hw4

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1 Assignment 4

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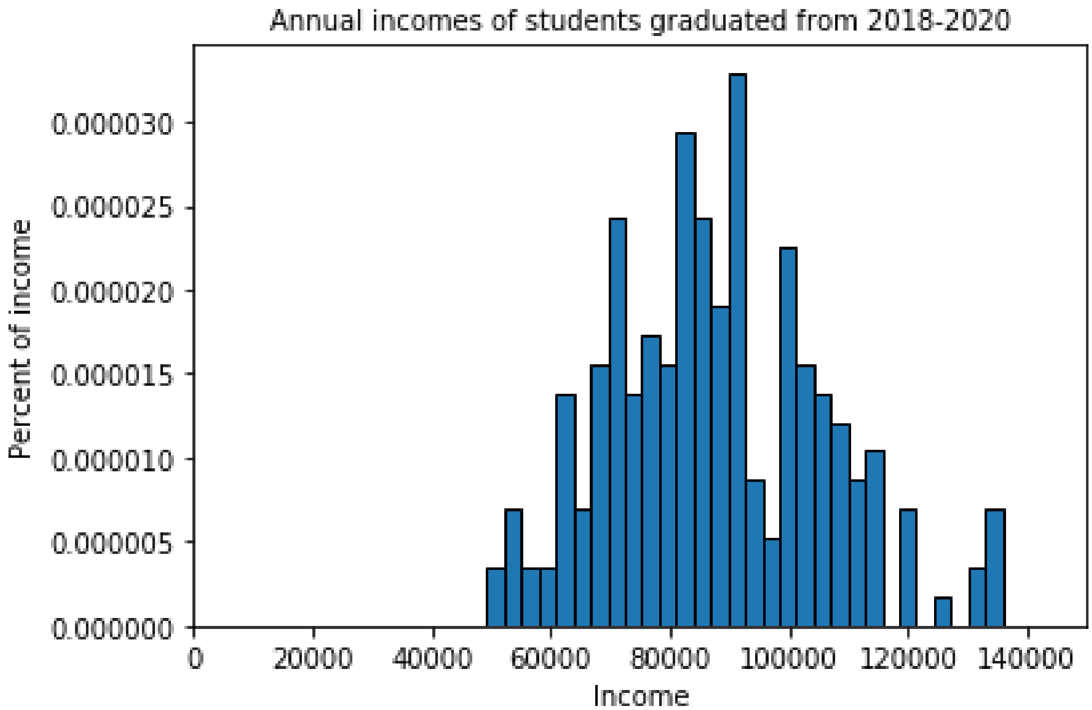
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy.optimize as opt
        from scipy.stats import chi2
        from scipy.stats import lognorm
        import pandas as pd
```

problem 1(a)

```
In [2]: pts = np.loadtxt('incomes.txt')
```

```
In [3]: num_bins = 30
        plt.hist(pts, num_bins,edgecolor='k',normed=True)
        plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
        plt.xlabel(r'Income')
        plt.ylabel('Percent of income')
        plt.xlim([0, 150000])  # This gives the xmin and xmax to be plotted"
```

