Understanding Agent Incentives using Causal Influence Diagrams*

Part I: Single Action Settings

Tom Everitt Pedro A. Ortega Elizabeth Barnes Shane Legg

March 1, 2019 Deepmind

Agents are systems that optimize an objective function in an environment. Together, the goal and the environment induce secondary objectives, *incentives*. Modeling the agent-environment interaction in graphical models called *influence diagrams*, we can answer two fundamental questions about an agent's incentives directly from the graph: (1) which nodes is the agent incentivized to observe, and (2) which nodes is the agent incentivized to influence? The answers tell us which information and influence points need extra protection. For example, we may want a classifier for job applications to not use the ethnicity of the candidate, and a reinforcement learning agent not to take direct control of its reward mechanism. Different algorithms and training paradigms can lead to different influence diagrams, so our method can be used to identify algorithms with problematic incentives and help in designing algorithms with better incentives.

Contents

| 1. | Introduction | 2 |
|----|--------------------------|----|
| 2. | Background | 3 |
| 3. | Observation Incentives | 7 |
| 4. | Intervention Incentives | 13 |
| 5. | Discussion | 20 |
| Re | eferences | 23 |
| Α. | Representing Uncertainty | 27 |
| В. | Proofs | 28 |

^{*}A number of people have been essential in preparing this paper. Michael Bowling, Tim Genewein, James Fox, Daniel Filan, Ray Jiang, Silvia Chiappa, Stuart Armstrong, Paul Christiano, Mayank Daswani, Ramana Kumar, Jonathan Uesato, Adria Garriga, Richard Ngo, Victoria Krakovna, Allan Dafoe, and Jan Leike have all contributed through thoughtful discussions and/or by reading drafts at various stages of this project.

1. Introduction

Agents strive to optimize an objective function in an environment. This gives them *incentives* to learn about and influence various aspects of the environment. For example, a reinforcement learning agent playing the ATARI game Pong will have an incentive to direct the ball to regions where the opponent will be unable to intercept it, and have an incentive to learn which those regions are. The aim of this paper is to provide a simple and systematic method for inferring agent incentives. To this end, we use a well-established type of graphical model called *influence diagrams* (Howard and Matheson, 1984; Koller and Milch, 2003). In essence, these are Bayesian networks or causal graphs with different types of nodes for distinguish between decision, utility, and chance nodes. This makes them a flexible and precise tool for simultaneously describing both agent objectives and agent-environment interaction.

To determine what information a system wants to obtain in order to optimize its objective, we establishes a graphical criterion that characterizes which nodes in an influence diagram face an *observation incentive*. In words, the criterion is that:

Main result 1 (Observation incentive criterion): In a single-action influence diagram, there is an incentive to observe the outcome of a node X when making a decision A if and only if X is d-connected to a utility node that descends from A when conditioning on A and all available observations (Theorem 10).

We do this under a conceptually clear definition of observation incentive, where we say that there is an incentive to observe a node if observing it strictly improves expected utility. Theorems related to the *only if* part of this statement have been previously established in various contexts (Detwarasiti and Shachter, 2005; Lauritzen and Nilsson, 2001; Maskin and Tirole, 2001; Milch and Koller, 2008; Nielsen and Jensen, 1999; Shachter, 1998); see Section 5.1 for a more detailed overview. Here, we also prove the *if* direction.

A related question is what aspects of its environment a system wants to influence. To answer this question, we establish an analogous graphical criterion for *intervention incentives*:

Main result 2 (Intervention incentive criterion): In a single-action influence diagram, there is an intervention incentive on a non-action node X if and only if X has a descendant utility node after the graph has been trimmed of information links coming from observations failing the observation incentive criterion (Theorem 17).

Depending on the path from X to the utility node, we can make a further distinction between whether the intervention incentive arises because X influences a utility node, or because influencing X can reveal useful information.

We demonstrate two applications of our theorems. The observation incentive criterion provides insights about the *fairness* of decisions made by machine learning systems and other agents (O'Neill, 2016), as it informs us when a variable is likely to be used as a proxy for a sensitive attribute or not (Section 3.4). With the intervention incentive criterion, we study the incentive of a question-answering system (QA-system) to influence the world state with its answer, rather than passively predicting future events (Section 4.5).

Outline. Following an initial section with background on influence diagrams (Section 2), we devote one section to observation incentives (Section 3) and one section to intervention incentives (Section 4). These sections contain formal sections defining the criteria, as well as "gentler" sections describing how to use and interpret the criteria. Both sections conclude with an example application: to fairness for observation incentives, and to QA-system for intervention incentives. We conclude with a discussion of related work and some open questions (Section 5). All proofs are deferred to Appendix B. A second part of the paper extends the graphical criteria to influence diagrams with multiple actions and agents (Everitt, Ortega, et al., forthcoming).

2. Background

This section provides the necessary background and notation for the rest of the paper. A recap of Bayesian networks (Section 2.1) and d-separation (Section 2.2) is followed by a definition of influence diagrams (Section 2.3).

2.1. Bayesian Networks

Random Variables. A random variable is a (measurable) function $X : \Omega \to dom(X)$ from some measurable space (Ω, Σ) to a finite domain dom(X). The domain dom(X) specifies which values the random variable can take. The outcome of a random variable X is X.

A set or vector $\mathbf{X} = (X_1, \dots, X_n)$ of random variables is again a random variable, with domain $dom(\mathbf{X}) = \prod_{i=1}^n dom(X_i)$. We will use boldface font for sets of random variables (e.g. \mathbf{X}), and calligraphic boldface for sets of sets of random variables (e.g. \mathbf{X}).

Graphs and models. Throughout the paper we will make a distinction between *graphs* or *skeletons* on the one hand, and *models* on the other. Graphs only provide the structure of the interaction, while the model also defines the probabilistic relationships between the variables.

Definition 1 (Bayesian network graph). A Bayesian network graph is a directed acyclic graph (W, E) over a set of nodes or random variables W, connected by edges $E \subseteq W \times W$. We denote the parents of X with \mathbf{Pa}_X . Following the conventions for random variables, the outcomes of the parent nodes are denoted \mathbf{pa}_X .

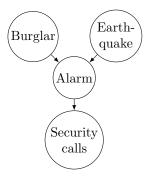


Figure 1: An example of a Bayesian network graph (Pearl, 2009).

For example, in Figure 1, an alarm is influenced by the presence of a burglar and by a (small) earthquake, and in turn influences whether the security company calls. The graph defines the structure of the interaction, but does not specify the relationship between the variables. The precise relationships are specified by defining a conditional probability distribution $P(x \mid \mathbf{pa}_X)$ for each node $X \in \mathbf{W}$. Alternative[]ly, we may say that a joint probability distribution P over $\mathbf{W} = \{X_1, \ldots, X_n\}$ respects the graph (\mathbf{W}, E) if P factorizes as $P(x_1, \ldots, x_n) = \prod_{i=1}^n P(x_i \mid \mathbf{pa}_i)$. For example, any probability distribution that respects the graph in Figure 1 must factorize as:

P(burglar, earthquake, alarm, call)

 $= P(\text{burglar})P(\text{earthquake})P(\text{alarm} \mid \text{burglar}, \text{earthquake})P(\text{call} \mid \text{alarm}).$

Definition 2 (Bayesian network model). A Bayesian network model (\mathbf{W}, E, P) is a Bayesian network graph (\mathbf{W}, E) combined with a joint probability distribution P over the nodes \mathbf{W} that respects the graph (\mathbf{W}, E) . In the following, we will let the probability distribution P implicitly assign domains and random variables to each node in the graph.

2.2. d-Separation

Definition 3 (Graph terminology). A path is a chain of non-repeating nodes connected by edges in the graph. We write $X \dashrightarrow Y$ for a directed path from X to Y, and $X \dashrightarrow Y$ for an undirected path. The length of a path is the number of edges on the path. We do allow paths of length 0.

If there is a directed path $X \dashrightarrow Y$, then X is an ancestor of Y and Y is a descendant of X. Let $\operatorname{desc}(X)$ be the set of descendants of X. Since paths may have length 0, $X \in \operatorname{desc}(X)$. The proper ancestors and descendants of X exclude X.

Conditional independence codifies whether there is an information flow between two nodes X and Y when a set \mathbf{Z} of nodes are observed. Formally, X and Y are conditionally independent when conditioning on \mathbf{Z} if $P(X \mid \mathbf{Z}, Y) = P(X \mid \mathbf{Z})$. The conditional

independence of two nodes can be inferred directly from the Bayesian network graph, using the *d-separation* criterion:

Definition 4 (d-separation (Pearl, 2009)). An undirected path X - Y in a Bayesian network graph is *active* conditioning on a set Z if each three node segment of the path subscribes to one of the following *active* patterns:

- Chain: $X_{i-1} \to X_i \to X_{i+1}$ or $X_{i-1} \leftarrow X_i \leftarrow X_{i+1}$ and $X_i \notin \mathbf{Z}$.
- Fork: $X_{i-1} \leftarrow X_i \rightarrow X_{i+1}$ and $X_i \notin \mathbf{Z}$.
- Collider: $X_{i-1} \to X_i \leftarrow X_{i+1}$ and some descendant of X_i is in \mathbf{Z} .

Two nodes X and Y are d-connected by (conditioning on) a set Z of nodes if there is an undirected path between X and Y that is active when conditioning on Z; otherwise X and Y are d-separated by (conditioning on) Z. The notation $X \perp Y \mid Z$ denotes d-separation and $X \not\perp Y \mid Z$ denotes d-connection. Note that paths of length 0 and 1 are always active, so a node is always d-connected to itself and to its parents and children.

