Name: Solutions

ECE 464 - Power Electronics, Fall 2012, Exam 1 Date: October 3

This exam is closed book, closed notes. No calculators are allowed (or needed). You may use one sheet (8.5x11 inch) of notes. Each student is allowed no more than one (1) clarifying question. Choose wisely!

Problem 1 35

Problem 2 30

Problem 3 35

Total ______

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Problem 1 (35 points).

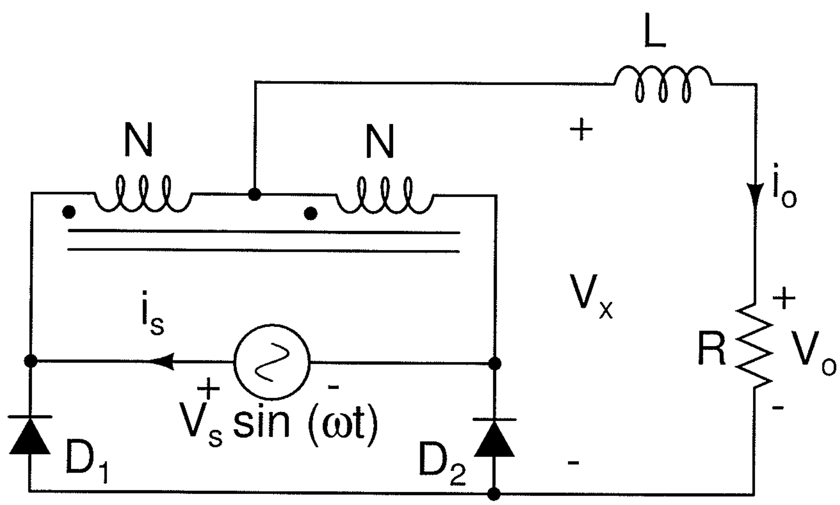


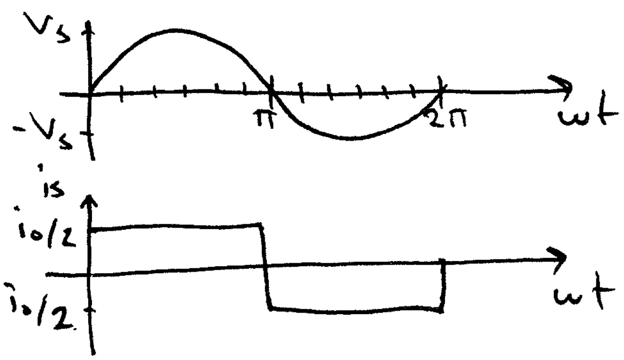
Figure 1: Rectifier circuit

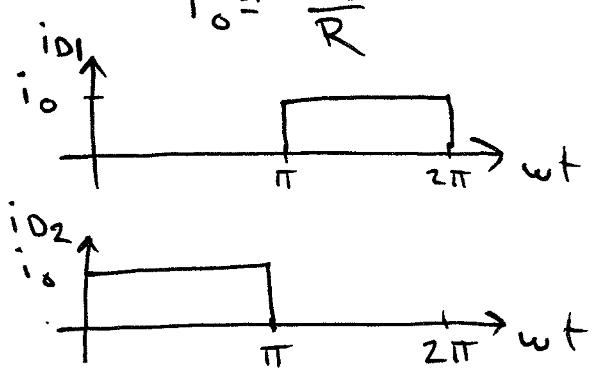
Consider the rectifier circuit of Fig.1. You may assume that the filter inductor L is suifficiently large such that R/L << ω, and that the transformer and diodes are ideal.

a) Sketch and dimension the input current i_s and the currents through diodes D_1 and D_2 under periodic steady-state conditions. Please label the magnitudes on the plot expressed in terms of V_o and R.

$i_o = \frac{V_o}{R}$

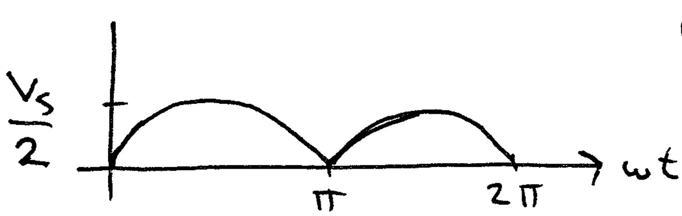
9 pts





b) Sketch and dimension the voltage labeled V_x.

