hw5_answer

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1 Assignment 5

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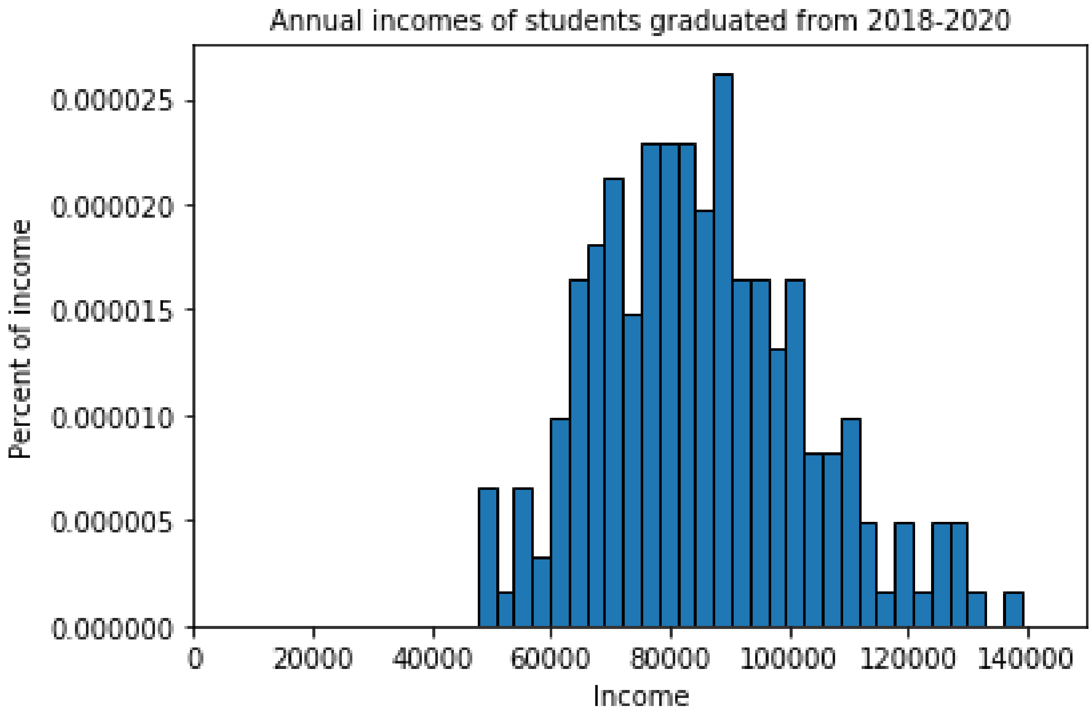
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy.optimize as opt
        from scipy.stats import lognorm
        import pandas as pd
        import scipy.integrate as intgr
        import numpy.linalg as lin
```

problem 1(a)

```
In [2]: pts = np.loadtxt('data\incomes.txt')
```

```
In [3]: num_bins = 30
        plt.hist(pts, num_bins,edgecolor='k',normed=True)
        plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
        plt.xlabel(r'Income')
        plt.ylabel('Percent of income')
        plt.xlim([0, 150000])  # This gives the xmin and xmax to be plotted"
```

