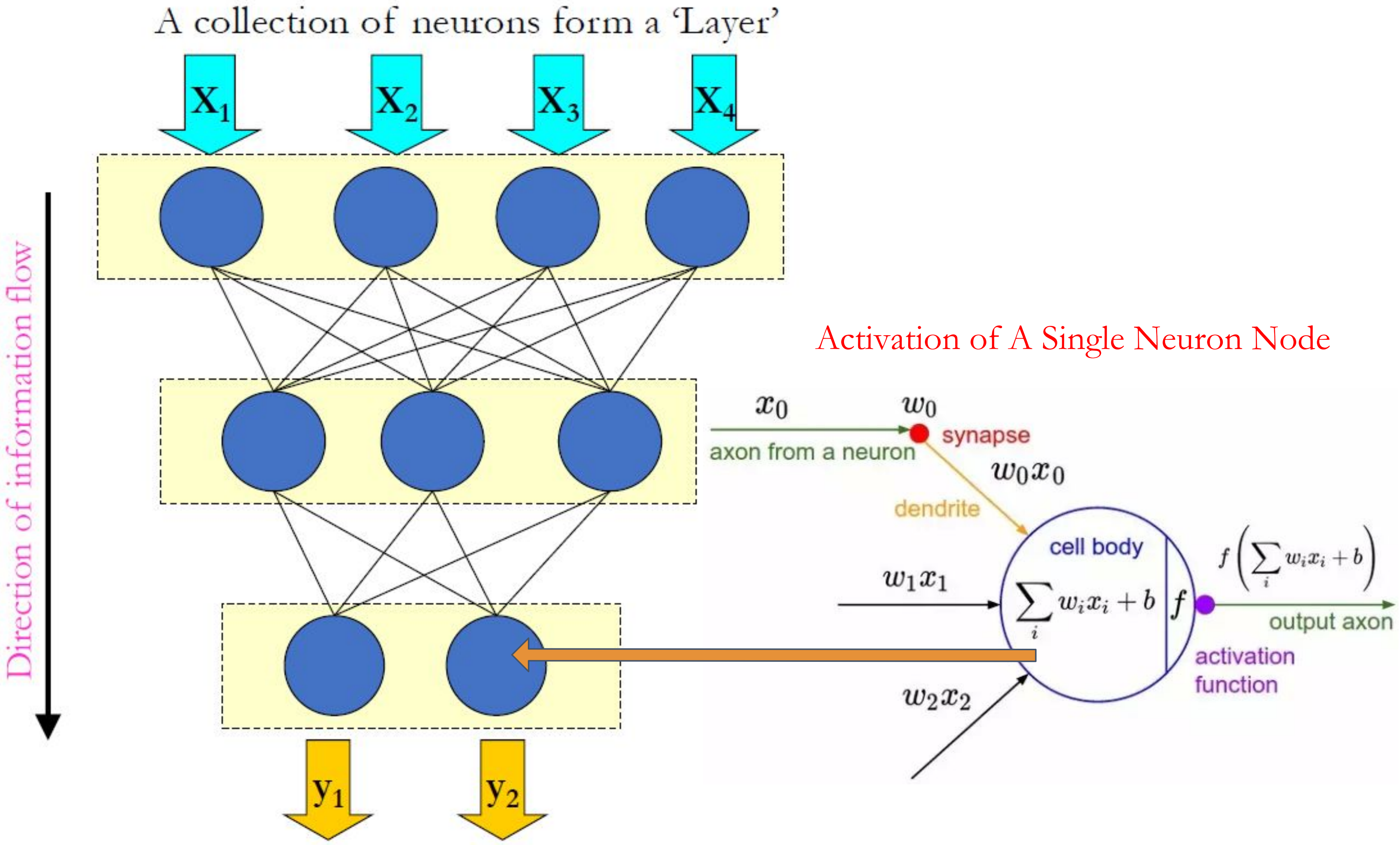
- Each neuron gets ONLY one input, directly from outside

Hidden Layer

- Connects Input and Output layers

Output Layer

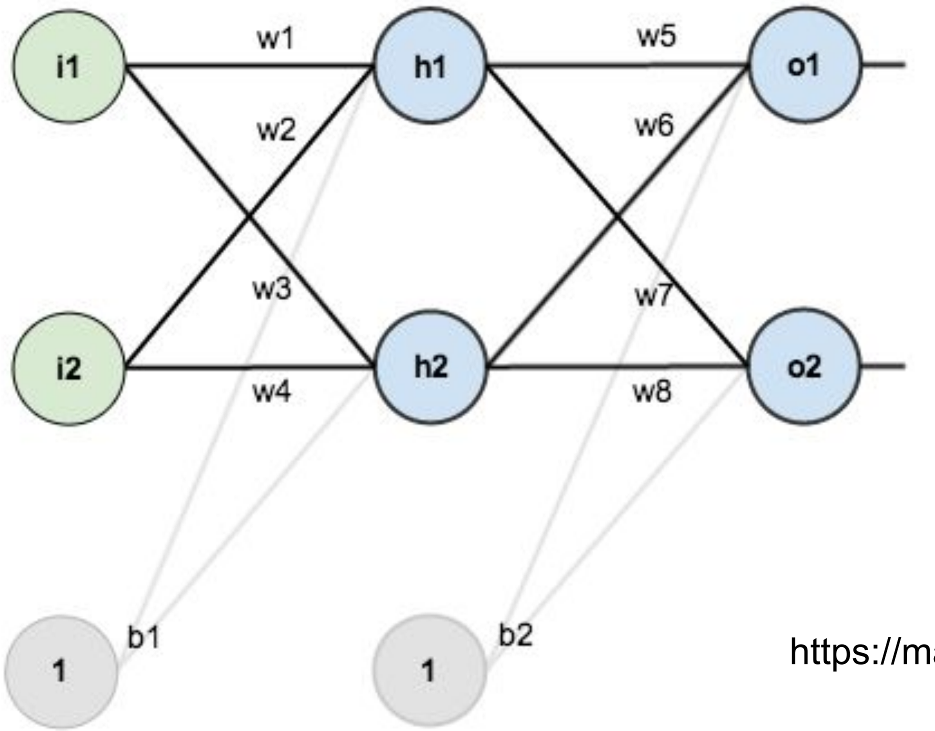
- Output of each neuron directly goes to outside

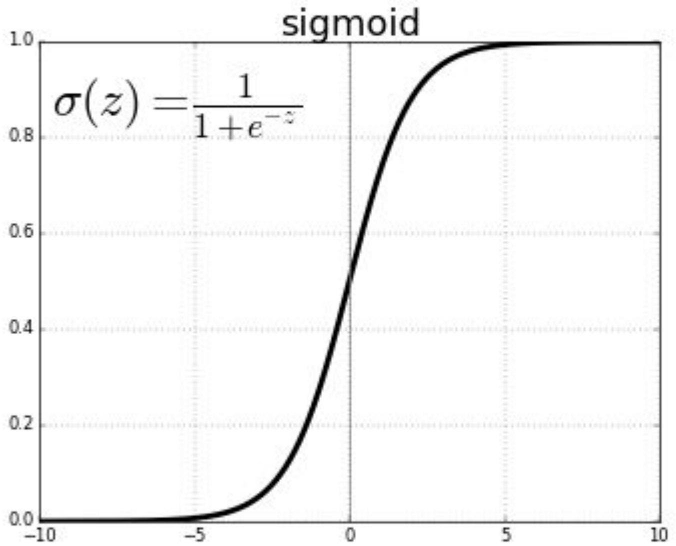


Feed Forward: Example

Suppose we have neural network. It has 2 inputs in the input layer, 1 hidden layer with 2 neurons, and 2 outputs in the output layer.

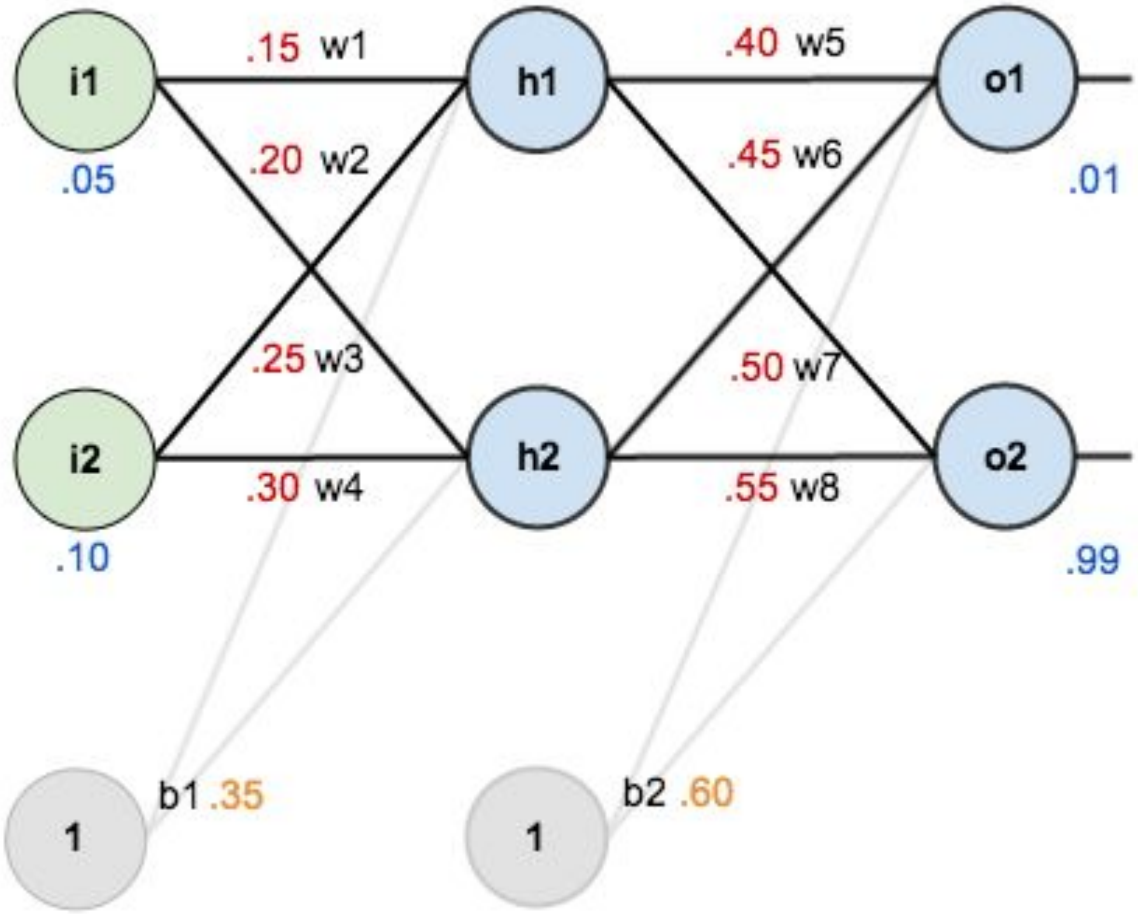
Suppose Activation function is sigmoid function (logistic function)





https://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/

Feed Forward: Example

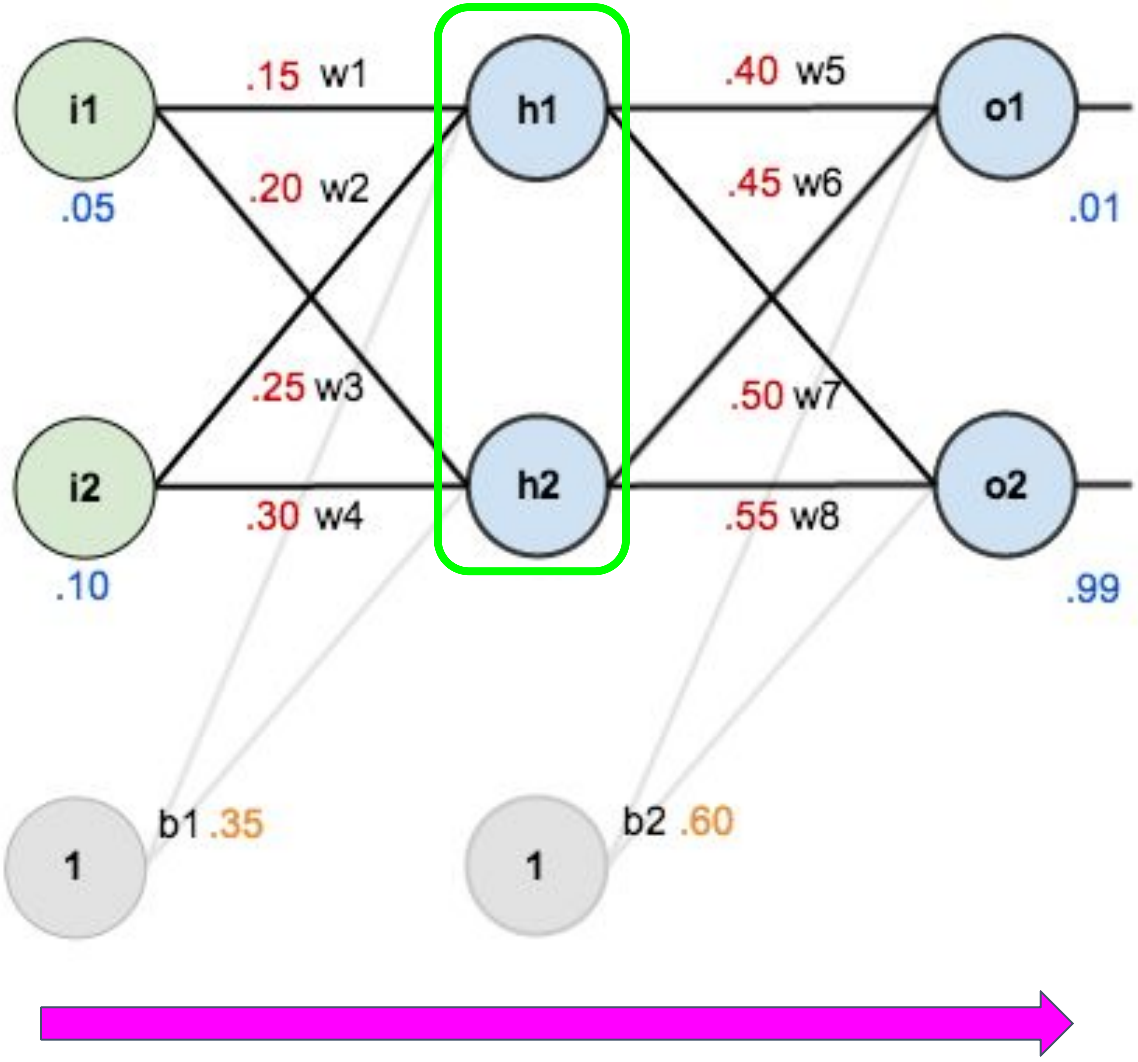


Step-1: Initialize weights/parameters

- In practice, set random initial weights
  - uniform distribution (0,1)
  - normal distribution N(0,1)

- Do not set initial weights too high (e.g., 100) or too low (e.g., all 0s), otherwise gradient descent is very slow, you may get poor solutions