5 pts



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c) Calculate the average power delivered to the load, and the power factor (seen by the ac voltage source) for this rectifier. (15 pts)

$$\langle P \rangle = \frac{1}{2\pi}\int_0^{2\pi} v_s i_s = \frac{1}{\pi}\int_0^{\pi} v_s \sin(\omega t)\cdot \frac{i_o}{2}\, d(\omega t)$$

ok.

$$= \frac{v_s i_o}{2\pi}\left(-\cos(\omega t)\Big|_0^{\pi}\right) = \boxed{\frac{V_s I_o}{\pi}} = \frac{V_s}{\pi}\cdot\frac{\langle V_o \rangle}{R} = \boxed{\frac{V_s^2}{\pi^2 R}}$$

$$\langle V_x \rangle = \langle V_o \rangle \qquad \langle V_x \rangle = \frac{1}{\pi}\int_0^{\pi} \frac{V_s}{2}\sin(\omega t)\, d(\omega t = \frac{V_s}{2\pi}\cdot 2$$

$$P.F. = \frac{\langle P \rangle}{V_{rms} I_{rms}} = \frac{\frac{V_s I_o}{\pi}}{\frac{V_s}{\sqrt{2}}\cdot\frac{I_o}{2}} = \boxed{\frac{2\sqrt{2}}{\pi}} \simeq 0.9$$

$$V_{rms} = \frac{V_s}{\sqrt{2}} \qquad I_{rms} = \frac{I_o}{2}$$

d) What is the distortion factor k_D and the displacement factor k_θ for this rectifier? (6 pts)

$$P.F. = k_D \cdot k_\theta \qquad \text{no phase shift} \Rightarrow k_\theta = 1$$

$$k_D = \boxed{\frac{2\sqrt{2}}{\pi}}$$

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Problem 2 (30 pts).

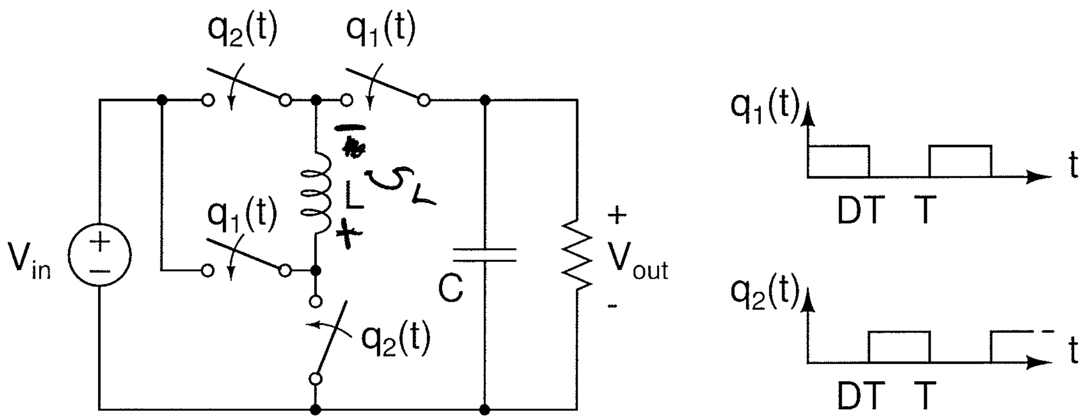

Figure 2: Watkins-Johnson converter

Figure 2 shows a power converter based on the Watkins-Johnson topology. You may assume that V_{in} is positive, and that all inductors and capacitors are large (i.e. they have small ripple). You may also assume that the converter operates in continuous conduction mode, and that the switches are ideal. The switches are controlled as shown in the figure, where each switch is on when its corresponding q(t) is high, and off when q(t) is low.

a) Find the voltage conversion ratio V_{out}/V_{in} in periodic steady state.

