Methodology: Converting the given r(u) into right and left wheel velocities:
Our first step was to define u in terms of time t and a parameter β, because in order to specify wheel velocities we need to define r in terms of time. We decided that we wanted the Neato to take twenty seconds to cross the Bridge of Doom, so we set the domain of t to be [0, 20], and the β value that satisfies is $u_{final} = \beta t_{final} \rightarrow 3.2 = \beta * 20 \rightarrow \beta = 0.16$.

$$u = \beta t$$
$$r(t) = 0.3960\, cos(2.65(\beta t + 1.4))\hat{\imath} - 0.99\, sin(\beta t + 1.4)\hat{\jmath},$$
$$t \in [0,\ 20]\ and\ \beta = 0.16$$

Our next step was to calculate the planned linear speed of the Neato vs time. To do this, we took the derivative of r(t) to get dr/dt. However, this gives the Neato's speed in each of the î and ĵ directions, so we took the magnitude of this vector, $||dr/dt||$, to find the resulting overall speed.

Next, we calculated the angular velocity vs. time. To do this, we normalized dr/dt by dividing it by its magnitude, giving us Ť, the unit tangent vector at each value of t. Following the equation

$$^{G}\vec{\omega}^{B} = \hat{T}^{B} \times \left.\frac{d\hat{T}^{B}}{dt}\right|_{G},$$

we took the cross product of Ť and its derivative, $d\check{T}/dt$ to find the angular velocity, ω.

Once we had calculated both the linear speed, dr/dt, and the linear velocity, ω, we used the equations for left and right wheel velocity, as well as d = 0.245, the distance in between the wheels and the center of mass, to calculate the velocities of the left and right wheels over time.

$$V_L = V_N - \omega\frac{d}{2} \quad \text{and} \quad V_R = V_N + \omega\frac{d}{2}$$, where $V_N = dr/dt$

Reconstructing a path from encoder data:
The encoder data from the Neato's drive is expressed discreetly, and for each data point it gives the time of measurement (from t=0 is the beginning of the drive), the left wheel's total travel, and the right wheel's total travel.

First, we found Δt, the amount of time in between each measurement, and used that to discreetly take the derivative of the left and right wheels' linear travel, giving us each wheel's velocity at each timestep.

$$v_{timestep} = \Delta x/\Delta t$$

Next, we used these equations to calculate the linear and angular velocities from the left & right wheel velocities:

$$V = \frac{V_L + V_R}{2} \qquad \omega = \frac{V_R - V_L}{d}$$

Finally we used those linear and angular velocities to calculate the Neato's xy-position and angle from the ground perspective using the following formulas:

$$\begin{aligned}\mathbf{r}(t+\Delta t) &= \mathbf{r}(t) + \frac{dr}{dt}\Delta t \\ &= \mathbf{r}(t) + V(t)\hat{\mathbf{T}}(t)\Delta t \\ &= \Big(x(t) + V(t)\cos\big(\theta(t)\big)\Delta t\Big)\hat{\imath} + \Big(y(t) + V(t)\sin\big(\theta(t)\big)\Delta t\Big)\hat{\jmath}\end{aligned}$$

$$\begin{aligned}\theta(t+\Delta t) &= \theta(t) + \frac{d\theta}{dt}\Delta t \\ &= \theta(t) + \omega(t)\Delta t.\end{aligned}$$

Our set of values for r(t + Δt) gives us the Neato's path.