Feed Forward: Example



Step-2: Forward pass to hidden layers

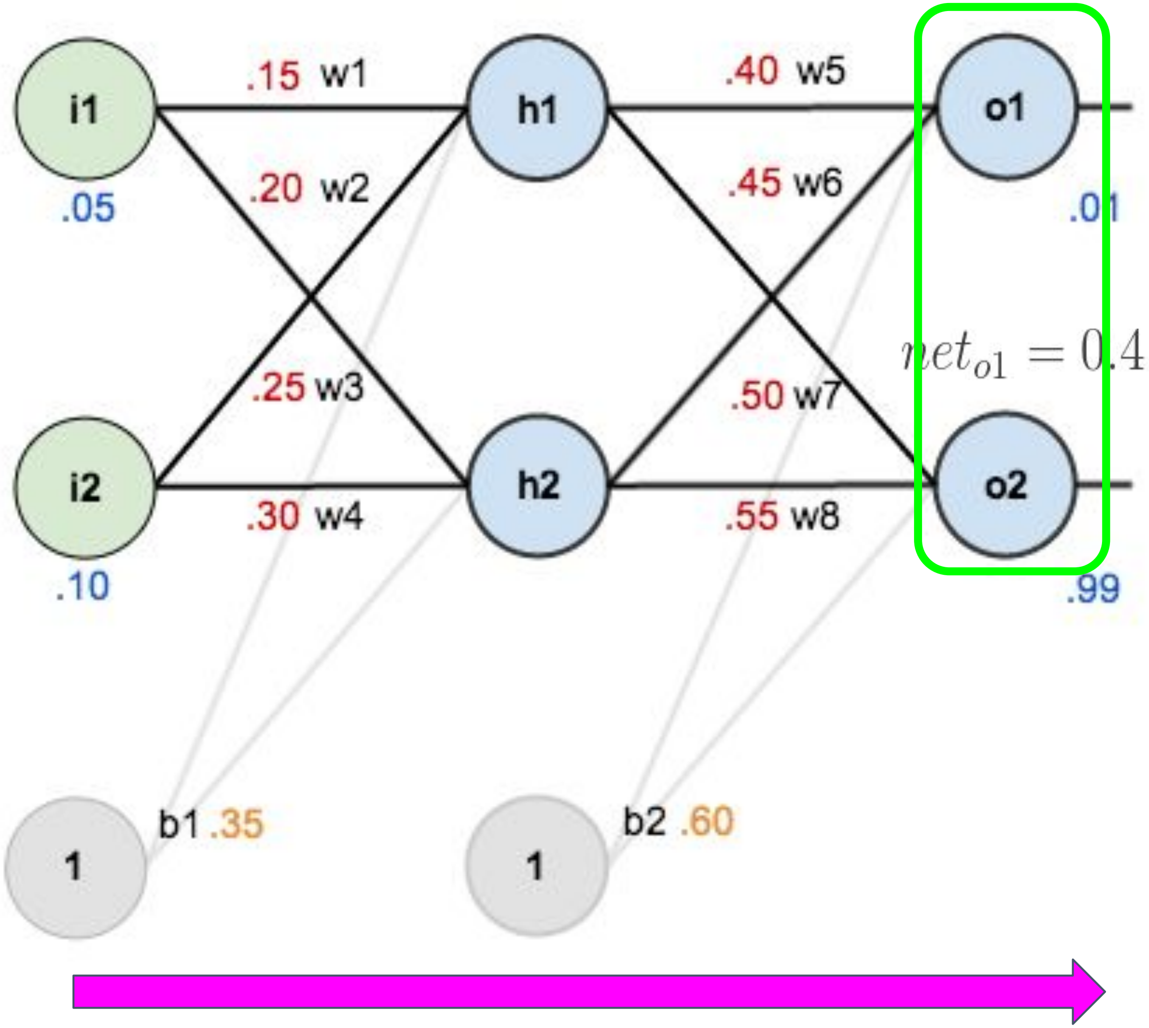
- Input layer: i1=0.05, i2=0.1
- Network input for neuron h1 in hidden layer:
- $$net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$
- $$net_{h1} = 0.15 * 0.05 + 0.2 * 0.1 + 0.35 * 1 = 0.3775$$
- Network output for neuron h1 in hidden layer:

$$out_{h1} = \frac{1}{1+e^{-net_{h1}}} = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

- Network output for neuron h2 in hidden layer:

$$out_{h2} = 0.596884378$$

Feed Forward: Example



Step-3: Forward pass to output layer

- Network input for neuron o1 in output layer:
- $net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$

$$net_{o1} = 0.4 * 0.593269992 + 0.45 * 0.596884378 + 0.6 * 1 = 1.105905967$$

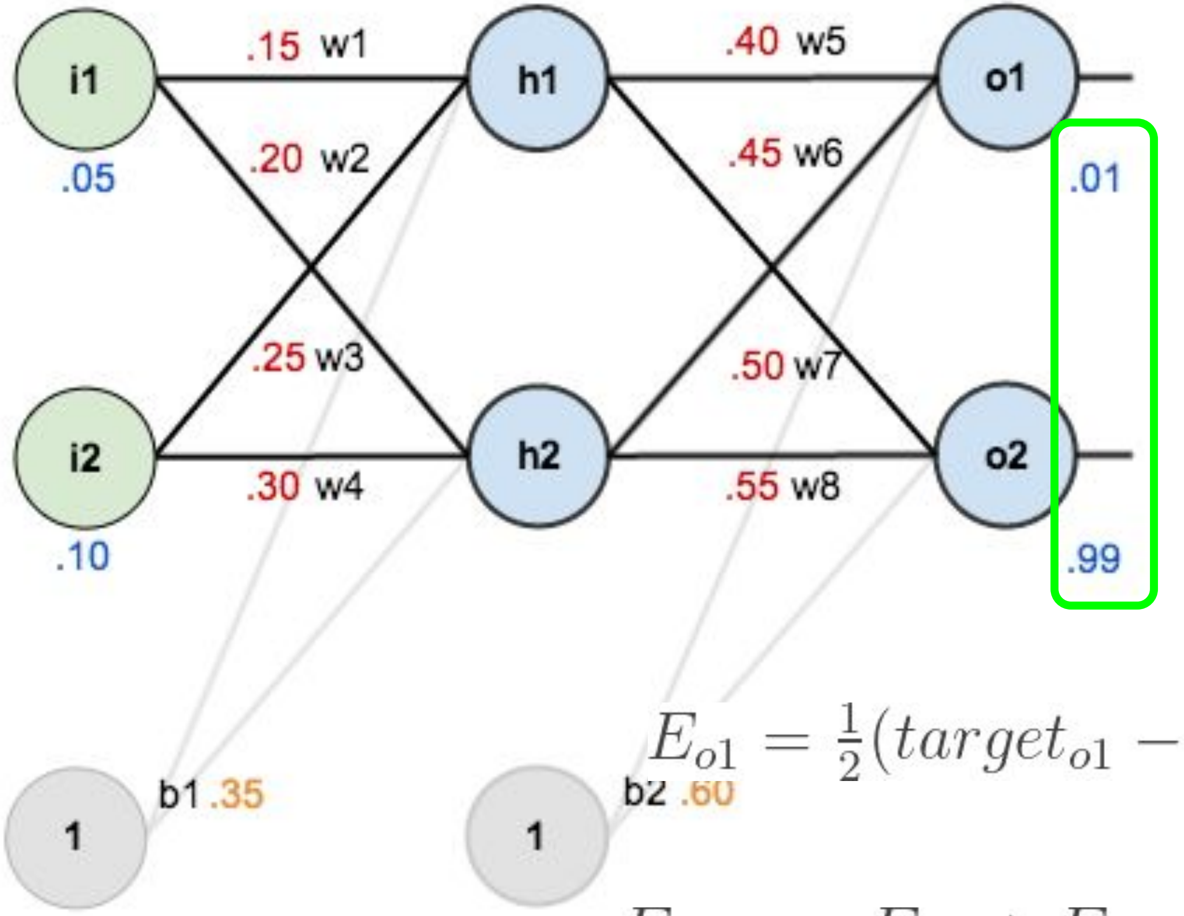
- Network output for neuron o1 in output layer:

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}} = \frac{1}{1+e^{-1.105905967}} = 0.75136507$$

- Network output for neuron o2 in output layer:

$$out_{o2} = 0.772928465$$

Feed Forward: Example



Step-4: Calculate errors

- Actual output for o1 is 0.01; Actual output for o2 is 0.99
- Predicted output for o1 is 0.75; Predicted output for o2 is 0.77
- We need to reduce errors
- Overall/Total error is:

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2 = \frac{1}{2}(0.01 - 0.75136507)^2 = 0.274811083$$

$$E_{o2} = 0.023560026$$

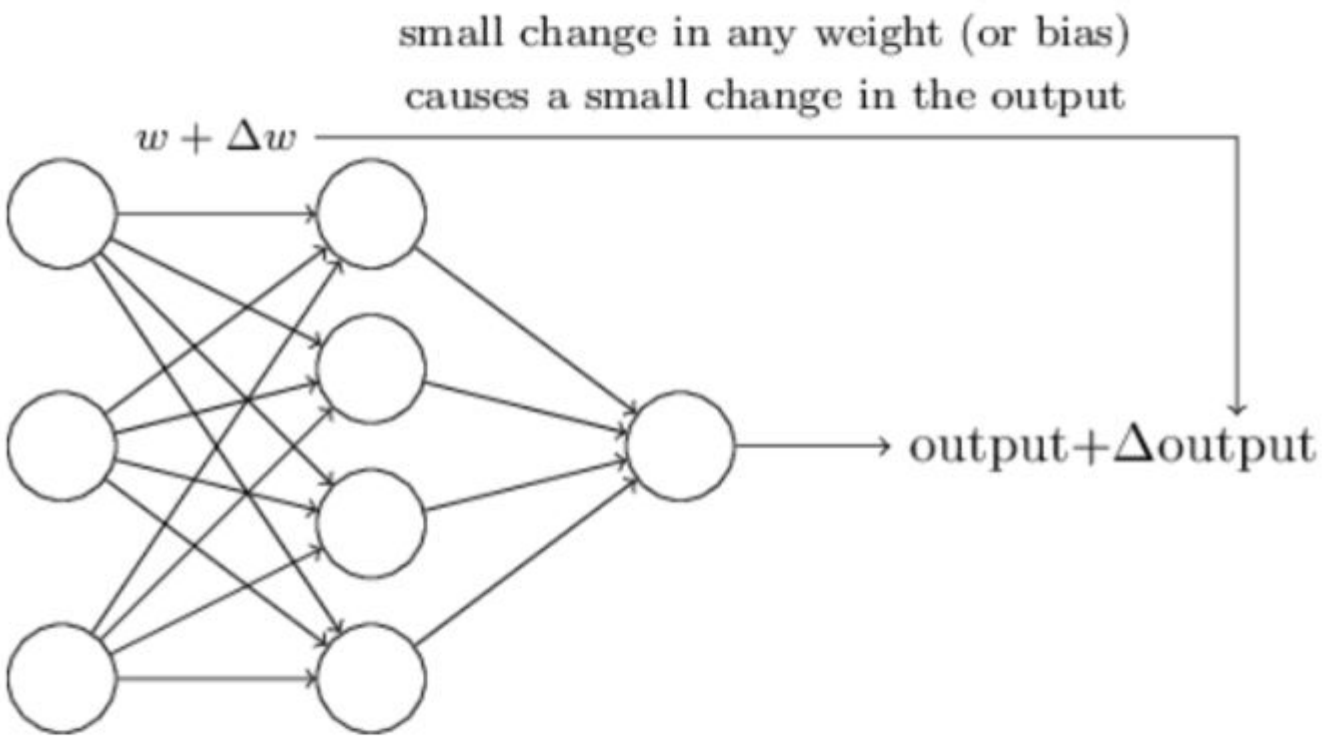
$$E_{total} = E_{o1} + E_{o2} = 0.274811083 + 0.023560026 = 0.298371109$$

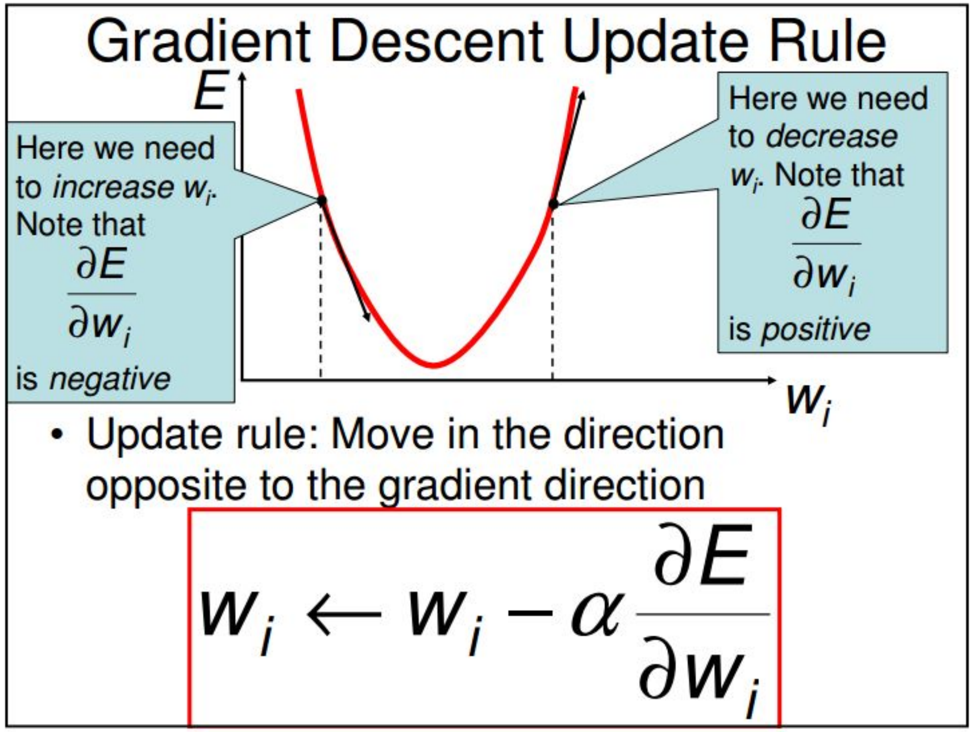
Backward Propagation (Backpropagation)

Why do we need backpropagation?

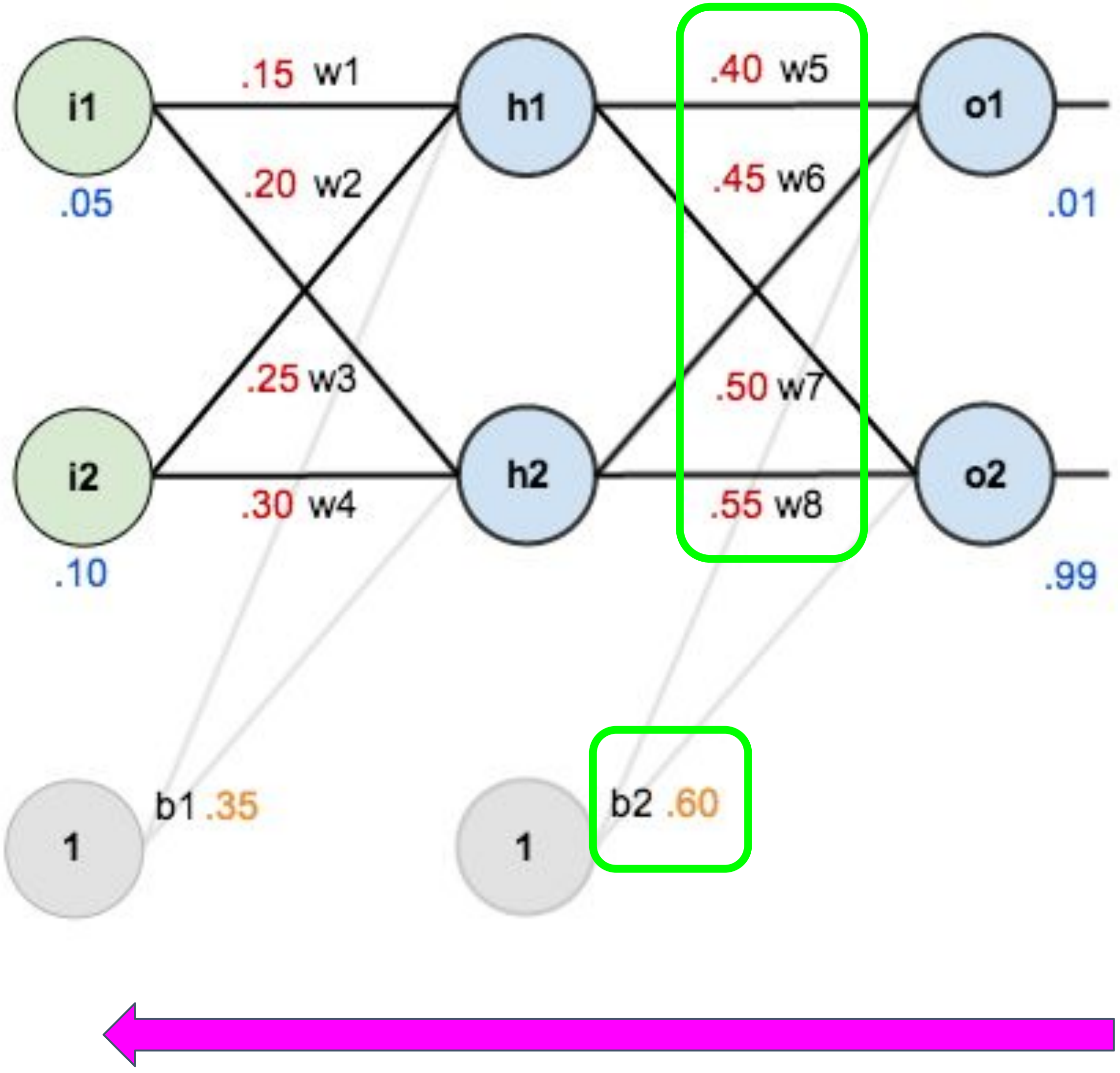
-Reduce overall/total errors after updating weights

