

Physics in Strong Magnetic Fields

Dong Lai

Cornell University

ISSI Workshop on “Strongest Magnetic Fields In the Universe”
Bern, Switzerland, Feb.2-7, 2014

How Strong is “Strong”?

Depends on

**Objects,
Questions/Issues...
Who you talk to...**

For atomic physicists

Critical Field:

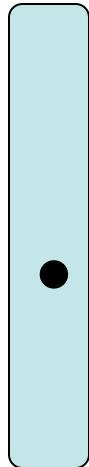
$$\hbar\omega_{ce} = \hbar \frac{eB}{m_e c} = \frac{e^2}{a_0} \implies B = B_0 = 2.35 \times 10^9 \text{ G}$$

Strong field: $B \gg B_0$

Property of atoms/molecules is very different from zero-field

Atoms and Molecules

Strong B field significantly increases the binding energy of atoms



For $b = \frac{B}{B_0} \gg 1$, $B_0 = 2.35 \times 10^9$ G

$$|E| \propto (\ln b)^2$$

E.g. $|E| = 160$ eV at 10^{12} G

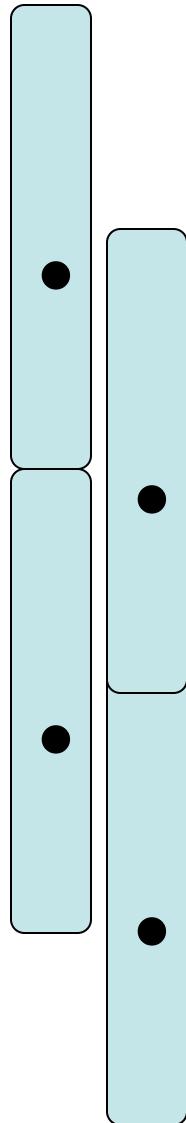
$$|E| = 540$$
 eV at 10^{14} G

Atoms combine to form molecular chains:

E.g. H_2, H_3, H_4, \dots

Condensed Matter

Chain-chain interactions lead to formation of 3D condensed matter



Binding energy per cell $|E| \propto Z^{9/5} B^{2/5}$

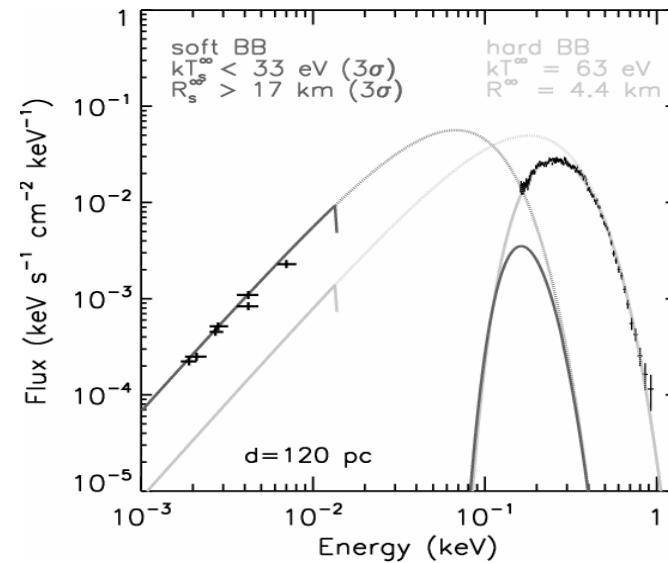
Zero-pressure density
 $\simeq 10^3 A Z^{3/5} B_{12}^{6/5} \text{ g cm}^{-3}$

Thermally Emitting Isolated NSs

“Perfect” X-ray blackbody:
RX J1856.5-3754

Spectral lines detected:
(e.g., van Kerkwijk & Kaplan 06; Haberl 06)

- RXJ1308+2127 (0.2-0.3 keV)
- RXJ1605+3249 (~0.45 keV)
- RXJ0720-3125 (~0.3 keV)
- RXJ0420-5022 (~0.3 keV)?
- RXJ0806-4123 (~0.5 keV)?
- RBS 1774 (~0.7 keV)?



Burwitz et al. (2003)

Atmosphere is cool for atoms to exists
 $\Rightarrow B \sim 10^{13-14} \text{G}$? magnetar descendant & off-beam radio pulsar?

Spectral line for 10^9G WD...

For Condensed Matter Physicists

B fields affect the transport properties

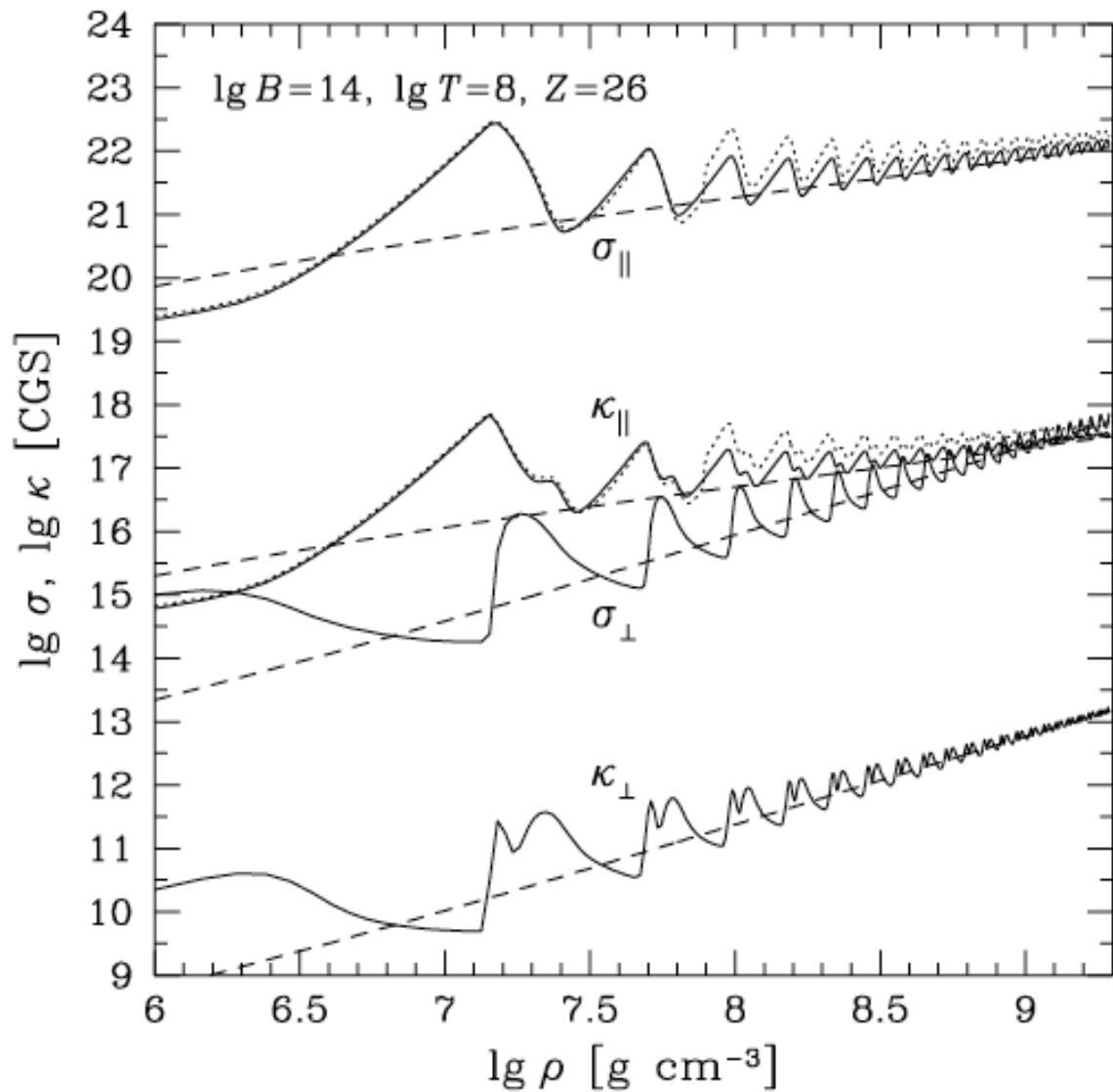
gyrofrequency $\omega_c^* = \frac{eB}{m_e^*c} \gg$ electron collision frequency τ_0^{-1}

→ transverse conductivity suppressed by a factor of $(\omega_c^* \tau_0)^{-2}$

Affect thermal structure of NS envelope and cooling

(e.g. Hernquist, van Riper, Page, Heyl & Hernquist, Potekhin & Yakovlev etc.)

Heat and electric conductivities

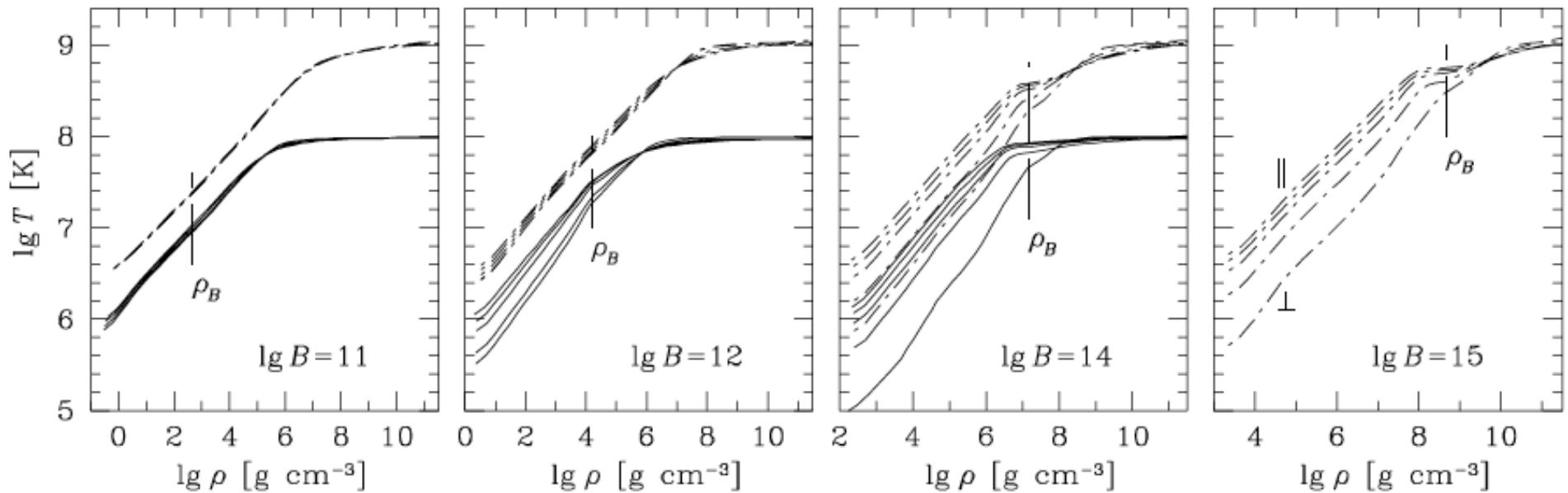


de Haas-van Alphen
oscillations

Potekhin 1999

Nonuniform surface T due to anisotropic heat transport

- Region where B perpendicular to r : heat flux is reduced
- Region where B parallel to r : heat flux remains or increases (due to quantization)



Potekhin & Yakovlev 2001

For Particle Physicists

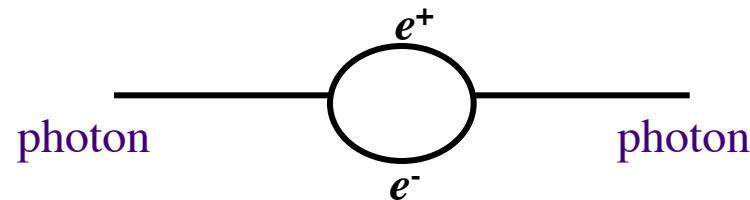
QED Processes in Superstrong B Fields

- **Photon splitting:**

$$\gamma \longrightarrow \gamma + \gamma$$

- **Vacuum birefringence:**

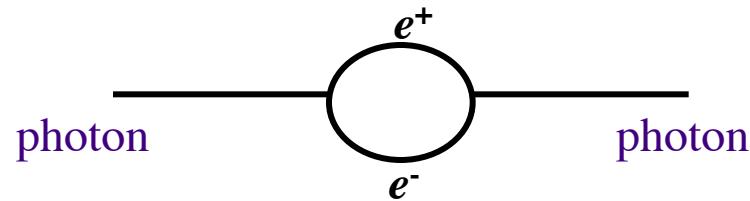
(photon propagation affected by B field)



Critical Field: $\hbar\omega_{ce} = \hbar \frac{eB}{m_e c} = m_e c^2$

$$\rightarrow B_Q = 4.4 \times 10^{13} \text{ G}$$

Vacuum Polarization in Strong B



Heisenberg & Euler,
Weisskopf, Schwinger,
Adler...

Dielectric tensor: $\boldsymbol{\epsilon} = \mathbf{I} + \Delta\boldsymbol{\epsilon}_{\text{vac}}$

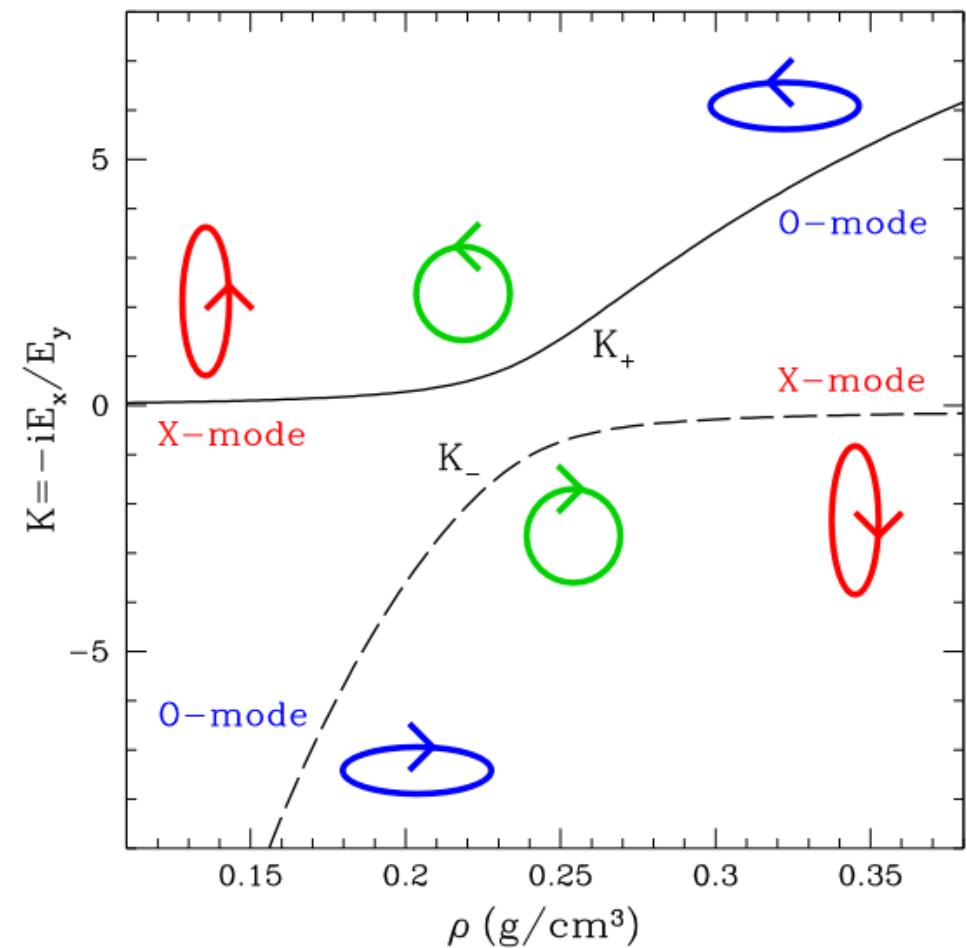
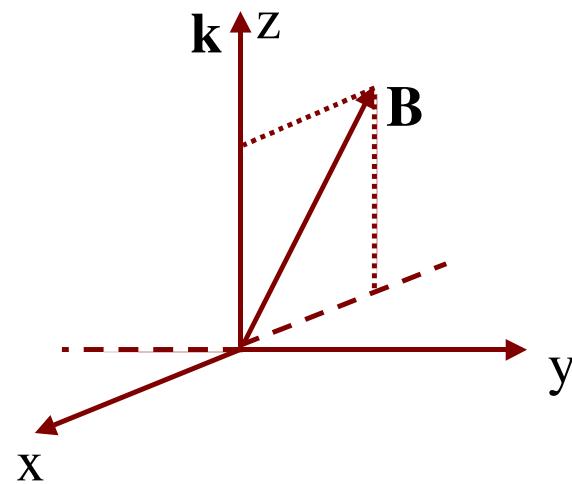
$$|\Delta\epsilon_{\text{vac}}| \sim 10^{-4}(B/B_Q)^2, \text{ with } B_Q = 4.4 \times 10^{13} \text{ G}$$

Two photon modes:

Ordinary mode (//)

Extraordinary mode (⊥)

“Plasma+Vacuum” ==> Vacuum resonance

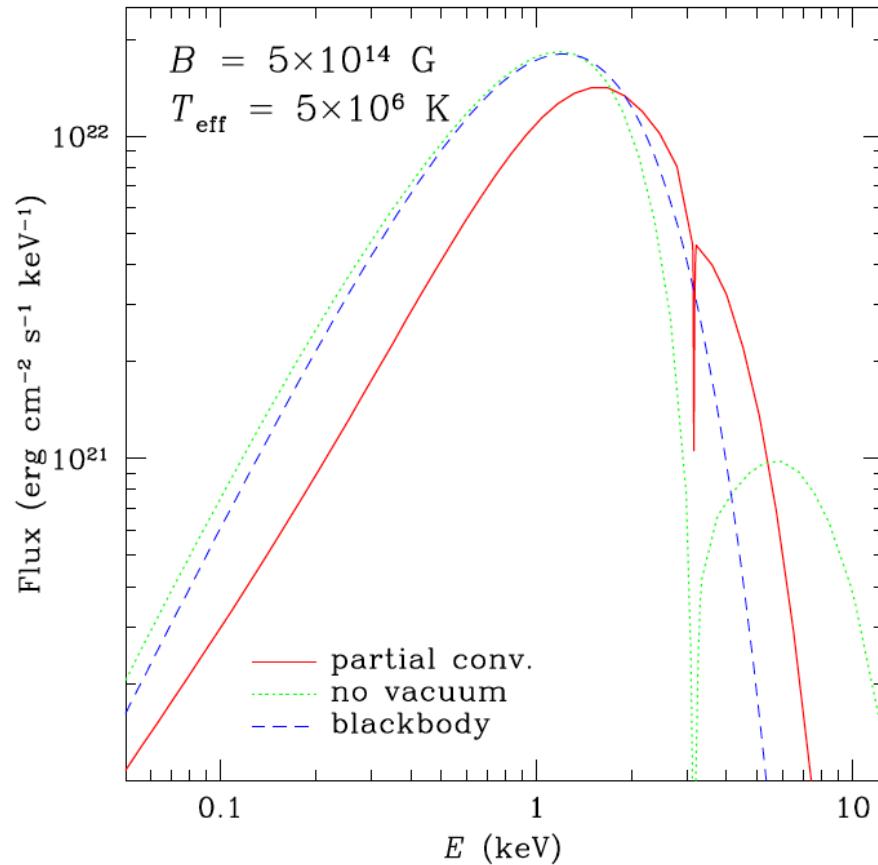


Astrophysical Consequence:

The two photon modes have very different opacities

=> Vacuum polarization can affect radiative transfer significantly

=> **Spectrum and polarization signal from the NS**



Van Adelsberg & DL 2006

For (Most) Astrophysicists

“Classical” B-fields can be “strong”

Their effects are important, interesting, rich...

“Clouds” and Stars/Compact Objects

“Clouds” and Stars/Compact Objects

Static equilibrium requires:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \sim \frac{1}{6\pi^2 G} \left(\frac{\Phi}{M} \right)^2 < 1$$

↑
If only large-scale poloidal fields

“Clouds” and Stars/Compact Objects

Static equilibrium requires:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \sim \frac{1}{6\pi^2 G} \left(\frac{\Phi}{M} \right)^2 < 1$$

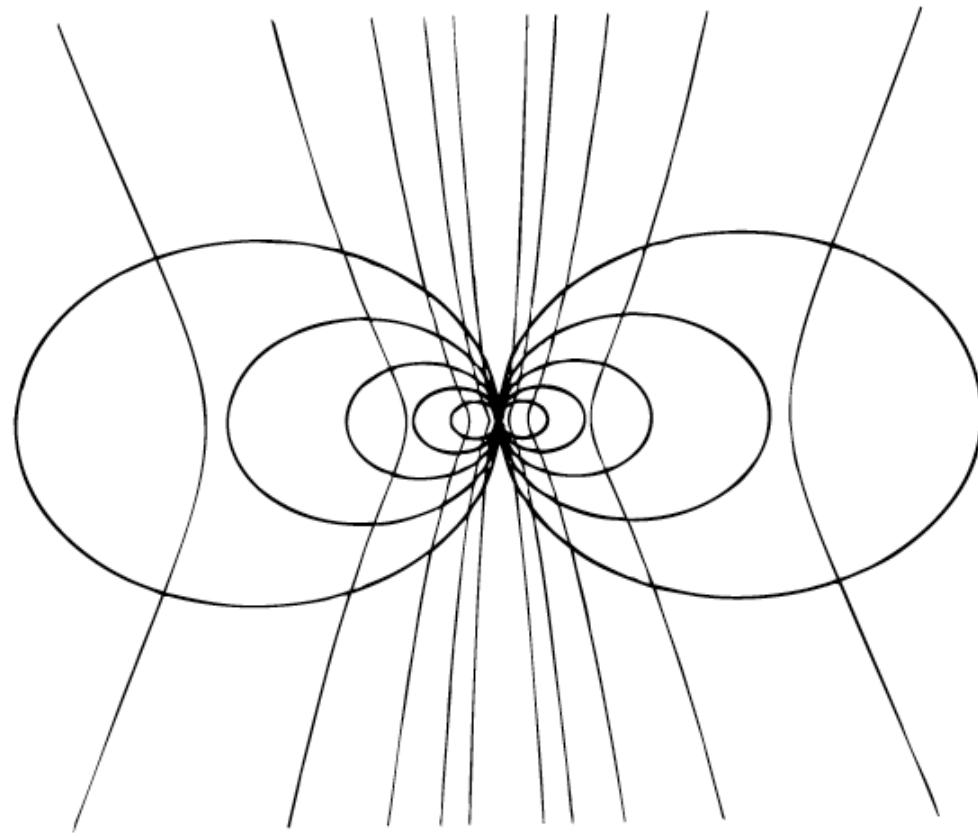
If only large-scale poloidal fields

Star formation:

Clouds with $E_{\text{mag}}/E_{\text{grav}} > 1$

Need ambipolar diffusion (low-mass star formation?)

