Numerical Optimization HW07

December 17, 2021

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1 Genetic Algorithm

Implement Genetic algorithm to seek the global minimum of the following functions :

$f(x) = 2(x-0.5)^2 + 1 \; on \; [0,1]$

$f(x) = |x-0.5|(cos(12\pi[x-0.5]) + 1.2) \; on \; [0,1]$

Discuss its performance depending on various control parameters such as length of genotype, population size, mutation probability, crossover probability and so on

2 Implementation

Define objective function

[ ]:
```
def f1(x):
    return 2*(x-0.5)**2+1


def f2(x):
    return np.abs(x-0.5)*(np.cos(12*np.pi*(x-0.5))+1.2)
```

Define parameters

[ ]:
```
L, N = 16, 50
p_mutate, p_cross = 1e-2, 0.8
factor = [float(2**i) for i in reversed(range(L))]
wgt = 0.3
selection_weight = [float(wgt*(1-wgt)**i) for i in range(N)]


def encode(val):  # Encode chromosome
    gene = [0 for _ in range(L)]
    val *= float(2**L-1)
    for i in range(L):
        gene[i], val = divmod(val, factor[i])
    return gene
```

```python
def decode(gene):  # Decode chromosome
    val = 0
    for i in range(L):
        val += gene[i]*factor[i]
    val /= float(2**L-1)
    return val


def fitness(p, f):  # Evaluate fitness
    key = []
    for chromosome in p:
        val = decode(chromosome)
        key.append(f(val))
    return key


def mutate(p):
    for chromosome in p:
        if r.uniform(0, 1) > p_mutate:
            continue
        i = r.randint(0, L-1)
        chromosome[i] = (chromosome[i]+1) % 2


def crossover(p, a, b):
    cross = []
    if r.uniform(0, 1) > p_cross:
        return 0, cross
    i = r.randint(0, L-1)
    cross = [p[a][:i]+p[b][i:],
             p[b][:i]+p[a][i:]]
    return 1, cross
```

[290]:
```python
def GA(f, gen):
    population = [[r.randint(0, 1) for _ in range(L)] for _ in range(N)]
    for i in range(gen):
        flag_mutate, flag_cross = 0, 0
        calculated_fitness = fitness(population, f)
        maintained_parents = [i for _, i in sorted(
            zip(calculated_fitness, population))]

        mutate(maintained_parents)
        [a, b] = r.choices([i for i in range(0, N)],
                           weights=selection_weight, k=2)
        flag_cross, cross = crossover(maintained_parents, a, b)
```

```
        population = maintained_parents[:N -
↪(flag_mutate+flag_cross)*len(cross)]+cross
    return decode(population[0])


def GAtest(f, l=L, n=N, pm=p_mutate, pc=p_cross, gen=100):
    factor = [float(2**i) for i in reversed(range(L))]
    wgt = 0.3
    selection_weight = [float(wgt*(1-wgt)**i) for i in range(N)]
    count = 0
    t = 500
    for i in range(t):
        if np.abs(GA(f, gen) - 0.5) < 0.01:
            count += 1
    return round(count*100.0/t, 2)
```

