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Asymmetric multi-fractality in the U.S. stock indices using index-based model of A-MFDFA

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ABSTRACT

We detect the asymmetric multi-fractality in the U.S. stock indices based on the asymmetric multi-fractal detrended fluctuation analysis (A-MFDFA). Instead using the conventional return-based approach, we propose the index-based model of A-MFDFA where the trend based on the evolution of stock index rather than stock price return plays a role for evaluating the asymmetric scaling behaviors. The results show that the multi-fractal behaviors of the U.S. stock indices are asymmetric and the index-based model detects the asymmetric multi-fractality better than return-based model. We also discuss the source of multi-fractality and its asymmetry and observe that the multi-fractal asymmetry in the U.S. stock indices has a time-varying feature where the degree of multi-fractality and asymmetry increase during the financial crisis.

1. Introduction

The multi-fractal analysis has been applied to investigate various stylized facts of the financial market including market efficiency [1–4], financial crisis [5], risk evaluation [6], stock markets [7–9] and crash prediction [10]. Specifically, the multi-fractal detrended fluctuation analysis (MF-DFA), a generalization of the detrended fluctuation analysis (DFA) [11], is a typical approach to measure the long-range autocorrelations and multi-fractality of a time-series [12]. Both DFA and MF-DFA also have been widely applied in various fields such as DNA sequences [13], heart rate dynamics [14], long-range weather records [15,16], exchange rate dynamics [17] and oil market [18].

Recently, there have been a number of studies in the asymmetric correlation in the financial market [19–21]. Longin and Solnik [19] show that the international market correlation increases at the extreme left-tail event based on extreme value theory. Ang and Chen [20] detect the strong asymmetric correlations between equity portfolios and the U.S. aggregate market. Ding et al. [21] examine potential fundamental of asymmetric correlation of stock portfolio. Therefore, the research on the asymmetric correlations within the financial market can provide an understanding of the asymmetric features of risk, which can be applied to enhance the portfolio in terms of diversification and risk management.

It is commonly accepted fact that there are two trends of stock market, namely bullish and bearish markets, and they should be treated differently in analyzing the multi-fractal scaling behavior and correlation. However, there are limited numbers of studies focusing on measuring the asymmetric multi-fractality. Alvarez-Ramirez et al. [22] introduce the asymmetric DFA (A-DFA) to examine asymmetric correlations in the scaling behavior of time-series. Based on A-DFA, Cao et al. [23] propose the asymmetric multi-fractal detrended fluctuation analysis (A-MFDFA) method to extend MF-DFA methods, whereas Zhang et al. [24] introduce the asymmetric multi-fractal detrending moving average analysis (A-MFDMA) method to extend MF-DMA [25] to quantify the long-term correlations of non-stationary time-series.

Interestingly, A-MFDFA method and A-MFDMA methods demonstrate the distinct scaling properties in two different market trends where the up- and down-trends are distinguished based on the linear regression of return dynamics. However, we claim that the gain of portfolio profit is achieved when the market index moves up and the loss of portfolio profit is realized when the market index moves down. That is, the index dynamics can be a better proxy of market trend. Based on this idea, we provide the new model named ‘index-based A-MFDFA’ which employs the index dynamics as more intuitive criterion for separating the market trends. In addition, to distinguish between our new model and conventional A-MFDFA method, we call conventional model as ‘return-based A-MFDFA’ in this paper. We employ ‘index-based A-MFDFA’ method for analyzing the stock indices of the United States so that the existence of asymmetric multi-fractal scaling

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behavior can be observed. Furthermore, we also analyze the stock indices using 'return-based A-MFDFA' to compare with our result as a reference. Based on two models, we discuss the empirical difference of two models and features of scaling behavior. Furthermore, we investigate the scaling asymmetries, source of the multi-fractality and asymmetry. Lastly, we explore the time-varying feature of asymmetric scaling behavior. This research is based on our initial work [26] and the contents of this paper is the upgrade and the completion of our previous work.

This paper is organized as follows: Chapter 2 proposes the definition and step-by-step scenario of the return- and index-based model for A-MFDFA; Chapter 3 describes the statistical features of data; Chapter 4 discusses the empirical findings of this paper; and Chapter 5 concludes.

2. Index-based A-MFDFA

We can investigate the asymmetric multifractal scaling behavior with different trends using the A-MFDFA method. Cao et al [23] proposed the 'return-based A-MFDFA' method. We modify 'return-based A-MFDFA' method and introduce the 'index-based A-MFDFA' method, which use index criterion for dividing the market trend. We have a time-series $\{x_t:\ t = 1, 2, \ldots, N\}$. Our proposed method has the following steps.

***Step 1**:* Define $\mathrm{y_t} = \sum_{j=1}^{t} (x_j - \bar{x})$, $t = 1, 2, \ldots, N$ where $\bar{x} = \sum_{j=1}^{N} x_j / N$.

***Step 2**:* Divide time-series into non-overlapping sub-time series

Let $I_t = I_{t-1} exp(x_t)$ for $t = 1, 2, \ldots, N$ where $I_0 = 1$ and I_t is an indexing proxy for return time-series. We divide $\{I_t : t = 1, 2, \ldots, N\}$ and $\{\mathrm{y_t} : t = 1, 2, \ldots, N\}$ into $N_n \equiv N/n$ non-overlapping sub-time series of equal length n, where x is the largest integer less than or equal to x. We repeat this procedure from the other end of $\{I_t\}$ and $\{\mathrm{y_t}\}$ respectively, resulting in $2N_n$ sub-time series. Suppose $G_j = \{g_{j,k},\ k = 1,\ 2,\ \ldots,\ n\}$ be the length n sub-time series of $\{I_t\}$ in the j^{th} time interval and $H_j = \{h_{j,k}, k = 1, 2, \ldots, n\}$ be the j^{th} sub-time series of $\{\mathrm{y_t}\}$ for $j = 1, 2, \ldots, 2N_n$. Then, we have $g_{j,k} = I_{(j-1)n+k}$ and $h_{j,k} = y_{(j-1)n+k}$ for $j = 1,\ 2,\ \ldots,\ N_n$, and for $j = N_n + 1,\ \ldots,\ 2N_n$ $g_{j,k} = I_{N-(j-N_n)n+k}$ and $h_{j,k} = y_{N-(j-N_n)n+k}$. Peng et al. [11] suggest that $5 \le n \le N/4$.

