Brain and Entropy

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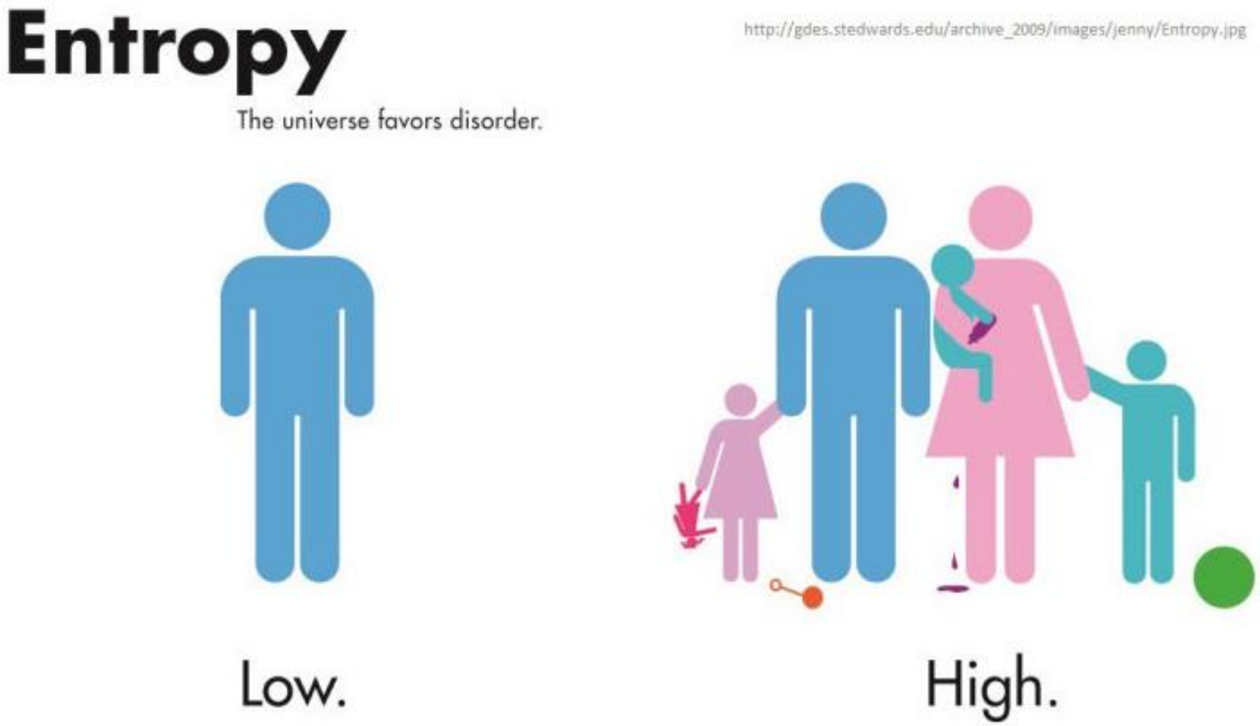
What is the Entropy

Entropy *is a scientific concept as well as a measurable physical property that is most associated with a state of disorder, randomness, or uncertainty.*

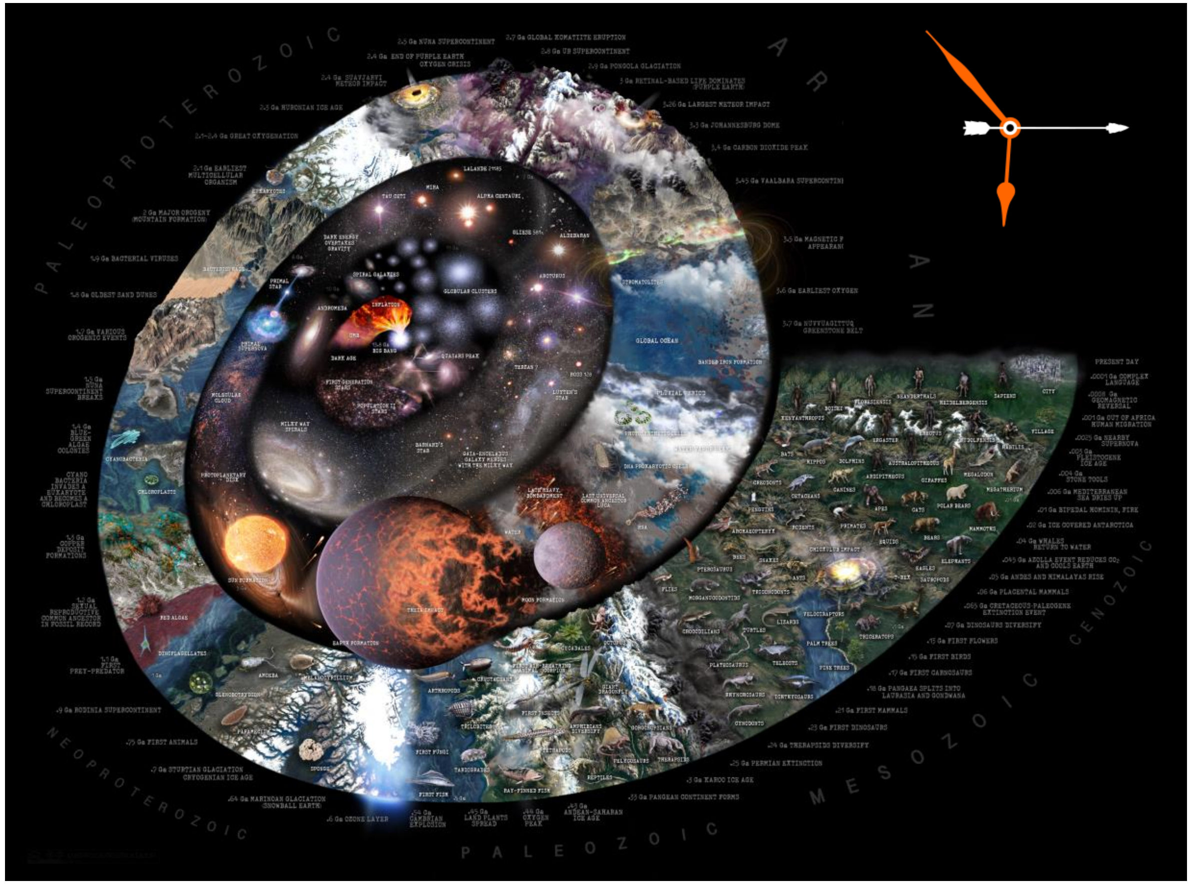
https://prateekvjoshi.files.wordpress.com/2014/12/1-main.png

Entropy from Everyday

Entropy is Increasing in the Universe



Arrow of time



https://medium.com/30-days-of-machine-learning/day-5-entropy-relative-entropy-and-cross-entropy-8369d67cc180

https://upload.wikimedia.org/wikipedia/commons/6/66/Nature_Timespiral.png

From Thermodynamic to Brain Science

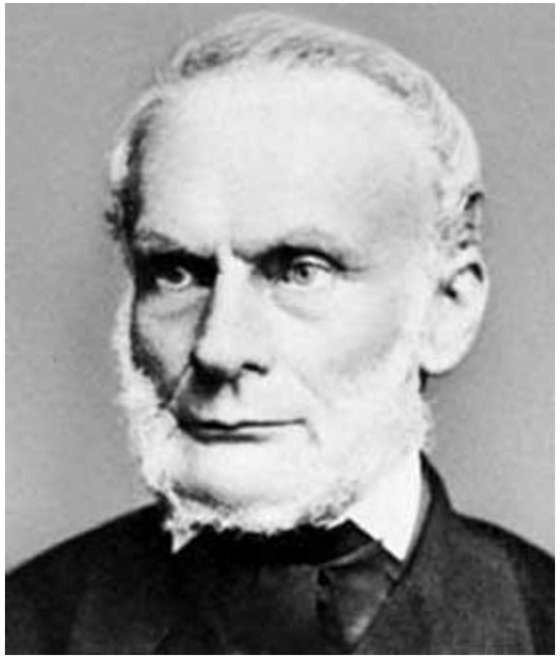
The Origin of Entropy

What does the word Thermodynamics bring to mind ?