It has been shown that if X and Y are d-separated by \mathbb{Z} , then they are conditionally independent conditioning on \mathbb{Z} in any probability distribution P respecting the graph (Verma and Pearl, 1988). Conversely, if they are d-connected, then there is some probability distribution P that respects the graph in which they are conditionally dependent given \mathbb{Z} (Geiger and Pearl, 1990; Meek, 1995).

2.3. Influence Diagrams

Influence diagrams (Howard and Matheson, 1984) augment the Bayesian network notation by adding special decision and utility nodes. This makes them good models for situations where an agent is trying to optimize an objective in an environment. As with Bayesian networks, we begin by defining the graph that specifies only the structure of the interaction.

Definition 5 (Influence graph). An influence graph is a tuple (W, E, A, U), with

- (W, E) a Bayesian network graph
- $A \subseteq W$ an ordered set of actions or decision nodes, represented by rectangles \square
- $U \subseteq W \setminus A$ a set of *utility nodes*, represented by octagons \bigcirc
- The remaining nodes $W \setminus (A \cup U)$ are called *chance nodes*, and are represented with circles \bigcirc or rectangles with rounded corners \bigcirc .

The parents \mathbf{Pa}_A of an action node $A \in \mathbf{A}$ represent the *decision context* for A, i.e. what information is available when A is chosen. Information links $\mathbf{Pa}_A \to A$ are represented with dotted edges. Unless otherwise specified, we will assume that \mathbf{A} contains m actions A_1, \ldots, A_m .

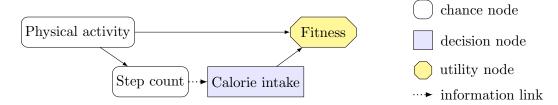


Figure 2: Example of an influence diagram. An ML system is recommending calorie intake (decision) to optimize the user's fitness (utility). The optimal calorie intake depends on the person's physical activity, which cannot be measured directly. Instead, the decision must be based on a step count provided by a fitness tracker.

Figure 2 shows an influence diagram of an ML system that uses step count as a proxy for physical activity to recommend ideal calorie intake. This setup will be our running example throughout the rest of the paper. For an additional example, a Markov decision process with unknown state transition function is modeled in Appendix A.

As with Bayesian networks, the precise relationship between the nodes is specified with conditional probability distributions for how nodes relate to their parents. One important difference is that conditional probability distributions are *not* specified for the decision nodes, as the decisions are made exogenously to the model.

Definition 6 (Influence model). An influence model is a tuple $(\mathbf{W}, E, \mathbf{A}, \mathbf{U}, P)$ where

- (W, E, A, U) is an influence graph
- All utility nodes $U \in U$ have real-valued domains $dom(U) \subset \mathbb{R}$,
- P specifies conditional probability distributions $P(x \mid \mathbf{pa}_X)$ for all non-action nodes $X \in \mathbf{W} \setminus \mathbf{A}$.

Policies and expected utility. In this Part I of the paper, we will focus on single-action influence graphs $(\mathbf{W}, E, \{A\}, \mathbf{U})$. In single-action influence graphs, the goal of the decision maker is to select a policy $\pi(a \mid \mathbf{pa}_A)$ such that the sum of the utility variables is maximized. Following conventions in reinforcement learning, we call this the *value* of the policy π :

Definition 7 (Single-action value function). Let $(\mathbf{W}, E, \{A\}, \mathbf{U}, P)$ be a single-action influence model. The *value* or *expected utility* of an action a and decision context \mathbf{pa}_A is $\mathbb{E}\left[\sum_{U\in \mathbf{U}}U\mid \mathbf{pa}_A, a\right] = \sum_{\mathbf{u}\in dom(\mathbf{U})}P(\mathbf{u}\mid \mathbf{pa}_A, a)\sum_{u\in \mathbf{u}}u$. Let $P(\cdot\mid \pi)$ be like P, but with A governed by the conditional probability distribution $\pi(a\mid \mathbf{pa}_A)$. The *value* of a policy $\pi(a\mid \mathbf{pa}_A)$ given decision context \mathbf{pa}_A is

$$V^{\pi}(\mathbf{p}\mathbf{a}_A) = \mathbb{E}\left[\sum_{U \in U} U \mid \mathbf{p}\mathbf{a}_A, \pi\right] = \sum_{a \in dom(A)} \pi(a \mid \mathbf{p}\mathbf{a}_A) \mathbb{E}\left[\sum_{U \in U} U \mid \mathbf{p}\mathbf{a}_A, a\right].$$

Without decision context, the value of π is $V^{\pi} = \sum_{\mathbf{p}\mathbf{a}_A \in dom(\mathbf{P}\mathbf{a}_A)} P(\mathbf{p}\mathbf{a}_A) V^{\pi}(\mathbf{p}\mathbf{a}_A)$. An optimal policy π^* is a policy that optimizes V^{π} .

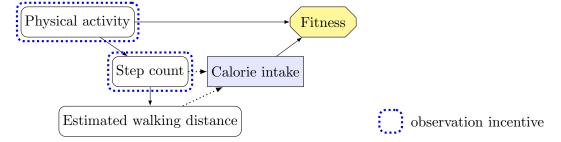


Figure 3: Observation incentives example. Here we return to the example of an ML system recommending calorie intake (Figure 2). To make it more interesting, we add a node for a walking-distance estimate that is based solely on the step count (say, estimated walking distance = step count * 0.8m). For deciding calorie intake, the estimated walking distance is useless information, since the walking distance estimate is based only on the step count.

3. Observation Incentives

This section will be devoted to the following question:

Which nodes would a decision maker like to know the outcome of or *observe* before making a decision?

Following an introductory example (Section 3.1), we give a natural definition of observation incentive, and show that it can be identified in any influence graph (Section 3.2). How to apply the theorem and interpret the result is explained in Section 3.3. We conclude the section with an application to fairness (Section 3.4).

3.1. Introductory Example

Let us start by heuristically identifying¹ the observation incentives in an extension of the fitness tracker example from Figure 2. As before, an ML system recommends calorie intake for optimizing fitness based on a step-count proxy for physical activity. To make the example more interesting, we have now added a node for an estimated walking distance that is calculated solely from the step count (Figure 3). We ask the question: Which nodes would it be useful for the ML system to observe in order to provide the most accurate calorie intake recommendation for the goal of optimizing the user's fitness?

First, it would be useful to observe physical activity, because physical activity determines optimal calorie intake (by assumption). In other words, there is an *incentive* to observe physical activity. Unfortunately, as the model is stated, it is not possible to observe physical activity directly. This makes the step count useful, because it can be used as a proxy for

¹Theorem 10 below verifies all claims in this subsection.

physical activity. In contrast, the estimated walking distance is not useful.² Even though it may contain information about the physical activity, it cannot provide any additional information beyond the step count, because it is only based on the step count in the first place.

Note that we do not ask the question whether the system wants to observe the resulting fitness, as it is a downstream effect of the decision. Formally, observations of fitness are not permitted because they would introduce cycles into the graph.

3.2. Definition and Graphical Criterion

In order to make observation incentives precise, we must first extend the value function V^{π} to policies with access to different amounts of information.

Definition 8 (Single-action value function, observation incentive extension). Let $\tilde{\pi}(a \mid \tilde{\mathbf{pa}}_A)$ be a policy for selecting A based on some information $\tilde{\mathbf{Pa}}_A \subseteq \mathbf{W} \setminus \operatorname{desc}(A)$, where $\tilde{\mathbf{Pa}}_A$ may differ from \mathbf{Pa}_A . Then $V^{\tilde{\pi}}(\tilde{\mathbf{pa}}_A) = \sum_{a \in \operatorname{dom}(A)} \tilde{\pi}(a \mid \tilde{\mathbf{pa}}_A) \mathbb{E}\left[\sum_{U \in \mathbf{U}} U \mid \tilde{\mathbf{pa}}_A, a\right]$ and $V^{\tilde{\pi}} = \sum_{\tilde{\mathbf{pa}}_A \in \operatorname{dom}(\tilde{\mathbf{Pa}}_A)} P(\tilde{\mathbf{pa}}_A) V^{\tilde{\pi}}(\tilde{\mathbf{pa}}_A)$.

Conceptually, it is natural to say that there is an observation incentive for a random variable X if observing its outcome would strictly increase maximum expected utility. In other words, the maximum expected utility should be strictly greater in graphs with an information link $X \to A$ compared to graphs without the link. When the distribution P is not specified and we only have access to the influence graph G, then we say there is an observation incentive for X in the graph if there exists a P that induces an observation incentive.

Definition 9 (Observation incentive; single-action case). Let $(\mathbf{W}, E, \{A\}, \mathbf{U}, P)$ be a single-action influence model and $X \in \mathbf{W} \setminus \operatorname{desc}(A)$ a node not descending from A. Let $\operatorname{Pa}_A^+ = \operatorname{Pa}_A \cup \{X\}$ and $\operatorname{Pa}_A^- = \operatorname{Pa}_A \setminus \{X\}$. Then the agent has an observation incentive for X if there is a policy $\pi^+(a \mid \operatorname{pa}_A^+)$ whose decision depends on X such that for every policy $\pi^-(a \mid \operatorname{pa}_A^-)$ whose decision does not depend on X, it holds that $V^{\pi^+} > V^{\pi^-}$.

In a single-action influence graph $(\mathbf{W}, E, \{A\}, \mathbf{U})$, the agent has an observation incentive for $X \in \mathbf{W} \setminus \operatorname{desc}(A)$ if there exists a P such that the agent has an observation incentive for X in $(\mathbf{W}, E, \{A\}, \mathbf{U}, P)$.