```
Out[3]: (0, 150000)
```



(b)

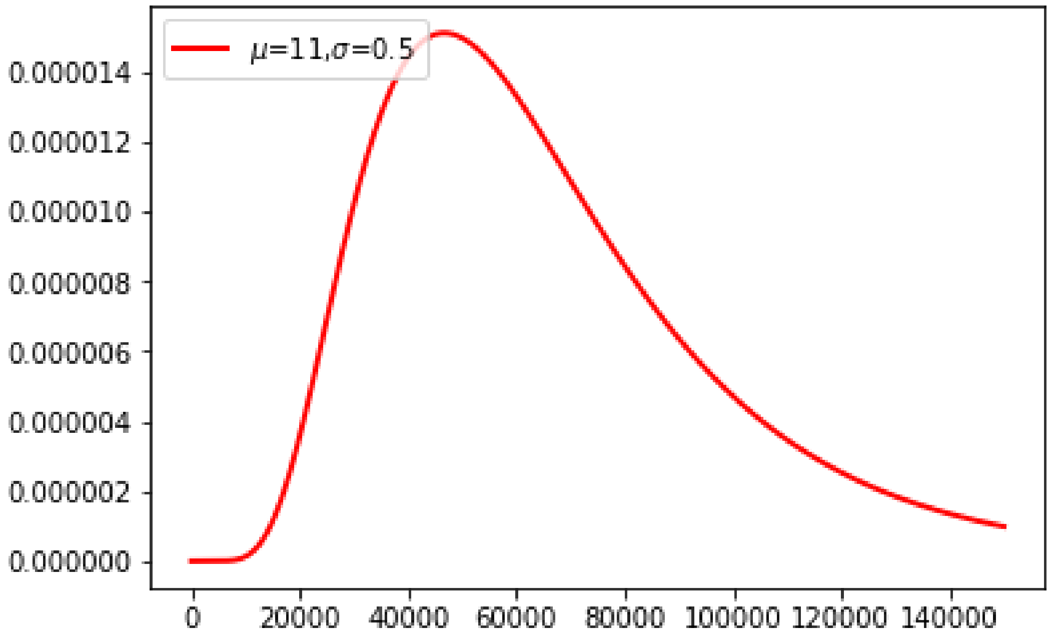
```
In [15]: def log_normal_pdf(xvals, mu, sigma):
             return 1/(xvals*sigma * np.sqrt(2 * np.pi))*np.e \
                     **(-(np.log(xvals) - mu)**2 / (2 * sigma**2))

         # Plot smooth line with distribution 1
         dist_pts = np.linspace(1e-9, 150000, 200000)
         mu_1 = 11
         sig_1 = 0.5
         plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_1, sig_1 ),
                  linewidth=2, color='r', label='$\mu$=11,$\sigma$=0.5')
         plt.legend(loc='upper left')

         def log_lik_log_normal(xvals, mu, sigma):
             pdf_vals = log_normal_pdf(xvals, mu, sigma)
             ln_pdf_vals = np.log(pdf_vals)
             log_lik_val = ln_pdf_vals.sum()
             return log_lik_val

         print('Log-likelihood: ', log_lik_log_normal(pts, mu_1, sig_1))

Log-likelihood:  -2385.856997808558
```



(c)

```
In [16]: def crit(params, *args):
             mu, sigma = params
             incomes = args
             pdf_vals = lognorm.pdf(incomes, s=abs(sigma), scale=np.exp(mu))
             ln_pdf_vals = np.log(pdf_vals)
             log_lik_val = ln_pdf_vals.sum()
             neg_log_lik_val = -log_lik_val
             return neg_log_lik_val

         mu_init = 11
         sig_init = 0.5
         params_init = np.array([mu_init, sig_init])
         mle_args = pts
         results = opt.minimize(crit, params_init, args=mle_args)
         mu, sigma = results.x
         Hess = results.hess_inv
         fval = -results.fun
         print('The optimized result is mu = {:.2f}, sigma = {:.2f}'.format(mu, sigma))
         print('The value of the likelihood function is ', fval)
         print('The inverse Hessian matrix is\n ', Hess)
```