```
Out[3]: (0, 150000)
```



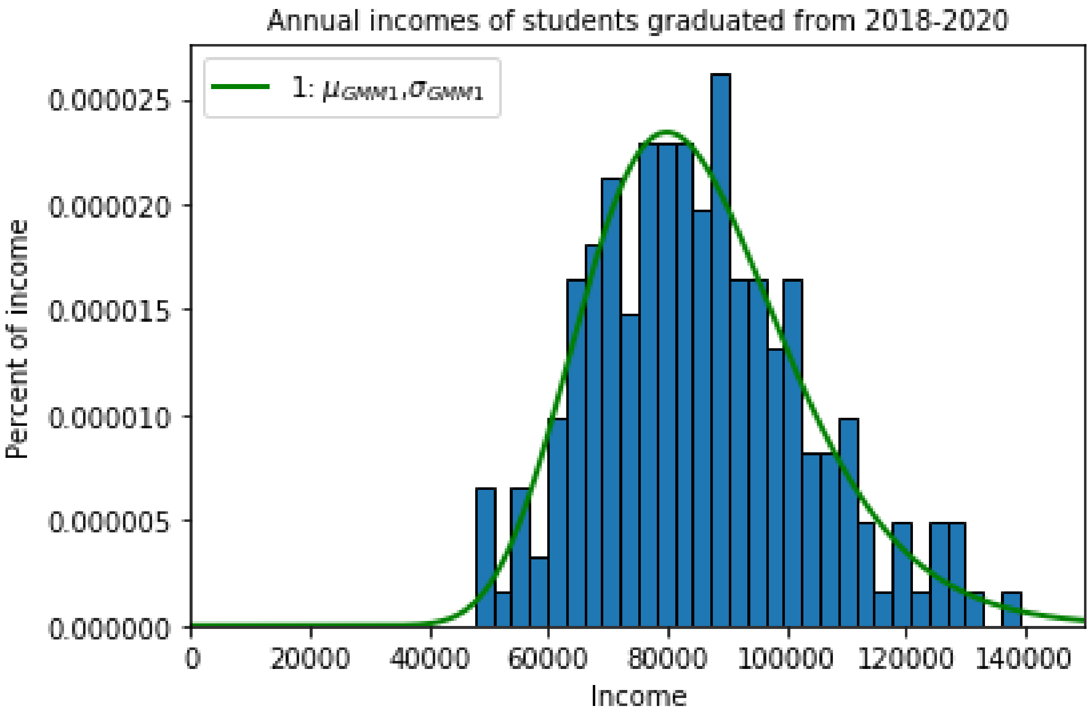
(b)

```
In [4]: def log_normal_pdf(xvals, mu, sigma):
            return lognorm.pdf(xvals,s = abs(sigma), scale = np.exp(mu))

        def data_moments(xvals):
            mean_data = xvals.mean()
            std_data = xvals.std()
            return mean_data, std_data

        def model_moments(mu, sigma):
            mean_model = np.exp(mu+(sigma**2)/2)
            std_model = np.sqrt(np.exp(2*mu+sigma**2)*(np.exp(sigma**2)-1))
            return mean_model, std_model

        def err_vec(xvals, mu, sigma, simple):

            mean_data, std_data = data_moments(xvals)
            moms_data = np.array([[mean_data], [std_data]])
            mean_model, std_model = model_moments(mu, sigma)
            moms_model = np.array([[mean_model], [std_model]])
            if simple:
                err_vec = moms_model - moms_data
            else:
```

```
            err_vec = (moms_model - moms_data) / moms_data

        return err_vec

    def criterion(params, *args):

        mu, sigma = params
        xvals, W = args
        err = err_vec(xvals, mu, sigma, simple=False)
        crit_val = err.T @ W @ err

        return crit_val

    mu_init = 11
    sig_init = 0.5
    params_init = np.array([mu_init, sig_init])
    W_hat = np.eye(2)
    gmm_args = (pts, W_hat)
    results = opt.minimize(criterion, params_init, args=(gmm_args), tol=1e-14,
                           method='L-BFGS-B', bounds=((1e-10, None), (1e-10, None)))
    mu_GMM1, sig_GMM1 = results.x
```

In [6]:
```
num_bins = 30
plt.hist(pts, num_bins,edgecolor='k',normed=True)
plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
plt.xlabel(r'Income')
plt.ylabel('Percent of income')
plt.xlim([1e-9, 150000])  # This gives the xmin and xmax to be plotted"

dist_pts = np.linspace(1e-9, 150000, 200000)
mu_1 = mu_GMM1
sig_1 = sig_GMM1
plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_1, sig_1 ),
         linewidth=2, color='g', label='1: $\mu_{GMM1}$,$\sigma_{GMM1}$')
plt.legend(loc='upper left')
```

Out[6]: <matplotlib.legend.Legend at 0x2850b01d68>




