Application of Recursive Methods for Parameter Estimation in Adaptive Control of DC Motor

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Abstract. The adaptive control uses a number of methods which provide a systematic approach for real-time automatic control. The recursive parameter estimation methods in particular are best applicable if simple parameter estimates are optainable using real-time identification algorithms. This is imposed by the fact that model adjustments after submission of new data from monitoring, require generation of adequate (adjusted) control signal within the sample time. The present article investigates an application of those methods for parameter estimation and their influence on the quality of adaptive control process of DC motor.

Keywords: Adaptive system · Instrumental variable method · Least squares method · Recursive methods for parameter estimation · DC motor

1 Introduction

System control by default requires deep understanding the dynamic features of the processes [3]. Recursive methods for parameter estimation are only used for real-time system identification. The real dynamics of the system should be reflected in the dynamic of the control signal synthesis and in the model in accordance with the new data received. This is of crucial importance especially when the parameters are estimated for time variant systems. These changes are analyzed in terms of values of input and output variables for the purpose of synthesis of control signal for adaptive systems. Those recursive methods are often used in transmission and of signal processing as independent real time control instrument [1, 2, 7, 8].

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2 Recursive Versions of Least Squares Method for Parameter Estimation

Short description of the methods used in the present paper will be presented below.

2.1 Recursive Weighted Least Squares (RWLS)

Recursive estimates using weighted least squares method can be obtained with:

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \left(y_{N+1} - f_{N+1}^T \hat{\theta}_N \right) \tag{1}$$

According to (1) the previous value $\hat{\theta}_N$ is adjusted proportionally to the difference $y_{N+1} - \hat{y}_{N+1}$ with a vector coefficient of proportionality:

$$\Gamma_{N+1} = \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \tag{2}$$

Where: $\hat{y}_{N+1} = f_{N+1}^T \hat{\theta}_N$ is predicted value for y_{N+1}, where $\hat{\theta}_N$ is the vector of the coefficients estimated in the previous iteration.

Instead of using prediction errors it is possible residuals to be used as follows:

The prediction error included in formula (1) can be written also:

$$e_{N+1} = y_{N+1} - \hat{y}_{N+1}\left(\hat{\theta}_N\right) = y_{N+1} - f_{N+1}^T \hat{\theta}_N \tag{3}$$

for improvement of the accuracy of prediction instead of e_{N+1} is used the residual:

$$r_{N+1} = y_{N+1} - f_{N+1}^T \hat{\theta}_{N+1} \tag{4}$$

combining (2) and (3) gives:

$$r_{N+1} - e_{N+1} = -f_{N+1}^T (\hat{\theta}_{N+1} - \hat{\theta}_N) \tag{5}$$

If difference $(\hat{\theta}_{N+1} - \hat{\theta}_N)$ is derived using (2) and substituted in (5) it follows:

$$r_{N+1} = \frac{\frac{1}{w_{N+1}} e_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \tag{6}$$

Calculation Procedure Requirement.

In order to obtain parameter estimates using the method of weighted least squares initial estimates of $\hat{\theta}_N$ and C_N should be available. This is only possible if $N \geq k$ observations for $\hat{\theta}_N$ and C_N are obtained with non-recursive method of weighted least-squares. Then the procedure continues following the calculation procedure described with (1) and (7).

$$C_{N+1} = C_N - \frac{C_N f_{N+1} f_{N+1}^T C_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \tag{7}$$

The algorithm of the recursive weighted least squares method is the corner stone for many other recursive procedures. It could be shortly presented by steps 1 to 4:

1. $N \geq k$ observations are collected and processed with non-recursive method of weighted least-squares and the initial estimates $\hat{\theta}_N$ and C_N are obtained.
2. The new estimates are calculated by the formula

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{C_N f_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}} \left(y_{N+1} - f_{N+1}^T \hat{\theta}_N \right)$$

3. Recalculation of matrix C_{N+1} to be submitted for next iteration

$$C_{N+1} = C_N - \frac{C_N f_{N+1} f_{N+1}^T C_{N+1}}{\frac{1}{w_{N+1}} - f_{N+1}^T C_N f_{N+1}}$$

4. The new estimates $\hat{\theta}_{N+1}$ and C_{N+1} start the next iteration from point 2 of the algorithm presented above [2–4].

