# Direct Torque Control of Permanent Magnet Synchronous Motor at Low Speed Using a Variable PI Feedback Flux Observer

**Suying Zhang, Wenshuai Cui, Yankai Shen and Huixian Liu**

**Abstract** In the study of direct torque control (DTC) of permanent magnet synchronous motor (PMSM), the stator resistance is easily affected by temperature variation and its value varies from time to time. What is more, it will lead to flux ripple. In order to solve the problem of error existing between the set value of the stator flux and the reality, an improved method based on variable parameter PI is proposed to compensate the stator resistance. By constructing the stator flux observer mathematical model of DTC of PMSM and analyzing the stator resistance variation influenced by the stator flux observation, it can be found that the stator resistance is compensated by adjusting PI parameters with time, the flux error will be reduced, and this new method will be testified by MATLAB software. The simulation result shows that the improved stator resistance compensation algorithm has effectively solved the flux ripple problem, decreased the flux error, and achieved the expected control effect.

**Keywords** PMSM · DTC · Stator resistance compensator · Torque ripple reduction

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# 1 Introduction

Direct torque control (DTC) was firstly proposed by Depenbrock Takahashi and Naguchi in 1980s. Its main idea is to control the stator flux space vector and electromagnetic torque, simplify the calculation, and reduce the influence of motor parameters. However, DTC is an unsophisticated control method which has lots of advantages such as quick response of torque, excellent dynamic performance, and less parameter dependence [1, 2]. But this method also has the disadvantages of increasing torque ripple and narrow speed range which are caused by drift in stator flux linkage estimation. What is more, the stator resistance variation, the test accuracy of rotor's initial position, and stator flux offset error in measurement are the reasons for why there is change in stator flux linkage estimation [3].

In recent years, an improved method based on PI controller to compensate stator resistance has been proposed [4]. However, the parameters of the traditional PI method are not adjustable. This paper introduces a modified method that the parameters are determined by the scope of the stator flux error, which has a better time-varying performance and a smaller computational complexity [5].

# 2 Mathematical Model Based on PMSM

DTC system is composed of inverter, permanent magnet synchronous motor (PMSM), the estimation of flux model, torque model, and rotor position model, the switching table model, and so on. The torque signal is the output of the regulator that takes the error between the given and actual rotating speeds as inputs. The flux value and torque value can be calculated by the flux linkage model and torque model. In addition, the motor rotor position, the flux error, and torque error between the estimated signal and the actual signal also can be known. The purpose of regulating speed can be achieved by selecting the appropriate switching vectors of the inverter and adjusting the output torque.

The stator voltages $\alpha - \beta$ are expressed as follows:

$$u_{\alpha} = R_{\mathrm{s}} i_{\alpha} + \frac{\mathrm{d}}{\mathrm{d}t} \Psi_{\alpha} \tag{2.1}$$

$$u_{\beta} = R_{\mathrm{s}} i_{\beta} + \frac{\mathrm{d}}{\mathrm{d}t} \Psi_{\beta} \tag{2.2}$$

$$\theta = \arctan \frac{\Psi_{\alpha}}{\Psi_{\beta}} \tag{2.3}$$

where $u_{\alpha}$ and $u_{\beta}$ represent the $\alpha - \beta$ axis components of voltage, respectively; $i_{\alpha}$ and $i_{\beta}$ stand for the $\alpha - \beta$ current components; $\Psi_{\alpha}$ and $\Psi_{\beta}$ are for the stator flux; $\theta$

for flux angle; and $R_s$ for the stator resistance. Torque equation is expressed as follows:

$$T_e = \frac{3}{2} n_p (\Psi_\alpha i_\beta - \Psi_\beta i_\alpha) \tag{2.4}$$

where $n_p$ is the number of pole pairs.