Cloud Contraction via Ambipolar Diffusion



F. Shu, Li... 1996

“Clouds” and Stars/Compact Objects

Static equilibrium requires:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \sim \frac{1}{6\pi^2 G} \left(\frac{\Phi}{M} \right)^2 < 1$$

↑
If only large-scale poloidal fields

“Clouds” and Stars/Compact Objects

Static equilibrium requires:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \sim \frac{1}{6\pi^2 G} \left(\frac{\Phi}{M} \right)^2 < 1$$

If only large-scale poloidal fields

E.g. For Neutron stars: $B_{\text{in}} < 10^{18}$ G

Magnetic Fields of Isolated Neutron Stars

Radio pulsars: $P, \dot{P} \Rightarrow$

Most pulsars : $B \sim 10^{12-13}$ G

Millisecond pulsars : $B \sim 10^{8-9}$ G

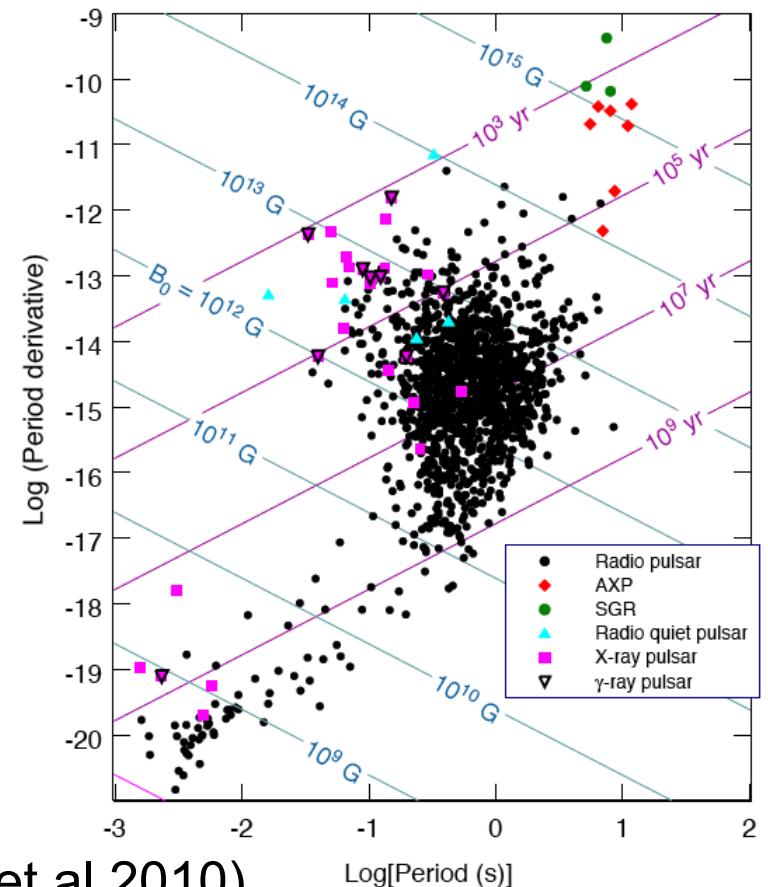
High – B radio pulsars : $B \sim 10^{14}$ G

Magnetars:

Neutron stars powered by B-Fields

--- Usually $>10^{14}$ G

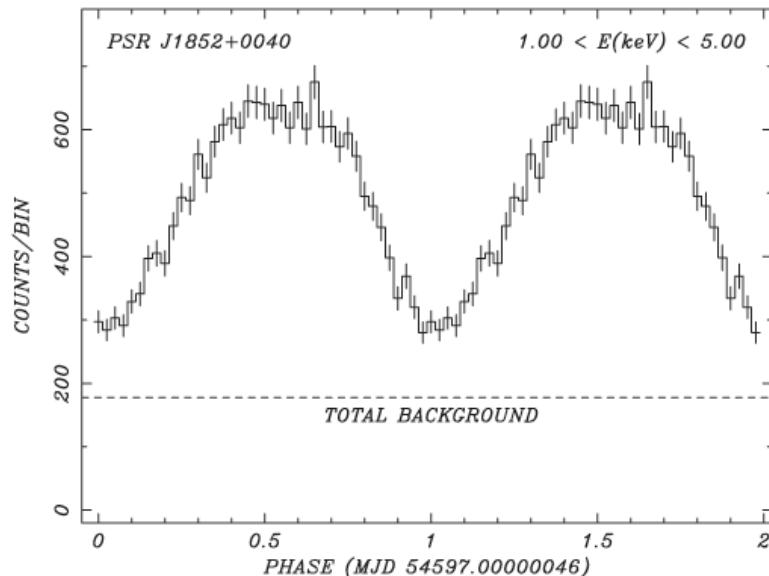
--- Low-field ($\sim 10^{13}$ G) magnetars (Rea et al 2010)



But internal fields could be higher

Hidden Magnetic Fields of Neutron Stars

- NS in Kes 79 SNR has $B_{\text{dipole}} \simeq 3 \times 10^{10}$ G, but large pulse fraction 60%



(Halpern & Gotthelf 2010)

$\Rightarrow B_{\text{crust}} \sim \text{a few} \times 10^{14}$ G

(Shabaltas & DL 2011)

- SGR 0418+5729, with $B_{\text{dipole}} \simeq 4 \times 10^{12}$ G (Rea et al. 2010)
==> Internal field is much larger (Turolla et al 2011)

B Fields: How Strong is “Strong”?

Energetics

Magnetars:

Even in quiescence, $L \sim 10^{34-36} \text{erg s}^{-1} \gg I\Omega\dot{\Omega}$

Giant flares in 3 SGRs

12/2004 flare of SGR1806-20 has $E > 10^{46} \text{erg}$

→ $B > \text{a few } 10^{14} \text{ G}$

Observed (nonconvective) Stars

| Star type | Upper main sequence | White dwarf | Neutron star |
|--|---------------------|-------------|--------------|
| Radius R [km] | $10^{6.5}$ | 10^4 | 10^1 |
| Maximum magnetic field B_{max} [G] | $10^{4.5}$ | 10^9 | 10^{15} |
| Maximum magnetic flux $\Phi_{max} \equiv \pi R^2 B_{max}$ [G km ²] | 10^{18} | $10^{17.5}$ | $10^{17.5}$ |

Reisenegger

$$\Phi_{max} \sim 10^{17.5-18} \text{ G km}^2$$

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} < 10^{-6}$$

If no hidden interior field

→ No effect on global static equilibrium of stars

Local “Static” Equilibrium

Neutron star crusts:

Crust breaking occurs when

$$B > 2 \times 10^{14} \left(\frac{\theta_{\max}}{10^{-3}} \right)^{1/2} \text{ G}$$

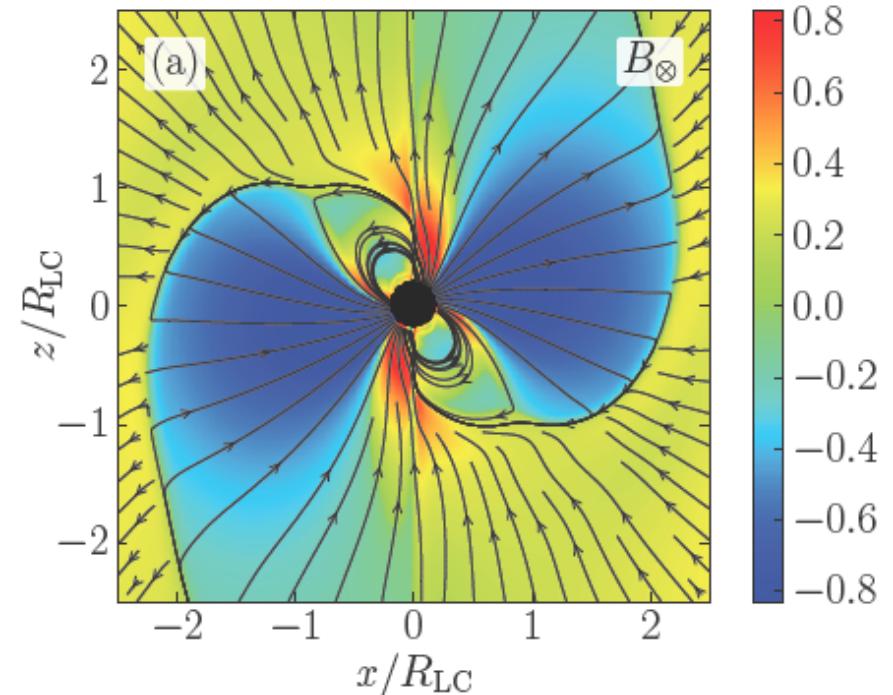
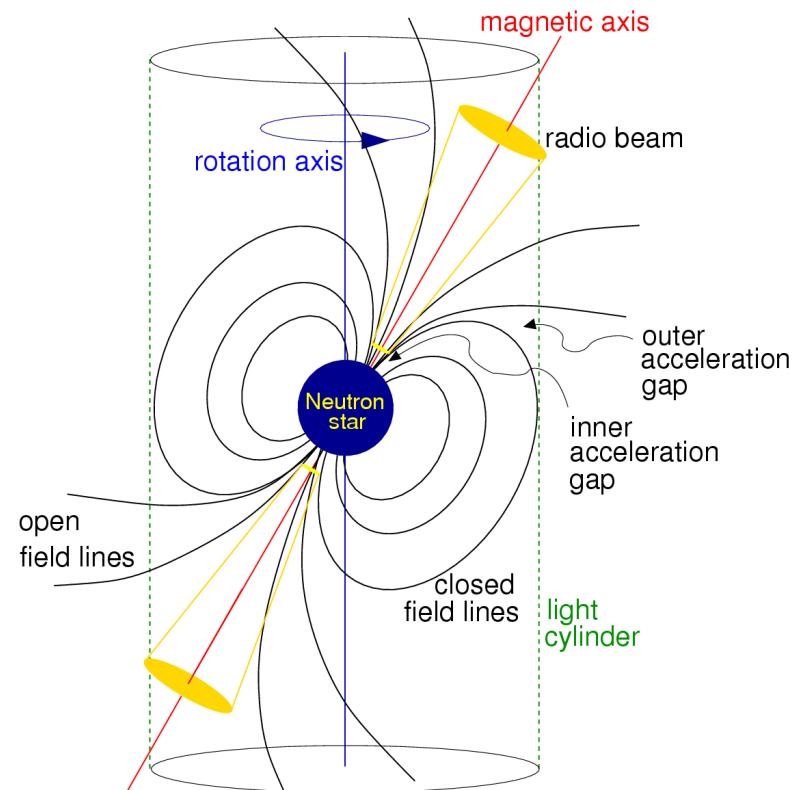
Magnetic field evolution (e.g. Ohmic and Hall) cause crust breaking
(A. Cumming, J. Pons)

Consequences of breaking:

- Fast or slow? (Y. Levin)
- Energy release?
- Can energy get out? (B. Link 2014; Blaes et al. 1989)
- Magnetar flares?

Even weak field can be “Strong” outside the star...
(e.g. magnetic braking of stars....)

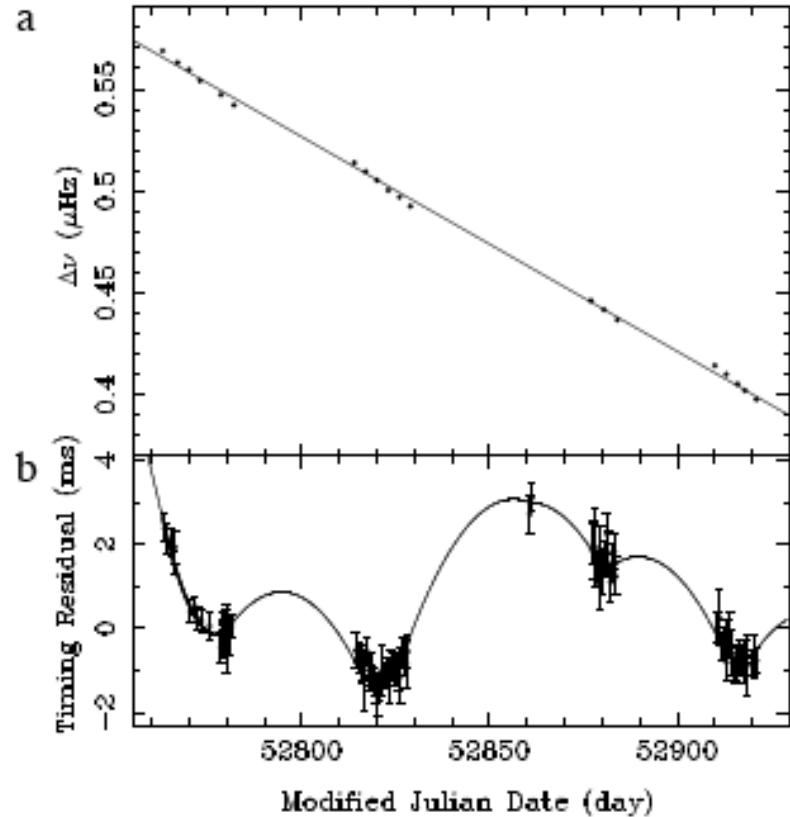
Magnetospheres: Radio Pulsars



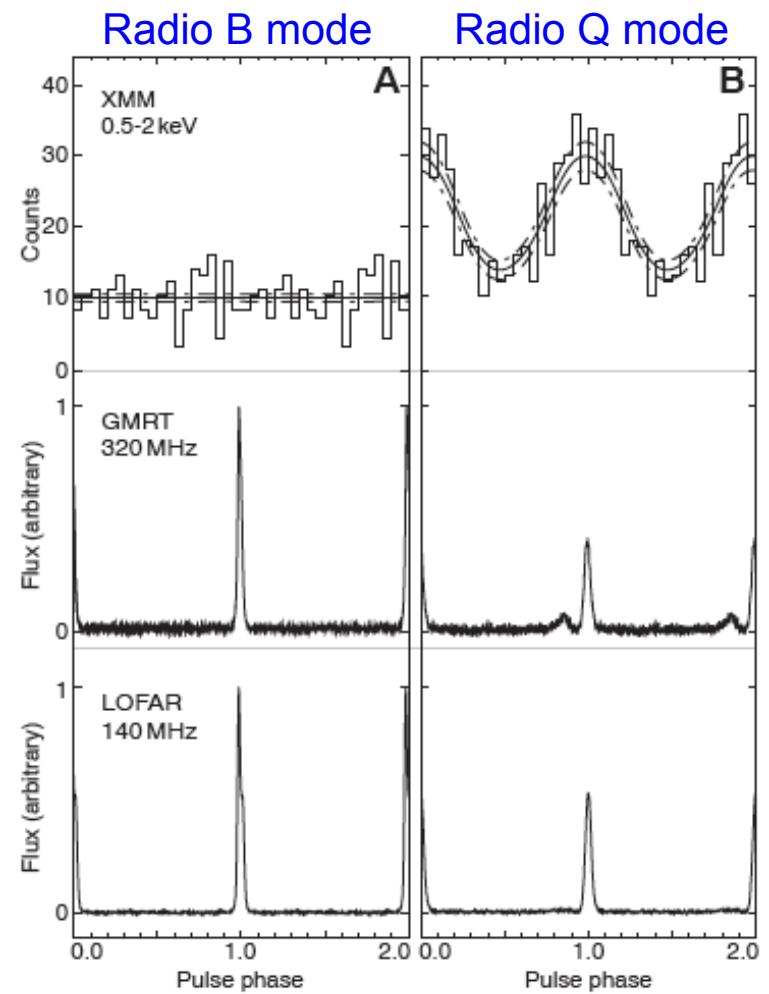
Tchekovskoy, Spitkovsky...

Magnetospheres: Radio Pulsars

Mode-switching pulsars

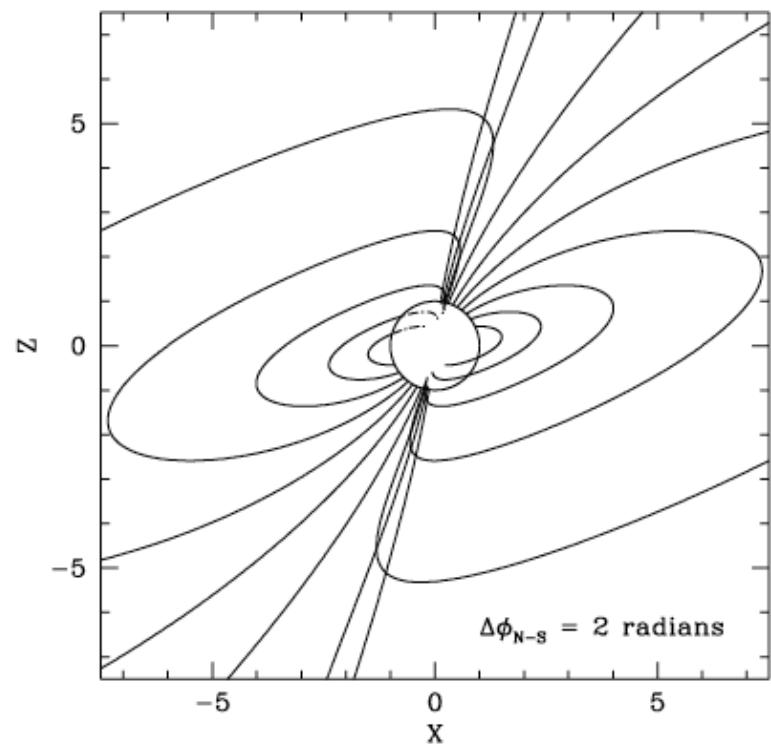


Kramer et al. 2006

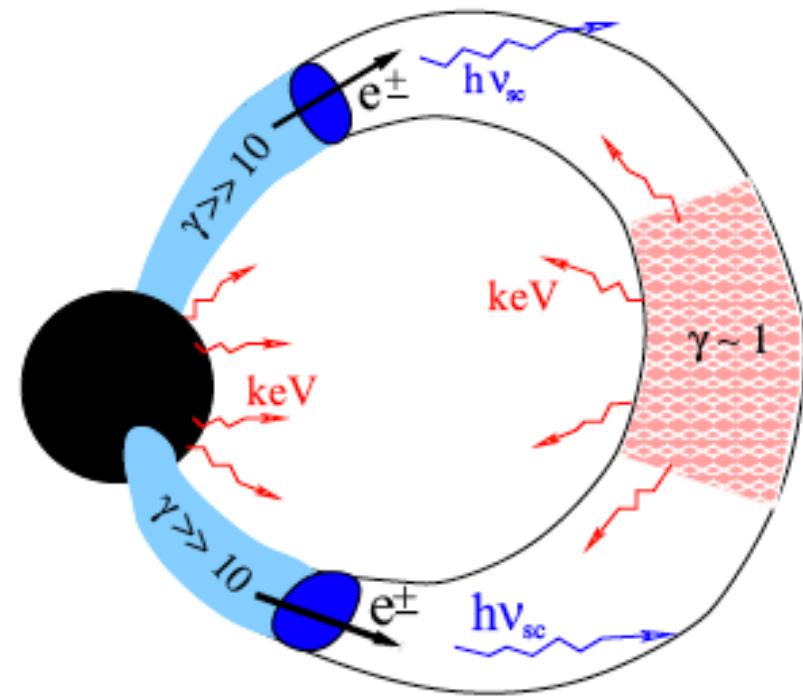


Hermsen et al 2013

Magnetospheres: Magnetars

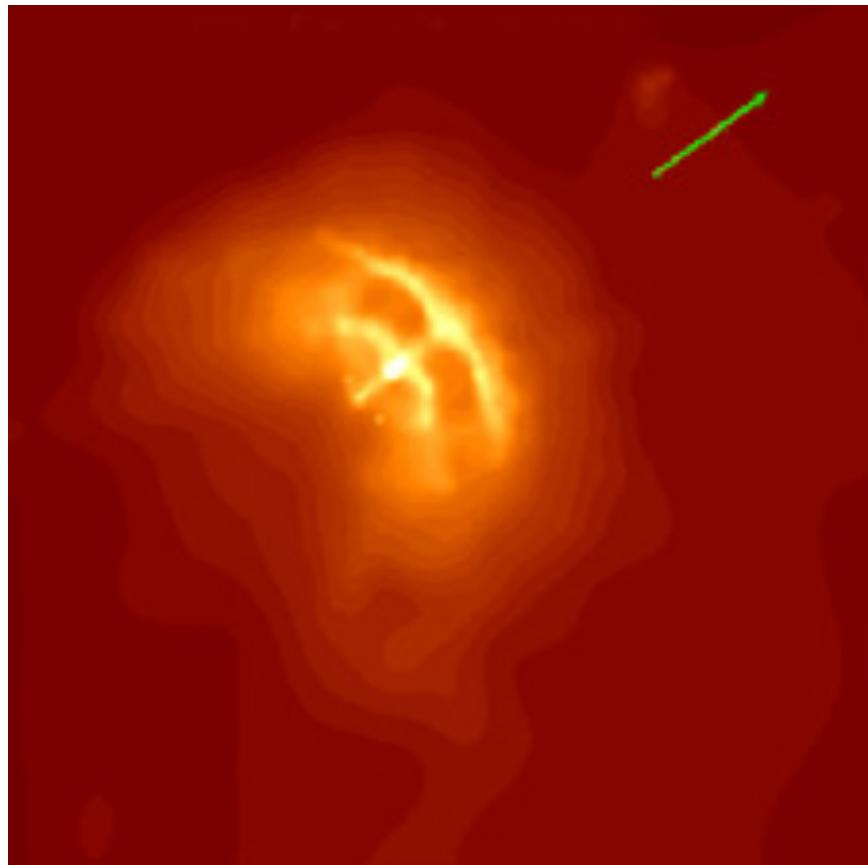


Thompson, Lyutikov & Kulkarni 2002



Beloborodov 2013

Pulsar Wind Nebulae

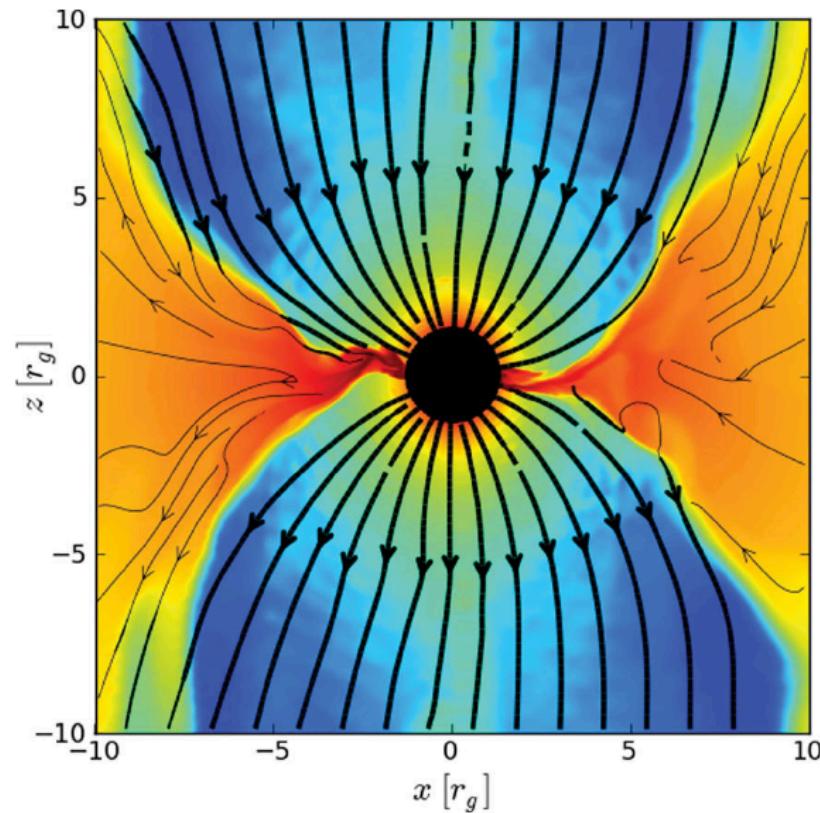


Accretion Disks

Accretion Disks

Magnetically Dominated Disks

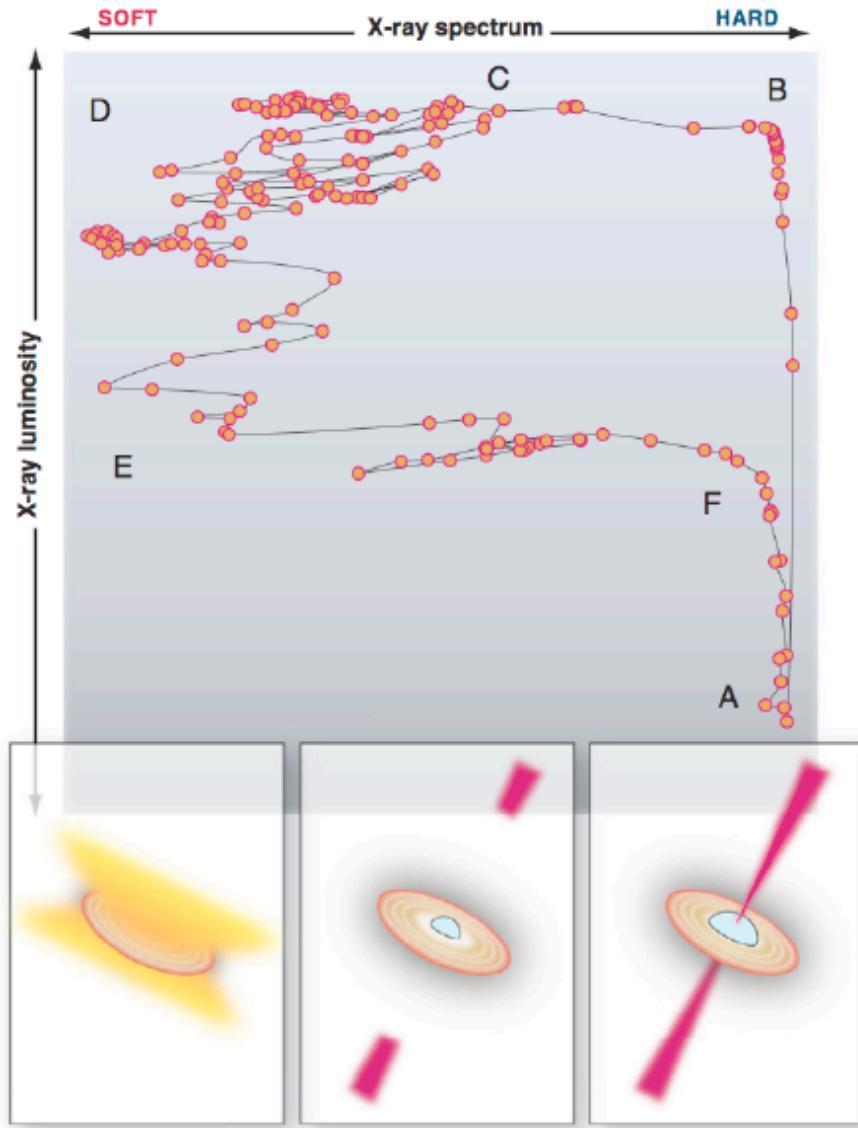
$$\frac{B^2}{8\pi} > \frac{1}{2}\rho v_K^2$$