-Go back to update weights/parameters (Gradient Descent)





Backpropagation: Example

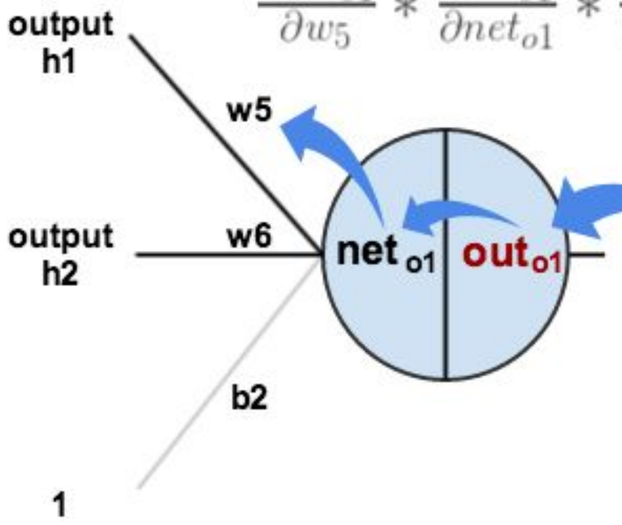


Step-1: Calculate derivatives on weights

- Gradient w5: Applying **Chain Rule**

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

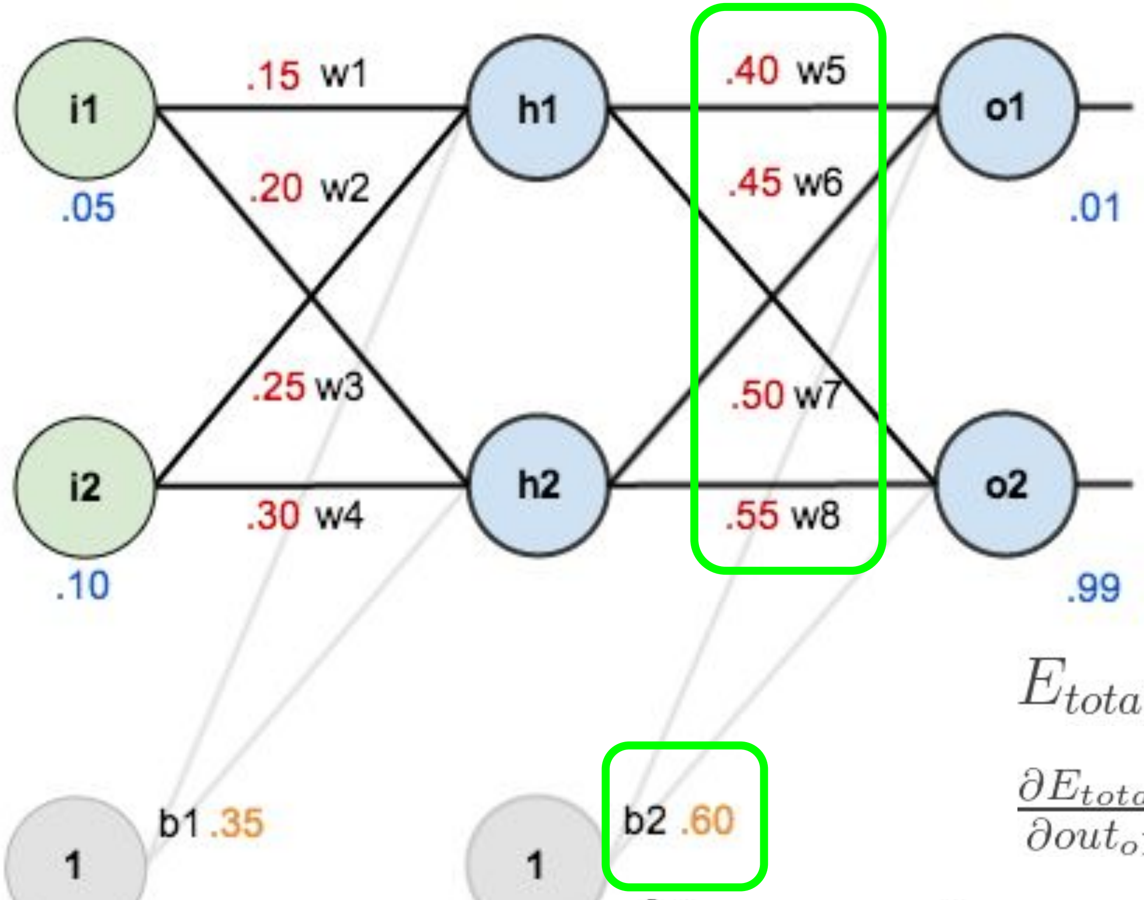
$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2$

$E_{total} = E_{o1} + E_{o2}$

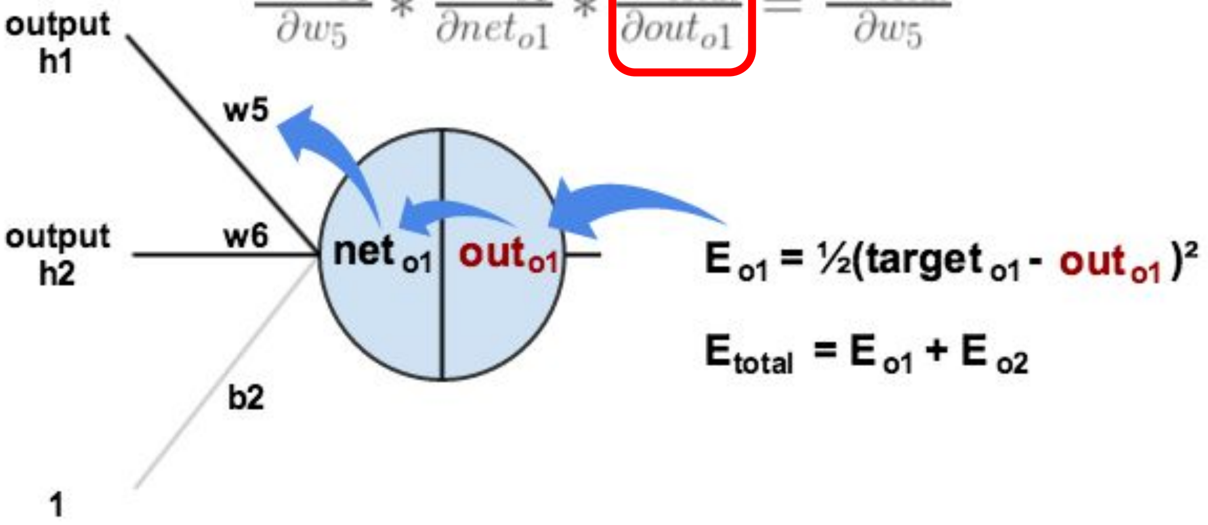
Backpropagation: Example



Step-1: Calculate derivatives on weights

- Gradient w5: Applying **Chain Rule**

$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \boxed{\frac{\partial E_{total}}{\partial out_{o1}}} = \frac{\partial E_{total}}{\partial w_5}$$

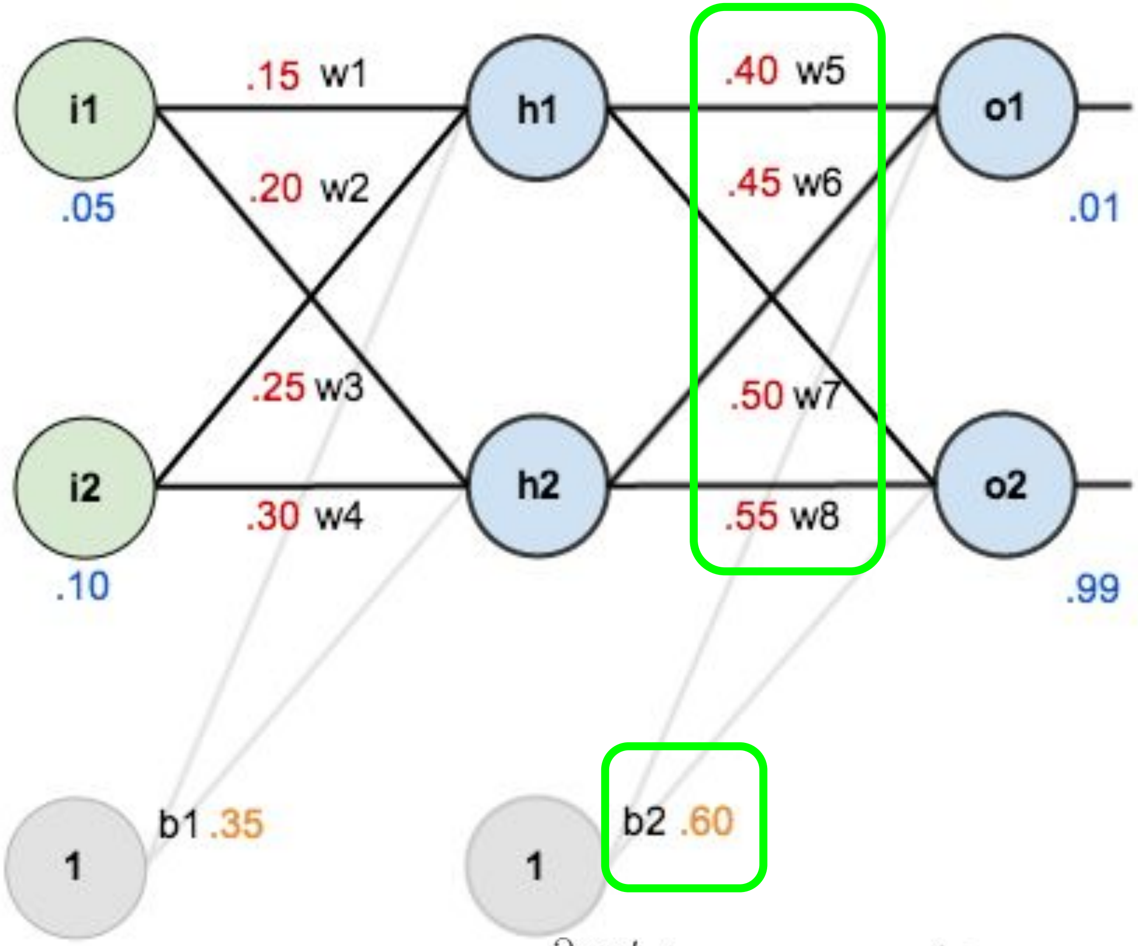


$$E_{total} = \frac{1}{2}(target_{o1} - out_{o1})^2 + \frac{1}{2}(target_{o2} - out_{o2})^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1}) = -(0.01 - 0.75136507) = 0.74136507$$

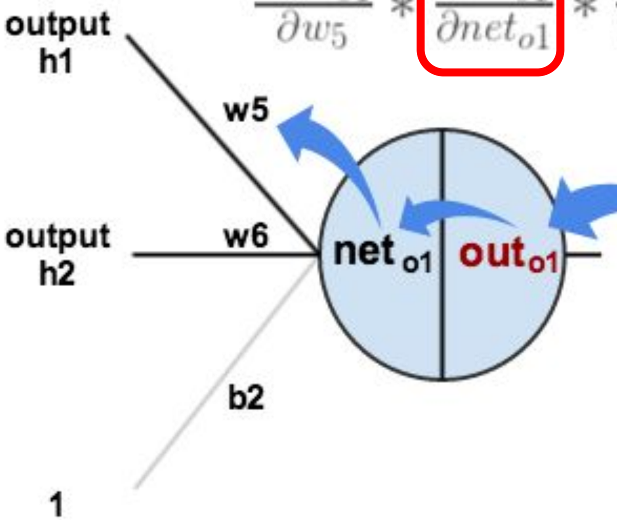
Backpropagation: Example



Step-1: Calculate derivatives on weights

- Gradient w5: Applying **Chain Rule**

$$\frac{\partial net_{o1}}{\partial w_5} * \boxed{\frac{\partial out_{o1}}{\partial net_{o1}}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



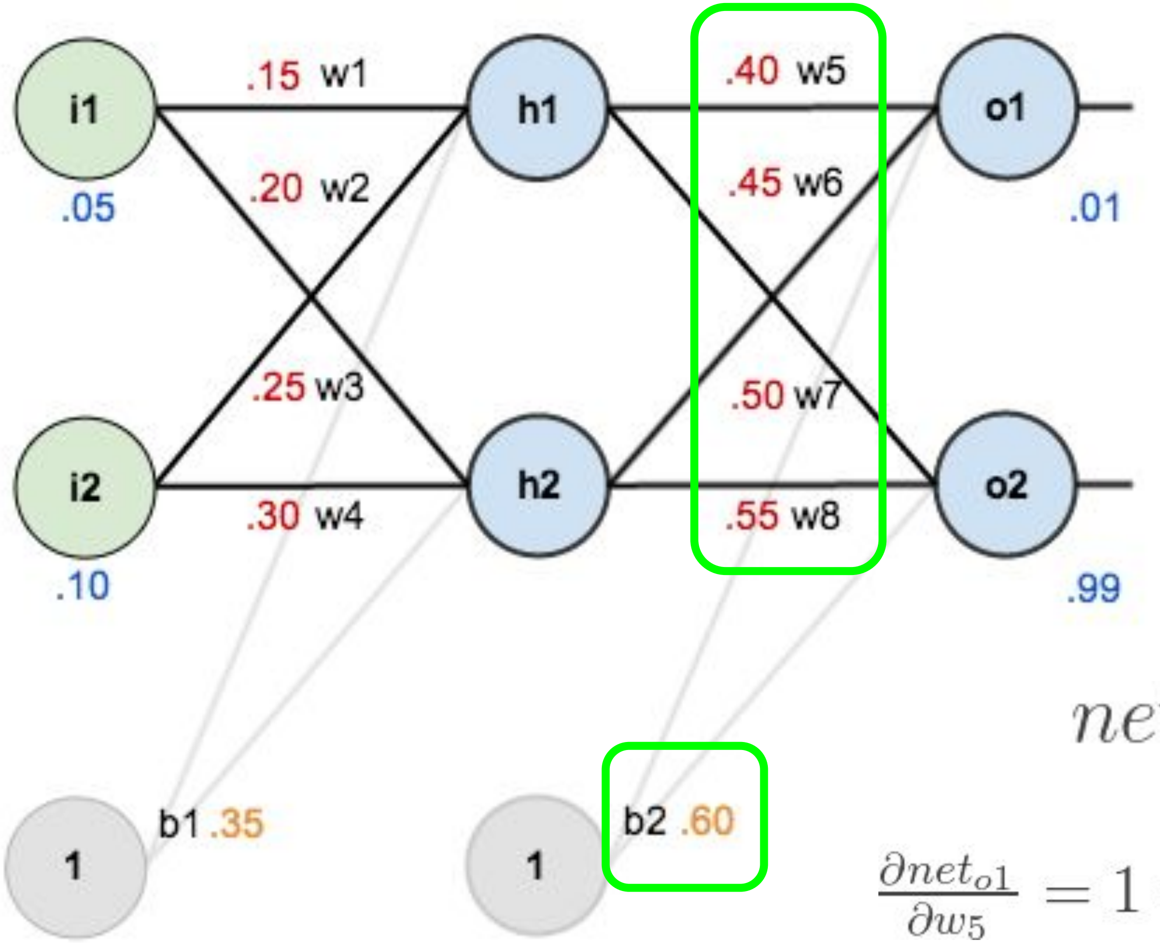
$E_{o1} = ½(target_{o1} - out_{o1})^2$

$E_{total} = E_{o1} + E_{o2}$

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}}$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1}) = 0.75136507(1 - 0.75136507) = 0.186815602$$