***Step 3**:* Construct the fluctuation function

For each sub-time series G_j and H_j, we calculate the local trend by least-squares fits $L_{G_j}(k) = a_{G_j} + b_{G_j} k$ and $L_{H_j}(k) = a_{H_j} + b_{H_j} k$, where k is for the horizontal coordinate. The slope of $L_{G_j}(k)$, b_{G_j}, is used to discriminate whether the trend of G_j is positive or negative. The linear fitting equation, L_{H_j} is used to detrend the integrated time-series H_j. We define the fluctuation function as $F_j(n) = \sum_{k=1}^{n} (y_{j,k} - L_{H_j}(k))^2 / n$ for $j = 1, 2, \ldots, 2N_n$.

***Step 4**:* Identify trend using index dynamics

Assuming that $\{I_t\}$ has piecewise positive and negative linear trends, the asymmetric cross-correlation scaling property of fluctuation functions can be assessed by the sign of the slope, b_{G_j}. When $b_{G_j} > 0$, the sub-time series G_j of $\{I_t\}$ has a positive trend. By contrast, $b_{G_j} < 0$ indicates that the sub-time series G_j of $\{I_t\}$ exhibits a negative trend.

***Step 5**:* Construct q-order average fluctuation functions

The directional q-order average fluctuation functions of index-based model (when $q \neq 0$) is computed by, $F_q^+(n) = (\sum_{j=1}^{2N_n} (1 + sgn(b_{G_j}))[F_j(n)]^{q/2} / M^+)^{1/q}$ and $F_q^-(n) = (\sum_{j=1}^{2N_n} (1 - sgn(b_{G_j}))[F_j(n)]^{q/2} / M^-)^{1/q}$ where $M^+ = \sum_{j=1}^{2N_n} (1 + sgn(b_{G_j}))$, $M^- = \sum_{j=1}^{2N_n} (1 - sgn(b_{G_j}))$, and $sgn(x)$ is the sign of x. Note that M^+ and M^- are twice the number of sub-time series with positive and negative trends, respectively. We assume that $b_{G_j} \neq 0$ and $M^+ + M^- = 4N_n$. The average fluctuation function of the traditional MF-DFA model also can be computed as $F_q(n) = (\sum_{j=1}^{2N_n} [F_j(n)]^{q/2} / (2N_n))^{1/q}$.

***Step 6**:* Calculating the generalized Hurst exponent

If a time-series has a long-range correlation, the following power-law relationship is observed. Let $H(q)$, $H^+(q)$, and $H^-(q)$ denote the overall, upward, and downward scaling exponents, which are called the generalized Hurst exponents, respectively. Specifically, the scaling satisfies, $F_q(n) \sim n^{H(q)}$, $F_q^+(n) \sim n^{H^+(q)}$, and $F_q^-(n) \sim n^{H^-(q)}$. Using the logarithmic form, $H(q)$, $H^+(q)$, and $H^-(q)$ can be obtained by the ordinary least square method. If $H(q)$ is constant for all q, the corresponding time series is mono-fractal. Otherwise, the time-series are multi-fractal. Note that the correlations in the time-series are persistent if $H(2) > 0.5$, whereas the correlations in the time-series are anti-persistent if $H(2) < 0.5$. If $H(2) = 0.5$, time-series follows random walk process [12].

Analogous to $H(q)$, the up-trend (down-trend) time-series are multi-fractal if the time-series shows positive (negative) trend. In addition, the correlations in the time-series are symmetric if $H^+(q) = H^-(q)$, whereas the correlations are asymmetric if $H^+(q) \neq H^-(q)$. The asymmetric scaling behavior means that the correlations are different between positive and negative trends.

Note that the 'return-based A-MFDFA' model, which is used as a benchmark for our 'index-based A-MFDFA' model, is construct using $\{x_t\}$ instead of $\{I_t\}$ in step 2 and analyzes the sub-time series trend of $\{x_t\}$ to separate the positive trend and negative trend in step 4.

3. Data

Our dataset consists of daily closing prices of the United States stock indices including the Dow Jones Industrial Average Index (DJIA), National Association of Securities Dealers Automated Quotations Composite Index (NASDAQ), New York Stock Exchange Composite Index (NYSE), and the Standard & Poor's 500 Index (S&P500). The experimental period of time-series is from 1991-01-01 to 2015-12-31. Then, we transform the price-series to the logarithmic return-series, $r_t = \log(P_t) - \log(P_{t-1})$, where P_t is the closing price of index at time t. Specifically, the sample sizes of DJIA, NASDAQ, NYSE, and S&P500 are 6290, 6293, 6291 and 6289 trading dates, respectively.

Fig. 1 and Table 1 demonstrate the evolutions of return series and their descriptive statistics for DJIA, NASDAQ, NYSE and S&P500, respectively. As shown in Fig. 1 and Table 1, the skewness of the entire return series is not zero where all series except for the case of NASDAQ are skewed left. Also, all series are fat-tailed and peaked since the kurtosis of them are greater than three. The JB statistics are all significant at 1% level, suggesting that the normality assumption of the distribution of all return series is rejected. Furthermore, the ADF test shows that the absence of unit root is rejected at the 1% significant level.

4. Empirical results of asymmetric scaling behavior

4.1. Asymmetric fluctuation functions and their dynamics

Fig. 2 illustrates the results of both return- (left) and index-based (right) models of A-MFDFA when q = 2, which describes how $log_2(F_2(n))$ changes with respect to $log_2(n)$. Note that the blue,

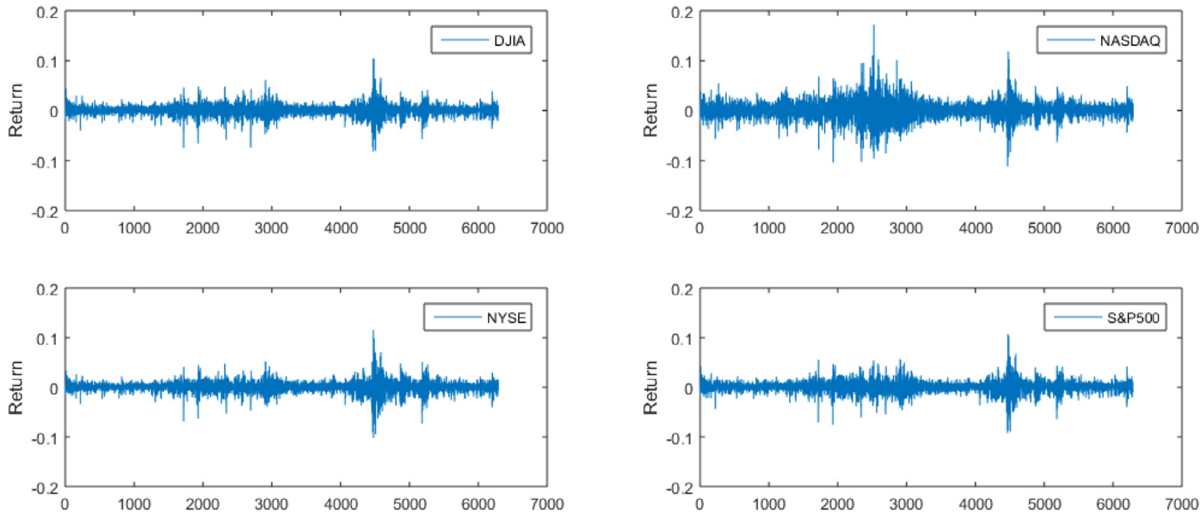

Fig. 1. Daily return series of indices.

