

1-(a) w_0, w_1, \dots, w_d
 \therefore size of vector w is $(d+1)$

y_1, y_2, \dots, y_n
 \therefore size of vector w is n

1-(b)
$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^d \\ 1 & x_2 & x_2^2 & \dots & x_2^d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^d \end{bmatrix}$$

 \therefore size of matrix A is $n \times (d+1)$

1-(c)
$$\det A = \begin{vmatrix} 1 & x_1 & x_1^2 & \dots & x_1^d \\ 1 & x_2 & x_2^2 & \dots & x_2^d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^d \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x_1 & x_1^2 & \dots & x_1^d \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 & \dots & x_2^d - x_1^d \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & x_n - x_1 & x_n^2 - x_1^2 & \dots & x_n^d - x_1^d \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & x_2 - x_1 & x_2(x_2 - x_1) & \dots & x_2^{d-1}(x_2 - x_1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & x_n - x_1 & x_n(x_n - x_1) & \dots & x_n^{d-1}(x_n - x_1) \end{vmatrix}$$

$$= \prod_{i=2}^n (x_i - x_1) \begin{vmatrix} 1 & x_1 & \dots & x_1^{d-1} \\ 1 & x_2 & \dots & x_2^{d-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \dots & x_n^{d-1} \end{vmatrix}$$

$$= \prod_{k=1}^{n-1} \left(\prod_{i=k+1}^n (x_i - x_k) \right) = \prod_{1 \leq j < i \leq n} (x_i - x_j)$$

$$1-(d) \det A = \prod_{1 \leq i < j \leq n} (\alpha_i - \alpha_j)$$

$\alpha_i \neq \alpha_j$ (즉, $i \neq j$) 이여야 non-zero 된다.

$$1-(e) \quad \begin{aligned} Aw &= y \\ A^T Aw &= A^T y \\ w &= A^+ y \end{aligned}$$

$$\begin{aligned} 2 \quad Aw &= y \\ (A^T A)^+ A^T Aw &= (A^T A)^+ A^T y \\ w &= (A^T A)^+ A^T y \\ W &= A^+ y \end{aligned}$$