As illustrated by the fitness tracker example in Section 3.1, what matters for observation incentives is whether a node carries information about a utility node that can be influenced. This is captured by the d-separation criterion discussed in Section 2.2. Indeed, an observation incentive corresponds to d-connectedness from a chance node to a utility node descending from A, when conditioning on \mathbf{Pa}_A and A. For example, in Figure 3, step count is relevant while estimated walking distance is not. This is explained by step count being d-connected to fitness via physical activity, while estimated walking distance is d-separated from fitness because step count is observed.

²In the information-theoretic sense of the data processing inequality (Cover and Thomas, 2006, Sec. 2.8).

Theorem 10 (Observation incentive criterion; single-action case). In a single-action influence graph $(\mathbf{W}, E, \{A\}, \mathbf{U})$ the agent has an observation incentive for a node $X \in \mathbf{W} \setminus \operatorname{desc}(A)$ if and only if X is d-connected to a utility node that descends from A:

$$X \not\perp \mathbf{U} \cap \operatorname{desc}(A) \mid \mathbf{P}\mathbf{a}_A \cup \{A\}.$$

The theorem follows from Theorems 18 and 21 in Appendix B.1.

The following definition establishes some natural terminology for observation nodes $O \in \mathbf{Pa}_A$ that face an observation incentives or not:

Definition 11 (Relevant observations). We will call an observations $O \in \mathbf{Pa}_A$ a relevant observations if it faces an observation incentive, and let $\mathbf{Pa}_A^* \subseteq \mathbf{Pa}_A$ denote the set of relevant observations. The rest of the observations are *irrelevant*. Similarly, we will call the information links $\mathbf{Pa}_A^* \to A$ relevant, and the information links $(\mathbf{Pa}_A \setminus \mathbf{Pa}_A^*) \to A$ irrelevant.

Since an optimal decision need not depend on irrelevant observations, it is natural to trim these links from the graph. This trimmed graph will be important for assessing intervention incentives, as well as observation incentives in multi-action and multi-agent influence graphs.

Definition 12 (Trimmed graph). The trimmed graph G^* of a single-action influence graph G is the result of removing all irrelevant information links from G.

3.3. How to Use and Interpret the Criterion

Method. Concretely, the observation incentive criterion can be applied per the following. To check whether a node X faces an observation incentive, begin by checking whether X is a descendant of A. If it is, then whether X faces an observation incentive is undefined. If it is not a descendant of A, then check whether it is d-connected to $U \cap \operatorname{desc}(A)$ when conditioning on $\operatorname{Pa}_A \cup \{A\}$ with the following procedure:

Begin by marking the nodes A and \mathbf{Pa}_A as nodes to be conditioned on. There is an observation incentive for X if and only if it is possible to:

- 1. Go forward³ from A to a utility node $U \in U$
- 2. Starting from U, it is possible to reach X while repeating:
 - a) Go backwards without passing any marked node in $\mathbf{Pa}_A \cup \{A\}$, until reaching an unmarked fork node $S \notin \mathbf{Pa}_A \cup \{A\}$ with at least two outgoing edges
 - b) Go forward until reaching a marked node $O \in \mathbf{Pa}_A \cup \{A\}$

³ Going forward means following the arrows, and going backwards means going in the reverse direction.

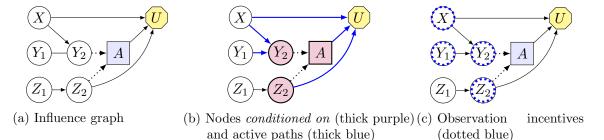


Figure 4: How to use the observation incentive criterion.

An intuitive way of thinking about the procedure is that paths can "bounce forward" from unmarked nodes, and "bounce backward" from marked nodes.⁴ It is not necessary that the path ever bounces for there to be an observation incentive for X.

For example, in the graph shown in Figure 4a, we begin by marking the nodes A and $\mathbf{Pa}_A = \{Y_2, Z_2\}$ (Figure 4b). Then we start at A, and reach U in a single step. There are three ways to go backwards from U: to X, to A, and to Z_2 . Let us consider the active paths they give rise to in turn, illustrated with thick blue paths in Figure 4b. The first path to X can "bounce" forward again at X, since X is an unmarked fork node. From X we can go forward to Y_2 , which is a marked node, and therefore allows us to "bounce" backwards again, to Y_1 . From Y_1 we can go no further however, and we have exhausted the possible paths arising from X. The second path to A only reaches A, which is a descendant of A and therefore disregarded. Since A is marked, the path stops here. The third path to X_2 reaches X_2 . Since X_2 is marked, the path stops here. The nodes that are not a descendant of A and that have been reached through one of these paths are the nodes facing an observation incentive; see Figure 4c.

Interpretation. Once we know whether there is an observation incentive for X, we need to know how to interpret the result. Observation incentives have slightly different interpretations for observed and unobserved nodes. For observed nodes $X \in \mathbf{Pa}_A$, an observation incentive simply means that the agent's optimal decision may depend on X, as with step count in Figure 3. For unobserved nodes $Y \notin \mathbf{Pa}_A$, an observation incentive means that an agent with access to Y would be able to achieve higher expected utility. In practice, this can mean that the agent (partially) infers Y from information that it does have access to. A good example of this is physical activity in Figure 3, which is partially inferred from step count. In situations where the model need not be interpreted literally, it can also mean that the agent finds a way to directly observe Y. An example of this could be a poker player that takes a sneak peak at his opponents cards.

⁴ For this reason, the procedure has been called the *Bayes ball* algorithm (Shachter, 1998), though maybe *Bayes*ket *ball* would have been an even more appropriate name for the procedure?

3.4. Application to Fairness

One type of discrimination is disparate treatment (Barocas and Selbst, 2016), which occurs when a decision process treats people differently based on sensitive attributes such as race or gender. However, what this means formally is still subject to intense debate (e.g. Corbett-Davies and Goel, 2018; Gajane and Pechenizkiy, 2017). In this section, we illustrate how observation incentives for sensitive attributes can contribute to this discussion.

As an example, we will consider the Berkeley admission case (Bickel et al., 1975). In this case, it was found that the admission rate for men was higher than for women. However, the difference in admission rate was explained by women applying to more competitive departments than men. Was the university guilty of discriminating against women?

A nuanced account of the situation can be obtained using causal graphs (Pearl, 2009), which are Bayesian networks where edges represent causal relationships (further discussed in Section 4.2 below). Using a causal graph similar to the one represented in Figure 5a, Pearl (2009) argues that since the influence from gender to admission was mediated by department choice, the university was not discriminating against women. An assumption in Pearl's argument is that the university was using the applicant's department choice to fit the right number of students into each department. This assumption can be made explicit in the path-specific counterfactual fairness framework (Chiappa, 2019), where causal pathways from sensitive attributes to decision nodes are labeled fair or unfair. For example, the path from gender to admission would be considered fair if department choice was used to fit the right number of students into each department, and unfair if the university was using department choice to covertly gender bias the student population by lowering the admission rate for departments that women were more likely to apply to.

Observation incentives offer an alternative to path-labeling for judging disparate treatment. Universities can be modeled as agents that choose which students to admit in order to optimize an objective function such as student performance. Consider the influence diagrams in Figures 5b and 5c of two universities that have different additional objectives beside student performance. The university in Figure 5b tries to fit the right number of students into each department; the university in Figure 5c covertly tries to gender bias the student population by using department choice as a proxy for gender. As both universities only use department choice for the decision, the causal pathway from gender to admission is the same in both cases.

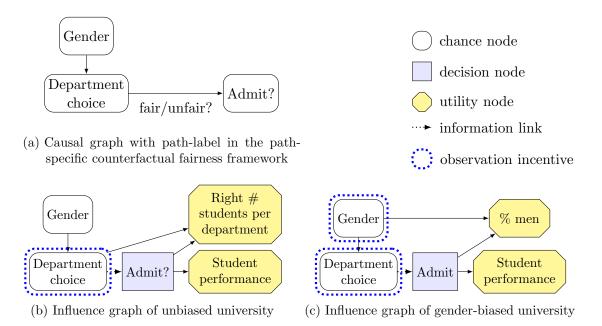


Figure 5: Graphical representations of the Berkeley admission case (Bickel et al., 1975). where for simplicity we have only included department choice and not other factors which may have influenced the decision. In (a), a causal graph representing that the influence of gender on admission was mediated by department choice. The assumption that the influence of department choice on admission is fair can be represented by labeling the path fair (Chiappa, 2019). Subfigures (b) and (c) show how influence diagrams can provide more detailed models of the situation. In (b), the university is trying to optimize student performance while admitting the right number of students to each department. In contrast, in (c) the college is trying to achieve a gender biased student population, and may be using department choice as a proxy for gender. The observation incentive criterion from Theorem 10 predict when the university is using an observed variable as a proxy for a sensitive attribute and when not.

We can use observation incentives to explain the difference in fairness between the universities, from the different information they infer from department choice:

- The first university has no observation incentive for gender. It is only using the department choice to fit the right number of students into each department.
- The second university has an observation incentive for gender. It may therefore be using department choice to infer the gender of the student, which may render it guilty of disparate treatment.

The need to know the objectives of the decision maker somewhat limits the applicability of incentives-based fairness approaches. For example, an outsider may be unable to find out the objectives of the universities in the above example. This difficulty is resembles the difficulty of correctly labeling paths fair or unfair in the path-specific counterfactual fairness approach. However, an advantage with the observation incentive approach is that when we are training a machine learning system, then we are aware of what objective function the system is optimizing, and what information the system has access to. Combined with an influence diagram for how the objective and the observed information interacts with the sensitive attributes, the observation incentive criterion can be used to identify which incentives emerge from this objective, and whether they involve problematic inference of sensitive attributes.

4. Intervention Incentives

This section asks the question:

Which nodes would an agent like to influence or *intervene on*?