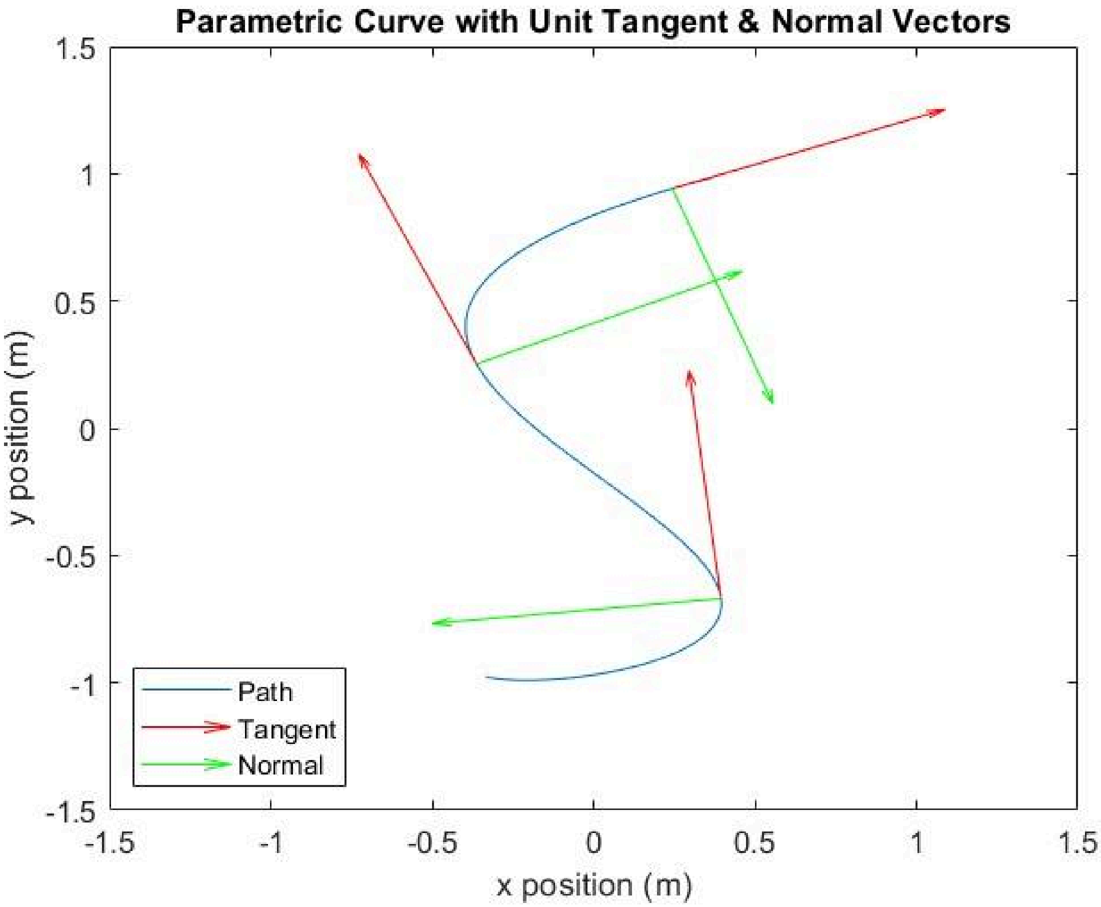

Figure 1. Plot of the parametric curve of Neato's path (r(u) = 0.3960cos(2.65(u+1.4)) î - 0.99sin(u+1.4) ĵ) with unit tangent and normal vectors at three points along the curve when u equals 1, 2, and 3.