```
The optimized result is mu = 11.36, sigma = 0.21
The value of the likelihood function is  -2241.7193013573587
The inverse Hessian matrix is
```

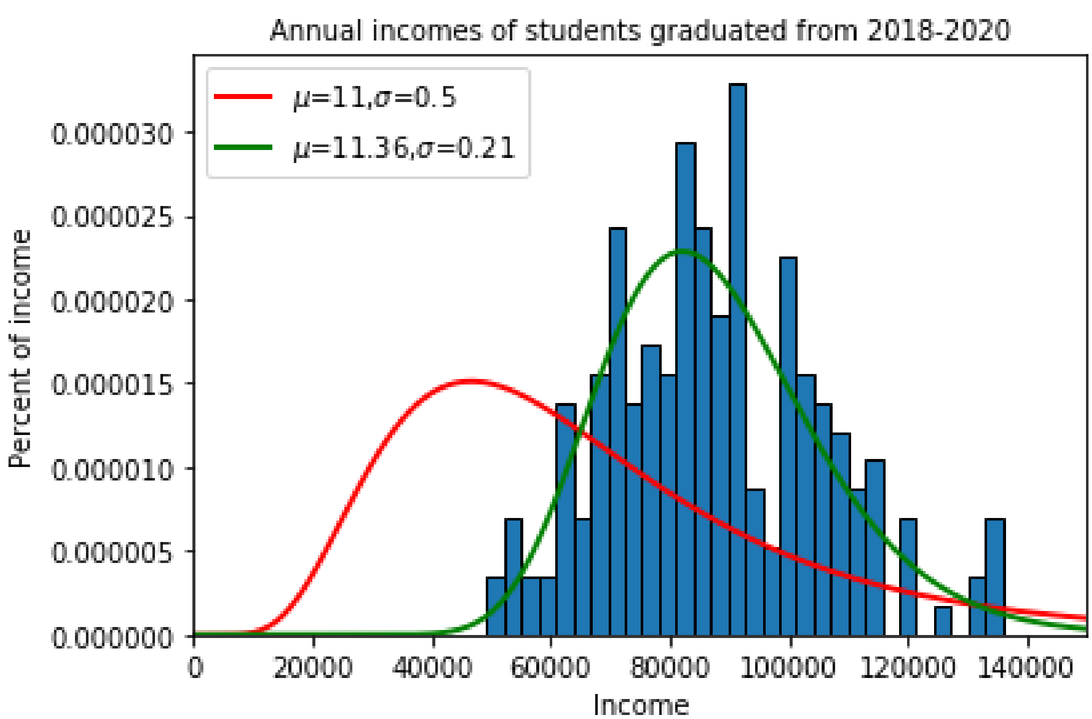
```
 [[ 2.20429404e-04 -9.56905596e-07]
 [-9.56905596e-07  1.08962777e-04]]
```

In [7]:
```
num_bins = 30
plt.hist(pts, num_bins,edgecolor='k',normed=True)
plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
plt.xlabel(r'Income')
plt.ylabel('Percent of income')
plt.xlim([1e-9, 150000])  # This gives the xmin and xmax to be plotted"

# Plot smooth line with distribution 1
dist_pts = np.linspace(1e-9, 150000, 200000)
mu_1 = 11
sig_1 = 0.5
plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_1, sig_1 ),
         linewidth=2, color='r', label='$\mu$=11,$\sigma$=0.5')
plt.legend(loc='upper left')

# Plot smooth line with MLE distribution
dist_pts = np.linspace(1e-9, 150000, 200000)
mu_2 = mu
sig_2 = sigma
plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_2, sig_2 ),
         linewidth=2, color='g', label='$\mu$=11.36,$\sigma$=0.21')
plt.legend(loc='upper left')
```

Out[7]: <matplotlib.legend.Legend at 0xa8c894a7f0>



(d)

```
In [8]: mu_new, sig_new = np.array([11, 0.5])
        log_lik_h0 = log_lik_log_normal(pts, mu_new, sig_new)
        print('hypothesis value log likelihood', log_lik_h0)
        log_lik_mle = log_lik_log_normal(pts, mu, sigma)
        print('MLE log likelihood', log_lik_mle)
        LR_val = 2 * (log_lik_mle - log_lik_h0)
        print('likelihood ratio value', LR_val)
        pval_h0 = 1.0 - chi2.cdf(LR_val, 2)
        print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2385.856997808558
MLE log likelihood -2241.7193013573587
likelihood ratio value 288.2753929023984
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