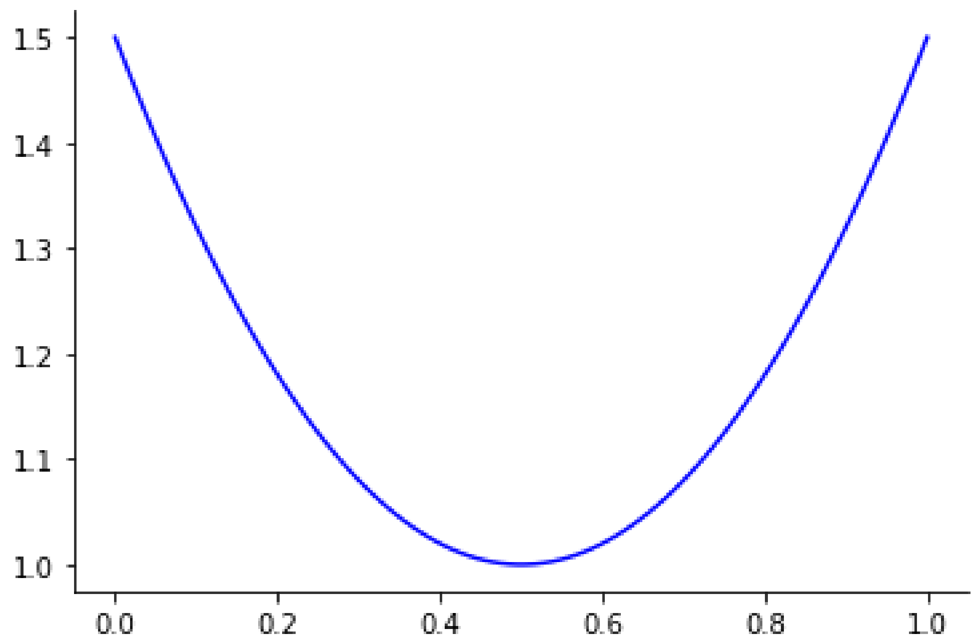
3 Result

We ran the genetic algorithm 500 times and measured the accuracy by the number of times the global minimum came out.

3.1 $f(x) = 2(x-0.5)^2 + 1 \; on \; [0, 1]$

the global minimum is 1 at $x = 0.5$

[313]:
```
plot(f1)
```



3.1.1 Along Population size

[293]:
```
print("L : {}, p_mutate : {}, p_cross : {}, gen : {}".format(16,0.01,0.8,50))
print("N : {:3d} -> Accuracy : {:.2f}%".format(20,GAtest(f1,16,20,0.01,0.8,50)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(50,GAtest(f1,16,50,0.01,0.8,50)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(100,GAtest(f1,16,100,0.01,0.
↪8,50)))
print("L : {}, p_mutate : {}, p_cross : {}, gen : {}".format(16,0.01,0.8,200))
print("N : {:3d} -> Accuracy : {:.2f}%".format(20,GAtest(f1,16,20,0.01,0.
↪8,200)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(50,GAtest(f1,16,50,0.01,0.
↪8,200)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(100,GAtest(f1,16,100,0.01,0.
↪8,200)))
```

```
L : 16, p_mutate : 0.01, p_cross : 0.8, gen : 50
N :  20 -> Accuracy : 89.20%
N :  50 -> Accuracy : 91.00%
N : 100 -> Accuracy : 90.00%
L : 16, p_mutate : 0.01, p_cross : 0.8, gen : 200
N :  20 -> Accuracy : 99.20%
N :  50 -> Accuracy : 99.40%
N : 100 -> Accuracy : 99.00%
```