```
In [7]: print('mu_GMM1=', mu_GMM1, ' sig_GMM1=', sig_GMM1)
        func_value = results.fun
        mean_data, std_data = data_moments(pts)
        mean_model, std_model = model_moments(mu_GMM1, sig_GMM1)
        print("The value of GMM criterion function at \
        the estimated parameter values:\n", func_value)
        print('Mean of points =', mean_data, ', Variance of points =', std_data)
        print('Mean of model =', mean_model, ', Variance of model =', std_model)
        err1 = err_vec(pts, mu_GMM1, sig_GMM1, False).reshape(2,)
        print('Error vector=', err1)
```

```
mu_GMM1= 11.331880866982898  sig_GMM1= 0.20869665246828492
The value of GMM criterion function at the estimated parameter values:
 [[1.00518646e-15]]
Mean of points = 85276.82360625811 , Variance of points = 17992.542128046523
Mean of model = 85276.82542230906 , Variance of model = 17992.541705444266
Error vector= [ 2.12959497e-08 -2.34876347e-08]
```

(c)

```
In [8]: def get_Err_mat2(pts, mu, sigma, simple=False):
            R = 2
            N = len(pts)
```

```
        Err_mat = np.zeros((R, N))
        mean_model, std_model = model_moments(mu, sigma)
        if simple:
            Err_mat[0, :] = pts - mean_model
            Err_mat[1, :] = np.sqrt(((mean_data - pts) ** 2)) - std_model
        else:
            Err_mat[0, :] = (pts - mean_model) / mean_model
            Err_mat[1, :] = (np.sqrt(((mean_data - pts) ** 2)) - std_model) / std_model
        return Err_mat

    Err_mat = get_Err_mat2(pts, mu_GMM1, sig_GMM1, False)
    VCV2 = (1 / pts.shape[0]) * (Err_mat @ Err_mat.T)
    print("The variance covariance matrix:\n", VCV2)
    W_hat2 = lin.inv(VCV2)
    print("The two-step estimator for \
    the optimal weighting matrix:\n", W_hat2)
```

```
The variance covariance matrix:
 [[0.0445167  0.02724079]
 [0.02724079 0.40593057]]
The two-step estimator for the optimal weighting matrix:
 [[23.42542697 -1.57201059]
 [-1.57201059  2.56896842]]
```

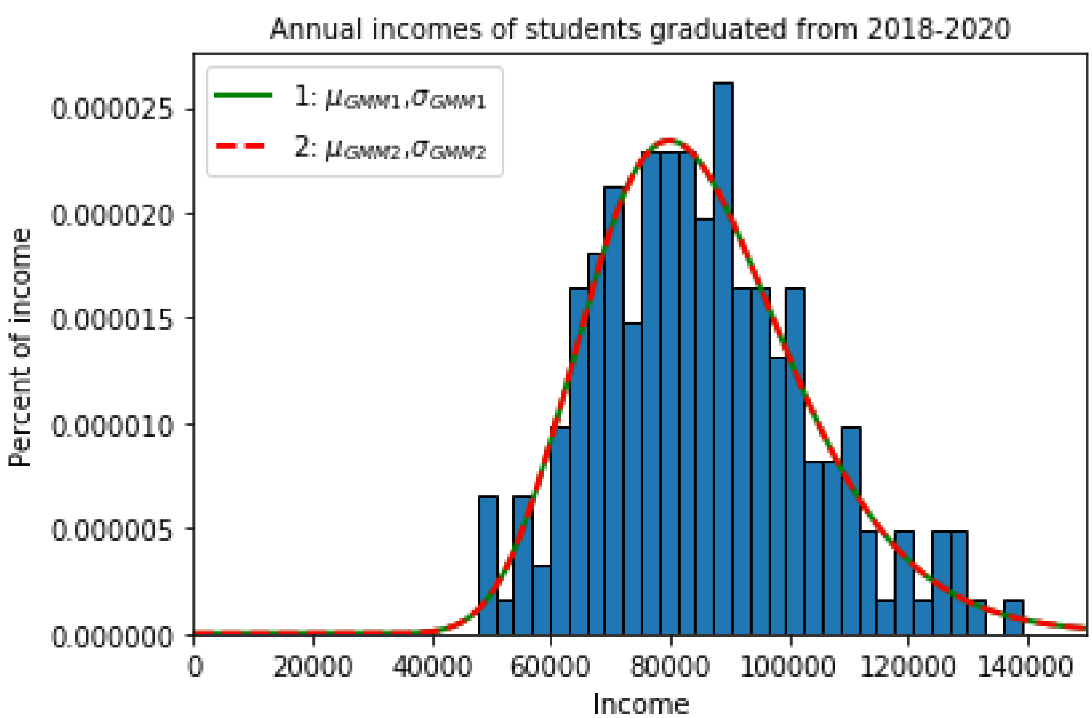
```
In [9]: mu_init = 11
        sig_init = 0.5
        params_init = np.array([mu_init, sig_init])
        gmm_args = (pts, W_hat2)
        results2 = opt.minimize(criterion, params_init, args=(gmm_args),tol=1e-14,
                               method='L-BFGS-B', bounds=((1e-10, None), (1e-10, None)))
        mu_GMM2, sig_GMM2 = results2.x
        print('mu_GMM2=', mu_GMM2, ' sig_GMM2=', sig_GMM2)

