ps2

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1 Assignment 2

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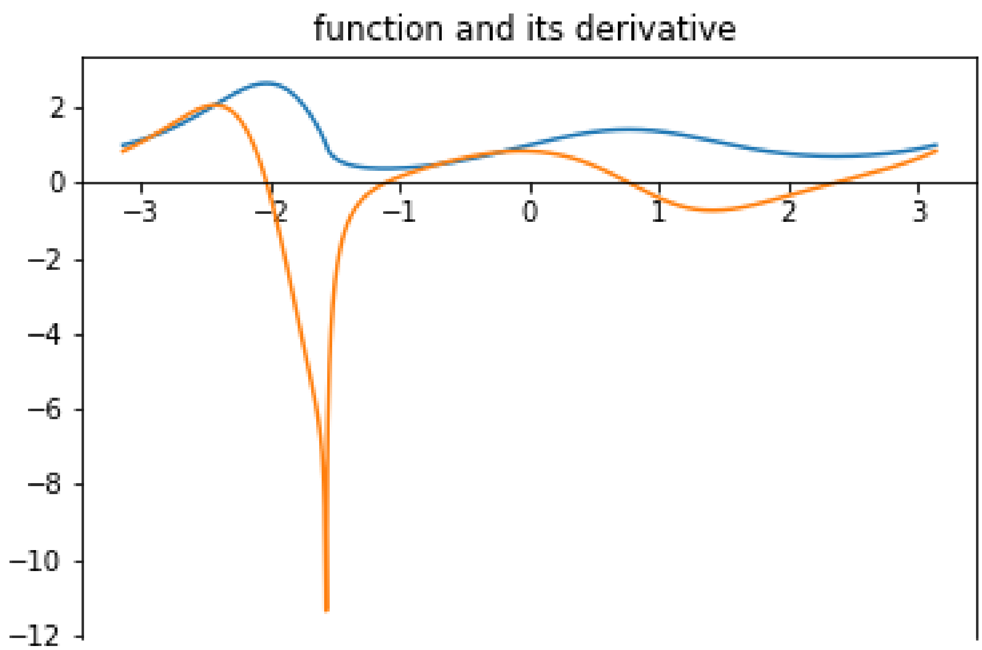
1.0.3 part 1 - ACME: Numerical Differentiation

problem1

```
In [1]: from sympy import *
        import sympy as sy
        x = sy.symbols('x')
        sy.diff((sin(x)+1)**(sin(cos(x))), x)
```

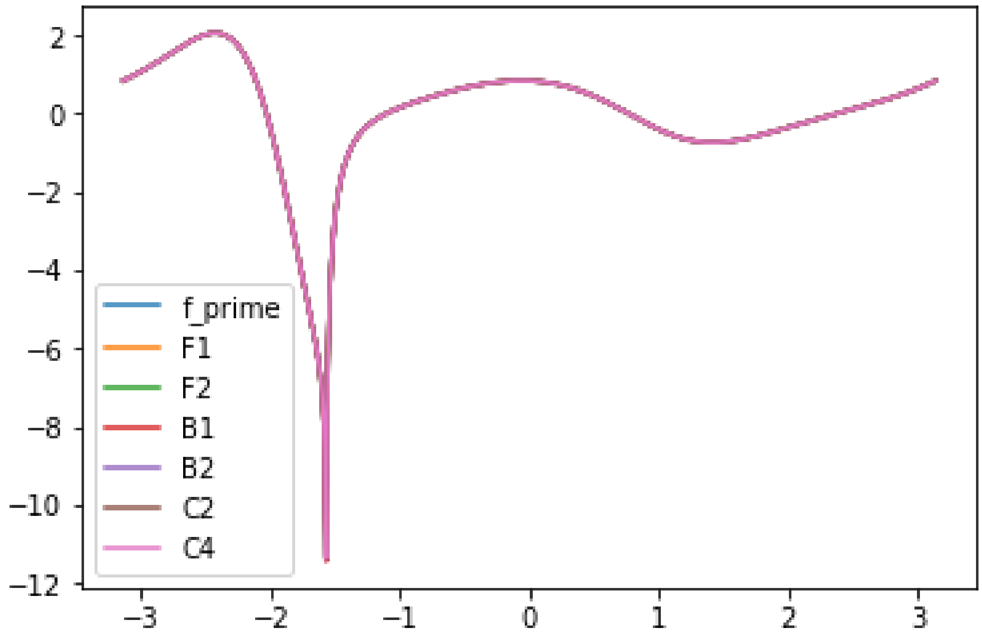
```
Out[1]: (-log(sin(x) + 1)*sin(x)*cos(cos(x)) + sin(cos(x))*cos(x)/(sin(x) + 1))*(sin(x) + 1)**si
```

```
In [3]: import numpy as np
        import math

        from matplotlib import pyplot as plt
        ax = plt.gca()
        ax.spines["bottom"].set_position("zero")
        ax.set_title('function and its derivative ')

        def f(x):
            return (np.sin(x)+1)**(np.sin(np.cos(x)))
        def f_prime(x):
            return (-np.log(np.sin(x) + 1)*np.sin(x)*np.cos(np.cos(x)) \
                    + np.sin(np.cos(x))*np.cos(x)/(np.sin(x) + 1))* \
                    (np.sin(x) + 1)**np.sin(np.cos(x))
        x = np.linspace(-math.pi,math.pi,1000)
        plt.plot(x,f(x))
        plt.plot(x, f_prime(x))
        plt.show()
```



