Second Overall Evaluation

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HalCoGen 설정에 한하여 단 하나도 빠짐없이
모두 수업중 진행했던 부분이기에 생략하고 코드에 해당하는 답안만 작성함
또한 펌웨어 시험 문제는 대부분을 복습 위주로 구성하였기 때문에
별도의 작업이 들어가는 부분에 대해서만 답안지 코드를 작성함

(수업중에 다루지 않은 별도의 답안에 해당하는 번호들: 5, 6, 7, 8, 22, 23)
나머지 문항들은 수업 내용 복습을 통해 확인하길 바람

1.

24. $y' - y = 0, y(0) = 3$

$$\mu(t) = e^{-t}$$
$$\frac{d}{dt}(e^{-t}y) = 0$$
$$e^{-t}y = C$$
$$y = Ce^{t}$$
$$3 = C$$
$$\therefore y = 3e^{t}$$

25. $y' - 3xy = 0, y(0) = 3$

$$\frac{dy}{dx} = 3xy$$
$$3x\,dx = \frac{1}{y}dy$$
$$\mathrm{C} + \frac{3}{2}x^2 = ln|y|$$
$$e^{\left(C+\frac{3}{2}x^2\right)} = y$$
$$\mathrm{y} = e^{C}e^{\frac{3}{2}x^2}$$
$$y = Ce^{\frac{3}{2}x^2}$$
$$3 = C$$
$$\therefore y = 3e^{\frac{3}{2}x^2}$$

26. $2x^3y^2 + x^4y\frac{dy}{dx} = 0$

$$\frac{\partial 2x^3y^2}{\partial y} = 4x^3y, \qquad \frac{\partial x^4y}{\partial x} = 4x^3y$$
$$u(x,y) = \int 2x^3y^2\,dx + h(y) = \frac{1}{2}x^4y^2 + h(y)$$
$$\frac{\partial u}{\partial y} = x^4y + h'(y) = x^4y$$
$$h'(y) = 0$$
$$h(y) = C$$
$$\therefore u(x,y) = \frac{1}{2}x^4y^2 + C$$

27. $y' + 3y = 2$

$$\mu(t) = e^{3t}$$
$$\frac{d}{dt}\left(e^{3t}y\right) = 2e^{3t} + C$$
$$\therefore y = 2 + Ce^{-3t}$$

28. $2y'' + 4y' + 2y = 0,$

$$y'' + 2y' + 1y = 0$$
$$(r+1)^2, \qquad r = -1$$
$$\therefore y = c_1e^{-t} + c_2xe^{-t}$$

29. $2y'' + 4y' + 2y = x^2 + x + 1$

$$y'' + 2y' + 1y = \frac{x^2}{2} + \frac{x}{2} + \frac{1}{2}$$
$$y_h = c_1e^{-t} + c_2xe^{-t}$$
$$y_p = Ax^2 + Bx + C$$
$$y_p' = 2Ax + B$$
$$y_p'' = 2A$$
$$y_p'' + 2y_p' + y_p = \frac{x^2}{2} + \frac{x}{2} + \frac{1}{2}$$
$$2A + 4Ax + 2B + Ax^2 + Bx + C = \frac{x^2}{2} + \frac{x}{2} + \frac{1}{2}$$
$$A = \frac{1}{2}$$
$$4A + B = \frac{1}{2},\ B = -\frac{3}{2}$$
$$2A + 2B + C = \frac{1}{2},\ C = \frac{5}{2}$$
$$\therefore y = c_1e^{-t} + c_2xe^{-t} + \frac{1}{2}x^2 - \frac{3}{2}x + \frac{5}{2}$$

30. $2y'' + 4y' + 2y = e^{3x}$

$$y'' + 2y' + y = \frac{1}{2}e^{3x}$$
$$y_p = Ae^{3x},\ y_p' = 3Ae^{3x},\ y_p'' = 9Ae^{3x}$$
$$9Ae^{3x} + 6Ae^{3x} + Ae^{3x} = \frac{1}{2}e^{3x}$$
$$16Ae^{3x} = \frac{1}{2}e^{3x},\ A = \frac{1}{32}$$
$$\therefore y = c_1e^{-t} + c_2xe^{-t} + \frac{1}{32}e^{3x}$$

31. $x^2y'' + 5xy' + 4y = 0$

$$y = x^r,\ y' = rx^{r-1},\ y'' = r(r-1)x^{r-2}$$
$$x^{r-2}\left(r^2 - r\right)x^2 + 5rx^{r-1}x + 4x^r = 0$$
$$x^r\left(r^2 - r\right) + x^r5r + 4x^r = 0$$
$$\left(r^2 - 4r + 4\right)x^r = 0$$
$$r = -2$$
$$\therefore y = c_1x^{-2} + c_2x^{-2}ln(x)$$

