Exam - Master 2 Econometrics-Big Data-Statistics

Lecture by Yoann Bourgeois 2019/2020

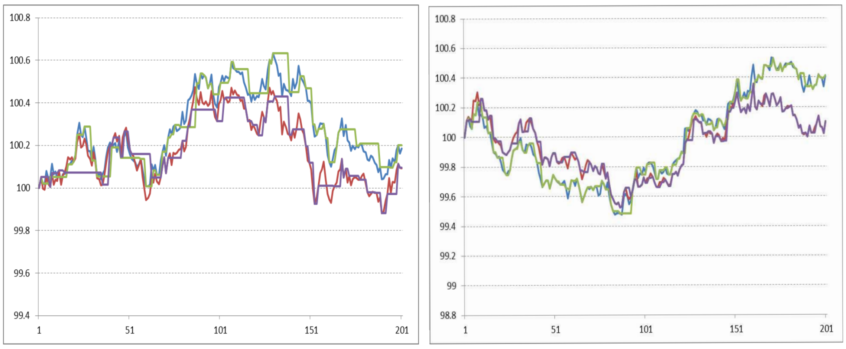
Choose one subject. Any software can be used (include the code to your document). You can work as a group of 2 or 3 maximum. For the first subject, you can focus only on one of the three questions if you conduct a deep analysis. For the paper comments, feel free to include some implementation examples. The maximum number of pages is 20pp without code. Bon courage.

1 Effect of asynchronous arrival time on realized correlation

We propose to observe the effect of asynchronous arrival time of log-returns on the realized covariance. This will highlight the existence of the Epps effect. Let's consider the two following processes defined on a probability space $(\Omega, \mathcal{F}, P)$:

$$d\log(X_t) = \mu_1 dt + \sigma_1 dW_t^1$$
$$d\log(Y_t) = \mu_2 dt + \sigma_2 dW_t^2$$

with W_t^i is a standard brownian motion, $d < W_t^1, W_t^2 >= \rho dt$, ρ is the constant instantaneous correlation between the two brownians.
We consider a maturity $T = \frac{1}{365}$ (ie. one day); we note $M > 1$ an integer



(a) $\lambda_X = \lambda_Y = 0.6$ (b) $\lambda_X = \lambda_Y = 0.2$

Figure 1: Paths for $\rho = 0.9$

for the discretization size such as $h = \frac{T}{M}$ is the discretization length. The realized covariance $V_h(\omega)$ is defined as:

$$V_h(\omega) = \sum_{i=1}^{M} \left(\log(X^h_{t_i}(\omega)) - \log(X^h_{t_{i-1}}(\omega))\right)\left(\log(Y^h_{t_i}(\omega)) - \log(Y^h_{t_{i-1}}(\omega))\right)$$

for $\omega \in \Omega$.
1 - We assume at first that we have synchronous arrival times for X and Y. Using Monte Carlo simulations, show the convergence of the MC estimator of $\mathbb{E}[V_h]$ for several value of M and N, the number of Monte Carlo experiments. If U, T, are two independent gaussian random variables then, U and $V = cU + \sqrt{1-c^2}\epsilon$, $\epsilon \sim N(0,1)$ have a correlation of c.
2 - We assume now arrival times of X and Y follow two independant Poisson processes with intensity λ_X and λ_Y respectively. Observation times are then different: for X:$0 = t_0, t_1, ..., t_N = T$ and for Y:$0 = s_0, s_1, ..., s_N = T$.
To simulate a random variable J following a Poisson process, use $J = J - \text{Integer}(\frac{\log(Z)}{\lambda})$... where Z is a Uniform random variable on $[0,1]$. At each jump of the process, we observe the price simulated at this moment; if there is no jump we observe the previous price observed at the previous jump moment.
To illustrate this, an example in Fig (1) shows two correlated geometric brownian motions ($\mu_1 = \mu_2 = 0, \sigma_1 = \sigma_2 = 15\%, \rho = 0.9$) and the two other series with different arrival times.
Give Monte Carlo estimators for $\mathbb{E}[V_h]$ with respect to several Poisson intensities and show the effect of asynchronicity.
3 - Compute the Hayashi-Yoshida estimator as explained during the lecture. Conclude.

2 Comment Paper 1

Tyler Manning, (2017) PCA and Autoencoders. This paper is available here.

3 Comment Paper 2

Lemieux, V. and al. (2015) Clustering Techniques And their Effect on Portfolio Formation and Risk Analysis. This paper is available here