Relativistic jets in
BH x-ray binaries/AGNs:

McKinney et al.2012

State Transition and Jets from BH x-ray Binaries



Jets:
episodic vs steady

Fender & Belloni 2012

Accretion Disks

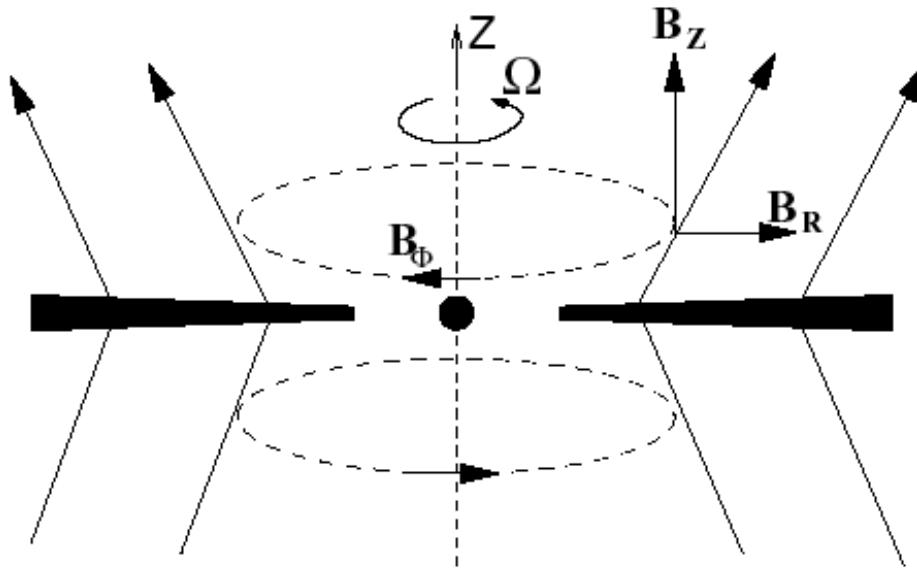
Super-thermal B Fields

$$\frac{B^2}{8\pi} > \frac{1}{2}\rho c_s^2$$

Magnetocentrifugal winds/outflows (e.g., Blandford-Payne)

X-ray binaries (thermal state)

Protostars



Magnetic Field Advection in Accretion Disks

Such large-scale strong field cannot be produced in disks (?)...

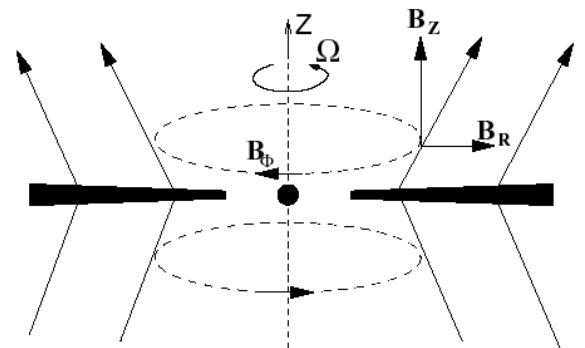
Need to be advected from outside

Radial advection inward:

$$|u_r| \sim \frac{\nu}{r}$$

Diffusion outward:

$$|u_{\text{diff}}| \sim \frac{\eta}{H} \frac{B_r}{B_z}$$



Depend on magnetic Prandtl number

$$P_r = \frac{\nu}{\eta}$$

~ 1 from MRI simulations

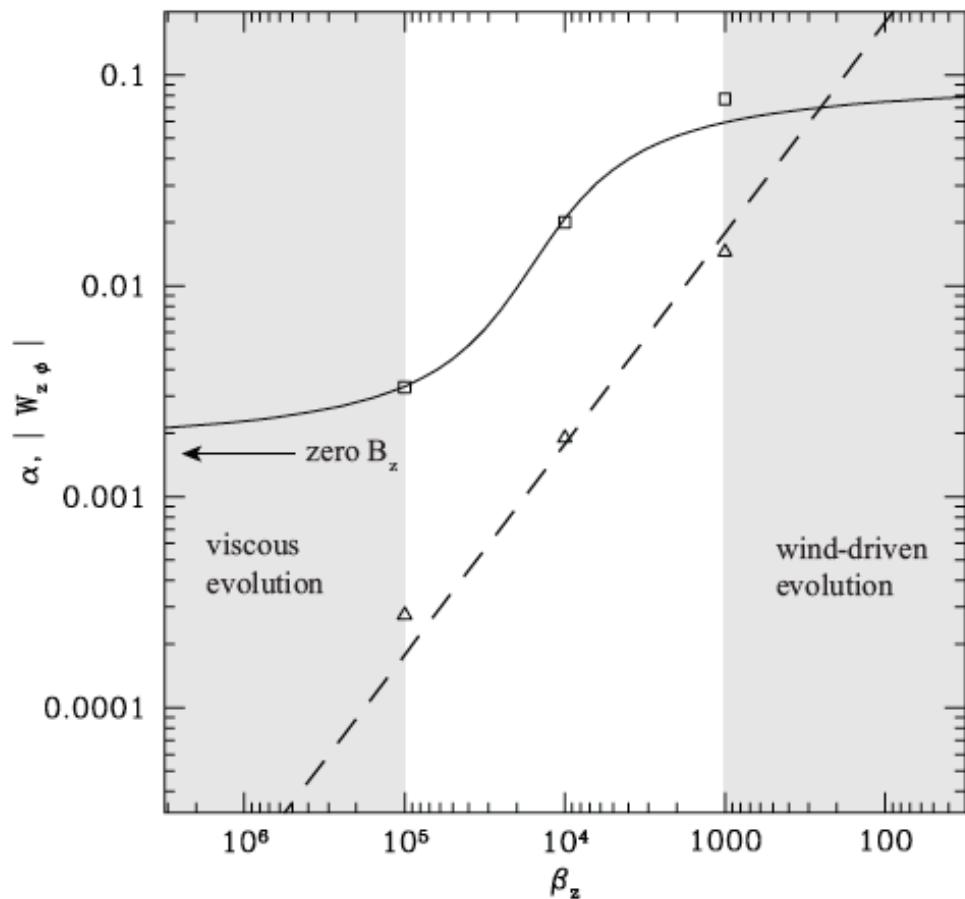
Vertical structure important (e.g. Guilet & Ogilvie 2013)

e.g., conductivity higher at disk surface → field advection faster than mass
 H_B is larger than H

Sub-Thermal Magnetic Fields in Disks

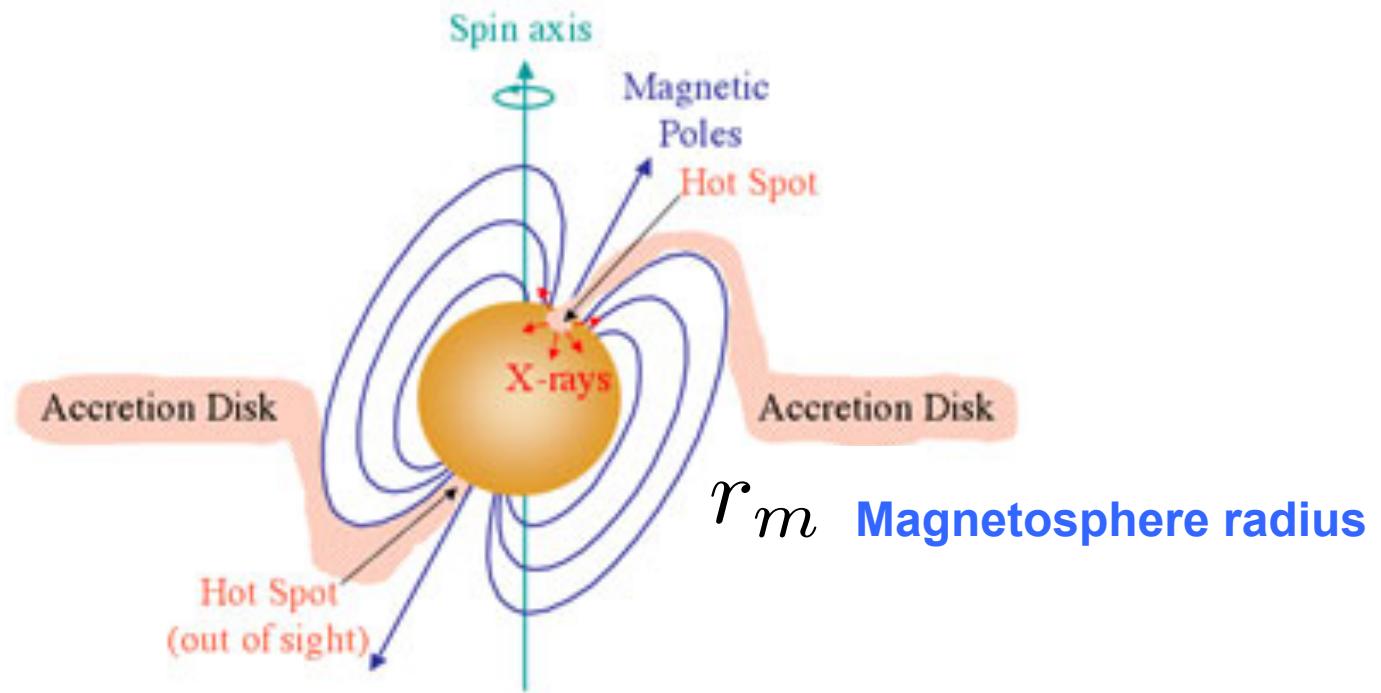
→ MRI → turbulence

Strength of turbulence depends on net vertical field (Hawley et al. 1995)

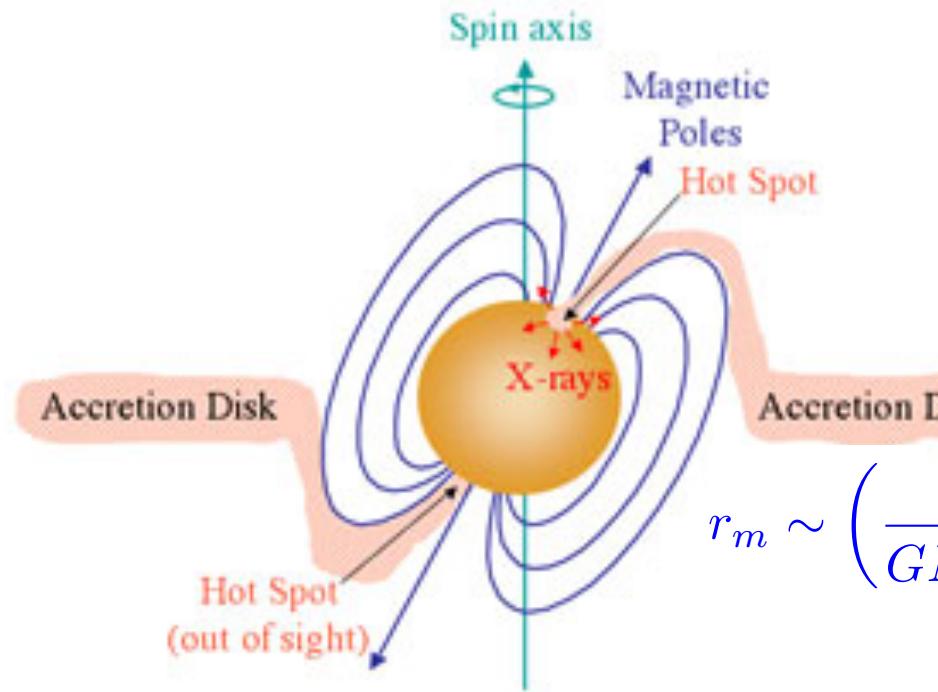


Simon et al. 2013
Armitage et al. 2013

Disk Accretion onto Magnetic Stars



Disk Accretion onto Magnetic Stars



$$r_m \sim \left(\frac{\mu^4}{G M \dot{M}^2} \right)^{1/7}$$
$$\frac{B(r)^2}{8\pi} \sim \frac{1}{2} \rho V_r V_\phi$$

Accreting x-ray pulsars:

$$B_\star \sim 10^{12} \text{ G}, \quad r_m \sim 10^2 R_\star$$

Accreting ms pulsars:

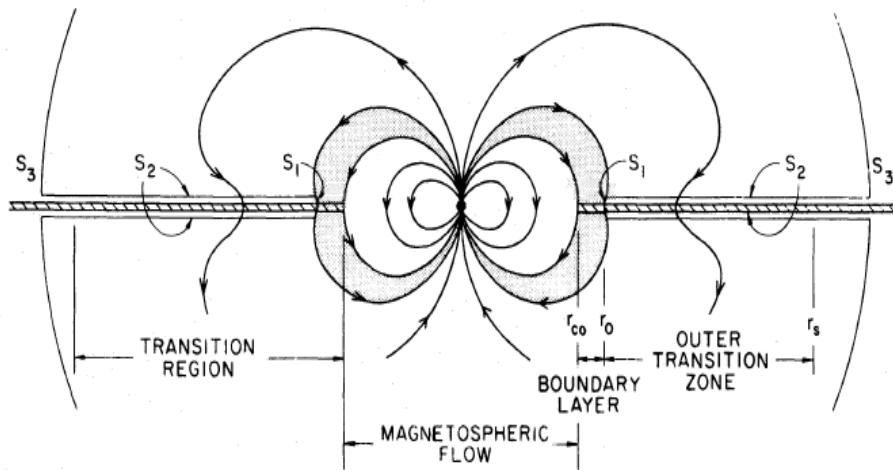
$$B_\star \sim 10^8 \text{ G}, \quad r_m \sim (\text{a few}) R_\star$$

Accreting WDs (Intermediate polars):

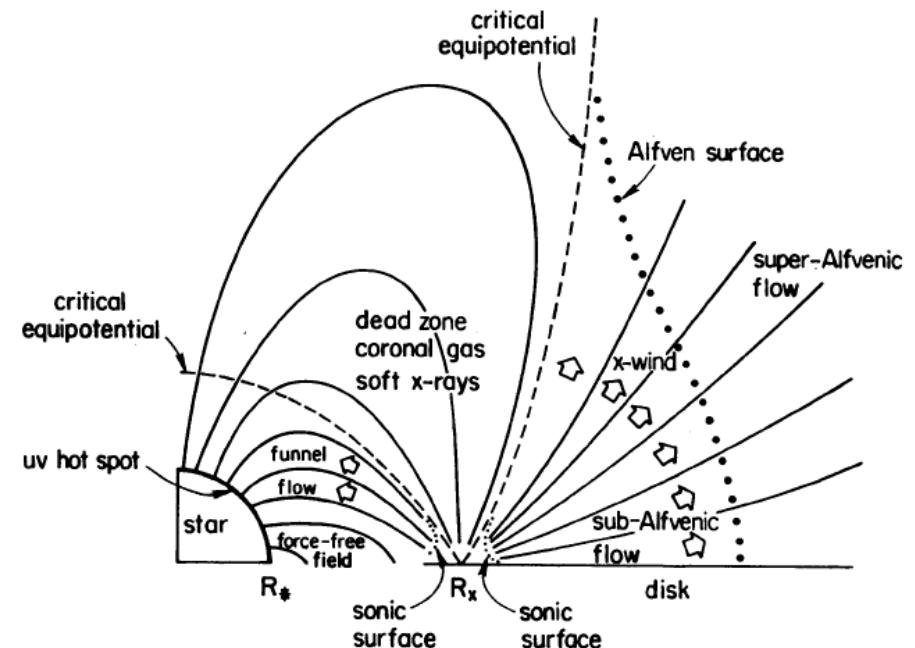
$$B_\star \sim 10^7 \text{ G}, \quad r_m \sim 10 R_\star$$

Protostars (Classical T Tauri stars):

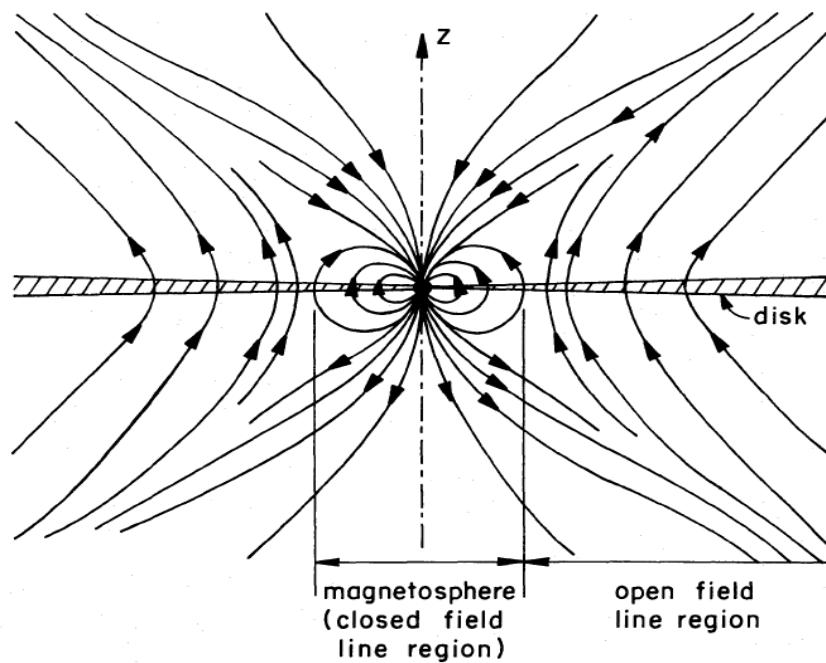
$$B_\star \sim 10^3 \text{ G}, \quad r_m \sim (\text{a few}) R_\star$$



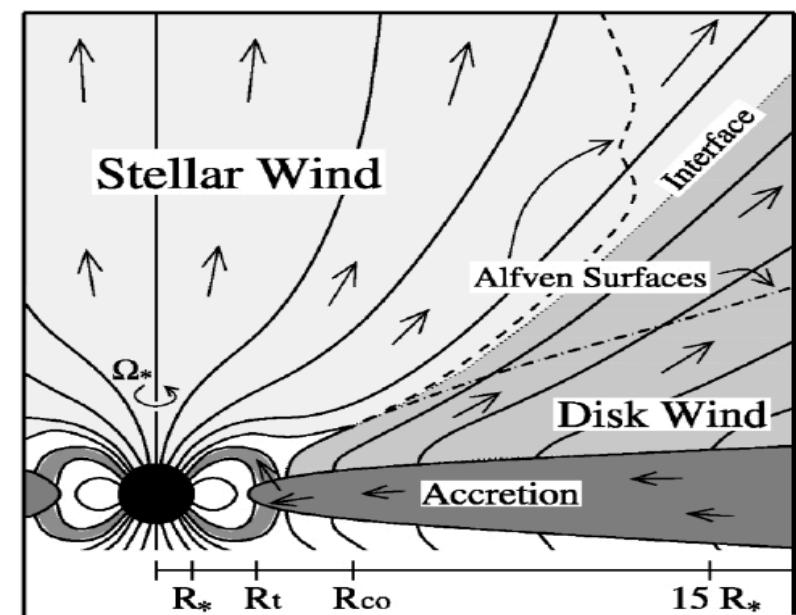
Ghosh & Lamb 1979



Shu et al. 1994



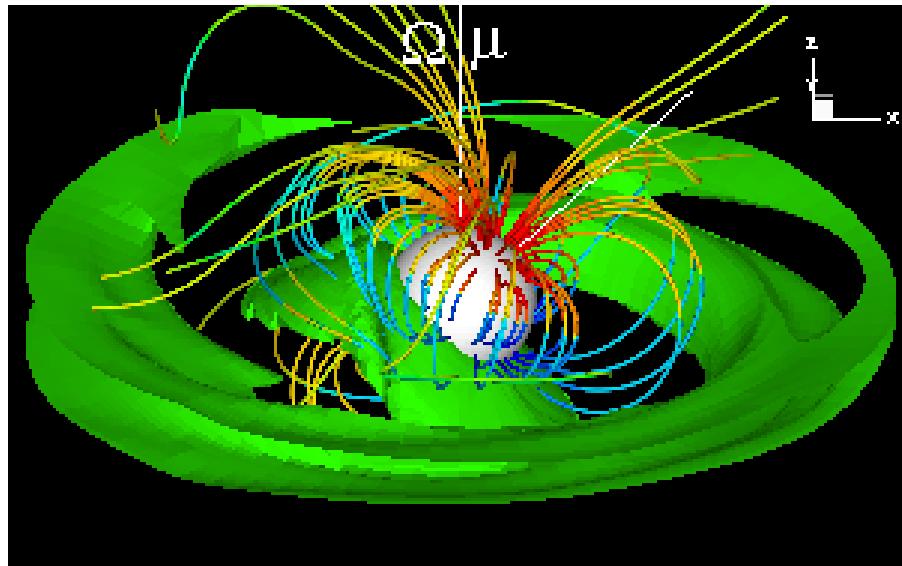
Lovelace et al. 1995



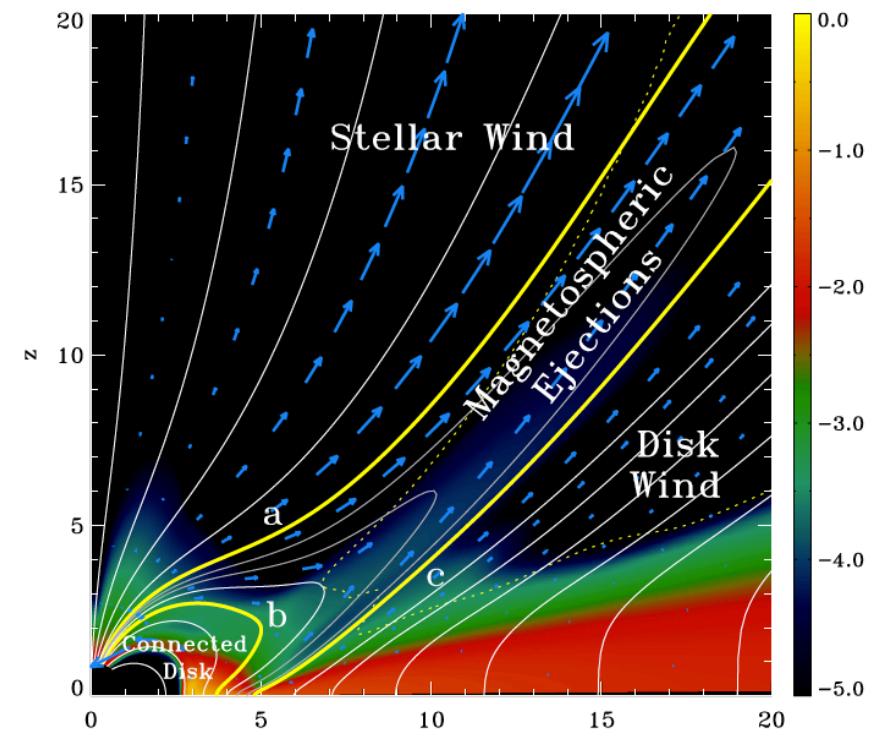
Matt & Pudritz 2005

Simulations...

