**Trajactory Simulation of Billiard Ball**

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We chose billiard ball's trajectory motion simulation as our team project title. In real world, billiard balls rarely proceed without spins. So, we decide to add a spin-considered model to express it more realistically. Basically, we want to configure the field through a 2-d array and assume a situation in which hitting the ball by cue stick. The physical concepts we will apply are impulse-momentum, momentum conservation, angular momentum conservation, energy conservation. And the variables to be considered include hitting position, hitting force, strike time, restitution coefficient on the wall, and friction. First, we will construct the model of the simulation for the theoretical situation without friction and spin, and secondly, we will construct the model of the simulations for situations with spin. In addition, we want to simulate the shape of billiard tables not only in rectangles but also regular-triangle, circle, ellipses, and case of with obstacles.

**Introduction**

We often find physical phenomena associated with collisions around us. One of the most common situations in which collisions occur is billiard games. In addition, billiards are a comfortable situation in which humans can actually manipulate the collision phenomenon in their daily lives. So we're going to target the simulation with billiard games and proceed with this project because we wanted to learn what physical phenomena are applied to billiard balls and how they are involved in the movement of balls by implementing the collision phenomenon that we had only learned from theory in code. Our simulations will simulate like the moment when a cue stick hits a billiard ball for similar a real situation. To do so, we thought billiard balls were not just a dot, but rather a ball of size and volume. Also, we want to describe the trajectory of billiard balls by considering the position of hitting, strength, and hitting time as variables. In addition, if there are trajectories, obstructions in various shapes of billiards, and if there is a spin in the billiard ball that is identical to the actual situation, we will simulate any trajectory.

In the Background study section, we will describe the initial conditions of billiard game. Our simulations are simulated with the assumption that the ball is moving by hitting it with a cue stick, and the initial conditions required will initiate the trajectory movement of the ball by specifying the initial position of the ball, the position of the ball hit with a cue stick, and the rate of strike as initial variables. And we will indicate the trajectory of the ball by taking into account the law of conservation of momentum, the friction between the ball and the billiard table. We will also describe the physical formulas and constants used in the code. And then in the code implementation section, We will describe the assumption that we used to code and explain where they were applied. We will also describe the structure of the code. Lastly the results section, show a comparison of ball trajectories with or without spin and a comparison of ball trajectories with table shape.

**Background study**

The most representative physical phenomenon in billiard games is the law of conservation of momentum.

The law of momentum conservation means the same amount of momentum as the initial amount of momentum and later, which we can calculate the speed after the collision through the law of momentum conservation when the billiard ball hits the billiard table or when the billiard ball hits an obstacle.

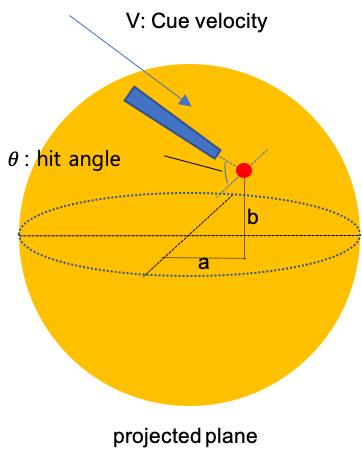


Figure 1. hitting the ball by cue stick

The figure 1 describes the initial conditions when the cue stick hits the ball. When the cue stick hit the ball, the variables about hitting are divided two part as hit point part - () and the cue properties- (). For preventing jump shot, the hitting angle should be limited by . And using the conservation of momentum, we can derive the ball’s state, linear/angular velocity right after hitting.

The next is an explanation of the situation when the ball proceeds. The motion of the ball is divided into sliding and rolling according to the relative velocity between the center and floor. Because the coefficients of friction by rolling and sliding are not same, we must construct the computation by different way.

The moment when the ball moves from sliding to rolling is when the ball's center velocity and the relative velocity of the ball's contact point are zero. As shown in the above equation, the coefficient of friction when sliding and rolling are different, requiring different equations. Also, the coefficient of friction between spin and sliding motion is different.

While recording trajectory, we should detect collision by obstacles (include rails). If there is an obstacle at the distance from the center to Rb, using the coefficient of restitution and friction, we must control the collision.

Through the above process, we will express the trajectory of the ball on a two-dimensional plane and analyze the distance of travel, the number of collisions, according to various variables.

**Conclusion**

We would like to understand in more detail the collision we had learned only from theory through billiard ball trajectory simulation. Our simulation has two steps: the first is the initialization process, which creates a billiard table, positions the initial billiard ball, and sets the hitting point and speed of the cue stick. The second is a record of trajectory motion. The ball's exercise is divided into sliding and rolling exercises, and each has different coefficient of friction, so we have to consider it separately.

Through this theoretical background, we will express the trajectory of the billiard ball movement in a two-dimensional plane, analyze the distance moved, and analyze the number of collisions.