32. $y'' + 4y' + 3y = 0$

$$(r+3)(r+1) = 0$$
$$\therefore y = c_1e^{-3x} + c_1e^{-x}$$

33. $y'' + 4y' + 5y = 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$
$$\therefore y = c_1e^{-2x}cos(x) + c_2e^{-2x}sin(x)$$

34. $y'' + 3y' + 2y = 0$

$$(r+2)(r+1) = 0$$
$$\therefore y = c_1e^{-2x} + c_2e^{-x}$$

35. $\int_0^\infty 1 \times e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_0^\infty = \frac{1}{s}$

36. $\int_0^\infty t \times e^{-st}\,dt$

$\int\{f(x)g(x)\}' = \int f'(x)g(x) + \int f(x)g'(x)$

$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$

$f(x) = t,\ g'(x) = e^{-st}$

$f'(x) = 1,\ g(x) = -\frac{1}{s}e^{-st}$

$\int_0^\infty t \times e^{-st}\,dt = \left[-\frac{1}{s}te^{-st}\right]_0^\infty - \int_0^\infty -\frac{1}{s}e^{-st}\,dt$

$\int_0^\infty t \times e^{-st}\,dt = 0 + \frac{1}{s}\int_0^\infty e^{-st}\,dt = \frac{1}{s^2}$

37. $\int_0^\infty t^2 \times e^{-st}\,dt$

$\int\{f(x)g(x)\}' = \int f'(x)g(x) + \int f(x)g'(x)$

$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$

$f(x) = t^2,\ g'(x) = e^{-st}$

$f'(x) = 2t,\ g(x) = -\frac{1}{s}e^{-st}$

$\int_0^\infty t^2 \times e^{-st}\,dt = \left[-\frac{1}{s}t^2e^{-st}\right]_0^\infty - \int_0^\infty -\frac{1}{s}2te^{-st}\,dt$

$\int_0^\infty t^2 \times e^{-st}\,dt = 0 + \frac{2}{s}\int_0^\infty te^{-st}\,dt = \frac{2}{s^3}$

38. $\int_0^\infty e^{2t} \times e^{-st}\,dt$

$\int_0^\infty e^{-(s-2)t}\,dt = \left[-\frac{1}{s-2}e^{-(s-2)t}\right]_0^\infty = \frac{1}{s-2}$

39. $\int_0^\infty cos(7t) \times e^{-st}\,dt$

$\int_0^\infty \frac{1}{2}\left(e^{7it} + e^{-7it}\right) \times e^{-st}\,dt = \frac{1}{2}\int_0^\infty \left(e^{7it} + e^{-7it}\right) \times e^{-st}\,dt$

$\frac{1}{2}\int_0^\infty e^{-(s-7i)t} + e^{-(s+7i)t}\,dt$

$\frac{1}{2}\left(\frac{1}{s-7i} + \frac{1}{s+7i}\right) = \frac{1}{2}\frac{2s}{s^2+49} = \frac{s}{s^2+49}$

40. $\int_0^\infty sin(2t) \times e^{-st}\,dt$

$\frac{1}{2i}\int_0^\infty \left(e^{2it} - e^{-2it}\right) \times e^{-st}\,dt = \frac{1}{2i}\int_0^\infty e^{-(s-2i)t} - e^{-(s+2i)t}\,dt$

$\frac{1}{2i}\left(\frac{1}{s-2i} - \frac{1}{s+2i}\right) = \frac{1}{2i}\frac{4i}{s^2+4} = \frac{2}{s^2+4}$

41. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots$

42. 무한번 미분이 가능해야함

43. $y'' + 3y' + 2y = 0,\ y(0) = 1,\ y'(0) = 1$

$s^2Y(s) - sy(0) - y'(0) + 3\{sY(s) - y(0)\} + 2Y(s) = 0$

$s^2Y(s) - s - 1 + 3sY(s) - 3 + 2Y(s) = 0$

$\left(s^2 + 3s + 2\right)Y(s) = s + 4$

$Y(s) = \frac{s+4}{s^2+3s+2} = \frac{s+2+2}{(s+2)(s+1)} = \frac{s+2}{(s+2)(s+1)} + \frac{2}{(s+2)(s+1)}$

$\frac{s+2}{(s+2)(s+1)} + \frac{2}{(s+2)(s+1)} = \frac{1}{s+1} + \frac{2}{(s+2)(s+1)}$