Thermo *- heat*

Dynamics *- motion or change*

The second law of thermodynamics states that the total ***entropy*** *of a system either increases or remains constant in any spontaneous process; it never decreases. An important implication of this law is that heat transfers energy spontaneously from higher- to lower-temperature objects, but never spontaneously in the reverse direction.*

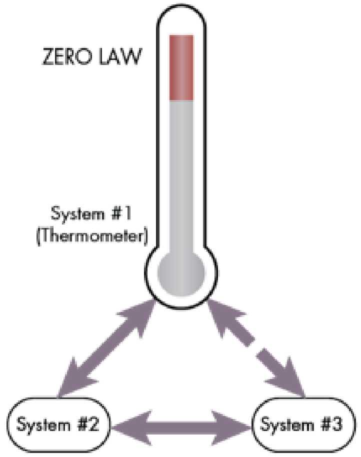


1822.01.02 –1888.08.24

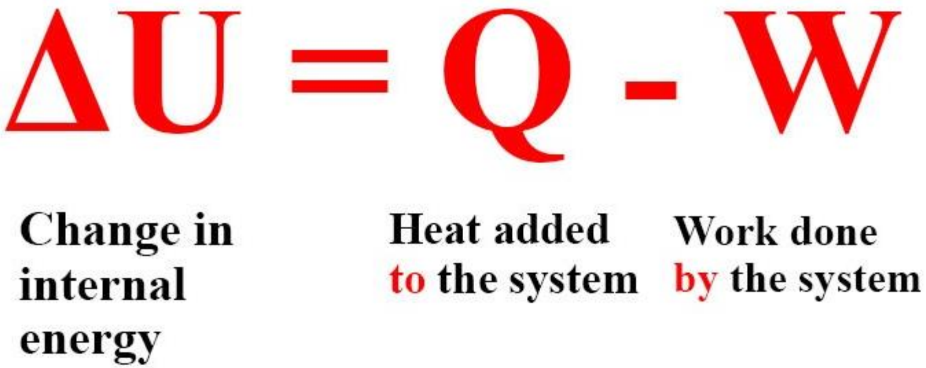
https://en.wikipedia.org/wiki/Rudolf_Clausius

Laws of Thermodynamics

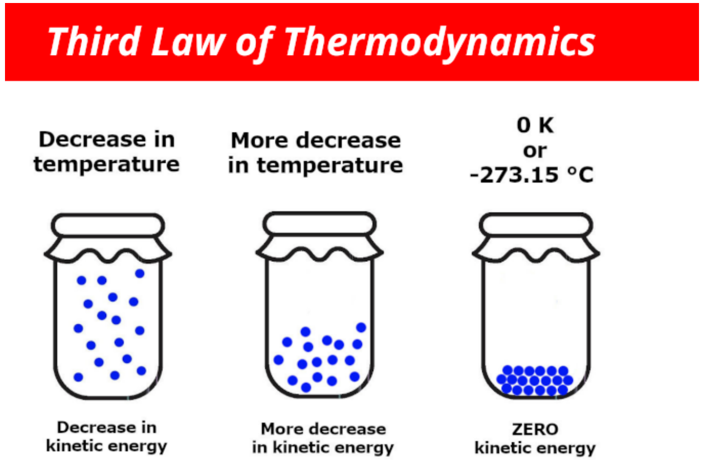
The zeroth law of thermodynamics *defines thermal equilibrium and forms a basis for the definition of temperature: If two systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.*



The first law of thermodynamics *states that, when energy passes into or out of a system (as work, heat, or matter), the system's internal energy changes in accord with the law of conservation of energy.*



The third law of thermodynamics *states that a system's entropy approaches a constant value as the temperature approaches absolute zero. With the exception of non-crystalline solids (glasses) the entropy of a system at absolute zero is typically close to zero*



Boltzmann Formula

$$S \propto \log W$$

$$S = k_{\mathrm{B}} \ln W$$

Where k_B *= Boltzmann's constant = 1.381 x* 10^{-23} *J/K*

W = number of microstates

In short, the Boltzmann formula shows the relationship between entropy and the number of ways the atoms or molecules of a certain kind of thermodynamic system can be arranged.

https://zh.wikipedia.org/wiki/File:Zentralfriedhof_Vienna_-_Boltzmann.JPG

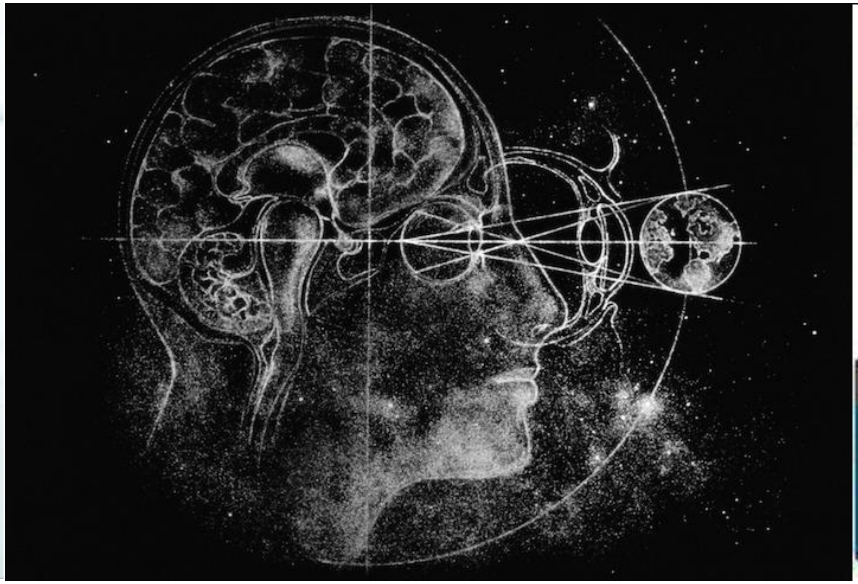
https://www.coursera.org/lecture/da-xue-hua-xue/9-9-shang-yu-re-li-xue-di-er-ding-lu-bo-er-zi-man-fang-cheng-re-li-xue-di-san-yHK7c

Boltzmann Brain

Infinite Monkey Theorem

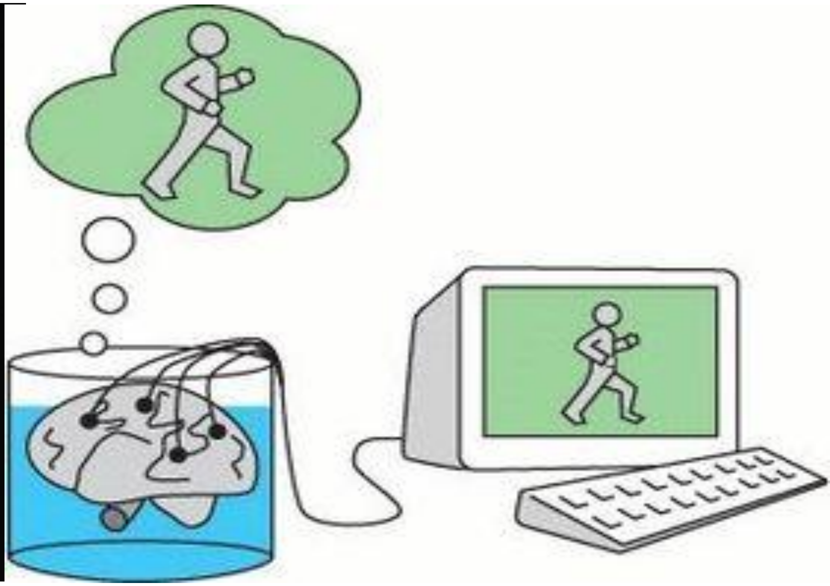
https://shrineofhiddenknowledge.wordpress.com/2016/09/14/280/

Boltzmann Brain



https://nautil.us/blog/can-many_worlds-theory-rescue-us-from-boltzmann-brains

Brain in a Vat



https://en.wikipedia.org/wiki/Brain_in_a_vat

Gibbs Free Energy

Gibbs free energy is the energy available in a substance to do work.

