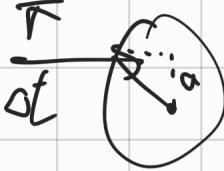


Physical Base

$$I_{\text{sphere}} : \frac{2}{5} m R^2$$

$$\frac{\tau}{\Delta t} = I \alpha \cdot \Delta t = I \omega = F \cdot a \Delta t$$

$$= \text{Sphere rotating with angular velocity } \omega \text{ and linear velocity } v = R\omega.$$
$$F_{\text{frict}} = m \Delta V$$

Constants.

μ_k , μ_r , e
 B_{mass} , B_{radius}
Cmass

For Coding:

Ball:

Current_Position. [P_x, P_y, P_z])

V [V_x, V_y, V_z])

W [W_x, W_y, W_z])

function
Hit_Ball ($\phi, \alpha, b, V_0, \theta$)

projection angle
hit point
cue property

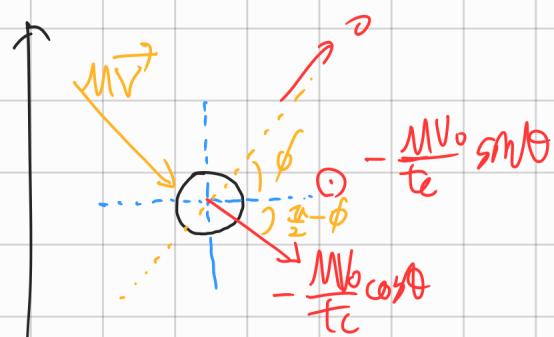
Collide (Ball_Pos, Coll_pos, V, W)

Proceed

Physics.

Impact

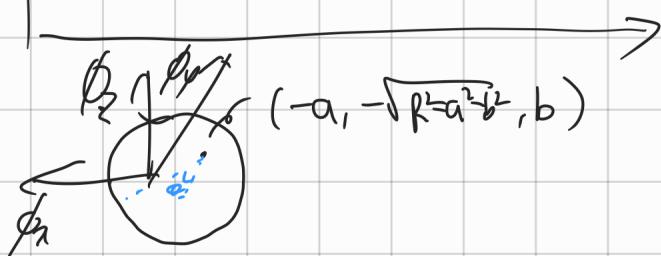
Cue property: V, M, θ, t_c
 Impact point: (ϕ, a, b)



$$MV = F t_c = m \vec{v}$$

$$\vec{V} = (0, |V| \cos \theta, -|V| \sin \theta)$$

$$\vec{F} = \left(0, \frac{MV_0}{t_c} \cos \theta, -\frac{MV_0}{t_c} \sin \theta \right)$$



$$\Rightarrow \vec{F}_{\text{conv}} = \left(F_{\phi_x} \cos \left(\frac{\pi}{2} - \phi \right), -F_{\phi_y} \sin \left(\frac{\pi}{2} - \phi \right), F_{\phi_z} \right)$$

$$= \left(F_{\phi_y} \sin \phi, -F_{\phi_y} \cos \phi, F_{\phi_z} \right)$$

$F_{\phi_z} = \begin{cases} F_{\phi_z} & \text{if } > 0 \\ 0 & \text{if } \leq 0. \end{cases}$

$$\Rightarrow \vec{r} = \frac{t_c}{m} \cdot \vec{F}_{\text{com}}$$



$$\vec{v} = \vec{r} \times \vec{F}$$

$$\vec{r} = (-a, -\sqrt{R^2 - a^2 - b^2}, b)$$

$$\vec{v}_{\text{com}} = (l_{\phi_x} \cos \phi + l_{\phi_y} \cos(\frac{\alpha}{2} - \phi), \\ l_{\phi_x} \sin \phi - l_{\phi_y} \sin(\frac{\alpha}{2} - \phi), l_{\phi_z})$$

$$= (l_{\phi_x} \cos \phi + l_{\phi_y} \sin \phi, \\ l_{\phi_x} \sin \phi - l_{\phi_y} \cos \phi, l_{\phi_z})$$