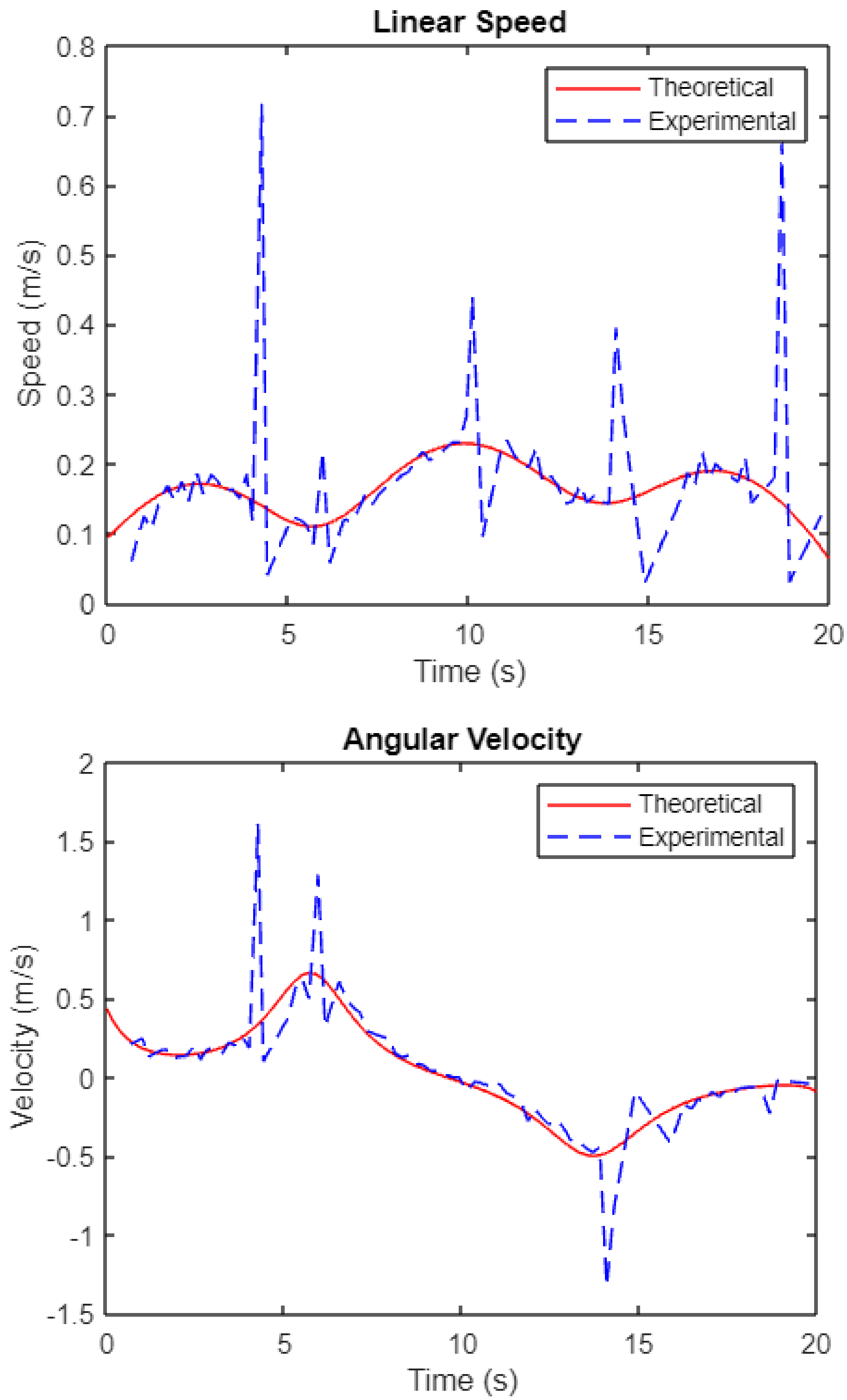

Figure 2. Plots of the theoretical and encoder-measured linear speed (top) and planned angular velocity (bottom) as the Neato traverses the curve from t = [0, 20] seconds.

