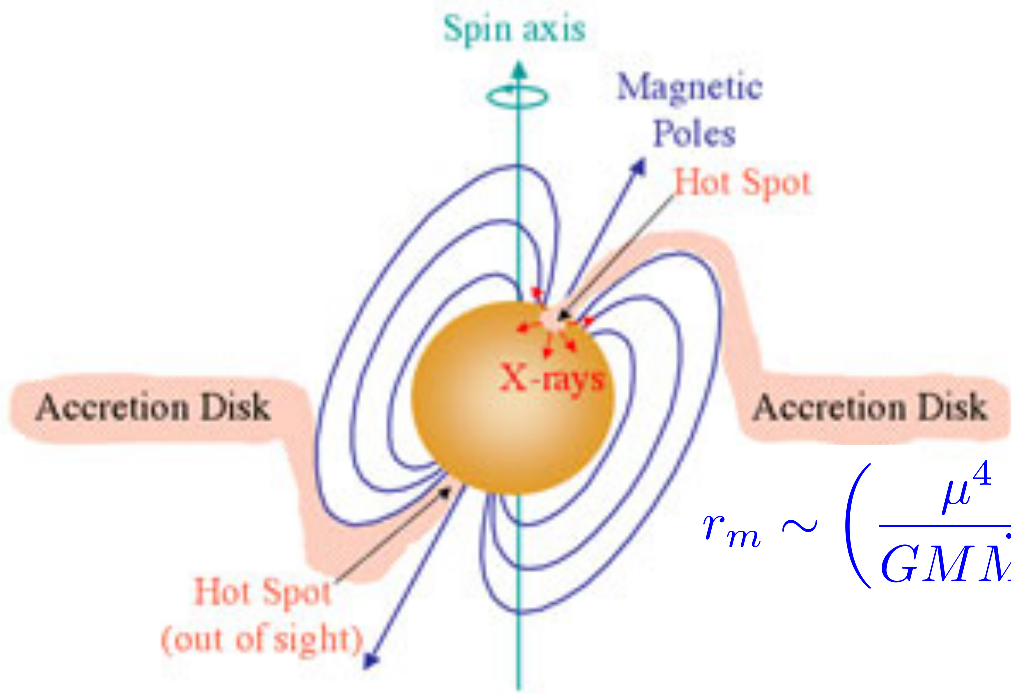
Theory of Disk Accretion onto Magnetic (Compact) Stars

Dong Lai
Cornell University

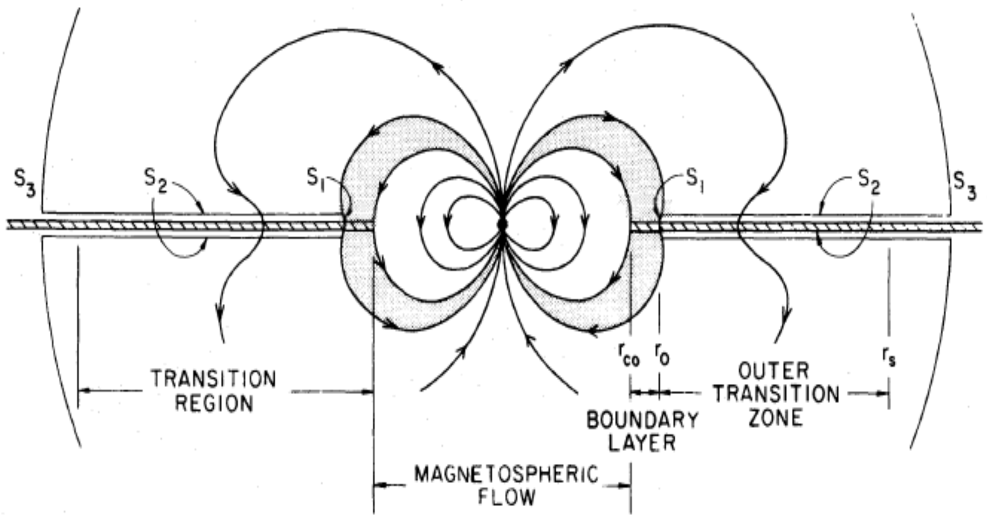
"Physics at the Magnetospheric Boundary", Geneva, 6/25/2013

Magnetosphere - Disk Interaction: Basic Picture

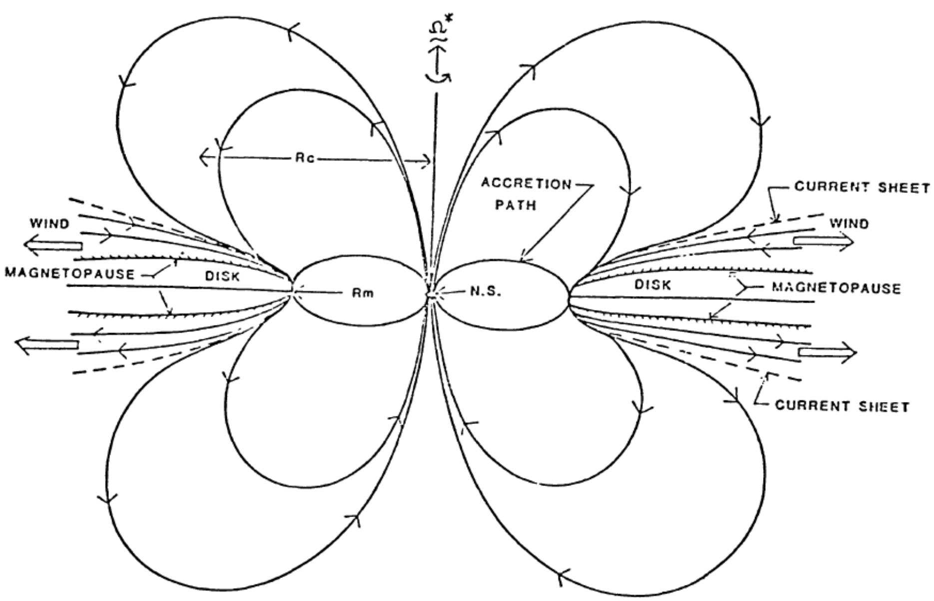


$$r_m \sim \left(\frac{\mu^4}{GM\dot{M}^2}\right)^{1/7}$$

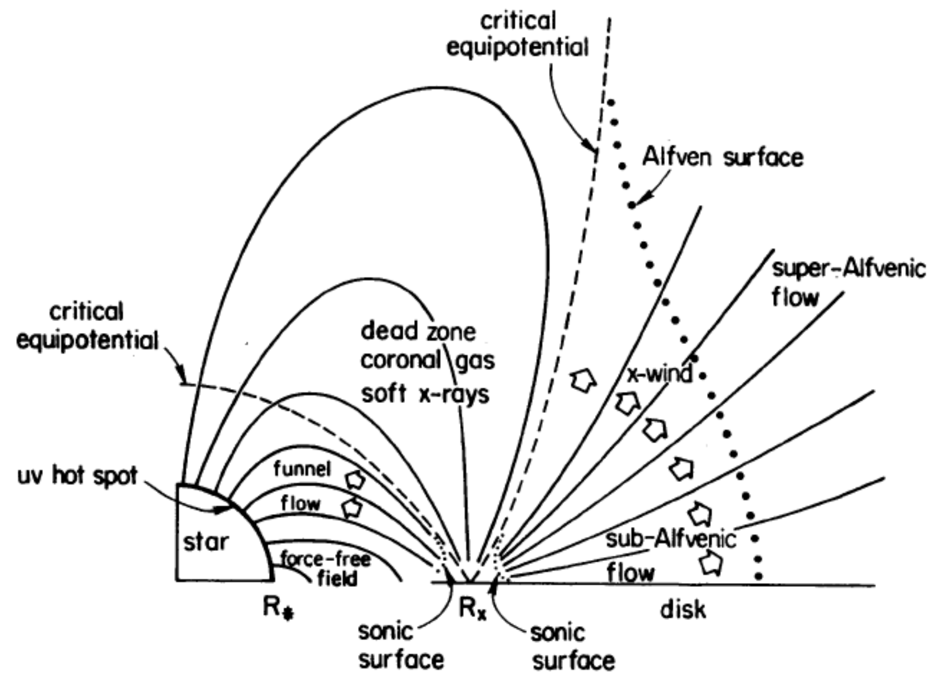
Accreting x-ray pulsars:	$B_\star \sim 10^{12}\mathrm{G}, \quad r_m \sim 10^2 R_\star$
Accreting ms pulsars (LMXBs):	$B_\star \sim 10^{8}\mathrm{G}, \quad r_m \sim (\mathrm{a\ few}) R_\star$
CV's (Intermediate polars):	$B_\star \sim 10^{7}\mathrm{G}, \quad r_m \sim 10 R_\star$
Classical T Tauri stars:	$B_\star \sim 10^{3}\mathrm{G}, \quad r_m \sim (\mathrm{a\ few}) R_\star$



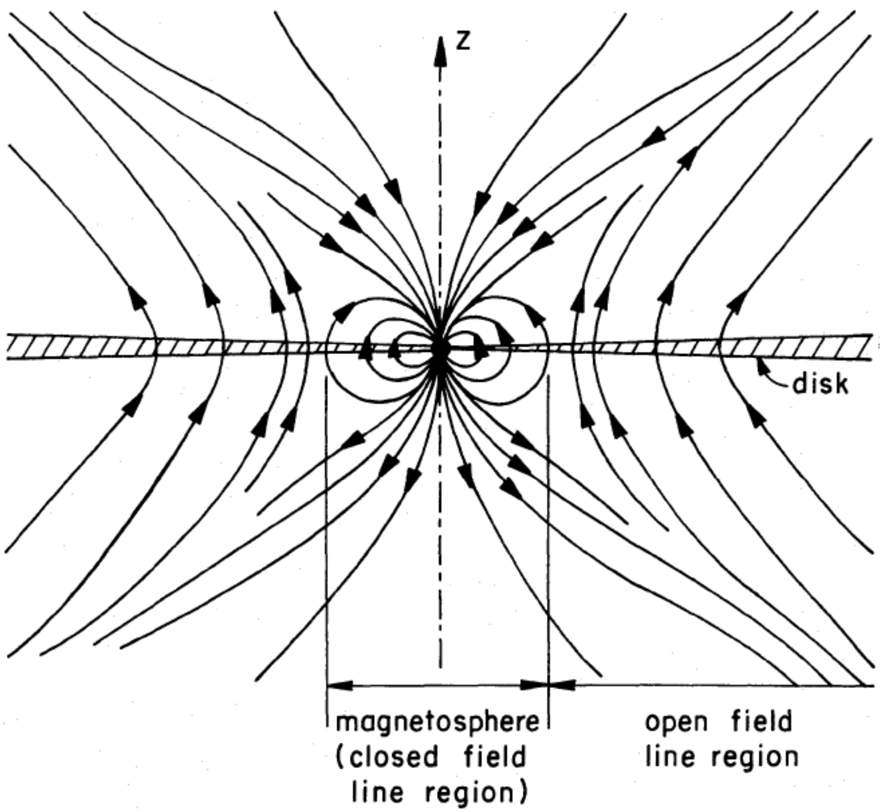
Ghosh & Lamb 1979



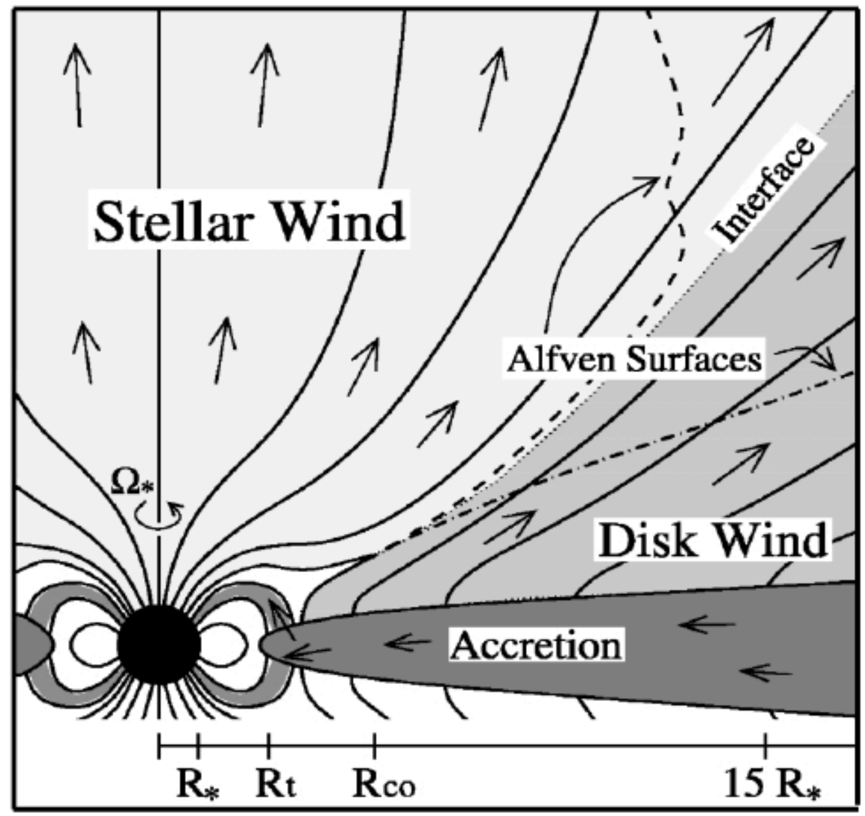
Arons, McKee, Pudritz (Arons 1986)



Shu et al. 1994



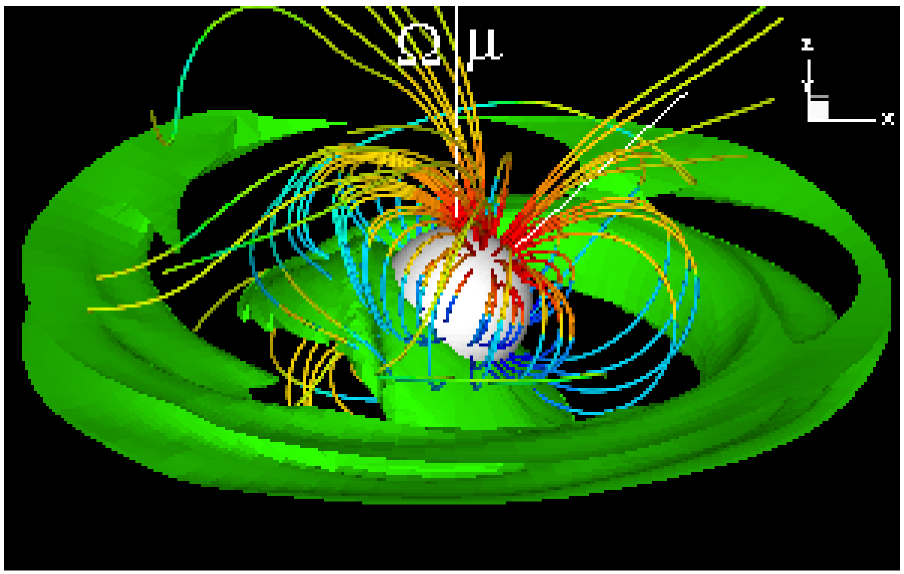
Lovelace et al. 1995



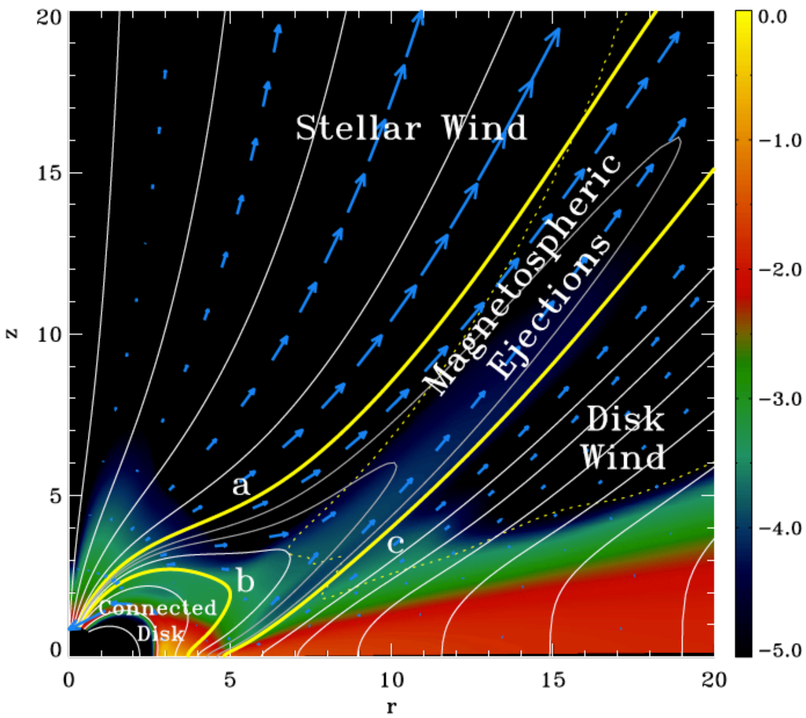
Matt & Pudritz 2005

Simulations…

Hayashi, Shibata & Matsumoto, Miller & Stone, Goodson, Winglee & Bohm, Fendt & Elastner, Matt et al, Romanova, Lovelace, Kulkarni, Long, Lii et al, Zanni & Ferreira, ……



Romanova et al.