3.1.2 Along crossover probability

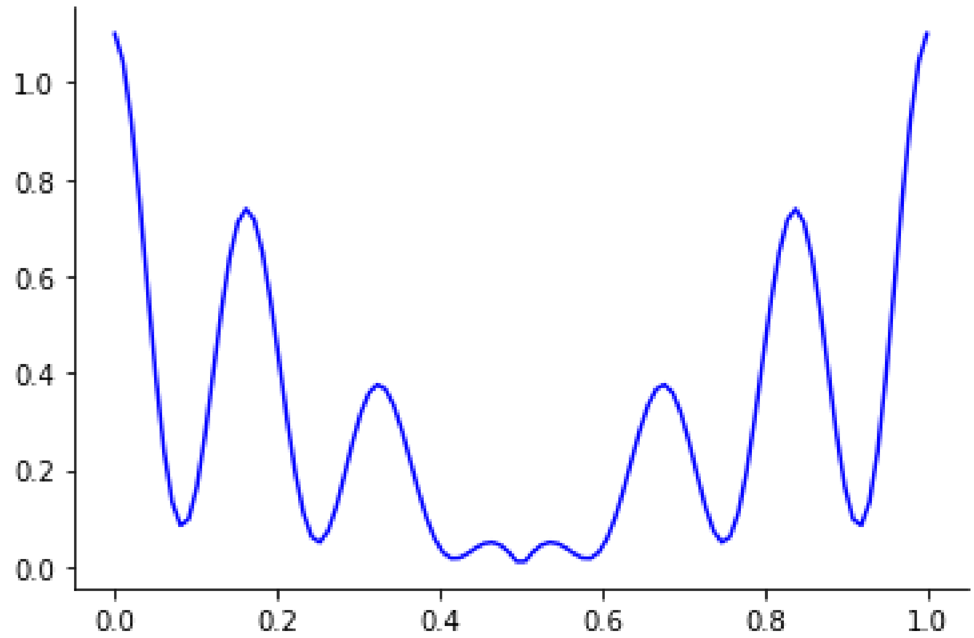
[298]:
```
print("L : {}, N : {:3d}, p_mutate : {}, gen : {}".format(16,50,0.01,50))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.5,GAtest(f1,16,50,0.01,0.
↪5,50)))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.7,GAtest(f1,16,50,0.01,0.
↪7,50)))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.9,GAtest(f1,16,50,0.01,0.
↪9,50)))
```

```
L : 16, N :  50, p_mutate : 0.01, gen : 50
p_cross : 0.5 -> Accuracy : 89.00%
p_cross : 0.7 -> Accuracy : 88.00%
p_cross : 0.9 -> Accuracy : 90.20%
```

3.2 $f(x) = |x - 0.5|(cos(12\pi[x - 0.5]) + 1.2)\ on\ [0, 1]$

the global minimum is 0 at $x = 0.5$

[314]:
```
plot(f2)
```



3.2.1 Along Population size

[294]:
```
print("L : {}, p_mutate : {}, p_cross : {}, gen : {}".format(16,0.01,0.8,50))
print("N : {:3d} -> Accuracy : {:.2f}%".format(20,GAtest(f2,16,20,0.01,0.8,50)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(50,GAtest(f2,16,50,0.01,0.8,50)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(100,GAtest(f2,16,100,0.01,0.
↪8,50)))
print("L : {}, p_mutate : {}, p_cross : {}, gen : {}".format(16,0.01,0.8,200))
print("N : {:3d} -> Accuracy : {:.2f}%".format(20,GAtest(f2,16,20,0.01,0.
↪8,200)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(50,GAtest(f2,16,50,0.01,0.
↪8,200)))
print("N : {:3d} -> Accuracy : {:.2f}%".format(100,GAtest(f2,16,100,0.01,0.
↪8,200)))
```

```
L : 16, p_mutate : 0.01, p_cross : 0.8, gen : 50
N :  20 -> Accuracy : 60.80%
N :  50 -> Accuracy : 61.40%
N : 100 -> Accuracy : 62.20%
L : 16, p_mutate : 0.01, p_cross : 0.8, gen : 200
N :  20 -> Accuracy : 63.40%
N :  50 -> Accuracy : 67.60%
N : 100 -> Accuracy : 65.20%
```

3.2.2 Along crossover probability

[299]:

```
print("L : {}, N : {:3d}, p_mutate : {}, gen : {}".format(16,50,0.01,200))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.5,GAtest(f2,16,50,0.01,0.
↪5,200)))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.7,GAtest(f2,16,50,0.01,0.
↪7,200)))
print("p_cross : {} -> Accuracy : {:.2f}%".format(0.9,GAtest(f2,16,50,0.01,0.
↪9,200)))
```

```
L : 16, N :  50, p_mutate : 0.01, gen : 200
p_cross : 0.5 -> Accuracy : 65.80%
p_cross : 0.7 -> Accuracy : 63.80%
p_cross : 0.9 -> Accuracy : 64.40%
```