Table 1
Descriptive statistics for the returns of indices.

	Mean	max	min	std	skew	kurt	JB	ADF
DJIA	0.0003	0.1051	−0.0820	0.0108	−0.15	11.32	18,173.9*	−83.4*
NASDAQ	0.0005	0.1720	−0.1111	0.0176	0.07	8.75	8674.5*	−82.5*
NYSE	0.0003	0.1153	−0.1023	0.0111	−0.39	13.92	31,429.5*	−82.2*
S&P500	0.0003	0.1066	−0.0919	0.0115	−0.19	11.07	17,109.8*	−84.8*

Note: “max”, “min”, “std.”, “skew” and “kurt” denote maximum, minimum, standard deviation, skewness and kurtosis, respectively. “JB” denotes Jarque-Bera statistics for normality test and “ADF” denotes the Augmented Dicky-Fuller(ADF) test for unit root test. * denotes 1% level of significance.

red, and yellow dots represent the overall, upwards, and downwards, respectively. It is well-known stylized fact that $log_2(F_2(n))$ vs. $log_2(n)$ possesses a power-law dependency where the straight dotted line indicates a decent power-law fit. In general, the asymmetry in fluctuation function is discovered within a single unit of time-scale where the distinctions between the values of uptrend and downtrend are observed through most of time-scale. Besides, the dynamics of fluctuation functions exhibit the symmetric evolution in accordance with the time-scale increment. In addition, the newly-suggested approach of index-based model clearly distinguishes the straight trends of upward and downward pivoting on the overall dots, whereas the conventional approach of return-based model shows the scattered dots with a weak straight trend. Hence, the results suggest that the index-based model provides more robust criterion of detecting the power-law scaling property. In other words, the index-based model performs better clustering of two different trends.

Furthermore, the fluctuation functions of trends show the reverse order of their values between the return- and index-based models. All cases of DJIA, NASDAQ, NYSE and S&P500, the descending order of the fluctuation functions of return-based model is upward, overall, and downward, whereas that of index-based model is downward, overall, and upward. Note that the higher value of fluctuation function implies the more volatility of the market.

Fig. 3 shows the plots of $Df = \log_2 F_2^+(n) - \log_2 F_2^-(n)$ versus n to visualize the asymmetry of fluctuation function. Based on Fig. 3, the shape of *Df* are similar among all indices in each model. Specifically, the return-based model shows many crossovers around zero, whereas the index-based model has much less cases of crossovers. Since $Df = 0$ indicates the symmetry between upward and downward, it is clear that the index-based model detects the asymmetry more explicitly than the return-based one. In addition, the mean *Df* values of return-based model for DJIA, NASDAQ, NYSE, S&P500 are 0.2320, −0.1793, 0.3293, 0.1466, respectively, whereas those of index-based one are −0.8415, −1.0045, −0.9424, −0.9187. Therefore, the downward trend has greater fluctuation function in index-based model as shown in Fig. 2.

4.2. Estimating the generalized Hurst exponent H(q)

Fig. 4 visualizes the values of generalized Hurst exponents $H(q)$, $H^+(q)$, and $H^-(q)$ with respect to q varying from −5 to 5 with interval of 0.1. The result shows that the values of $H(q)$, $H^+(q)$, and $H^-(q)$ decrease when q increases for the most of cases except for NASDAQ, whose values changing regardless of q orders. It refers that each series possess the multi-fractal feature regardless of the trend. In case of the return-based model, the gap between the uptrend and downtrend is small when q is small (i.e. small fluctuation), whereas the gap becomes larger as q increases. The large gap is analogous to the significance of asymmetry. In case of the index-based model, the coupling of overall and uptrend is observed, while the downward shows different trend. Furthermore, the gap between the upward and downward decreases as q increases.

4.3. Source of multi-fractality

In general, there are two major sources of multi-fractality [12,27]: (1) different long-range correlations for small and large fluctuations, and (2) fat-tailed probability distributions. It is also well known that the contribution of each source can be evaluated by comparing the multi-fractality of original and modified series. At first, the long-range correlation can be tested by comparing the multi-fractality between the original and randomly shuffled series. The step-by-step scenario of creating the randomly shuffled series is as follows:

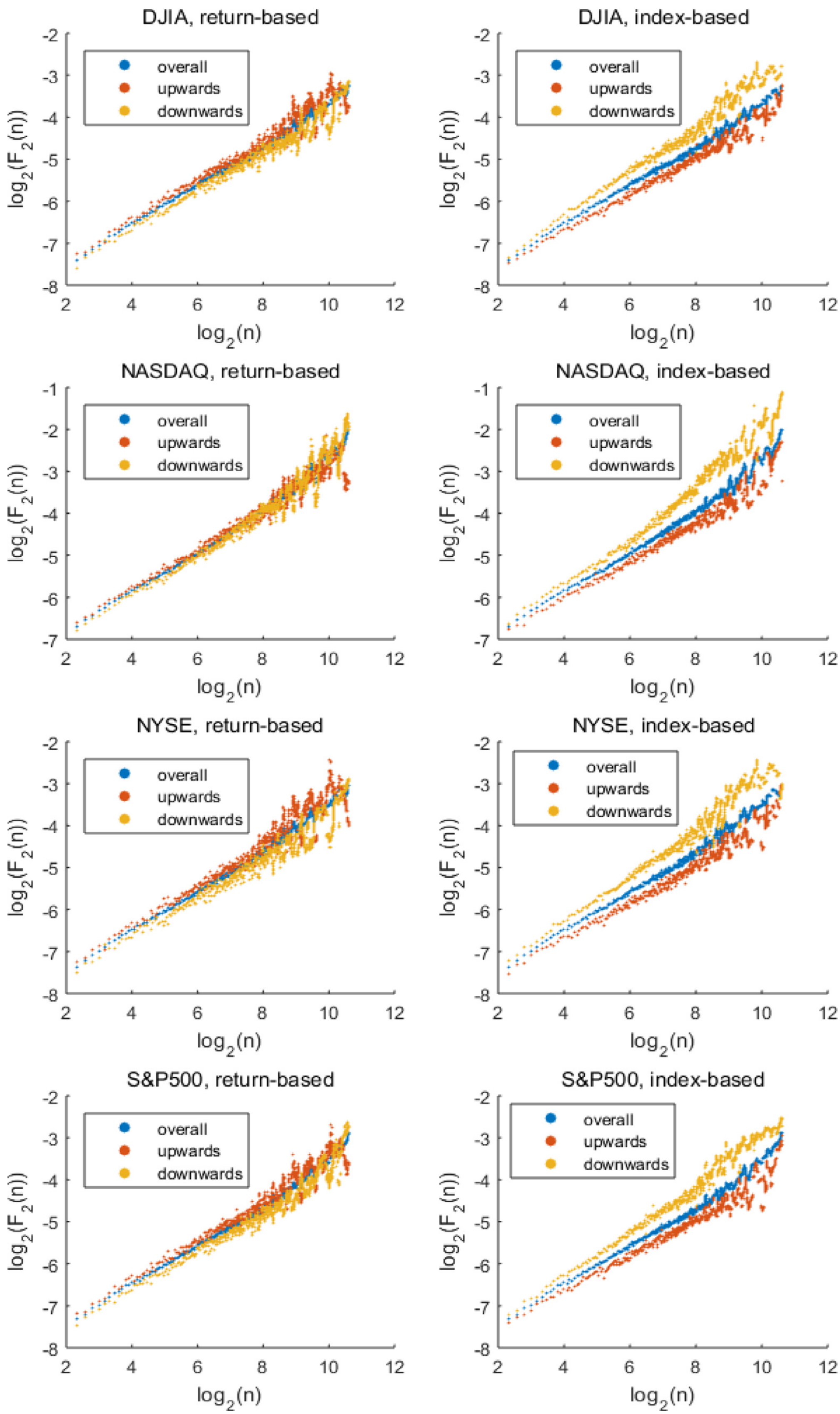

Fig. 2. Plots of $\boldsymbol{log}_2(\boldsymbol{F}_2(\boldsymbol{n}))$ vs. $\boldsymbol{log}_2(\boldsymbol{n})$ for DJIA, NASDAQ, NYSE and S&P500.

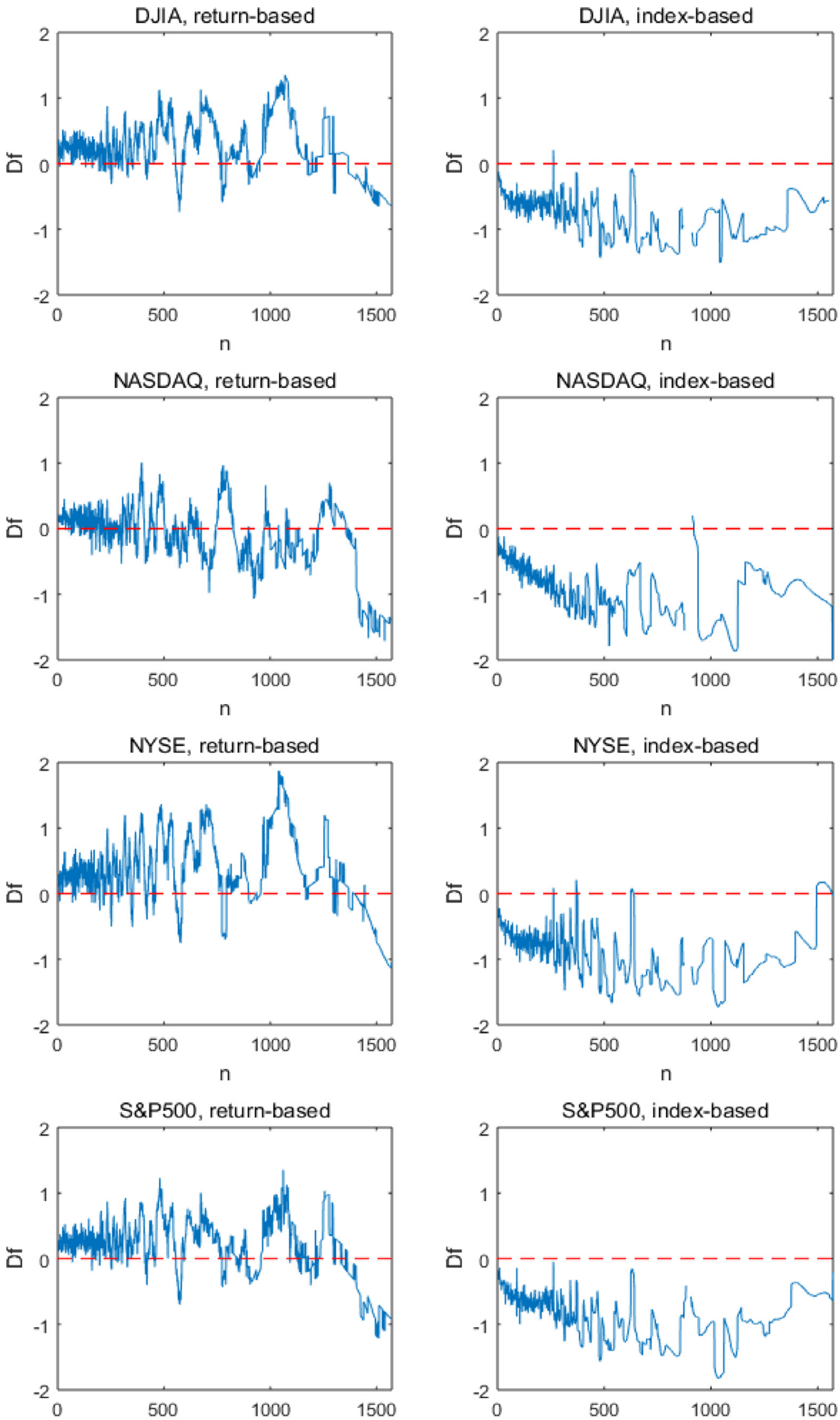

Fig. 3. Plots of ***Df*** for DJIA, NASDAQ, NYSE and S&P500.