$\Rightarrow \frac{2}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$

$2 = As + A + Bs + 2B$

$A + B = 0$

$A + 2B = 2$

$B = 2,\ A = -2$

$Y(s) = \frac{1}{s+1} - \frac{2}{s+2} + \frac{2}{s+1}$

$y(t) = e^{-t} - 2e^{-2t} + 2e^{-t} = 3e^{-t} - 2e^{-2t}$

44. $y' + 4y = 3,\ y(0) = 1$

$$sY(s) - y(0) + 4Y(s) = \frac{3}{s}$$
$$s^2Y(s) - s + 4sY(s) = 3$$
$$(s^2 + 4s)Y(s) = s + 3$$
$$Y(s) = \frac{s+3}{s(s+4)} = \frac{s+4-1}{s(s+4)}$$
$$\frac{s+4-1}{s(s+4)} = \frac{s+4}{s(s+4)} - \frac{1}{s(s+4)}$$
$$\frac{1}{s} - \frac{1}{s(s+4)}$$
$$\Rightarrow \frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$
$$1 = As + Bs + 4B$$
$$A + B = 0$$
$$4B = 1$$
$$B = \frac{1}{4},\ A = -\frac{1}{4}$$
$$Y(s) = \frac{1}{s} - \frac{1}{4}\frac{1}{s} + \frac{1}{4}\frac{1}{s+4}$$
$$y(t) = 1 - \frac{1}{4} + \frac{1}{4}e^{-4t} = \frac{3}{4} + \frac{1}{4}e^{-4t}$$

45. $\mathrm{Y}(\mathrm{s}) = \frac{9\mathrm{s}e^{-3s}}{s^2+3}$

$$\mathcal{L}[cos(at)] = \frac{s}{s^2+a^2}$$
$$\mathcal{L}[H(t-a)] = e^{-as}$$
$$Y(s) = 9\frac{s}{s^2+3}e^{-3s}$$
$$y(t) = 9cos\{\sqrt{3}(t-3)\}H(t-3)$$

46. $f(x) = \begin{cases} 0 & (-\pi < x < 0) \\ x & (0 < x < \pi) \end{cases}$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left\{a_n cos\left(\frac{n\pi}{T}x\right) + b_n sin\left(\frac{n\pi}{T}x\right)\right\}$$
$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x\,dx\right)$$
$$a_0 = \frac{1}{2\pi}\left(0 + \left[\frac{1}{2}x^2\right]_0^{\pi}\right) = \frac{1}{2\pi}\frac{1}{2}\pi^2 = \frac{\pi}{4}$$
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) cos\left(\frac{n\pi}{\pi}x\right)dx$$
$$a_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x\,cos(nx)dx\right\}$$
$$\int\{f(x)g(x)\}' = \int f'(x)g(x) + \int f(x)g'(x)$$
$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$$
$$f(x) = x,\ g'(x) = cos(nx)$$
$$f'(x) = 1,\ g(x) = \frac{1}{n}sin(nx)$$
$$\int_0^{\pi} x\,cos(nx) = \left[\frac{x}{n}sin(nx)\right]_0^{\pi} - \int_0^{\pi}\frac{1}{n}sin(nx)$$
$$\int_0^{\pi} x\,cos(nx) = 0 + \frac{1}{n}\left[\frac{1}{n}cos(nx)\right]_0^{\pi} = \frac{cos(n\pi)-1}{n^2}$$
$$a_n = \frac{cos(n\pi)-1}{\pi n^2}$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) sin\left(\frac{n\pi}{\pi}x\right)dx$$
$$b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x\,sin(nx)dx\right\}$$
$$f(x) = x,\ g'(x) = sin(nx)$$
$$f'(x) = 1,\ g(x) = -\frac{1}{n}cos(nx)$$
$$\int_0^{\pi} x\,sin(nx) = \left[-\frac{x}{n}cos(nx)\right]_0^{\pi} + \int_0^{\pi}\frac{1}{n}cos(nx)$$
$$\int_0^{\pi} x\,sin(nx) = -\frac{\pi cos(n\pi)}{n}$$
$$b_n = -\frac{cos(n\pi)}{n}$$
$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{cos(n\pi)-1}{\pi n^2}cos(nx) - \frac{cos(n\pi)}{n}sin(nx)\right\}$$

47. $40q_1' + 120(q_1 - q_2) = 10$

$60q_2' + 120q_2 = 120(q_1 - q_2)$

$40sQ_1(s) + 120Q_1(s) - 120Q_2(s) = \frac{10}{s}$

$60sQ_2(s) + 120Q_2(s) = 120Q_1(s) - 120Q_2(s)$

$4sQ_1(s) + 12Q_1(s) - 12Q_2(s) = \frac{1}{s}$

$sQ_2(s) + 2Q_2(s) = 2Q_1(s) - 2Q_2(s)$

$(s+4)Q_2(s) = 2Q_1(s)$

$Q_2(s) = \frac{2}{s+4}Q_1(s)$

$4sQ_1(s) + 12Q_1(s) - \frac{24}{s+4}Q_1(s) = \frac{1}{s}$

$\{(4s)(s+4) + 12(s+4) - 24\}Q_1(s) = \frac{1}{s}$

$(4s^2 + 16s + 12s + 48 - 24)Q_1(s) = \frac{s+4}{s}$

$Q_1(s) = \frac{1}{s}\frac{s+4}{4s^2+28s+24} = \frac{s+4}{4s(s+1)(s+6)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+6}$