$q_1(t) = 1$: $v_L = V_{in} - V_{out}$

$q_2(t) = 1$: $v_L = -V_{in}$

3 pts

4 pts

Volt-second balance: $D(V_{in} - V_{out}) + (1-D)(-V_{in}) = 0$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{2D-1}{D}}$$

8 pts total

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b) Now assume that converter is operating with a duty ratio of 0.75, the input voltage is 15 V, and the output power is 30 W. Calculate the maximum switch voltage stress observed in this converter. How many switches see this maximum voltage? (You may ignore any voltage ripple on the capacitor for this calculation.)

$$V_{out} = 15 \cdot \left(\frac{2 \cdot 0.75 - 1}{0.75}\right) = 10\,V$$

1 pt

$q_1(t) = 1$ bottom $q_2(t)$ sees V_{in} across it

top $q_2(t)$ sees $V_{in} - V_{out}$

$q_2(t) = 1$ bottom $q_1(t)$ sees V_{in}

top $q_1(t)$ sees $V_{in} - V_{out}$

$$\boxed{V_{sw,max} = V_{in} = 15\,V}$$

4 pts total

$\boxed{\text{2 switches see this voltage}}$

4 pts total

c) What is the peak current going through the switch that is connected to the positive output voltage node for the operating conditions of (b)? (You may assume that L is so large that you can ignore any current ripple.)

$P_{out} = 30\,W$, $V_{out} = 10\,V \Rightarrow I_{out} = 3\,A$

1 pt

$\langle i_{sw} \rangle = I_{out} = 3\,A$

$D \cdot i_{sw} = 3\,A$

$$i_{sw} = \frac{3A}{0.75} = \frac{3A}{3/4} = 4\,A$$

$$\boxed{i_{sw} = 4\,A}$$

8 pts total

d) Is this a direct of indirect converter? Justify your answer.

It is a <u>direct</u> converter. When $q_1(t) = 1$, energy is transfered directly from the source to the load

6 pts total

3pts - right answer

3pts - explanation

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Problem 3 (35 points)

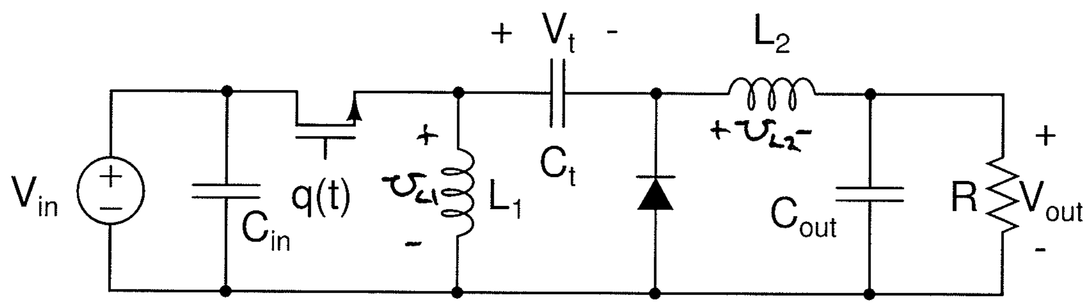


Figure 3: Zeta converter

Figure 3 shows a power converter known as a Zeta converter. You may assume that V_{in} is positive, and that all inductors and capacitors are large (i.e. they have small ripple). You may also assume that the converter operates in continuous conduction mode, and that the MOSFET and diode are ideal. The transistor is on for a period DT over a full switching cycle T.

a) What is the average voltage, $< V_t >$ across the capacitor?

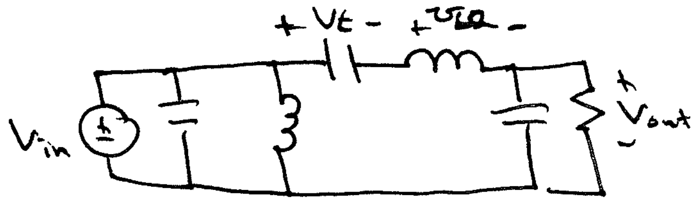
$$\underbrace{\cancel{\langle v_{L1} \rangle}}_{=0} - \langle V_t \rangle - \underbrace{\cancel{\langle v_{L2} \rangle}}_{=0} - V_{out} = 0$$

$$\langle V_t \rangle = -V_{out}$$

5 pts

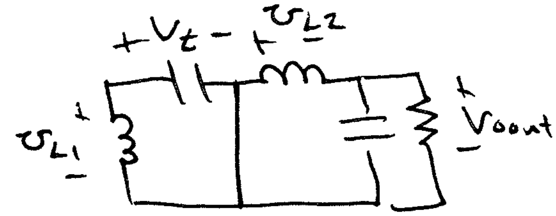
b) Find an expression for V_{out} in terms of V_{in} and D.