        func_value2 = results2.fun
        mean_data, std_data = data_moments(pts)
        mean_model2, std_model2 = model_moments(mu_GMM2, sig_GMM2)
        print("The value of GMM criterion function at \
        the estimated parameter values:\n", func_value2)
        print('Mean of points =', mean_data, ', Variance of points =', std_data)
        print('Mean of model2 =', mean_model2, ', Variance of model2 =', std_model2)
        err2 = err_vec(pts, mu_GMM2, sig_GMM2, False).reshape(2,)
        print('Error vector=', err2)
```

```
mu_GMM2= 11.33188084234173  sig_GMM2= 0.20869665693060432
The value of GMM criterion function at the estimated parameter values:
 [[1.59457809e-15]]
Mean of points = 85276.82360625811 , Variance of points = 17992.542128046523
```

```
Mean of model2 = 85276.82340040439 , Variance of model2 = 17992.541671995466
Error vector= [-2.41394687e-09 -2.53466716e-08]
```

```
In [11]: num_bins = 30
         plt.hist(pts, num_bins,edgecolor='k',normed=True)
         plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
         plt.xlabel(r'Income')
         plt.ylabel('Percent of income')
         plt.xlim([1e-9, 150000])  # This gives the xmin and xmax to be plotted"

         dist_pts = np.linspace(1e-9, 150000, 200000)
         mu_1 = mu_GMM1
         sig_1 = sig_GMM1
         plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_1, sig_1),
                  linewidth=2, color='g', label='1: $\mu_{GMM1}$,$\sigma_{GMM1}$')
         plt.legend(loc='upper left')

         dist_pts = np.linspace(1e-9, 150000, 200000)
         mu_2 = mu_GMM2
         sig_2 = sig_GMM2
         plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_2, sig_2 ),
                  linewidth=2, color='r', label='2: $\mu_{GMM2}$,$\sigma_{GMM2}$',ls ='--')
         plt.legend(loc='upper left')
```