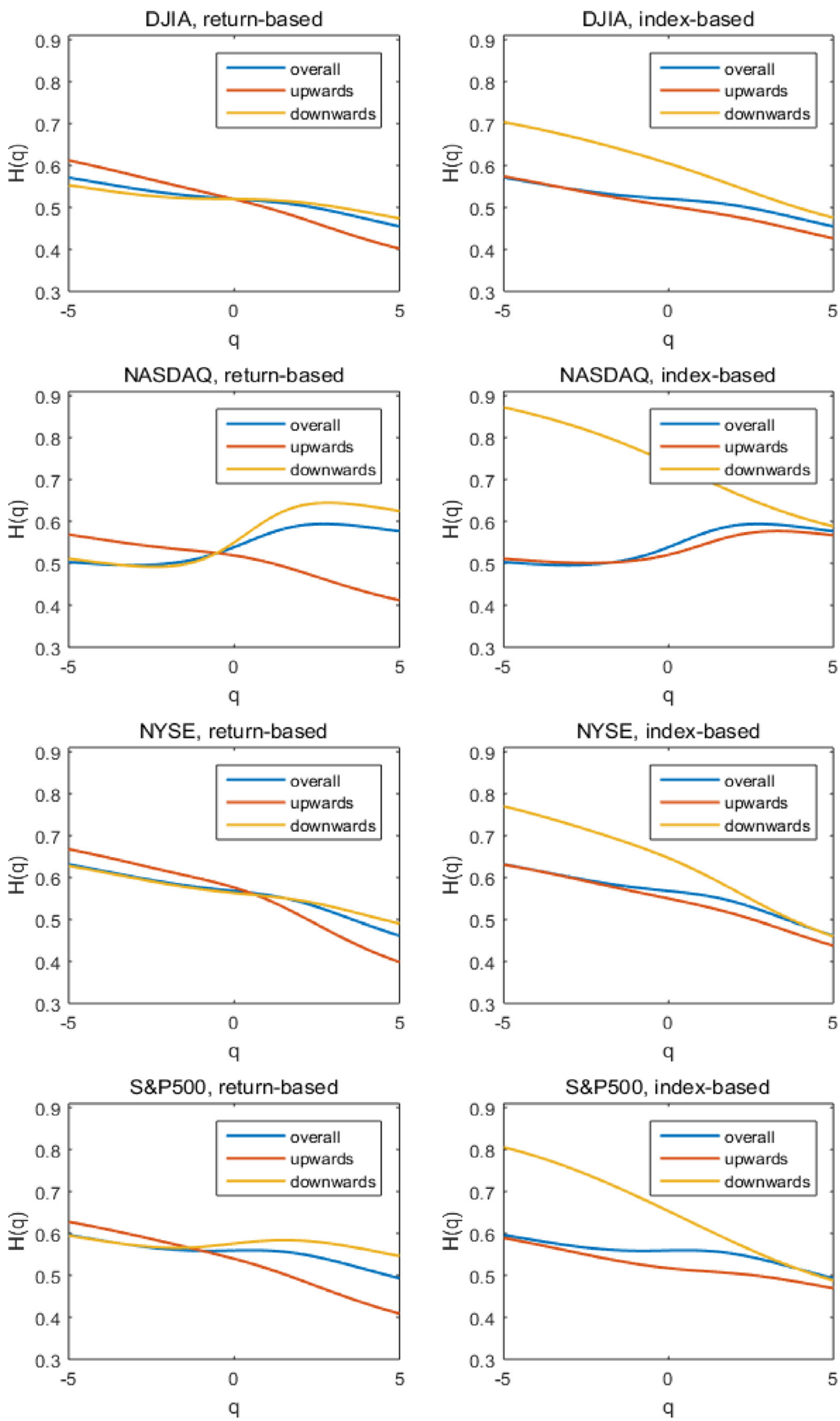

Fig. 4. Plots of $H(q)$, $H^{+}(q)$, and $H^{-}(q)$ versus q.

Table 2
ΔH of the original, shuffled, and surrogated series using return-based A-MFDFA model.

Return-based A-MFDFA	DJIA			NASDAQ		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1164	0.0826(29.00%)	**0.0558(52.09%)**	0.0983	0.0979(0.35%)	0.1360(−38.39%)
Upward	0.2103	**0.1034(50.82%)**	0.1256(40.27%)	0.1567	**0.1095(30.17%)**	0.1490(4.92%)
Downward	0.0790	0.1069(−35.42%)	**0.0367(53.51%)**	0.1527	**0.1152(24.57%)**	0.2218(−45.27%)
	NYSE			S&P500		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1700	0.1247(26.66%)	**0.1094(35.69%)**	0.1031	0.0961(6.79%)	**0.0297(71.17%)**
Upward	0.2690	**0.1300(51.67%)**	0.1625(39.60%)	0.2187	**0.1171(46.46%)**	0.1291(40.99%)
Downward	0.1375	0.1431(−4.11%)	**0.0959(30.26%)**	0.0487	0.1141(−134.31%)	0.1008(−106.98%)

Note for Table 2 and 3: The value in parentheses is the change in the ΔH value for the shuffled (resp. surrogated) data to that of the original data, $(\Delta H_{orig} - \Delta H_{shuf})\,/\Delta H_{orig}$ (resp. $(\Delta H_{orig} - \Delta H_{surr})/\Delta H_{orig}$)

Table 3
ΔH of the original, shuffled, and surrogated series using index-based A-MFDFA model.