Hayashi, Shibata & Matsumoto, Miller & Stone, Goodson, Winglee & Bohm,
Fendt & Elastner, Matt et al, Romanova, Lovelace, Kulkarni, Long, Lii et al,
Zanni & Ferreira,

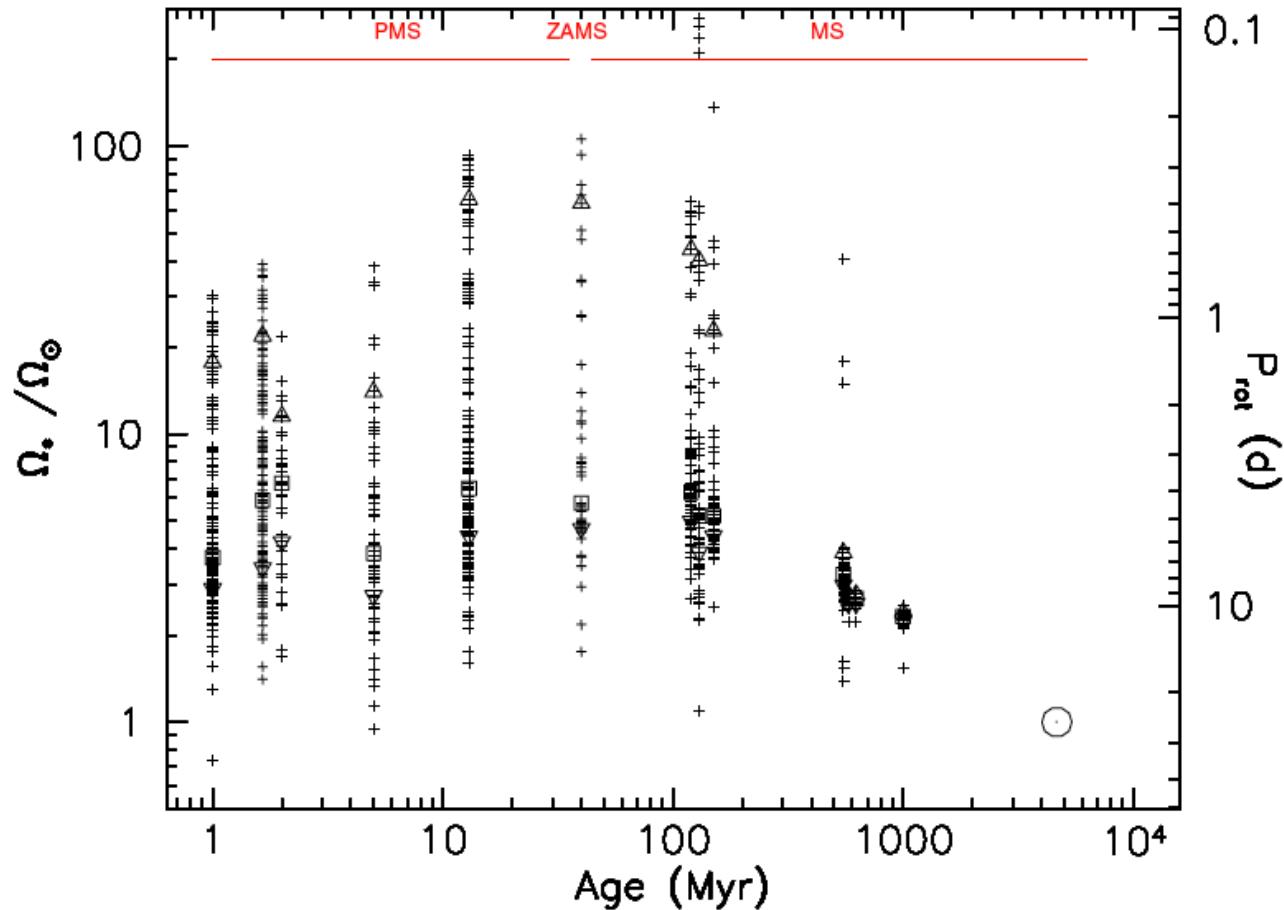


Romanova et al. 12



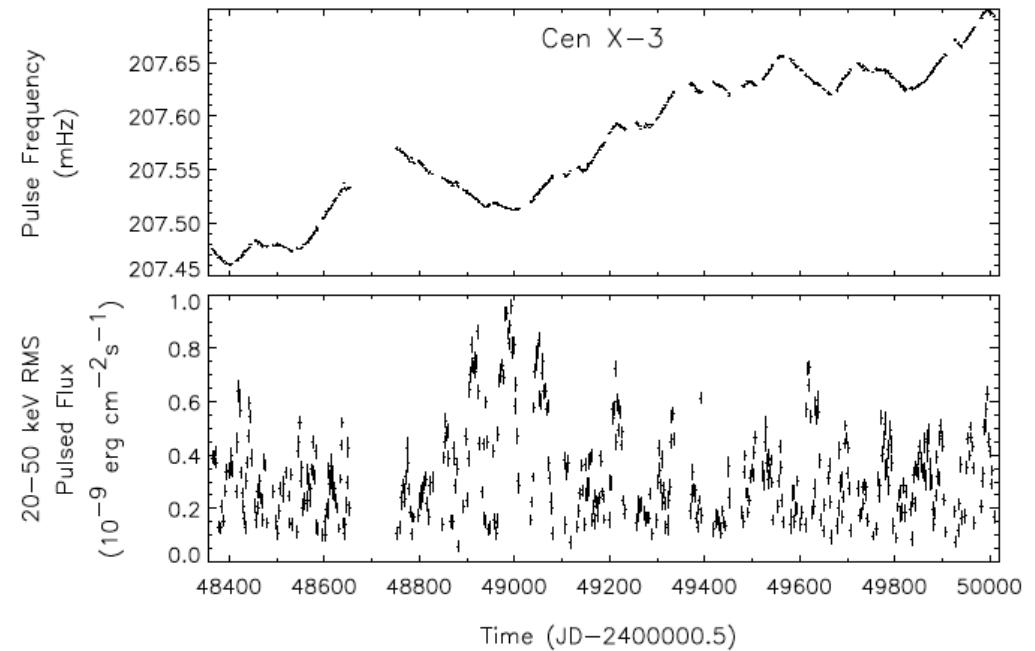
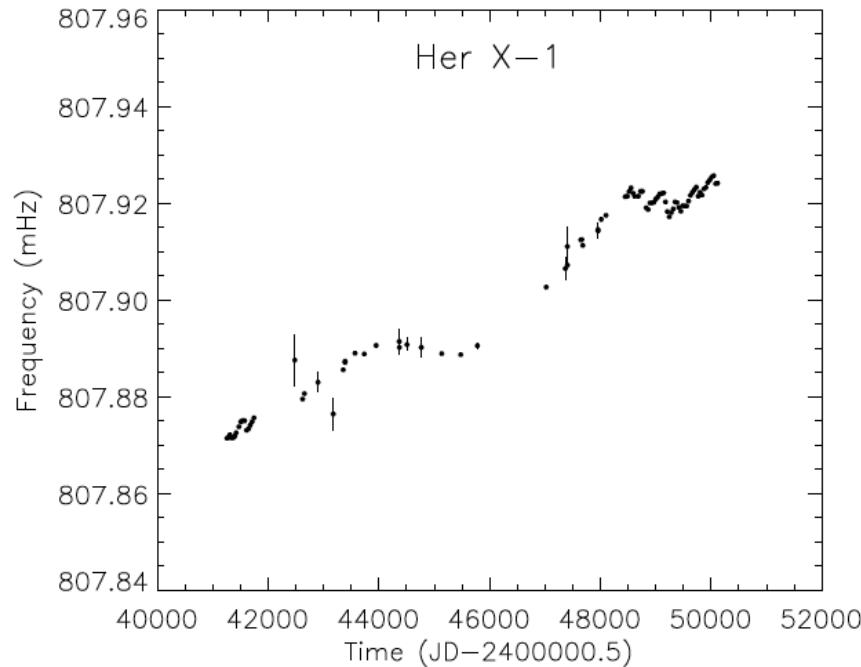
Zanni & Ferreira '13

Application: Rotation of Protostars: why 10% of breakup?

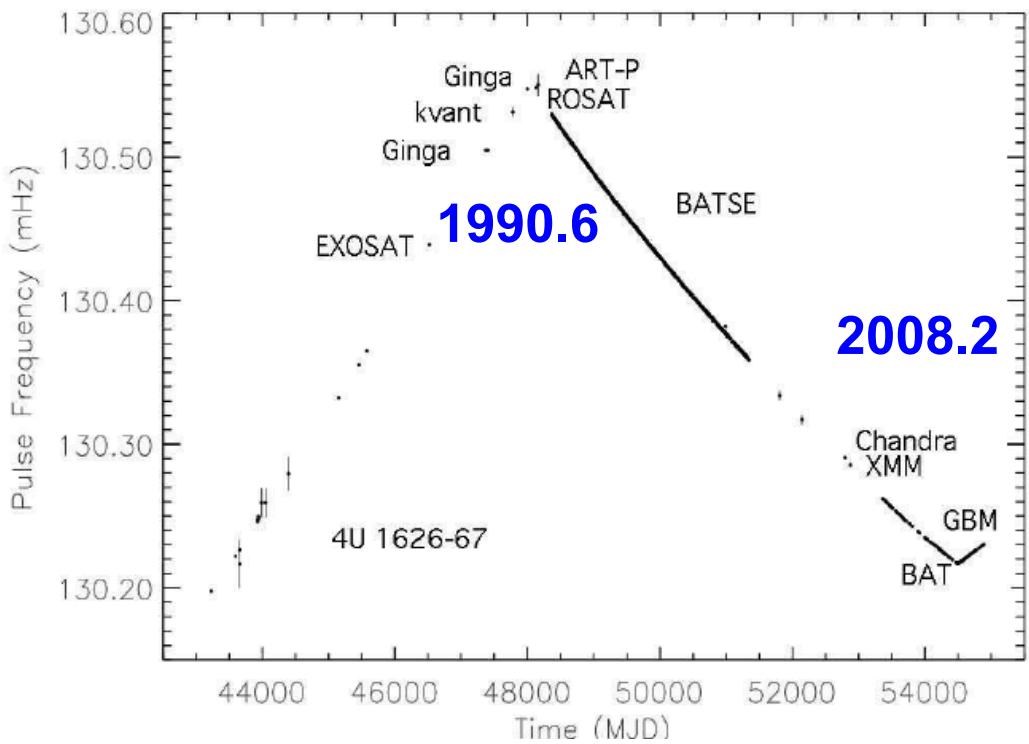


Gallet & Bouvier 2013

Application: Spinup/Spindown of Accreting X-ray pulsars

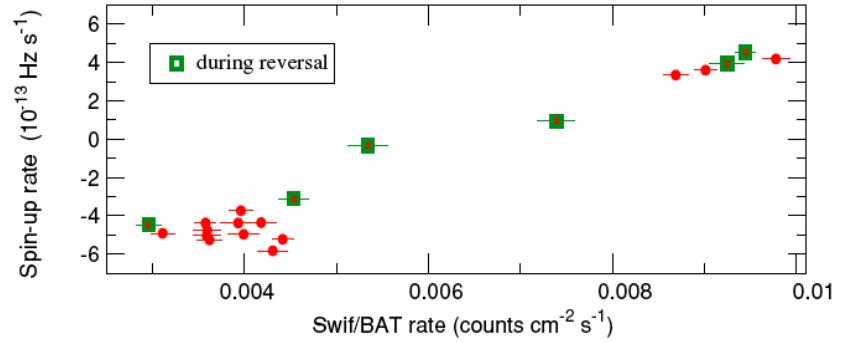
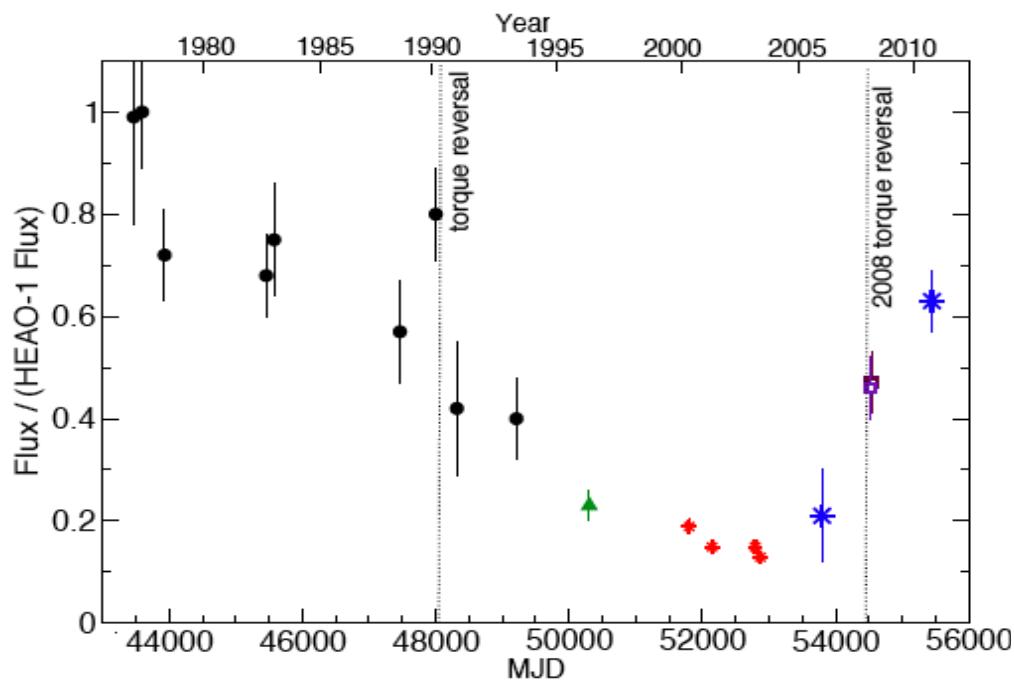


Bildsten et al. 1997



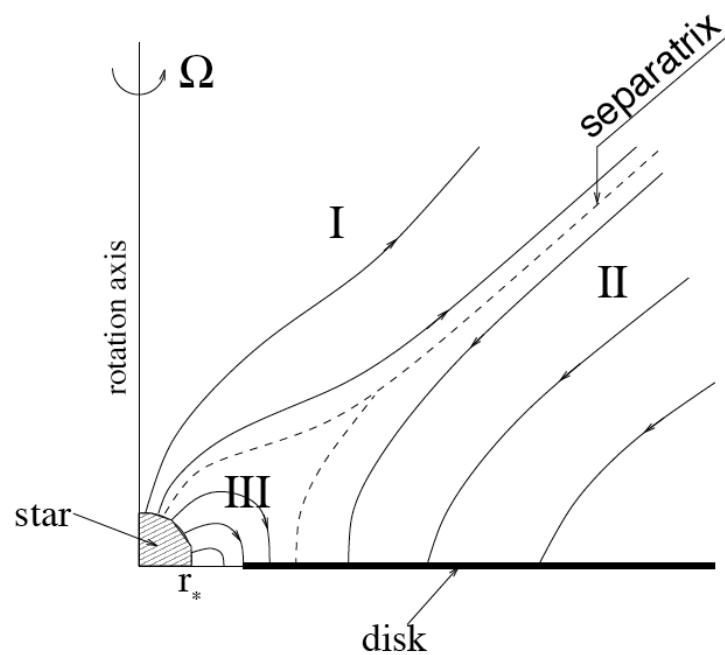
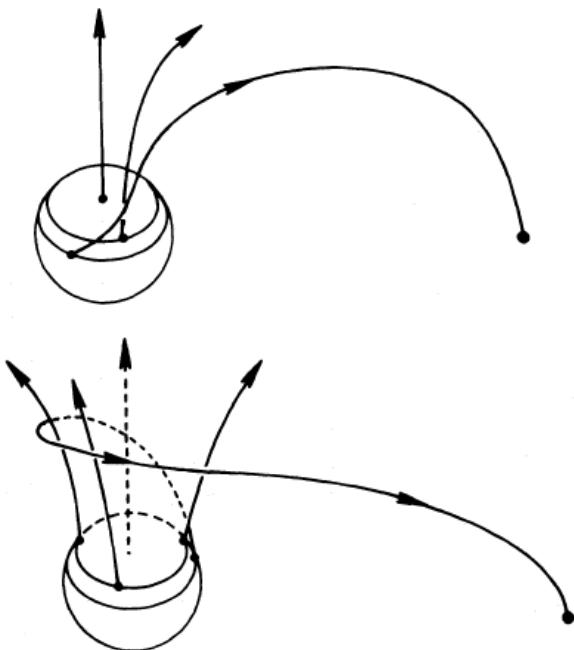
4U1626-67
7.66s
Transition lasted 150 days

Camero-Arranz et al. 2010,2012

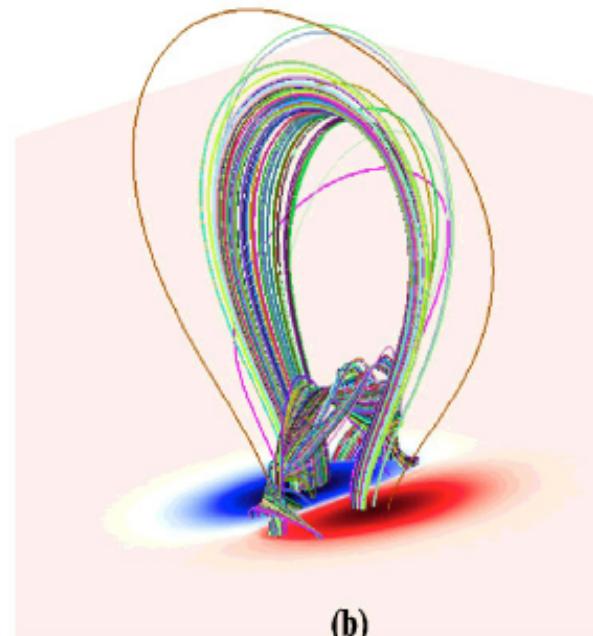
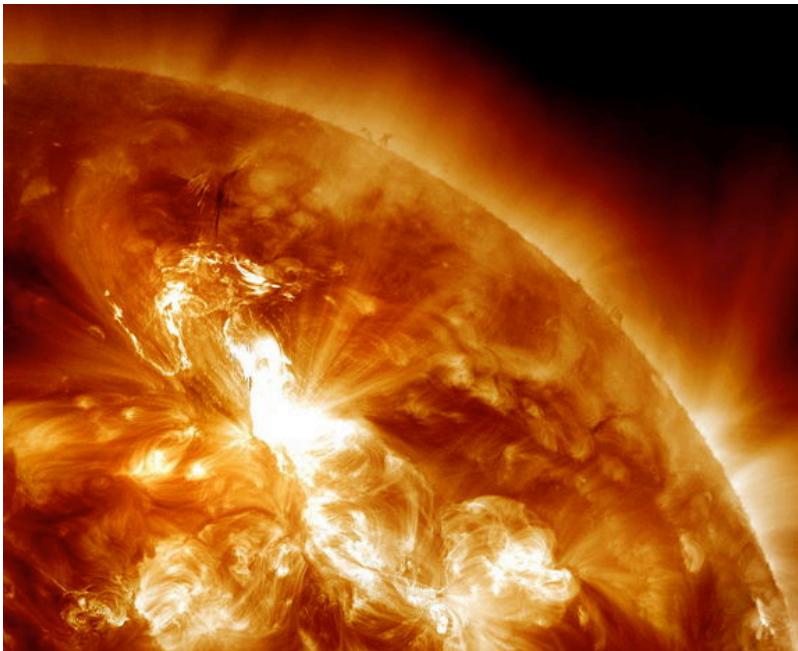


Key Physics: Star-Disk Linkage

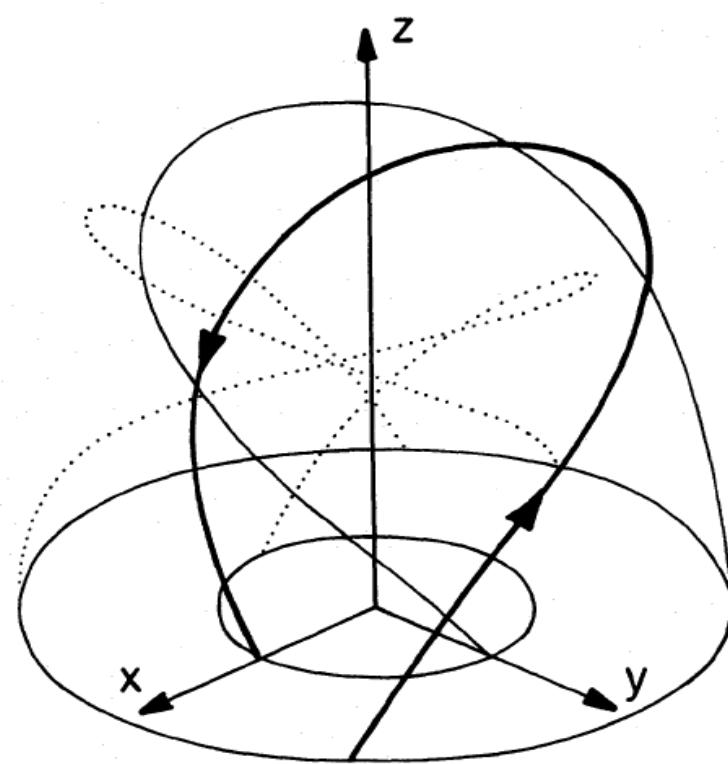
Linked fields are twisted by differential rotation...



Aly; Lovelace et al.; Uzdensky,...