Zanni & Ferreira

Issues:
(uncertainties, possible applications…)

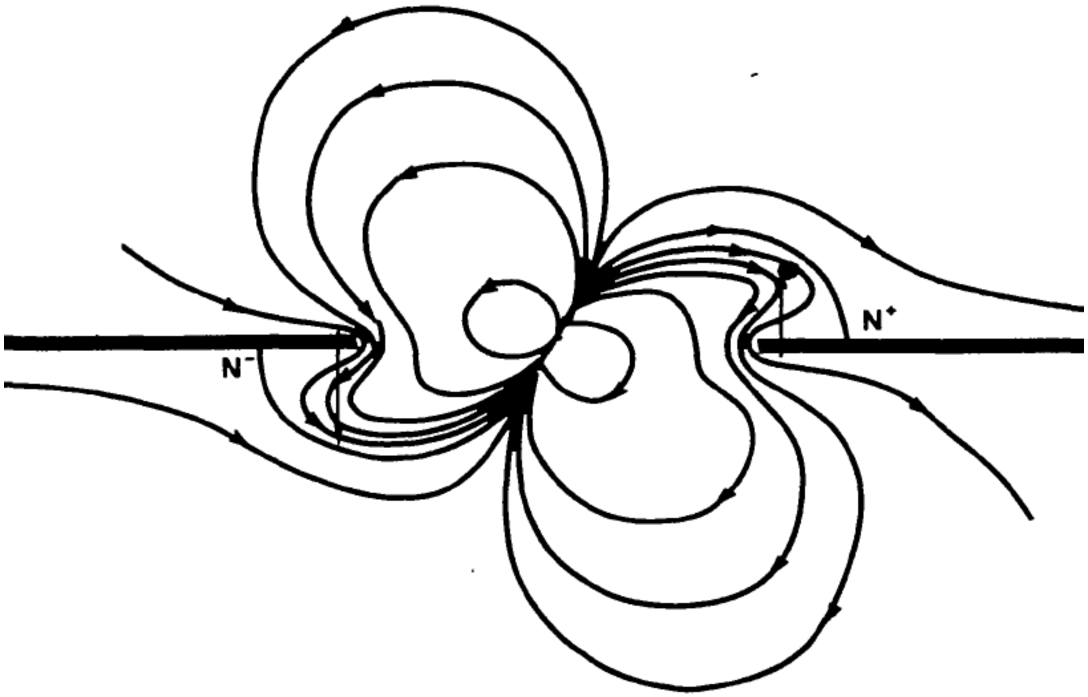
- Magnetosphere boundary vs disk inner radius
- Magnetic linkage between star and disk:
  Width of interaction zone, steady vs cyclic behaviors
- Magnetosphere outflows
- Propeller effect
- Torque on the star: Spinup/spindown
- Misaligned dipole: Effect on disks
- Spin-disk misalignment

Dipole Field Invaded by a Conducting Disk

Disk is a good conductor:
Diffusion of B into disk:

$$t_{\rm diff} \sim \frac{H^2}{\eta} \sim \frac{1}{\alpha\Omega}$$

$$\eta \sim \nu = \alpha H c_s \qquad \text{(MRI)}$$



Exact solution (Aly 1980)

$$B_z(r_{\rm in}) \simeq B_\star(r_{\rm in}) \left(\frac{r_{\rm in}}{H}\right)^{1/2}$$

Istomin's talk

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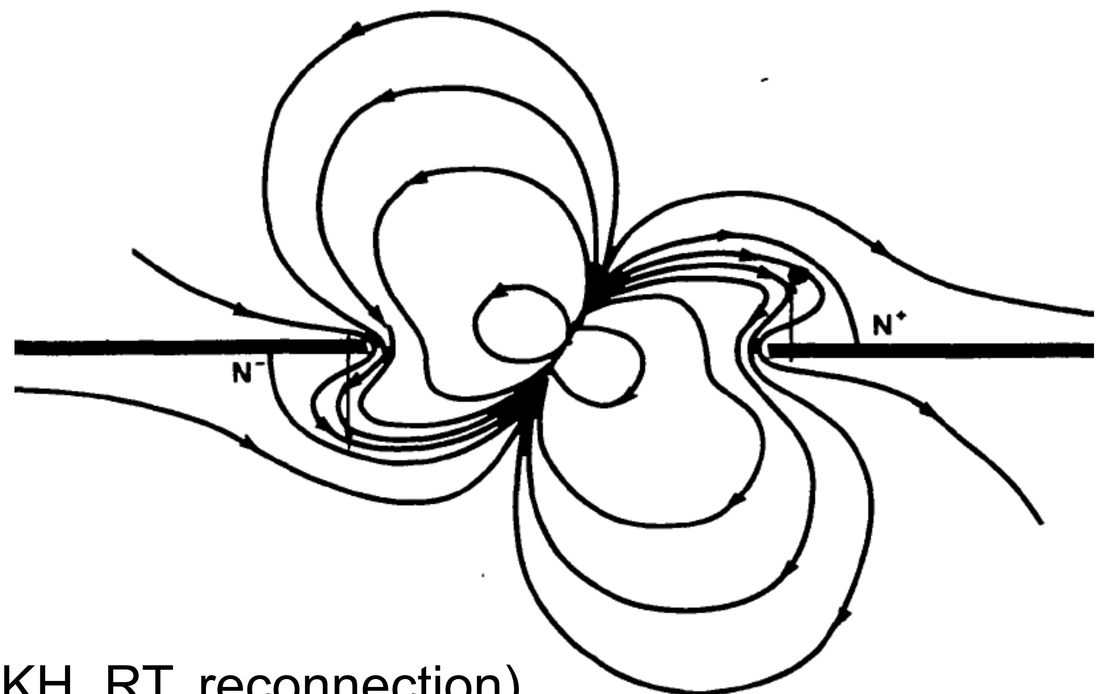


Instabilities at inner edge (KH, RT, reconnection)
➔ **Boundary layer**

Exact solution (Aly 1980)

$$B_z(r_{\rm in}) \simeq B_\star(r_{\rm in}) \left(\frac{r_{\rm in}}{H}\right)^{1/2}$$

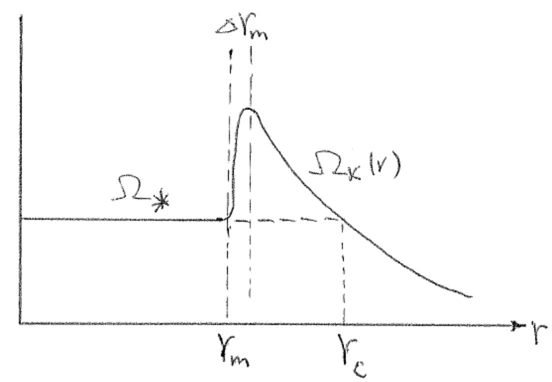
Istomin's talk

Magnetosphere Boundary Layer

$$r_m \to r_m + \Delta r_m$$

Transition from Ω_K to $\Omega_\star$

$$B_{\phi+} = -\zeta B_z$$



Magnetic torque on BL (per unit area)

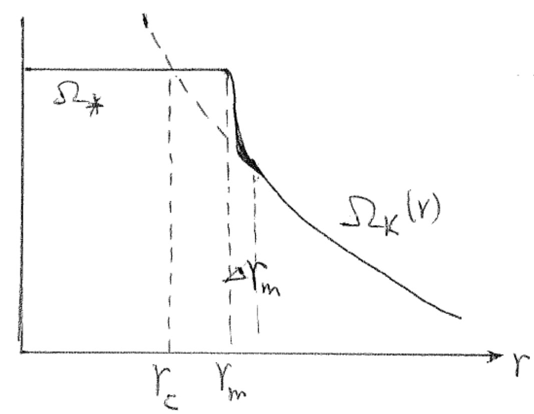
$$r\frac{B_z B_{\phi+}}{2\pi} = \Sigma\frac{\mathrm{d}(r^2\Omega)}{\mathrm{d}t} = \Sigma\, v_r \frac{\partial(r^2\Omega)}{\partial r}$$

$$\Rightarrow \quad r_m \sim \left(\frac{\mu^4}{GM\dot{M}^2}\right)^{1/7}$$

depends on ζ, Δr_m.

Note : For $r_c < r_m$:

$$\Rightarrow \quad r_m \sim \left(\frac{\mu^2}{\Omega_\star \dot{M}}\right)^{1/5}$$



$r_m = r_{in}$?

-- Disk-like structure could exist inside

-- The r_m expression assumes steady state.

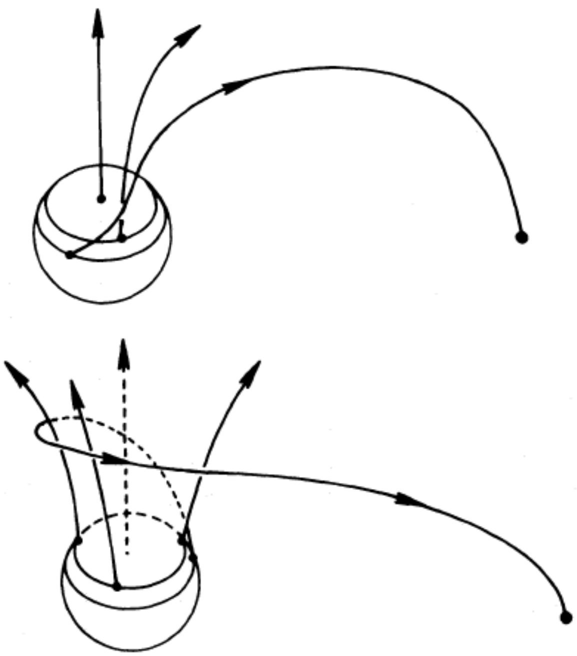
If mass accumulates at boundary ➔ episodic accretion

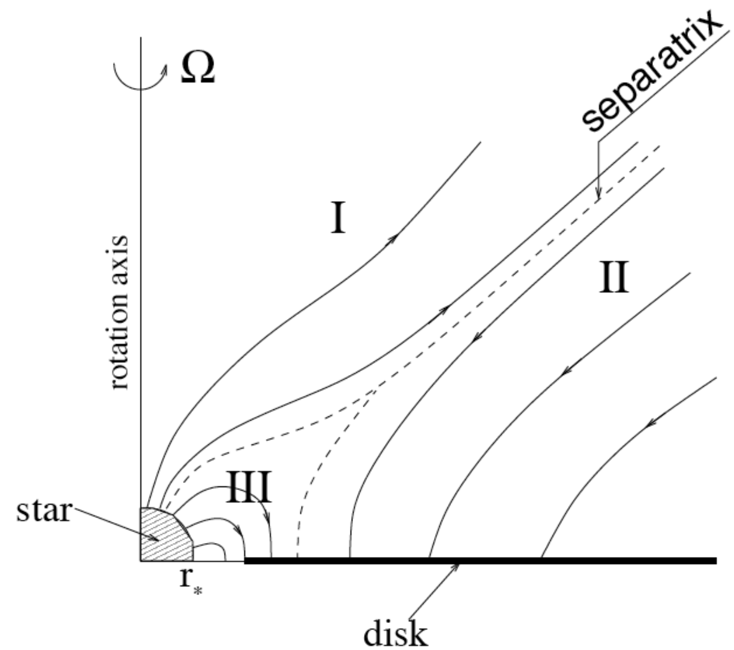
e.g., when Mdot is small, r_m larger than r_c: r_{in} could be << r_m

(Spruit & Taam 1993; D'Angelo & Spruit)

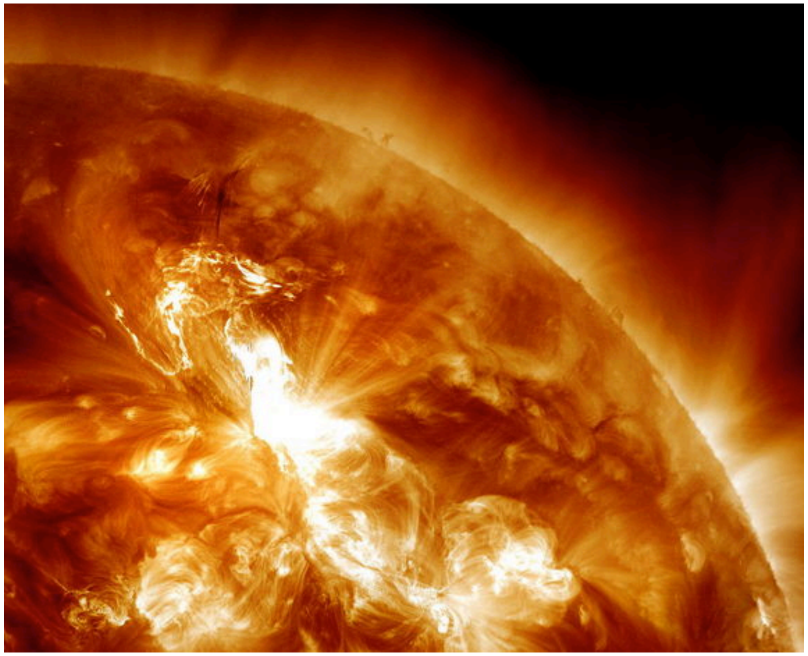
Star-Disk Linkage
(Width, Time-dependence…)

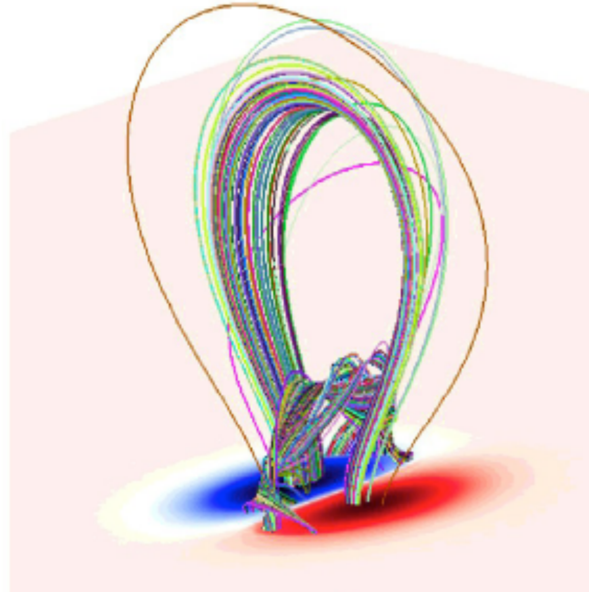
Linked fields are twisted by differential rotation…
➔Field inflates, breaks the linkage



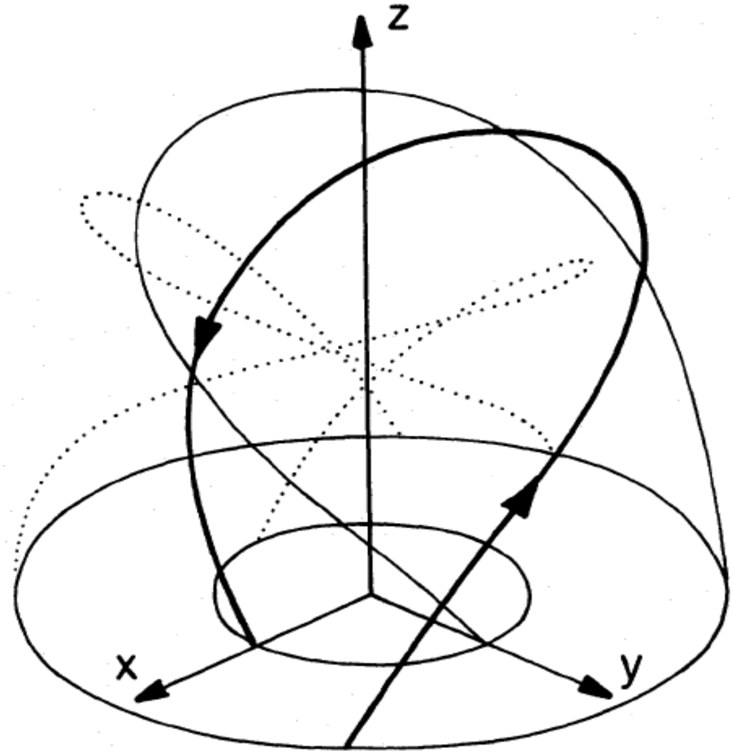


Aly; Lovelace et al.; Uzdensky,…





(b)



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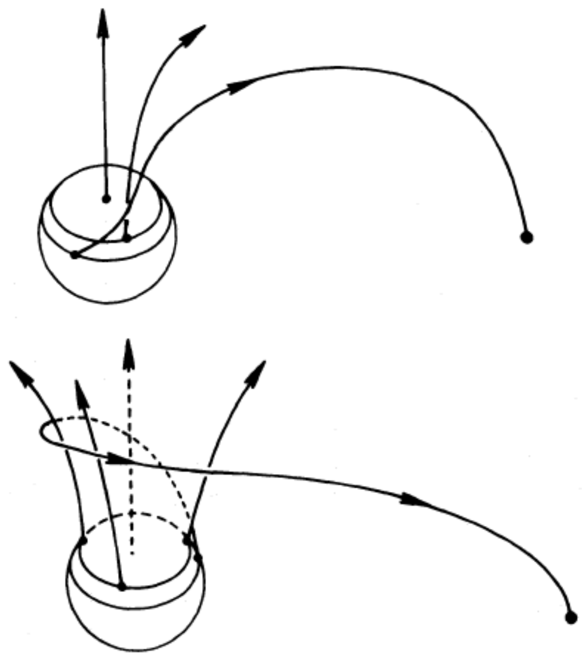
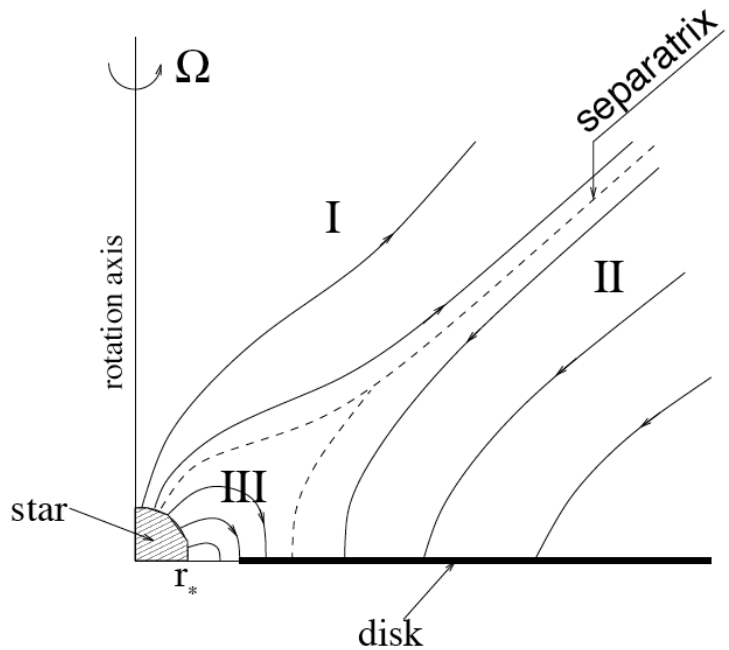




Maximum twist: $\left|\frac{B_{\phi+}}{B_z}\right|_{\max} \sim 1$

Aly; Lovelace et al.; Uzdensky,…

Star-Disk Linkage
(Width, Time-dependence…)

Steady-state?

$$\frac{\partial B_\phi}{\partial t} \sim B_z\left(\Omega - \Omega_s\right) - \frac{B_\phi}{t_{\rm diff}} = 0$$

$$\Rightarrow \left|\frac{B_\phi}{B_z}\right| \sim \left|\Omega - \Omega_s\right| t_{\rm diff}$$

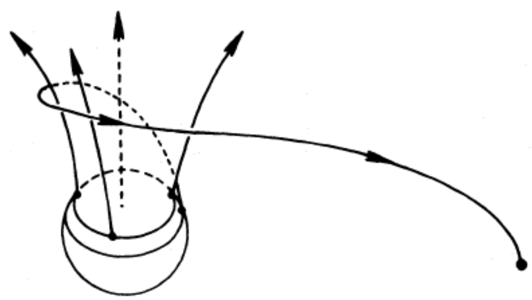
Steady-state linkage possible only very near corotation