## 3 Stator Flux Algorithm

In this system, the stator flux of AC motor is observed by the flux model. In static coordinate $\alpha - \beta$, the stator flux of PMSM are expressed as follows:

$$\begin{cases} \Psi_\alpha = \int e_\alpha \mathrm{d}t = \int (u_\alpha - R_s i_\alpha)\mathrm{d}t \\ \Psi_\beta = \int e_\beta \mathrm{d}t = \int (u_\beta - R_s i_\beta)\mathrm{d}t \\ \Psi_s = \sqrt{\Psi_\alpha^2 + \Psi_\beta^2} \end{cases} \tag{3.1}$$

where $e_\alpha$ and $e_\beta$ mean the components of counter-electromotive force on $\alpha - \beta$ axis, respectively; $\Psi_s$ is for the flux amplitude.

The accuracy of the flux observer will be affected by the variation of stator resistance when the motor is in low-speed running, and at that time, the pressure drop of the stator resistance and the power supply voltage will be at the same level which makes the running environment worse. At the same time, there are many factors that affect the resistance value, such as temperature and voltage. The value will vary with temperature seriously. However, the change of temperature is the main reason causing the flux linkage error.

Not only does the value change of stator resistance influence the accuracy of flux values, but also it leads to the torque variation [6]. When the motor speed is low, the errors of sector judgment and voltage space vector judgment will make the motor unstable.

In the process of analyzing the flux and torque equations, it can be noticed that the accuracy of flux observation can be affected by the variation of stator resistance. The influence caused by the stator resistance change is more serious especially if the power supply voltage drop on the stator resistance is at the same order when the motor is in low-speed running. In addition, there are many factors that affect the stator resistance, such as the change of the temperature and voltage frequency. After all, the temperature is the main reason, because the resistance value varies with temperature seriously during operation.

Assume that the changes of stator resistance and stator current are, respectively, $\Delta R_s$, $\Delta i_s$, and then, the actual relationship between the flux and torque can be expressed as follows [7]:

$$\Psi_\alpha = \int (u_\alpha - (R_s + \Delta R_s)(i_\alpha + \Delta i_\alpha))\mathrm{d}t \tag{3.2}$$

$$\Psi_\beta = \int (u_\beta - (R_s + \Delta R_s)(i_\beta + \Delta i_\beta))\mathrm{d}t \tag{3.3}$$

$$\Psi_s = \int (u_s - (R_s + \Delta R_s)(i_s + \Delta i_s))\mathrm{d}t \tag{3.4}$$

$$T_e = \frac{3n_p}{2}\left[\Psi_\alpha\left(i_\beta + \Delta i_\beta\right) - \Psi_\beta(i_\alpha + \Delta i_\alpha)\right] \tag{3.5}$$

Considering that the change of stator resistance cannot be detected directly, the equations with no change in stator resistance are expressed as follows:

$$\hat{\Psi}_\alpha = \int (u_\alpha - R_s(i_\alpha + \Delta i_\alpha))\mathrm{d}t \tag{3.6}$$

$$\hat{\Psi}_\beta = \int (u_\beta - R_s(i_\beta + \Delta i_\beta))\mathrm{d}t \tag{3.7}$$

$$\hat{\Psi}_s = \int (u_s - R_s(i_s + \Delta i_s))\mathrm{d}t \tag{3.8}$$

$$\hat{T}_e = \frac{3n_p}{2}\left[\hat{\Psi}_\alpha\left(i_\beta + \Delta i_\beta\right) - \hat{\Psi}_\beta(i_\alpha + \Delta i_\alpha)\right] \tag{3.9}$$

The errors between the actual value and the observed value of the stator flux and electromagnetic torque are expressed as follows [8, 9]:

$$\Delta\Psi_\alpha = \Psi_\alpha - \hat{\Psi}_\alpha = \int -\Delta R_s(i_\alpha + \Delta i_\alpha)\mathrm{d}t \tag{3.10}$$

$$\Delta\Psi_\beta = \Psi_\beta - \hat{\Psi}_\beta = \int -\Delta R_s(i_\beta + \Delta i_\beta)\mathrm{d}t \tag{3.11}$$

$$\Delta\Psi_s = \Psi_s - \hat{\Psi}_s = \int -\Delta R_s(i_s + \Delta i_s)\mathrm{d}t \tag{3.12}$$

$$\Delta T_e = T_e - \hat{T}_e = \frac{3n_p}{2}\left[\Delta\Psi_\alpha\left(i_\beta + \Delta i_\beta\right) - \Delta\Psi_\beta(i_\alpha + \Delta i_\alpha)\right] \tag{3.13}$$