$$\vec{I} t_c = I \alpha \cdot t_c \\ = I \vec{\omega}$$

$$\Rightarrow \vec{\omega} = \frac{t_c}{I} \vec{v}$$

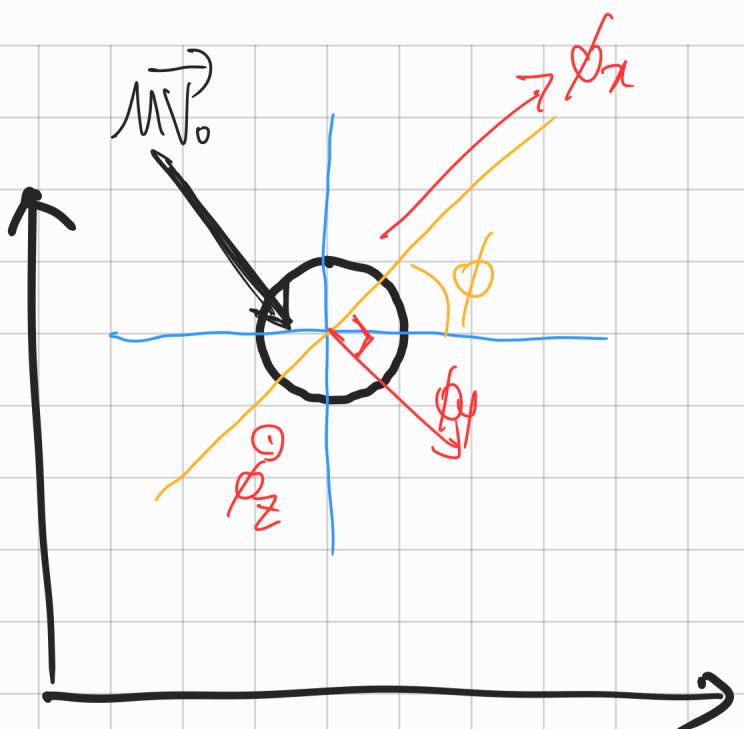
1. Hit the ball.

$$M\vec{V}_0 = \vec{F}t_c = m\vec{V}$$

$$\vec{V}_0 = (0, |V_0|\cos\theta, -|V_0|\sin\theta)$$

$$\vec{F} = \frac{M}{t_c} \vec{V}_0$$

$$= \left(0, \frac{M|V_0|}{t_c} \cos\theta, -\frac{M|V_0|}{t_c} \sin\theta\right)$$



$$\vec{F}_{com} = (F_{\phi y} \cos(\frac{\pi}{2} - \phi), -F_{\phi y} \sin(\frac{\pi}{2} - \phi), F_{\phi z}) \quad (a, -\sqrt{a^2 + b^2}, b)$$

$$= (F_{\phi y} \cdot \sin\phi, -F_{\phi y} \cdot \cos\phi, F_{\phi z})$$

$$a^2 + b^2 \leq R^2$$

$$\vec{v} = \frac{t_c}{m} \vec{F}_{com}$$

$$F_{\phi z} = \begin{cases} F_{\phi z} & \text{if } > 0 \\ 0 & \text{if } \leq 0 \end{cases}$$

$$\vec{Z} = \vec{F} \times \vec{F}, \quad \vec{R} = (a, -\sqrt{a^2 + b^2}, b)$$

$$\begin{aligned}\vec{Z}_{\text{conv}} &= (Z_{\phi_x} \cos \phi + Z_{\phi_y} \cos(\frac{\pi}{2} - \phi), \\ &\quad Z_{\phi_x} \sin \phi - Z_{\phi_y} \sin(\frac{\pi}{2} - \phi), Z_{\phi_z})\end{aligned}$$

$$= (Z_{\phi_x} \cos \phi + Z_{\phi_y} \sin \phi, Z_{\phi_x} \sin \phi - Z_{\phi_y} \cos \phi, Z_{\phi_z})$$

$$\vec{Z} t_c = J \alpha t_c = \vec{J \omega}$$

$$\therefore \vec{\omega} = \frac{t_c}{J} \vec{Z}$$