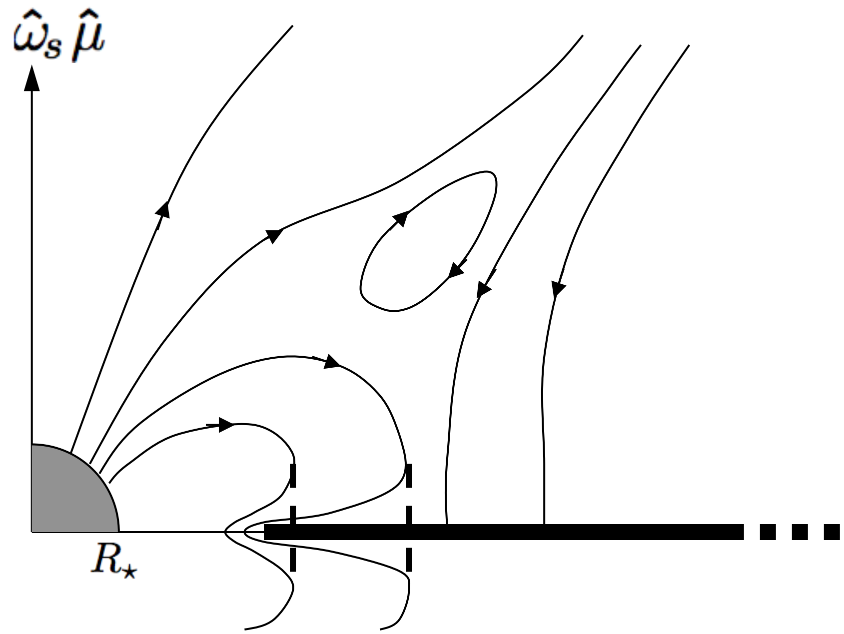
$$\frac{\Delta r}{r_c} \sim \alpha \quad \text{for } \eta = \alpha H c_s$$

Star-Disk Linkage
(Width, Time-dependence…)

Quasi-cyclic behavior

Stellar field penetrates the inner region of disk;
Field lines linking star and disk are twisted --> toroidal field --> field inflation
Reconnection of inflated fields restore linkage





Inevitable…

Star-Disk Linkage
(Width, Time-dependence…)

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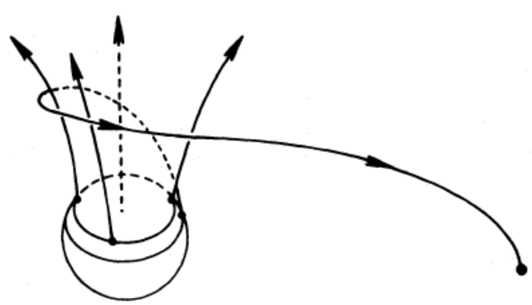
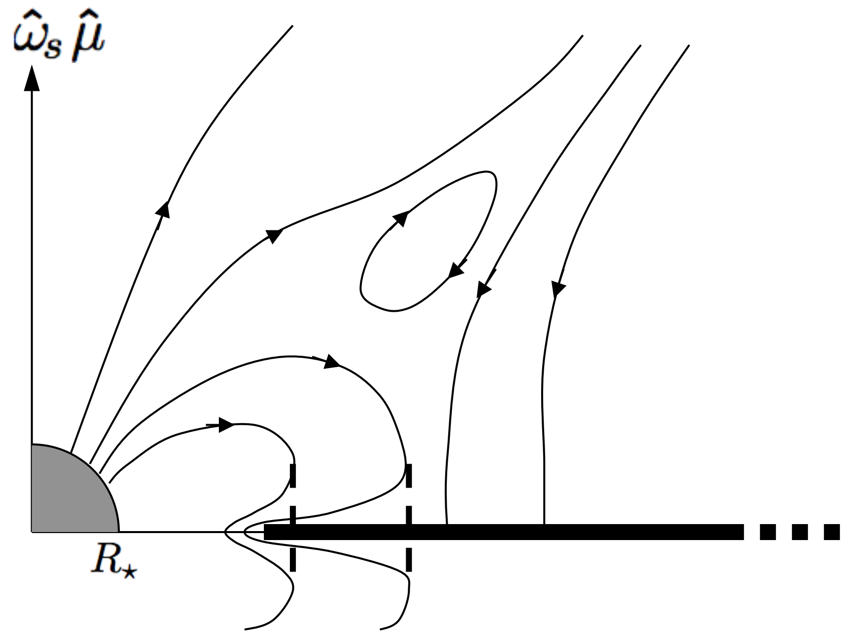
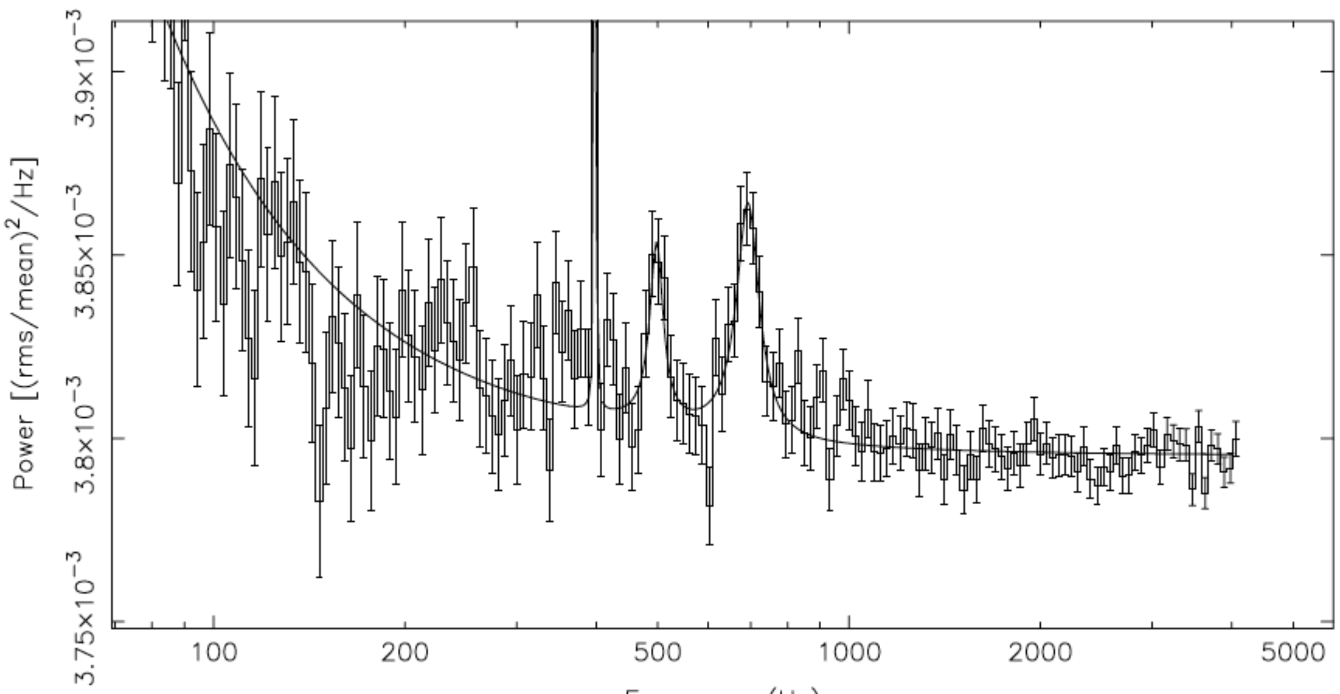




QUESTION: Connection with QPOs in LMXBs (and other systems) ?

Quasi-Periodic Oscillations (QPOs)

Power density spectrum of x-ray flux variations of accreting millisecond pulsars



Van der Klis 2005

Star-Disk Linkage
(Width, Time-dependence…)

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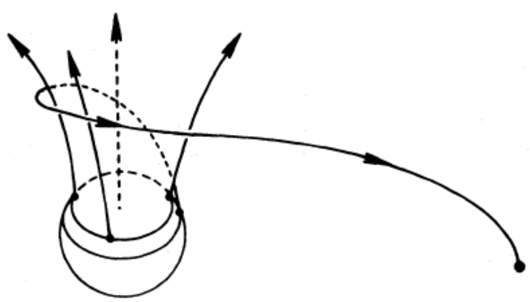
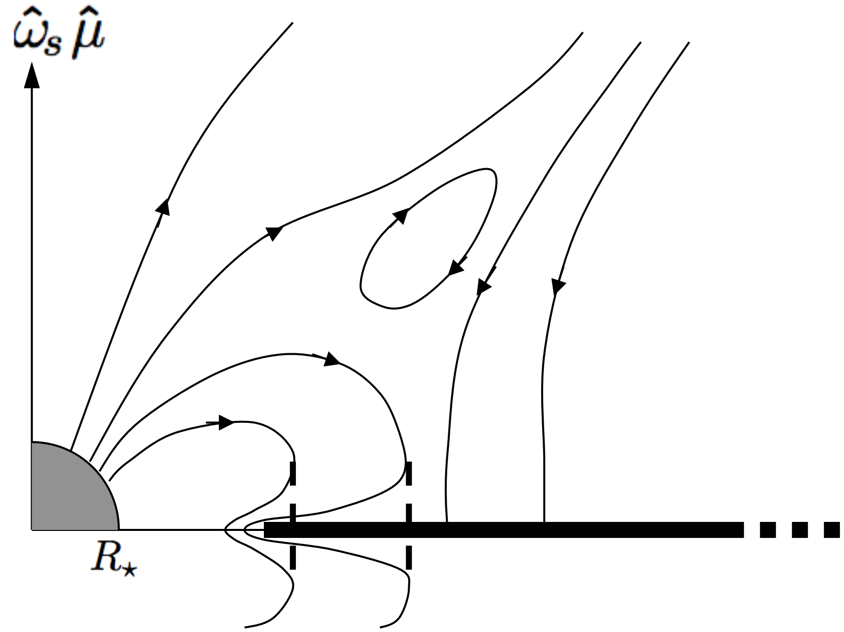
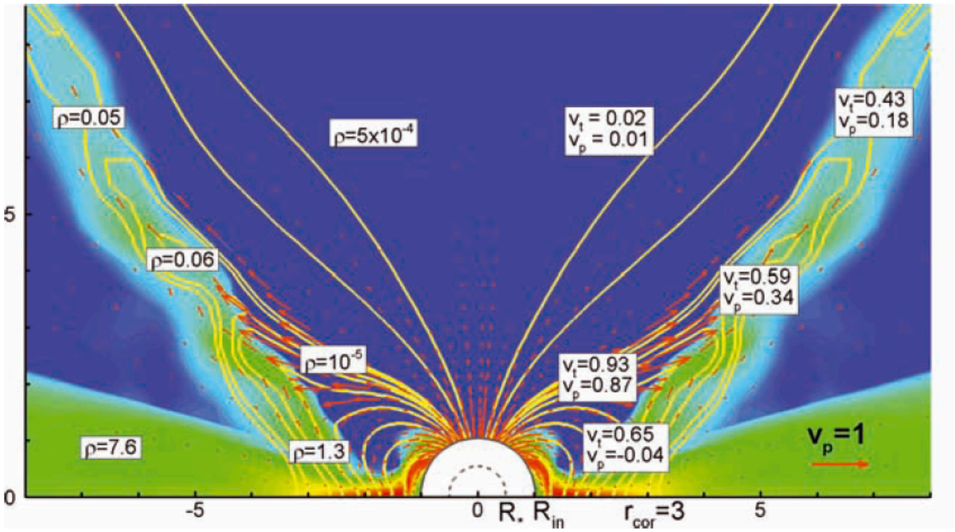


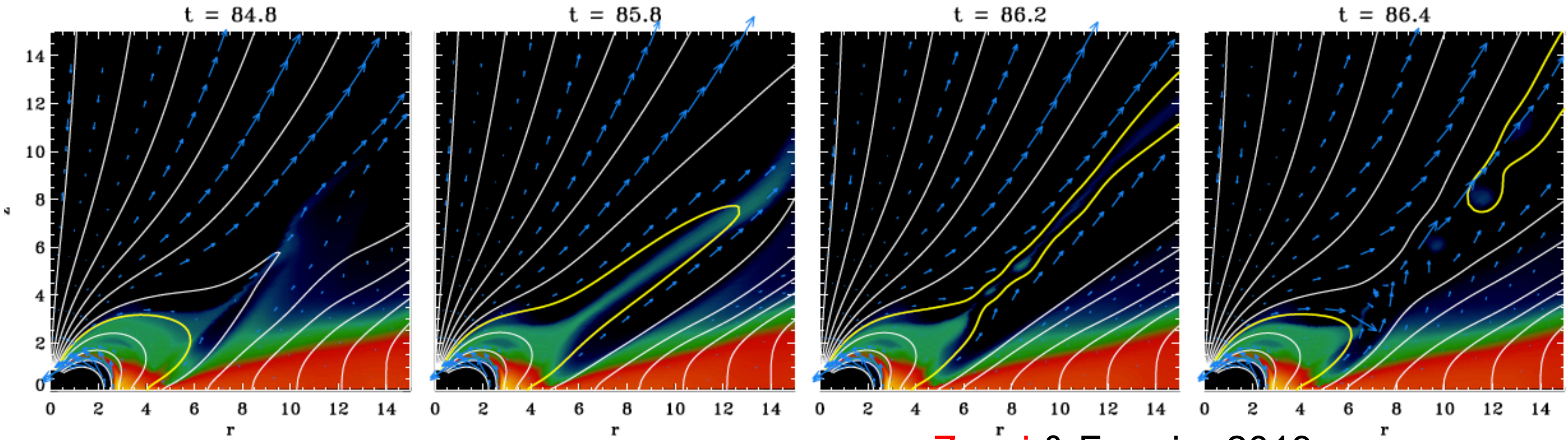


QUESTION: Episodic outflow (X-wind?)… Connection with observations?

Ejection from Magnetospheric Boundary

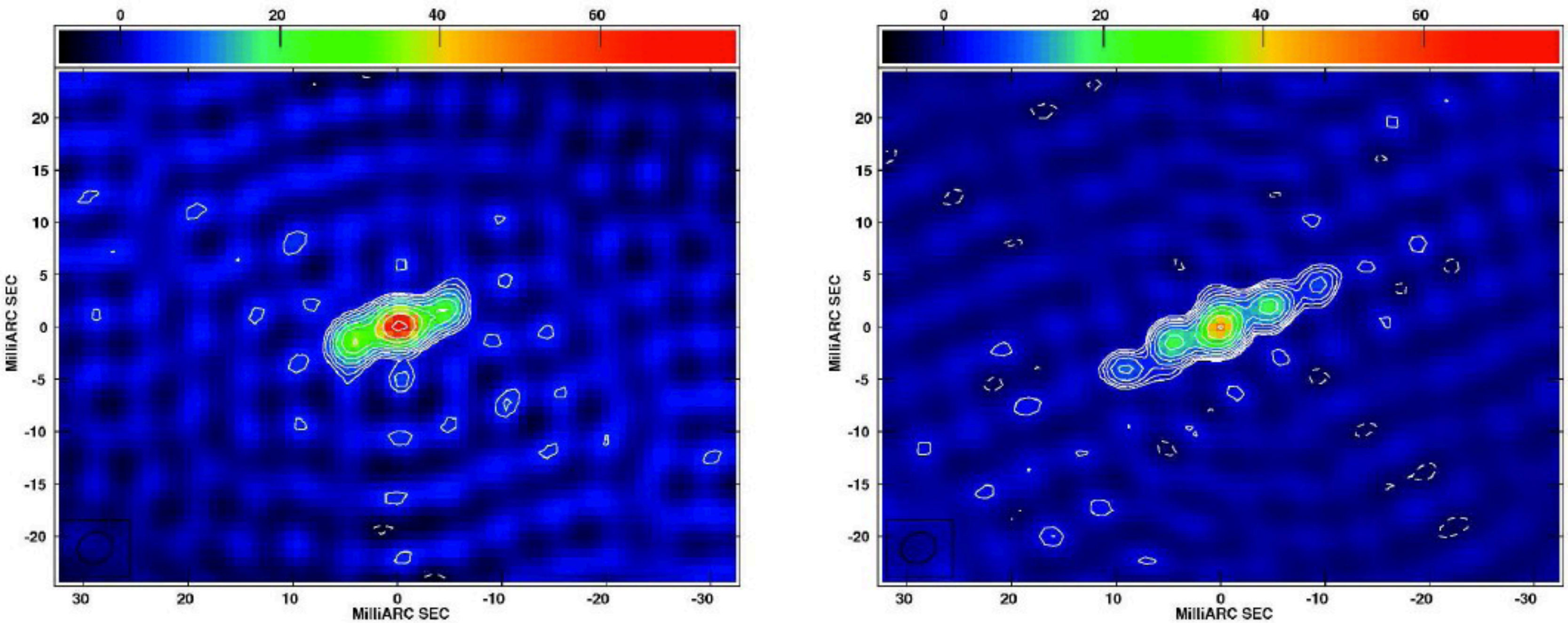


Romanova et al. 2009



Zanni & Ferreira 2013

Jets from Accreting Neutron Stars



Cir X-1: Miller-Jones et al.2011

Jets observed in

-- Atolls (Aql X-1), Z-sources (Sco X-1), AMXPs (SAX J1808)

-- In soft x-ray (banana) states (Ser X-1, 4U1820-30) (Migliari et al. 2011)

(cf. BHs: no jet in thermal state)