Index-based A-MFDFA	DJIA			NASDAQ		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1164	0.0826(29.00%)	**0.0558(52.09%)**	0.0983	0.0979(0.35%)	0.1360(−38.39%)
Upward	0.1476	**0.0921(37.60%)**	0.1191(19.29%)	0.0759	0.1064(−40.13%)	0.0980(−29.12%)
Downward	0.2276	0.1533(32.66%)	**0.0824(63.82%)**	0.2839	0.1514(46.69%)	**0.0778(72.60%)**
	NYSE			S&P500		
	Original series	Shuffled series	Surrogated series	Original series	Shuffled series	Surrogated series
Overall	0.1700	0.1247(26.66%)	**0.1094(35.69%)**	0.1031	0.0961(6.79%)	**0.0297(71.17%)**
Upward	0.1932	**0.1302(32.60%)**	0.1527(20.97%)	0.1199	0.0971(19.04%)	**0.0385(67.89%)**
Downward	0.3099	0.1723(44.41%)	**0.1202(61.23%)**	0.3175	0.1572(50.50%)	**0.0963(69.68%)**

(1) Generate pairs $(a,\ b)$ of random integer numbers with $a,\ b \le N$, where N is the length of the time-series.
(2) Change the value in a-th order with b-th order
(3) Repeat (1) and (2) for $20N$ times

Secondly, the fat-tailed distribution can be investigated by comparing the multi-fractality of original and surrogated series [28]. The algorithm to create the surrogated series is as follows:

(1) Generate a sequence of random numbers $\{\tilde{x}_t : t = 1, 2, \ldots, N\}$ with the Gaussian distribution
(2) Rearrange $\{\tilde{x}_t\}$ in the same order of $\{x_t\}$ so that two time series can have the same rank patterns

The degree of multi-fractality can be defined as $\Delta H = \max(H(q)) - \min(H(q))$ [29]. When ΔH is zero, the time-series is called as mono-fractal, and the degree of multi-fractality is stronger as ΔH increases. Let $\Delta H_{orig}, \Delta H_{shuf}$ and ΔH_{surr} represent the degree of multi-fractality for the original, shuffled, and surrogated series, respectively [23]. To achieve the robust result, we use the mean of 30 repeated values for ΔH_{shuf} and ΔH_{surr}.

Tables 2 and 3 summarize the degree of multi-fracatality for the original, shuffled and surrogated series using A-MFDFA. The bold numbers indicate the significance of multi-fractality. Note that either ΔH_{shuf} or ΔH_{surr} is significant if its value is smaller than others given that it is also significantly smaller than ΔH_{orig}. For the most cases of overall, ΔH_{shuf} and ΔH_{surr} are smaller than ΔH_{orig}. This implies that the multi-fractality in the U.S. indices is affected by both long-range correlation and fat-tailed distribution. In case of original series, ΔH_{orig} is larger in the upward than downward for the return-based model. However, ΔH_{orig} is significantly larger in the downward than upward for the index-based model. That is, the strong multi-fractality is presented in the downward for the index-based model, while that is observed in the upward for the return-based.

When ΔH_{shuf} is much smaller than not only ΔH_{orig} but also ΔH_{surr}, we can claim that the main source of multi-fractality is long-range correlation. In Table 2 for a return-based A-MFDFA model, ΔH_{shuf} values for upward stock market are significant smaller in DJIA, NASDAQ, NYSE, and S&P500 so that we can claim that the main source of multi-fractality in these upward stock market is fat-tail distribution.

When ΔH_{surr} is much smaller than not only ΔH_{orig} but also ΔH_{surr} , we can claim that the main source of multi-fractality is fat-tail distribution. In Table 3 for index-based A-MFDFA model, ΔH_{surr} values for downward stock market are significant smaller in DJIA, NASDAQ, NYSE, and S&P500 so that we can claim that the main source of multi-fractality in these downward stock market is long-range correlation.

4.4. Source of asymmetry

Alvarez-Ramirez et al. [22] suggest that the asymmetric scaling behavior also can be produced by the long-range correlation and fat-tailed distribution. Hence, we re-apply the method in Section 4.3 to discover the source of the asymmetry scaling behavior. The degree of asymmetric scaling behavior can be defined as $\Delta H^{\pm}(q) = |H^{+}(q) - H^{-}(q)|$ [30]. Note that the time-series has symmetric scaling behavior if $\Delta H^{\pm}(q)$ is close to 0, whereas it has stronger asymmetry as $\Delta H^{\pm}(q)$ increases.

Fig. 5 shows the value of $\Delta H^{\pm}(q)$ for the original, shuffled and surrogated series. If $\Delta H^{\pm}(q)$ for the shuffled and surrogated series are smaller than those of the original series, then the long-range correlation and fat-tailed distribution can be possible sources of the asymmetric scaling behavior. Analogous to the source of multi-fractality, the smallest value of $\Delta H^{\pm}(q)$ is the main source. In case of return-based model, $\Delta H^{\pm}(q)$ of shuffled and surrogated series are not significantly smaller than that of original series. In other words, the source of asymmetry is not distinguishable. In contrast, the index-based model clearly detects the source of asymmetry. The source of asymmetry in DJIA and S&P500 is the long-range correlation based on $\Delta H^{\pm}(q)$ being close to zero for shuffled series. The result of NASDAQ shows that the long-range correlation and fat-tailed distribution are the main sources of asymmetry for