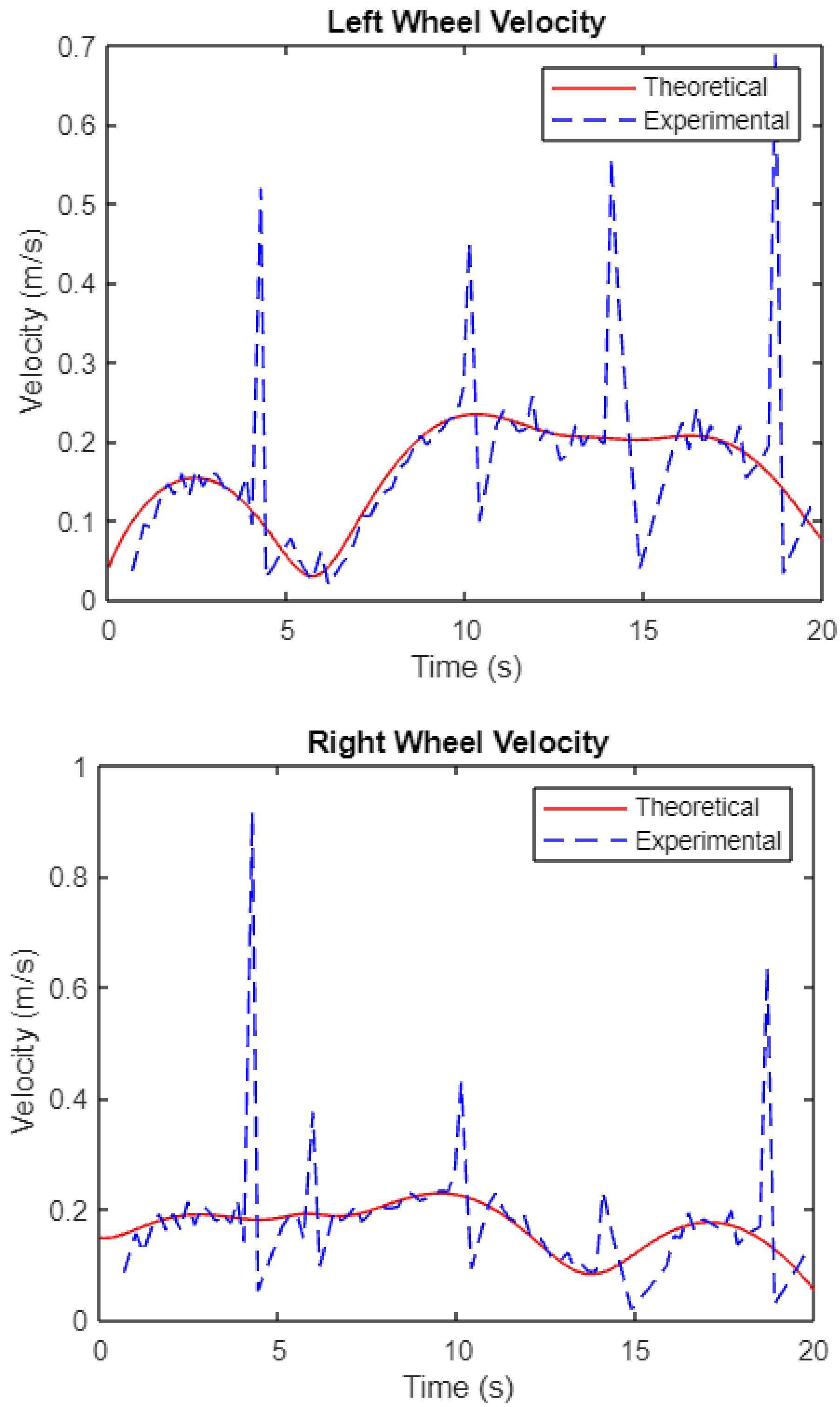

Figure 3. Plots of the Neato’s theoretical and encoder-measured left (top) and right (bottom) wheel velocities.

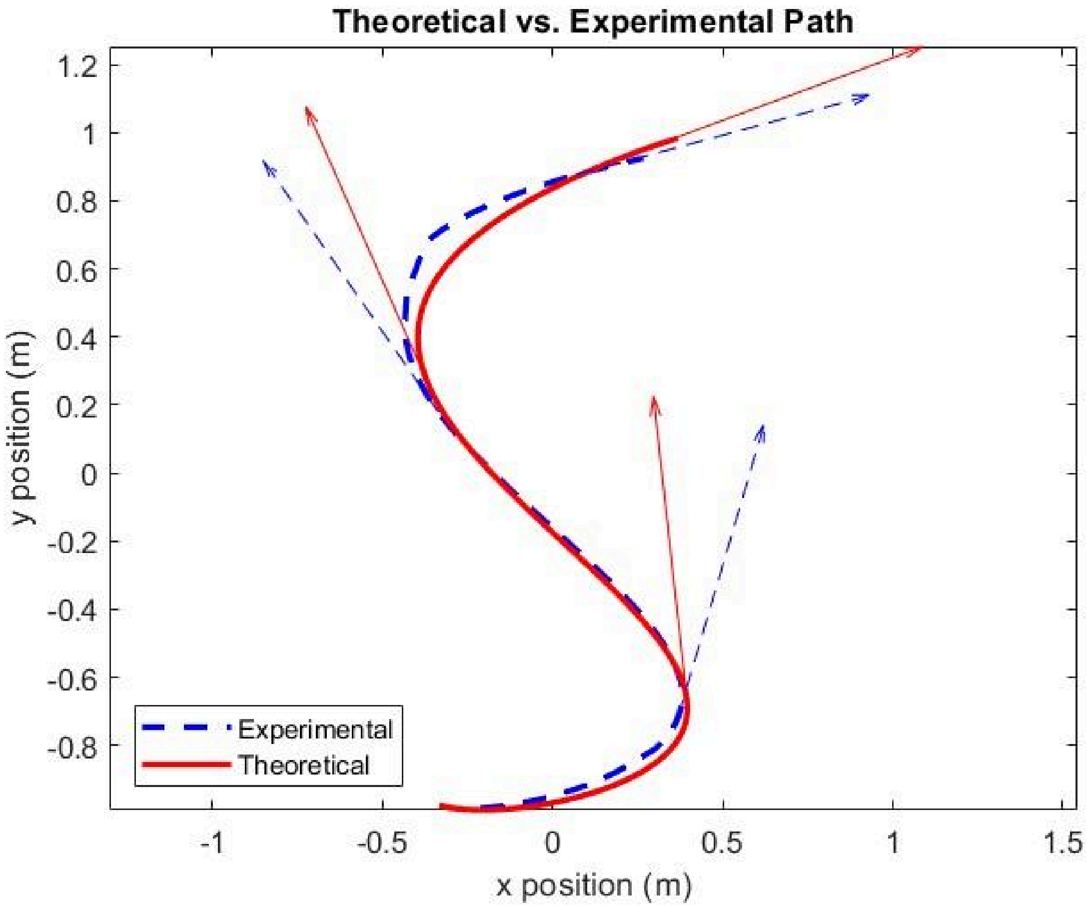

Figure 4. Plot of the Neato's theoretical and experimental path with unit tangent vectors at three points along the curve when u equals 1, 2, and 3. The experimental path was derived using the encoder data while our Neato traveled across the bridge.

Link to our Neato's epic journey across the Bridge of Doom:
https://youtu.be/ap_Qg2qdv1Y?si=JnScbKW1yfbUfPnh