Star-Disk Linkage
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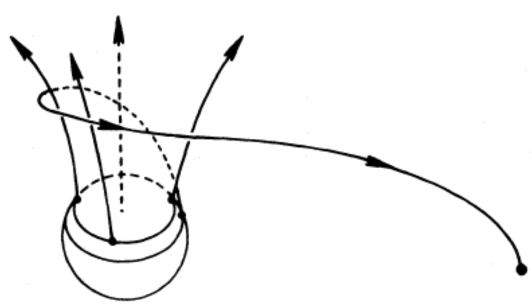
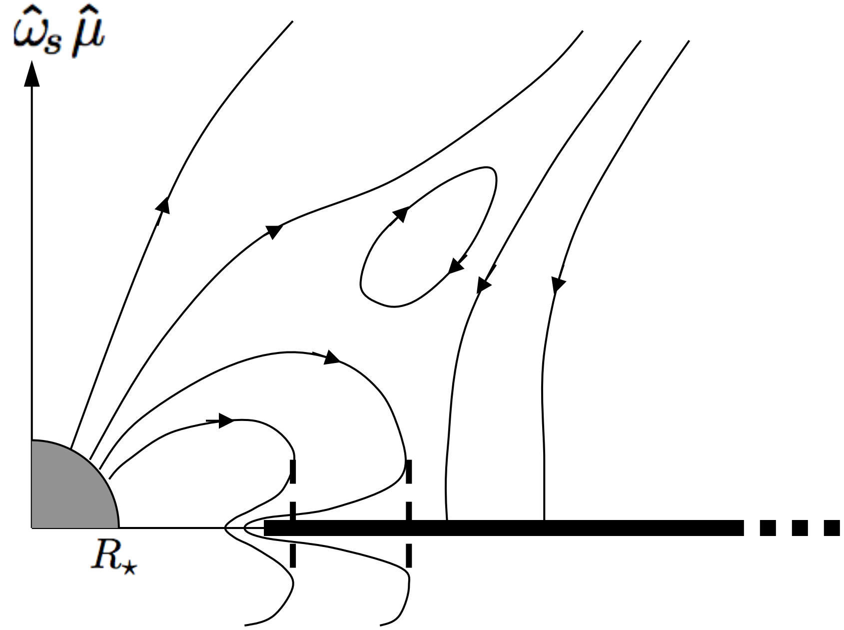
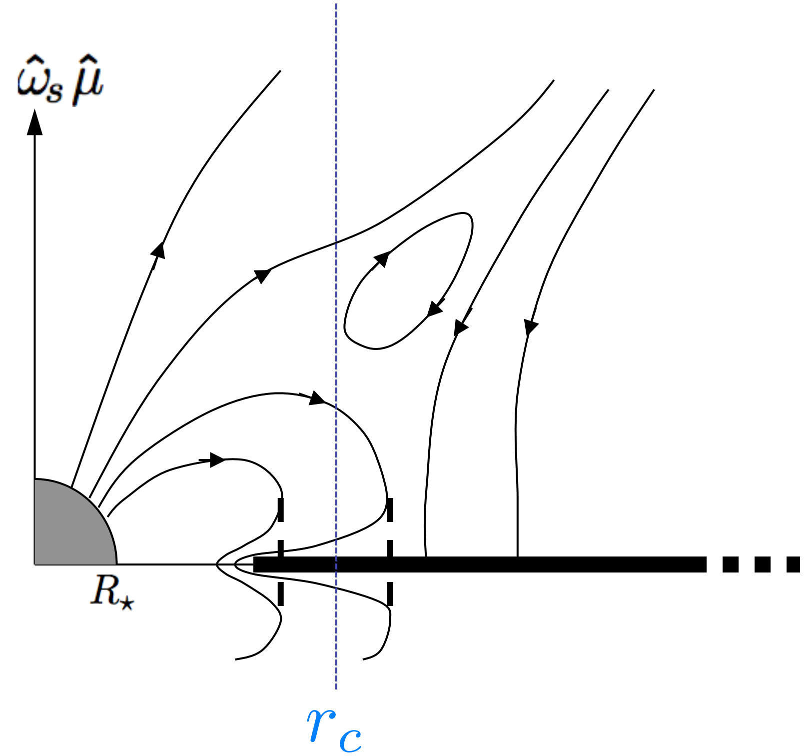




QUESTION: On average, what is the width of the linked region? Δr

Torque on Star



$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\rm acc} r_m^2 \Omega(r_m) + T_m$$

$$T_m \simeq -\left(r^2 B_z B_{\phi+}\right)_{r_m} \Delta r = \zeta \left(r^2 B_z^2\right)_{r_m} \Delta r \quad \text{for } B_{\phi+} = -\zeta B_z$$

Note:

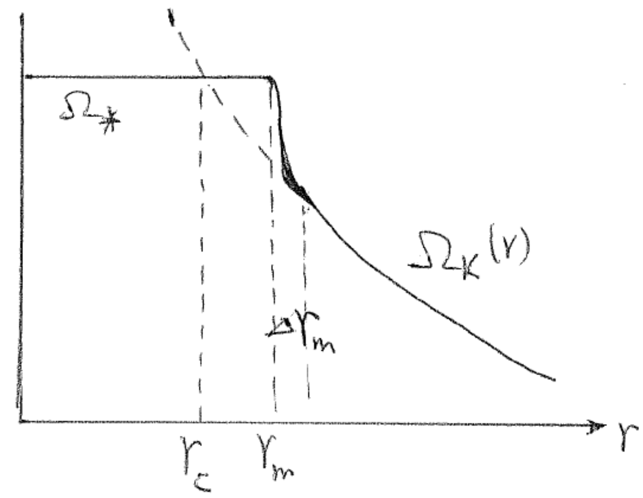
$|T_m| \sim \dot{M} r_m^2 \Omega(r_m)$ if $\Delta r \sim$ width of BL

$|\zeta| \sim 1$: $\zeta > 0$ when $r < r_c$ and $\zeta < 0$ when $r > r_c$

Propeller Effect

When $r_m \Omega_\star > v_{\rm esc}(r_m)$ or $r_m > 2^{1/3} r_c$

accreted plasma in atmosphere **may** be ejected



Torque on star:

$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\rm acc} r_m^2 \Omega(r_m) + T_m - \dot{M}_{\rm eject} r_m^2 \Omega_\star$$

Lii's talk

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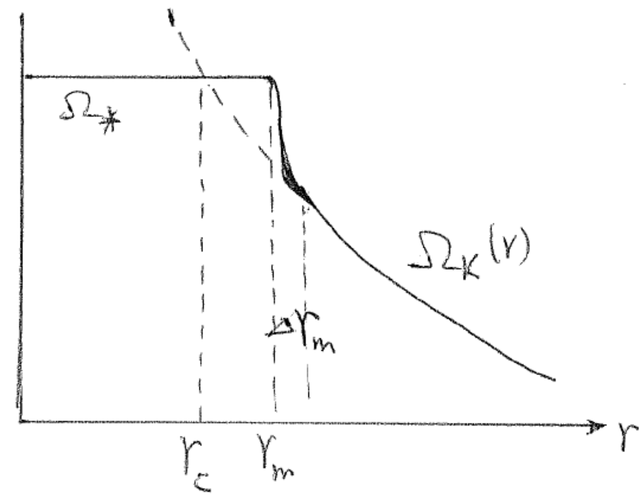

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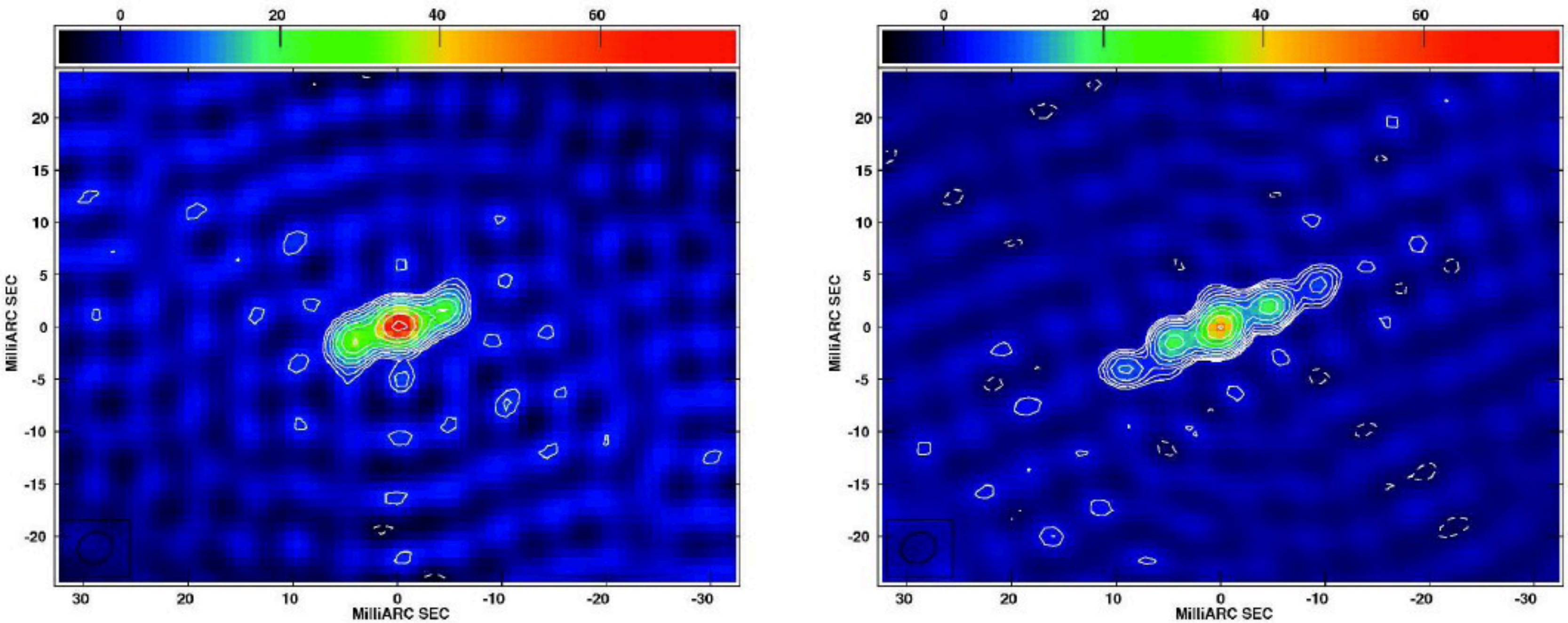
Question:

Propeller outflow/jet: Connection with observations?

(cf. boundary layer ejection)

Jets from Accreting Neutron Stars



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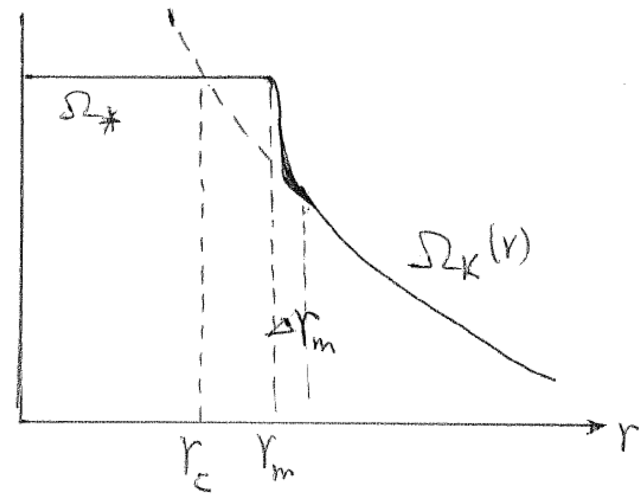
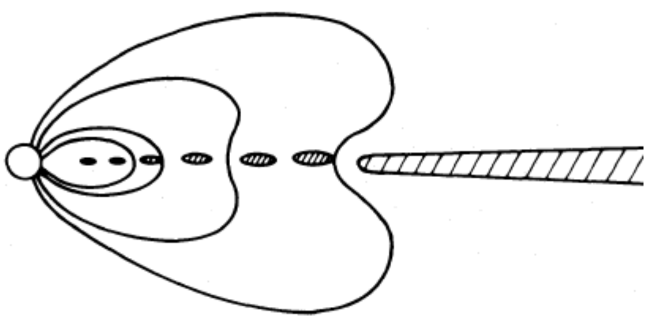


Torque on star:

$$\frac{dJ_\star}{dt} \simeq \dot{M}_{\rm acc} r_m^2 \Omega(r_m) + T_m - \dot{M}_{\rm eject} r_m^2 \Omega_\star$$

Question:
Is there accretion onto star?
Dead disk? (D'Angelo & Spruit)



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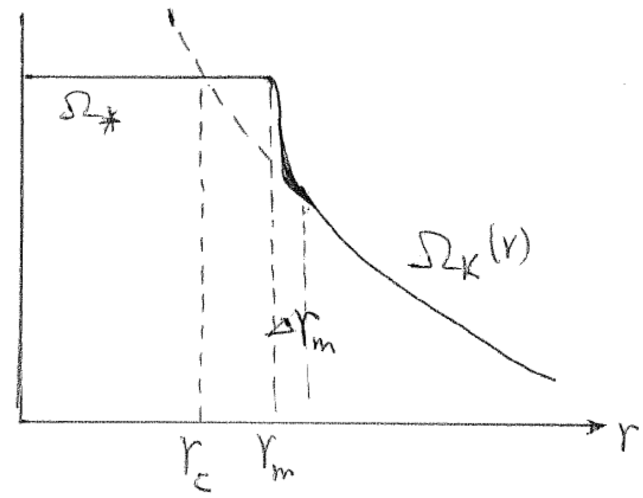

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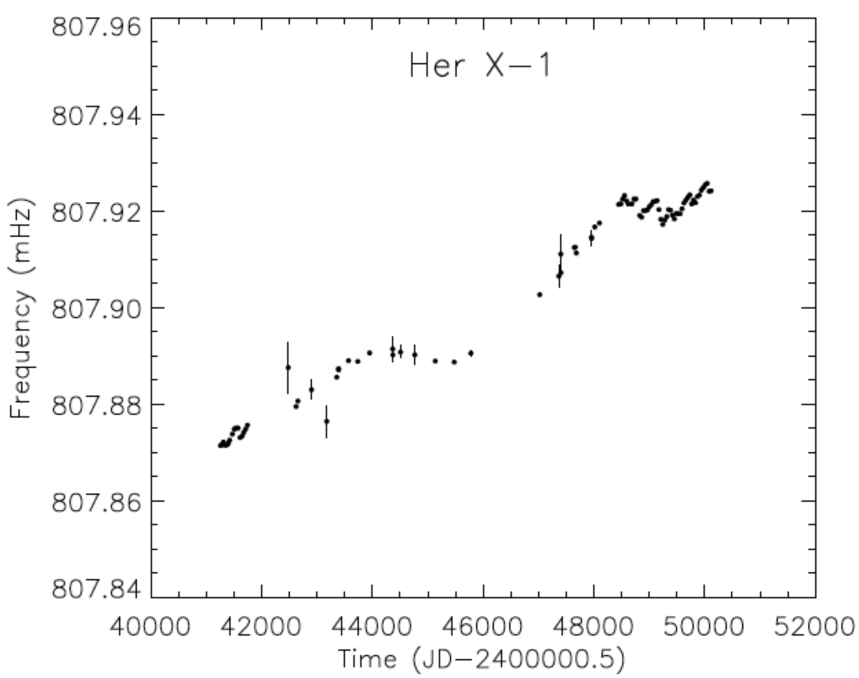
Question:

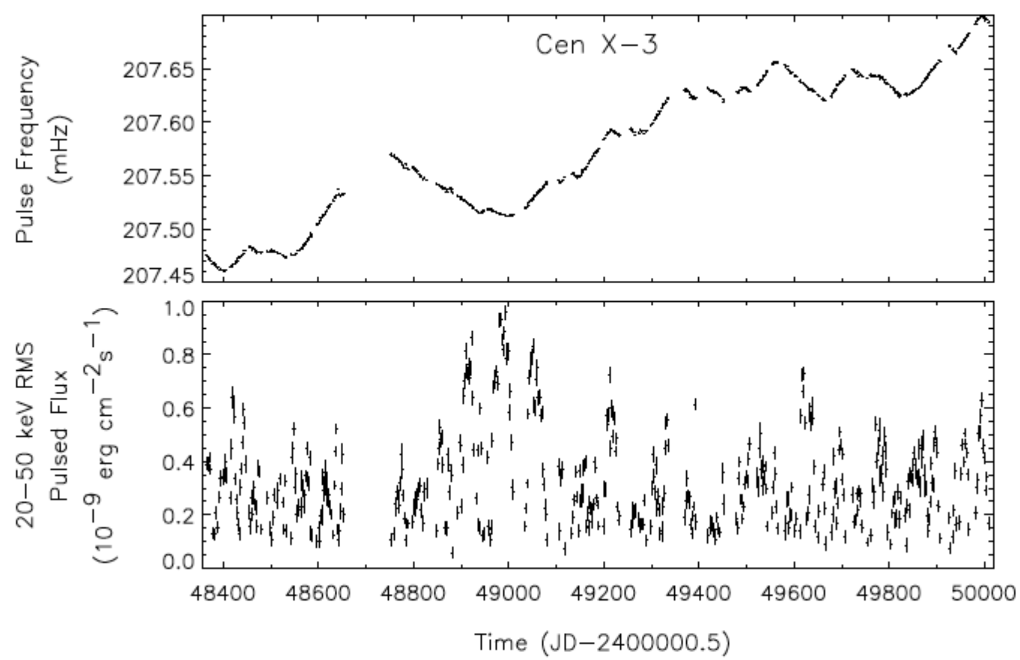
Equilibrium spin: T Tauri stars, millisecond pulsars,

Long-period pulsars (→ magnetar field?)

Can we understand spinup/spindown of X-ray pulsars?