```
Out[11]: <matplotlib.legend.Legend at 0x285003b518>
```

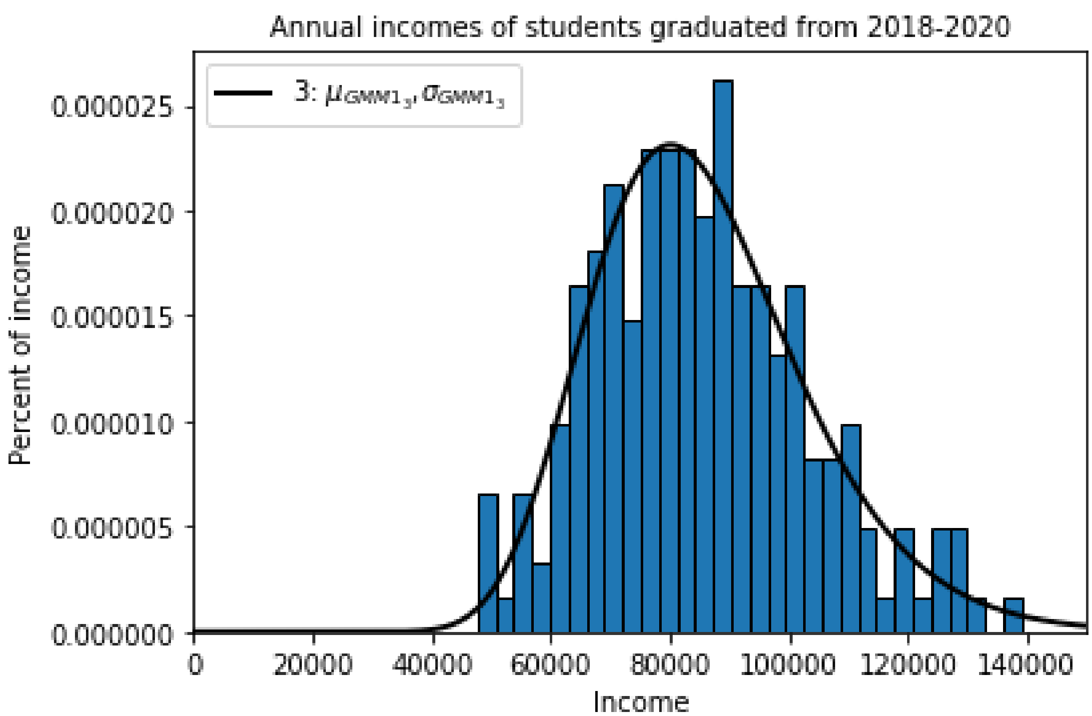


(d)

In [12]:
```python
def data_moments3(xvals):
    bpct_1_dat = xvals[xvals < 75000].shape[0] / xvals.shape[0]
    bpct_2_dat = (xvals[(xvals >=75000) & (xvals<=100000)].shape[0] /
                  xvals.shape[0])
    bpct_3_dat = xvals[xvals>100000].shape[0] / xvals.shape[0]
    return bpct_1_dat, bpct_2_dat, bpct_3_dat

def model_moments3(mu, sigma):
    xfx = lambda x: log_normal_pdf(x, mu, sigma)
    (bpct_1_mod, bp_1_err) = intgr.quad(xfx, 0.0, 75000)
    (bpct_2_mod, bp_2_err) = intgr.quad(xfx, 75000, 100000)
    (bpct_3_mod, bp_3_err) = intgr.quad(xfx, 100000, np.inf)
    return bpct_1_mod, bpct_2_mod, bpct_3_mod

def err_vec3(xvals, mu, sigma,simple):

    bpct_1_dat, bpct_2_dat, bpct_3_dat = data_moments3(xvals)
    moms_data = np.array([[bpct_1_dat], [bpct_2_dat], [bpct_3_dat]])
    bpct_1_mod, bpct_2_mod, bpct_3_mod = model_moments3(mu, sigma)
    moms_model = np.array([[bpct_1_mod], [bpct_2_mod], [bpct_3_mod]])
    if simple:
        err_vec = moms_model - moms_data
    else:
        err_vec = (moms_model - moms_data) / moms_data

    return err_vec


def criterion3(params, *args):
    mu, sigma = params
    xvals, W = args
    err = err_vec3(xvals, mu, sigma, simple=False)
    crit_val = err.T @ W @ err
    return crit_val


mu_init = 11
sig_init = 0.5
params_init = np.array([mu_init, sig_init])
W_hat1_3 = np.eye(3)
gmm_args = (pts, W_hat1_3)
results_3 = opt.minimize(criterion3, params_init, args=(gmm_args),tol=1e-14,
                       method='L-BFGS-B', bounds=((1e-10, None), (1e-10, None)))
mu_GMM1_3, sig_GMM1_3 = results_3.x
```