2.2 Recursive Ordinary Least Squares

Recursive least squares is also a special case of RWLS when $W = I$. This means that all weights are significant and unbiased estimates when equal values are assigned (1), which is only possible if ρ = 1. subsequently, the recursive least squares method (RLS) can be applied using the same procedure, as described in paragraph 1.1. to substitute ρ = 1 where possible [3–5].

3 Recursive Method of Instrumental Variables

Obtaining estimates by the method of the instrumental variables is using similar procedure to the one used for the least squares method.

The Instrumental vector v_{N+1} is choosen from (8), or (9)

$$v_i^T == \left[-\hat{y}_{i-1} \quad -\hat{y}_{i-2} \ldots - \hat{y}_{i-n} \quad u_{i-1} \quad u_{i-2} \ldots u_{i-m} \right] \tag{8}$$

$$v_i^T = \left[u_{i-1} \quad u_{i-2} \ldots u_{i-m} \right] \tag{9}$$

The Recursive Algorithm used When the Method of Instrumental Variable is Applied is:

1. The calculations start with RWLS. The higher is the number of iterations the higher is the accuracy of the initial estimates. Once a certain number $N > k$ observations is obtained the calculations continue using the recursive method of instrumental variable (RIV). N is usually chosen to satisfy the following condition – to be equal to 3 to 5 times the number of coefficients.
2. The estimated coefficients of $N + 1^{-\text{st}}$ iteration are calculated by the formula:

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{C_N v_{N+1}}{\rho + f_{N+1}^T C_N v_{N+1}} \left(y_{N+1} - f_{N+1}^T \hat{\theta}_N \right) \tag{10}$$

3. Recalculation of matrix C_{N+1} to be submitted for the next iteration by the formula:

$$C_{N+1} = \frac{1}{\rho} \left(C_N - \frac{C_N v_{N+1} f_{N+1}^T C_N}{\rho + f_{N+1}^T C_N v_{N+1}} \right) \tag{11}$$

4. The calculation procedure continues with repetition of step 2 with the new estimates obtained. If weighing of observations (10) and (11) is not needed then it is simply substituted $\rho = 1$ [4–6].

4 DC Motor Control System

DC motors are widely used in terms of automation, robotics and various control systems. They are suitable where a precise regulation of torque and velocity is needed, while maintaining the torque at low and zero velocity [1, 6, 7].

4.1 DC Motor

The dynamics of the DC motor may be expressed by the following equation:

$$\left| \begin{array}{l} U_a = R_a(T_a p + 1) i_a + c\Phi\omega \\ M_e = c\Phi i_a \\ M_e - M_c = Jp\omega \end{array} \right. \tag{12}$$

where: $T_a = L_a / R_a$ and U_a – armature voltage, i_a – armature current, R_a – armature winding active resistance, L_a - armature induction -ω – angular velocity, M_e – electromagnetic torque, M_c – load torque, Φ – magnetic flux, c – armature constant, J – moment of inertia. The block diagram of the DC motor used in the present study is shown in Figs. 1, 6, and 7:

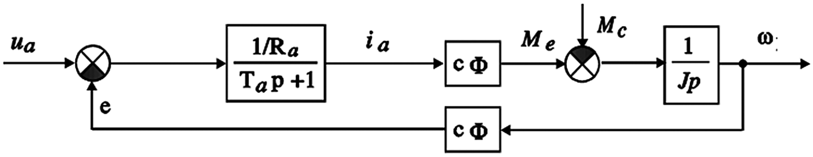

Fig. 1. Block diagram of the DC motor

4.2 DC Motor Control Using Pulse-Width Modulation

Modern DC motor control systems use pulse-width modulation, which is found to be characterized with better energetic parameters, smaller current and velocity pulsations, which in turn leads to reductuion of energetic losses and expansion of the control span.