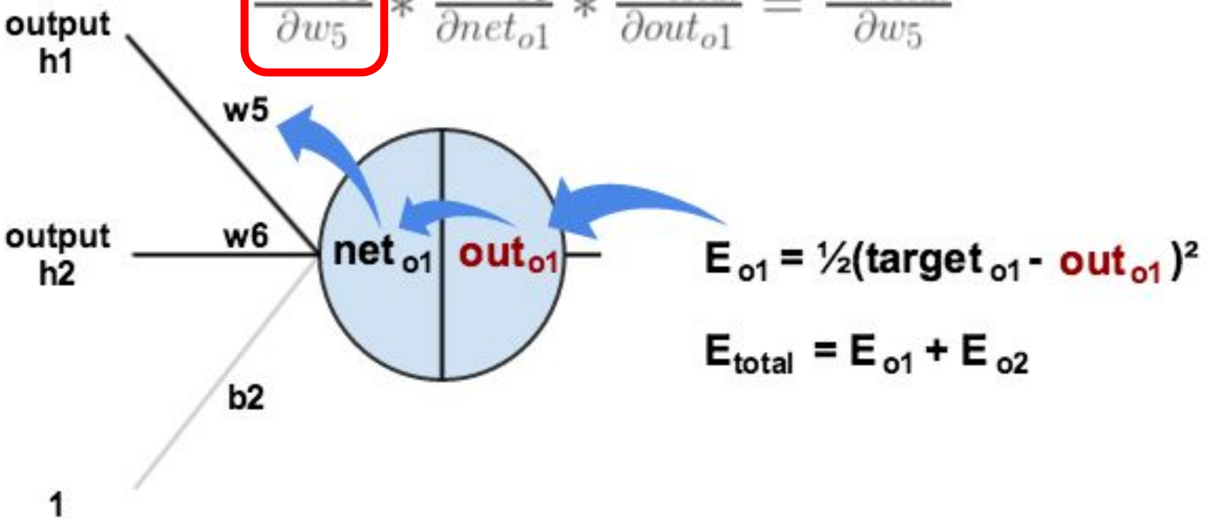
Backpropagation: Example



Step-1: Calculate derivatives on weights

- Gradient w5: Applying **Chain Rule**

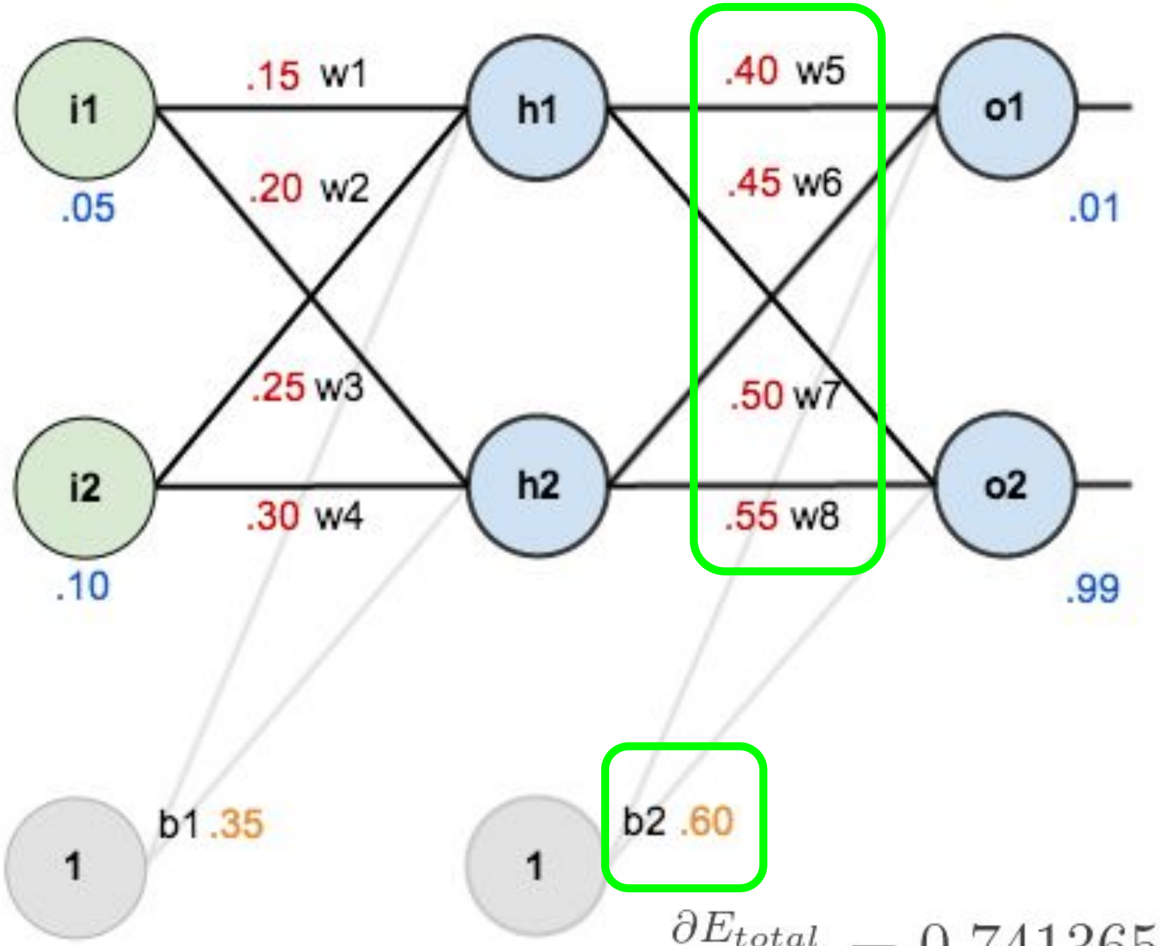
$$\boxed{\frac{\partial net_{o1}}{\partial w_5}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \frac{\partial E_{total}}{\partial w_5}$$



$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial w_5} = 1 * out_{h1} * w_5^{(1-1)} + 0 + 0 = out_{h1} = 0.593269992$$

Backpropagation: Example



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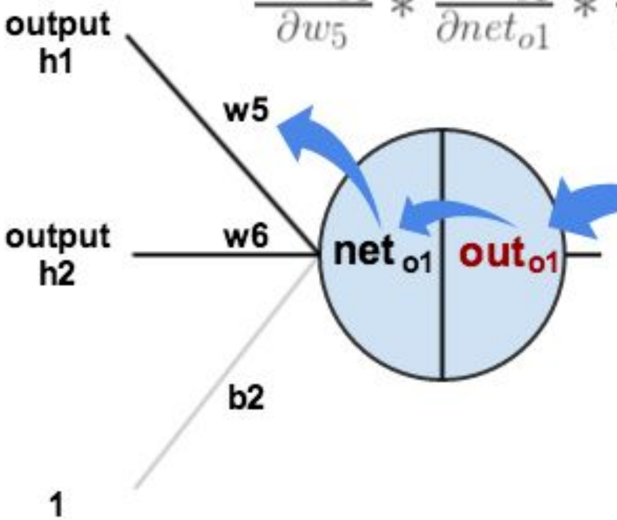


$$\frac{\partial net_{o1}}{\partial w_5} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial E_{total}}{\partial out_{o1}} = \boxed{\frac{\partial E_{total}}{\partial w_5}}$$

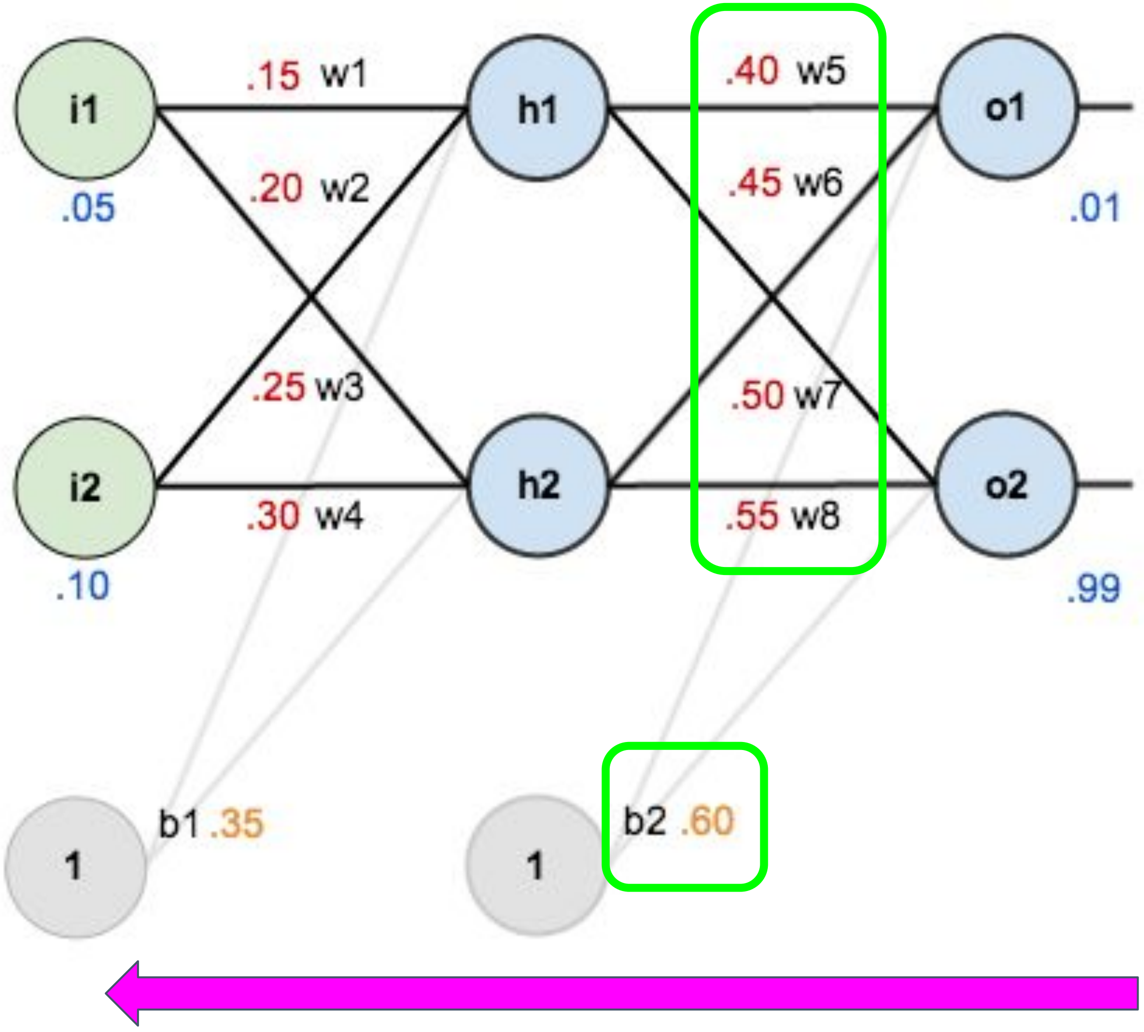
$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2$

$E_{total} = E_{o1} + E_{o2}$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

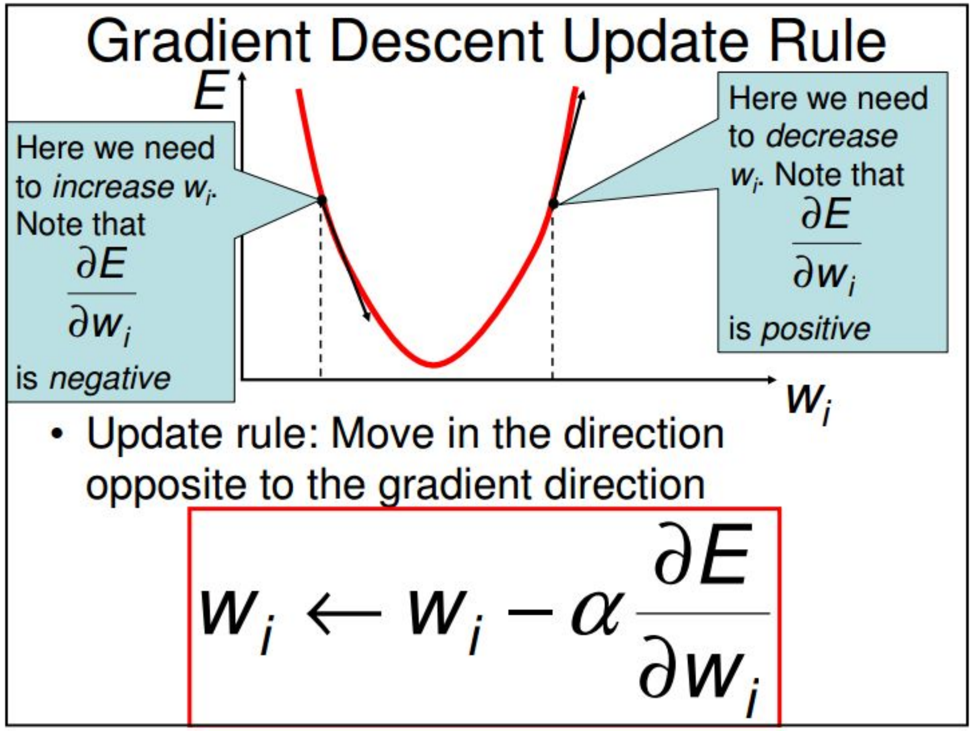
$$\frac{\partial E_{total}}{\partial w_5} = 0.74136507 * 0.186815602 * 0.593269992 = 0.082167041$$

Backpropagation: Example



Step-2: Update weights

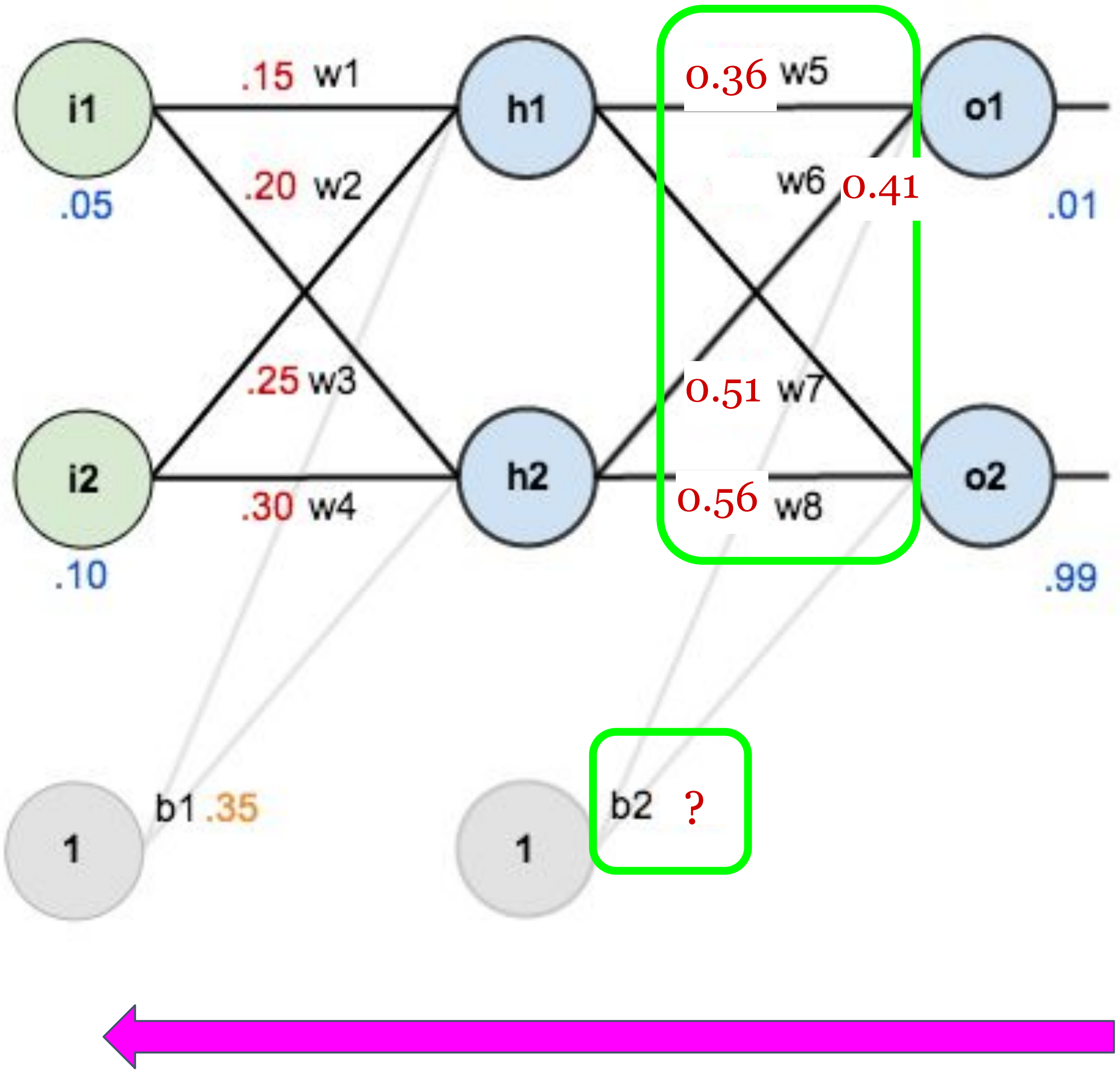
- Update w5: **Gradient Descent**
- Remember Tutorial 2:



- $w5 \leftarrow w5$ - stepsize $\times(\partial E_{total}/\partial w5)$

$$w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 * 0.082167041 = 0.35891648$$

Backpropagation: Example



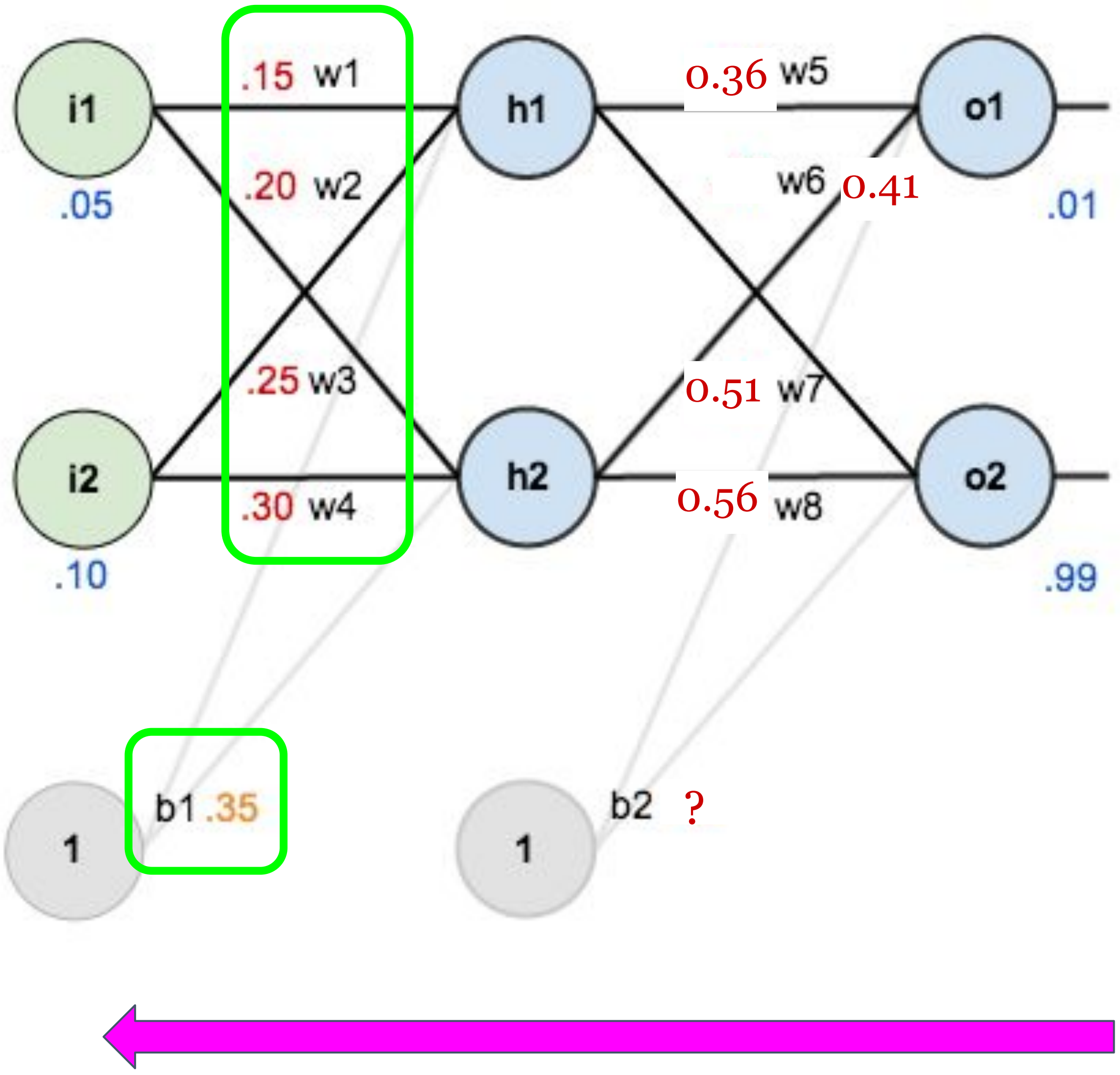
Step-2: Update weights

- Update w6, w7, w8, b2: **Gradient Descent**

$$w_6^+ = 0.408666186$$
$$w_7^+ = 0.511301270$$
$$w_8^+ = 0.561370121$$

b2: You can try to update b2 by yourself

Backpropagation: Example

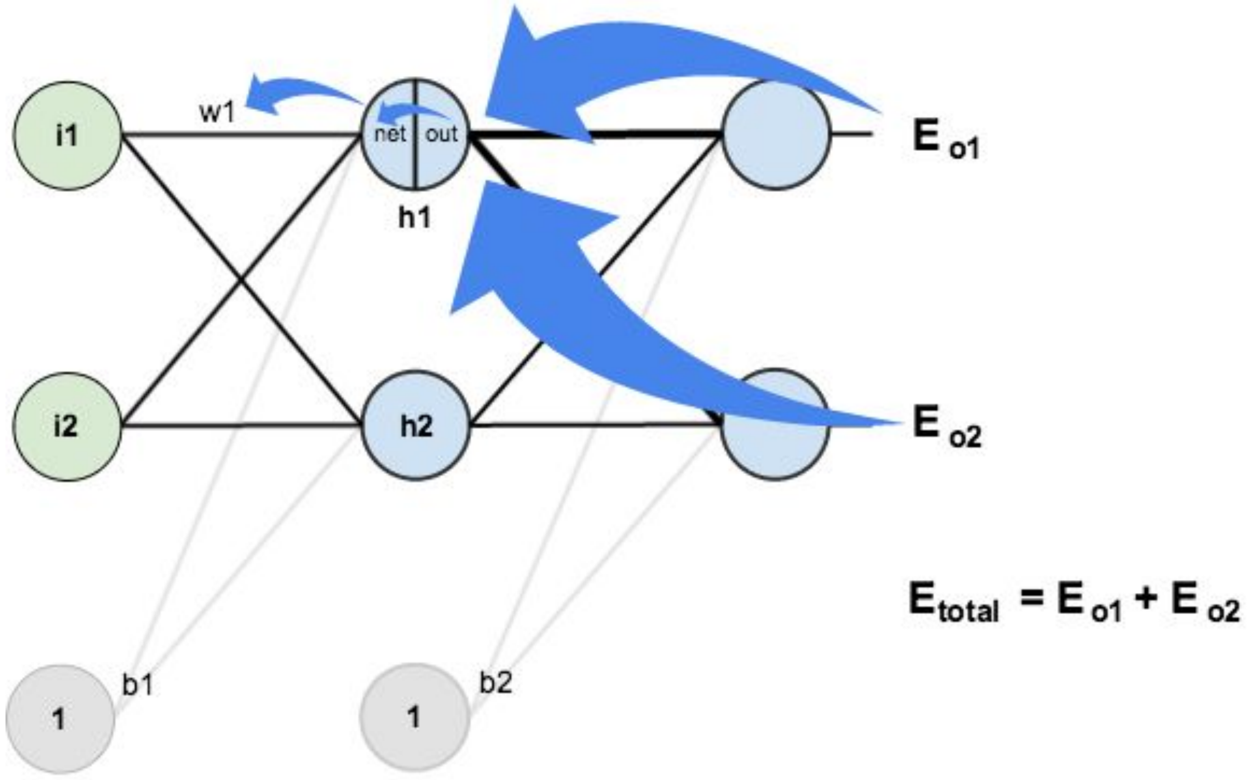


Step-3: Repeat updating weights

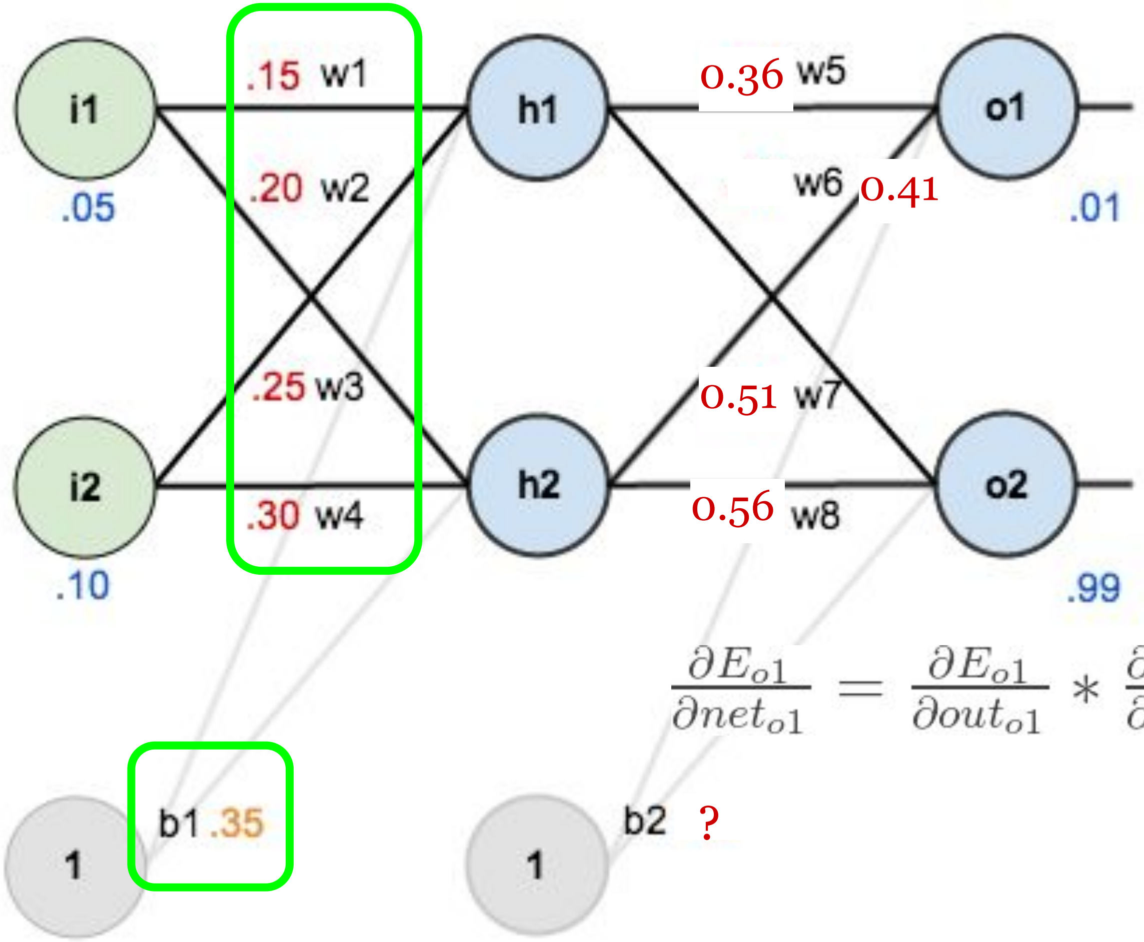
- Gradient w1: **Chain Rule**

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$



Backpropagation: Example



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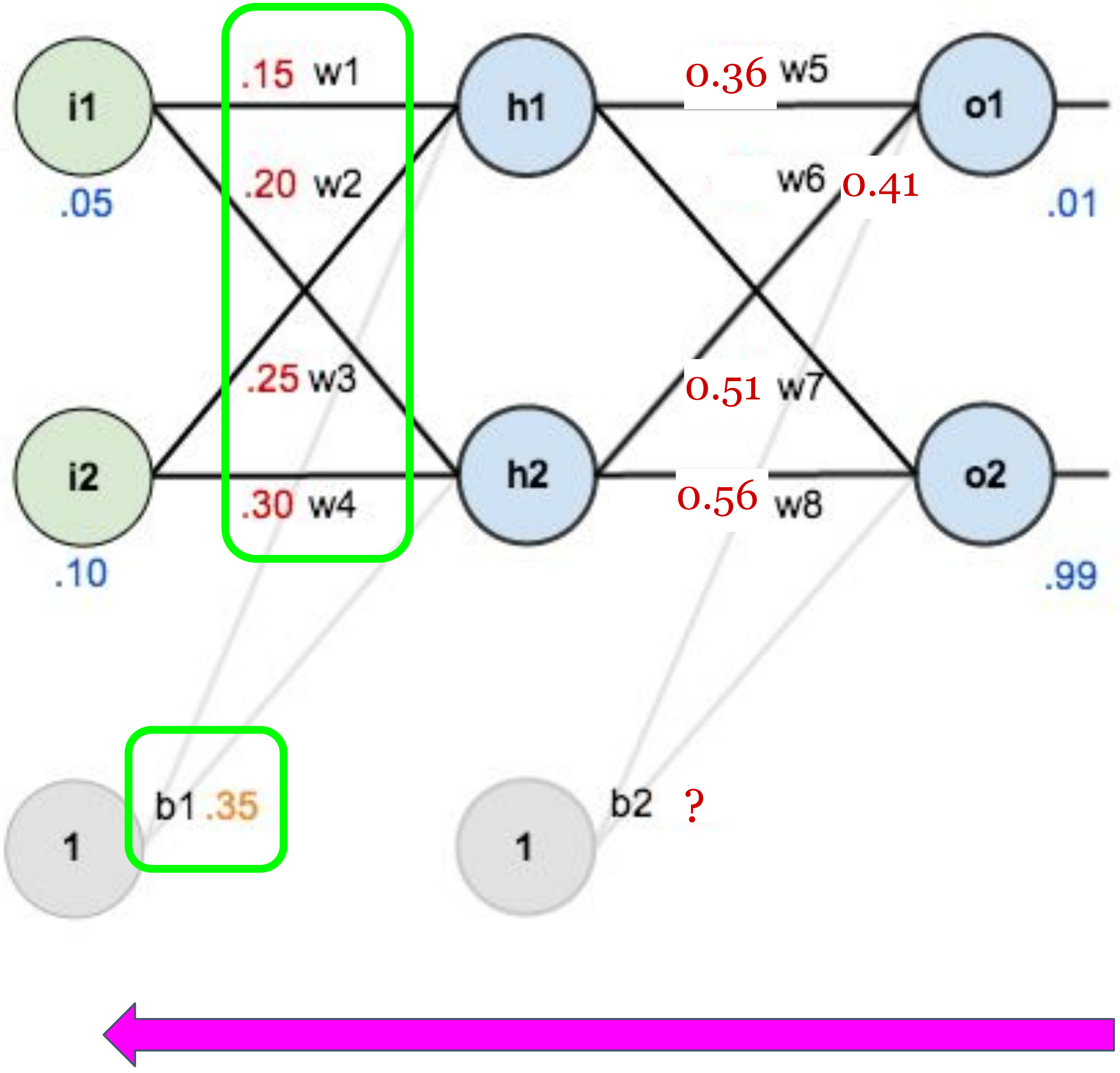
$$\frac{\partial E_{o1}}{\partial out_{h1}} = \boxed{\frac{\partial E_{o1}}{\partial net_{o1}}} * \frac{\partial net_{o1}}{\partial out_{h1}}$$

$$\frac{\partial E_{o1}}{\partial net_{o1}} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} = 0.74136507 * 0.186815602 = 0.138498562$$