$$G(p,T) = U + pV - TS,$$

$$G(p,T) = H - TS,$$

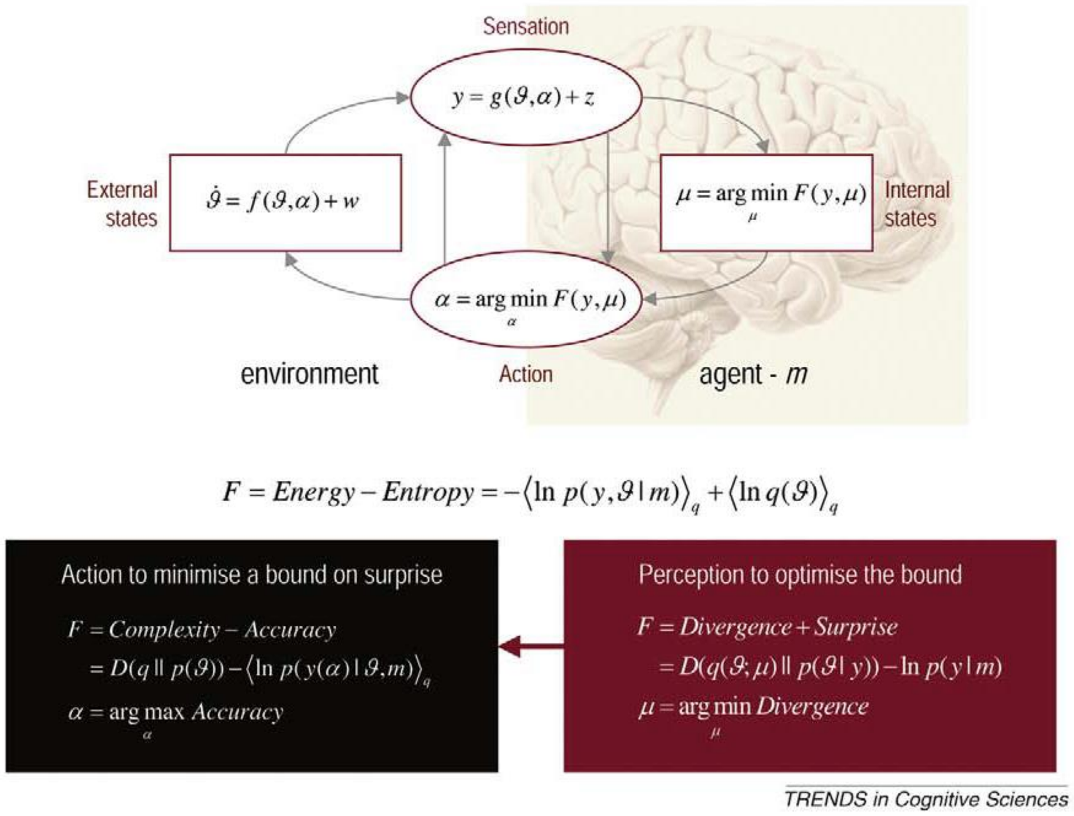
where:

- U is the internal energy (SI unit: joule),
- p is pressure (SI unit: pascal),
- V is volume (SI unit: m^3),
- T is the temperature (SI unit: kelvin),
- S is the entropy (SI unit: joule per kelvin),
- H is the enthalpy (SI unit: joule).

According to the second law of thermodynamics, for systems reacting at standard conditions for temperature and pressure (or any other fixed temperature and pressure), there is a general natural tendency to achieve a minimum of the Gibbs free energy.

Free Energy Principle (FEP)

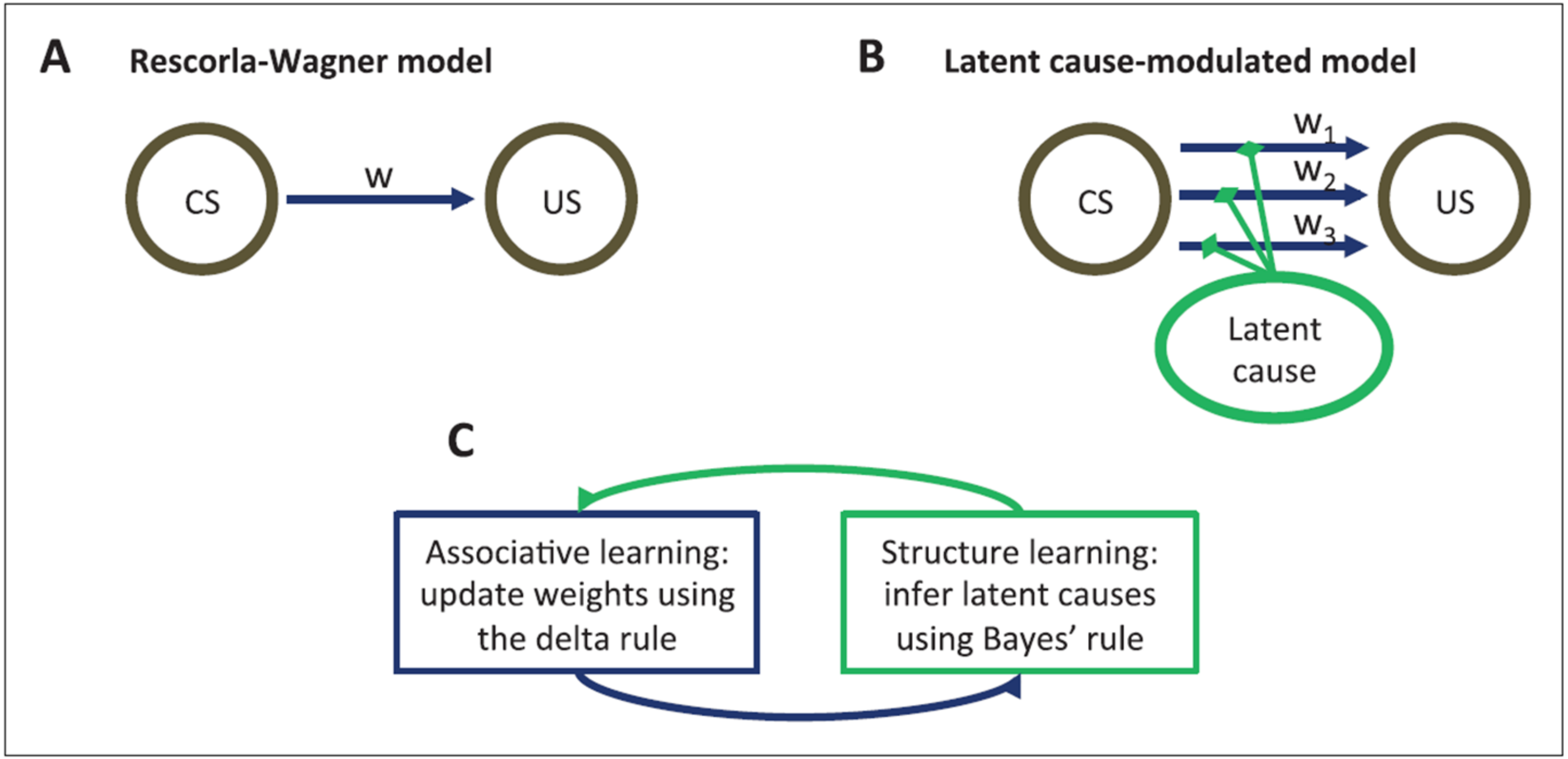
The free energy principle (FEP) states, in a nutshell, that the brain seeks to minimize surprise. (Gershman,2019)



Friston, K. (2009). The free-energy principle: a rough guide to the brain?. Trends in cognitive sciences, 13(7), 293-301.

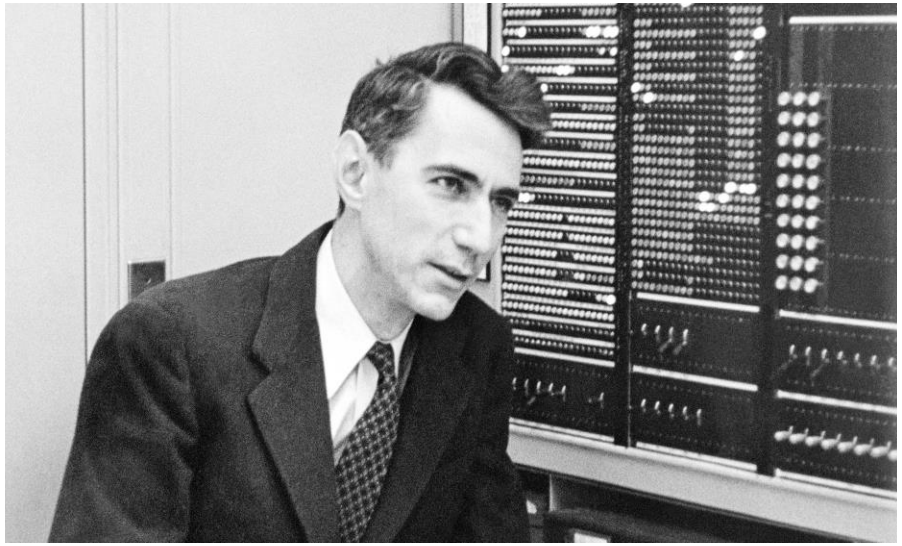
Karl Friston

The Computational Nature of Memory Modification



Gershman, S. J., Monfils, M. H., Norman, K. A., & Niv, Y. (2017). The computational nature of memory modification. Elife, 6, e23763.