$q(t)=1$:
switch on, diode off: $V_{in} - V_t - v_{L2} - v_{out} = 0$



$\Rightarrow v_{L2} = V_{in}$

$q(t)=0$
switch off,
diode on



$v_{L2} = -V_{out}$

Volt-second balance on L_2: $DV_{in} + (1-D)(-V_{out}) = 0$

$$\boxed{V_{out} = \frac{D}{1-D} V_{in}}$$

10 pts

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c) Now consider the following operating parameters: $V_{in} = 20$ V, D = 0.75, $P_{out} = 60$ W, $f_{sw} = 200$ kHz, and $L_1 = 75$ μH. Carefully sketch the current through the inductor L_1 and find the maximum and minimum values, as well as the peak-to-peak current ripple. You may ignore any ripple on the components C_t and L_2 in this analysis.

$$V_{out} = \frac{0.75}{1-0.75} \cdot 20V = 60V \qquad I_{out} = \frac{P_{out}}{V_{out}} = 1A$$

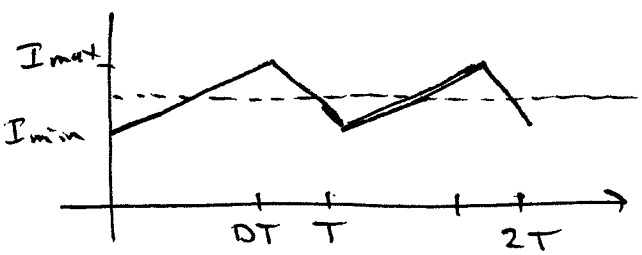
$$I_{in} = \frac{D}{1-D} I_{out} = 3 \cdot I_{out} = 3A \qquad I_{in} = \overbrace{\cancel{\langle i_{C_1} \rangle}}^{=0} + \langle i_{sw} \rangle$$

$$\langle i_{sw} \rangle = \langle i_{L_1} \rangle + \overbrace{\cancel{\langle i_{C_T} \rangle}}^{=0}$$

$$\langle i_{L_1} \rangle = \boxed{I_{in} = 3A}$$

$$\Delta i_{L1p\text{-}p} = \frac{V_{in} DT}{L_1}$$

$$= \frac{20 \cdot 0.75}{75 \cdot 10^{-6} \cdot 200 \cdot 10^3} = \frac{2 \cdot 7.5}{7.5 \cdot 10^{-5} \cdot 2 \cdot 10^5} = \boxed{1\ A = \Delta i_{L_1 p\text{-}p}}$$



$$\boxed{\begin{array}{l} I_{max} = 3.5A \\ I_{min} = 2.5A \end{array}}$$

10 pts

note: D=0.75, so triangle is not symmetric!

d) Find the load resistance for which the converter is at the boundary between CCM and DCM operation in terms of D, L_1, and f_{sw}. Also, please numerically calculate the resistance value assuming the same converter parameters as given in (c).

$$\frac{\Delta i_{L1p\text{-}p}}{2} = I_{in} \Rightarrow \frac{V_{in} DT}{2L_1} = \frac{D}{1-D} I_{out} = \frac{D}{1-D} \frac{V_{out}}{R}$$

$$= \frac{D}{1-D} \cdot \frac{D}{1-D} \frac{V_{in}}{R}$$

$$\boxed{R = \frac{D}{(1-D)^2} \cdot 2 L_1 f_{sw}}$$

$$= \frac{0.75}{(1-0.75)^2} \cdot 2 \cdot 75 \cdot 10^{-6} \cdot 200 \cdot 10^3 = \frac{0.75}{0.25^2} \cdot 2 \cdot 75 \cdot 10^{-6} \cdot 0.2 \cdot 10^6$$

$$= \frac{3}{0.25} \cdot 2 \cdot 75 \cdot 0.2 = \frac{90}{1/4} = \boxed{360\ \Omega}$$

10 pts