$s + 4 = 4(s+1)(s+6)A + 4s(s+6)B + 4s(s+1)C$

$s + 4 = (4s^2 + 28s + 24)A + (4s^2 + 24s)B + (4s^2 + 4s)C$

$4A + 4B + 4C = 0$

$28A + 24B + 4C = 1$

$24A = 4$

$A = \frac{1}{6} \Rightarrow 4B + 4C = -\frac{2}{3},\ 24B + 4C = -\frac{11}{3}$

$B = -\frac{3}{20},\ C = -\frac{1}{60}$

$Q_1(s) = \frac{1}{6}\frac{1}{s} - \frac{3}{20}\frac{1}{s+1} - \frac{1}{60}\frac{1}{s+6}$

$q_1(t) = \frac{1}{6} - \frac{3}{20}e^{-t} - \frac{1}{60}e^{-6t}$

$Q_2(s) = \frac{2}{s+4}\frac{s+4}{4s(s+1)(s+6)} = \frac{1}{2s(s+1)(s+6)}$

$Q_2(s) = \frac{1}{2s(s+1)(s+6)} = \frac{\mathrm{A}}{\mathrm{s}} + \frac{\mathrm{B}}{s+1} + \frac{C}{s+6}$

$1 = 2(s+1)(s+6)A + 2s(s+6)B + 2s(s+1)C$

$1 = (2s^2 + 14s + 12)A + (2s^2 + 12s)B + (2s^2 + 2s)C$

$2A + 2B + 2C = 0$

$14A + 12B + 2C = 0$

$12A = 1$

$A = \frac{1}{12}$

$2B + 2C = -\frac{1}{6}$

$12B + 2C = -\frac{7}{6}$

$B = -\frac{1}{10}$

$C = \frac{1}{60}$

$Q_2(s) = \frac{1}{12}\frac{1}{s} - \frac{1}{10}\frac{1}{s+1} + \frac{1}{60}\frac{1}{s+6}$

$q_2(t) = \frac{1}{12} - \frac{1}{10}e^{-t} + \frac{1}{60}e^{-6t}$

$i_1(t) = \frac{dq_1(t)}{dt} = \frac{3}{20}e^{t} + \frac{1}{10}e^{-6t}$

$i_2(t) = \frac{dq_2(t)}{dt} = \frac{1}{10}e^{-t} - \frac{1}{10}e^{-6t}$

48. $V_{C1}(t) = \frac{1}{C_1}\{q_1(t) - q_2(t)\}$ *or* $10 - i_1(t)R_1$

$120\left\{\left(\frac{1}{6} - \frac{3}{20}e^{-t} - \frac{1}{60}e^{-6t}\right) - \left(\frac{1}{12} - \frac{1}{10}e^{-t} + \frac{1}{60}e^{-6t}\right)\right\} = 20 - 18e^{-t} - 2e^{-6t} - 10 + 12e^{-t} - 2e^{-6t} = 10 - 6e^{-t} - 4e^{-6t}$

$V_{C2}(t) = \frac{1}{C_2}q_2(t) = 120\left(\frac{1}{12} - \frac{1}{10}e^{-t} + \frac{1}{60}e^{-6t}\right) = 10 - 12e^{-t} + 2e^{-6t}$

same as $V_{C1}(t) - i_2(t)R_2$

49. $10\{H(t)-H(t-1)\}=q_1'R_1+\frac{q_1}{C_1}$

$\left(\frac{q_2}{C_1}+q_2'R_2\right)H(t-1)=0$

$\frac{10}{s}(1-e^{-s})=sQ_1(s)10^3+10^6Q_1(s)$

$\frac{1}{s}(1-e^{-s})=10^2(s+10^3)Q_1(s)$

$Q_1(s)=10^2\frac{1}{s}\frac{1}{s+10^3}(1-e^{-s})$

$\frac{10^2}{s(s+10^3)}=\frac{A}{s}+\frac{B}{s+1000}$

$100=As+1000A+Bs$

$A+B=0,\ 1000A=100$

$A=\frac{1}{100000},\ B=-\frac{1}{100000}$

$\mathrm{Q_1(s)}=\left(\frac{1}{100000}\frac{1}{s}-\frac{1}{100000}\frac{1}{s+1000}\right)(1-e^{-s})$

$q_1(t)=\left(\frac{1}{100000}-\frac{1}{100000}e^{-1000t}\right)H(t)-\left\{\frac{1}{100000}-\frac{1}{100000}e^{-1000(t-1)}\right\}H(t-1)$