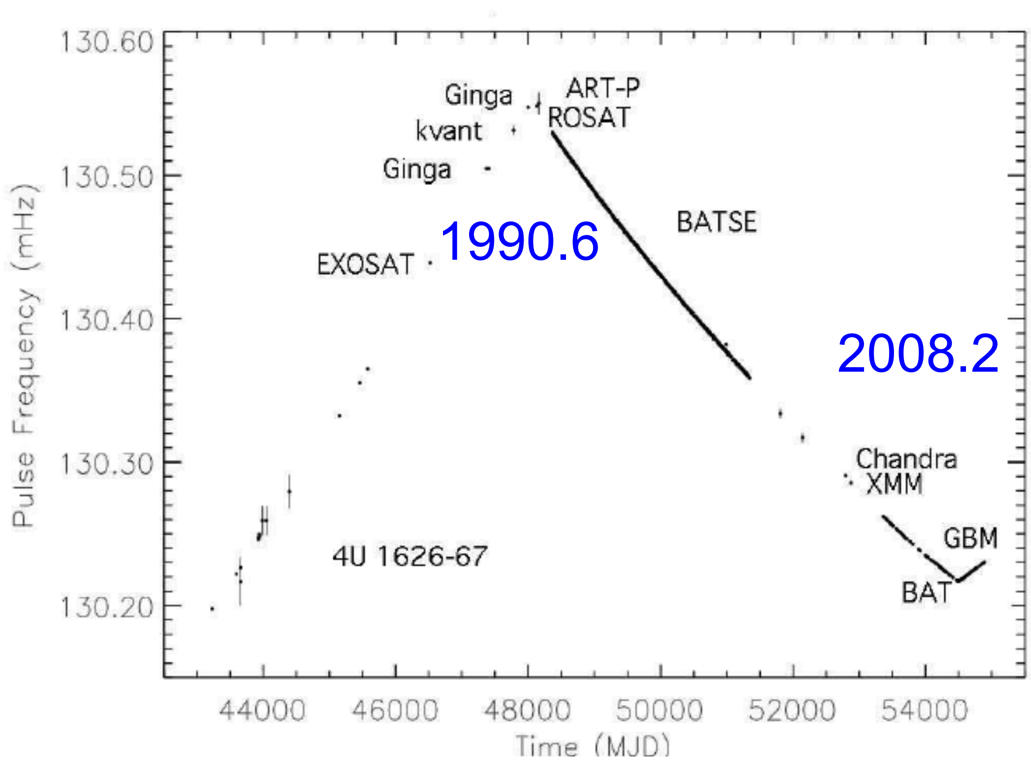
Spinup/Spindown of Accreting X-ray pulsars





Bildsten et al. 1997

Postnov's talk

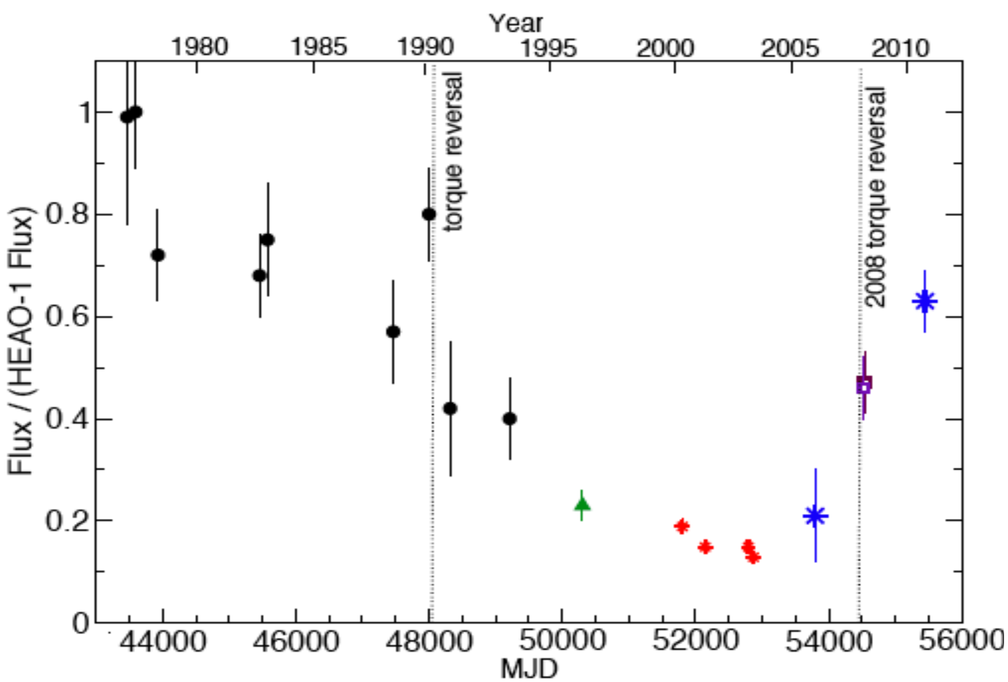


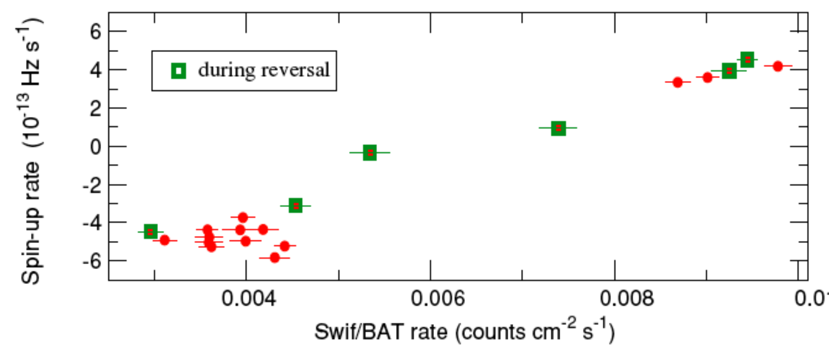
4U1626-67

7.66s

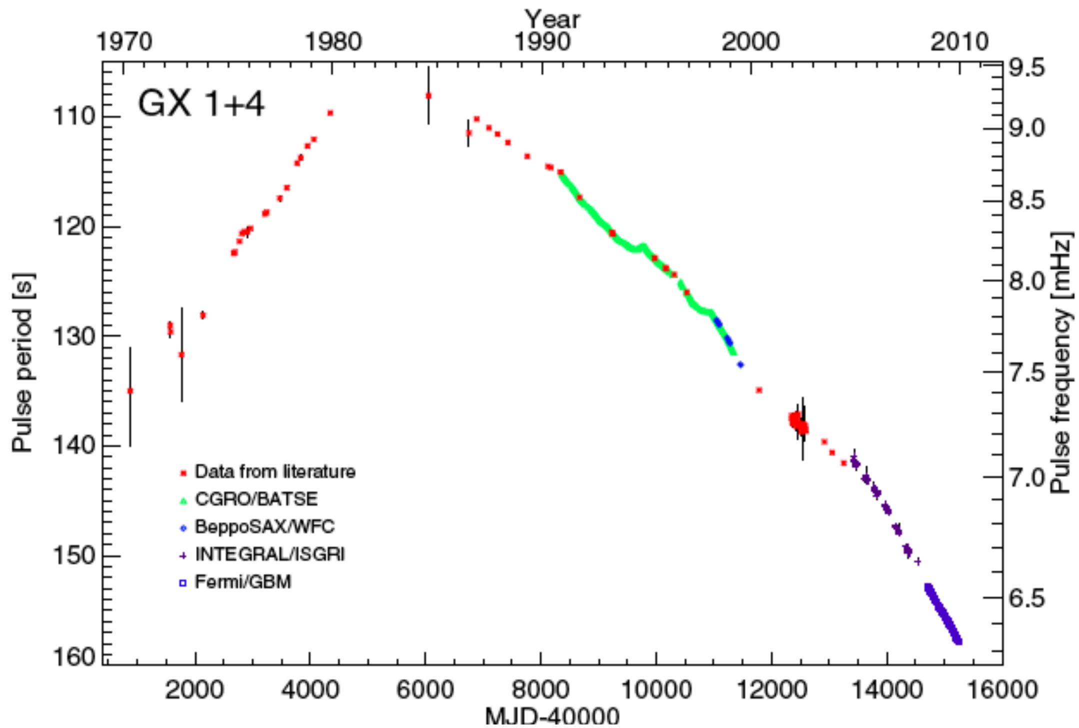
Transition lasted 150 days

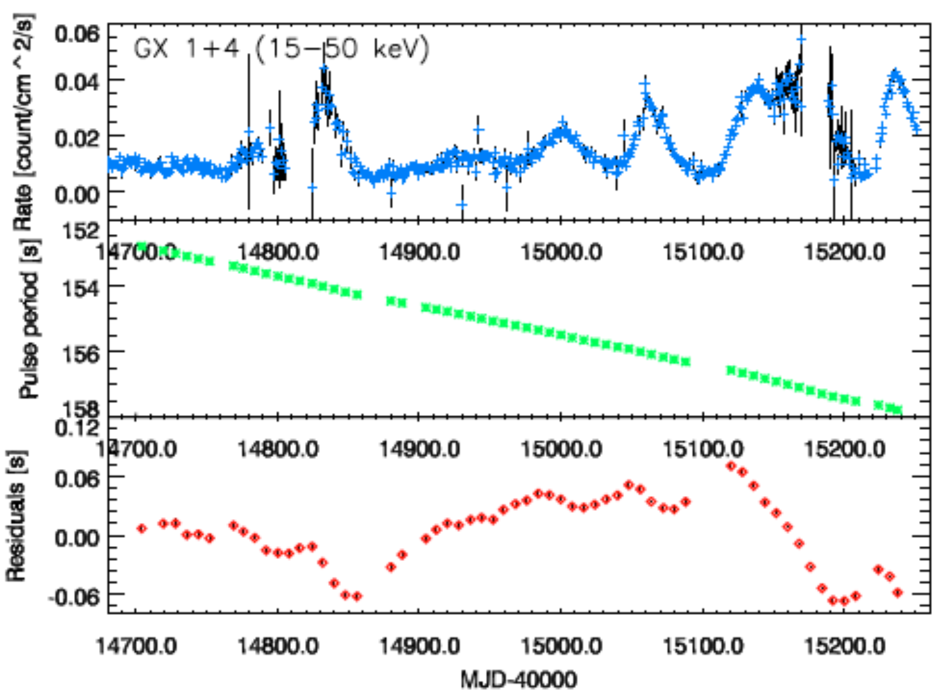
Camero-Arranz et al. 2010,2012





GX 1+4





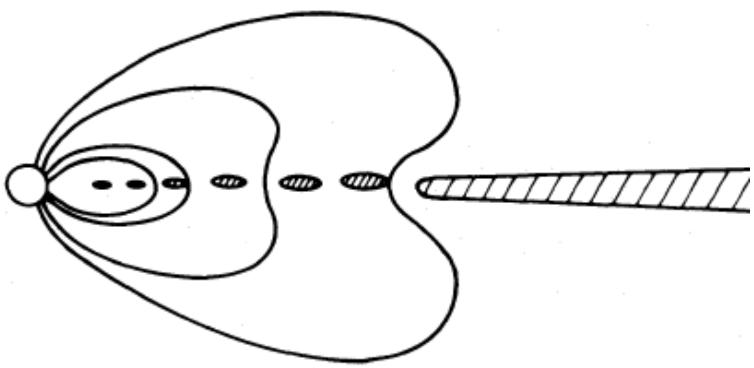
$$-\dot{\nu} \propto F_x^{(0.30 \pm 0.07)}$$

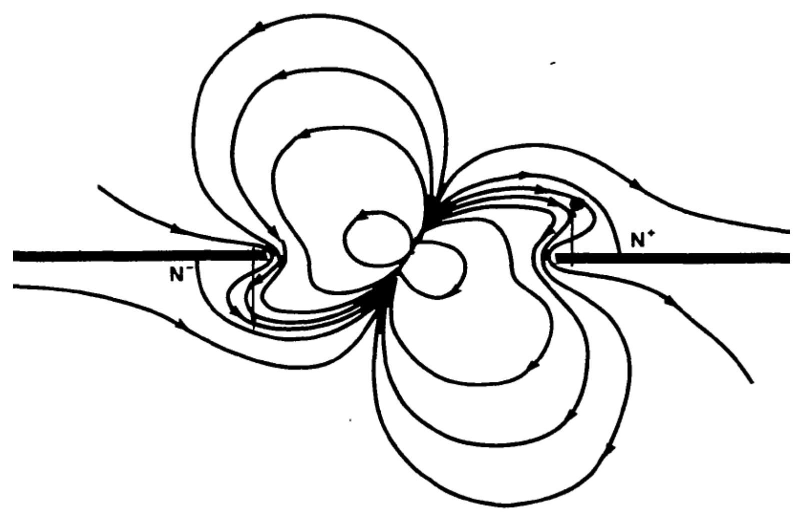
Chakrabarty et al.1997;
Gonzalez-Galan et al 2012

So far, physics of aligned dipole…

Misaligned Dipole

-- Funnel flow to polar caps
-- Accretion through instabilities





Simulations by Romanova et al…

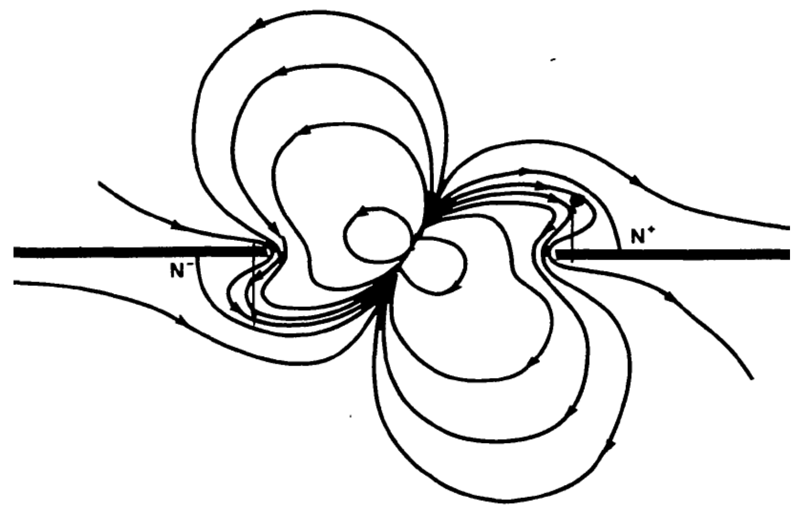
Misaligned Dipole: Exciting Waves in Disks

Vertical force on disk:

$$F_z(r,\varphi,t) = F_\omega(r)\exp(im\varphi - i\omega t)$$

$$m = 1, \quad \omega = \Omega_\star, 2\Omega_\star$$

➔ Excitation of Bending waves in the disk



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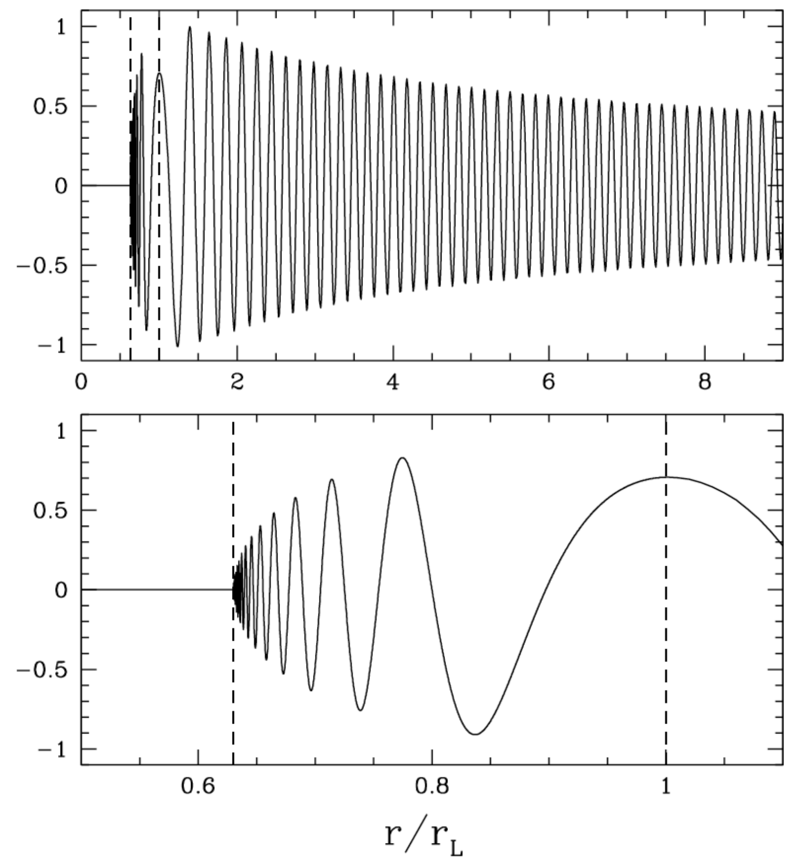
➔ Excitation of Bending waves in the disk

Perturbations most "visible" at Lindblad/Vertical Resonance

$$\omega - \Omega = \Omega_\perp \simeq \Omega$$

➔ $$\Omega(r_L) = \frac{\omega}{2} = \frac{\Omega_\star}{2}, \Omega_\star$$

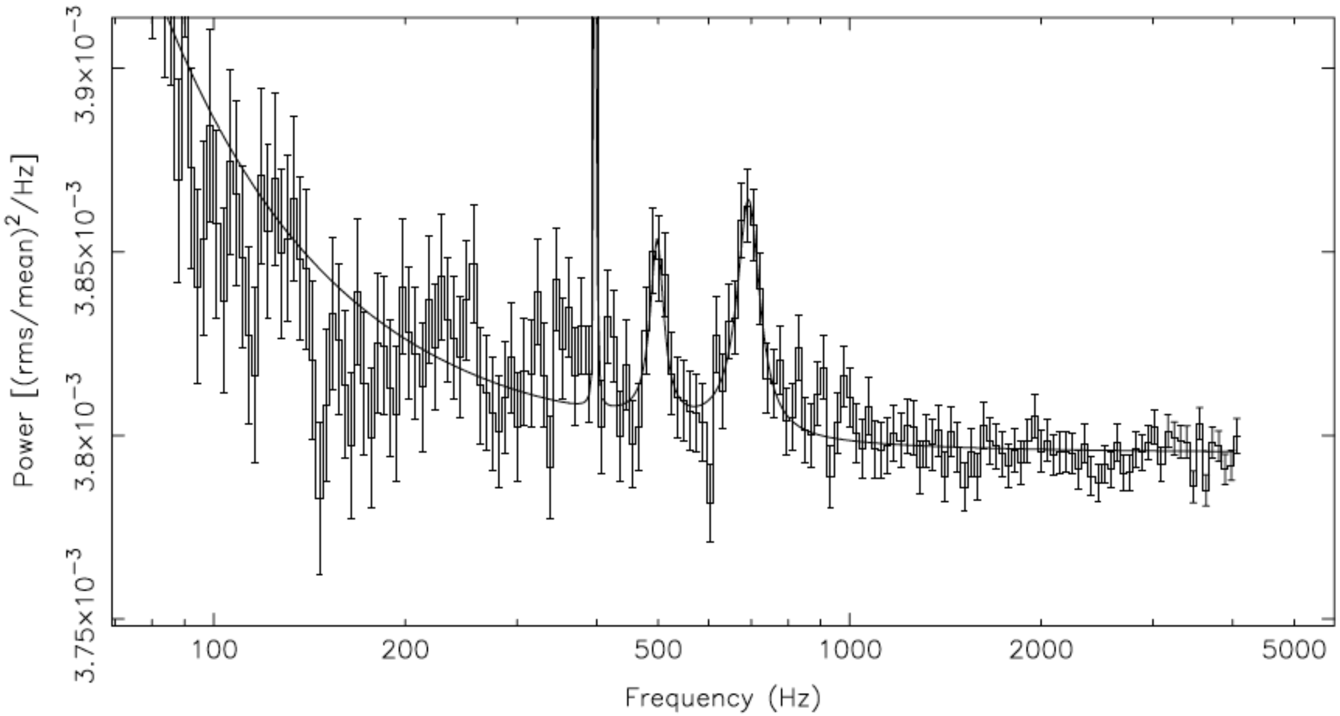


DL & Zhang 08

Question: QPOs….