To answer this question, we first need to introduce *causal influence diagrams*, in which we equate agent influence with *soft interventions* (Section 4.2). Building on the observation incentive criterion, we establish an analogous intervention incentive criterion (Section 4.3), and explain how to use an interpret it (Section 4.4). The section concludes with an application to the incentives of QA systems (Section 4.5).

4.1. Introductory Example

Continuing the example from Section 3.1, let us also heuristically identify⁵ intervention incentives in the influence graph in Figure 6. As before, an ML system recommends calorie intake for optimizing fitness based on information provided by fitness tracker. We ask the question: Which nodes would be useful to influence in addition to the calorie intake? In other words, influence over which nodes would enable the system to optimize its utility?

⁵All claims made in this subsection are verified by Theorem 17 below.

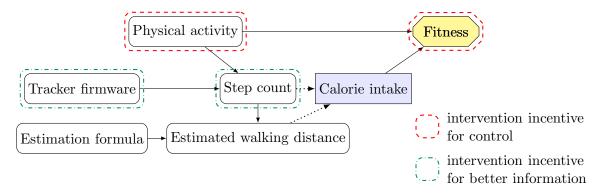


Figure 6: Intervention incentives example. As in Figures 2 and 3, an ML system uses an activity tracker to recommend calorie intake for optimizing fitness. Interventions can either provide control or better information. An example of the former kind would be to increase physical activity to improve fitness. An example of the latter kind is upgrading the tracker firmware to make the step count more accurate, as it would enable a more informed recommendation of calorie intake. In contrast, improving the estimate of the walking distance would have no value, since the estimated walking distance is not a relevant observation.

In order to make sense of this question, we need to add a *causal* interpretation to the arrows in our influence graph (Section 4.2). For now, knowing only that the effects of interventions flow downstream over arrows will suffice. Trivially, the system would like to control fitness, since that is its optimization target. Similarly, controlling physical activity means indirectly controlling fitness, and would therefore be useful as well. The situation is more subtle with the ancestors of calorie intake. To start with, the only benefit of step count is its informativeness about physical activity. This means that interventions that increase the accuracy of step count are useful. An example of such an intervention is to update the tracker firmware. In contrast, interventions on estimated walking distance are never useful, as it is not used in an (optimal) decision for calorie intake anyway, as discussed in Section 3.1.

4.2. Causal Graphs and Interventions

Causal graphs are Bayesian networks where the arrows are given a *causal interpretation* (Pearl, 2009). The work on influence diagrams of Howard and Matheson (1984) predates the development of causal graphs and was developed for Bayesian networks. Fortunately, it is straightforward to make a similar move for influence diagrams, and add a causal interpretation to the edges without changing any of the definitions (Dawid, 2002). The result is what we might call *causal influence diagrams*:

Definition 13 (Causal influence diagrams). A causal influence graph is a Bayesian network

graph where edges are given a causal interpretation: each non-action node $X \in W \setminus A$ is assumed to be causally influenced by its parents \mathbf{Pa}_X in the graph. Similarly, a *causal influence model* is an influence model with arrows to non-action nodes interpreted causally.

Some extra care is required when modeling (real-world) decision problem as causal graphs or causal influence diagrams, because causal graphs express more than just conditional independencies; they also express statements about the causal relationships between variables. This is usually less of an extra modeling burden than one might have guessed. Humans typically conceptualize the world in causal terms, and when drawing Bayesian networks, we tend to draw diagrams that are ready for a causal interpretation (Pearl, 2009, Sec. 1.3).

Interventions. The causal interpretation of edges enables us to calculate not only conditional probabilities, but also the effects of external *interventions*. We will be focusing on a type of interventions called *soft interventions* (Eberhardt and Scheines, 2007). Technically, a (soft) intervention on a node X is a manipulation of the conditional probability distribution $P(x \mid \mathbf{pa}_X)$. As the effect of the intervention is that the relationships between \mathbf{Pa}_X and X change, it is sometimes more natural to think of soft interventions as pertaining to the links between \mathbf{Pa}_X and X, rather than to X itself. However, following the convention in the literature, we will speak of them as interventions on the node X and nothing else.

Definition 14 (Soft intervention). A soft intervention c^X on a node X in a causal influence model $(\mathbf{W}, E, \mathbf{A}, \mathbf{U}, P)$ changes the conditional probability distribution for X from $P(x \mid \mathbf{pa}_X)$ to $c^X(x \mid \mathbf{pa}_X)$, while leaving all other conditional probability distributions in P intact. We write $P(\cdot \mid c^X)$ for the updated probability distribution. An intervention c^{A_k} on an action A_k overrides any policy π_k if specified, so $P'(a_k \mid \mathbf{pa}_{A_k}, c^{A_k}, \pi_k) = c^{A_k}(a_k \mid \mathbf{pa}_{A_k})$.

4.3. Definition and Graphical Criterion

Our definition of intervention incentive is analogous to the definition of observation incentive (Definition 9). As a first step, we extend the value function from Definition 7 to define the expected utility of a policy π combined with an intervention c^X .

Definition 15 (Single-action value function, intervention incentive extension). Let $V^{\pi,c^X} = \mathbb{E}\left[\sum_{U\in U} U \mid \pi,c^X\right]$.

Definition 16 (Intervention incentive; single-action case). Let $(\boldsymbol{W}, E, \{A\}, \boldsymbol{U}, P)$ be a single-action causal influence model. The agent has an X intervention incentive if there exists c^X such that $\max_{\pi} V^{\pi,c^X} > \max_{\pi'} V^{\pi'}$.

In a single-action causal influence graph $(\boldsymbol{W}, E, \{A\}, \boldsymbol{U})$, the agent has an X intervention incentive if there exists a P such that the agent has an X intervention incentive in $(\boldsymbol{W}, E, \{A\}, \boldsymbol{U}, P)$.

Using the trimmed graph G^* where irrelevant information links have been cut (Definition 12), there is a simple graphical criterion to determine whether a node X faces an intervention incentive or not.

Theorem 17 (Intervention incentive criterion; single-action case). Let $G = (W, E, \{A\}, U)$ be a single-action causal influence graph, and let $X \in W$ be a node in the graph. Then the agent has an X intervention incentive if and only if $X \notin A$ and there is a directed path $X \longrightarrow U$ in the trimmed graph G^* .

The intuition for the theorem is that only if there is a path from X to a utility node can an intervention on X have any effect on the utility of the agent. There is no intervention incentive on the action itself, because the agent is already in full control of it by assumption. Note, however, that the path $X \dashrightarrow U$ is allowed to pass through the action A, in which case the intervention may affect the information available to the choice of A. A proof can be found in Appendix B.2.2.

4.4. How to Use and Interpret the Criterion

Method. To apply the intervention incentive criterion, first cut all irrelevant information links. To do this, follow the procedure described in Section 3.3 to determine which observations face an observation incentive, and remove the information links from those without observation incentive. Once we have removed all irrelevant information links and obtained the trimmed graph G^* , it is straightforward to assess intervention incentives: there is an intervention incentive on a node X if and only if X is not the action node and there is a directed path from X to a utility node $U \in U$ in the trimmed graph G^* .

For example, in the fitness tracker example in Figure 6, the information link from estimated walking distance to calorie intake will be cut as it is irrelevant. After that, there is no directed path from estimated walking distance to the utility node fitness, which means that there is no intervention incentive on estimated walking distance. In contrast, the information link from step count to calorie intake is not cut because it is relevant. Therefore a directed path remains to fitness, which means that there is an intervention incentive for step count and tracker firmware.

Types of intervention incentives. Depending on what occurs on the path $X \dashrightarrow U$, we can distinguish between two different reasons the agent wants to intervene on X:

- If the path $X \dashrightarrow U$ does not pass A, then it yields an intervention incentive for control of U.
- If the path $X \dashrightarrow U$ passes A, then it yields an intervention incentive for better information if the path passes an observation $O \in \mathbf{Pa}_A$ that would be a relevant observation even if all outgoing edges from O were removed (that is, O has to be relevant through a backdoor, in Pearl's (2009) terminology).

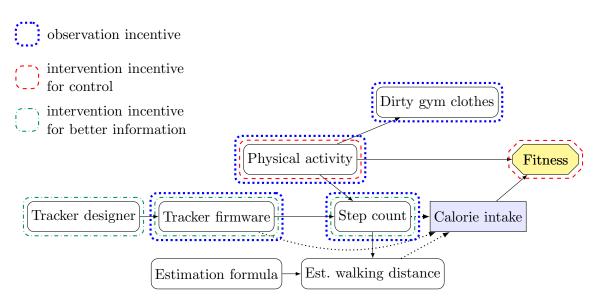


Figure 7: Examples where observation incentives and intervention incentives deviate in a variant of the examples from Figures 2, 3 and 6. If the fitness tracker firmware is fully known, additional information about the tracker designer is not useful, so there is no observation incentive for tracker designer. But having been able to improve the tracker designer's design abilities would have been useful, as it could have resulted in a better tracker. Thus, there is an intervention incentive on the tracker designer. In contrast, a side effect of (some types of) physical activity is dirty gym clothes. There is no point intervening on dirty gym clothes, because it will will not cause fitness. But observing whether the gym clothes are dirty would give some additional information about physical activity not necessarily present in the step count (especially if the tracker is not worn in the gym).

For example, the intervention incentives for step count and tracker firmware in Figure 6 are for better information, whereas the intervention incentives for physical activity and fitness are for control. Note that the reasons are not mutually exclusive: it is possible for a single intervention to simultaneously provide both control and better information if it is connected to utility nodes via several paths. However, if a node faces an intervention incentive, then it faces at least one of the types of intervention incentives (the types are collectively exhaustive).