Entropy and Information Theory

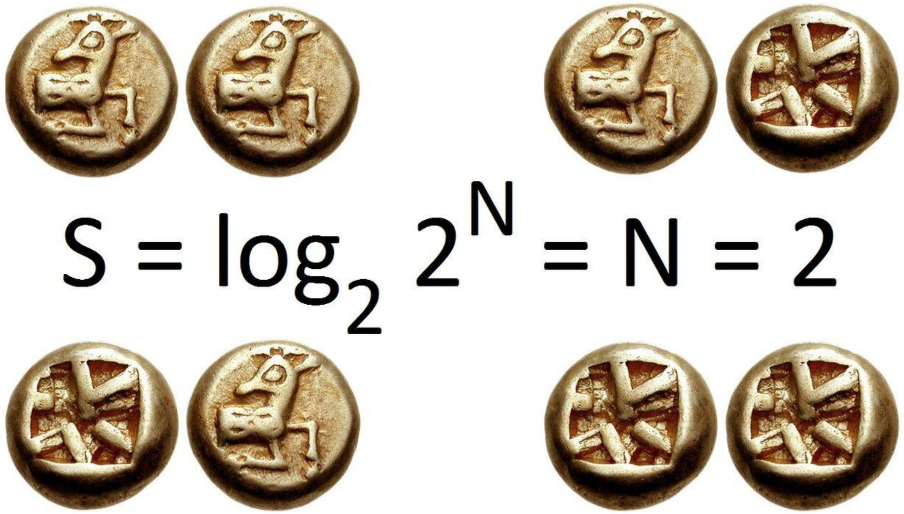


https://www.quantamagazine.org/how-claude-shannons-information-theory-invented-the-future-20201222/

Given a discrete random variable, with possible outcomes $x_1, \ldots, x_n$,which occur with probability $\mathrm{P}(x_1), \ldots, \mathrm{P}(x_n)$, the entropy of X, is formally defined as:

$$\mathrm{H}(X) = -\sum_{i=1}^{n} \mathrm{P}(x_i) \log \mathrm{P}(x_i)$$

$$H(X) = -(\tfrac{1}{4} \times log_2(\tfrac{1}{4}) \times 4)$$



Entropy and Biological Signal Measure

$$C_i^m(r) = \frac{num[d_m(X(i), X(j) < r)]}{(N-m+1)}$$

$$\phi^m(r) = \frac{\sum_{i=1}^{N-m+1} log(C_i^m(r))}{(N-m+1)^{-1}}$$

$$ApEn = \phi^m(r) - \phi^{m+1}(r)$$

Pincus (1991) introduced **Approximate Entropy (ApEn)** , a set of measures of system complexity closely related to entropy, which is easily applied to clinical cardiovascular and other time series.

$$B_i^m(r) = \frac{num[d_m(X(i), X(j) < r)]}{(N-m)}$$

$$B^m(r) = \frac{\sum_{i=1}^{N-m+1} B_i^m(r)}{N-m+1}$$

$$SampEn(m,r) = -ln[\frac{B^{m+1}(r)}{B^m(r)}]$$

Sample Entropy (SampEn) is a modification of approximate entropy (ApEn), used for assessing the complexity of physiological time-series signals, diagnosing diseased states.

Pincus, S. M. (1991). Approximate entropy as a measure of system complexity. Proceedings of the National Academy of Sciences, 88(6), 2297-2301.

Richman, J. S., & Moorman, J. R. (2000). Physiological time-series analysis using approximate entropy and sample entropy. American Journal of Physiology-Heart and Circulatory Physiology, 278(6), H2039-H2049.

Calculation of Entropy

Shannon Entropy

Approximate Entropy

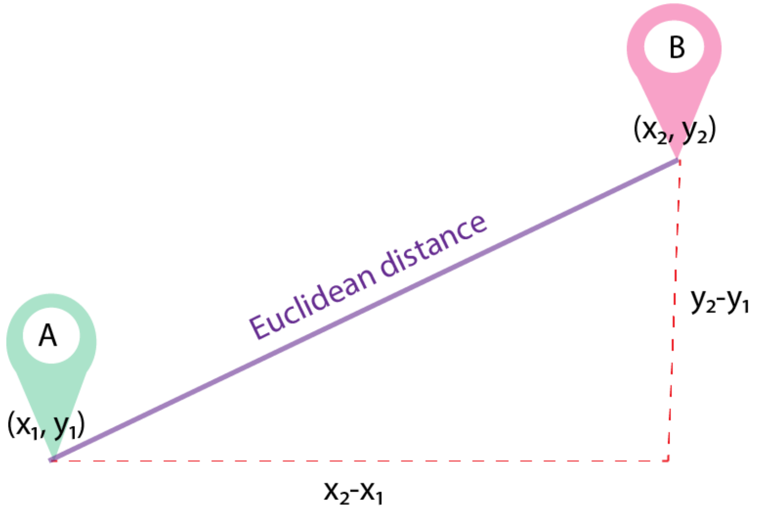
Sample Entropy

Calculating Entropy : https://colab.research.google.com/github/donghui1119/SEW_Brain_Entropy/blob/gh-pages/SEW_Entropy%20Calculation.ipynb