Lovelace's talk

kHz QPOs in Accreting Millisecond Pulsars



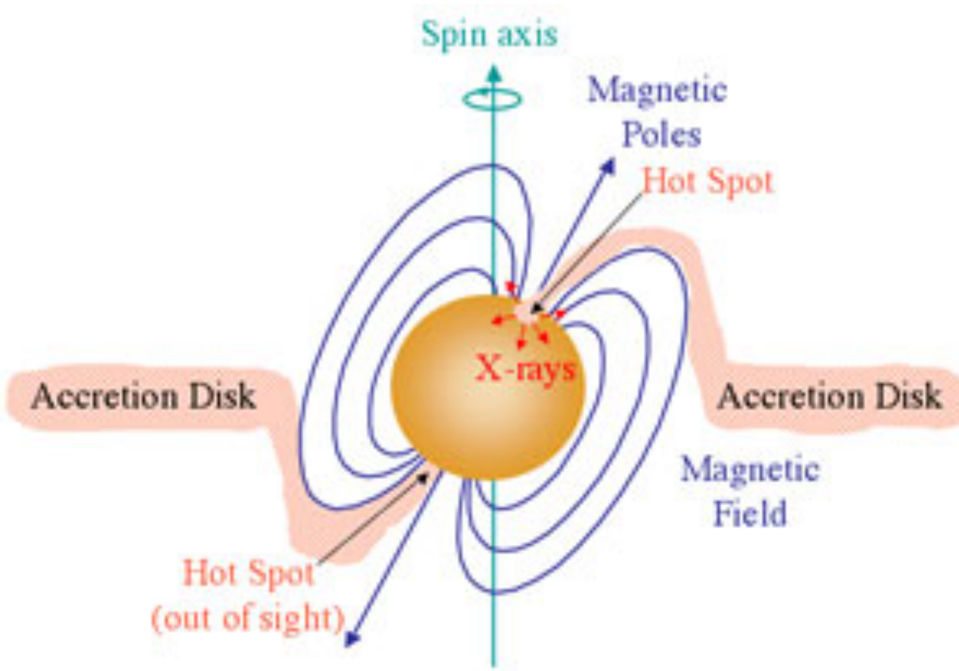
Van der Klis 2005

SAX J1808.4-3658: $\nu_s = 401$ Hz, $\nu_h - \nu_l \simeq \nu_s/2$ ($\pm$a few Hz)

XTE J1807.4-294: $\nu_s = 191$ Hz, $\nu_h - \nu_l \simeq \nu_s$

Beating of high-freq. QPO with perturbed fluid at L/VR ?

Stellar Spin – Disk Misalignment



Standard story: **S//L**

There are magnetic torques which tend to make the inner disk

-- warp

-- precess

on timescale >> dynamical time (rotation/orbital period)

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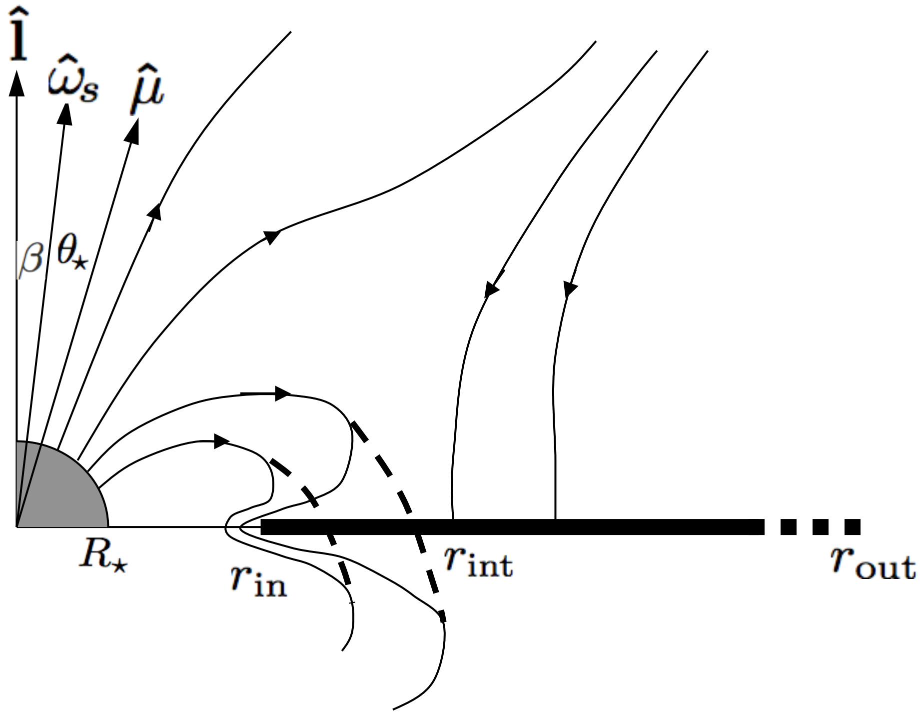
-- precess

on timescale >> dynamical time (rotation/orbital period)

Consider general geometry.

Two limiting cases…

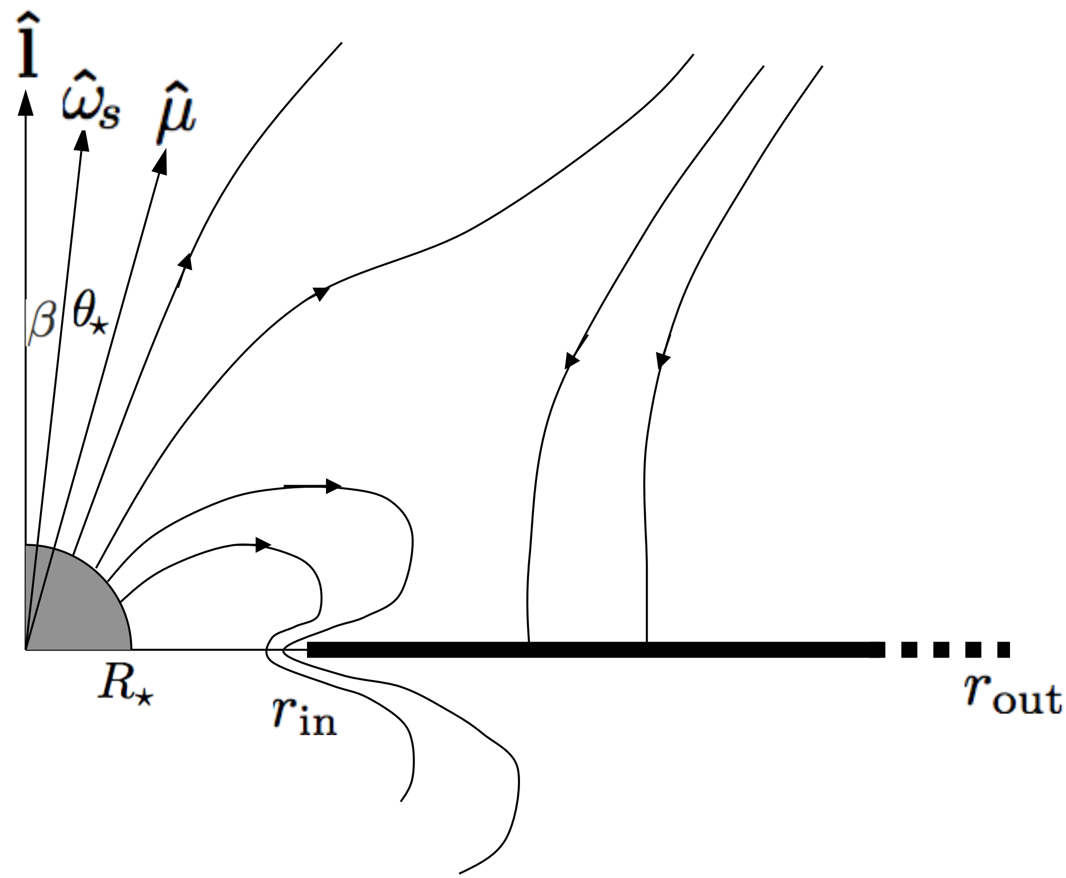


Perfect conducting disk:

Torque on disk (per unit area): $\mathbf{N} \propto \hat{\mu} \times \hat{\mathbf{l}}$

Averaging over stellar rotation: $\mathbf{N} \propto \hat{\omega}_s \times \hat{\mathbf{l}}$

Precessional Torque

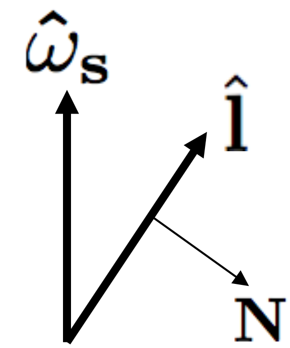


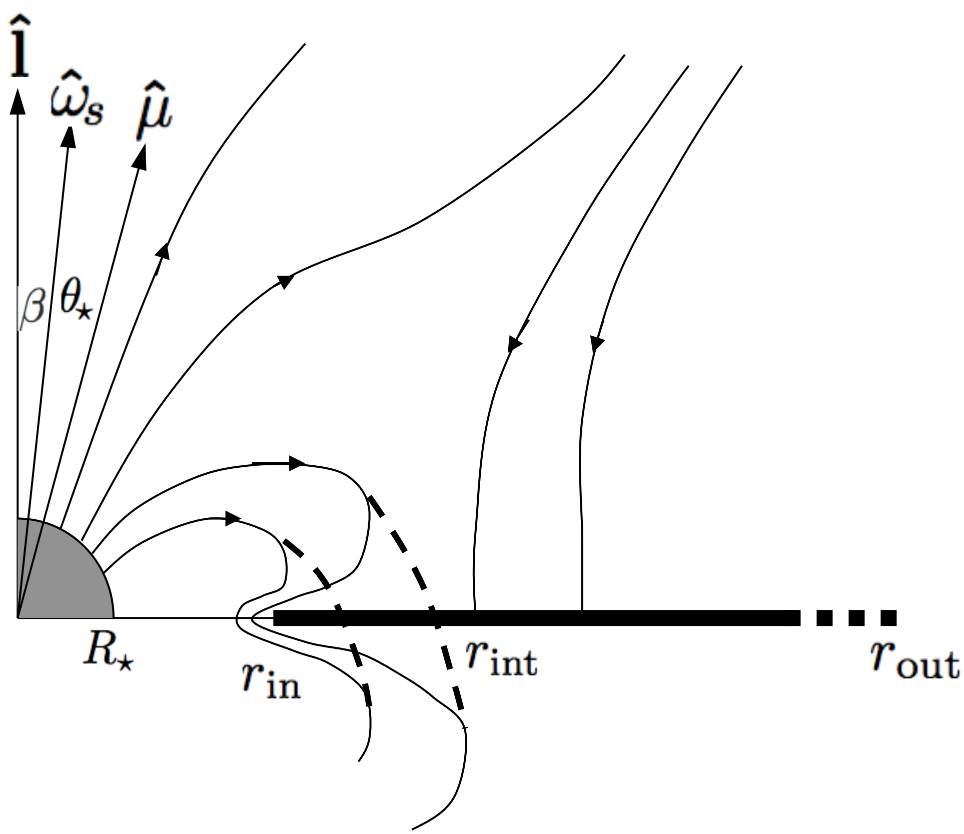
Magnetically Threaded disk:

Torque on disk (per unit area): $\mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\mu} \times \hat{\mathbf{l}})$

Averaging over stellar rotation: $\mathbf{N} \propto -\hat{\mathbf{l}} \times (\hat{\omega}_s \times \hat{\mathbf{l}})$

Warping torque





B_z threads the disk

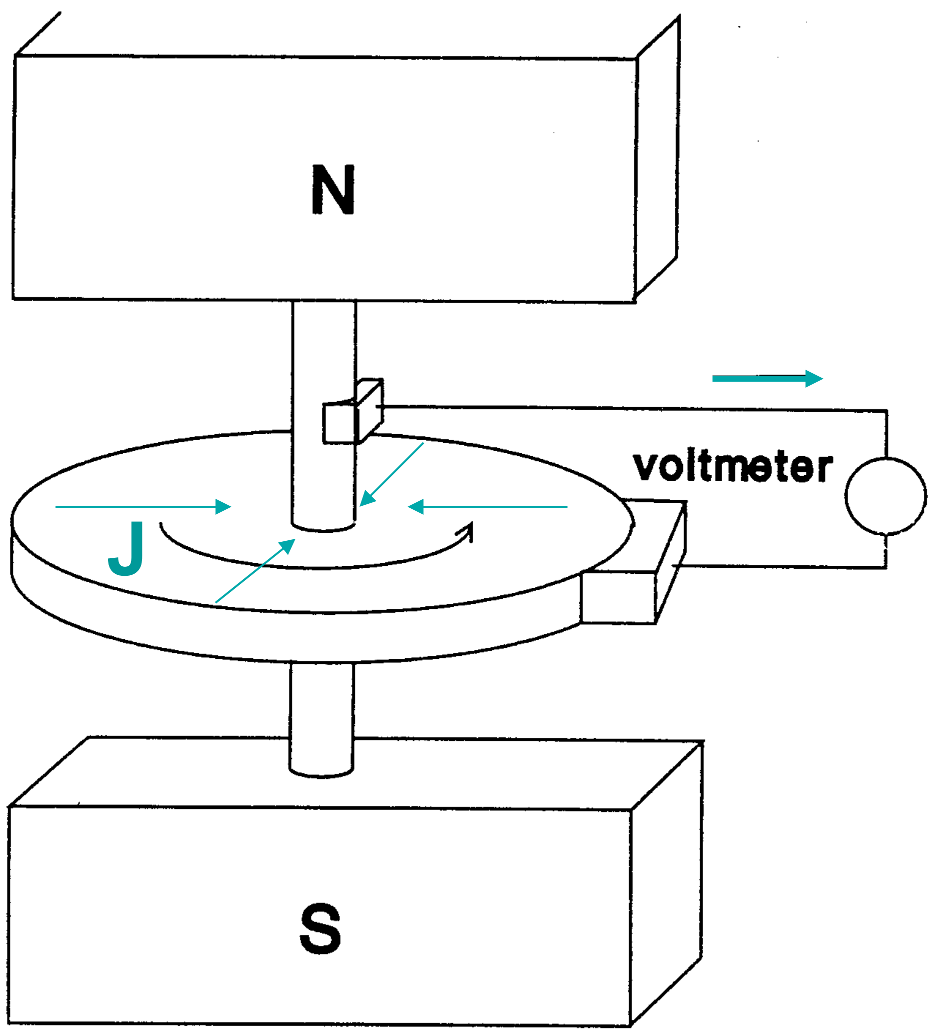
$$\Longrightarrow \quad \Delta B_\phi = \mp \zeta B_z$$

$$\Longrightarrow \quad B_\phi^+ = B_\phi^{(s)} - \zeta B_z$$

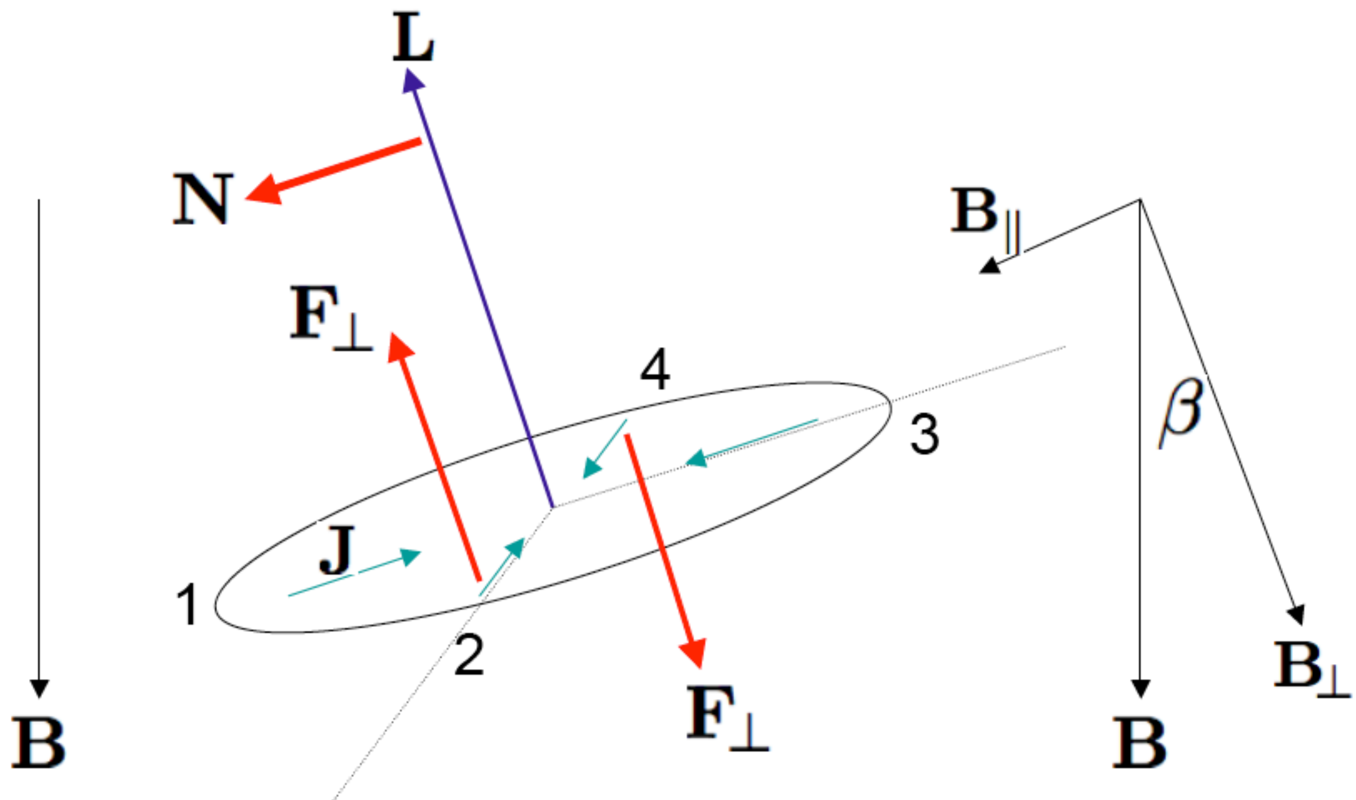
$$B_\phi^- = B_\phi^{(s)} + \zeta B_z$$

$$\Longrightarrow \quad F_z(\phi)$$

A Laboratory Experiment



A Laboratory Experiment



DL, Foucart & Lin 2011

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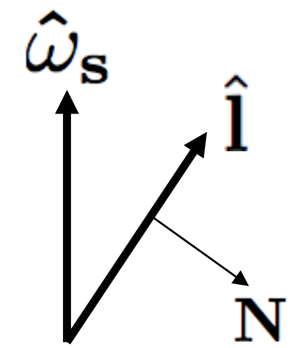
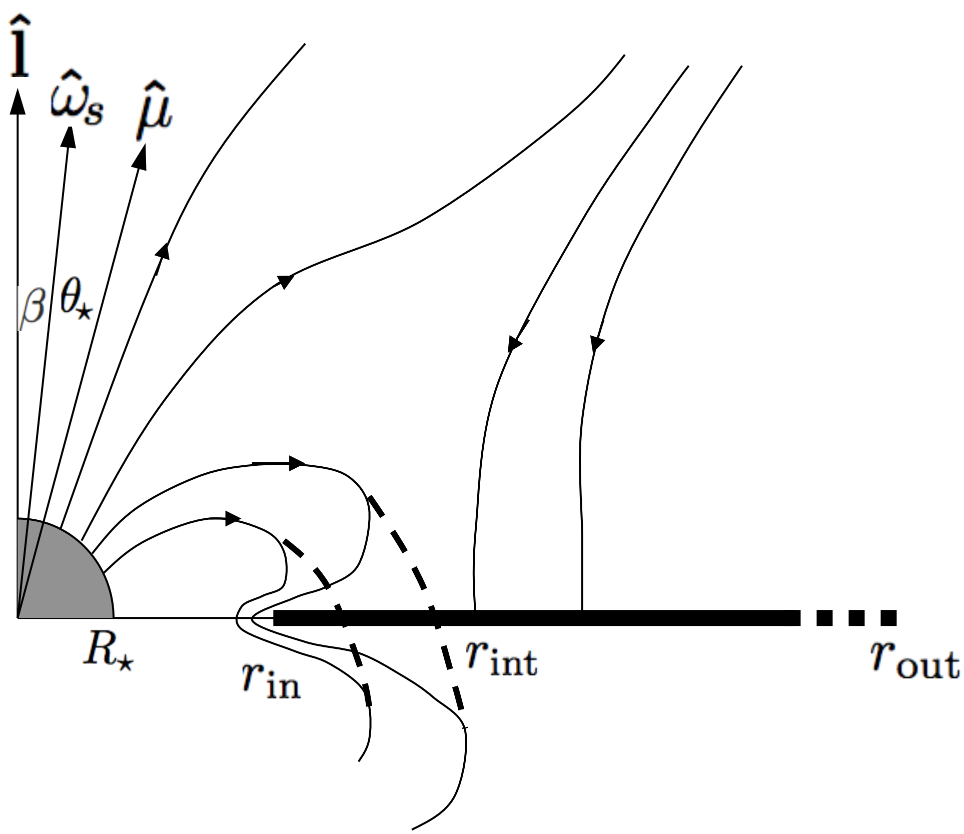
Warping torque