In Fig. 2 is presented block diagram of DC motor control with PWM, where: TWG- Triangle Wave Generator, C – comparator, TDB – time delay block, IS– Impulse shaper, UR – uncontrolled rectifier, PTC – power transistor commutator, M– DC motor [1, 6, 7].

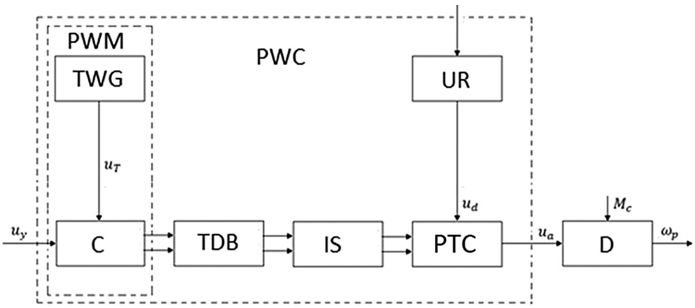

Fig. 2. Block diagram of PWM - DC motor

5 Self-tuning Controller

Self-tuning controllers (STC) use a combination of recursive process identification based on selected model process and controller synthesis based on apriory knowledge for control process system features, parameters estimates and ranges of variability.

Block diagram of STC (with direct identification) is shown in Fig. 3.

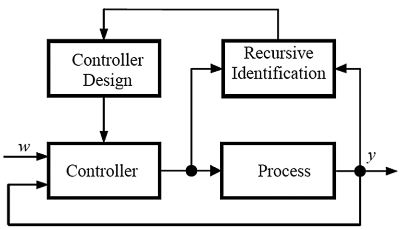

Fig. 3. Block diagram of STC

Self-tuning control (STC) is one of the control methods which have been developed considerably over the past years. Self-tuning control is focused on discrete or sampled models of processes. Computation of appropriate control algorithms is then realized using discrete model of representation of the system.

STC has three main elements (Fig. 3). The first element presents classic feedback system. The second shows implementation of identification block. The third realizes the algorithm of parameters adjustment of controller, based on estimates of the process parameters. This block diagram can be used in both stationary systems with unknown parameters and is also applicable for systems with distributed parameters which are expected to vary within certain limits. The present study investigates application of direct STC with minimal variance (STC-MV) [1, 8, 9].

Algorithm for Direct STC - MV.

1. Based on input/output data for the process at a given $k^{-\text{th}}$ time sample, the estimates $f_i(k)$ and $g_i(k)$ of the polynomials $F(z^{-1})$ and $G(z^{-1})$ are obtained using one of the methods described in paragraphs 2 and 3
2. The estimates derived at step 1 form the control signal $u(k)$:

$$
\begin{aligned}
u(k) = -\frac{1}{g_0}[&f_0 y(k) + f_1 y(k-1) + \ldots + f_{n-1} y(k-n+1) + \\
&+ g_1 u(k-1) + g_2 u(k-2) + \ldots + g_{m+d-1} u(k-m-d+1)]
\end{aligned}
\tag{13}
$$

3. Once the new data is received and processed the procedure is repeated from steps 1 to 2.

It is clear that when the direct method of estimation of $F(z^{-1})$ and $G(z^{-1})$ is applied, the use of more complicated algorithms as well as solving the Diophantine equation is not needed and could be skipped [1, 8, 9].

6 Experimental Research and Results

The present research is mostly focused on the performance capabilities of the described recursive methods for parameter estimation used in real time adaptive control of DC motor. For this purpose the study is completed using random input signal which is simulating noise at the input of the system under investigation.