problem2

```
In [4]: fig, ax = plt.subplots()
        def F1(f, x, h):
            return (f(x+h)-f(x))/h
        def F2(f, x, h):
            return (-3*f(x)+4*f(x+h)-f(x+2*h))/(2*h)
        def B1(f, x, h):
            return (f(x)-f(x-h))/h
        def B2(f, x, h):
            return (3*f(x)-4*f(x-h)+f(x-2*h))/(2*h)
        def C2(f, x, h):
            return (f(x+h)-f(x-h))/(2*h)
        def C4(f, x, h):
            return (f(x-2*h)-8*f(x-h)+8*f(x+h)-f(x+2*h))/(12*h)
        ax.plot(x,f_prime(x),label = "f_prime")
        ax.plot(x,F1(f,x,0.0001),label = "F1")
        ax.plot(x,F2(f,x,0.0001),label = "F2")
        ax.plot(x,B1(f,x,0.0001),label = "B1")
        ax.plot(x,B2(f,x,0.0001),label = "B2")
        ax.plot(x,C2(f,x,0.0001),label = "C2")
        ax.plot(x,C4(f,x,0.0001),label = "C4")
        ax.legend()

Out[4]: <matplotlib.legend.Legend at 0xd6ab568160>
```

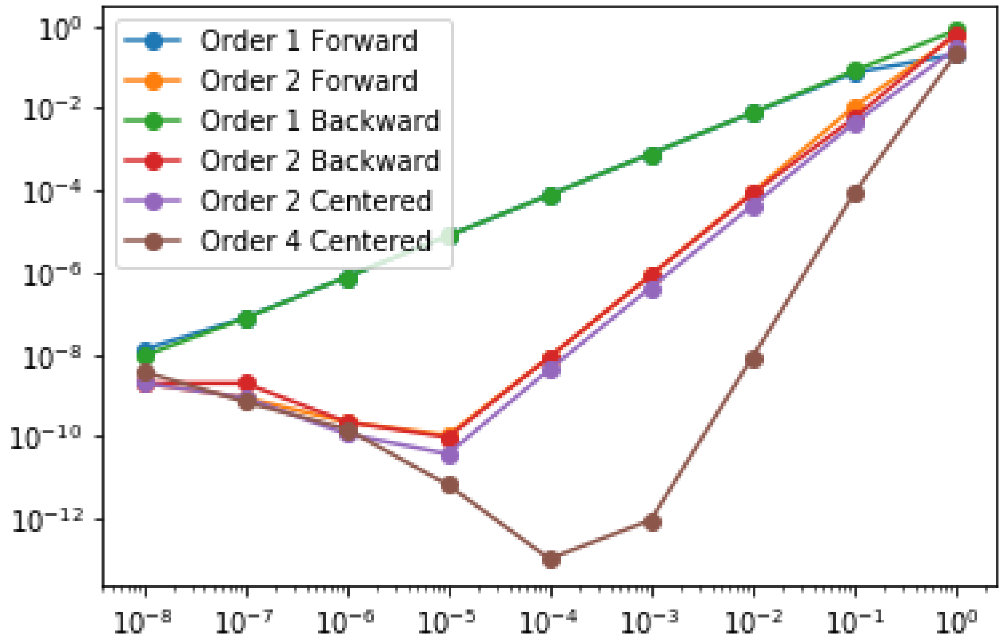


In the plot above, we can see that when h is set as 0.0001, these altogether seven plots overlap with each other, which shows that when h is small, the six approximation all have good convergence properties.

problem3

In [5]:
```
def absolute_error(func,x,h):
    return np.absolute(f_prime(x)-func(f,x,h))
x0=1
h = np.logspace(-8, 0, 9)
fig, ax = plt.subplots()
ax.loglog(h,absolute_error(F1,x0,h), marker='o',label = "Order 1 Forward")
ax.loglog(h,absolute_error(F2,x0,h), marker='o',label = "Order 2 Forward")
ax.loglog(h,absolute_error(B1,x0,h), marker='o',label = "Order 1 Backward")
ax.loglog(h,absolute_error(B2,x0,h), marker='o',label = "Order 2 Backward")
ax.loglog(h,absolute_error(C2,x0,h), marker='o',label = "Order 2 Centered")
ax.loglog(h,absolute_error(C4,x0,h), marker='o',label = "Order 4 Centered")
ax.legend(loc = "upper left")
```

Out[5]:
```
<matplotlib.legend.Legend at 0xd6ab709d30>
```



problem4

```
In [6]: input_data = np.load('plane.npy')
        alpha = np.deg2rad(input_data[:,1])