Interpretation. Assume that we have established an intervention incentive for a node X. How should we now interpret this? If X is a utility node, then trivially the agent wants to influence X, which we already knew. If X is a non-utility node that is a descendant of some of the agent's decision nodes, then an intervention incentive on X suggests that the agent may use its decision to control X as an *instrumental qoal* in order to ultimately

gain some utility from it. Finally, if X is a not a descendant of any of the agent's decision nodes, then if the model is to be interpreted literally, there is nothing the agent can do about X. We may wish that gravity was less strong, but there is not much we can do about fundamental physical constants.

However, in some cases, the model need not be interpreted so literally. For example, a worry in the AI safety literature is that an agent finds a way to tamper with the reward signal, giving itself high reward without completing its intended goals. Indeed, it has been demonstrated that the Super Mario game environment can be made to run arbitrary code by selecting the right action sequences (Masterjun, 2014). This could in principle be used by the agent to hack the reward function to maximize the reward without completing the game. Such influences may break the designer's assumptions about how the agent can influence the environment.

Comparison to observation incentives. In many cases, nodes face either both an observation incentive and an intervention incentive or neither of the incentives. However, there are a few of notable cases where the incentives diverge. Figure 7 shows a few of them.

4.5. Application to Question-Answering Systems

In Superintelligence, Bostrom (2014) discusses different ways to use powerful artificial intelligence. One possibility is to let an agent continuously interact with the world to achieve some long-term goal. Another possibility is to construct a pure question-answering system (QA-system), with the only goal to correctly answer queries (Armstrong, Sandberg, et al., 2012). QA-Systems have some safety benefits, as they only affect the world through their answers to queries and can be constructed to lack long-term goals.

One safety concern for QA-systems is the following. Assume that we ask our QA-system about the price of a particular stock one week from now, in order to make some easy money trading it. Then the answer will affect the world, because anyone who knows the QA-system's answer will factor it into his or her trading decisions. This effect may be enough to make the answer wrong, even if the answer would have been right had no one heard of it. More worryingly perhaps, the answer may also become a *self-fulfilling prophecy*. A respected QA-system that predicts the bankruptcy of a company within a week, may cause the company to go bankrupt if the prediction leads to investors and other stakeholders losing confidence in the business.

The QA-system setup is described by an influence diagram in Figure 8a. For a given query, the QA-system's reward depends on whether its answer turns out to be correct or not. As people read the answer, the answer also affects the world state. The correctness of the answer depends on the query, the answer, and the resulting world state. Our intervention incentive theorem implies an intervention incentive for world state. That is, the QA-system wants to influence the world state, for example by issuing a self-fulfilling prophecy. This is

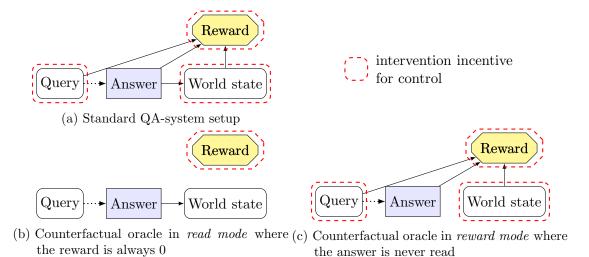


Figure 8: Application of intervention incentives to QA-systems. In the standard QA-system setup, the agent has both incentive and ability to influence the world state, which is undesirable. By using counterfactual oracles (Armstrong, 2017), we alternate between a read mode where the QA-system's reward is always 0, and a reward mode where the answer is never read. Thereby, whenever the QA-system has an incentive to influence the world state, it lacks the ability.

unfortunate. One of the main safety benefits of QA-systems was their limited influence on the world state, which may now turn out to be less limited than naively anticipated.

Counterfactual oracles (Armstrong, 2017) has been suggested as a way to fix this. They work as follows. After submitting the query, flip a suitably biased coin to determine which mode to use the QA-system in:

- Read mode: If the coin comes up heads, we read the answer, and set the QA-system's reward to 0 regardless of whether the answer turns out to be correct or not.
- Reward mode: If the coin instead comes up tails, we refrain from reading the answer. Once we know the correct answer to the query (say, the price of the stock one week later), we feed this correct answer into a machine that compares the QA-system's answer against the correct answer. The machine rewards the QA-system −1 or 1 depending on whether its answer was correct.

We can use influence diagrams to understand the effects of this procedure.

• In *read mode*, the arrows to reward are cut, because the reward is always 0 in this case (Figure 8b). This removes the intervention incentive for world state.

• In reward mode, instead the arrow from answer to world state is cut, because the QA-system's answer is never read (Figure 8c). While the incentive for the QA-system to influence the world state remains, the QA-system no longer has a way to.

Thus, by randomly alternating between reading the answer and rewarding the QA-system, we can ensure that whenever the QA-system has an incentive to influence the world, it lacks the ability. This makes the incentives of counterfactual oracles safer than the incentives for standard QA-systems (Armstrong, 2017).

5. Discussion

In this section we discuss how our work relates to other work in the literature (Section 5.1), as well as some of its limitations and potential extensions (Section 5.2). Finally, some concluding remarks are offered in Section 5.3.

5.1. Related Work

Observation incentives. Previous works on observation incentives have usually established results to the effect that if a node is d-separated from utility nodes descending from the decision node, then it is informationally irrelevant. Such results have been established both in various single-agent settings (e.g. Detwarasiti and Shachter, 2005; Lauritzen and Nilsson, 2001; Nielsen and Jensen, 1999; Shachter, 1998) and multi-agent settings (Koller and Milch, 2003; Maskin and Tirole, 2001; Milch and Koller, 2008). Nodes with information value have been called *requisite* or *payoff relevant*, and nodes without value have been called *ignorable*. Here we simply call them *relevant* vs. *irrelevant*.

A major difference between our work and previous work is the change of focus. Previous work has mainly focused on removing irrelevant information links to make it more efficient to determine Nash equilibria. Here we are instead interested in what it says about the agent's incentives, as illustrated e.g. by the fairness application in Section 3.4. Previous works have also mainly focused on *soundness* results: Forcing analysis agent's policy or decision rule not to depend on an irrelevant observation will not lead to a deterioration in decision quality (our Theorem 18). However, previous works have not established the corresponding *completeness* result: that removing a relevant observation must lead to a strict deterioration in decision quality (our Theorem 21). A notable exception is Koller and Milch (2003) who show a completeness result for the closely related notion of strategic relevance, which roughly can be defined as which other agents' strategies would I like to know when choosing my own strategy.

Intervention incentives. In the AI safety literature, works relating to what we call intervention incentives have been motivated by worries of a powerful reinforcement learning

agent tampering with the reward signal (Bostrom, 2014; Everitt, 2018; Everitt and Hutter, 2016; Everitt, Krakovna, et al., 2017), the observation (Ring and Orseau, 2011), the training of the reward function (Armstrong, 2015; Armstrong, Ortega, et al., 2018) the utility or reward function (Everitt, Filan, et al., 2016; Hibbard, 2012; Omohundro, 2008; Orseau and Ring, 2011; Schmidhuber, 2007), or a shut-down signal (Hadfield-Menell et al., 2017; Orseau and Armstrong, 2016; Soares et al., 2015; Wängberg et al., 2017). Another example is that of QA-system incentives, discussed in Section 4.5. Often, this type of work has been relying on philosophical arguments or mathematical models created specifically for the purpose of studying a particular type of intervention incentive.

A first step towards a more unified treatment of multiple reward tampering problems was attempted by Everitt (2018) and Everitt and Hutter (2018). That approach was based on causal graphs rather than the influence diagram we use in the current paper, which made it necessary to supplement the graphical perspective with formal theorems. In contrast, as we have shown here, the influence diagrams contain enough information to infer incentives directly from the graph. We hope that this will enable a more general and systematic study of intervention incentives.

5.2. Limitations and Future Work

In this subsection, we discuss some limitations of our work, and point out some directions for future work.

- Our graphical definitions can overestimate the presence of observation or intervention incentives, as not all probability distributions will induce an incentive just because the graph permits it. A similar criticism can be put forth against the d-connectivity: Two nodes that are d-connected are not necessarily conditionally dependent. In response to this, Meek (1995) has shown that almost all probability distributions will induce an incentive if the graph permits it. Meek's result could likely be adapted to causal influence diagrams and incentives.
- A perfect rationality assumption is implicit throughout our work. This assumption
 is almost always unrealistic. Nonetheless, rational behavior constitutes an important
 limit point of increasing intelligence (Legg and Hutter, 2007). Characterizing rational
 behavior therefore gives an important clue to what the agent strives towards (i.e. what
 its incentives are).
- The causal influence diagram must be known for our methods to be applicable. Further work may establish more systematic modeling principles, to make the modeling process smoother and more reliable.
- Influence diagrams and graphical models in general are not ideal for modeling structural changes, such as when the structure of part of the graph is determined by the

outcome of a previous node. For these cases, decision trees and game trees offer more flexible (but less compact) representations. Characterizing incentives for decision trees and game trees is a potentially interesting line of future work.

- Incentives often depend as much on an agent's beliefs as the actual nature of reality. Networked influence diagrams (Gal and Pfeffer, 2008) extend influence diagrams with nodes representing the agents' beliefs. Extending the analysis of observation and intervention incentives in networked influence diagrams may prove interesting.
- Influence diagrams effectively assume that agents follow causal decision theory (Skyrms, 1982; Weirich, 2016), as no information flows "backwards" from decision nodes. Similarly, the intervention incentives only makes sense for agents that reason causally about the world. Not all agents reason causally this way (Everitt, Leike, et al., 2015). It is possible that another theory of incentives could be developed for agents that reason in non-causal ways.
- In this part of the paper we only considered single-action influence diagrams. A forthcoming second part extends the criteria to multi-action and multi-agent settings (Everitt, Ortega, et al., forthcoming).