```
In [13]: num_bins = 30
         plt.hist(pts, num_bins,edgecolor='k',normed=True)
         plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
         plt.xlabel(r'Income')
         plt.ylabel('Percent of income')
         plt.xlim([1e-9, 150000])  # This gives the xmin and xmax to be plotted"

         dist_pts = np.linspace(1e-9, 150000, 200000)
         mu_3 = mu_GMM1_3
         sig_3 = sig_GMM1_3
         plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_3, sig_3 ),
                  linewidth=2, color='k', label='3: $\mu_{GMM1_3}$,$\sigma_{GMM1_3}$')
         plt.legend(loc='upper left')
```

```
Out[13]: <matplotlib.legend.Legend at 0x2850164b00>
```




```
In [14]: params = np.array([mu_GMM1_3, sig_GMM1_3])
         print('mu_GMM1_3=', mu_GMM1_3, ' sig_GMM1_3=', sig_GMM1_3)
         func_value3 = results_3.fun
         print("The value of GMM criterion function at \
         the estimated parameter values:\n", func_value3)
         print("Data moments: ", data_moments3(pts))
         print("Model moments:", model_moments3(mu_GMM1_3, sig_GMM1_3))
```

```
mu_GMM1_3= 11.335681327424783  sig_GMM1_3= 0.21059845372069547
The value of GMM criterion function at the estimated parameter values:
 [[2.53378859e-15]]
Data moments:  (0.3, 0.5, 0.2)
Model moments: (0.30000000363266727, 0.5000000058543161, 0.19999999051301676)
```

(e)

```
In [15]: def get_Err_mat3(pts, mu, sigma, simple=False):
             R = 3
             N = len(pts)
             Err_mat = np.zeros((R, N))
             pct_1_mod, pct_2_mod, pct_3_mod = model_moments3(mu, sigma)
             if simple:
                 pts_in_grp1 = pts < 75000
                 Err_mat[0, :] = pts_in_grp1 - pct_1_mod
                 pts_in_grp2 = (pts >= 75000) & (pts <= 100000)
                 Err_mat[1, :] = pts_in_grp2 - pct_2_mod
                 pts_in_grp3 = pts > 100000
                 Err_mat[2, :] = pts_in_grp3 - pct_3_mod
             else:
                 pts_in_grp1 = pts < 75000
                 Err_mat[0, :] = (pts_in_grp1 - pct_1_mod) / pct_1_mod
                 pts_in_grp2 = (pts >= 75000) & (pts <= 100000)
                 Err_mat[1, :] = (pts_in_grp2 - pct_2_mod) / pct_2_mod
                 pts_in_grp3 = pts > 100000
                 Err_mat[2, :] = (pts_in_grp3 - pct_3_mod) / pct_3_mod
             return Err_mat

         Err_mat3 = get_Err_mat3(pts, mu_GMM1_3, sig_GMM1_3, False)
         VCV2_3 = (1 / pts.shape[0]) * (Err_mat3 @ Err_mat3.T)
         print("The variance covariance matrix:\n", VCV2_3)
         # I use the pseudo-inverse command here because the VCV matrix is poorly conditioned
         W_hat2_3 = lin.pinv(VCV2_3)
         print("The two-step estimator for \
         the optimal weighting matrix:\n", W_hat2_3)
```