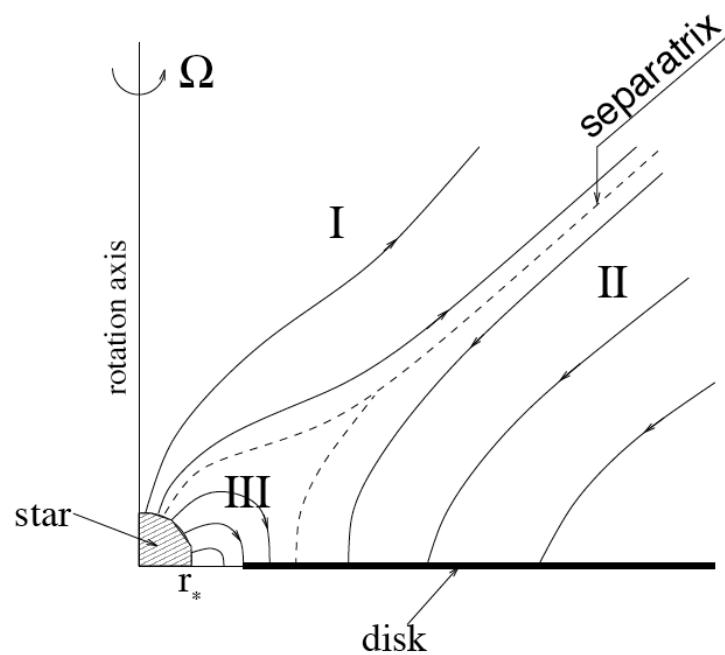
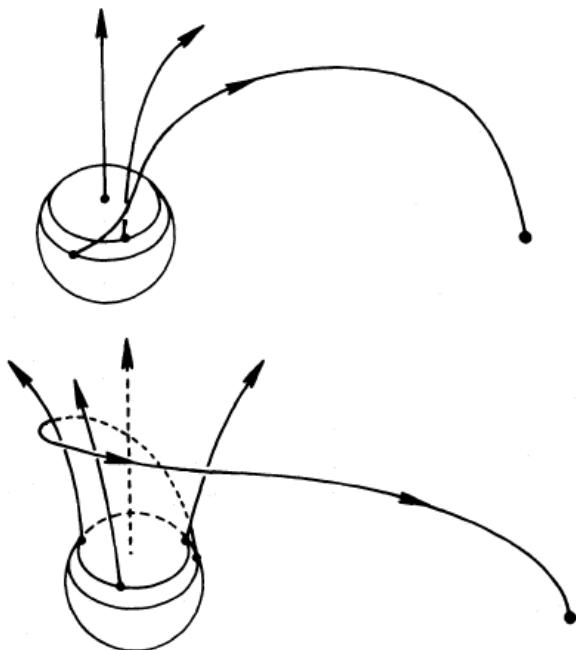


(b)



Star-Disk Linkage

Linked fields are twisted by differential rotation...
→ Field inflates, breaks the linkage



Maximum twist: $\left| \frac{B_{\phi+}}{B_z} \right|_{\max} \sim 1$

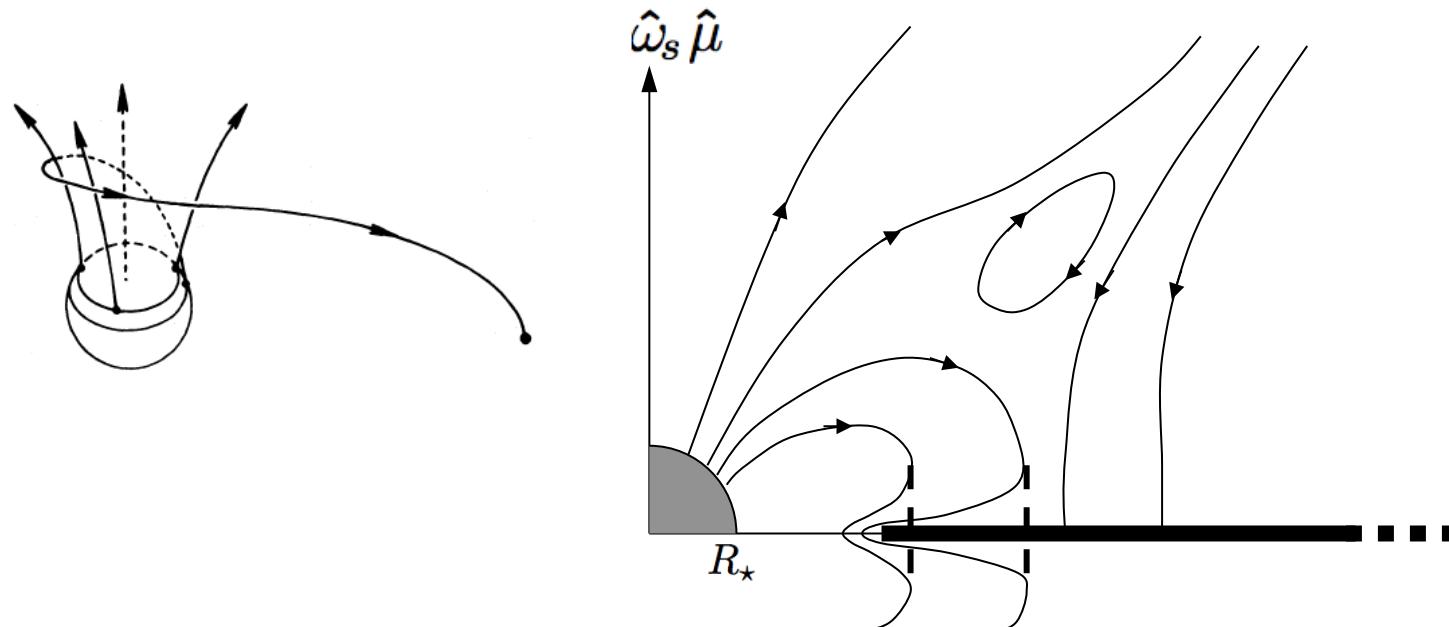
Aly; Lovelace et al.; Uzdensky,...

Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

Stellar field penetrates the inner region of disk;

Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**

Reconnection of inflated fields restore linkage



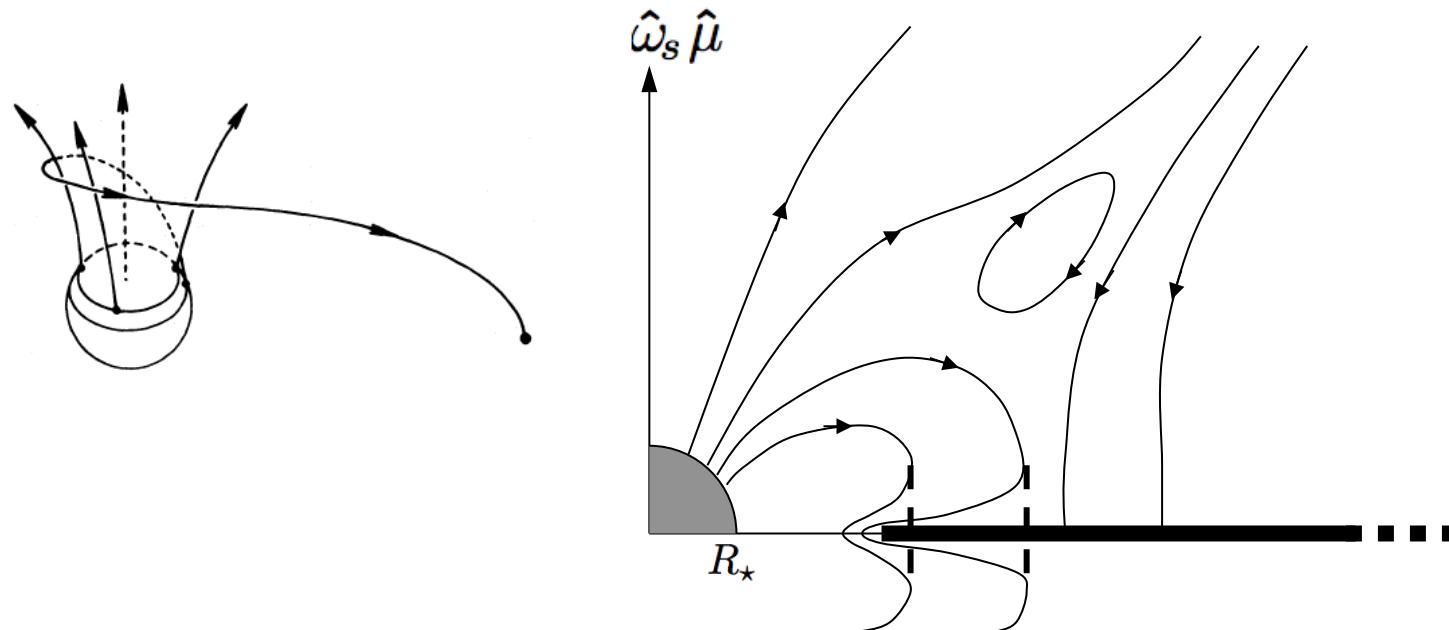
Inevitable...

Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

Stellar field penetrates the inner region of disk;

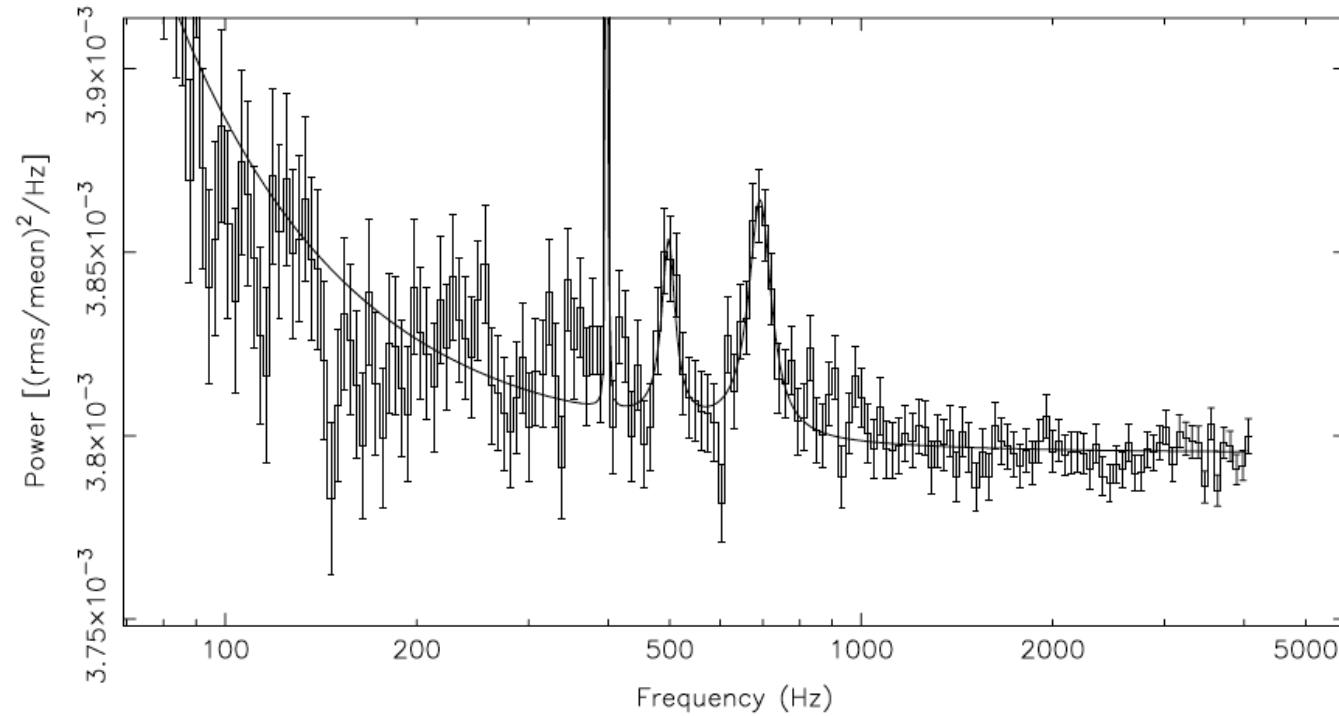
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**

Reconnection of inflated fields restore linkage



Application: Connection with QPOs in LMXBs (and other systems) ?

kHz QPOs in Accreting Millisecond Pulsars



Van der Klis 2005

SAX J1808.4-3658: $\nu_s = 401$ Hz, $\nu_h - \nu_l \simeq \nu_s/2$ (\pm a few Hz)

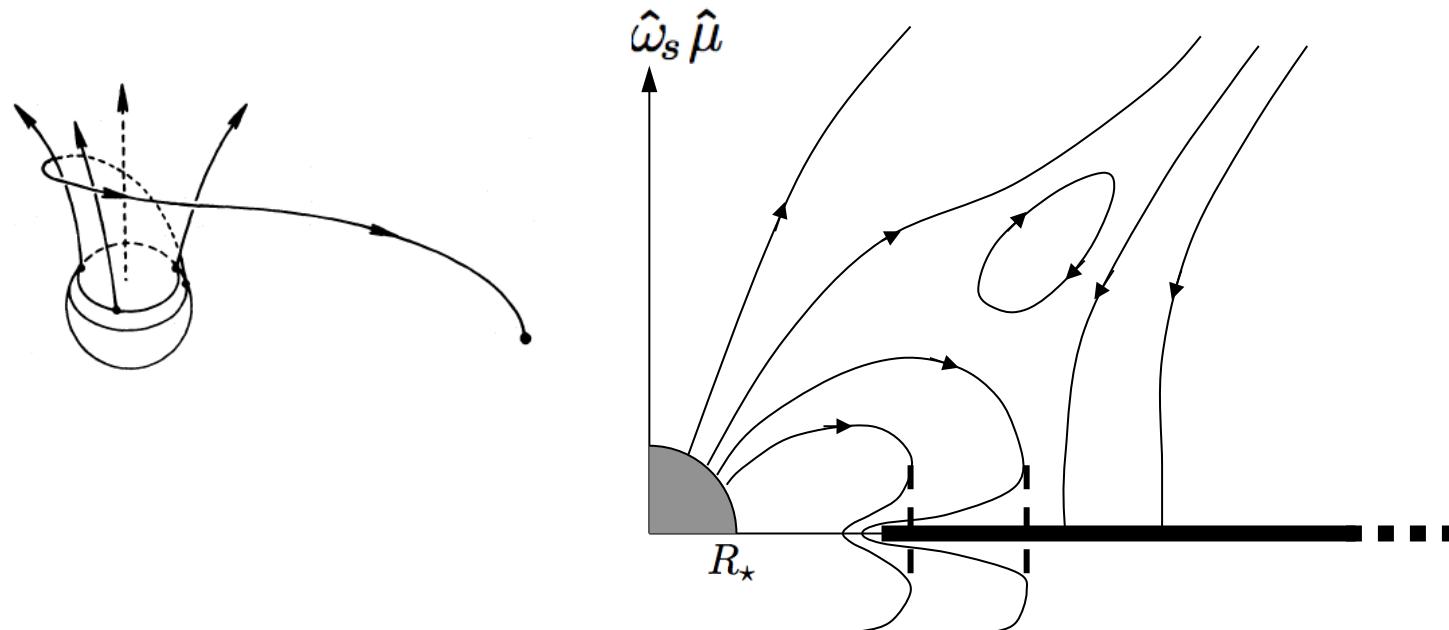
XTE J1807.4-294: $\nu_s = 191$ Hz, $\nu_h - \nu_l \simeq \nu_s$

Star-Disk Linkage: Quasi-cyclic behavior (Width, Time-dependence...)

Stellar field penetrates the inner region of disk;

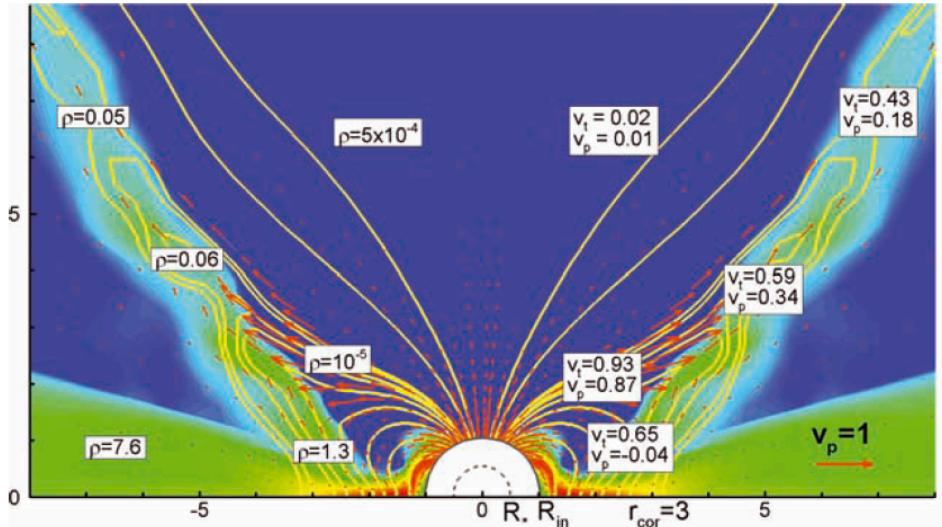
Field lines linking star and disk are **twisted** --> toroidal field --> field **inflation**

Reconnection of inflated fields restore linkage

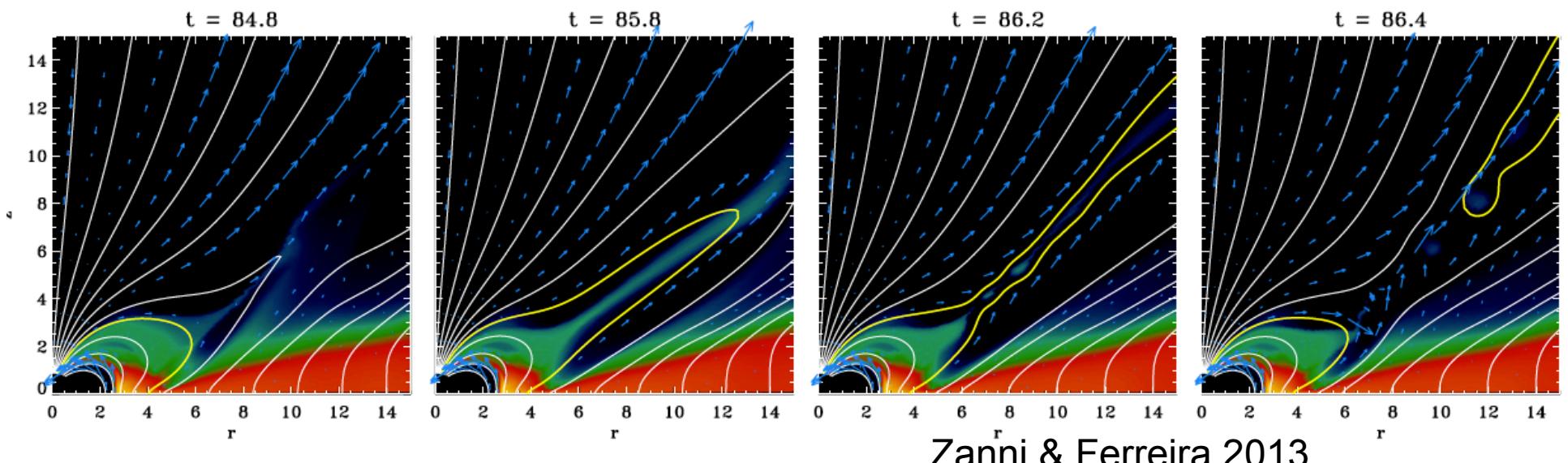


Application: Episodic outflow ...

Ejection from Magnetospheric Boundary

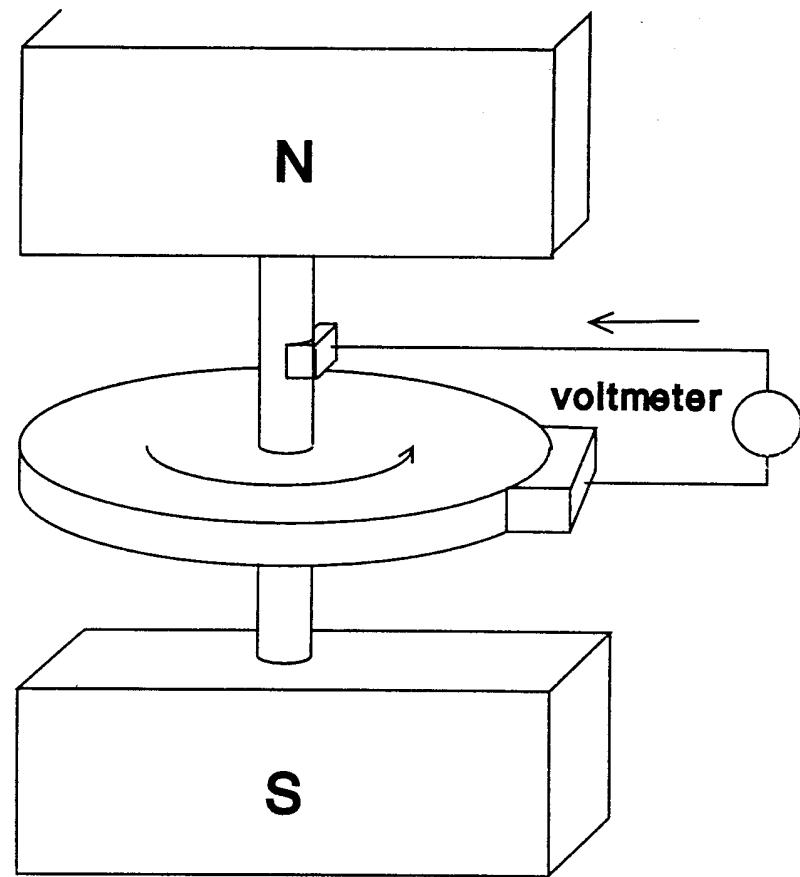


Romanova et al. 2009

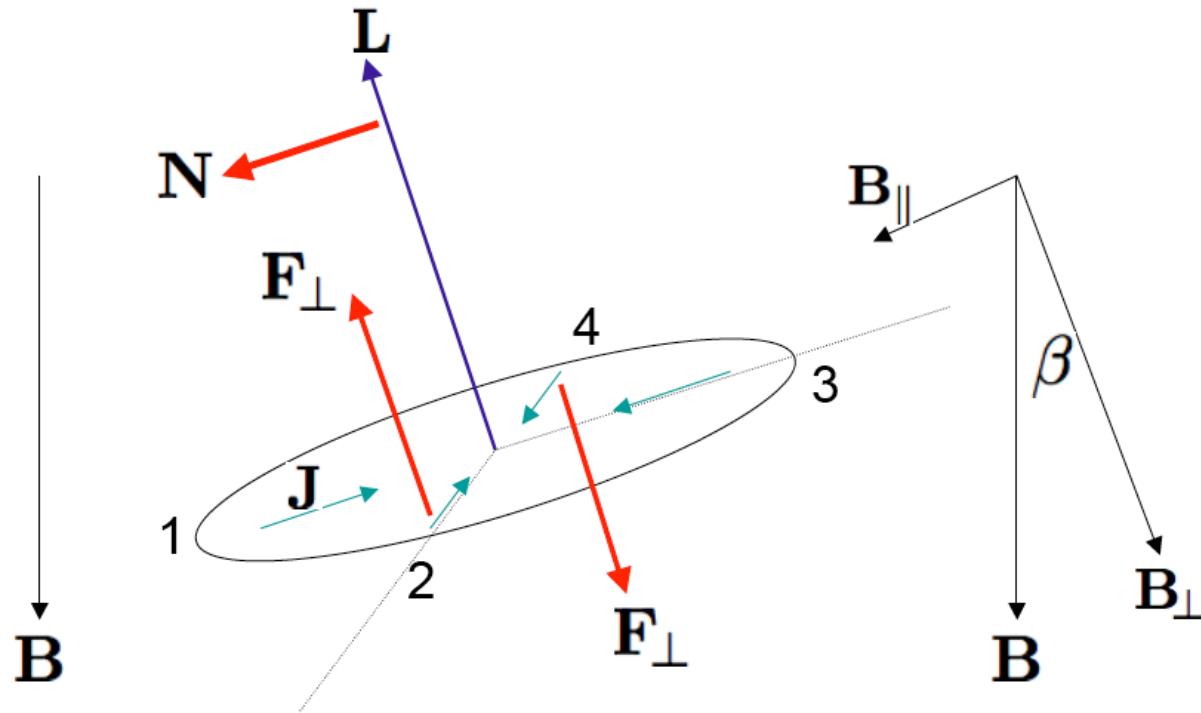


Zanni & Ferreira 2013

Star-Disk Linkage → Spin-Disk Misalignment (?)