$V_{C1}(t)=10^6q_1(t)=10\left[\left(1-e^{-1000t}\right)H(t)-\left\{1-e^{-1000(t-1)}\right\}H(t-1)\right]$

$i_1(t)=\frac{1}{100}e^{-1000t}H(t)-\frac{1}{100}e^{-1000(t-1)}H(t-1)$

$10\left[\left(1-e^{-1000t}\right)H(t)-\left\{1-e^{-1000(t-1)}\right\}H(t-1)\right]+10^4i_2=0$

$i_2(t)=10^{-3}\left[\left(1-e^{-1000t}\right)H(t)-\left\{1-e^{-1000(t-1)}\right\}H(t-1)\right]$

50. $10\{H(t)-H(t-1)\} = L\frac{di_1}{dt} + V_{C1}$

실제 현실에 맞게 콘덴서 값을 1mF, 인덕터 값을 100uF 로 수정하여 해석

$$V_{C1} + i_2 R_2 = 0$$

$$10\{H(t)-H(t-1)\} = L\frac{d^2(q_1-q_2)}{dt^2} + \frac{(q_1-q_2)}{C_1}$$

$$\frac{q_1}{C_1} + \frac{d(q_2-q_1)}{dt}R_2 = 0$$

$$10\{H(t)-H(t-1)\} = 10^{-4}(q_1'' - q_2'') + 10^3 q_1$$

$$10^3 q_1 + 10^4(q_2' - q_1') = 0$$

$$\{H(t)-H(t-1)\} = 10^{-5}(q_1'' - q_2'') + 10^2 q_1$$

$$q_1 + 10(q_2' - q_1') = 0$$

$$\frac{(1-e^{-s})}{s} = 10^{-5}s^2\big(Q_1(s) - Q_2(s)\big) + 10^2 Q_1(s)$$

$$Q_1(s) + 10s\big(Q_2(s) - Q_1(s)\big) = 0$$

$$10sQ_2(s) = (10s-1)Q_1(s)$$

$$Q_2(s) = \frac{10s-1}{10s}Q_1(s)$$

$$\frac{(1-e^{-s})}{s} = 10^{-5}s^2 Q_1(s) - 10^{-5}s^2\frac{10s-1}{10s}Q_1(s) + 10^2 Q_1(s)$$

$$\big(10^{-5}s^2 - 10^{-6}s(10s-1) + 10^2\big)Q_1(s) = \frac{(1-e^{-s})}{s}$$

$$\big(10^{-5}s^2 - 10^{-7}s^2 + 10^{-6}s + 10^2\big)Q_1(s) = \frac{(1-e^{-s})}{s}$$

$$\big(10^2 s^2 - s^2 + 10s + 10^9\big)Q_1(s) = \frac{(1-e^{-s})}{s}10^7$$

$$\big(99s^2 + 10s + 10^9\big)Q_1(s) = \frac{(1-e^{-s})}{s}10^7$$

$$Q_1(s) = \frac{10^7}{s(99s^2+10s+10^9)}(1-e^{-s})$$

$$\frac{10^7}{(99s^2+10s+10^9)} = \frac{10^7}{\left\{s-\left(\frac{-5+i\times5\sqrt{3959999999}}{99}\right)\right\}\left\{s-\left(\frac{-5-i\times5\sqrt{3959999999}}{99}\right)\right\}}$$

$$s = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-10\pm\sqrt{100-4\times99\times10^9}}{198}$$

$$\sqrt{100-4\times99\times10^9} = i\times10\sqrt{3959999999}$$

$$s = \frac{-10\pm i\times10\sqrt{3959999999}}{198}$$

$$\therefore s = \frac{-5\pm i\times5\sqrt{3959999999}}{99}$$

Alternate forms: More forms

$$\frac{(99s+i5\sqrt{3959999999}+5)(99s-5i\sqrt{3959999999}+5)}{9801}$$

$$\frac{(99s+5)^2}{9801} + \frac{98999999975}{9801}$$

$$s\left(s+\frac{10}{99}\right) + \frac{1000000000}{99}$$

다음 페이지에 이어서 해석을 진행한다.