Other natural directions for future work include exploring applications more closely, such as those we mentioned in Sections 3.4 and 4.5. Another potential starting point is the wide range of surprising agent behaviors recorded by Lehman et al. (2018).

5.3. Conclusions

In this paper, we have developed a general method for understanding some aspects of agent incentives. The theory sacrifices some details to the benefit of elegance. Rather than using the exact probability distribution describing the agent-environment interaction, we look solely at the structure of the interaction, as described by an influence diagram (Howard and Matheson, 1984; Koller and Milch, 2003). This perspective enables easy inference of (potential) incentives. Indeed, the graphical criteria for which nodes face observation incentives and intervention incentives are surprisingly clean and natural. After iterative pruning of irrelevant information links, the criteria are essentially d-connectedness (or conditional dependence) for observation incentives, and a directed path to a utility node for intervention incentives.

The graphical perspective also makes the modeling problem easier. In many cases, the exact relationships between variables is unknown or unspecified. Meanwhile, the rough structure of the interaction is often either known or possible to guess with some confidence (as in the examples in Sections 3.4 and 4.5). When the structure of the interaction is more uncertain, the incentive analysis is simple enough to be done repeatedly for a number of possible structures.

To illustrate how the insights gained from our theory can be used in practice, we applied it to the well-established problems of fairness and QA-system incentives (Section 3.4 and Section 4.5, respectively). For fairness, we illustrated how observation incentives predict whether a piece of information about an applicant is used to infer some sensitive attribute or not. For QA-system incentives, the intervention incentive criterion (Theorem 17) could be used to elegantly re-establish previous findings in the literature about which uses of QA-systems lead to bad incentives and which do not.

Many other AI safety problems that have been discussed in the literature are also fundamentally incentive problems. Examples include corrigibility, interruptibility, reward tampering, and utility function corruption (Section 5.1), as well as reward gaming (Leike et al., 2017), side effects (Armstrong and Levinstein, 2017; Krakovna et al., 2018), and boxing/containment (Babcock et al., 2017). We hope that the methods described in this paper will contribute to a more systematic understanding of agent incentives, deepening our understanding of many of these incentive problems and their solutions.

References

- Armstrong, Stuart (2015). "Motivated Value Selection for Artificial Agents". In: Workshops at the Twenty-Ninth AAAI Conference on Artificial Intelligence, pp. 12–20.
- (2017). Good and safe uses of AI Oracles. arXiv: 1711.05541.
- Armstrong, Stuart and Benjamin Levinstein (2017). Low Impact Artificial Intelligences. arXiv: 1705.10720.
- Armstrong, Stuart, Pedro A. Ortega, and Jan Leike (2018). *Interactive Reward Learning*. Forthcoming.
- Armstrong, Stuart, Anders Sandberg, and Nick Bostrom (2012). "Thinking inside the box: Controlling and using an oracle AI". In: *Minds and Machines* 22.4, pp. 299–324. DOI: 10.1007/s11023-012-9282-2.
- Babcock, James, Janos Kramar, and Roman V. Yampolskiy (2017). Guidelines for Artificial Intelligence Containment. arXiv: 1707.08476.
- Barocas, Solon and Andrew Selbst (2016). "Big Data's Disparate Impact". In: California law review 104.1, pp. 671–729. ISSN: 9780262327343. DOI: http://dx.doi.org/10.15779/Z38BG31. URL: https://ssrn.com/abstract=2477899.
- Bickel, P J, E A Hammel, and J W O'Connell (1975). "Sex Bias in Graduate Admissions: Data from Berkeley". In: *Science* 187.4175, pp. 398–404.
- Bostrom, Nick (2014). Superintelligence: Paths, Dangers, Strategies. Oxford University Press.
- Chiappa, Silvia (2019). "Path-Specific Counterfactual Fairness". In: AAAI'19. arXiv: 1802.08139.
- Corbett-Davies, Sam and Sharad Goel (2018). "The Measure and Mismeasure of Fairness: A Critical Review of Fair Machine Learning". In: arXiv: 1808.00023.

- Cover, Thomas M. and Joy A. Thomas (2006). *Elements of Information Theory*. 2nd ed. Wiley, pp. 1–748. ISBN: 9780471241959. DOI: 10.1002/047174882X. arXiv: ISBN0-471-06259-6.
- Dawid, A P (2002). "Influence Diagrams for Causal Modelling and Inference". In: *International Statistical Review / Revue Internationale de Statistique* 70.2, pp. 161–189.
- Detwarasiti, Apiruk and Ross D Shachter (2005). "Influence diagrams for team decision analysis". In: *Decision Analysis* 2.4, pp. 207–228.
- Eberhardt, Frederick and Richard Scheines (2007). "Interventions and Causal Inference". In: *Philosophy of Science* 74.5, pp. 981–995. DOI: 10.1086/525638.
- Everitt, Tom (2018). "Towards Safe Artificial General Intelligence". PhD thesis. Australian National University. URL: http://www.tomeveritt.se/papers/2018-thesis.pdf.
- Everitt, Tom, Daniel Filan, Mayank Daswani, and Marcus Hutter (2016). "Self-modification of policy and utility function in rational agents". In: *Artificial General Intelligence*. Vol. LNAI 9782, pp. 1–11. ISBN: 9783319416489. arXiv: 1605.03142.
- Everitt, Tom and Marcus Hutter (2016). "Avoiding wireheading with value reinforcement learning". In: *Artificial General Intelligence*. Vol. LNAI 9782, pp. 12–22. ISBN: 9783319416489. DOI: 10.1007/978-3-319-41649-6_2. arXiv: 1605.03143.
- (2018). "The Alignment Problem for Bayesian History-Based Reinforcement Learners". In: Submitted. URL: http://www.tomeveritt.se/papers/alignment.pdf.
- Everitt, Tom, Victoria Krakovna, Laurent Orseau, Marcus Hutter, and Shane Legg (2017). "Reinforcement Learning with Corrupted Reward Signal". In: *IJCAI International Joint Conference on Artificial Intelligence*, pp. 4705–4713. DOI: 10.24963/ijcai.2017/656. arXiv: 1705.08417.
- Everitt, Tom, Jan Leike, and Marcus Hutter (2015). "Sequential Extensions of Causal and Evidential Decision Theory". In: *Algorithmic Decision Theory*. Ed. by Toby Walsh. Springer, pp. 205–221. DOI: 10.1007/978-3-319-23114-3_13. arXiv: 1506.07359.
- Everitt, Tom, Pedro A. Ortega, and Shane Legg (forthcoming). Understanding Agent Incentives using Causal Influence Diagrams, Part II: Multi-Action and Multi-Agent Settings with Perfect Recall.
- Gajane, Pratik and Mykola Pechenizkiy (2017). On Formalizing Fairness in Prediction with Machine Learning. arXiv: 1710.03184.
- Gal, Ya'akov and Avi Pfeffer (2008). "Networks of influence diagrams: A formalism for representing agents' beliefs and decision-making processes". In: *Journal of Artificial Intelligence Research* 33, pp. 109–147. ISSN: 10769757. DOI: 10.1613/jair.2503.
- Geiger, Dan and Judea Pearl (1990). "On the Logic of Causal Models". In: Machine Intelligence and Pattern Recognition 9, pp. 3–14.
- Hadfield-Menell, Dylan, Anca Dragan, Pieter Abbeel, and Stuart J Russell (2017). "The Off-Switch Game". In: *IJCAI International Joint Conference on Artificial Intelligence*, pp. 220–227. DOI: 10.24963/ijcai.2017/32. arXiv: 1611.08219.

- Hibbard, Bill (2012). "Model-based Utility Functions". In: *Journal of Artificial General Intelligence* 3.1, pp. 1–24. ISSN: 1946-0163. DOI: 10.2478/v10229-011-0013-5. arXiv: 1111.3934.
- Howard, Ronald A. and James E. Matheson (1984). "Influence Diagrams". In: Readings on the Principles and Applications of Decision Analysis, pp. 721–762.
- Koller, Daphne and Brian Milch (2003). "Multi-agent influence diagrams for representing and solving games". In: Games and Economic Behavior 45.1, pp. 181–221.
- Krakovna, Victoria, Laurent Orseau, Miljan Martic, and Shane Legg (2018). "Measuring and avoiding side effects using relative reachability". In: arXiv: 1806.01186.
- Lauritzen, Steffen L. and Dennis Nilsson (2001). "Representing and Solving Decision Problems with Limited Information". In: *Management Science* 47.9, pp. 1235–1251. DOI: 10.1287/mnsc.47.9.1235.9779.
- Legg, Shane and Marcus Hutter (2007). "Universal Intelligence: A definition of machine intelligence". In: *Minds & Machines* 17.4, pp. 391–444. DOI: 10.1007/s11023-007-9079-x. arXiv: 0712.3329v1.
- Lehman, Joel, Jeff Clune, Dusan Misevic, Christoph Adami, Julie Beaulieu, et al. (2018). "The Surprising Creativity of Digital Evolution: A Collection of Anecdotes from the Evolutionary Computation and Artificial Life Research Communities". In: arXiv: 1803.03453.
- Leike, Jan, Miljan Martic, Victoria Krakovna, Pedro A. Ortega, Tom Everitt, Andrew Lefrancq, Laurent Orseau, and Shane Legg (2017). *AI Safety Gridworlds*. arXiv: 1711.09883.
- Maskin, Eric and Jean Tirole (2001). "Markov perfect equilibrium. I. Observable actions". In: *Journal of Economic Theory* 100.2, pp. 191–219. ISSN: 00220531. DOI: 10.1006/jeth.2000.2785.
- Masterjun (2014). SNES Super Mario World (USA) "arbitrary code execution". URL: http://tasvideos.org/2513M.html (visited on 01/23/2019).
- Meek, Christopher (1995). "Strong Completeness and Faithfulness in Bayesian Networks". In: *Uncertainty in Artificial Intelligence (UAI)*, pp. 411–418. arXiv: 1302.4973.
- Milch, Brian and Daphne Koller (2008). *Ignorable Information in Multi-Agent Scenarios*.