```
The variance covariance matrix:
 [[ 2.33333328 -0.99999998 -1.00000004]
 [-0.99999998  0.99999998 -1.00000004]
 [-1.00000004 -1.00000004  4.00000038]]
The two-step estimator for the optimal weighting matrix:
 [[ 0.25761773 -0.14958449 -0.01246537]
 [-0.14958449  0.11911357 -0.07340719]
 [-0.01246537 -0.07340719  0.20221605]]
```

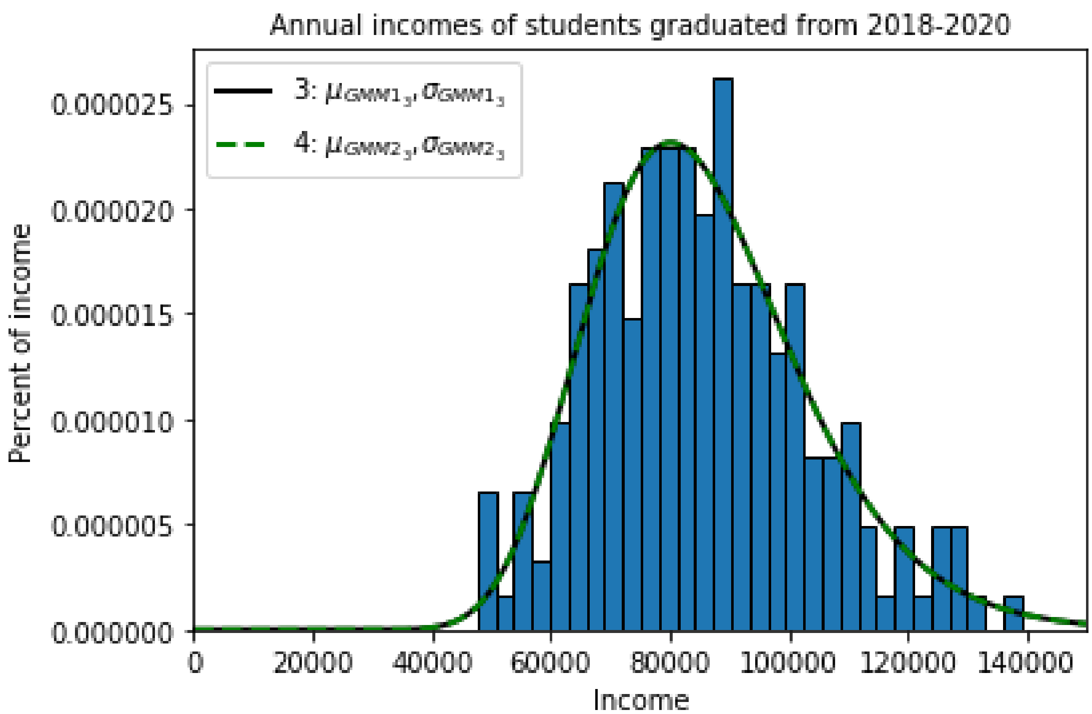
```
In [16]: mu_init = 11
         sig_init = 1
```

```
params_init = np.array([mu_init, sig_init])
gmm_args = (pts, W_hat2_3)
results2_3 = opt.minimize(criterion3, params_init, args=(gmm_args),tol=1e-14,
                          method='L-BFGS-B', bounds=((1e-10, None), (1e-2, None)))
mu_GMM2_3, sig_GMM2_3 = results2_3.x
print('mu_GMM2_3=', mu_GMM2_3, ' sig_GMM2_3=', sig_GMM2_3)
params = np.array([mu_GMM2_3, sig_GMM2_3])
func_value2_3 = results2_3.fun
print("The value of GMM criterion function at \
the estimated parameter values:\n", func_value2_3)
print("Data moments: ", data_moments3(pts))
print("Model moments:", model_moments3(mu_GMM2_3, sig_GMM2_3))
```