$$Q_1(s) = \frac{10^7}{s(99s^2 + 10s + 10^9)}(1 - e^{-s}) = \frac{10^7}{s\left\{s - \left(\frac{-5 + i \times 5\sqrt{3959999999}}{99}\right)\right\}\left\{s - \left(\frac{-5 - i \times 5\sqrt{3959999999}}{99}\right)\right\}}(1 - e^{-s})$$

$$\frac{-5 + i \times 5\sqrt{3959999999}}{99}, \qquad \frac{-5 - i \times 5\sqrt{3959999999}}{99}$$

$$(a - b + ic)(a - b - ic) = (a - b)^2 + c^2$$

$$(a + b + ic)(a + b - ic) = (a + b)^2 + c^2$$

$$10^7 \frac{1}{s}\left\{\frac{1}{\left(s + \frac{5}{99}\right)^2 + \left(\frac{5\sqrt{3959999999}}{99}\right)^2}\right\} = \frac{A}{s} + \frac{Bs + C}{99s^2 + 10s + 10^9}$$

$$10^7 = \left(99s^2 + 10s + 10^9\right)A + Bs^2 + Cs$$

$$99A + B = 0$$

$$10A + C = 0$$

$$10^9 A = 10^7$$

$$A = 10^{-2}, \qquad B = -\frac{99}{100}, \qquad C = -\frac{1}{10}$$

$$10^7 \frac{1}{s}\left\{\frac{1}{\left(s + \frac{5}{99}\right)^2 + \left(\frac{5\sqrt{3959999999}}{99}\right)^2}\right\} = 10^{-2}\frac{1}{s} - \frac{99}{100}\left(\frac{s}{99s^2 + 10s + 10^9}\right) - \frac{1}{10}\left(\frac{1}{99s^2 + 10s + 10^9}\right)$$

우리는 위의 켤레복소수 형태의 분모가 sin 과 cos 형태로 나타날 것임을 알고 있다.
또한 위의 형태는 Time Shifting 이 발생한 형태임을 알 수 있다.
이를 적용하도록 한다.

$$Q_1(s)=\frac{10^7}{s(99s^2+10s+10^9)}(1-e^{-s})=\left\{10^{-2}\frac{1}{s}-\frac{99}{100}\left(\frac{s}{99s^2+10s+10^9}\right)-\frac{1}{10}\left(\frac{1}{99s^2+10s+10^9}\right)\right\}(1-e^{-s})$$

$$\int_0^\infty sin(at)e^{-st}\,dt=\frac{a}{s^2+a^2},\qquad (a-b+ic)(a-b-ic)=(a-b)^2+c^2$$

$$\int_0^\infty sin(at)e^{pt}e^{-st}\,dt=\frac{1}{2i}\int_0^\infty\left(e^{iat}-e^{-iat}\right)e^{pt}e^{-st}\,dt=\frac{1}{2i}\int_0^\infty e^{-(s-p-ia)t}-e^{-(s-p+ia)t}\,dt$$

$$\frac{1}{2i}\left[-\frac{1}{s-p-ia}e^{-(s-p-ia)t}+\frac{1}{s-p+ia}e^{-(s-p+ia)t}\right]_0^\infty=\frac{1}{2i}\left[\frac{1}{s-p-ia}-\frac{1}{s-p+ia}\right]=\frac{1}{2i}\left[\frac{s-p+ia-s+p+ia}{(s-p)^2+a^2}\right]$$

$$\frac{1}{2i}\left[\frac{s-p+ia-s+p+ia}{(s-p)^2+a^2}\right]=\frac{1}{2i}\left[\frac{2ia}{(s-p)^2+a^2}\right]=\frac{a}{(s-p)^2+a^2}$$

$$\int_0^\infty cos(at)e^{pt}e^{-st}\,dt=\frac{1}{2}\int_0^\infty\left(e^{iat}+e^{-iat}\right)e^{pt}e^{-st}\,dt=\frac{1}{2}\int_0^\infty e^{-(s-p-ia)t}+e^{-(s-p+ia)t}\,dt$$

$$\frac{1}{2}\left[-\frac{1}{s-p-ia}e^{-(s-p-ia)t}-\frac{1}{s-p+ia}e^{-(s-p+ia)t}\right]_0^\infty=\frac{1}{2}\left[\frac{1}{s-p-ia}+\frac{1}{s-p+ia}\right]=\frac{1}{2}\left[\frac{s-p+ia+s-p-ia}{(s-p)^2+a^2}\right]$$

$$\frac{1}{2}\left[\frac{s-p+ia+s-p-ia}{(s-p)^2+a^2}\right]=\frac{1}{2}\left[\frac{2(s-p)}{(s-p)^2+a^2}\right]=\frac{s-p}{(s-p)^2+a^2}$$

$$Q_1(s)=\left[10^{-2}\frac{1}{s}-\frac{99}{100}\left\{\frac{s}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5\sqrt{3959999999}}{99}\right)^2}\right\}-\frac{1}{10}\frac{1}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5\sqrt{3959999999}}{99}\right)^2}\right](1-e^{-s})$$

$$\sqrt{\frac{99\times10^9-25}{99^2}}=\frac{5\sqrt{3959999999}}{99}\Rightarrow k=\sqrt{3959999999}$$

식이 너무 지저분 하기에 치환을 통해서 정리하도록 한다.