        beta = np.deg2rad(input_data[:,2])
        a = 500
        x = a*np.tan(beta)/(np.tan(beta)-np.tan(alpha))
        y = a*np.tan(beta)*np.tan(alpha)/(np.tan(beta)-np.tan(alpha))
        x_prime = np.zeros(8)
        y_prime = np.zeros(8)
        x_prime[0] = x[1]-x[0]
        y_prime[0] = y[1]-y[0]
        for i in range(1,7):
            x_prime[i] = (x[i+1]-x[i-1]) / 2
            y_prime[i] = (y[i+1]-y[i-1]) / 2
        x_prime[7] = x[7]-x[6]
        y_prime[7] = y[7]-y[6]
        speed_value = np.sqrt(x_prime**2+y_prime**2)
        for i,k in enumerate(speed_value):
            print("when t is {}, speed is {}".format(i+7,k))
```

```
when t is 7, speed is 46.424200622134585
when t is 8, speed is 47.00103938095283
when t is 9, speed is 48.998805140367324
when t is 10, speed is 50.09944162965303
when t is 11, speed is 48.290350838205164
```

```
when t is 12, speed is 51.56455904927243
when t is 13, speed is 53.923033545053535
when t is 14, speed is 51.51480056963612
```

problem5

```
In [7]: def Jacobian(function,x0,h):
            jacob = np.zeros((len(function),len(x0)))
            variable_set = set()
            for func in function:
                variable_set = variable_set.union(func.atoms(Symbol))
            for i, func in enumerate(function):
                for j, v in enumerate(variable_set):
                    func_prime = (func.subs(v, v + h) \
                                  - func.subs(v, v - h)) \
                                  / (2 * h)
                    fx_prime = lambdify(v, func_prime)
                    jacob[i,j] = fx_prime(x0[j])
            return jacob

        x, y = symbols('x y')
        func1=x**2
        func2=x**3-y
        func=[func1,func2]
        x=[1,1]
        h=1e-5
        Jacobian(func,x,h)
```

```
Out[7]: array([[ 0.,  2.],
               [-1.,  3.]])
```

The result is the same as what I have calculated by scratch work.