(e)

```
In [9]: prob1 = 1 - lognorm.cdf(100000, s=sigma, scale=np.exp(mu))
        prob2 = lognorm.cdf(75000, s=sigma, scale=np.exp(mu))
        print('Probability of earning more than $100,000 is {:.3f}'.format(prob1))
        print('Probability of earning less than $75,000 is {:.3f}'.format(prob2))
```

```
Probability of earning more than $100,000 is 0.230
Probability of earning less than $75,000 is 0.260
```

problem 2 (a)

```
In [10]: df=pd.read_csv("sick.txt").astype('float64')
```

```
In [11]: def norm_pdf(xvals, sig):
             pdf_vals = (1/(sig*np.sqrt(2*np.pi)))*np.exp(-(xvals)**2 / (2*sig**2))
             return pdf_vals

         def log_lik_norm(y, x1, x2, x3, beta0, beta1, beta2, beta3, sig):
             epsilon = y-beta0-beta1*x1-beta2*x2-beta3*x3
             pdf_vals = norm_pdf(epsilon, sig)
             ln_pdf_vals = np.log(pdf_vals)
             log_lik_val = ln_pdf_vals.sum()
             return log_lik_val

         def crit2(params,*args):
             beta0, beta1, beta2, beta3, sig = params
```

```python
        y, x1, x2, x3 = args
        log_lik_val = log_lik_norm(y, x1, x2, x3, beta0, beta1, \
                                   beta2, beta3, sig)
        neg_log_lik_val = -log_lik_val
        return neg_log_lik_val
```

```python
In [12]: b0_init, b1_init, b2_init, b3_init, sig_init = (0.2,0,0,0,1)
         y, x1, x2, x3=df['sick'],df['age'], df['children'], df['avgtemp_winter']
         params_init = np.array([b0_init, b1_init, b2_init, b3_init, sig_init])
         results = opt.minimize(crit2, params_init,(y, x1, x2, x3))
         b0_MLE, b1_MLE, b2_MLE, b3_MLE, sig_MLE = results.x
         print('beta 0=', b0_MLE)
         print('beta 1=', b1_MLE)
         print('beta 2=', b2_MLE)
         print('beta 3=', b3_MLE)
         print('sigma=', sig_MLE)
         print("value of the log likelihood function:",-results.fun)
```

```
beta 0= 0.2516462863009143
beta 1= 0.012933350241651839
beta 2= 0.4005020598812094
beta 3= -0.009991671522641493
sigma= 0.0030177004489735067
value of the log likelihood function: 876.8650468414168
```

```python
In [13]: vcv_mle = results.hess_inv
         print('estimated variance covariance matrix of the estimates:\n ', vcv_mle)
```

```
estimated variance covariance matrix of the estimates:
  [[ 1.08579401e-06  4.52195731e-09 -1.47000525e-07 -2.20502350e-08
   5.57249528e-09]
 [ 4.52195731e-09  4.20615273e-09 -3.71445246e-08 -2.57941093e-09
  -1.28317610e-10]
 [-1.47000525e-07 -3.71445246e-08  3.77442634e-07  2.33178375e-08
   6.82198342e-10]
 [-2.20502350e-08 -2.57941093e-09  2.33178375e-08  1.99610060e-09
  -2.42837085e-11]
 [ 5.57249528e-09 -1.28317610e-10  6.82198342e-10 -2.42837085e-11
   2.31215973e-08]]
```

(b)

```python
In [14]: b0_new, b1_new, b2_new, b3_new, sig_new = np.array([1,0,0,0,0.1])
         log_lik_h0 = log_lik_norm(y, x1, x2, x3, b0_new, \
                                   b1_new, b2_new, b3_new, sig_new)
         print('hypothesis value log likelihood', log_lik_h0)
         print('MLE log likelihood', -results.fun)
```

```python
LR_val = 2 * ((-results.fun)-log_lik_h0)
print('likelihood ratio value', LR_val)
pval_h0 = 1.0 - chi2.cdf(LR_val, 2)
print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 876.8650468414168
likelihood ratio value 6261.131469767083
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

The results shows that we could reject the null hypothesis that age, number of children and average winter temperature have no effect on the number of sick days.