$$\sqrt{\frac{99\times 10^{9}-25}{99^{2}}}=\frac{5\sqrt{3959999999}}{99}\Rightarrow k=\sqrt{3959999999}$$

$$Q_1(s)=\left[10^{-2}\frac{1}{s}-\frac{99}{100}\left\{\frac{s}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right\}-\frac{1}{10}\frac{1}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right](1-e^{-s})$$

$$\left[10^{-2}\frac{1}{s}-\frac{99}{100}\left\{\frac{s+\frac{5}{99}-\frac{5}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right\}-\frac{1}{10}\frac{99}{5k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right]$$

$$10^{-2}\frac{1}{s}-\frac{99}{100}\left\{\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}-\frac{\frac{5}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right\}-\frac{1}{10}\frac{99}{5k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}$$

$$10^{-2}\frac{1}{s}-\frac{99}{100}\left\{\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}-\frac{1}{k}\frac{\frac{5}{99}k}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}\right\}-\frac{1}{10}\frac{99}{5k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^{2}+\left(\frac{5k}{99}\right)^{2}}$$

$$10^{-2}-\frac{99}{100}\left\{e^{-\frac{5}{99}t}cos\left(\frac{5k}{99}t\right)-\frac{1}{k}e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)\right\}-\frac{99}{50k}e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)$$

$$10^{-2}-\frac{99}{100}e^{-\frac{5}{99}t}cos\left(\frac{5k}{99}t\right)+\frac{99}{100k}e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)-\frac{99}{50k}e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)$$

$$10^{-2}-\frac{99}{100}e^{-\frac{5}{99}t}cos\left(\frac{5k}{99}t\right)+e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)\left(\frac{99}{100k}-\frac{99}{50k}\right)$$

$$10^{-2}-\frac{99}{100}e^{-\frac{5}{99}t}cos\left(\frac{5k}{99}t\right)-e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)\left(\frac{99}{100k}\right)$$

$$q_1(t)=\left[10^{-2}-\frac{99}{100}e^{-\frac{5}{99}t}\left\{cos\left(\frac{5k}{99}t\right)-\frac{1}{k}sin\left(\frac{5k}{99}t\right)\right\}\right]H(t)-\left[10^{-2}-\frac{99}{100}e^{-\frac{5}{99}(t-1)}\left\{cos\left(\frac{5k}{99}(t-1)\right)-\frac{1}{k}sin\left(\frac{5k}{99}(t-1)\right)\right\}\right]H(t-1)$$

$$\boldsymbol{Q_2(s)=\frac{10s-1}{10s}Q_1(s)=\frac{10s-1}{10s}\frac{10^7}{s(99s^2+10s+10^9)}(1-e^{-s})}$$
$$\boldsymbol{Q_2(s)=\frac{10^7s-10^6}{s^2(99s^2+10s+10^9)}(1-e^{-s})=\frac{10^6(10s-1)}{s^2(99s^2+10s+10^9)}(1-e^{-s})}$$
$$\boldsymbol{\frac{10^6(10s-1)}{s^2(99s^2+10s+10^9)}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{99s^2+10s+10^9}}$$
$$\boldsymbol{10^6(10s-1)=99As^3+10As^2+10^9As+99Bs^2+10Bs+10^9B+Cs^3+Ds^2}$$
$$\boldsymbol{99A+C=0}$$
$$\boldsymbol{10A+99B+D=0}$$
$$\boldsymbol{10^9A+10B=10^7\to 10^9A=10^7-10\left(-10^{-3}\right)\to A=10^{-2}+10^{-11}=\frac{1}{10^2}+\frac{1}{10^{11}}=\frac{10^9+1}{10^{11}}}$$
$$\boldsymbol{10^9B=-10^6}$$
$$\boldsymbol{A=\frac{1000000001}{10^{11}},\qquad B=-10^{-3}}$$
$$\boldsymbol{C=-99A}$$
$$\boldsymbol{\frac{1000000001}{10^{10}}-\frac{99}{10^3}=-D}$$
$$\boldsymbol{D=\frac{99\times10^7-\left(10^9+1\right)}{10^3\times10^7}}$$
$$\boldsymbol{C=-\frac{99000000099}{10^{11}},\qquad D=-\frac{10000001}{10^{10}}}$$
$$\boldsymbol{Q_2(s)=\left(\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{99s^2+10s+10^9}\right)(1-e^{-s})}$$
$$\boldsymbol{\frac{1000000001}{10^{11}}\frac{1}{s}-10^{-3}\frac{1}{s^2}-\frac{99000000099}{10^{11}}\frac{s}{99s^2+10s+10^9}-\frac{10000001}{10^{10}}\frac{1}{99s^2+10s+10^9}}$$
$$\boldsymbol{\sqrt{\frac{99\times10^9-25}{99^2}}=\frac{5\sqrt{3959999999}}{99}\Rightarrow k=\sqrt{3959999999}}$$
$$\boldsymbol{\frac{1000000001}{10^{11}}\frac{1}{s}-10^{-3}\frac{1}{s^2}-\frac{99000000099}{10^{11}}\frac{s}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-\frac{10000001}{10^{10}}\frac{1}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}}$$
$$\boldsymbol{\frac{1000000001}{10^{11}}\frac{1}{s}-10^{-3}\frac{1}{s^2}-\frac{99000000099}{10^{11}}\frac{s+\frac{5}{99}-\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-\frac{10000001}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}}$$