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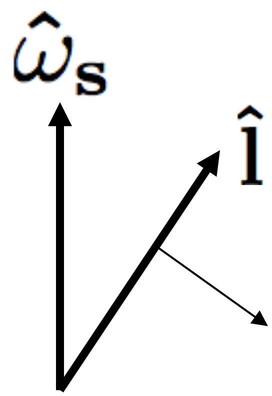
$$\Longrightarrow \quad F_z(\phi)$$

Recap:

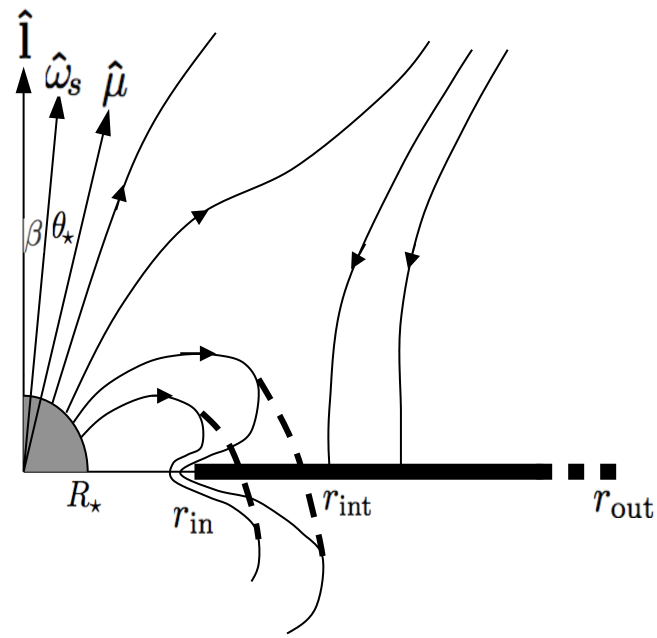
Magnetic precessional torque and warping torque on disk (per area)

$$\mathbf{N}_p = -\frac{\mu^2}{\pi^2 r^5 D(r)} \sin^2\theta_\star \cos\beta\, \hat{\boldsymbol{\omega}}_s \times \hat{\boldsymbol{l}}$$

$$\mathbf{N}_w = -\frac{\zeta\mu^2}{4\pi r^5} \cos\beta \cos^2\theta_\star\, \hat{\boldsymbol{l}} \times (\hat{\boldsymbol{\omega}}_s \times \hat{\boldsymbol{l}})$$



(Instability)



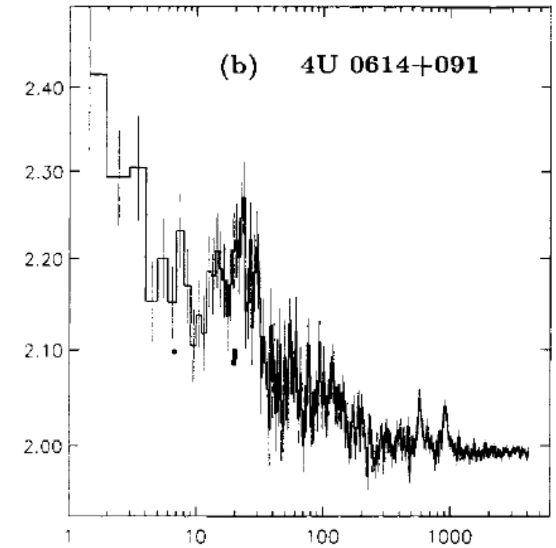
Low-Frequency (10-50 Hz) QPOs in LMXBs

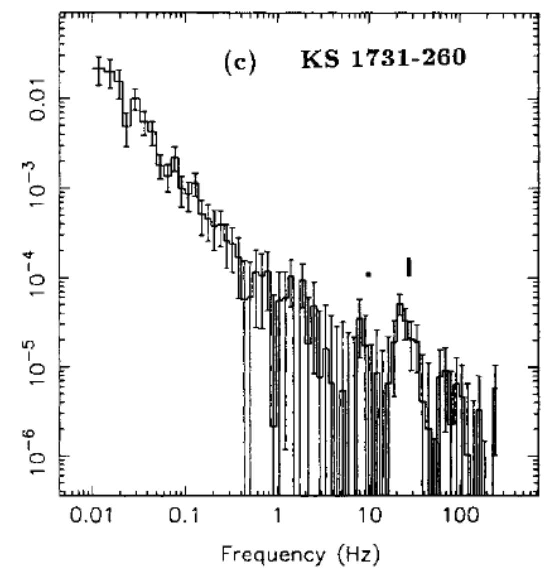
Lense-Thirring prcession of warped disk ?
(e.g. Stella & Vietri 1998) $\nu_{\rm LT} \propto \nu_K^2$

Magnetic effects induce warp and contribute to precession (Shirakawa & DL 02; Pfeiffer & DL 04)

Other systems:

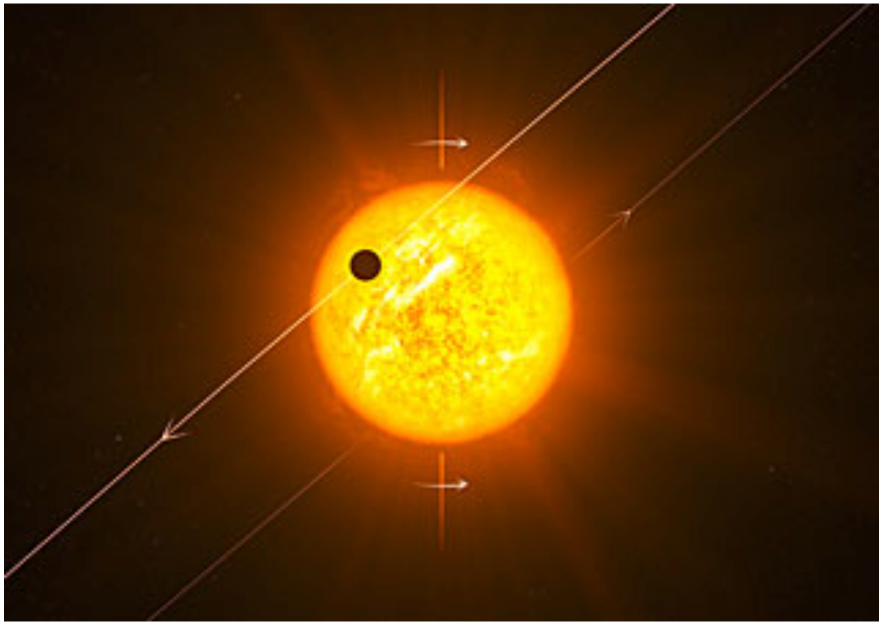
mHz QPOs in x-ray pulsars?
Photometric variabilities of AA Tau? (Bouvier+)

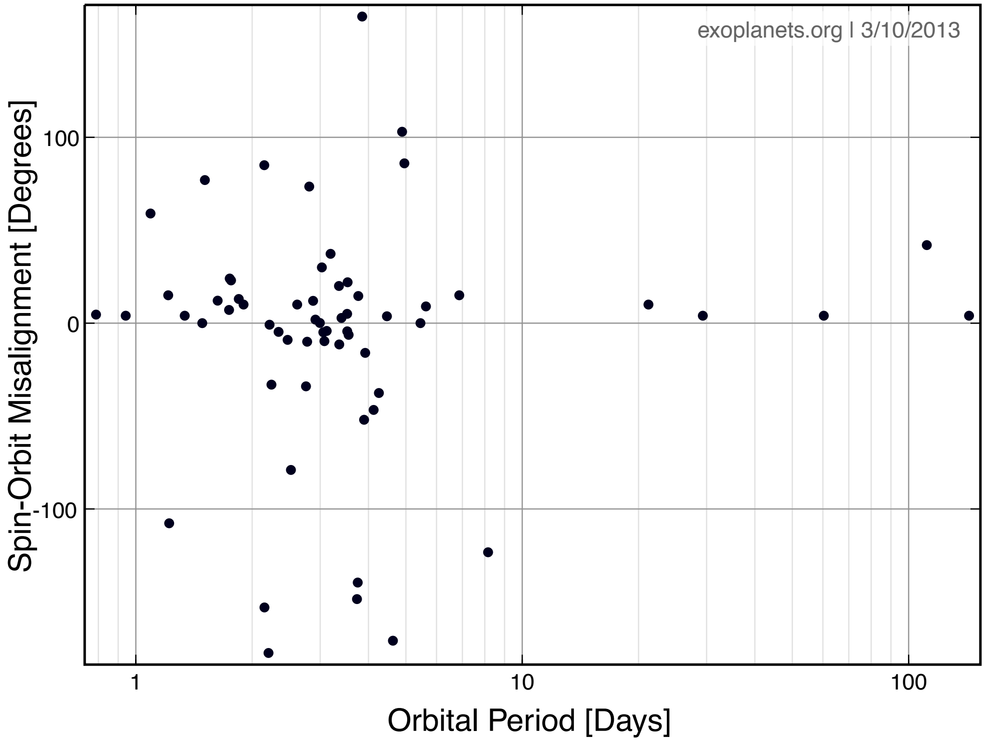




Possible Connection to (Exo)Planetary Systems

Many “hot Jupiters” have Misaligned S^*-L_p





S^*-L_p misalignment in "Hot Jupiter" Systems
➔ The Importance of few-body Interactions

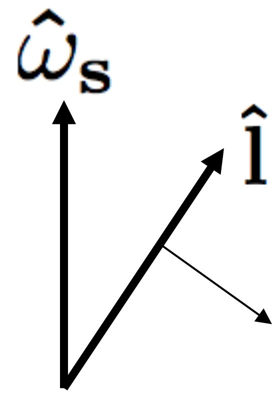
1. Kozai + Tide migration by a distant companion star/planet
2. Planet-planet Interactions (Strong scatterings, secular chaos etc)

Alternative possibility:

Misaligned protostar - protoplanetary disk ?

e.g.,
-- Solar system: orbital plane misaligned from solar equator by 7 deg.
-- Two multi-planet systems (KOI-1241) with common orbital plane misaligned with the stellar equator (J. Carter et al.)

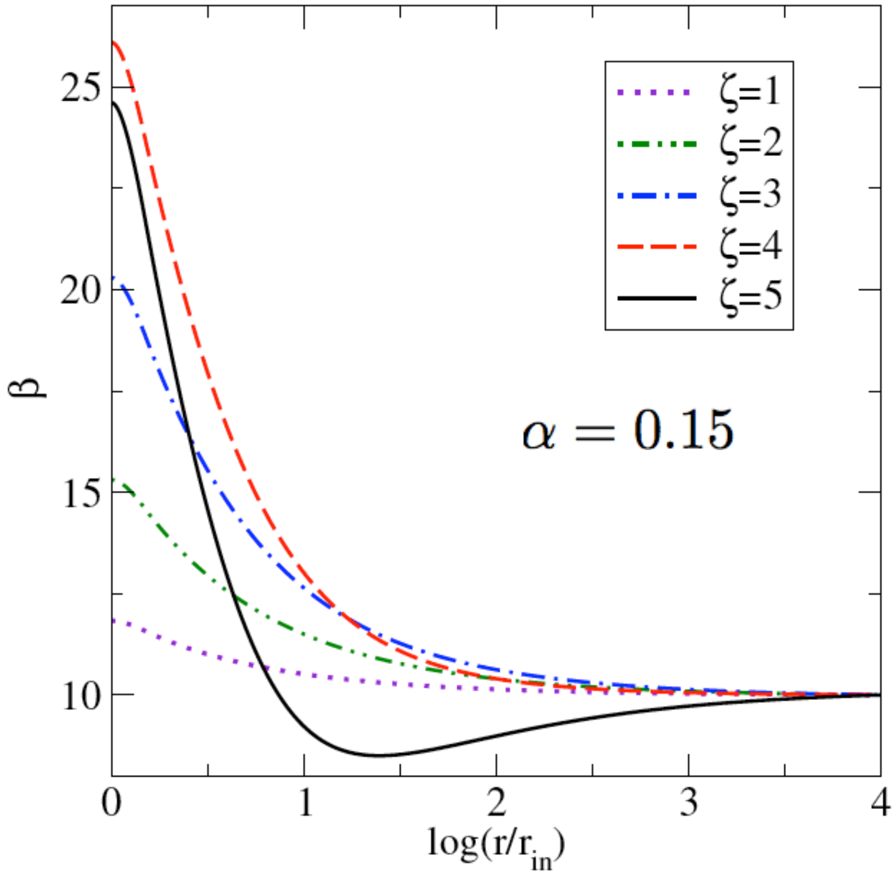
Recall: Magnetic toques from the star **want** to make the inner disk warp and precess…



But disk will **want** to resist it by internal stresses (viscosity or bending wave propagation)

$$\frac{\partial}{\partial t}\left(\Sigma r^2 \Omega \hat{\boldsymbol{l}}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\Sigma V_R r^3 \Omega \hat{\boldsymbol{l}}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(Q_1 I r^2 \Omega^2 \hat{\boldsymbol{l}}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(Q_2 I r^3 \Omega^2 \frac{\partial \hat{\boldsymbol{l}}}{\partial r} + Q_3 I r^3 \Omega^2 \hat{\boldsymbol{l}} \times \frac{\partial \hat{\boldsymbol{l}}}{\partial r}\right) + \mathbf{N}_m$$

Steady-state Disk Warp:



Foucart & DL 2011

For protoplanetary disk/star parameters, the disk warp is small

Back-reaction Torque on Star:
What is happening to the stellar spin direction?
(Is there secular change to the spin direction?)

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What is happening to the stellar spin direction?
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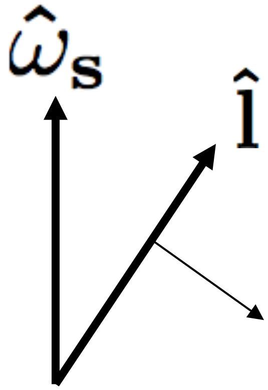

Warping torque

Back-reaction Torque on Star:
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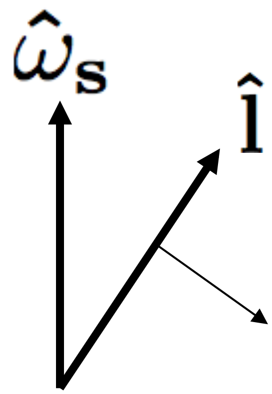
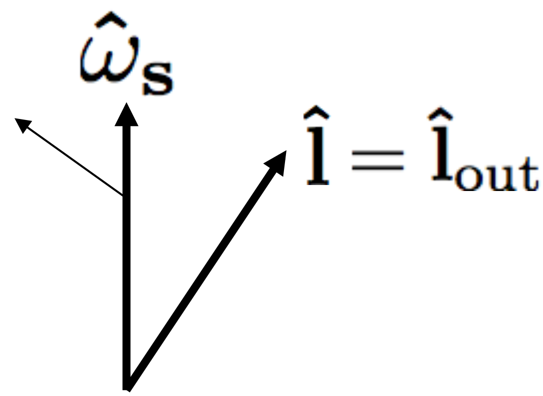
(Is there secular change to the spin direction?)