Distance Measures

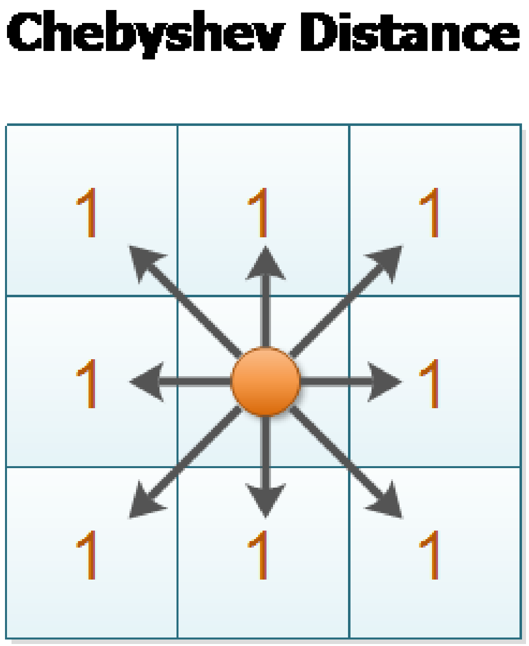
Euclidean Distance

$$EuclideanDistance = \sqrt{(x_2^2 - x_1^2) + (y_2^2 - y_1^2)}$$



Chebyshev distance

$$ChebyshevDistance = max(|x_2 - x_1|, |y_2 - y_1|)$$

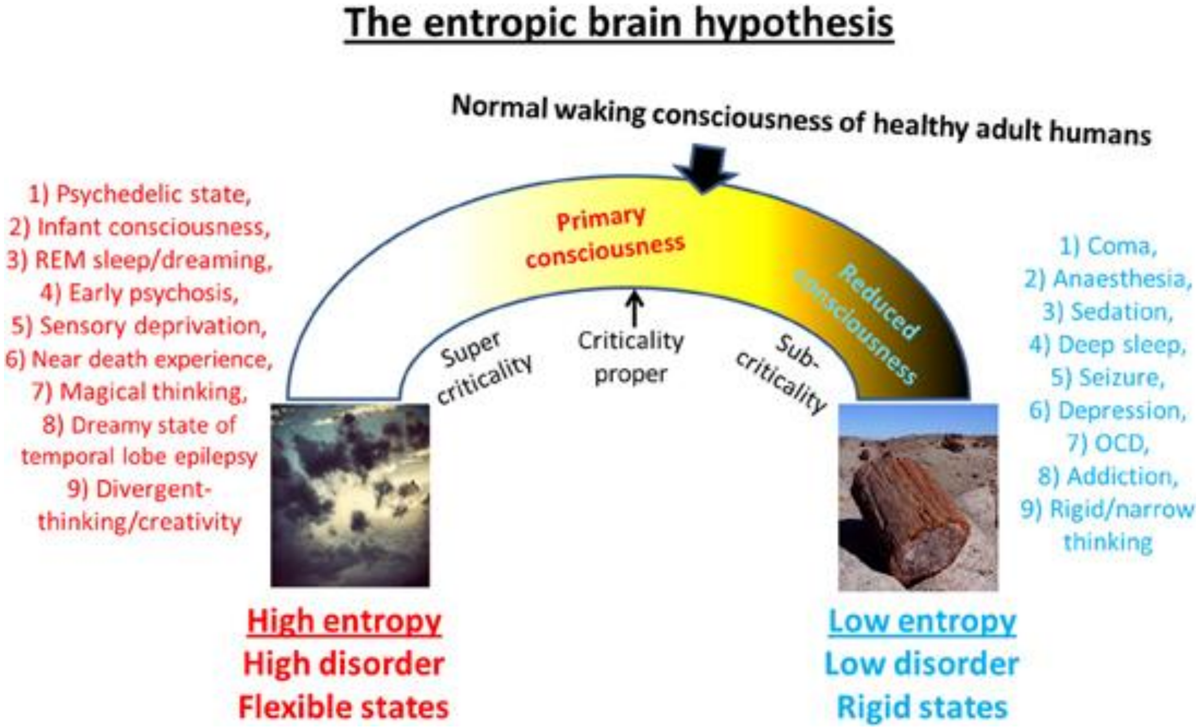


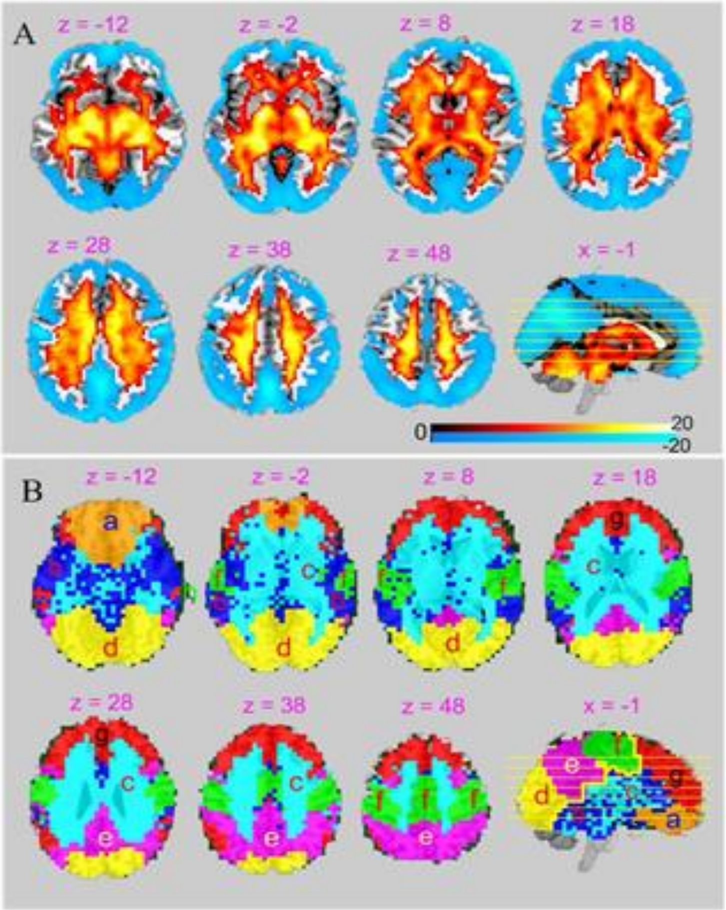
Manhattan Distance

$$ManhattanDistance = |x_2 - x_1| + |y_2 - y_1|$$

Calculating Distance https://colab.research.google.com/drive/1Ftq0slcUJCf_KaKbHlJJ7spJaTEp-Etn?usp=sharing

Entropy in Brain Science





Carhart-Harris et al.(2014). The entropic brain: a theory of conscious states informed by neuroimaging research with psychedelic drugs. Frontiers in human neuroscience, 8, 20.

Wang, Z., Li, Y., Childress, A. R., & Detre, J. A. (2014). Brain entropy mapping using fMRI. PloS one, 9(3), e89948.

Entropy in MRI

Denote the rsfMRI data of one voxel by $\mathrm{x}=[x_1,x_2,\ldots x_N]$, where N is the number of time points. SampEn starts with forming a series of vectors, the so called embedded vectors, each with m consecutive points extracted from x: $u_i=[x_i,x_i+1,\ldots x_i+m-1]$, where $\mathrm{i}=1$ to N-m+1, and m is the pre-defined dimension. Using a pre-specified distance threshold r, $\mathrm{B_i^m(r)}$ counts the number of u_j $(\mathrm{j}=1$,to N-m, and $\mathrm{j}\neq\mathrm{i})$ whose distances (Chebyshev distance is generally used though any other distance can be used as well) to u_i are less than r, so does $\mathrm{B_i^m+1(r)}$ for the dimension of m+1. By averaging across all possible vectors, we have

$$\mathrm{B^m(r)}=\frac{1}{\mathrm{(N-m)(N-m-1)}}\sum_{\mathrm{i=1}}^{\mathrm{N-m}}\mathrm{B_i^m(r)} \tag{1}$$

$$A^m(r)=\frac{1}{(N-m)(N-m-1)}\sum_{i=1}^{N-m}\mathrm{B}_i^{m+1}(r) \tag{2}$$

And SampEn is calculated as:

$$\mathrm{SampEn}(m,r,N,x)=-\ln\left[\frac{A^m(r)}{B^m(r)}\right] \tag{3}$$

https://github.com/zewangnew/BENtbx https://cfn.upenn.edu/zewang/BENtbx.php

BENtbx

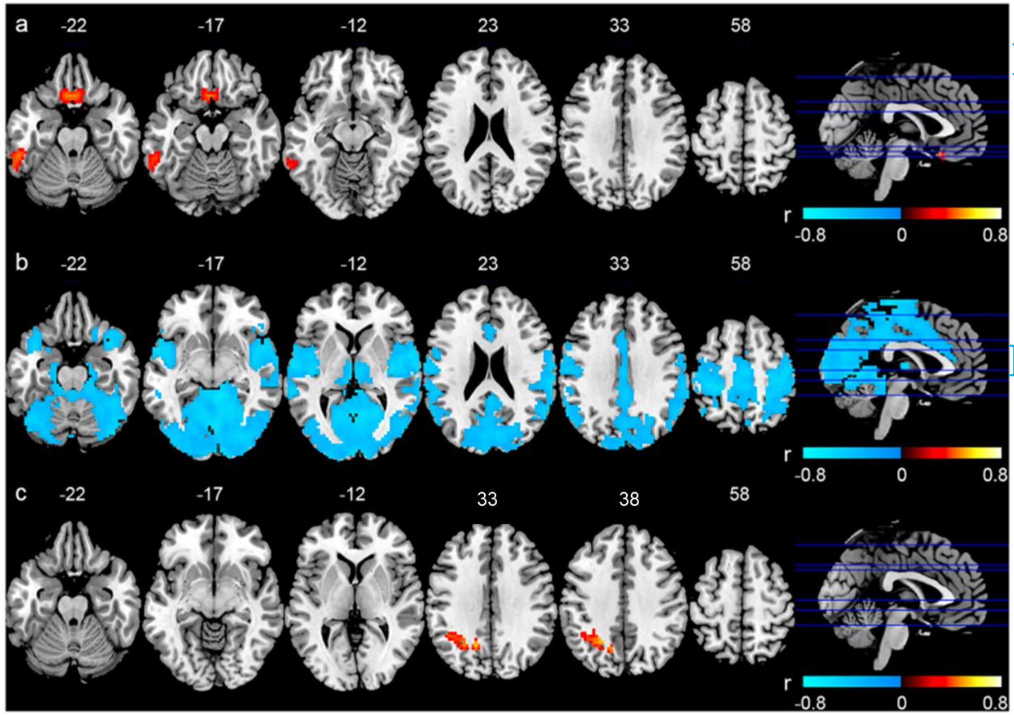
```
% File: batch_run.m.  pipeline file containing all processing steps
% Reference for the code can be
%   1. Ze Wang, Geoffrey K. Aguirre, Hengyi Rao, Jiongjiong Wang, Maria A. Fernandez-Seara, Anna R. Childres
%   2. Ze Wang, Yin Li, Anna Rose Childress, John A Detre, Brain Entropy Mapping using fMRI, Plos One, 2014,

batch_reset_orientation;   % resetting image origin to the center of the image volume. This step is required

batch_nsegment;     % segmenting T1 images. The c1, c2, c3 images (grey matter, white matter, and CSF segmen
batch_slicetiming;  % correcting acquisition time difference between slices
                                % You should change line 5 in batch_slicetiming.m to the correct number of
batch_realign;         % motion correction.  This code will be updated later to consider the global signal e
% batch_create_msk; % creating brain mask. You need fsl installed.  This file may not run in Windows. Then y
batch_create_msk2;
batch_tstd;               % calculating tsnr and temporal variance image
batch_coreg_t12EPI;    % registering T1 to EPI image. This is to resample the c2 and c3 images into the BOLD
batch_filtering;       % temporal nuisance filtering. A bandpass filter is used.
batch_smooth;        % smoothing the filtered images
batch_calc_BEN;   % calculating BEN maps
batch_coreg;            % coregistering EPI images and BEN map to T1
batch_norm_spm12; % normalization using spm8 nsegmentation
batch_smooth_BENmaps;   % smoothing BEN maps
```