$$\frac{1000000001}{10^{11}}\frac{1}{s}-10^{-3}\frac{1}{s^2}-\frac{99000000099}{10^{11}}\frac{s+\frac{5}{99}-\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-\frac{10000001}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}$$

$$A\frac{1}{s}-B\frac{1}{s^2}-11\frac{9000000009}{10^{11}}\left\{\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\right\}-11\frac{909091}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}$$

$$A\frac{1}{s}-B\frac{1}{s^2}-11\frac{9000000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{9000000009}{10^{11}k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-11\frac{909091}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}$$

cos, sin 파트만 집중적으로 분석

$$-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{900000009}{10^{11}k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-11\frac{909091}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}$$

$$-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{900000009}{10^{11}k}-\frac{909091}{10^{10}}\frac{1}{k}\right)$$

$$-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{900000009}{10^{11}k}-\frac{909091}{10^{10}}\frac{1}{k}\frac{10}{10}\right)$$

$$-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{900000009}{10^{11}k}-\frac{9090910}{10^{11}k}\right)$$

$$-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{890909099}{10^{11}k}\right)$$

$$A\frac{1}{s}-B\frac{1}{s^2}-11\frac{9000000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{9000000009}{10^{11}k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}-11\frac{909091}{10^{10}}\frac{1}{k}\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}$$

$$A\frac{1}{s}-B\frac{1}{s^2}-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{890909099}{10^{11}k}\right)$$

$$Q_2(s)=\left\{\frac{1000000001}{10^{11}}\frac{1}{s}-10^{-3}\frac{1}{s^2}-11\frac{900000009}{10^{11}}\frac{s+\frac{5}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}+11\frac{\frac{5k}{99}}{\left(s+\frac{5}{99}\right)^2+\left(\frac{5k}{99}\right)^2}\left(\frac{890909099}{10^{11}k}\right)\right\}(1-e^{-s})$$

$$q_2(t)=\left[\begin{array}{c}\left\{\frac{1000000001}{10^{11}}-\frac{t}{10^3}-11\frac{900000009}{10^{11}}e^{-\frac{5}{99}t}cos\left(\frac{5k}{99}t\right)+11\frac{890909099}{10^{11}k}e^{-\frac{5}{99}t}sin\left(\frac{5k}{99}t\right)\right\}H(t)-\\ \left\{\frac{1000000001}{10^{11}}-\frac{t-1}{10^3}-11\frac{900000009}{10^{11}}e^{-\frac{5}{99}(t-1)}cos\left(\frac{5k}{99}(t-1)\right)+11\frac{890909099}{10^{11}k}e^{-\frac{5}{99}(t-1)}sin\left(\frac{5k}{99}(t-1)\right)\right\}H(t-1)\end{array}\right]$$

이를 기반으로 전류와 전압을 해석할 수 있게 된다.
현재 상황에서는 회로 상에서 공진이 발생하고 있는 것을 볼 수 있다.
즉 이와 같이 DC – DC Converter 를 설계하면 안된다!

51. $f(x) = \begin{cases} 0 & (-\pi < x < 0) \\ 1 & (0 < x < \pi) \end{cases}$

$f(x) = a_0 + \sum_{n=1}^{\infty}\left\{a_n cos\left(\frac{n\pi}{T}x\right) + b_n sin\left(\frac{n\pi}{T}x\right)\right\}$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} 1\,dx\right)$

$a_0 = \frac{1}{2\pi}(0 + [x]_0^{\pi}) = \frac{1}{2\pi}\pi = \frac{1}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) cos\left(\frac{n\pi}{\pi}x\right)dx$

$a_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} cos(nx)\,dx\right\}$

$\int_0^{\pi} cos(nx) = \left[\frac{1}{n} sin(nx)\right]_0^{\pi} = 0$

$a_n = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) sin\left(\frac{n\pi}{\pi}x\right)dx$

$b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} sin(nx)\,dx\right\}$

$\int_0^{\pi} sin(nx) = \left[-\frac{1}{n} cos(nx)\right]_0^{\pi}$

$\int_0^{\pi} sin(nx) = \frac{1-cos(n\pi)}{n}$

$b_n = \frac{1-cos(n\pi)}{n\pi}$

$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{1-cos(n\pi)}{n\pi} sin(nx)\right\}$

52. 코드 참고