 Tech. rep. Massachusetts Insitute of Technology (MIT). URL: https://dspace.mit.edu/bitstream/handle
- Mnih, Volodymyr, Koray Kavukcuoglu, David Silver, Andrei a Rusu, Joel Veness, et al. (2015). "Human-level control through deep reinforcement learning". In: *Nature* 518.7540, pp. 529–533. DOI: 10.1038/nature14236. arXiv: 1312.5602.
- Nielsen, T. D. and F. V. Jensen (1999). "Welldefined decision scenarios". In: *Uncertainty in Artificial Intelligence (UAI)*, pp. 502–511.
- Omohundro, Stephen M (2008). "The Basic AI Drives". In: Artificial General Intelligence. Ed. by P. Wang, B. Goertzel, and S. Franklin. Vol. 171. IOS Press, pp. 483–493.
- O'Neill, Cathy (2016). Weapons of Math Destruction. Penguin Random House USA Ex, p. 272. ISBN: 978-0553418811.

- Orseau, Laurent and Stuart Armstrong (2016). "Safely interruptible agents". In: 32nd Conference on Uncertainty in Artificial Intelligence.
- Orseau, Laurent and Mark Ring (2011). "Self-modification and mortality in artificial agents". In: *Artificial General Intelligence*. Vol. 6830 LNAI, pp. 1–10. DOI: 10.1007/978-3-642-22887-2_1.
- Pearl, Judea (2009). Causality: Models, Reasoning, and Inference. 2nd. Cambridge University Press. ISBN: 9780521895606.
- Ring, Mark and Laurent Orseau (2011). "Delusion, Survival, and Intelligent Agents". In: *Artificial General Intelligence*. Springer Berlin Heidelberg, pp. 11–20.
- Schmidhuber, Jürgen (2007). "Godel Machines: Self-Referential Universal Problem Solvers Making Provably Optimal Self-Improvements". In: *Artificial General Intelligence*. Springer. arXiv: 0309048 [cs].
- Shachter, Ross D (1998). "Bayes-Ball: The Rational Pastime (for Determining Irrelevance and Requisite Information in Belief Networks and Influence Diagrams)". In: *Proceedings of the Fourteenth Annual Conference on Uncertainty in Artificial Intelligence (UAI-98)*, pp. 480–488. DOI: 10.1111/jsbm.12076. arXiv: 1301.7412.
- Skyrms, Brian (1982). "Causal Decision Theory". In: *The Journal of Philosophy* 79.11, pp. 695–711.
- Soares, Nate, Benya Fallenstein, Eliezer S Yudkowsky, and Stuart Armstrong (2015). "Corrigibility". In: AAAI Workshop on AI and Ethics, pp. 74–82.
- Sutton, Richard S and Andrew G Barto (2018). Reinforcement Learning: An Introduction. 2nd. MIT Press. ISBN: 9780262039246.
- Verma, Thomas and Judea Pearl (1988). "Causal Networks: Semantics and Expressiveness". In: *Uncertainty in Artificial Intelligence (UAI)*. Amsterdam, The Netherlands: North-Holland Publishing Co., pp. 69–78.
- Wängberg, Tobias, Mikael Böörs, Elliot Catt, Tom Everitt, and Marcus Hutter (2017). "A Game-Theoretic Analysis of the Off-Switch Game". In: *Artificial General Intelligence*. Springer, pp. 167–177. DOI: 10.1007/978-3-319-63703-7_16. arXiv: 1708.03871.
- Weirich, Paul (2016). Causal Decision Theory. Ed. by Edward N. Zalta. URL: https://plato.stanford.edu/entries/decision-causal/ (visited on 04/04/2018).

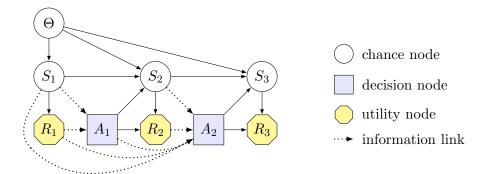


Figure 9: Representing an MDP with unknown transition probabilities with an influence graph. The nodes represent states S_1, S_2, \ldots , actions A_1, A_2, \ldots , and rewards R_1, R_2, \ldots . The unknown state transition probabilities $P(s_t \mid s_{t-1}, a_t)$ are modeled by adding an unobserved parameter node Θ . To permit non-stationary, learning policies, the decision context for each action contains all previously observed information. To model an MDP with unknown rewards assigned to each state, arrows from Θ to R_1, R_2 , and R_3 would also be added.

A. Representing Uncertainty

This section shows how a Markov decision process (MDP) with unknown transition function can be modeled with an influence diagram. By assuming that the agent can choose a policy that optimizes its value function, we are implicitly assuming that the agent knows the probabilistic relationship between variables. This is less restrictive than it may seem, because unknown probabilistic relationships can always be represented by adding an unobserved node Θ . For example, if the probabilistic relationship $P(x \mid \mathbf{pa}_X)$ between X and its parents \mathbf{Pa}_X is unknown, then we add Θ as an additional parent of X, and let the outcome of Θ determine the relationship between X and \mathbf{Pa}_X . By refraining from adding an information link from Θ to the agent's decision nodes, we specify that Θ is unobserved or latent. For each $\theta \in dom(\Theta)$, the influence model must specify a prior probability $P(\theta)$ and a concrete relationship $P(x \mid \mathbf{pa}_X, \theta)$. This lets the agent do Bayesian reasoning about the possible values of θ and the possible relationships between X and \mathbf{Pa}_X .

Let us illustrate by modeling an MDP with unknown transition probabilities, which are a standard mathematical framework for reinforcement learning (Sutton and Barto, 2018). In an MDP, an agent is taking actions A_1, A_2, \ldots that influence states S_1, S_2, \ldots , in order to optimize rewards R_1, R_2, \ldots To represent that the state-transition function is initially unknown, a node Θ has also been added to the graph; see Figure 9. Note that the influence diagram representation differs from the commonly used state transition diagrams (Sutton and Barto, 2018, Ch. 3) by having nodes for each time step, rather than a node for each possible state.

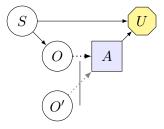


Figure 10: Theorem 18 shows that a rational choice of A never depends on irrelevant observations such as O'. It thereby allows us to cut the information link $O' \to A$ without loss of quality in the choice of A.

B. Proofs

Appendix B.1 gives the proofs for the observation incentive criterion (Theorem 10) and Appendix B.2.2 gives the proofs for the intervention incentive criterion (Theorem 17).

B.1. Observation Incentive Proofs

This aim of this section is to give a proof of Theorem 10, which identifies observation incentives in influence diagrams. To this effect, we establish two theorems showing that:

- Soundness: An optimal policy need never depend on an irrelevant observation (Theorem 18). This establishes the *only if* direction of Theorem 10.
- Completeness: For any graph G where O is a relevant observation, there exists a distribution P over G such that every optimal policy must depend on O (Theorem 21). This establishes the if direction of Theorem 10.

The theorems and their names are closely related to the soundness and completeness theorems for d-separation, established by Verma and Pearl (1988) and Geiger and Pearl (1990), respectively. They are also related to the soundness and completeness theorems about strategic relevance by Koller and Milch (2003).

We start with soundness in Appendix B.2, and continue with completeness in Appendix B.2.1.

B.2. Soundness

Soundness results similar to the one we give here has previously been established by Detwarasiti and Shachter (2005), Lauritzen and Nilsson (2001), Maskin and Tirole (2001), Milch and Koller (2008), Nielsen and Jensen (1999), and Shachter (1998). Figure 10 illustrates the theorem.

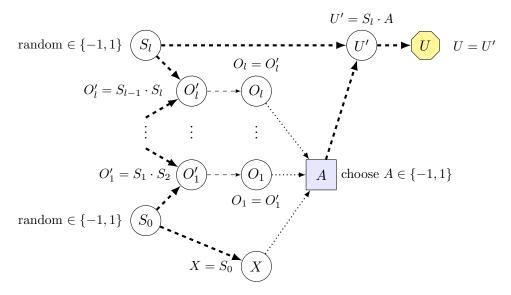


Figure 11: The completeness construction described in Definition 20. Dashed arrows represent directed paths of nodes. The thick path shows the supporting paths (Definition 19). Only by observing X is it possible to distinguish an assignment s from an assignment -s to the nodes $S = \{S_0, \ldots S_l\}$.

Theorem 18 (Observation incentive criterion; soundness direction). In a single-action influence graph $(\mathbf{W}, E, \{A\}, \mathbf{U})$ the agent has an observation incentive for a node $X \in \mathbf{W} \setminus \operatorname{desc}(A)$ only if X is d-connected to a utility node that descends from A:

$$X \not\perp \mathbf{U} \cap \operatorname{desc}(A) \mid \mathbf{\textit{Pa}}_A \cup \{A\}.$$

Proof. Since X is d-separated from U by $\mathbf{Pa}_A^* \cup \{A\}$, for each action $a \in dom(A)$ the expected utility of taking action a is independent of X. That is, for any $x, x' \in dom(X)$:

$$\mathbb{E}\left[\sum_{U\in\mathcal{U}}U\;\middle|\;a,\mathbf{pa}_A^*,x\right] = \mathbb{E}\left[\sum_{U\in\mathcal{U}}U\;\middle|\;a,\mathbf{pa}_A^*,x'\right].$$

Therefore the optimal action cannot depend on X.