Warping torque



Back-reaction torque

Evolution of the stellar spin

$$\frac{d}{dt}\left(J_s\hat{\boldsymbol{\omega}}_s\right)=\boldsymbol{\mathcal{N}}=\boldsymbol{\mathcal{N}}_{\rm acc}+\boldsymbol{\mathcal{N}}_m+\boldsymbol{\mathcal{N}}_{\rm sd}$$

$$\boldsymbol{\mathcal{N}}_{\rm acc}=\lambda\dot{M}\sqrt{GMr_{\rm in}}\;\hat{\boldsymbol{l}}_{\rm in},\quad \lambda\sim 1\ \text{(or less)}$$

$\boldsymbol{\mathcal{N}}_m$ =backreaction of magnetic (warping & precessional) torques

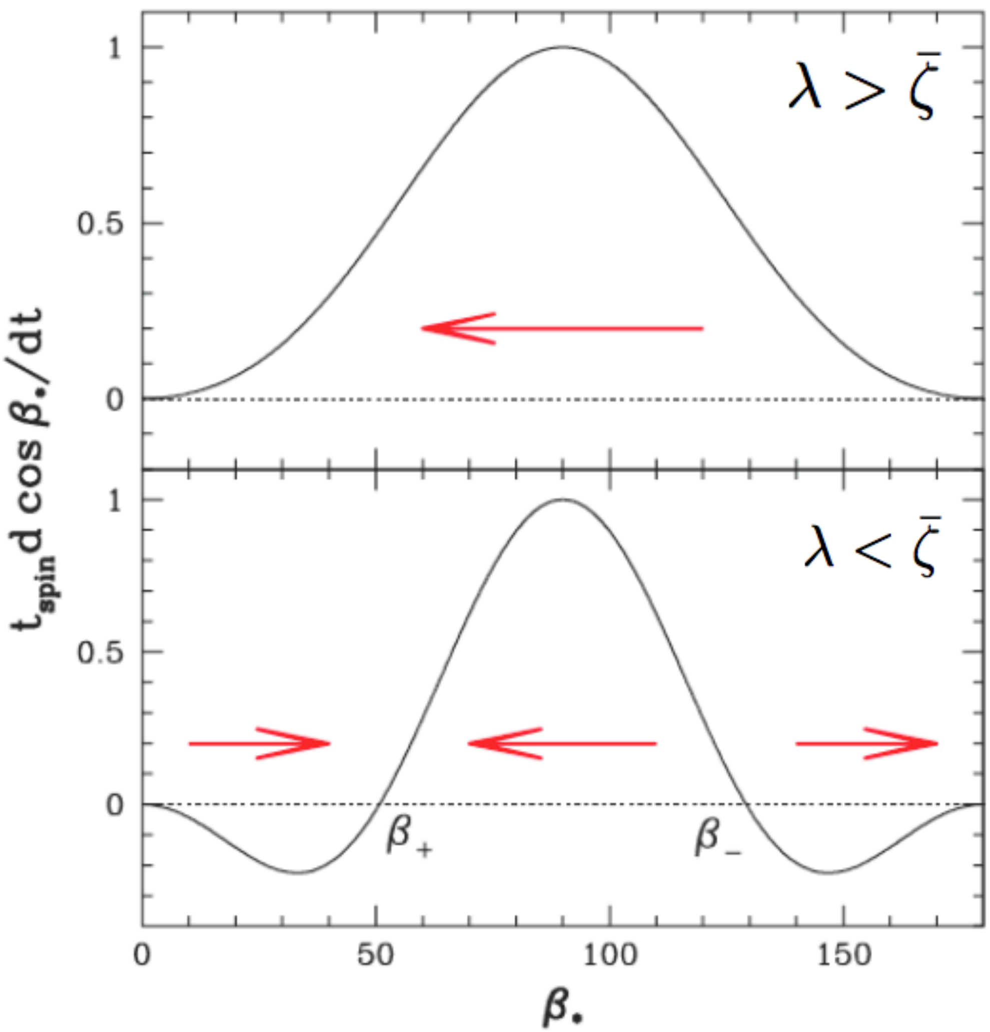
$$\boldsymbol{\mathcal{N}}_{\rm sd}=-|\mathcal{N}_{\rm sd}|\;\hat{\boldsymbol{\omega}}_s$$

(Each term is of order $\mathcal{N}_0=\dot{M}\sqrt{GMr_{\rm in}}$)

$$\Longrightarrow\quad \frac{d}{dt}\cos\beta=\frac{\mathcal{N}_0}{J_s}\sin^2\beta\left(\lambda-\tilde{\zeta}\cos^2\beta\right)\qquad \bar{\zeta}=\frac{\zeta\cos^2\theta_\star}{6\eta^{7/2}}\ (\sim 1)$$

Evolution of the stellar spin

$$\frac{d}{dt}\cos\beta = \frac{\mathcal{N}_0}{J_s}\sin^2\beta\left(\lambda - \tilde{\zeta}\cos^2\beta\right) \qquad \bar{\zeta} = \frac{\zeta\cos^2\theta_\star}{6\eta^{7/2}}\ (\sim 1)$$



"weak" warping torque

"strong" warping torque

A hierarchy of time scales:

(1) Orbital period of inner disk, spin period (days)
==> short… Averaged out already

(2) Warp growth time and precession period of inner disk

$$t_w \sim \Gamma_w^{-1} = (92\,\mathrm{days}) \left(\frac{1\ \mathrm{kG}}{B_\star}\right)^2 \left(\frac{2R_\odot}{R_\star}\right)^6 \left(\frac{M_\star}{1\,M_\odot}\right)^{1/2} \left(\frac{r_{\rm in}}{8R_\odot}\right)^{11/2} \left(\frac{\Sigma}{10\,\mathrm{g\,cm^{-2}}}\right) (\zeta \cos\theta_\star)^{-1}$$

(3) Disk warp evolution time: e.g., due to viscosity

$$t_{\rm vis} \sim (3000\,\mathrm{yrs}) \left(\frac{\alpha}{0.1}\right) \left(\frac{\delta}{0.1}\right)^{-2} \left(\frac{r}{100\ \mathrm{AU}}\right)^{3/2}$$

(4) Timescale to change the spin (longest!)

$$J_s = 0.2 M_\star R_\star^2 \omega_s \qquad \text{torque } \mathcal{N} \sim \dot{M}\sqrt{GMr_{\rm in}}$$

$$\Longrightarrow \quad t_{\rm spin} \sim \frac{J_s}{\mathcal{N}} = (1.25\,\mathrm{Myr}) \left(\frac{M_\star}{1\,M_\odot}\right) \left(\frac{\dot{M}}{10^{-8}\,M_\odot \mathrm{yr}^{-1}}\right)^{-1} \left(\frac{r_{\rm in}}{4R_\star}\right)^{-2} \frac{\omega_s}{\Omega(r_{\rm in})}$$

How to test this?

- Measuring spin-orbit angles for systems with 2 transiting planets

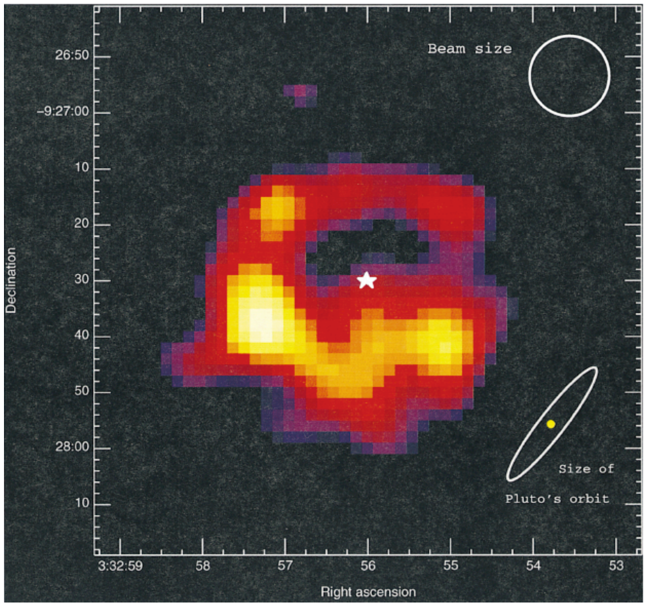
  e.g., Two multi-planet systems (KOI-1241) with common orbital plane misaligned with star

- Measuring the orientation of stellar spin and disk

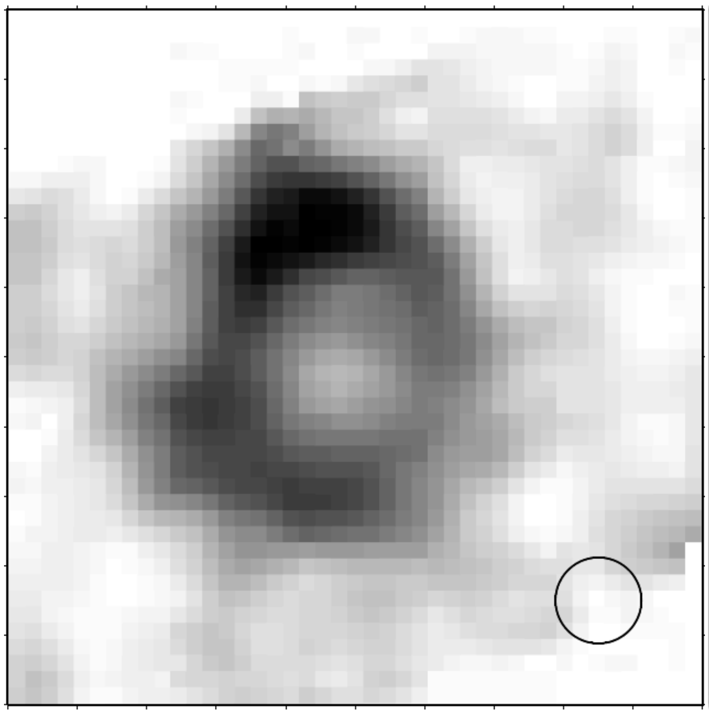
  Young star and disk (with jets)?
  MS stars with debris disks?

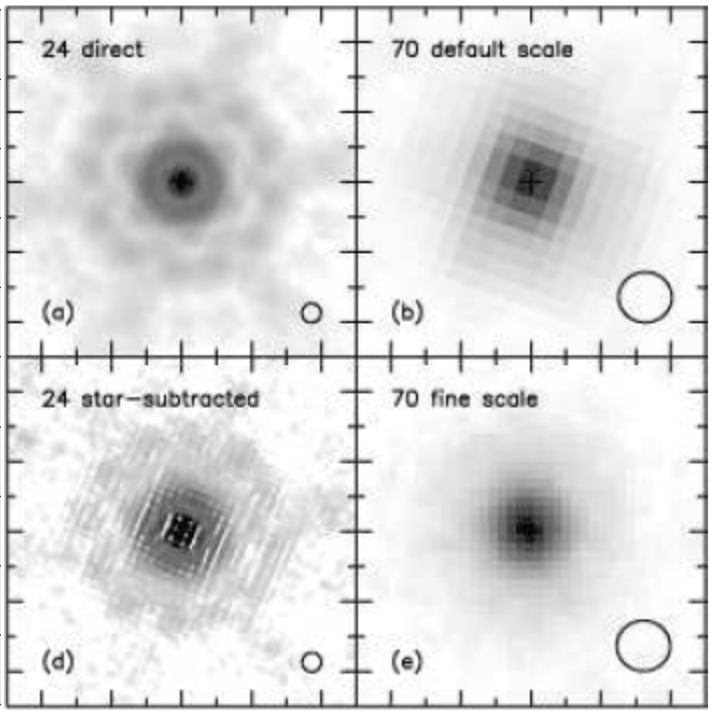
Watson et al 2011

HD	i_* (°)	i_{disk} (°)	ref.
10647	49^{+17}_{-11}	$\geqslant 52$	(Liseau et al. 2008)
10700	45^{+24}_{-15}	60–90	(Greaves et al. 2004)
22049	31^{+5}_{-5}	25	(Greaves et al. 1998)
61005	90^{+0}_{-26}	80	(Maness et al. 2009)
92945	65^{+21}_{-10}	70	(Krist et al. 2005)
107146	21^{+8}_{-9}	25 ± 5	(Ardila et al. 2004)
197481	90^{+0}_{-20}	90	(Krist et al. 2005)
207129	47^{+22}_{-13}	60 ± 3	(Krist et al. 2010)



Greaves et al. 1998





CSO and Spitzer (MIPS) image
Backman et al 2009

Consistent with face-on (Stapelfeldt 2010)

Summary

Issues, uncertainties, possible applications

- r_m vs $_{\text{rin}}$
- Star-disk linkage: width, steady vs cyclic
- Magnetosphere boundary outflow
- Propeller effect/outflow
- Torque on the star: spinup/spindown
- Misaligned dipole: effect on infall, effect on disk
- Spin-disk misalignment: disk warp/precession, exoplanets

Observations:

Variabilities (QPOs), NS jets (2 different types?),
spin equilibrium (infer B?), spinup/down observations,
warp/prcession, exoplanets, protoplanetary disks…

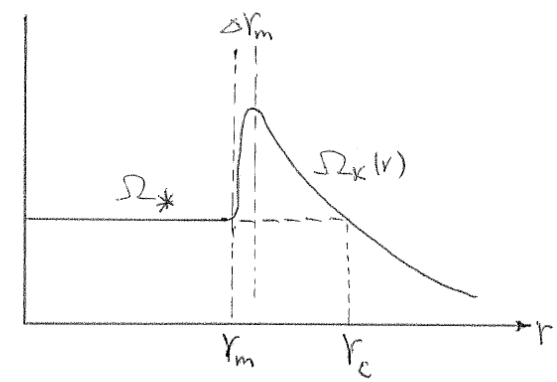
THANKS

Magnetosphere Boundary Layer

$$r_m \rightarrow r_m + \Delta r_m$$

Transition from Ω_K to Ω_s

$$B_{\phi +} = -\zeta B_z$$



Magnetic torque on BL (per unit area)

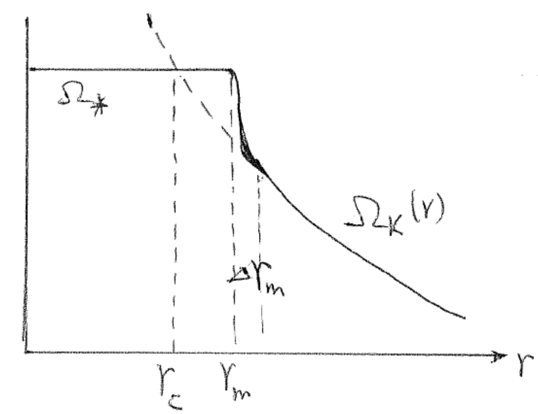
$$r\frac{B_z B_{\phi +}}{2\pi} = \Sigma \frac{\mathrm{d}(r^2\Omega)}{\mathrm{d}t} = \Sigma\, v_r \frac{\partial (r^2\Omega)}{\partial r}$$

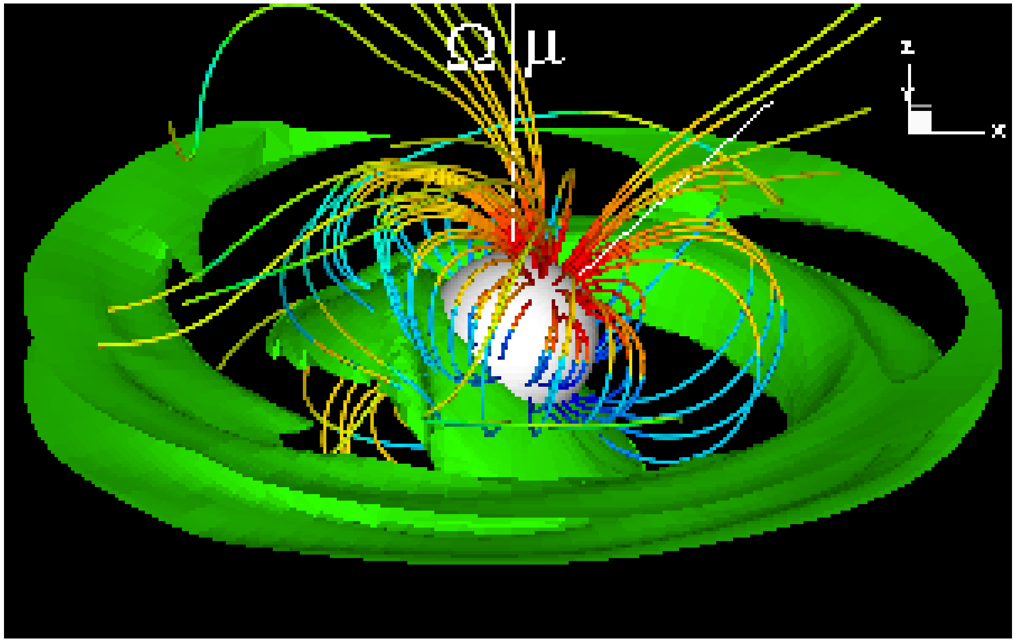
Mdot is local

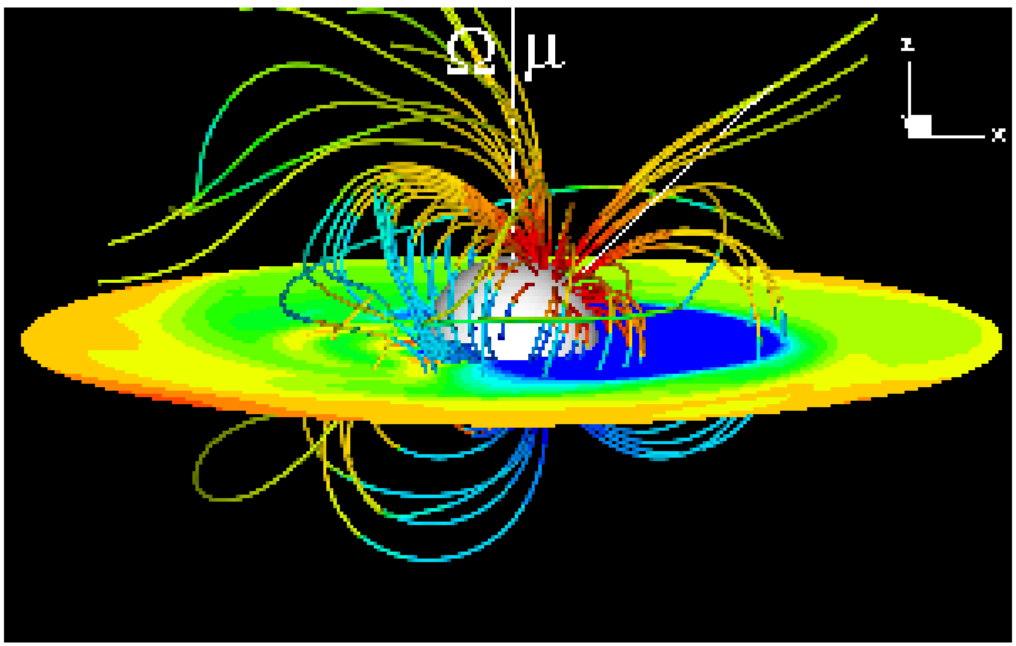
$$\Rightarrow \quad r_m \simeq \left(\zeta \frac{\Delta r_m}{H}\right)^{2/7} \left(\frac{\mu^4}{G M \dot{M}^2}\right)^{1/7}$$

$$\text{Note}: \quad \text{For } r_c < r_m:$$

$$r_m \simeq \left(\zeta \frac{\Delta r_m}{H}\right)^{1/5} \left(\frac{\mu^2}{\Omega_s \dot{M}}\right)^{1/5}$$







Simulations by Cornell group:
M. Romanova, Lovelace, etc