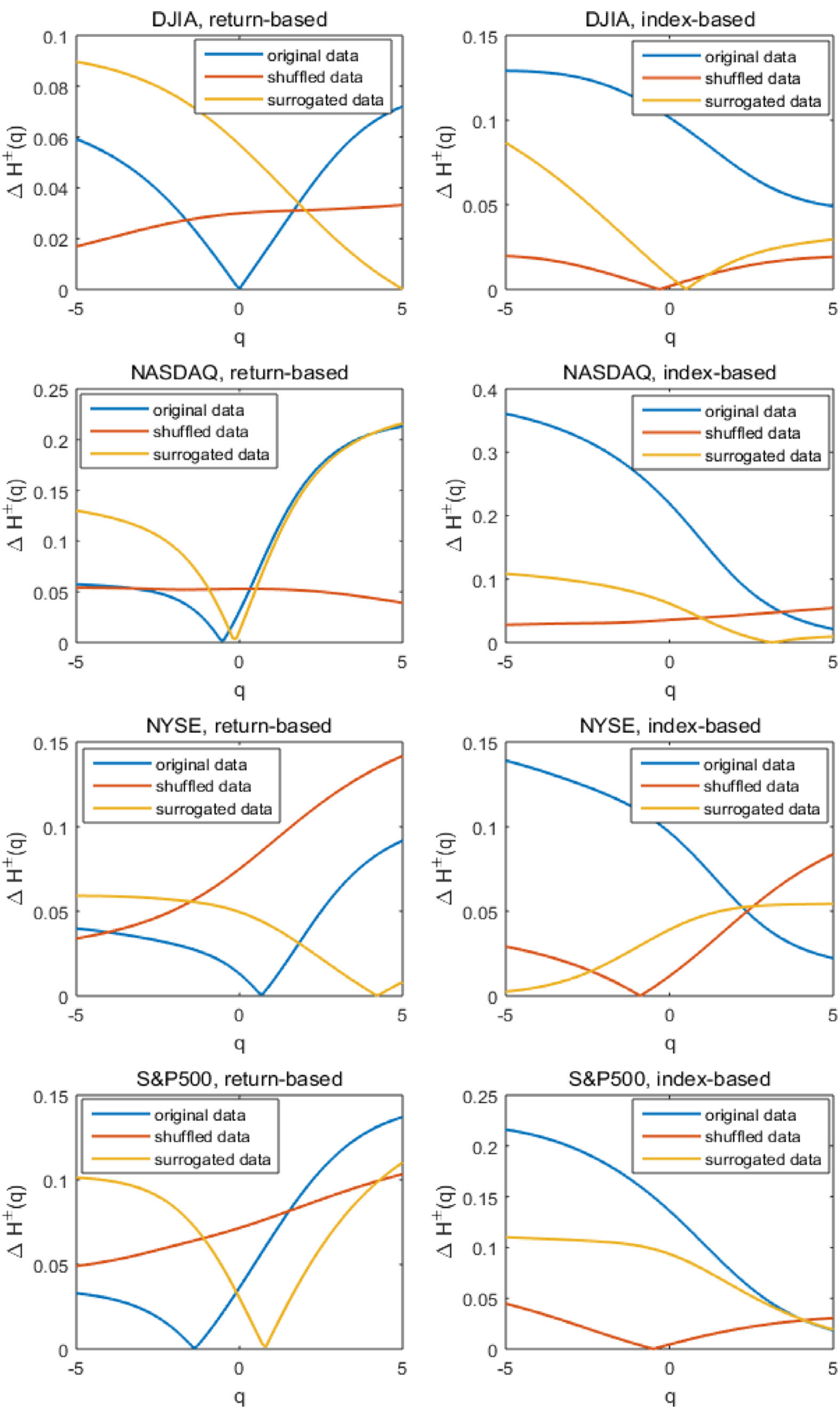

Fig. 5. Plots of $\Delta H^{\pm}(q)$ for the original, shuffled and surrogated data.

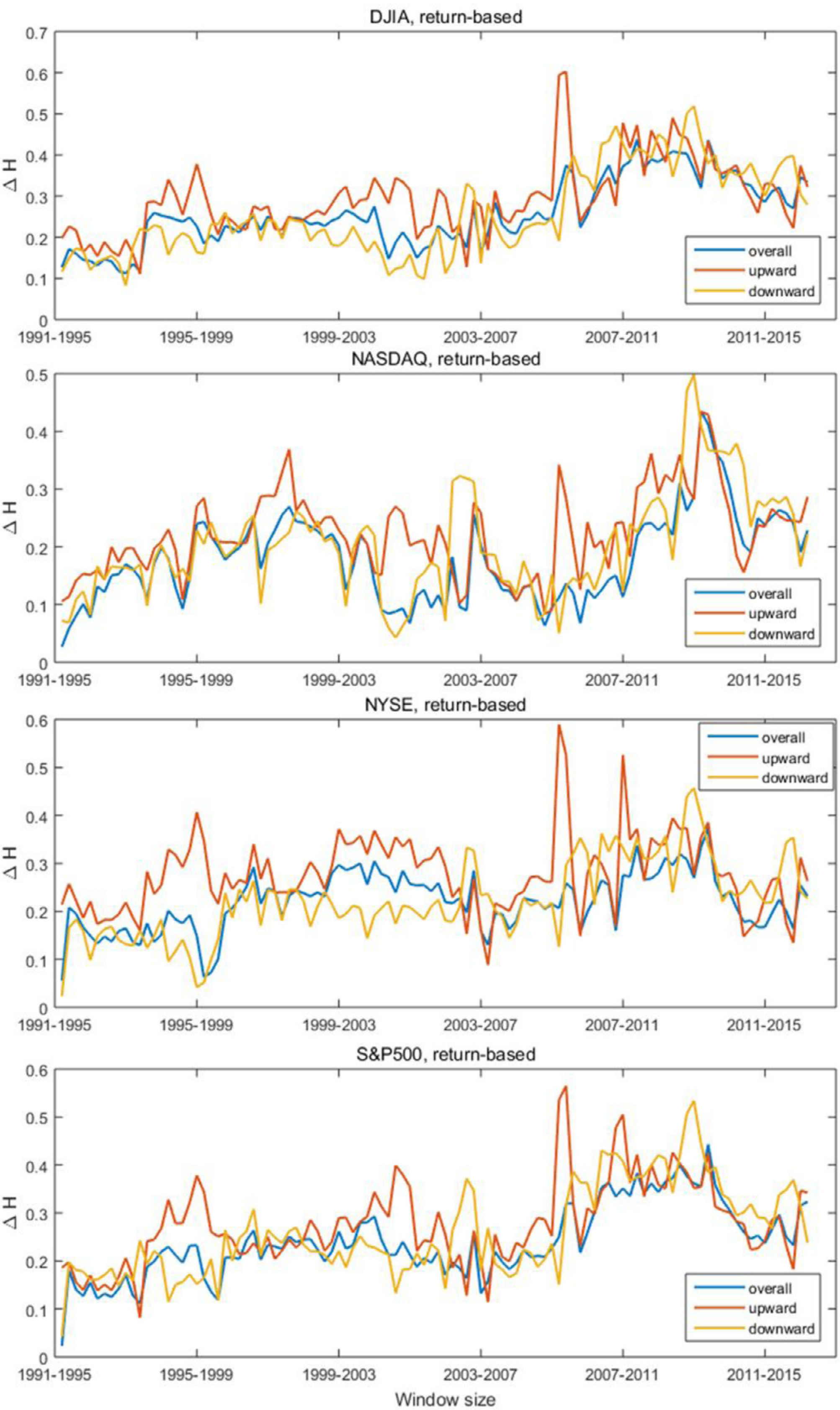

Fig. 6. Time evolution of ΔH with a slide step of 50 days for the overall, upward and downward for DJIA, NASDAQ, NYSE and S&P500, respectively, using return-based A-MFDFA model.