From (3.12) and (3.13), it is obvious that the change of stator resistance not only affects the accuracy of flux values, but also leads to the torque variation. When the motor is in low-speed running, the change of resistance will lead to the errors of sector judgment and errors of voltage space vector judgment which causes the motor to run unstably [10]. Equation (3.14) can be calculated according to (3.12):

$$\frac{d\Psi_s}{dt} = (i_s + \Delta i_s)dR_s \tag{3.14}$$

$$\frac{d\Psi_s}{dt(i_s + \Delta i_s)} = dR_s \tag{3.15}$$

PI stator resistance compensator can be designed as follows (3.16):

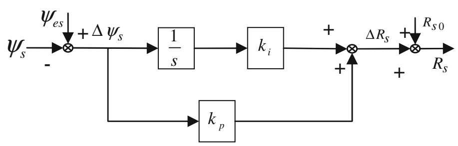
$$dR_s = \left(k_p + \frac{k_i}{s}\right)d\Psi_s \tag{3.16}$$

where $k_p, k_i$ stand for the proportional gain and integral of the PI compensator, respectively. The structure diagram is shown in Fig. 1.

When analyzing the PI controller parameters, the $k_p$ can decrease the response time and reduce the static error. In addition, it will lead to excessive overshoot and system oscillation if the value is too big. The change of $k_i$ mainly affects the static error, and it will prolong regulating time if the value is too big. Based on the above problems, an improved method to adjust the parameters $k_p$ and $k_i$ is proposed. The change trends of $k_p$ and $k_i$ are shown in Fig. 2, and the parameter change law can be summarized as follows:

(1) $k_p$ changes appropriately with the same trend of the input signal error.
(2) $k_i$ changes appropriately with the contrary trend of the input signal error.

**Fig. 1** Compensation of stator resistance



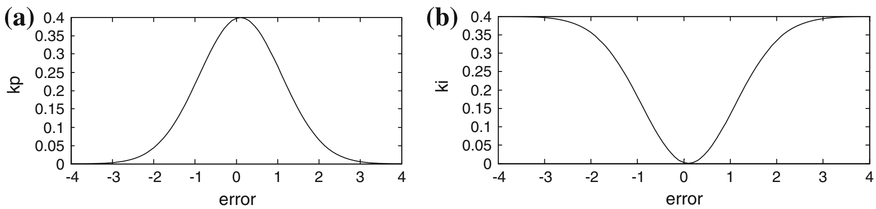




**Fig. 2** The rules of $k_p$ and $k_i$ with the different signal deviation

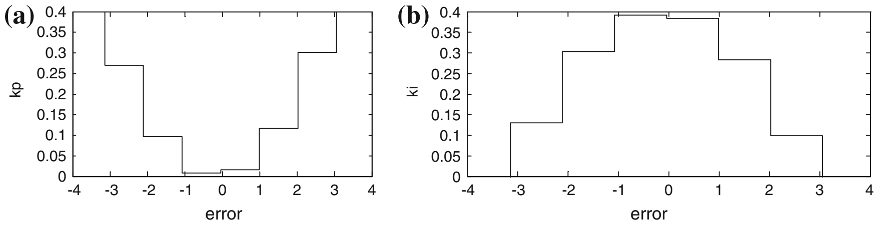



**Fig. 3** The simplified rules of $k_{\rm p}$ and $k_{\rm i}$ with the different signal deviation

$$\begin{cases} k_{\rm p} = k_1 \frac{1}{\sqrt{2\pi}\sigma_1}\left(1 - \exp\left(-\frac{e^2}{2\sigma_1^2}\right)\right) \\ k_i = k_2 \frac{1}{\sqrt{2\pi}\sigma_1}\left(1 - \exp\left(-\frac{e^2}{2\sigma_1^2}\right)\right) \end{cases} \tag{3.17}$$

In order to get the precise values of $k_{\rm p}k_{\rm i}$, the normal function forms of the parameters are expressed as follows.