```
mu_GMM2_3= 11.335681328964448  sig_GMM2_3= 0.2105984532557043
The value of GMM criterion function at the estimated parameter values:
 [[4.24195251e-16]]
Data moments:  (0.3, 0.5, 0.2)
Model moments: (0.3000000006881451, 0.5000000072723064, 0.19999999203954869)
```

In [17]:
```
num_bins = 30
plt.hist(pts, num_bins,edgecolor='k',normed=True)
plt.title('Annual incomes of students graduated from 2018-2020', fontsize=10)
plt.xlabel(r'Income')
plt.ylabel('Percent of income')
plt.xlim([1e-9, 150000])  # This gives the xmin and xmax to be plotted"

dist_pts = np.linspace(1e-9, 150000, 200000)
mu_3 = mu_GMM1_3
sig_3 = sig_GMM1_3
plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_3, sig_3 ),
         linewidth=2, color='k', label='3: $\mu_{GMM1_3}$,$\sigma_{GMM1_3}$')
plt.legend(loc='upper left')

dist_pts = np.linspace(1e-9, 150000, 200000)
mu_4 = mu_GMM2_3
sig_4 = sig_GMM2_3
plt.plot(dist_pts, log_normal_pdf(dist_pts, mu_4, sig_4 ),
         linewidth=2, color='g', label='4: $\mu_{GMM2_3}$,$\sigma_{GMM2_3}$',ls ='--')
plt.legend(loc='upper left')
```

Out[17]: <matplotlib.legend.Legend at 0x2854e85940>



(f)

In [22]:
```python
mu = [mu_GMM1,mu_GMM2,mu_GMM1_3,mu_GMM2_3]
sig = [sig_GMM1,sig_GMM2,sig_GMM1_3,sig_GMM2_3]
funcv = [func_value, func_value2, func_value3, func_value2_3]
pd.DataFrame({"mu":mu,"sigma":sig,"Value of GMM func":funcv}
             ,index=['2 moments with Identity Matrix','2 moments with Weighted Matrix',
                    '3 moments with Identity Matrix','3 moments with Weighted Matrix'])
              .astype(float)
```

Out[22]:

	mu	sigma	Value of GMM func
2 moments with Identity Matrix	11.331881	0.208697	1.005186e-15
2 moments with Weighted Matrix	11.331881	0.208697	1.594578e-15
3 moments with Identity Matrix	11.335681	0.210598	2.533789e-15
3 moments with Weighted Matrix	11.335681	0.210598	4.241953e-16

From the perspective of comparing criterion function value, the method 4 which use 3 moments and the two-step estimator fits the data best.

problem 2(a)

```
In [19]: df = pd.read_csv("data/sick.txt").astype('float64')
         df.head()
```

Out[19]:

	sick	age	children	avgtemp_winter
0	1.67	57.47	3.04	54.10
1	0.71	26.77	1.20	36.54
2	1.39	41.85	2.31	32.38
3	1.37	51.27	2.46	52.94
4	1.45	44.22	2.72	45.90

```
In [20]: def err_vec(y, x1, x2, x3, b0, b1, b2, b3, simple):
             y_hat = b0 + b1*x1 + b2*x2 + b3*x3
             if simple:
                 return y - y_hat
             else:
                 return (y - y_hat)/ y_hat

         def crit(params, *args):
             b0, b1, b2, b3 = params
             y, x1, x2, x3, W = args
             err = err_vec(y, x1, x2, x3, b0, b1, b2, b3, True)
             crit_val = err.T @ W @ err
             return crit_val
```

```
In [21]: params_init = np.array([1, 0, 0, 0])
         W_hat = np.eye(200)
         gmm_args = (df.sick, df.age, df.children, df.avgtemp_winter, W_hat)
         res = opt.minimize(crit, params_init, args=(gmm_args),tol=1e-14, method='L-BFGS-B')
         b0, b1, b2, b3 = res.x
         val_crit = res.fun
         print("Estimate of b0:",b0)
         print("Estimate of b1:",b1)
         print("Estimate of b2:",b2)
         print("Estimate of b3:",b3)
         print('Value of GMM criterion function:',val_crit)
```

```
Estimate of b0: 0.2516448636612042
Estimate of b1: 0.012933470965564249
Estimate of b2: 0.40050098470289774
Estimate of b3: -0.009991709711286762
Value of GMM criterion function: 0.0018212898060782808
```