https://cfn.upenn.edu/zewang/BENtbx.php

Brain Entropy and fALFF, CBF

·

Associations of BEN to cerebral blood flow and fALFF in the resting brain

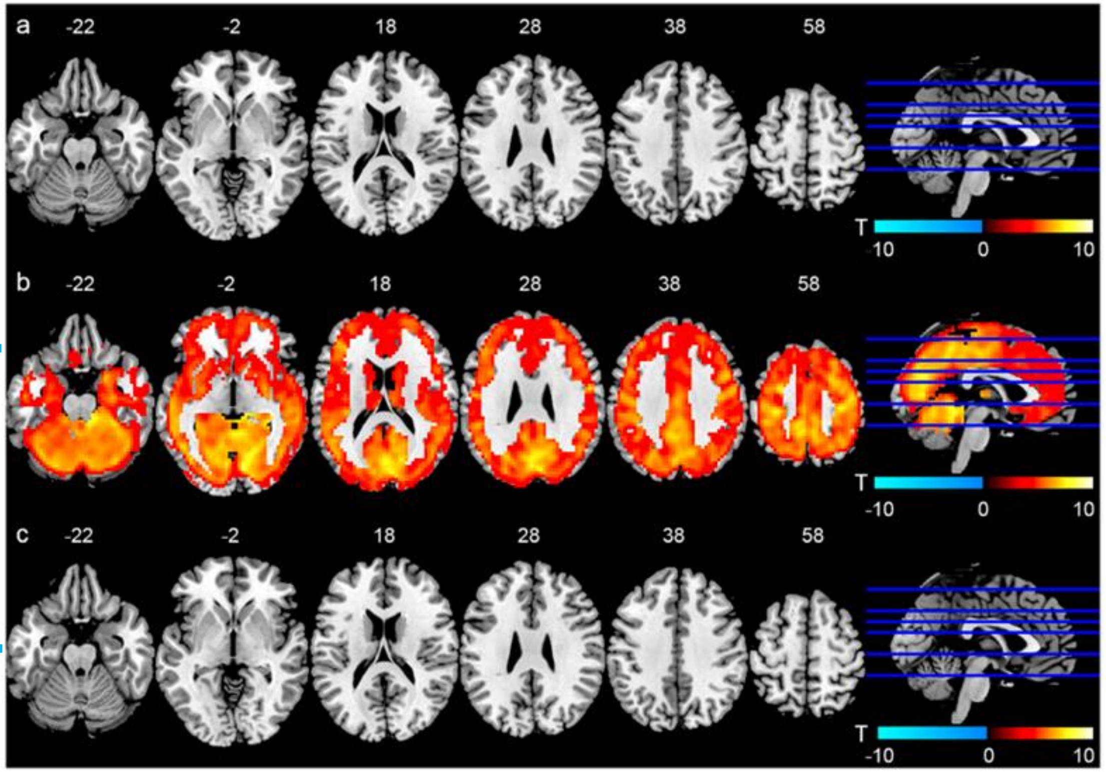


BEN-CBF

BEN-fALFF

CBF-fALFF

Sex differences to BEN, CBF and fALFF in the resting brain



BEN

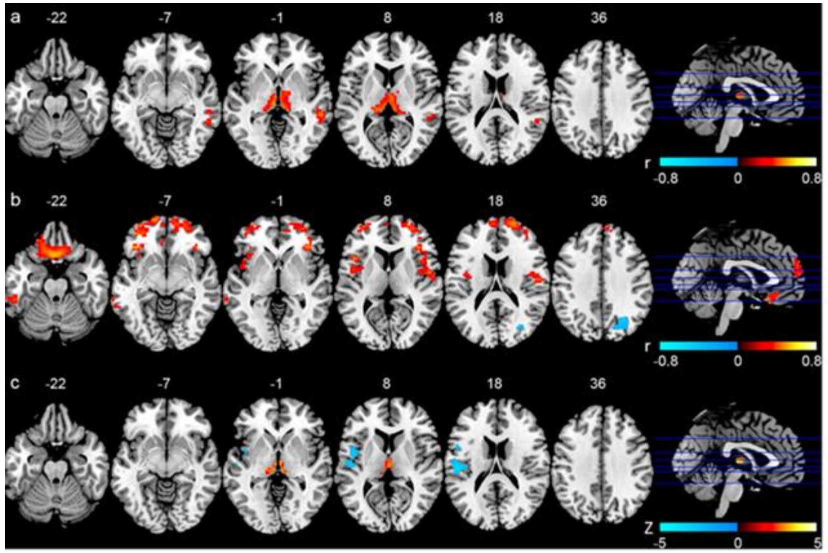
CBF

fALFF

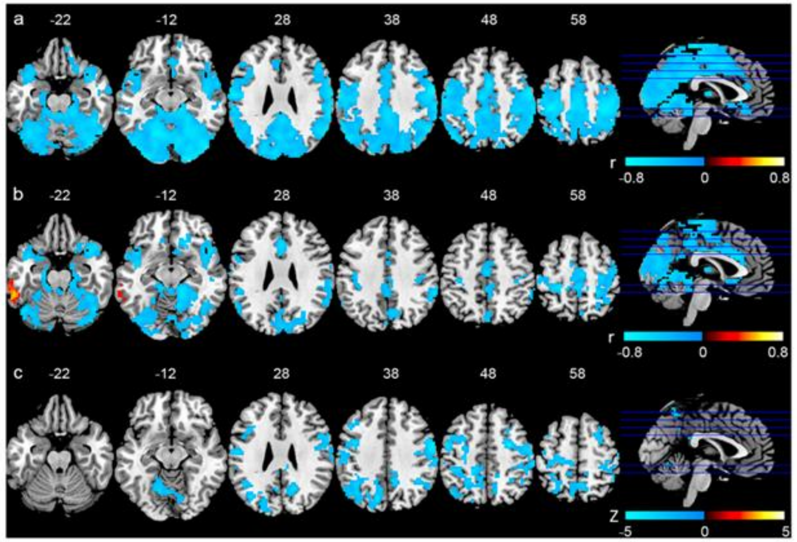
Donghui Song et al (2019b), Brain imaging and behavior