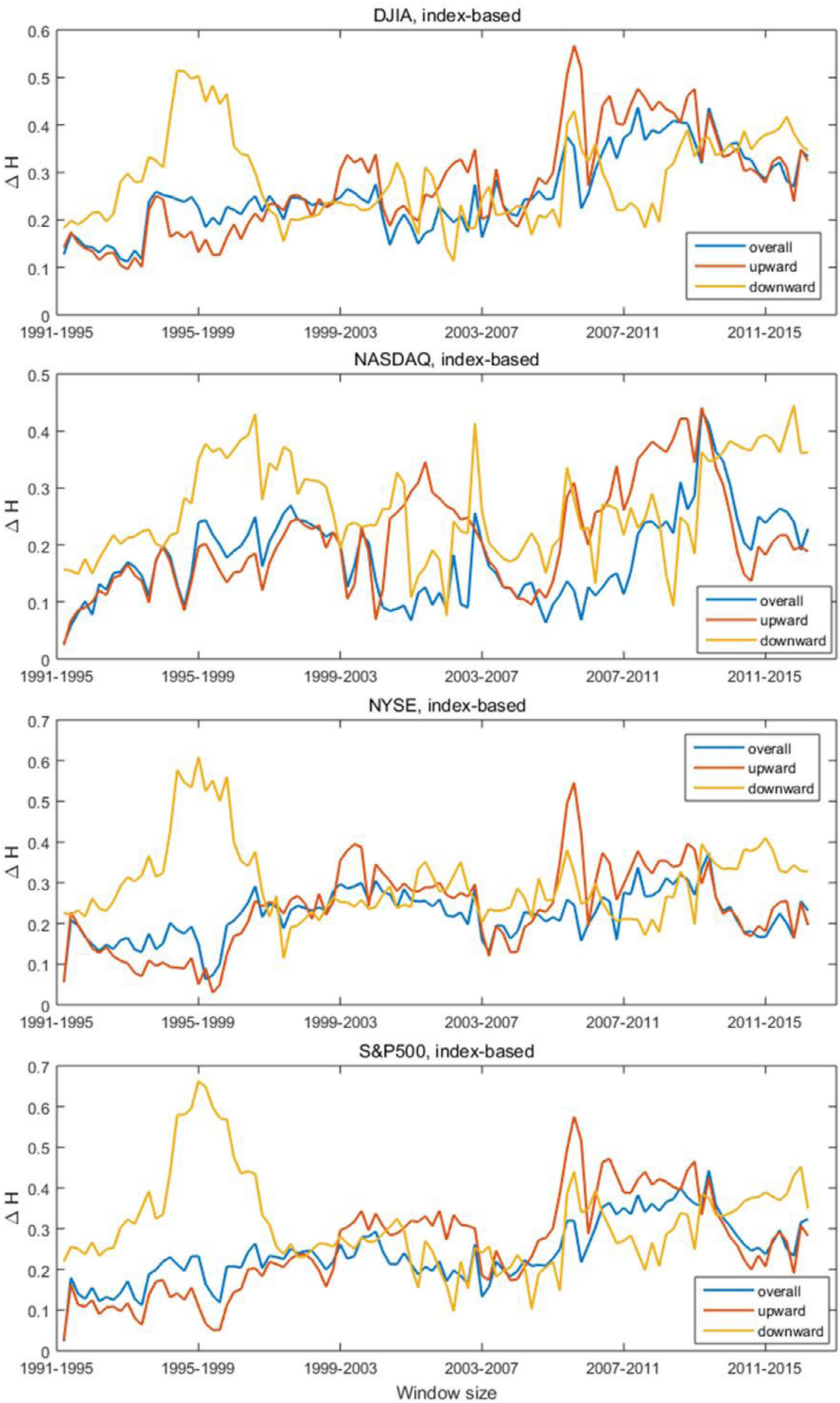

Fig. 7. Time evolution of ΔH with a slide step of 50 days for the overall, upward and downward for DJIA, NASDAQ, NYSE and S&P500, respectively, using index-based A-MFDFA model.

q < 1 (small fluctuation) and 1 < q (large fluctuation), respectively. Lastly, the result of NYSE reveals that the fat-tailed distribution and long-range correlation are the main sources of asymmetry for q < −2 and −2 < q < 2, respectively.

4.5. Time-varying multi-fractal asymmetry

The time-varying feature of the multi-fractal asymmetry can be studied based on the rolling window method. We set the size of window and slide to be 1000 trading dates (roughly 4 years) and 50 dates, respectively, to achieve the reliable result [7,31]. The corresponding results of return- and index-based model are illustrated in Fig. 6 and Fig. 7, respectively. In general, each model shows the similar evolutionary patterns of ΔH, ΔH^{+}, and ΔH^{-} among most of indices excluding NASDAQ. The difference between two models can be observed when the evolutions of upward and downward are compared. The return-based model shows the similar trend in the evolutions of overall, upward, and downward, whereas the index-based model shows the different evolution between the upward and downward. It is also noticeable that the time-varying ΔHs of upward and downward are correlated in the return-based model, whereas those in the index-based model are uncorrelated. That is, the index-based model is more suitable in discriminating the time-varying multi-fractal asymmetry. Furthermore, the peaks of ΔH, ΔH^{+} and ΔH^{-} in both models are observed in the window period from 1994–1998 to 1996–2000 and from 2005–2009 to 2008–2012, which include the Asian financial crisis in 1997 and the Sub-prime mortage crisis in 2008, respectively. It refers that the strong multi-fractality is the phenomenon of the financial crisis. As the market efficiency is measured by the degree of multi-fractality [2], the result of time-varying multi-fractal asymmetry provide the evidence of inefficient market during the financial crisis.

5. Conclusion

In this paper, we propose A-MFDFA with new criterion for separating the market trend. Originally, A-MFDFA method distinguishes the market trend based on the coefficient of regression in the return dynamics. Considering that the coefficient of regression in the index dynamics is more intuitive criterion for the market trend, we provide the index-based model of A-MFDFA so that asymmetric multi-fractal feature in the U.S. stock indices can be investigated.

At first, we discover that the existence of multi-fractality in all U.S. stock indices, whose feature is revealed to be asymmetric. Also, the index-based model can detect the asymmetric multi-fractality more distinctly than the return-based model since its fluctuation function shows the clear-cut between the positive and negative trends. Secondly, we find the source of multi-fractality and asymmetry by comparing the original series with the shuffling and surrogated series. Specifically, the long-range correlation is discovered to be the main source for the upward trend, whereas the fat-tailed distribution is the main source for the downward trend. The source of asymmetry is ambiguous in the return-based model, but the index-based model indisputably identifies the source where the source of asymmetry differs in each index and q. Lastly, we explore the time-varying feature of asymmetric multi-fractality based on the moving window method. The time-varying feature of uptrend and downtrend are correlated in the return-based model, whereas the features are clearly distinguished in index-based model. Furthermore, we detect that the degree of multi-fractality is high during the financial crisis period where the asymmetry between two trends are significantly elevated in the index-based model. Thus, we claim that the index-based model performs better in detecting the asymmetric multi-fractality.

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