Star-Disk Linkage → Spin-Disk Misalignment (?)

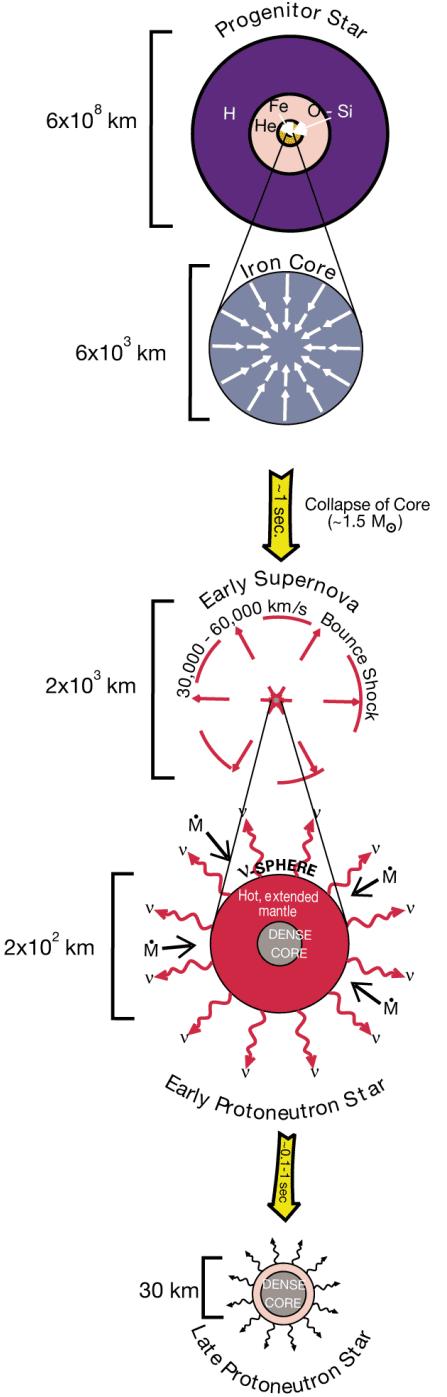
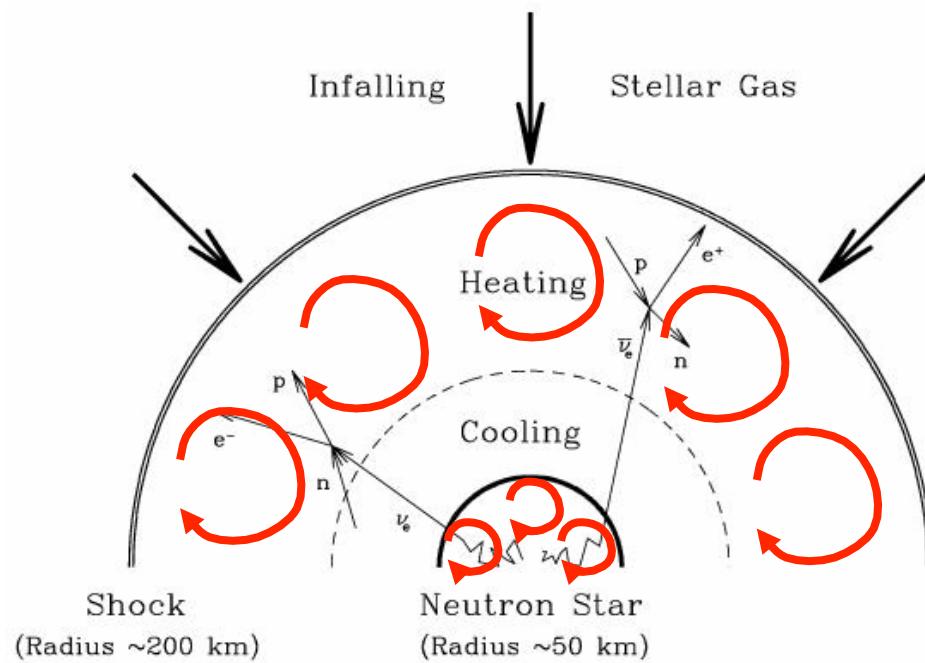


Lai et al 2011

7 degree misalignment in solar system...; exoplanetary systems...

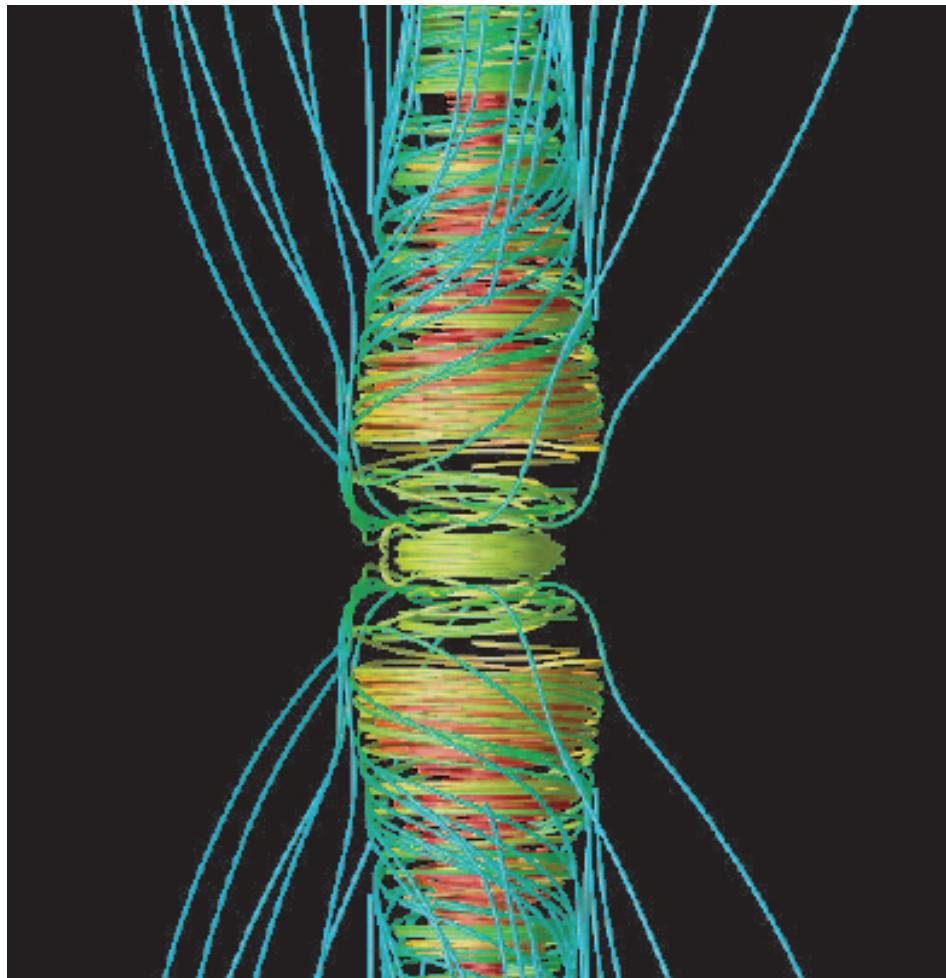
Magnetic Fields in Formation of Compact Objects

Core-Collapse Supernova → NS



Magnetically Driven Supernova/Hypernova

LeBlanc & Wilson 1970; Bisnovatyi-Kogan et al. 1976;....
Wheeler, Yamada, Thompson, Shibata, Moiseenko, Burrows,...



- Require rapid rotation
(uncertain preSN rotation)
- MRI scale usually not resolved unless put in $>10^{15}\text{G}$

Burrows et al. 2007

Roles of Magnetars in Supernovae

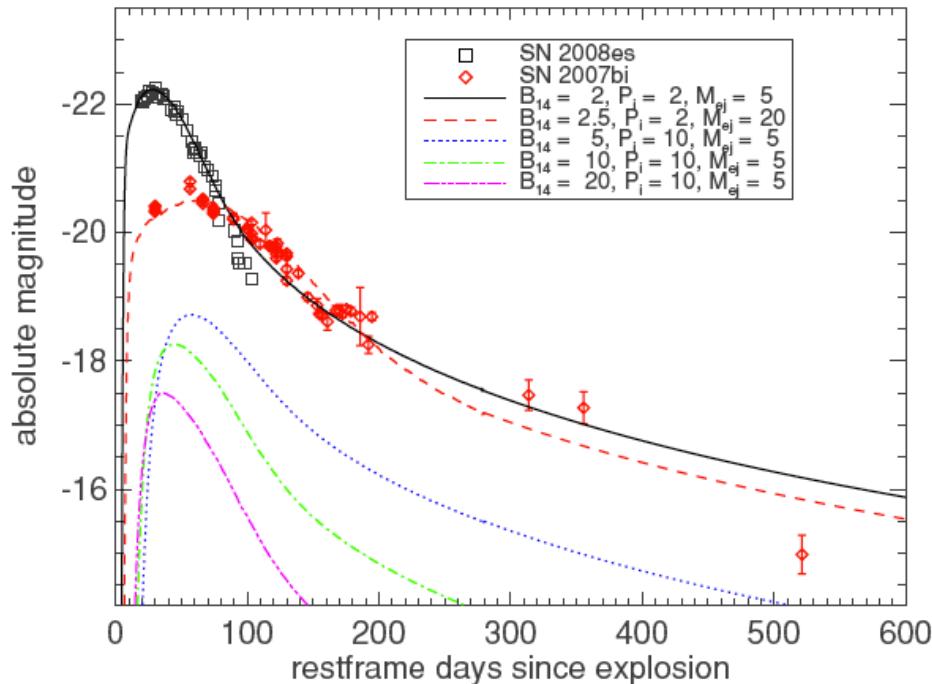
-- Power explosion: 1-3 ms, $\sim 10^{15}$ G

(Bodenheimer & Ostriker 1974; Wheeler et al. 2000; T. Thompson et al. 2004....)

-- For ~ 10 ms, not affect explosion, but still impact lightcurves:

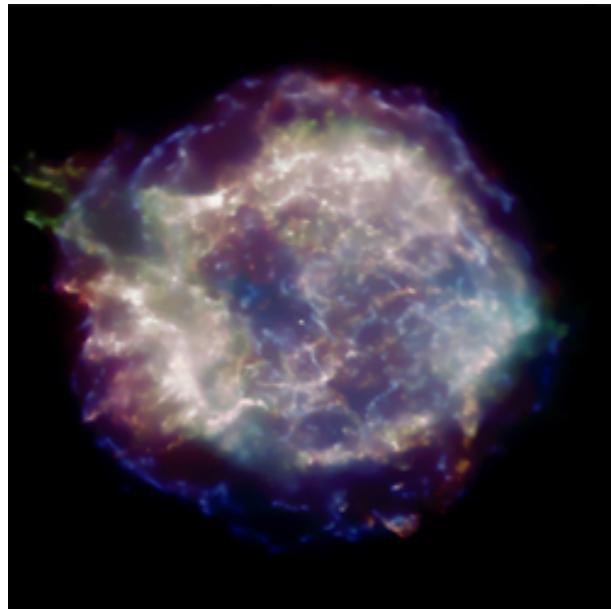
Spindown time ~days-weeks (~photon diffusion through remnant)

May explain SLSNe with $L > 10^{44}$ erg/s (Kasen & Bildsten 2010; Woosley 2010);
But Dado & Dar 2014



Kasen & Bildsten 2010

Central Compact Objects (CCOs) in SNRs



| CCO | SNR | Age (kyr) | d (kpc) | P (s) | f_p^a (%) | B_s (10^{11} G) |
|-------------------------|------------------|--------------|--------------|------------|----------------|-------------------------|
| RX J0822.0 – 4300 | Puppis A | 3.7 | 2.2 | 0.112 | 11 | <9.8 |
| CXOU J085201.4 – 461753 | G266.1 – 1.2 | 1 | 1 | ... | <7 | ... |
| 1E 1207.4 – 5209 | PKS 1209 – 51/52 | 7 | 2.2 | 0.424 | 9 | <3.3 |
| CXOU J160103.1 – 513353 | G330.2 + 1.0 | $\gtrsim 3$ | 5 | ... | <40 | ... |
| 1WGA J1713.4 – 3949 | G347.3 – 0.5 | 1.6 | 1.3 | ... | <7 | ... |
| CXOU J185238.6 + 004020 | Kes 79 | 7 | 7 | 0.105 | 64 | 0.31 |
| CXOU J232327.9 + 584842 | Cas A | 0.33 | 3.4 | ... | <12 | ... |

Halpern & Gotthelf

→ Small surface dipole field (although internal field could be higher)

Central Engine of (Long) GRBs

1. Hyper-accreting black hole

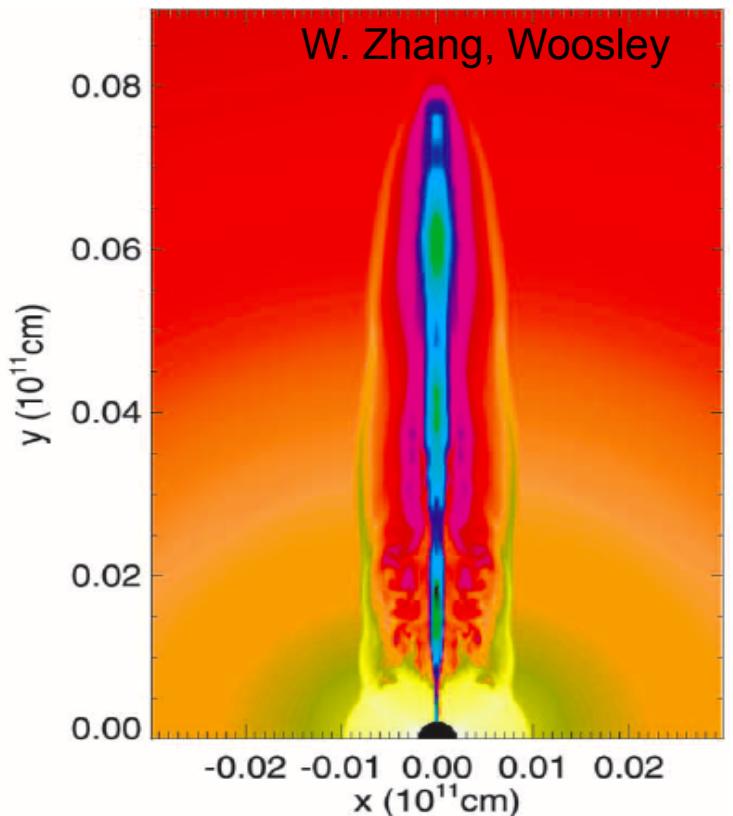
Neutrino annihilation, BZ → jets

2. Millisecond Magnetars

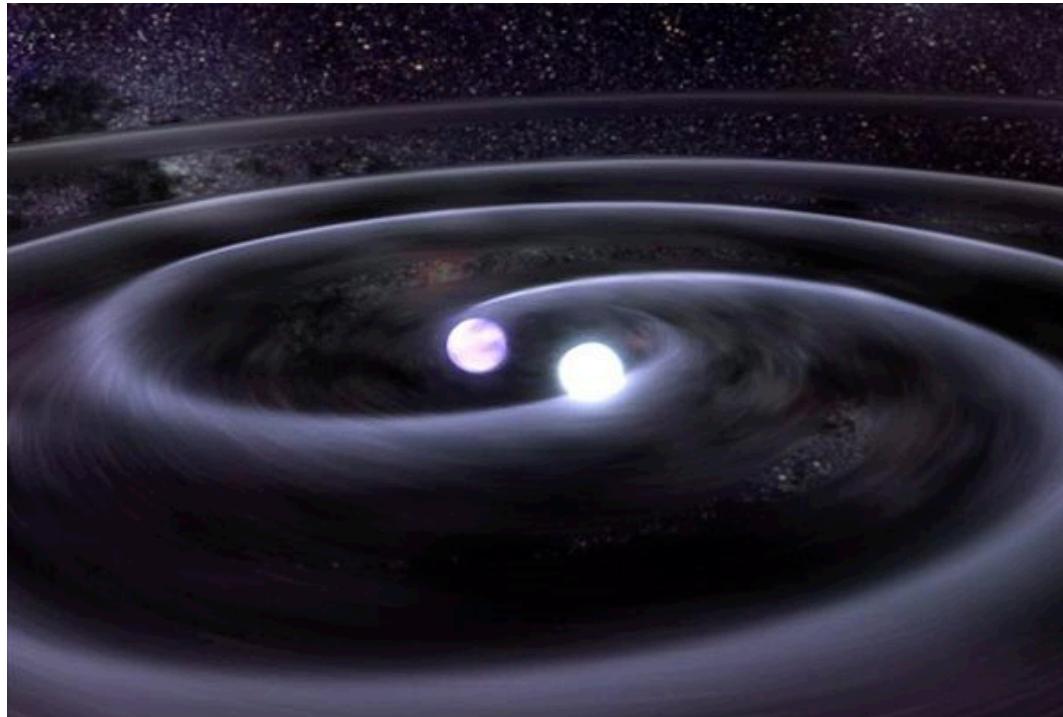
Rotational energy

Observational constraints:

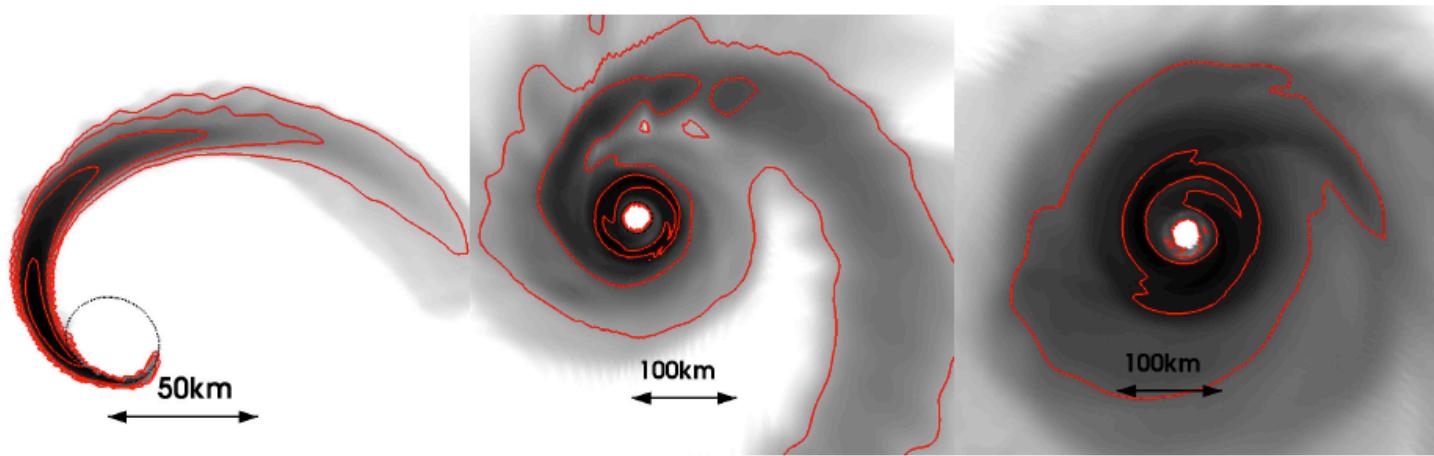
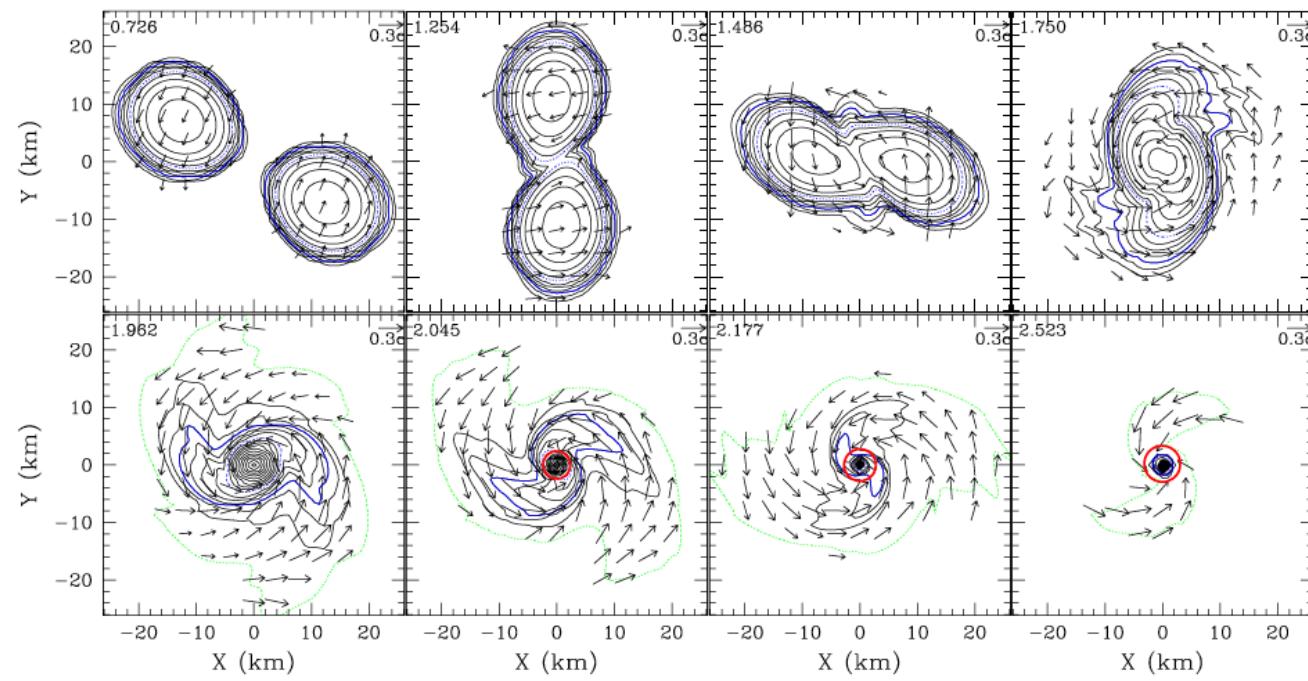
- long-lasting (10^4 s) x-ray emission/flares
- high polarization in reverse-shock emission
→ large-scale B field in GRB jets (Mundell et al. 2013)



Magnetic Fields in Merging Compact Binaries



- NS/NS and NS/BH binaries:** GWs for LIGO/VIRGO
EM counterparts (short GRBs, kiloNova)
- Compact WD/WD Binaries:** GWs for eLISA/NGO, R CrB stars,
AM CVn binaries, transients, AIC, SN Ia