But the computation procedure based on normal distribution function is complexity and will affect the system response rate, and a simple change trend of $k_{\rm p}$ and $k_{\rm i}$ is shown in Fig. 3.

## 4 Simulation

This part mainly introduces the simulation based on MATLAB software, and an improved method of the flux observer with resistance compensation is proposed. The motor parameters set as follows: $n_{\rm p} = 4$; $R_{\rm s} = 2.875\Omega$; $\Psi_{\rm s} = 1{\rm Wb}$ motor speed $wm = 80\,{\rm rad/s}$; inductance on axis $d$ $L_d = 42.44\,{\rm mH}$; inductance on axis

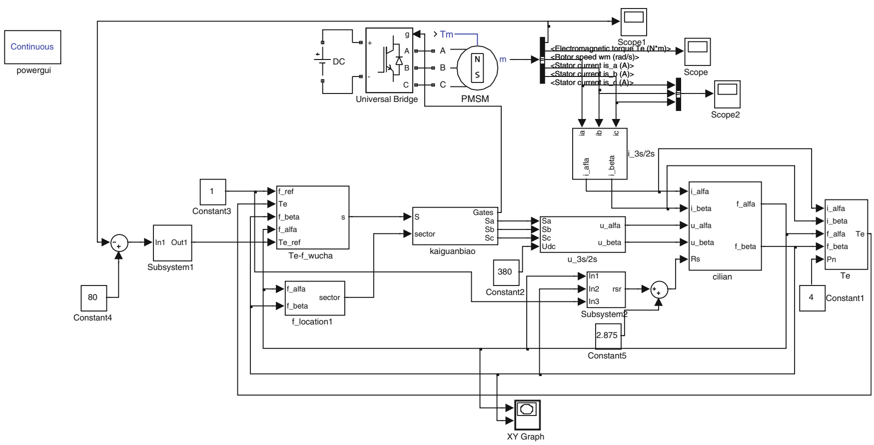



**Fig. 4** The improved block diagram of the PMSM DTC

$q$ $L_q = 79.57\,\text{mH}$; moment of inertia $J = 0.003\,\text{kg} \cdot \text{m}^2$; dc voltage $U_{dc} = 400\,\text{V}$, and the Simulink diagram is shown in Fig. 4.

This paper intercepted a waveform when the value of resistance is divorced from the estimated value and keeps for a period of time, and the advantage of the improved method can be proved in Fig. 4.

Figures 5 and 6 show the traditional flux observer waveform of flux linkage and the improved flux observer waveform of flux linkage. It is obvious that the flux is closer to the estimated value in Fig. 6, and the waveform is smoother.