Associations of BEN to CBF and fALFF in different gender

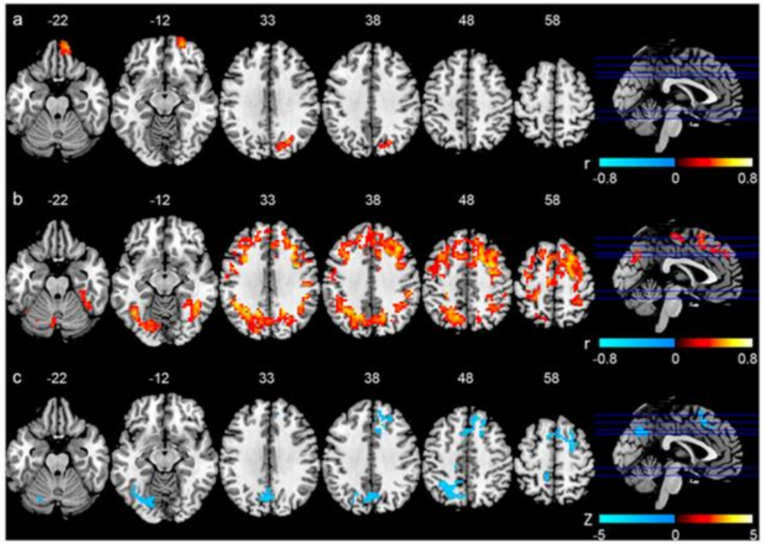
BEN-CBF



BEN-fALFF



CBF-fALFF

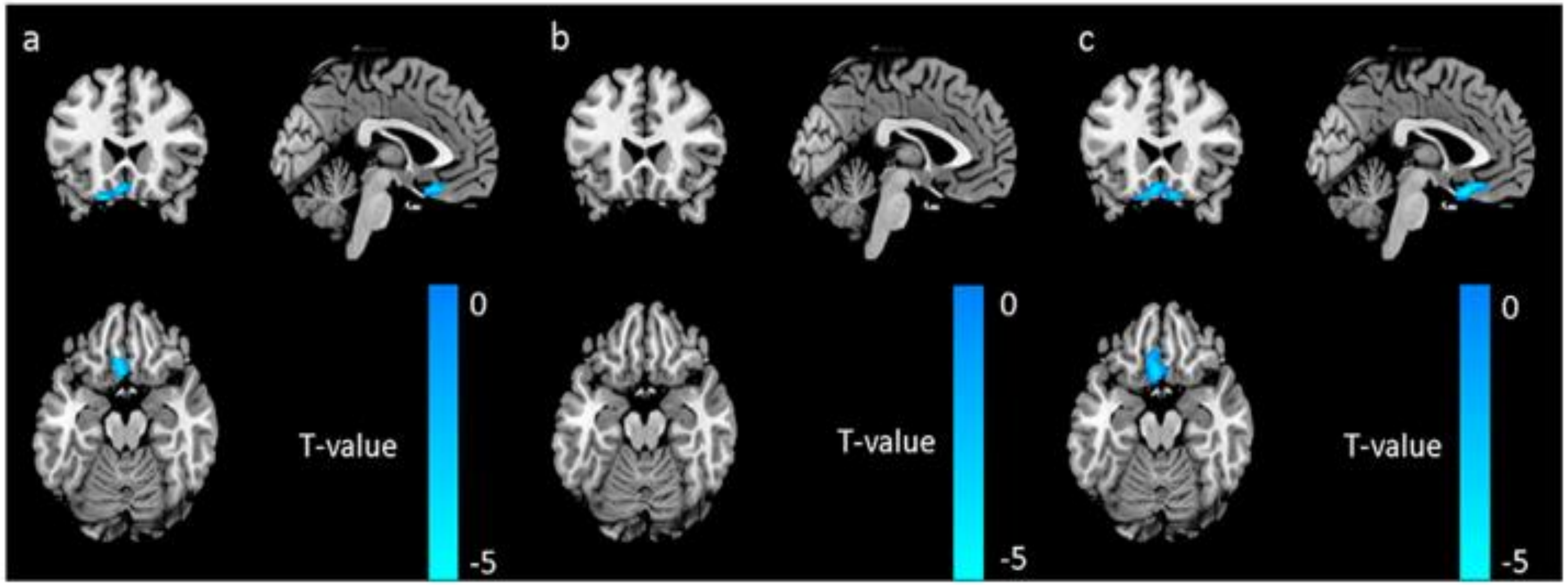


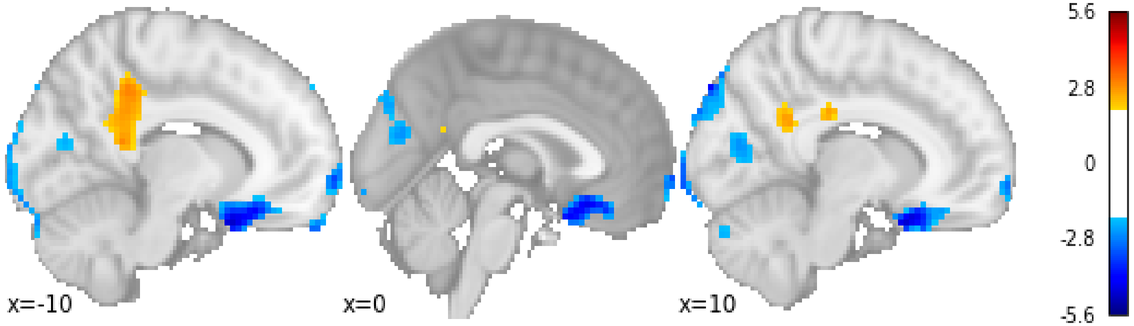
Donghui Song et al(2019b), Brain imaging and behavior

Brain Entropy and rTMS

Reduced Brain Entropy by rTMS on the left DLPFC in healthy young adults

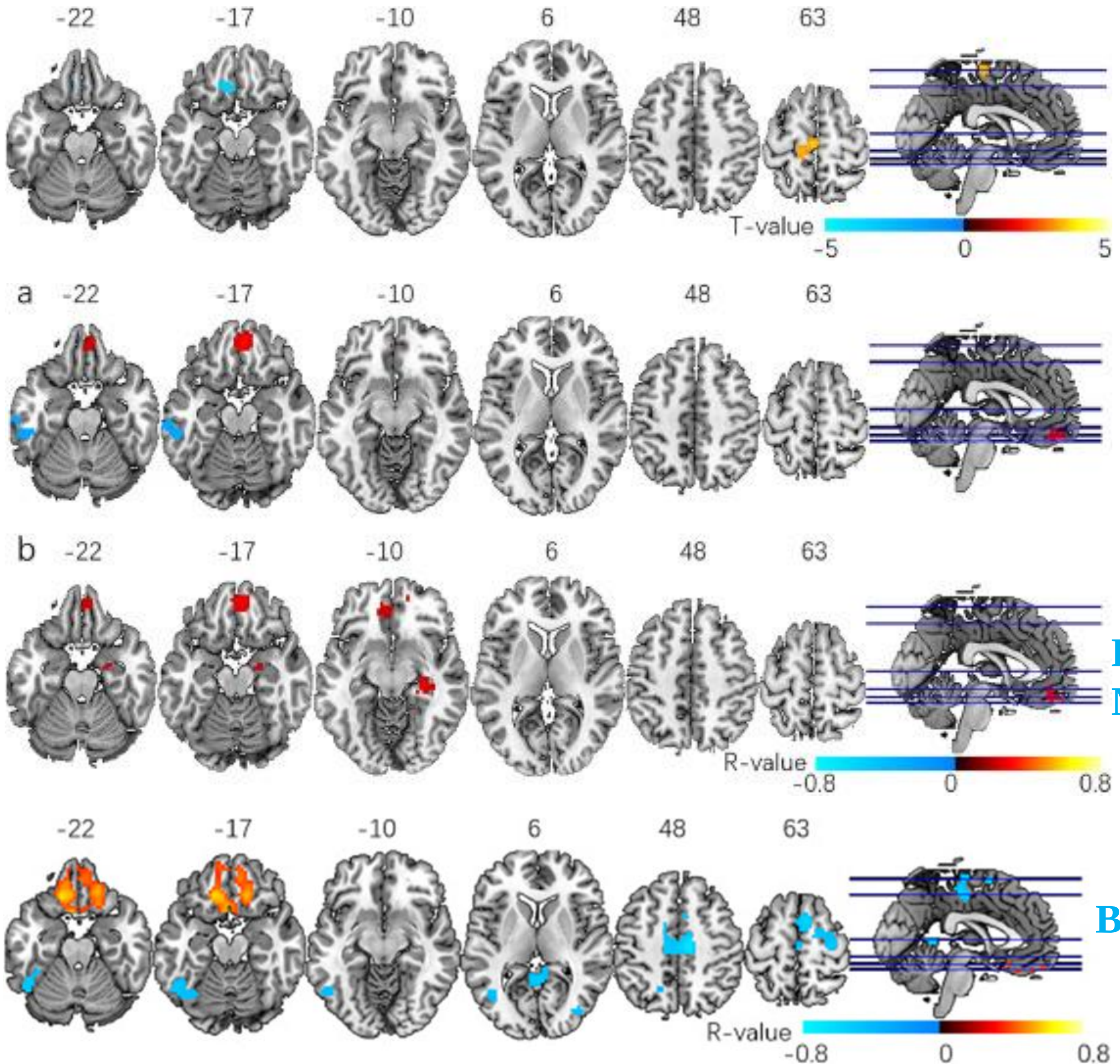
a. Post-rTMS vs pre-rTMS b. Post-SHAM vs pre-SHAM c. rTMS vs SHAM