Backpropagation: Example



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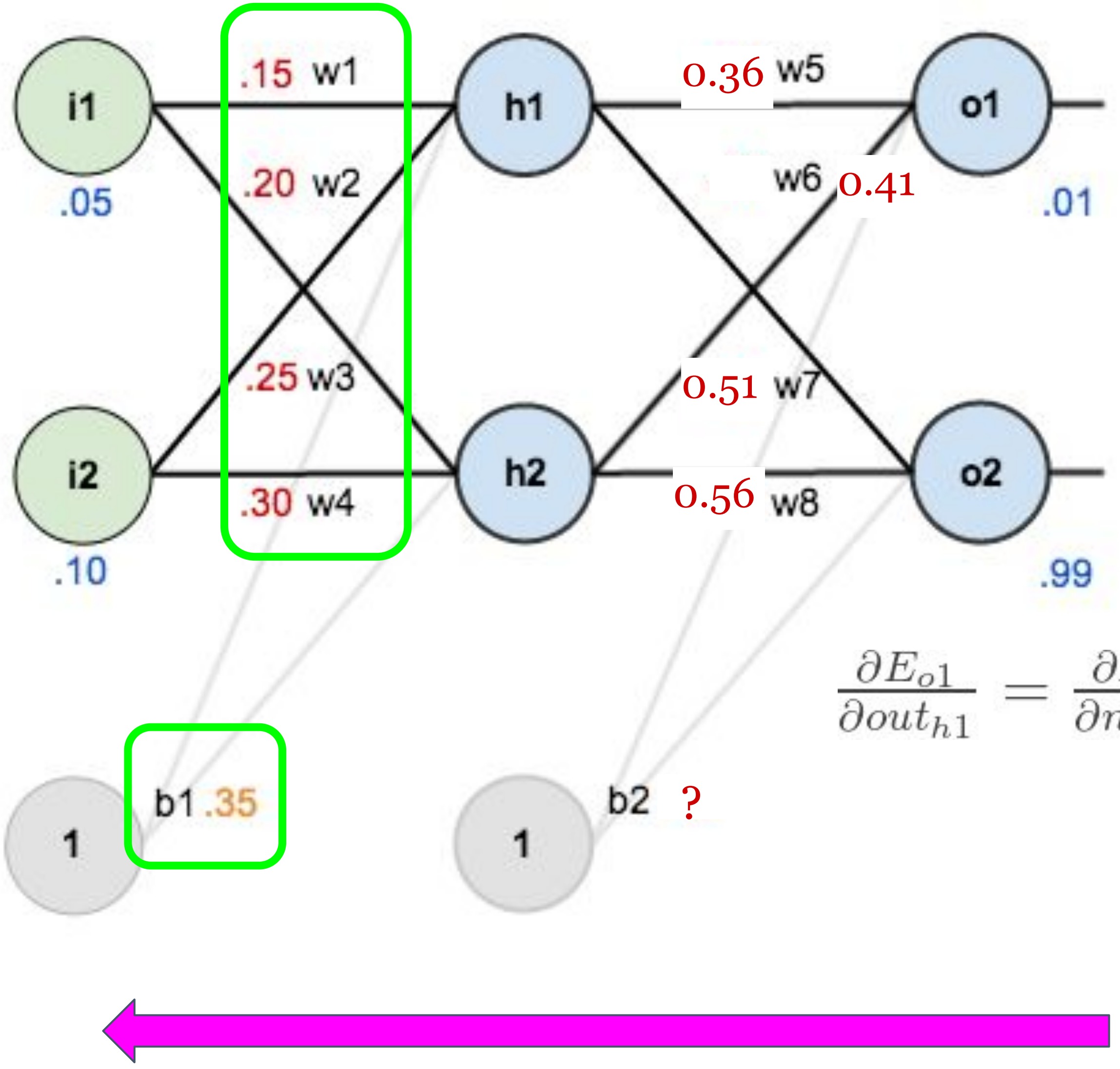
$$\frac{\partial E_{total}}{\partial out_{h1}} = \boxed{\frac{\partial E_{o1}}{\partial out_{h1}}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \boxed{\frac{\partial net_{o1}}{\partial out_{h1}}}$$

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$

Backpropagation: Example



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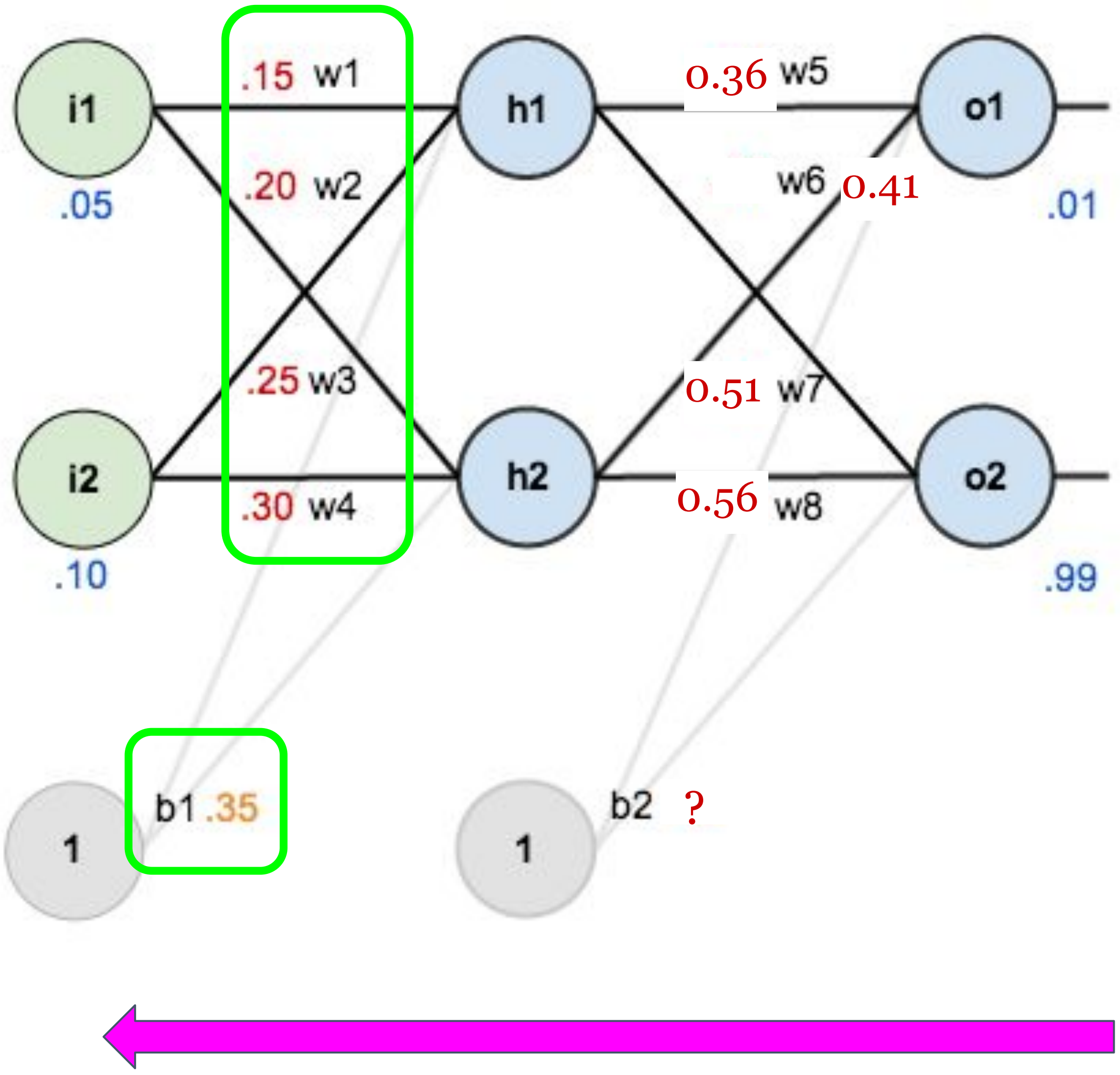
$$\frac{\partial E_{total}}{\partial out_{h1}} = \boxed{\frac{\partial E_{o1}}{\partial out_{h1}}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$\boxed{\frac{\partial E_{o1}}{\partial out_{h1}}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}}$$

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}} = 0.138498562 * 0.40 = 0.055399425$$

Backpropagation: Example



Step-3: Repeat updating weights

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$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \boxed{\frac{\partial E_{o2}}{\partial out_{h1}}}$$

$$\frac{\partial E_{o2}}{\partial out_{h1}} = -0.019049119$$

Backpropagation: Example

Step-3: Repeat updating weights

- Gradient w1: **Chain Rule**

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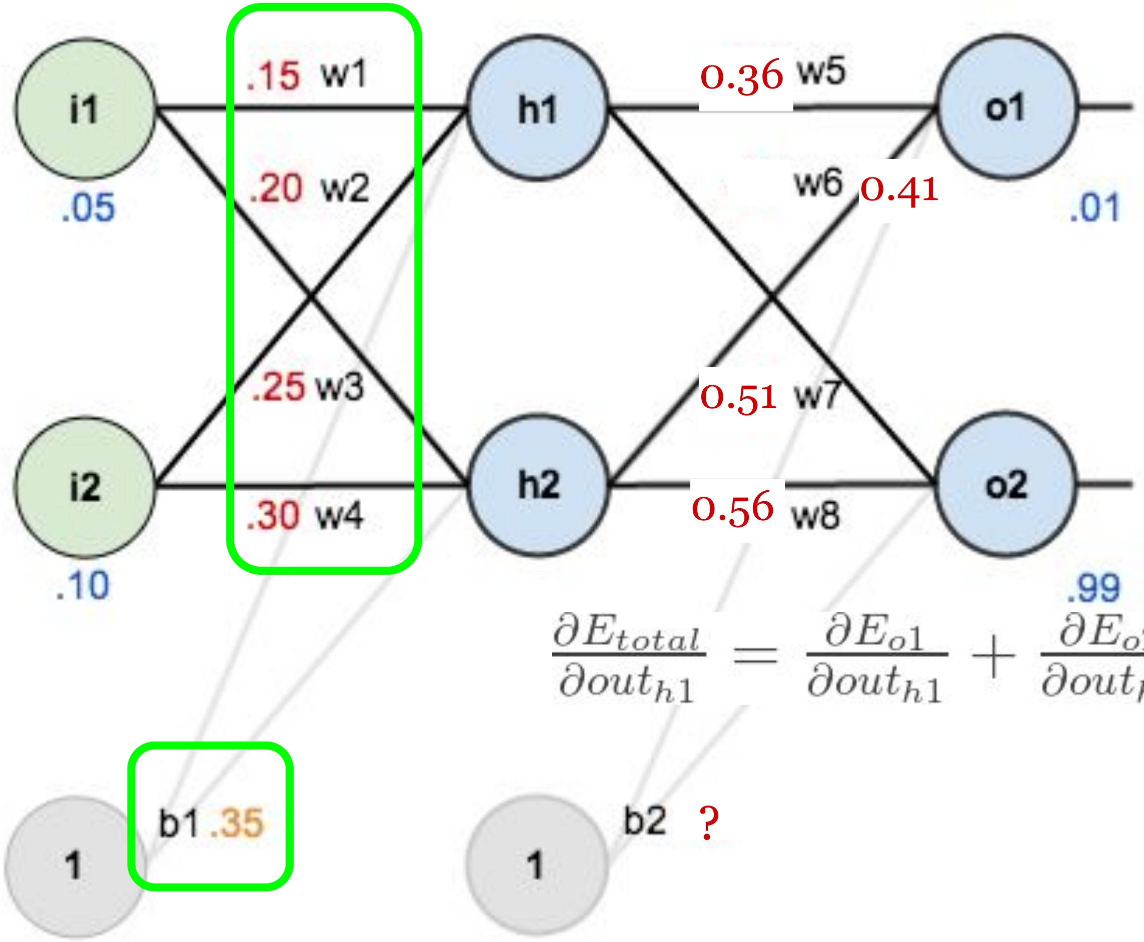
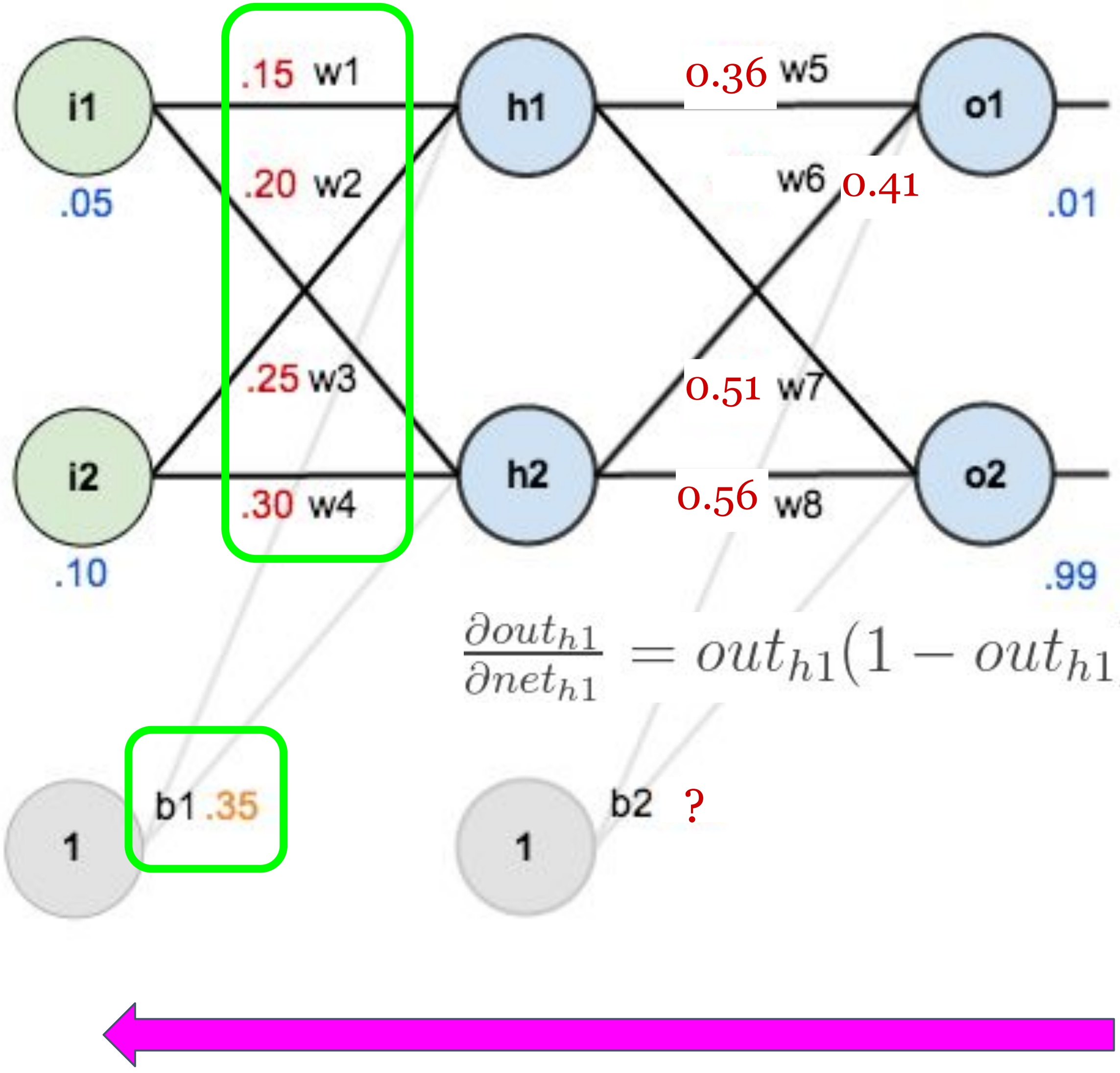


$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}} = 0.055399425 + -0.019049119 = 0.036350306$$