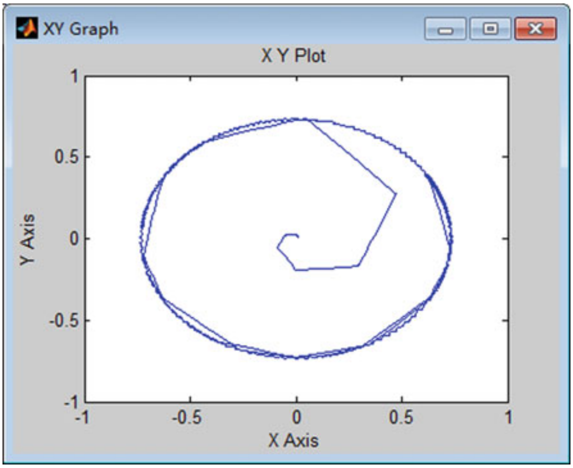



**Fig. 5** The traditional flux observer waveform of flux linkage

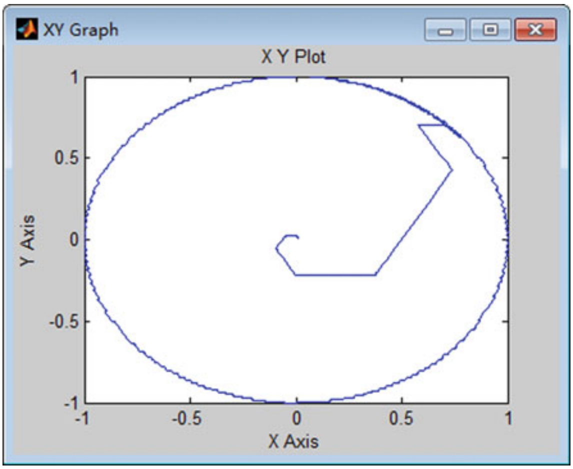



**Fig. 6** The improved flux observer waveform of flux linkage

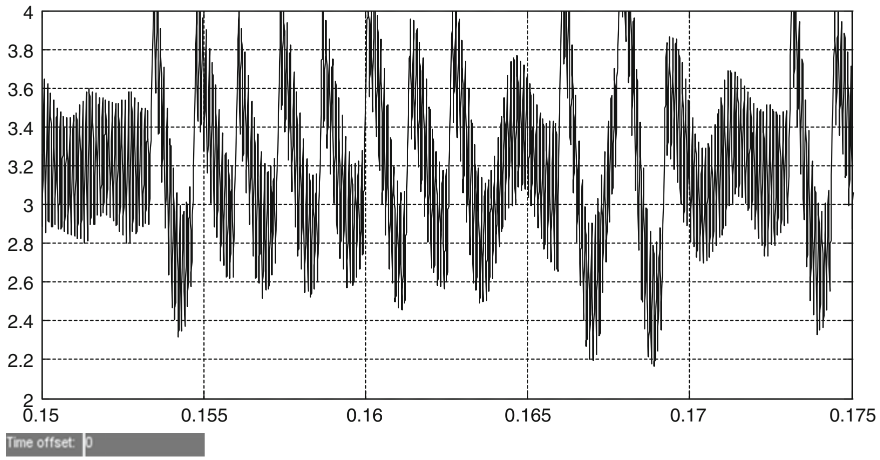



**Fig. 7** The traditional flux observer waveform of torque

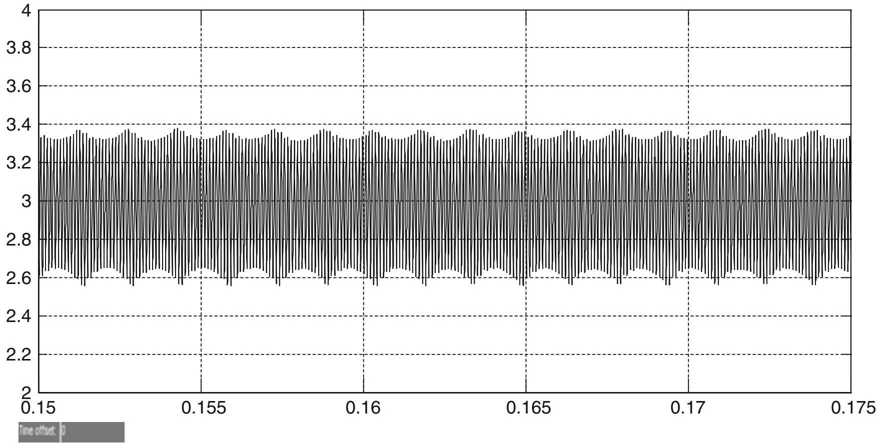



**Fig. 8** The improved flux observer waveform of torque

Figures 7 and 8 show the traditional flux observer waveform of torque and the improved flux observer waveform of torque. It is obvious that with the feedback compensation, the torque ripple reduced, and the error of stator resistance is compensated.

The results prove that when compared with the original system, the improved system reduces torque ripple and performs more stable. Besides, the stator flux is closer to the value than before, and the torque and speed have better tracking characteristics. In addition, the error of stator flux between the actual value and the estimated in the improved system will be less than before. In general, the experiment verifies the feasibility and effectiveness of this method.

## 5 Conclusion

In this paper, a modified algorithm to deal with the problems of increasing torque ripple is proposed, and the stator resistance compensation algorithm based on variable parameter PI has improved the control accuracy. The simulation results prove that not only does the adaptive PI control method compensate the stator resistance, but also it gets a good speed–torque performance and achieves the desired control effect.

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