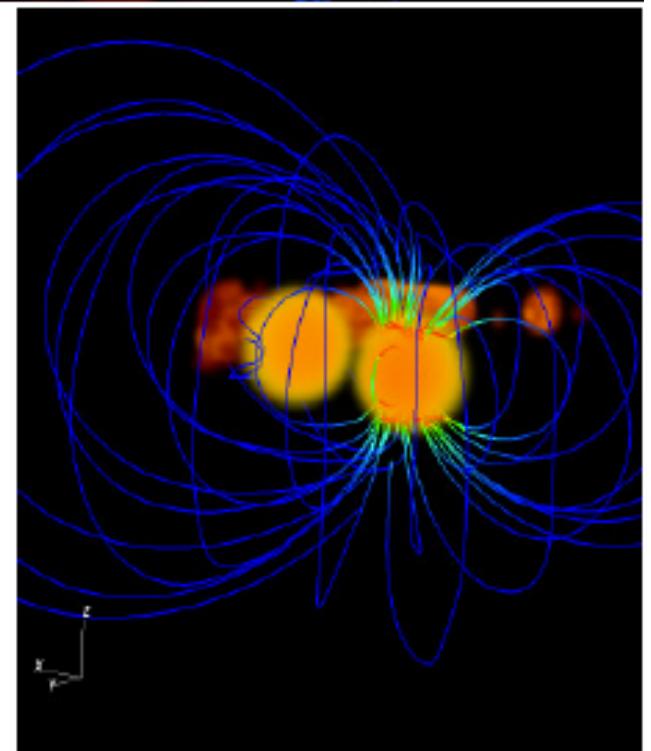
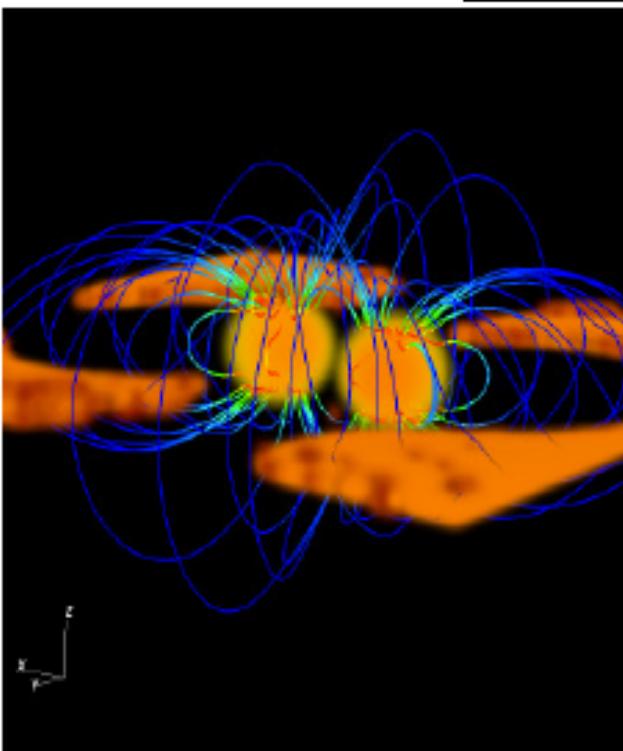
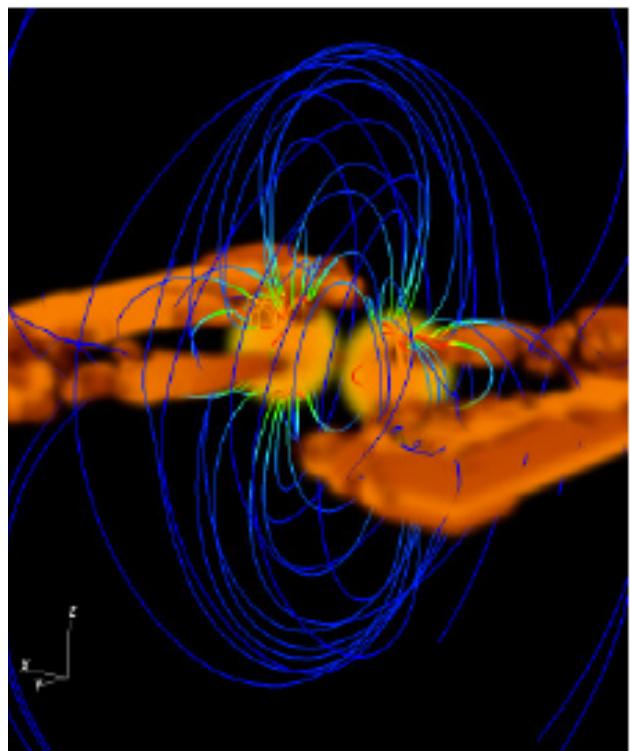
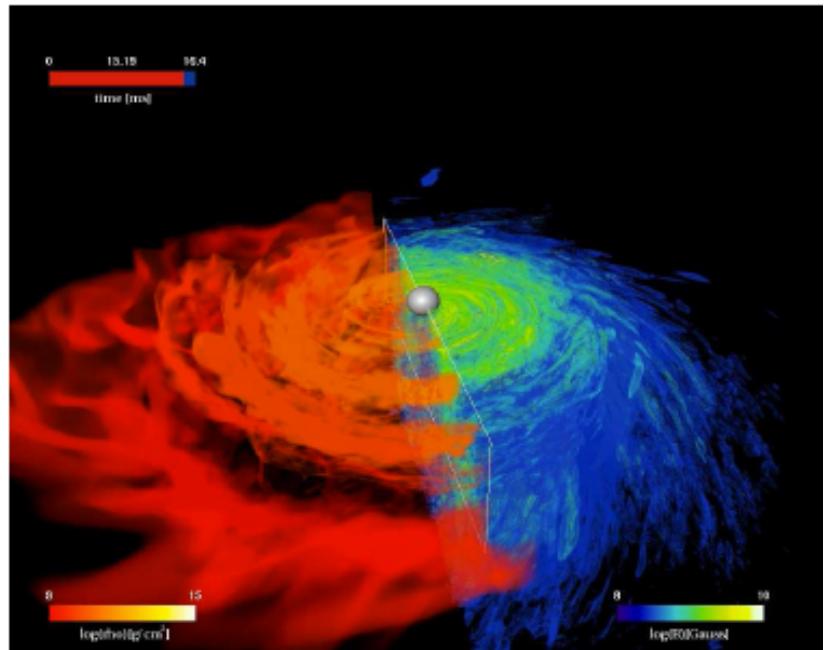


Foucart et al. (Cornell) 2011,13

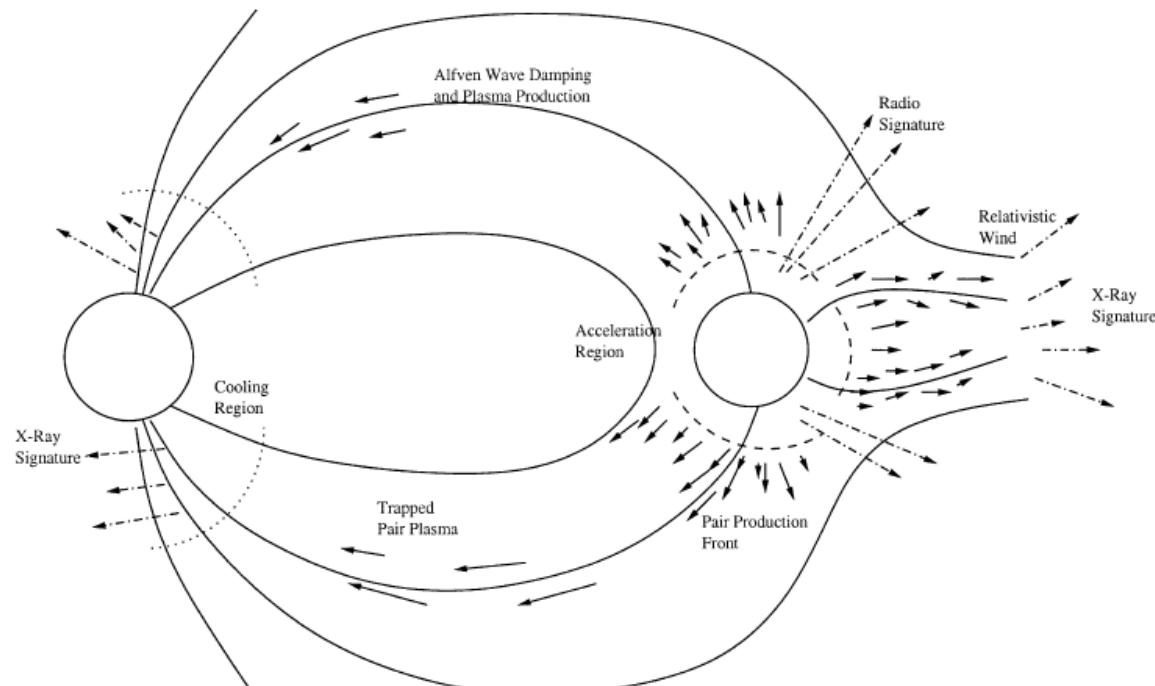
Merger Simulation with B Fields

Giacomazzo,
Rezzolla et al 2011

Palenzuela, Lehner et al. 2013



Merger of Magnetospheres



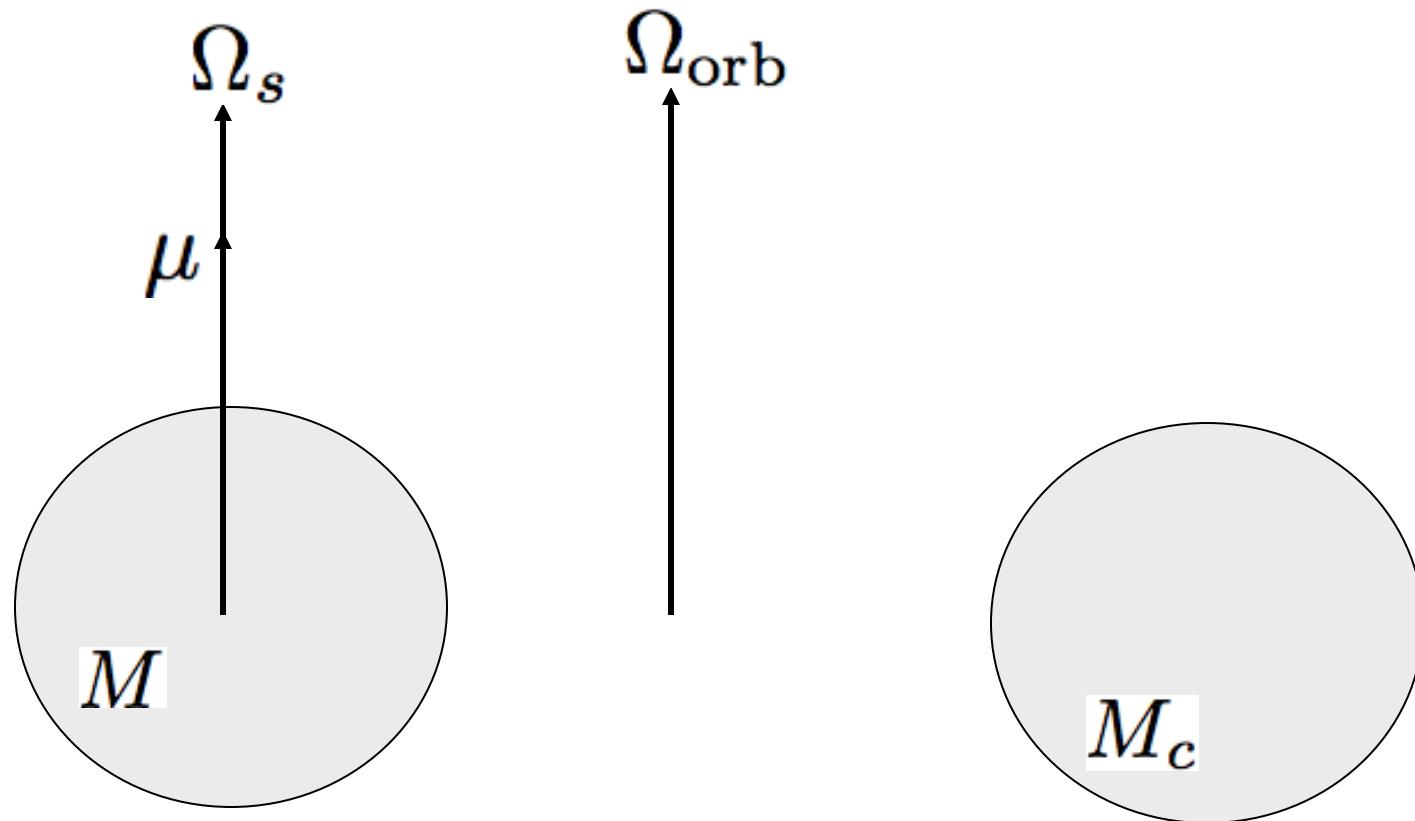
Hansen & Lyutikov 2001

Pre-Merger Phase: Magnetic NSs

Cf. Double Pulsars: PSR J0737-3039

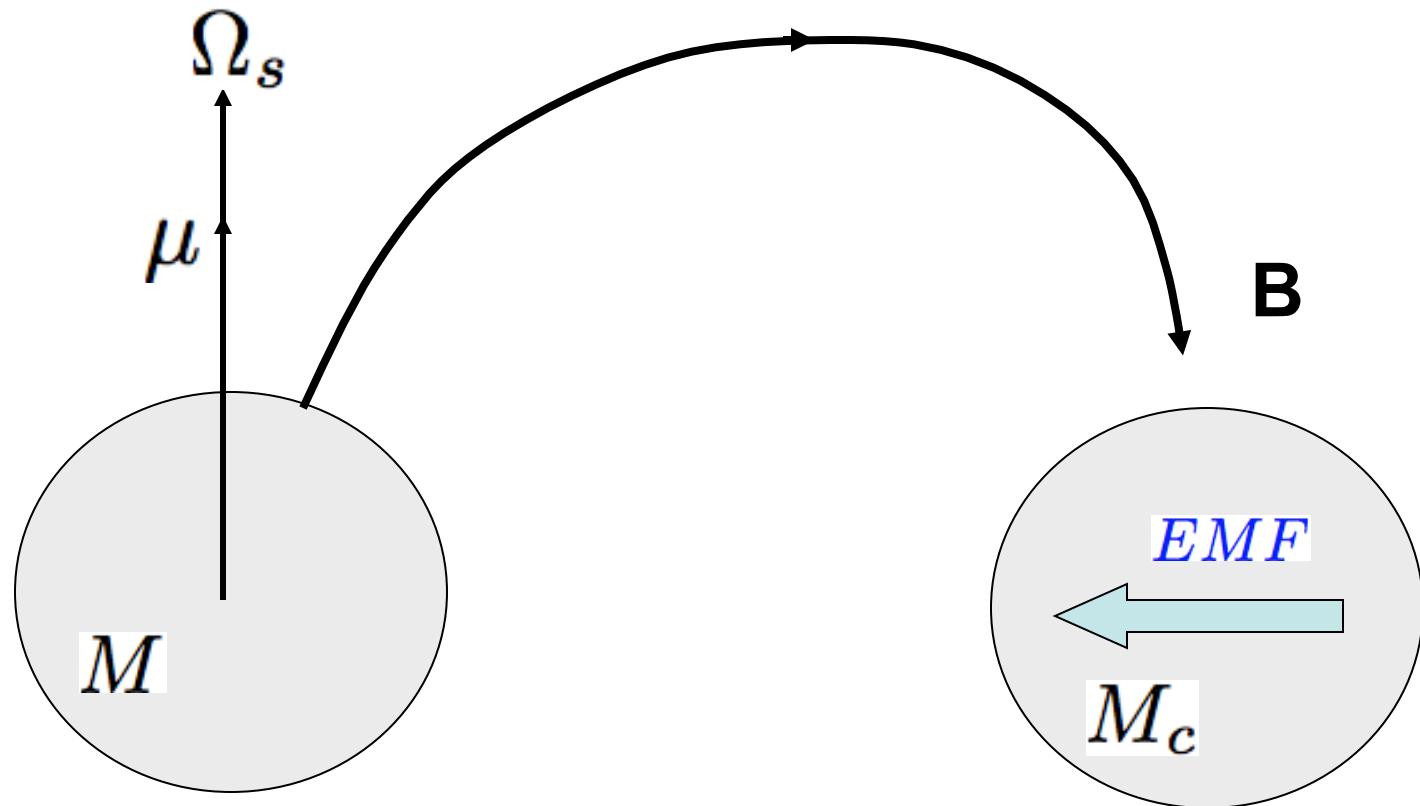
pulsar A: $\sim 10^{10}$ G

pulsar B: \sim a few $\times 10^{12}$ G



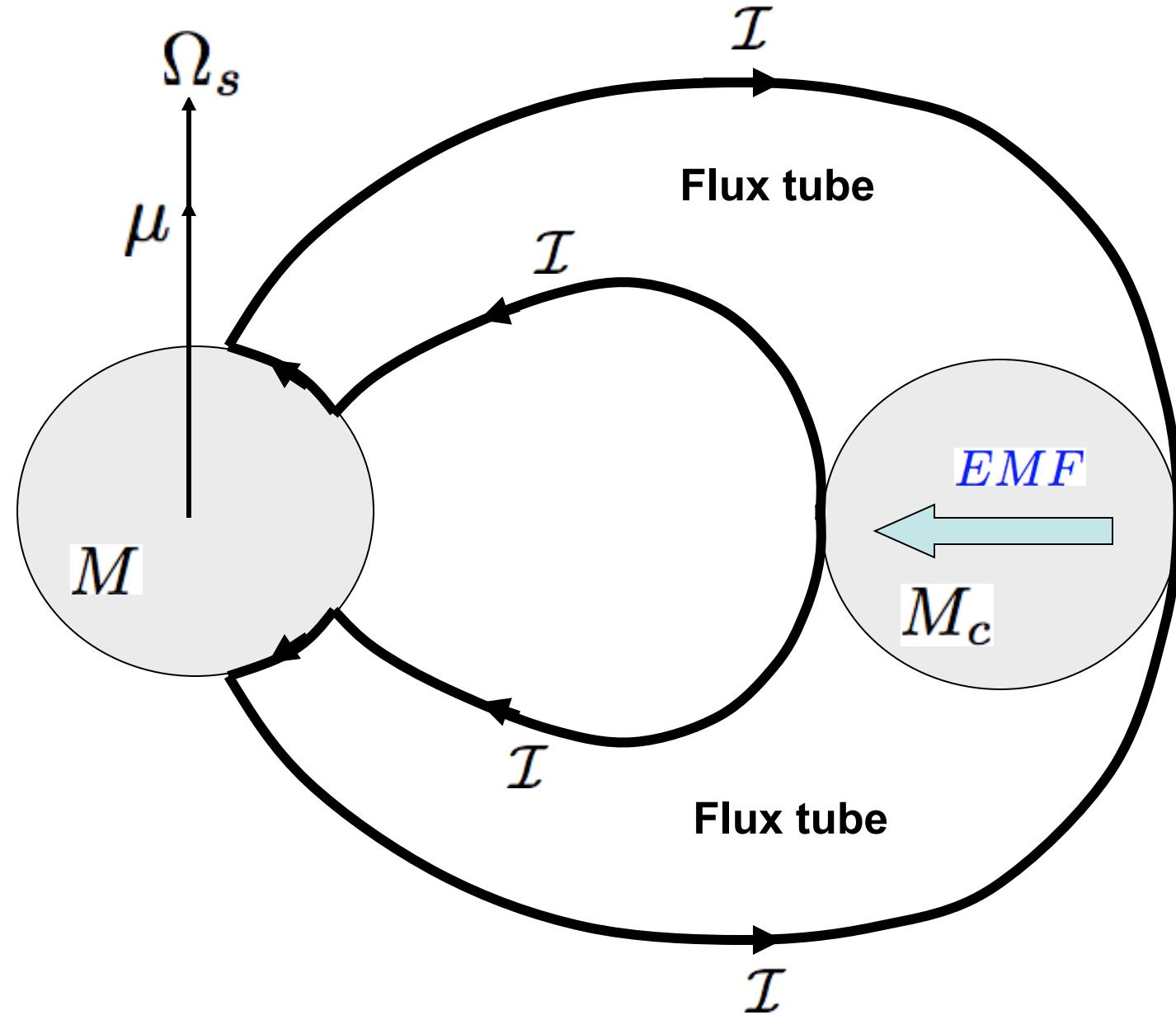
Consider a binary with

- magnetic NS ($>10^{12}$ G) + non-magnetic NS
- embedded in a tenuous plasma (magnetosphere)

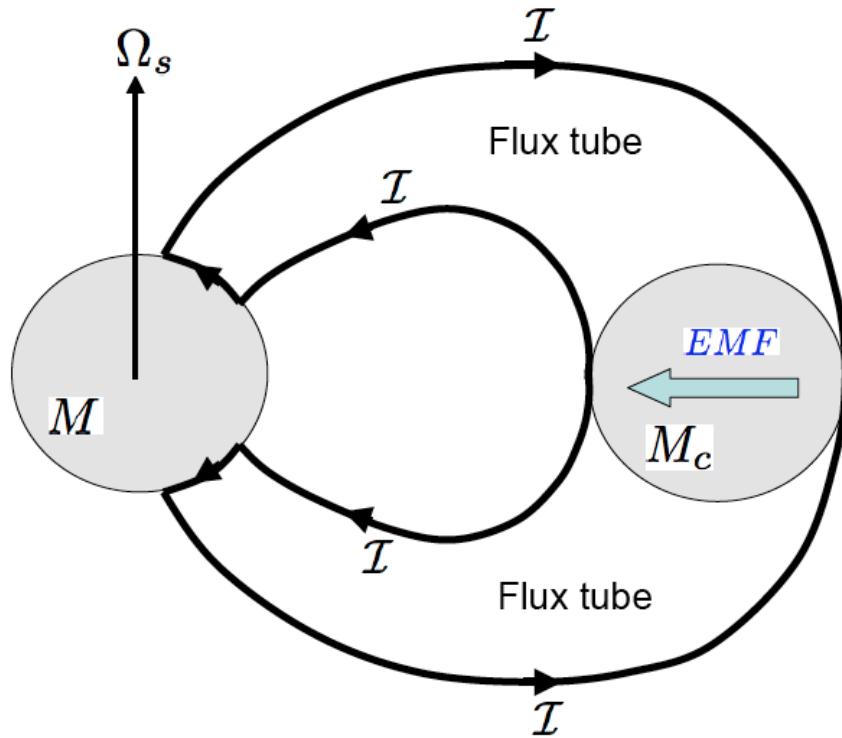


$$\text{EMF} : \Phi = 2R_c \left| \frac{\mathbf{v}}{c} \times \mathbf{B} \right|$$

e.g. $\Phi \sim 10^{13}$ Volt at $f_{\text{orb}} = 20$ Hz



DC Circuit Powered by Orbital Motion



$$\text{EMF} : \Phi = \frac{2\mu R_c}{ca^2}(\Omega_{\text{orb}} - \Omega_s)$$

$$\text{Current} : \mathcal{I} = \frac{\Phi}{\mathcal{R}}$$

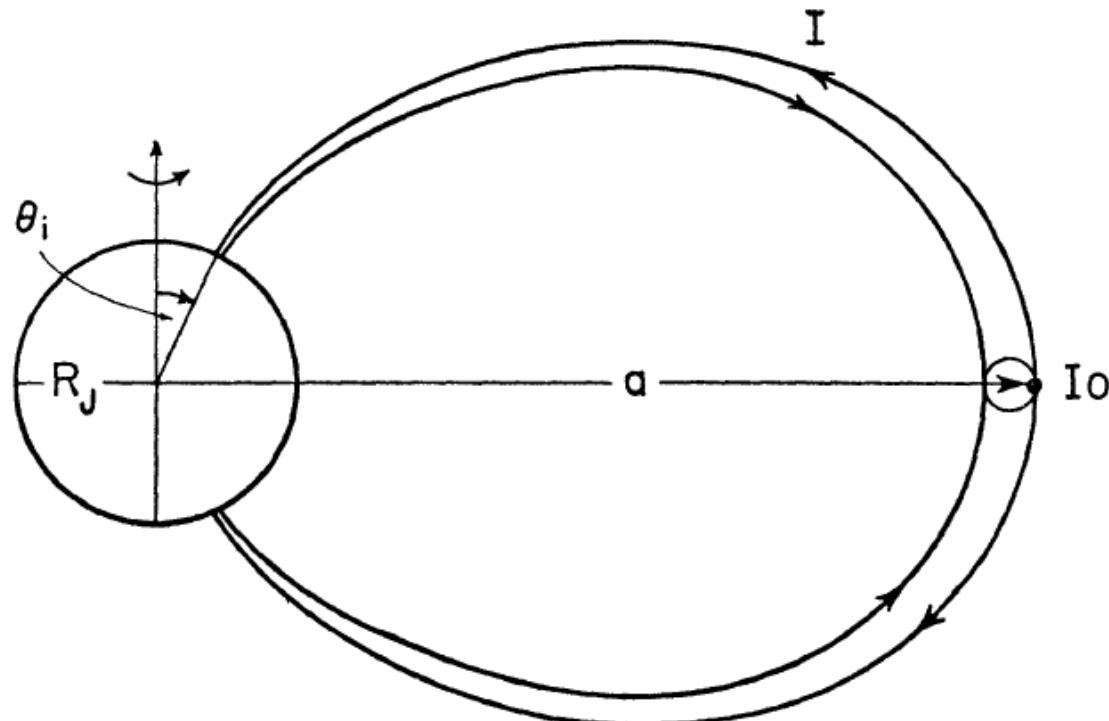
$$\text{Dissipation} : \dot{E}_{\text{diss}} = \frac{\Phi^2}{\mathcal{R}}$$

THE ASTROPHYSICAL JOURNAL, Vol. 156, April 1969
© 1969. The University of Chicago. All rights reserved. Printed in U.S.A.