The research has been done using the System Identification Toolbox in Matlab \Simulink. For simulation purposes custom blocks in Simulink are developed each one corresponding to a certain recursive parameter estimation algorithm as follows: recursive least squares (RLS), recursive least squares with using the residuals modification instead of the prediction errors (RLSr) and recursive instrumental variable method(IVu). (Figs. 4, 5 and 8)

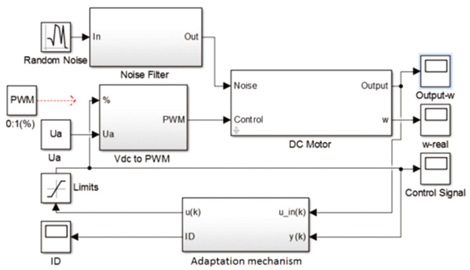

Fig. 4. Block diagram of adaptive DC motor control with STC-MV in Simulink

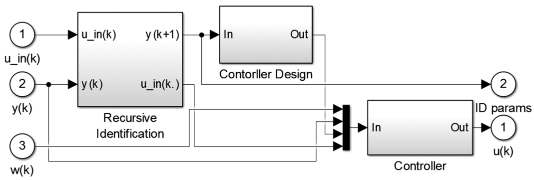

Fig. 5. Block diagram of the subsystem "Adaptation mechanism"

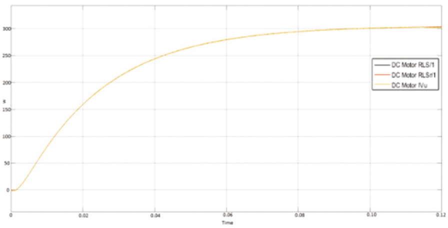

Fig. 6. Output with $\omega = 304,7$ rad/s

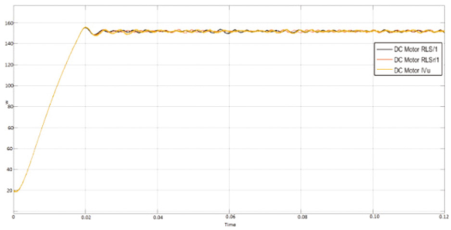

Fig. 7. Output with $\omega = 152,35$ rad/s

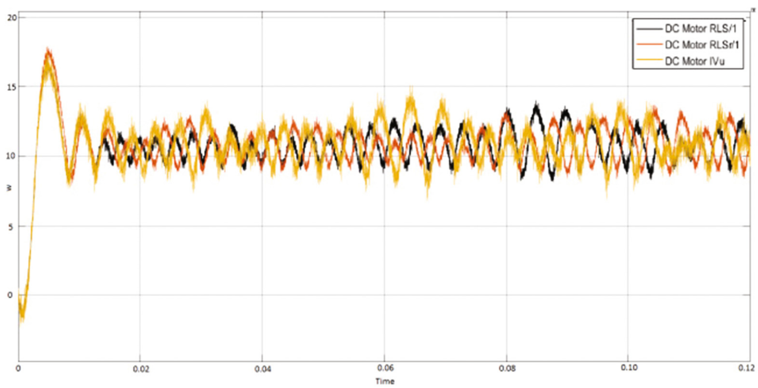

Fig. 8. Output with $\omega = 10,1567$ rad/s

7 Conclusions

The simulation results prove the applicability of described recursive methods for parameter estimation in adaptive control of DC motor. The results show that in the low speed interval the variation of the signal from the reference is the highiest even if it still belongs to acceptable limits. As it was formulated the recursive instrumental variable method require a second (external) noise signal in the output of the object for the purpose of better estimates of parameters however it strongly affects the quality of the control process. The described methods for parameter estimation can be further modified for better performance and process quality in other adaptive control systems. If there is an option for direct selection of the estimates weights the algorithms described can be used to facilitate a variety of robust estimates that can be noise resistant. Future research will be focused on this issue.

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