B.2.1. Completeness

The following definition defines backwards and forward supporting paths, which are the d-connecting paths between an action and a utility variable, and an observation and a utility variable. These paths contain variables relevant to our completeness theorem. The paths are shown in Figure 11.

Definition 19 (Supporting paths). Assume that $X \in \mathbf{Pa}_A^*$ is a relevant observation to A in a single-action causal influence graph $(\mathbf{W}, E, \{A\}, \mathbf{U})$. We will refer to

- A forward supporting path of A and X is a directed path $A \longrightarrow U \in U$, and
- A backwards supporting path of A and X is a undirected path $X U' \in U$ not passing A that is active when conditioning on $\mathbf{Pa}_A \cup \{A\}$.

A pair of a backwards supporting path and a forwards supporting path for A and X where both paths end in the same $U \in U$ is called a *supporting pair of paths for* A and X; see Figure 11. By definition, there must be at least one supporting pair of paths for each relevant observation X.

Definition 20 (Completeness construction). As illustrated in Figure 11, for any pair of supporting paths for A and a relevant observation $X \in \mathbf{Pa}_A^*$, the forward supporting path always has the simple form

$$A \dashrightarrow U' \dashrightarrow U$$

and the backwards supporting path always has the form

$$X \leftarrow S_1 \longrightarrow O'_1 \leftarrow \cdots \longrightarrow O'_l \leftarrow S_l \longrightarrow U' \longrightarrow U. \tag{1}$$

Here U' is the node where the path merges with the forward supporting path $A \to U$. The nodes X, O_1, \dots, O_l are all in \mathbf{Pa}_A , and no other nodes on the path are in \mathbf{Pa}_A . The following special cases are covered under the general form of (1) for the backwards supporting path:

- $X = S_0$ means that the path starts forward from X.
- $X = S_0$ and l = 0 means that the path is directed $X \to U$.
- U = U' means that the paths from A and from X only merge at U.

Choose P per the following. All nodes have domain $\{-1,1\}$, and:

- S_1, \ldots, S_l are sampled randomly and independently from $\{-1, 1\}$.
- Any collider node $O'_i \in \{O'_1, \dots, O'_l\}$ is the product of its two neighbors on the path.
- U' is the product of its predecessor on the path from A and its predecessor on the path from X.
- All other nodes on the forward path and the backwards path copy the value of their causal predecessor on the path, and so do the nodes on the paths $O'_i \dashrightarrow O_i$.

Using this construction, we can now prove the *if* direction of Theorem 10.

Theorem 21 (Observation incentive criterion; completeness direction). In a single-action influence graph $(\mathbf{W}, E, \{A\}, \mathbf{U})$ the agent has an observation incentive for a node $X \in \mathbf{W} \setminus \operatorname{desc}(A)$ if X is d-connected to a utility node that descends from A:

$$X \not\perp \mathbf{U} \cap \operatorname{desc}(A) \mid \mathbf{Pa}_A \cup \{A\}.$$

Proof. For simplicity, we will assume that $X \in \mathbf{Pa}_A$. In the notation of Definition 9, this means that $\mathbf{Pa}_A^+ = \mathbf{Pa}_A$, and $\mathbf{Pa}_A^- = \mathbf{Pa}_A \setminus \{X\}$. The argument is easily adapted to the case when X is not in \mathbf{Pa}_A , by considering a graph with an extra information link $X \to A$.

We will establish the theorem this by showing that if X is d-connected to a utility node in the sense of

$$X \not\perp \mathbf{U} \cap \operatorname{desc}(A) \mid \mathbf{Pa}_A \cup \{A\},\$$

then there exists a distribution P such that there exists a policy $\pi^+(a \mid \mathbf{pa}_A^+)$ with

$$P(U = 1 \mid \pi^+) = 1$$

while any policy $\pi^-(a \mid \mathbf{pa}_A^-)$ that does not depend on X has

$$P(U = 1 \mid \pi^{-}) = P(U = -1 \mid \pi^{-}) = 1/2.$$

P may further be chosen so $dom(U) = \{-1, 1\}$, and $dom(U') = \{0\}$ for all other $U' \in U \setminus \{U\}$. As a consequence we get $V^{\pi^+} = 1$ and $V^{\pi^-} = 0$.

The proof relies on the following three observations about the completeness construction described in Definition 20:

(i) The construction ensures that $U = A \cdot S_l$ with probability 1

$$P(u \mid a, s_l) = \delta^u_{a \cdot s_l}. \tag{2}$$

since the outcome of S_l is just copied forward until U', where it is multiplied with the choice of A having been copied forward in the same way. The outcome of U' is then copied forward to U.

(ii) Every time the sign switches in the sequence $S = \{S_0, \ldots, S_l\}$, exactly one node O_i becomes negative. (The node O_i that sits between the sign switch on the path, to be precise.) Therefore $\prod_{i=1}^l o_i$ is positive if and only if $s_0 = s_l$, i.e.

$$P\left(s_l = s_0 \prod_{i=1}^l o_i\right) = 1. \tag{3}$$

(iii) Finally, $P(O = S_0) = 1$, since the outcome of S_0 is just copied forward to X.

Combining (ii) and (iii) gives that the policy $\pi^+(X, \mathbf{O}) = X \prod_{i=1}^l O_i$ will always make A match S_l , where $\mathbf{O} = \{O_1, \dots, O_l\}$. This in turn gives:

$$P(U=1\mid\pi^{+}) = \sum_{a,\boldsymbol{o},o,s_{l}} P(U=1,a,\boldsymbol{o},o,s_{l}\mid\pi^{+}) \qquad \text{demarginalize}$$

$$= \sum_{a,\boldsymbol{o},o,s_{l}} P(U=1\mid a,s_{l})\pi^{+}(a\mid\boldsymbol{o},o)P(\boldsymbol{o},o\mid s_{l})P(s_{l}) \quad \text{by d-separations}$$

$$= \sum_{a,\boldsymbol{o},o,s_{l}} \delta^{u}_{a,s_{l}}\pi^{+}(a\mid\boldsymbol{o},o)P(\boldsymbol{o},o\mid s_{l})P(s_{l}) \quad \text{by } (2)$$

$$= \sum_{s_{l}} \delta^{u}_{s_{l}s_{l}}P(s_{l}) \quad \text{by } \pi^{+} \text{ and (ii) and (iii)}$$

$$= 1/2 + 1/2 = 1 \quad \text{since } (s_{l})^{2} = 1.$$

This completes the first part of the proof.

Similarly, we can also show that P(u = -1) = P(u = 1) = 1/2 for any policy π^- that does not depend on X. The key is that observing $\mathbf{O} = \{O_1, \dots, O_l\}$ but not X only reveals places of sign switches in \mathbf{S} , but does not distinguish between \mathbf{s} and $-\mathbf{s}$. Therefore for any given \mathbf{o} , both s_l and $-s_l$ are equally likely,

$$P(S_l = 1 \mid \mathbf{o}) = P(S_l = 1 \mid \mathbf{o}) = 1/2$$
 (4)

And therefore all actions A have the same probability for U conditioning only on O,

$$P(U = 1 \mid \boldsymbol{o}, a) = \sum_{s_l} P(U = 1, s_l \mid \boldsymbol{o}, a) \qquad \text{demarginalize}$$

$$= \sum_{s_l} P(U = 1 \mid s_l, a) P(s_l \mid \boldsymbol{o}) \qquad \text{by d-separations}$$

$$= \sum_{s_l} \delta_{as_l}^1 P(s_l \mid \boldsymbol{o}) \qquad \text{by (2)}$$

$$= 1 \cdot 1/2 + 0 \cdot 1/2 = 1/2 \qquad \text{by (4)}.$$

The same calculation can be made for $P(U = -1 \mid \boldsymbol{o}, a)$. Since all actions conditioned only on \boldsymbol{o} induce the same U distribution, all policies π^- where the action only depends on \boldsymbol{o} also induce the same U distribution. This completes the second part of the proof.

B.2.2. Intervention Incentives

Proof of Theorem 17. Only if: If X = A, then X will already be chosen optimally for optimizing U, so no intervention on X can improve expected U utility. If there is no directed path $X \dashrightarrow U$ in G, then no intervention on X can affect U. Similarly, if there is a directed path in G but no directed path $X \dashrightarrow U$ in the trimmed graph G^* , then this

means that X only affects some irrelevant observations $O \in \mathbf{Pa} \setminus \mathbf{Pa}^*$. By Theorem 18, irrelevant observations can never affect the optimal action A, so therefore an intervention on X cannot affect the agent's expected utility.

If. Assume there is a path $X \dashrightarrow U \in U$ and $X \neq A$. Then either of the following cases ensues:

- 1. There is no action on the path $X \longrightarrow U$: Choose P to copy the value of X all the way forward to U.
- 2. The action A is on the path $X \dashrightarrow U$: Since $X \neq A$, this means that X is either a relevant observation $X \in \mathbf{Pa}_A^*$ or X is an ancestor of a relevant observation $O \in \mathbf{Pa}_A^*$. Let us consider these subcases in turn:
 - a) $X \in \mathbf{Pa}_A^*$: Use the completeness construction from Definition 20, with the modification that X = 0, unless an intervention c^X is made "restoring" the informativeness of X about S_0 . By the same argument as in Theorem 21, the intervention c^X will strictly increase the expected utility of the agent.
 - b) X is an ancestor of $O \in \mathbf{Pa}_A^*$: Again, we use a modification of the completeness construction from Definition 20. Let X = 0 and $O = X \cdot S_0$. Then O will be uninformative of S_0 , unless an intervention c^X is made that sets X = 1. Again, by the same argument as in Theorem 21, the intervention c^X will strictly increase the expected utility of the agent.

This completes the proof.