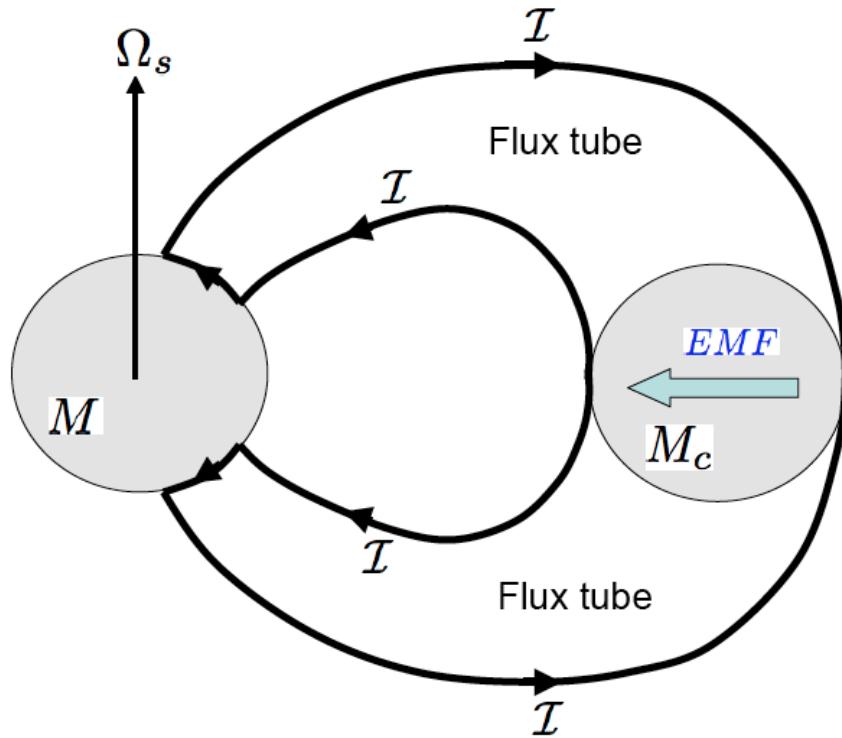
IO, A JOVIAN UNIPOLAR INDUCTOR

PETER GOLDREICH*
California Institute of Technology

AND
DONALD LYNDEN-BELL
Royal Greenwich Observatory



DC Circuit Powered by Orbital Motion



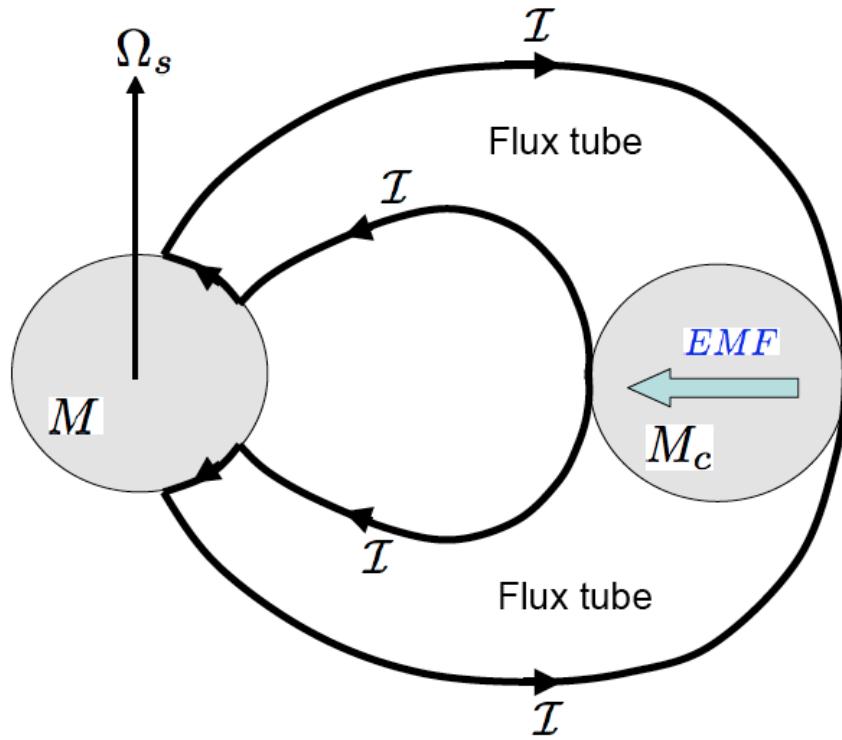
$$\text{EMF} : \Phi = \frac{2\mu R_c}{ca^2}(\Omega_{\text{orb}} - \Omega_s)$$

$$\text{Current} : \mathcal{I} = \frac{\Phi}{\mathcal{R}}$$

$$\text{Dissipation} : \dot{E}_{\text{diss}} = \frac{\Phi^2}{\mathcal{R}}$$

Results depend on the resistance: \mathcal{R}

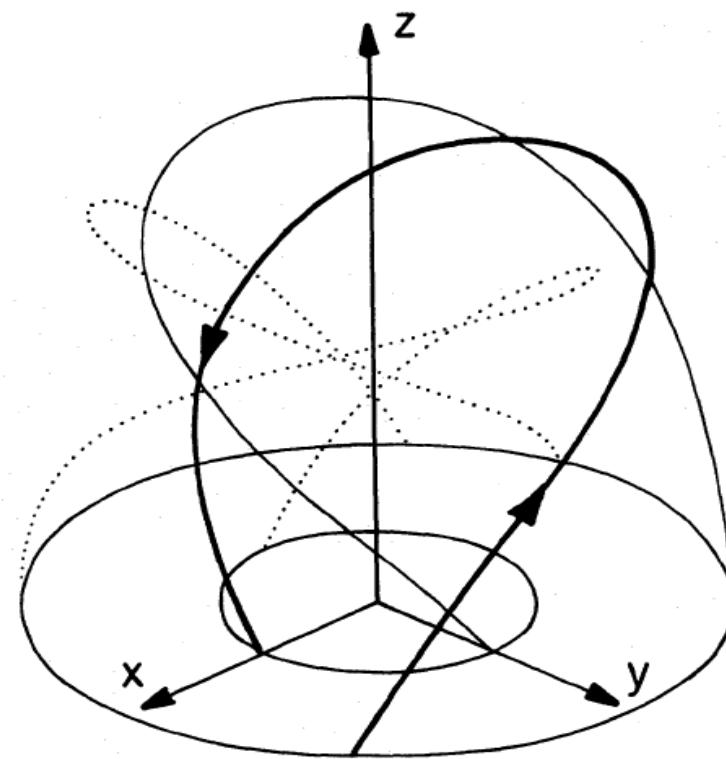
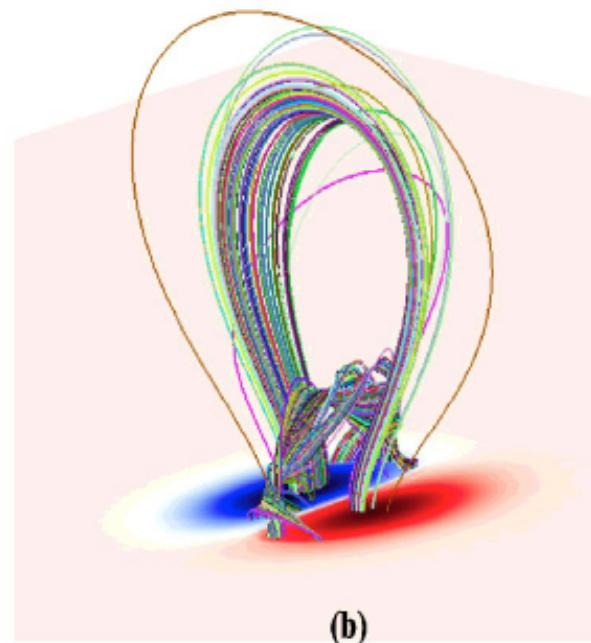
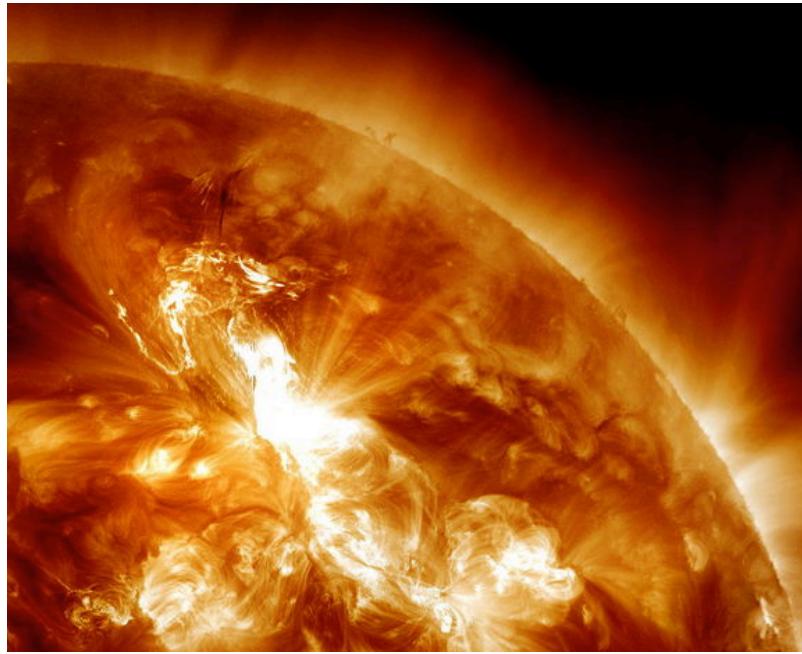
DC Circuit Powered by Orbital Motion



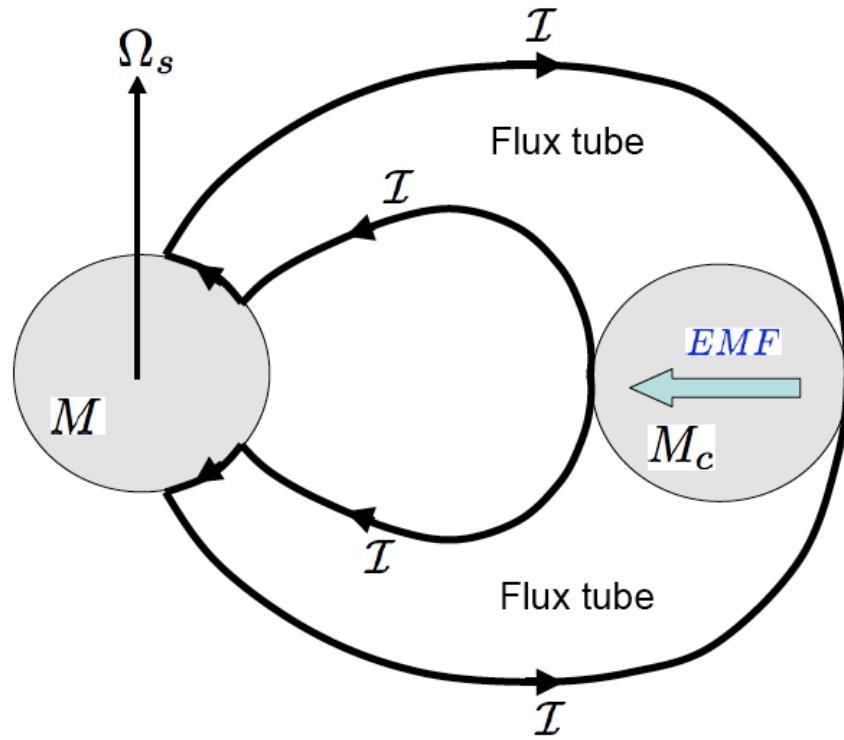
Problems with small \mathcal{R}
(\rightarrow large \mathcal{I}):

Flux tube is twisted

$$\frac{|B_\phi|}{|B_z|} \sim \frac{16 v/c^2}{\mathcal{R}}, \quad v = (\Omega_{\text{orb}} - \Omega_s)a$$



DC Circuit Powered by Orbital Motion



$$\frac{|B_\phi|}{|B_z|} \sim \frac{16v/c^2}{\mathcal{R}}, \quad v = (\Omega_{\text{orb}} - \Omega_s)a$$

Circuit will break when $|B_\phi|/|B_z| \gtrsim 1$

Energy Dissipation in the Magnetosphere of Pre-merging NS Binary

DL 2012

$$\dot{E}_{\max} \simeq 7 \times 10^{44} \left(\frac{B_{\text{NS}}}{10^{13} \text{ G}} \right)^2 \left(\frac{a}{30 \text{ km}} \right)^{-13/2} \text{ erg s}^{-1}$$

Actual dissipation rate:

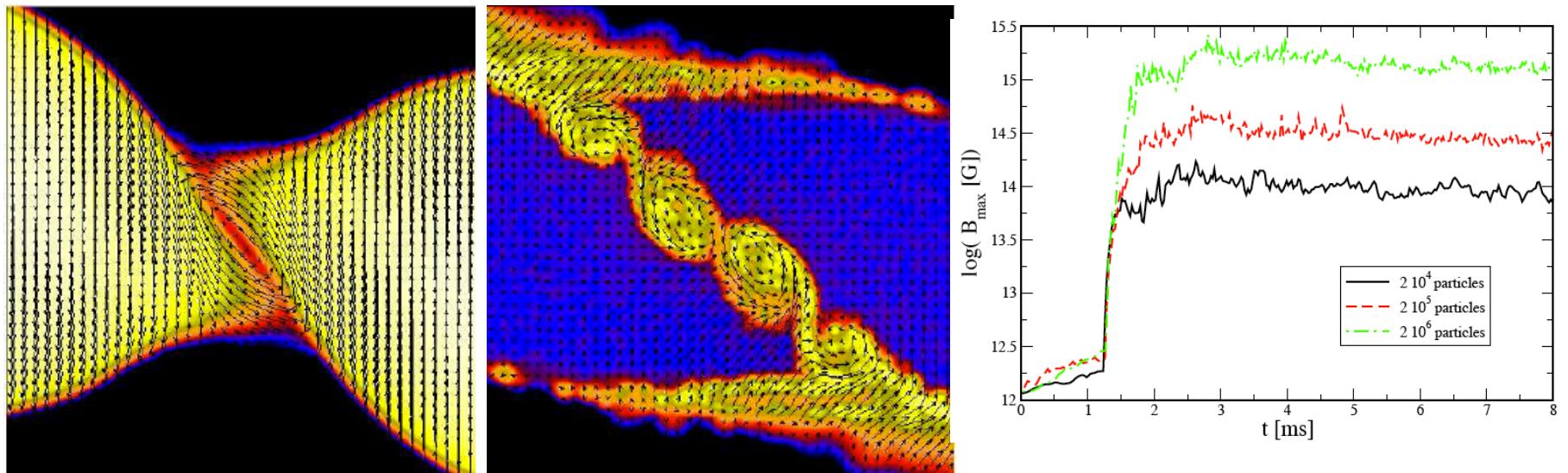
$$\dot{E} \sim 2 \times 10^{44} \left(\frac{B_{\text{NS}}}{10^{13} \text{ G}} \right)^2 \left(\frac{a}{30 \text{ km}} \right)^{-7} \text{ erg s}^{-1}$$

- This Edot will not affect orbital decay rate (GW signal)
- Radio emission prior to binary merger (?) cf. Vietri 96; Hansen & Lyutikov 01

cf. isolated pulsars: $\dot{E} \simeq 10^{33} \left(\frac{B_{\text{NS}}}{10^{13} \text{ G}} \right)^2 \left(\frac{P}{1 \text{ s}} \right)^{-4} \text{ erg s}^{-1}$

During the NS/NS Binary Merger

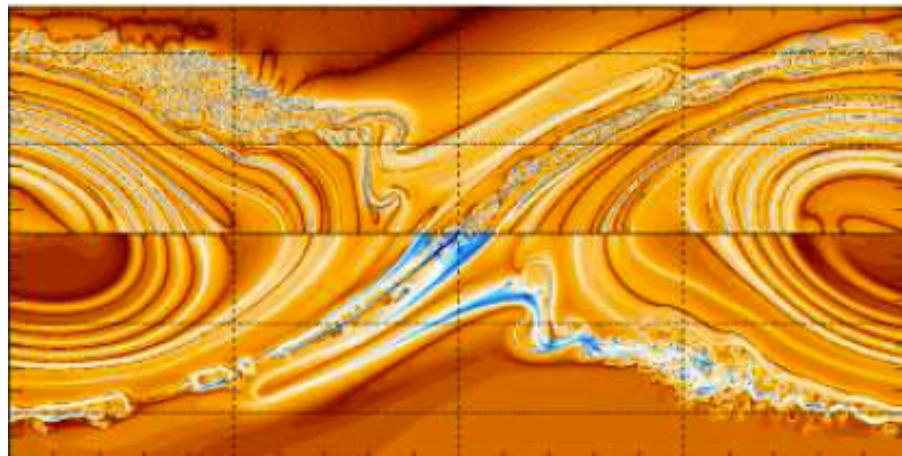
KH instability at interface → Generate strong B field?



Price & Rosswog 2006

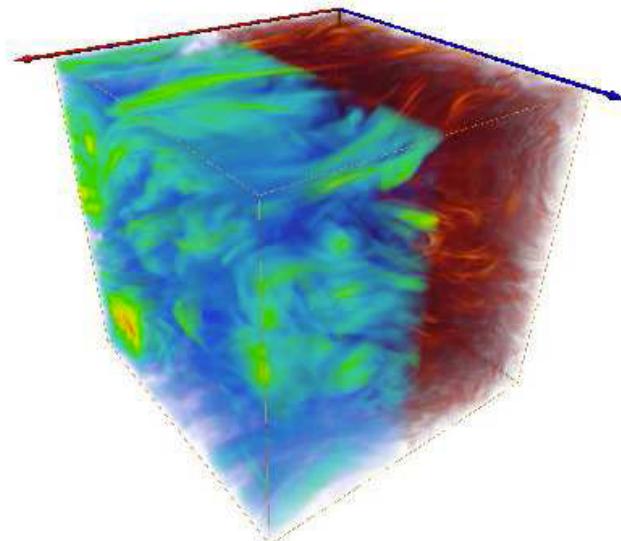
During the NS/NS Binary Merger

KH instability at interface → Generate strong B field?

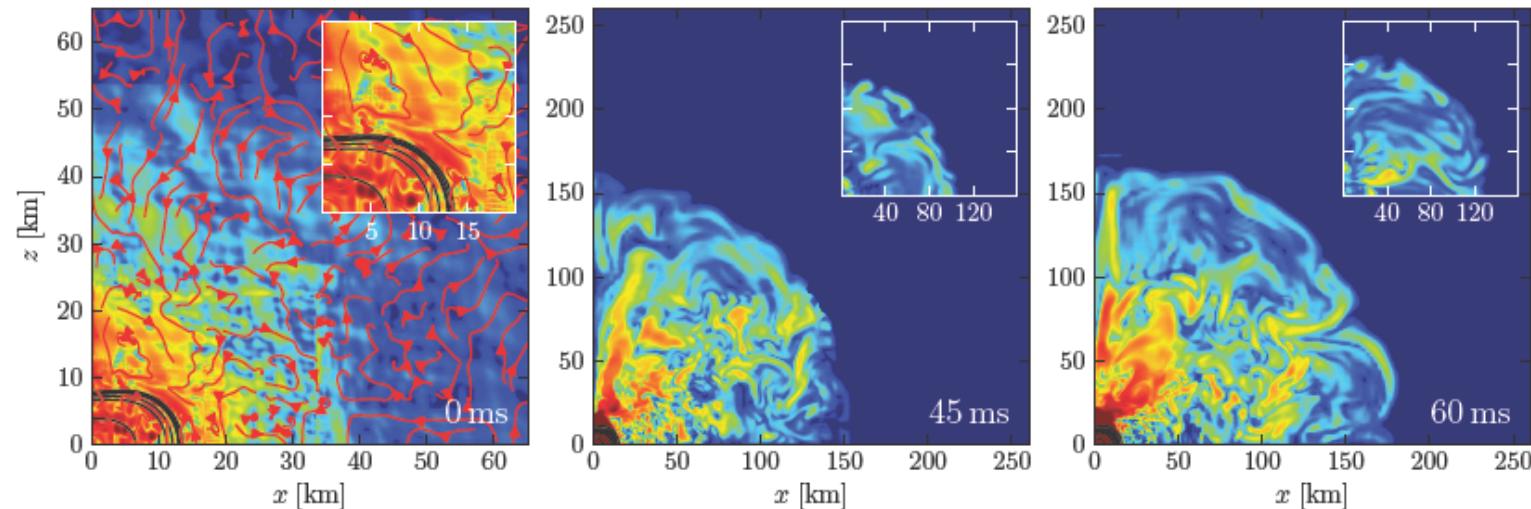


Obergaulinger, Aloy & Muller 2010

→ Yes, but dynamical impact of the field is limited to shear layer



Magnetic Fields in the Merger Remnant



Siegel, Ciolfi & Rezzolla 2014

- Field amplification by differential rotation (MRI resolved?)
- Wind/outflow
- Formation of ms magnetar?

Summary

Physicists' strong magnetic fields:

- Atomic physics
- Condensed matter: Transport property
- Particle physics: QED effects, radiative transfer

Astrophysicists' strong fields:

mostly “classical”, but effects important, interesting and rich

- **Stars & Compact Objects**: Static equilibrium, local equilibrium (crusts), Field evolution, Magnetospheres.
- **Disk**: MAD disks, Jets/outflows, field advection, turbulence
- **Disk accretion onto magnetic stars**: star-disk linkage, QPOs, outflows, misalignments
- **Core collapse and Formation**: MHD supernova, ms magnetars, GRBs
- **Binary Mergers**: pre-merger interaction, interface B-field generation, GRBs