Donghui Song et al(2019a), Brain imaging and behavior

Brain Entropy and Depression



Baseline MDD-BEN vs NC-BEN

Baseline BEN – HAMD in MDD

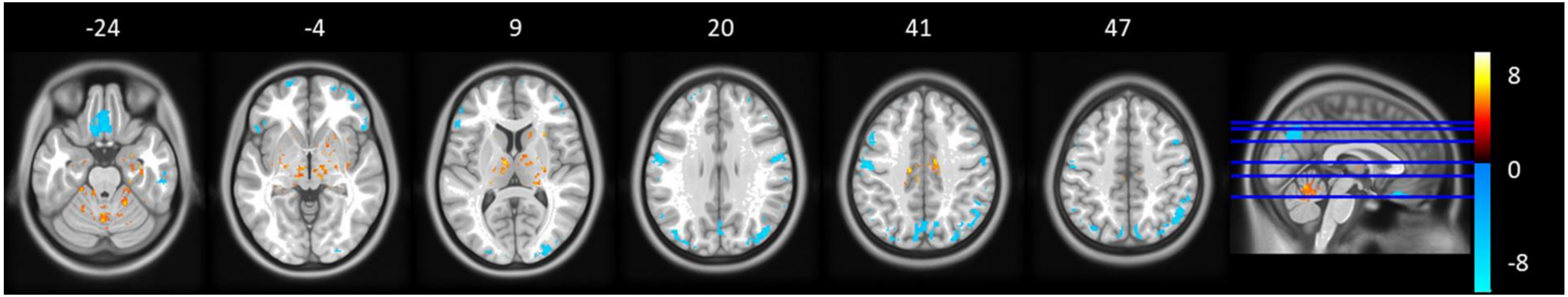
Baseline BEN-HAMD (8-week – baseline in MDD

BEN(diff)-HAMD (diff) after 8-week in MDD

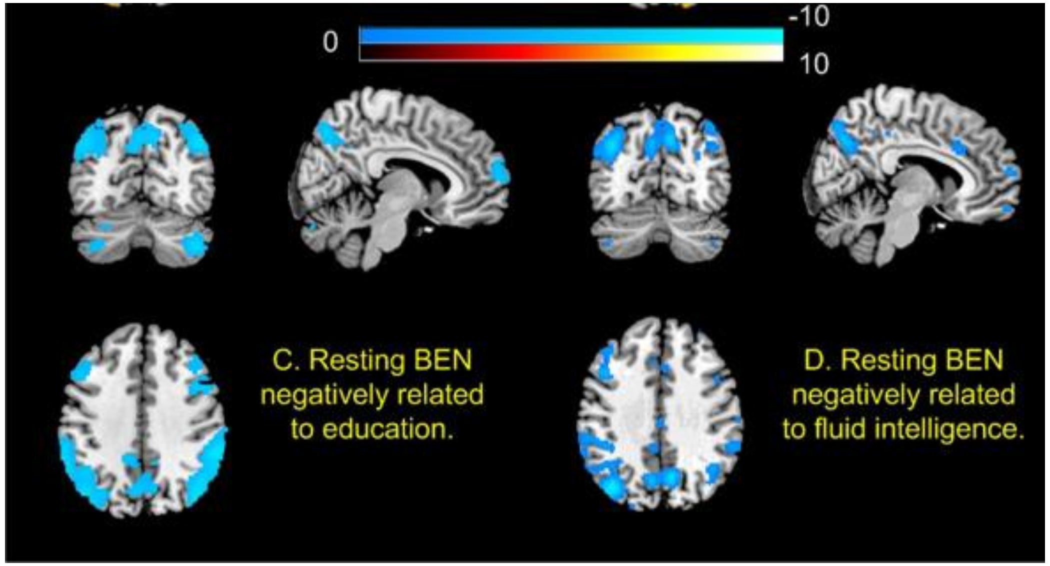
Xiaoyun Liu#, Donghui Song# et al (2020), Journal of affective disorders

BEN and Fluid Intelligence

Association between BEN and Fluid Intelligence in adolescent brain from ABCD study



Donghui Song et al (2021), poster on OHBM2021



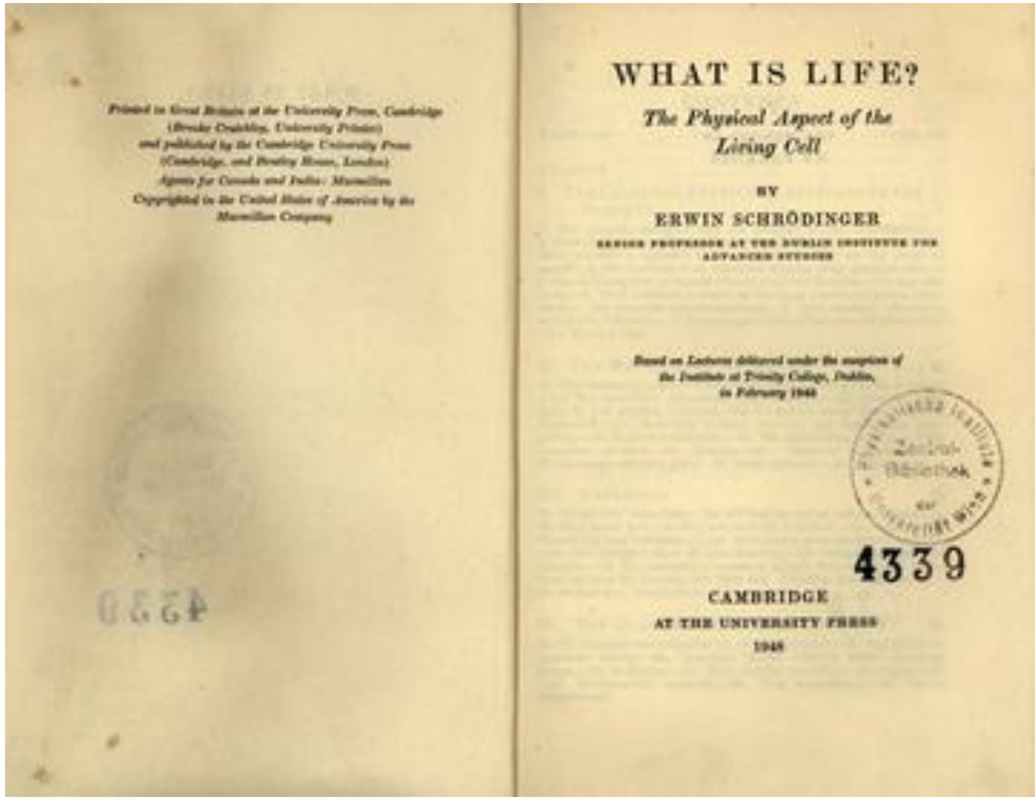
Ze Wang (2021), NeuroImage

Association between BEN and Fluid Intelligence in young adults' brain from HCP

Entropy and Life

Erwin Schrödinger

Karl Friston



WHAT IS LIFE?

The Physical Aspect of the Living Cell

BY

ERWIN SCHRÖDINGER

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Physics of Life Reviews

Volume 24, March 2018, Pages 1-16

Review

Answering Schrödinger's question: A free-energy formulation

Maxwell James Désormeau Ramstead [a, b], Paul Benjamin Badcock [c, d, e], Karl John Friston [f, 1]

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Acknowledgements

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Ze Wang, PhD

Biosketch

Dr. Wang received his PhD from Shanghai Jiao Tong University. His major research interests are in MR imaging, neuroimaging signal processing, and imaging-based translational research in Alzheimer's disease and addiction. He has published over 112 journal papers in these research topics. Regarding MRI, he focuses on arterial spin labeling (ASL) perfusion MRI and image reconstruction. His ASL work includes a 3D background suppressed spiral readout ASL MRI sequence, a series of ASL MRI data processing methods, as well as the first open-source software package for processing ASL data: ASLtbx.

https://www.medschool.umaryland.edu/pi/Ze-Wang-PhD/

https://www.bitrix24.com/about/blogs/webinars/bitrix24-live-qa-sessions-every-thursday.php