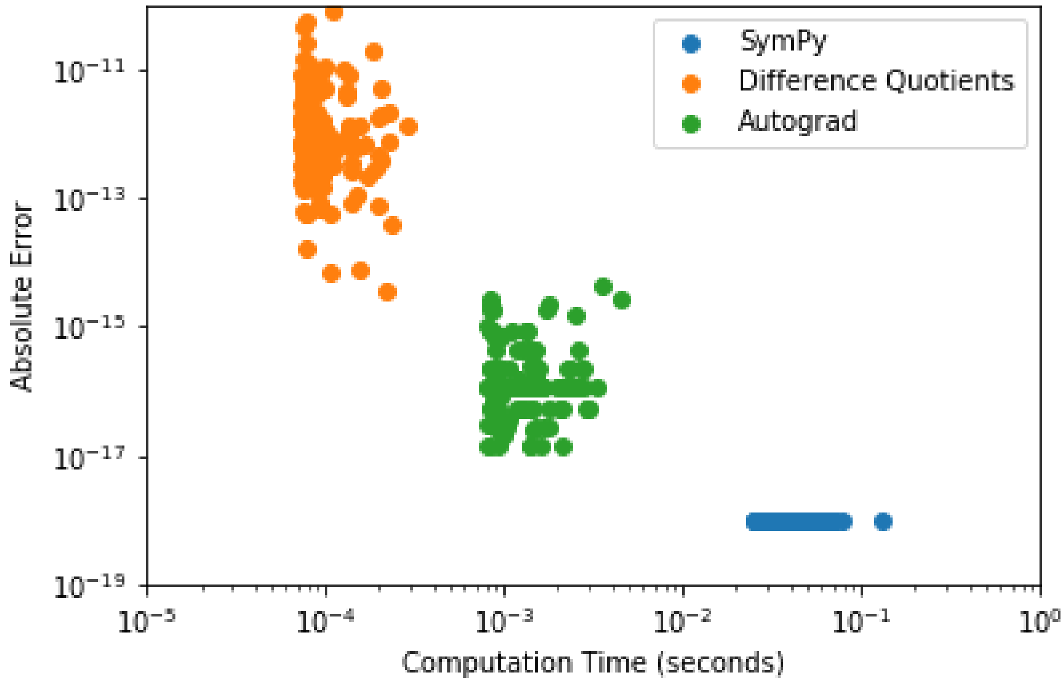
problem7

```
In [8]: from autograd import numpy as anp
        from autograd import grad
        import time
```

```
In [9]: g=lambda x: (anp.sin(x)+1)**(anp.sin(anp.cos(x)))
        dg=grad(g)
```

```
In [10]: def experiment(N):
             global T1,T2,T3,abse1,abse2
             T1,T2,T3=[],[],[]
             error1,error2=[],[]
```

```
    for i in range(N):
        x0=np.random.uniform(-math.pi,math.pi)
        t0=time.clock()
        x = Symbol('x')
        y=(sin(x)+1)**(sin(cos(x)))
        sympy=lambda x: y.diff(x)
        f_prime=lambdify(x,sympy(x),'numpy')
        y0=f_prime(x0)
        t1=time.clock()
        T1.append(t1-t0)
        t2=time.clock()
        y1=C4(f,x0,0.0001)
        t3=time.clock()
        T2.append(t3-t2)
        error1.append(abs(y1-y0))
        t4=time.clock()
        y2=dg(x0)
        t5=time.clock()
        T3.append(t5-t4)
        error2.append(abs(y2-y0))
    plt.scatter(np.array(T1),np.array([1e-18] * N),label="SymPy")
    plt.scatter(np.array(T2),np.array(error1),label="Difference Quotients")
    plt.scatter(np.array(T3),np.array(error2),label="Autograd")
    plt.legend(loc='upper right')
    plt.xlabel("Computation Time (seconds)")
    plt.ylabel("Absolute Error")
    plt.xlim(10**-5,1)
    plt.ylim(10**-19,10**-10)
    plt.loglog()
experiment(200)
```


SymPy
Difference Quotients
Autograd
Absolute Error
Computation Time (seconds)
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}