Backpropagation: Example



Step-3: Repeat updating weights

- Gradient w1: **Chain Rule**

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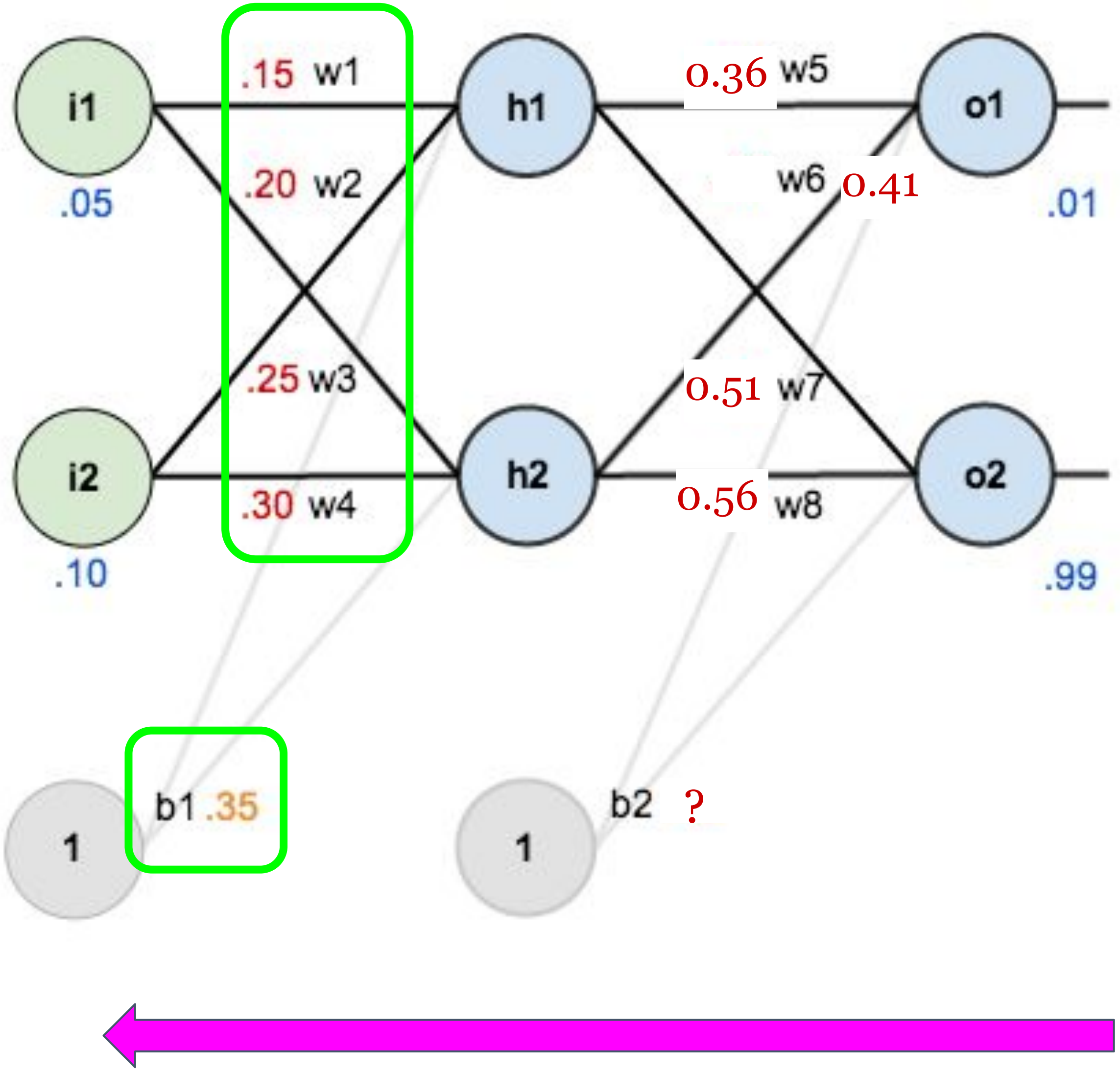
$$out_{h1} = \frac{1}{1+e^{-net_{h1}}}$$

$$\frac{\partial out_{h1}}{\partial net_{h1}} = out_{h1}(1 - out_{h1}) = 0.59326999(1 - 0.59326999) = 0.241300709$$

Backpropagation: Example



Step-3: Repeat updating weights

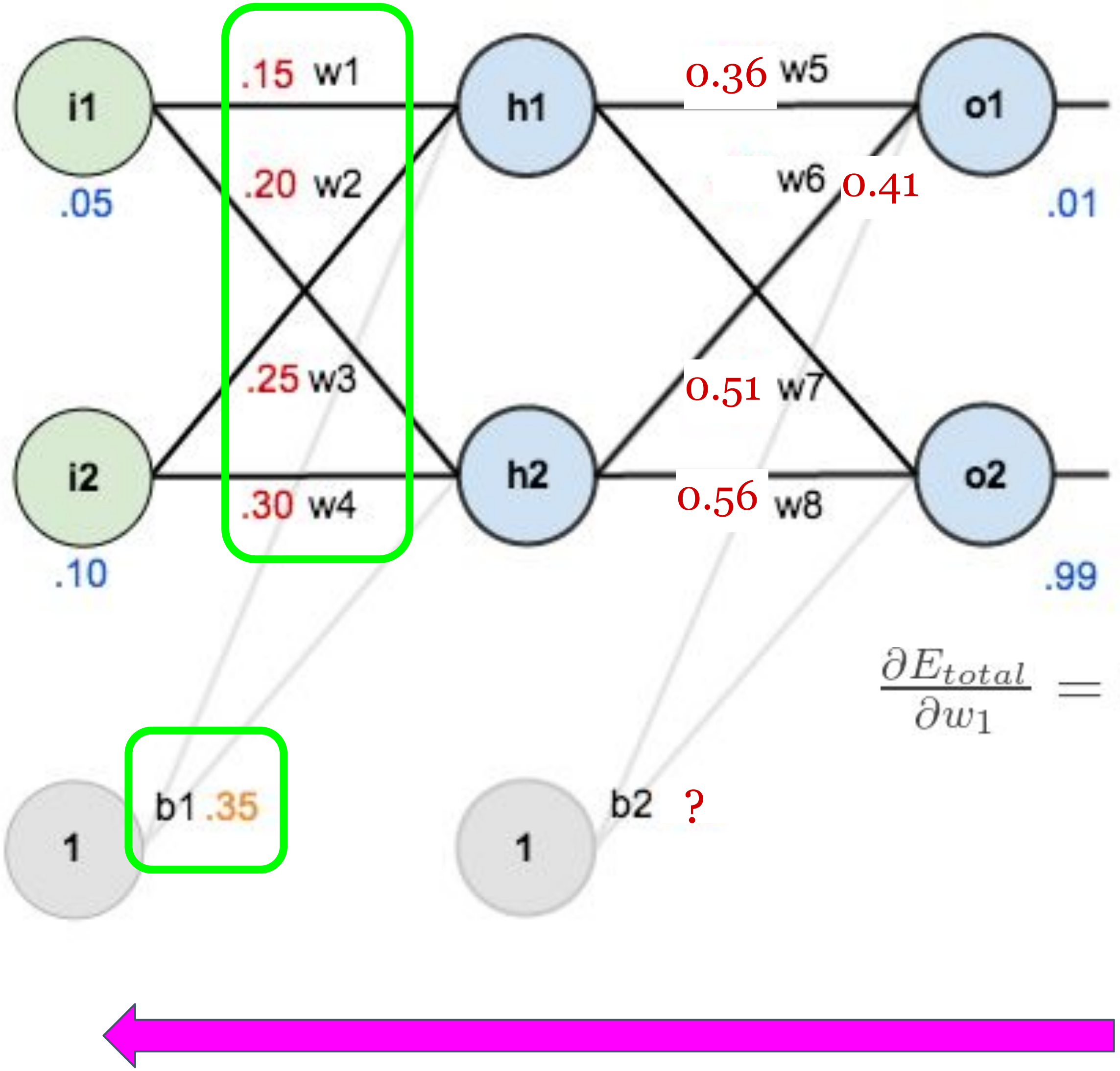
- Gradient w1: **Chain Rule**

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \boxed{\frac{\partial net_{h1}}{\partial w_1}}$$

$$net_{h1} = w_1 * i_1 + w_3 * i_2 + b_1 * 1$$

$$\frac{\partial net_{h1}}{\partial w_1} = i_1 = 0.05$$

Backpropagation: Example



Step-3: Repeat updating weights

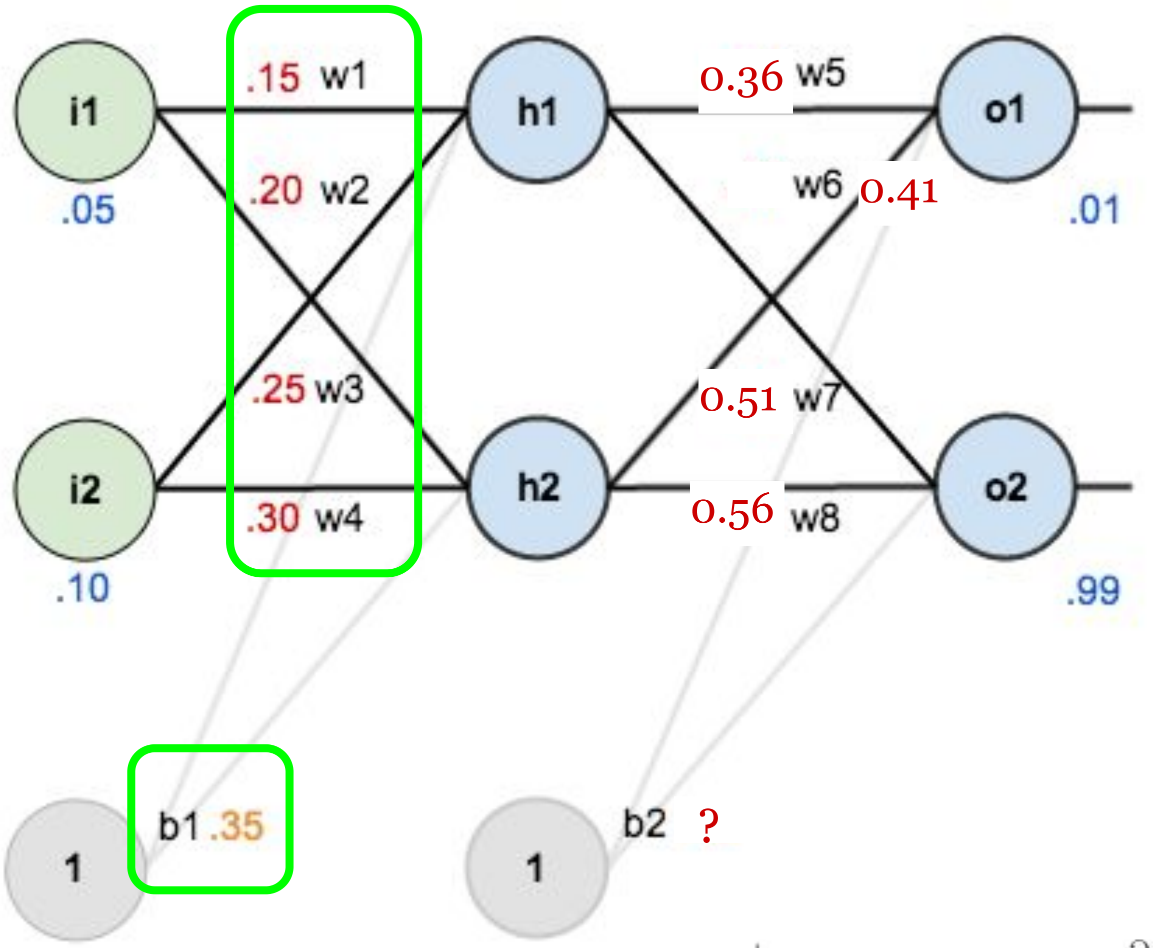
- Gradient w1: **Chain Rule**

$$\boxed{\frac{\partial E_{total}}{\partial w_1}} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

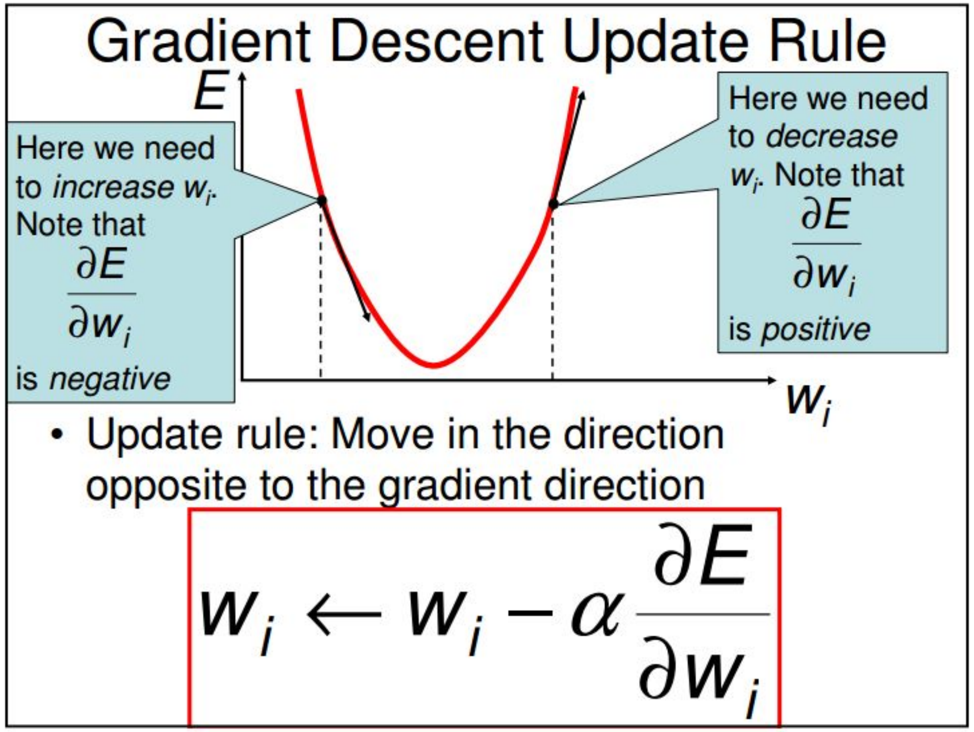
$$\frac{\partial E_{total}}{\partial w_1} = 0.036350306 * 0.241300709 * 0.05 = 0.000438568$$

Backpropagation: Example



Step-3: Repeat updating weights

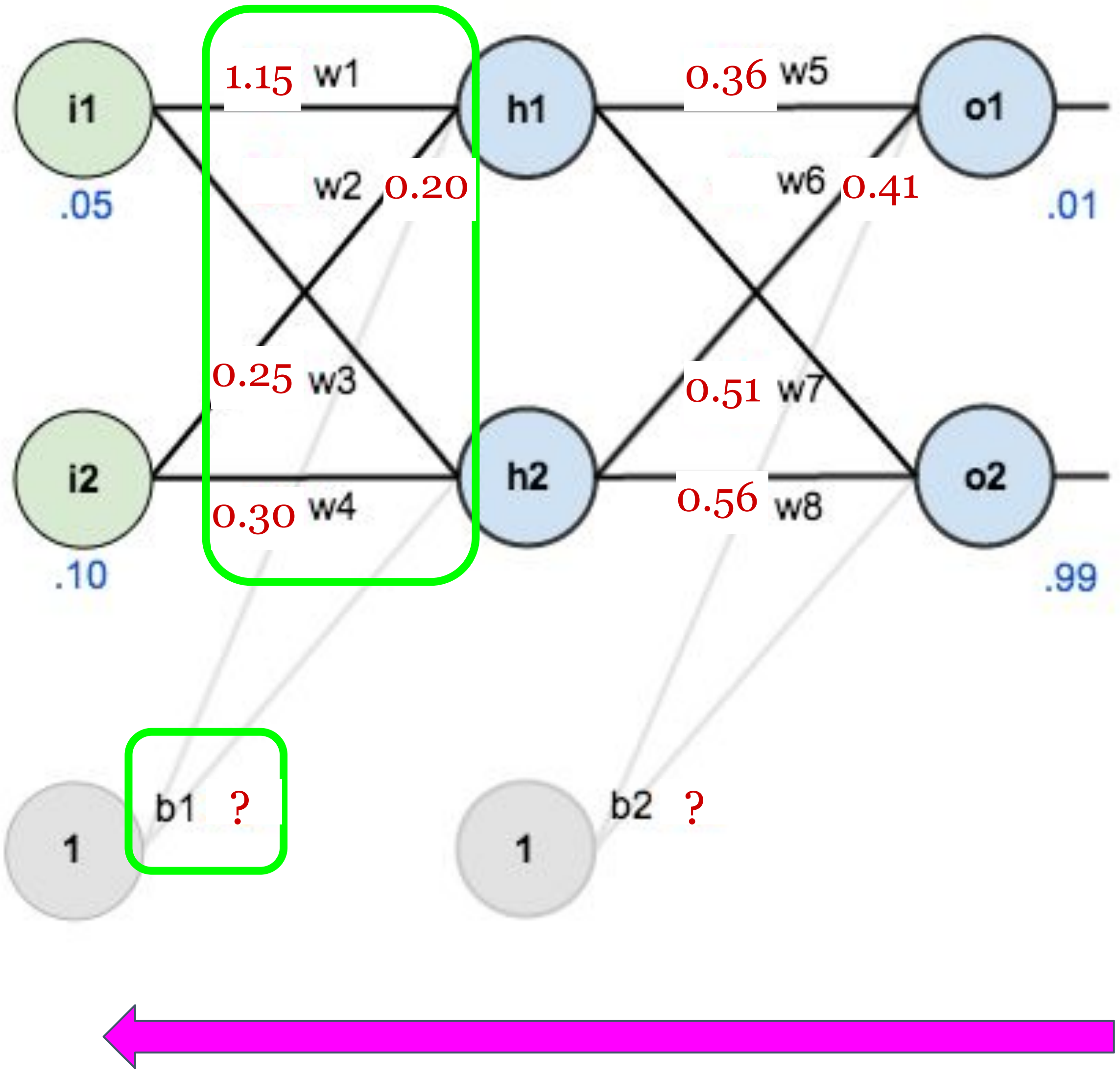
- Update w1: **Gradient Descent**



- w1 ← w1 - stepsize ×(∂E$_{total}$/∂w1)

$$w_1^+ = w_1 - \eta * \frac{\partial E_{total}}{\partial w_1} = 0.15 - 0.5 * 0.000438568 = 0.149780716$$

Backpropagation: Example



Step-3: Repeat updating weights

- Update w2, w3, w4, b1: **Gradient Descent**

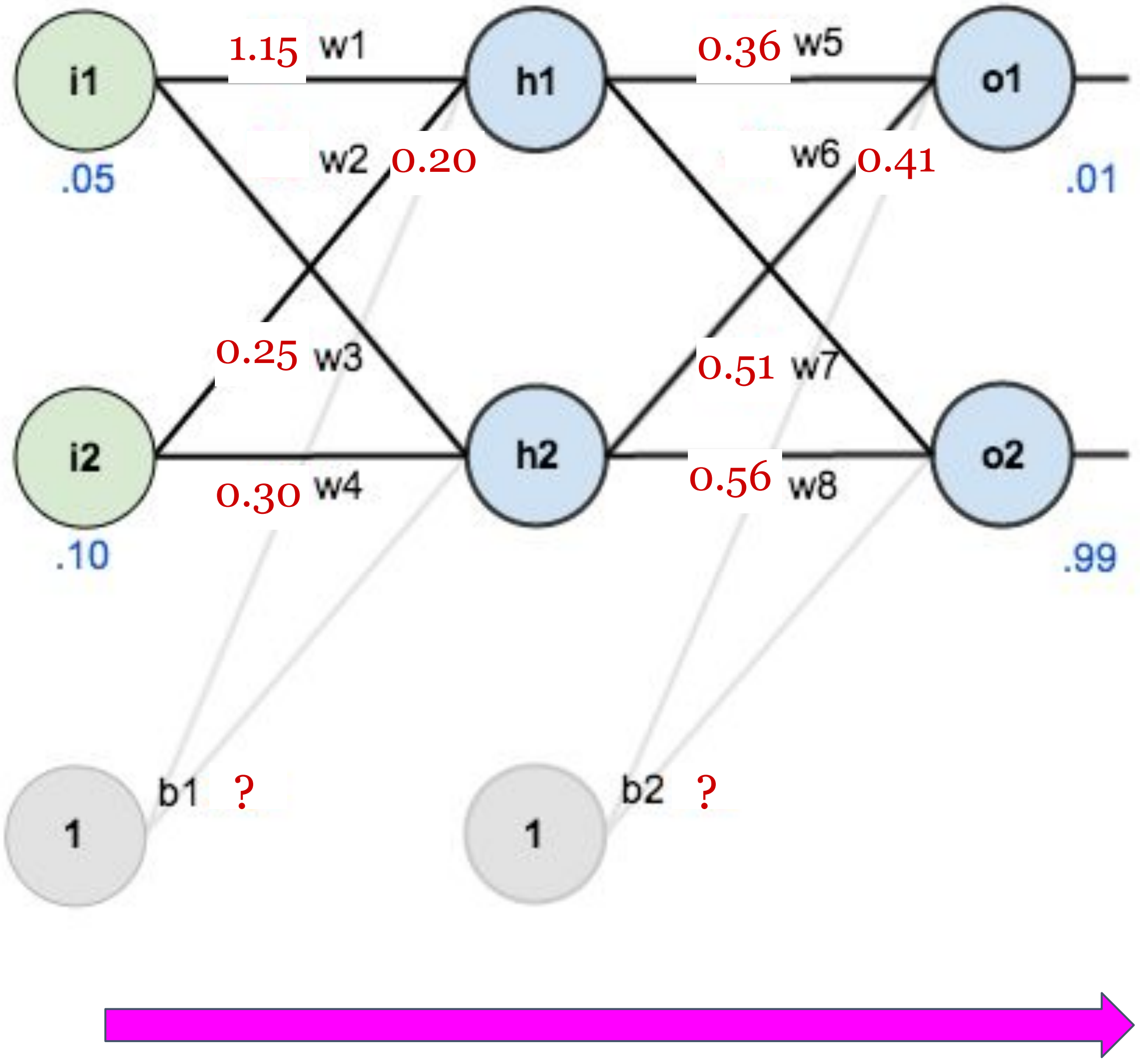
$$w_2^+ = 0.19956143$$

$$w_3^+ = 0.24975114$$

$$w_4^+ = 0.29950229$$

b1: You can try to update it by yourself

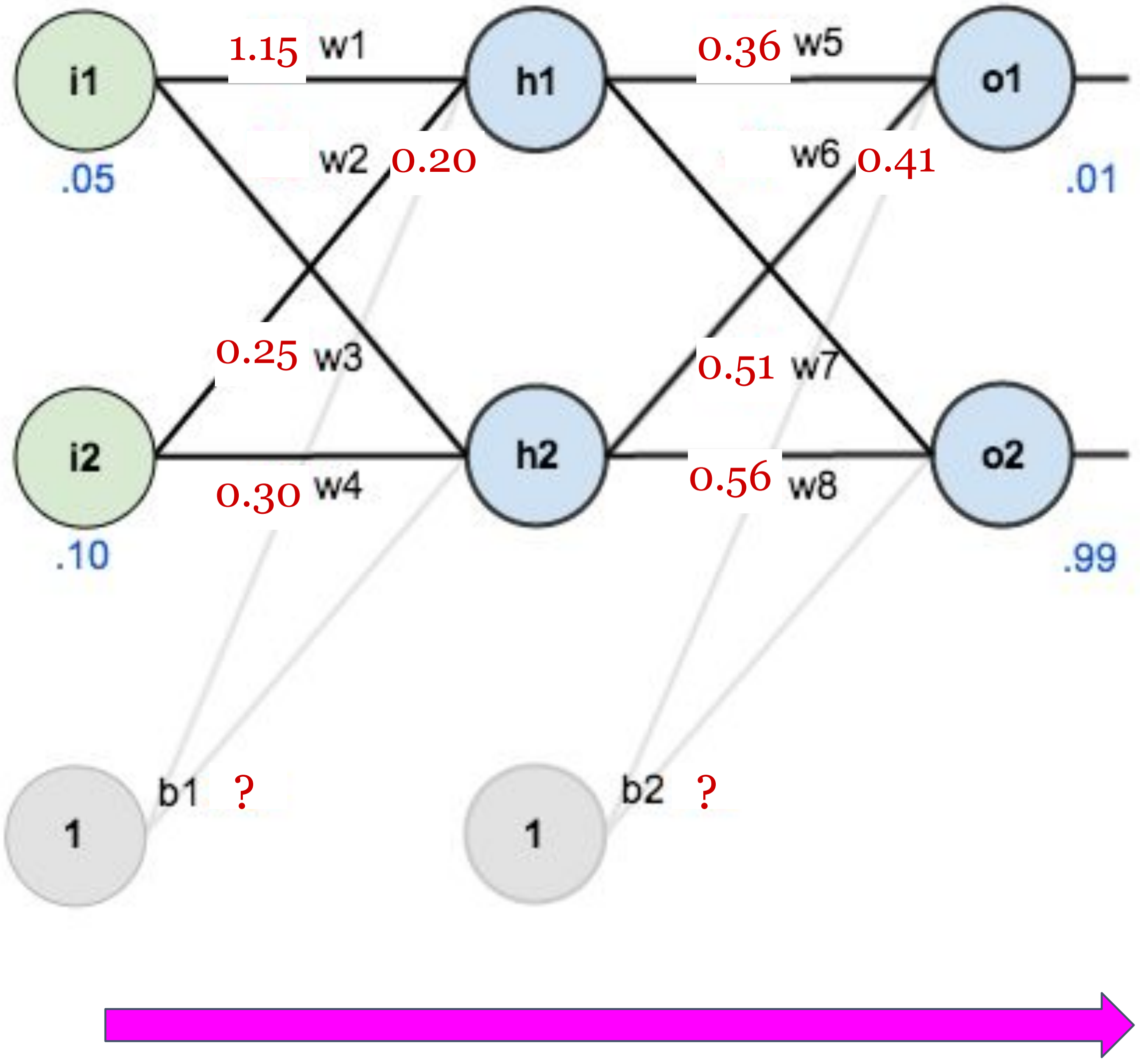
Repeat FeedForward and Backpropagation



Then Feed Forward Again

- Original error is about 0.298
- After 1st round update, error is about 0.291

Repeat FeedForward and Backpropagation



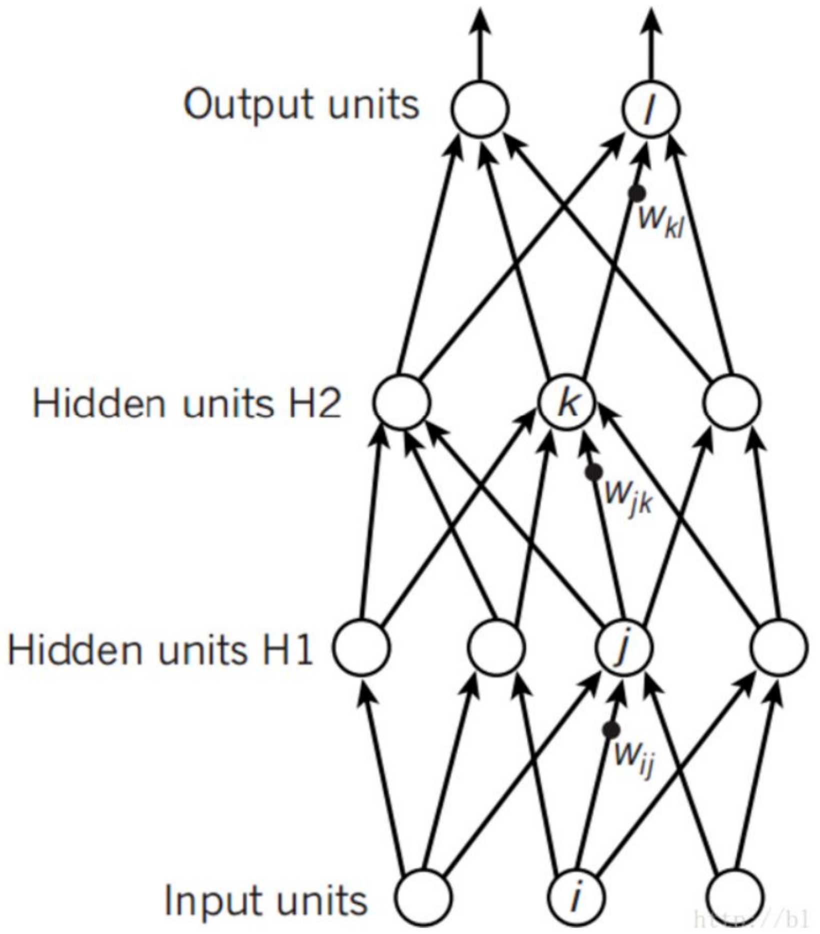
Repeat FeedForward and Backpropagation

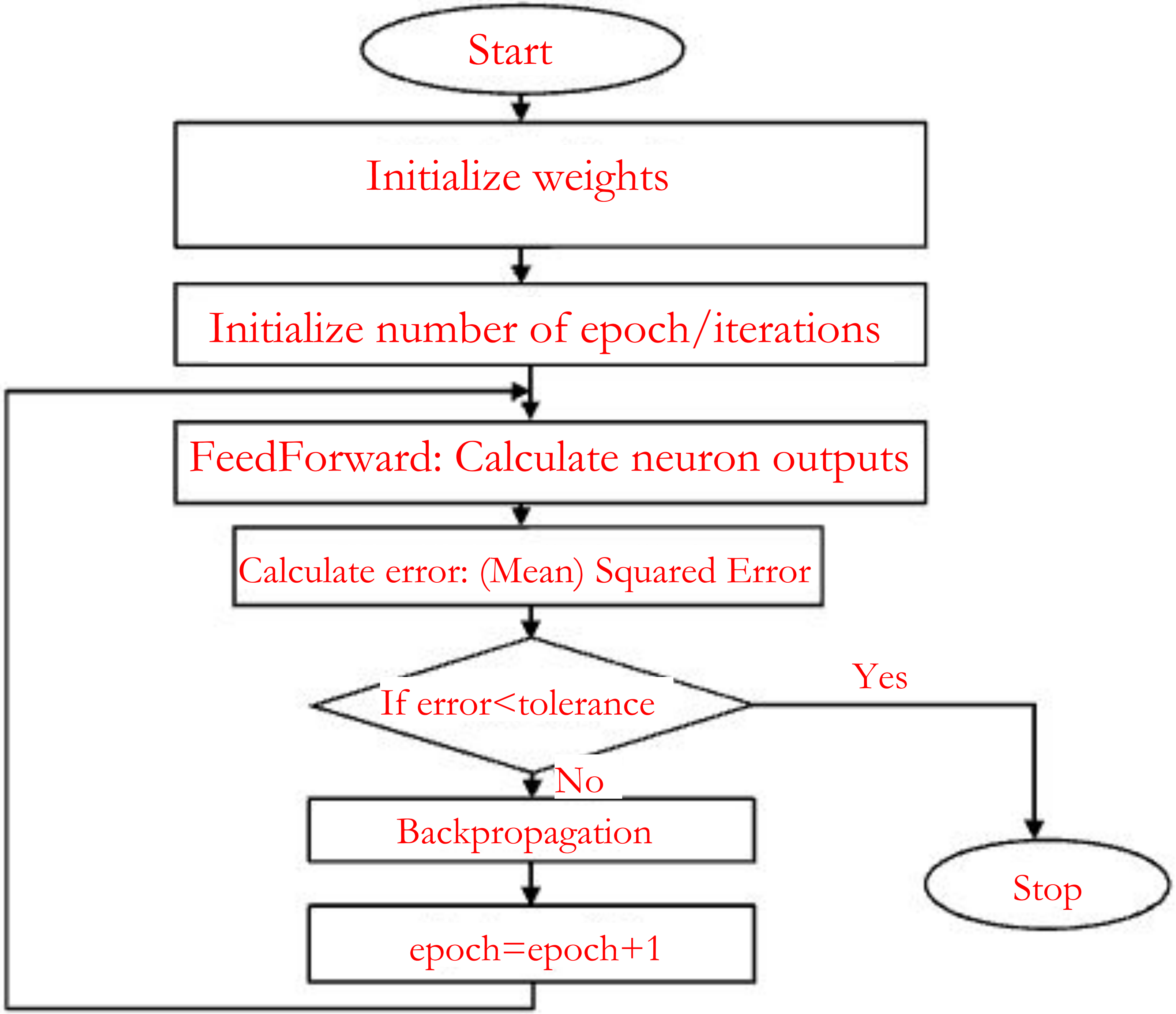
- After 10,000 rounds update, error is only about 4e-5
- Actual output for o1 is 0.01;

  Actual output for o2 is 0.99
- Predicted output for o1 is 0.016;

  Predicted output for o2 is 0.984

Neural Network: Feed Forward and Backpropagation





Programming Assignment 5

Make sure you install tensorflow and keras correctly in Python 3.5+ environment.

Programming Assignment 5

Using the BT2101 Tutorial 5 Notebook (Deep Learning with Tensorflow and Keras.ipynb), please answer the questions in the jupyter notebook

Answer all in the jupyter notebook.

Instructions

Submit Python Notebook to the submission folder and Named:
AXXXX_T5_program.ipynb

Include your answers in the jupyter notebook

- You need to show outputs, instead of just showing functions.

Submit by Tuesday OCT-09 (by 12:00pm noon)

- Based on Deep Learning with Tensorflow and Keras.ipynb

Thank You!