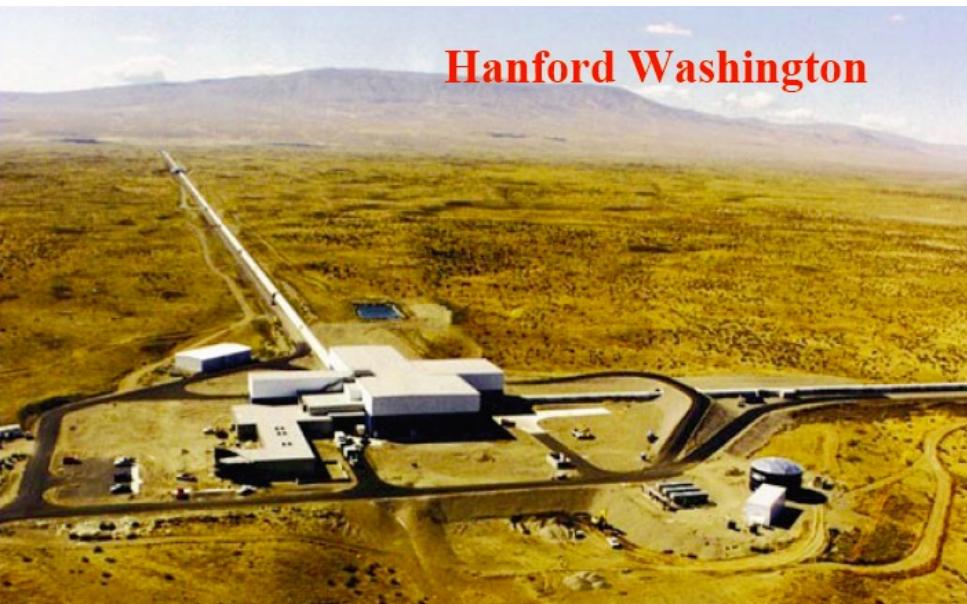


Formation of Merging Compact Binaries

Dong Lai
Cornell University

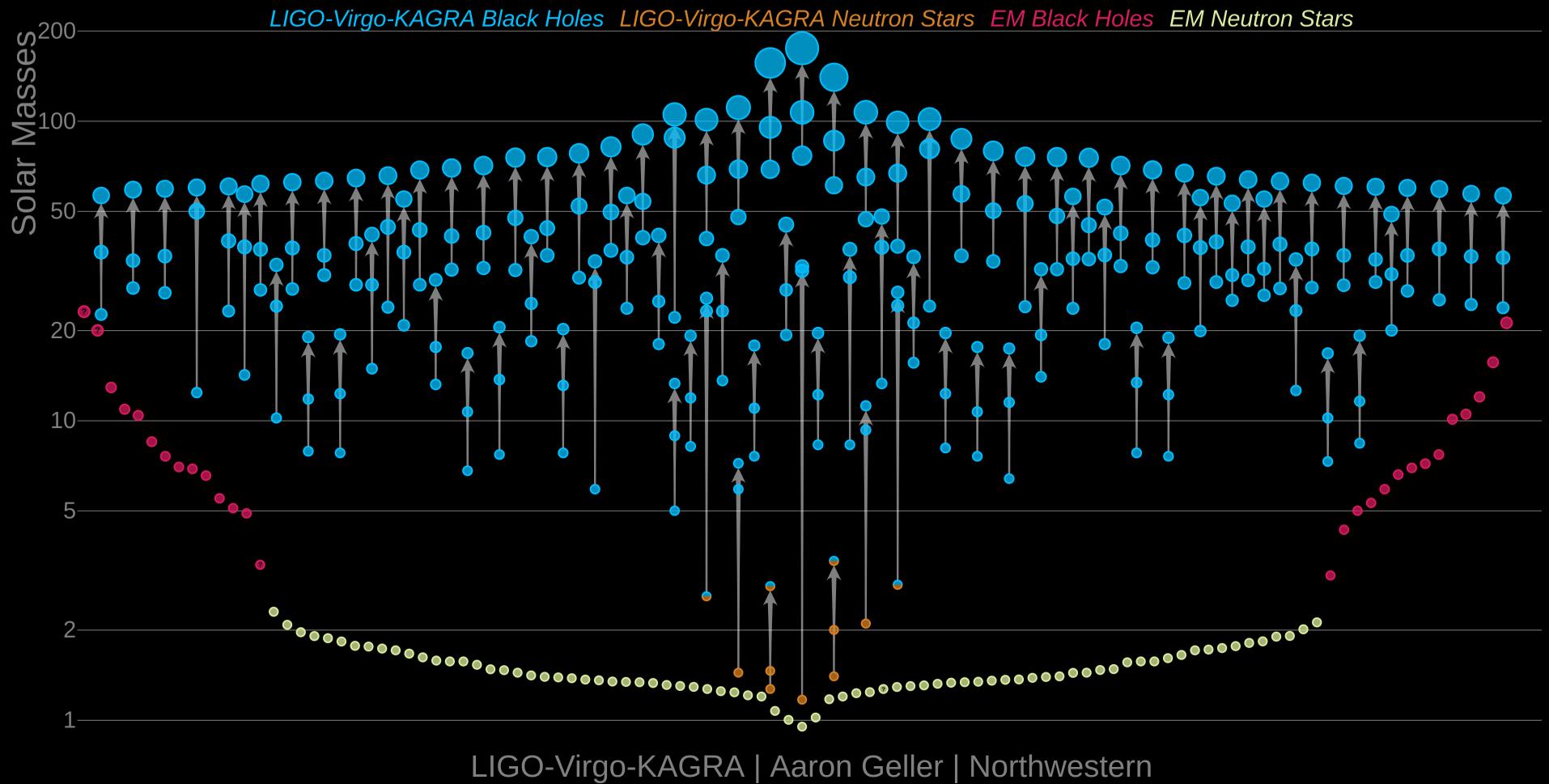
Gravitational Wave Astronomy



LIGO
VIRGO

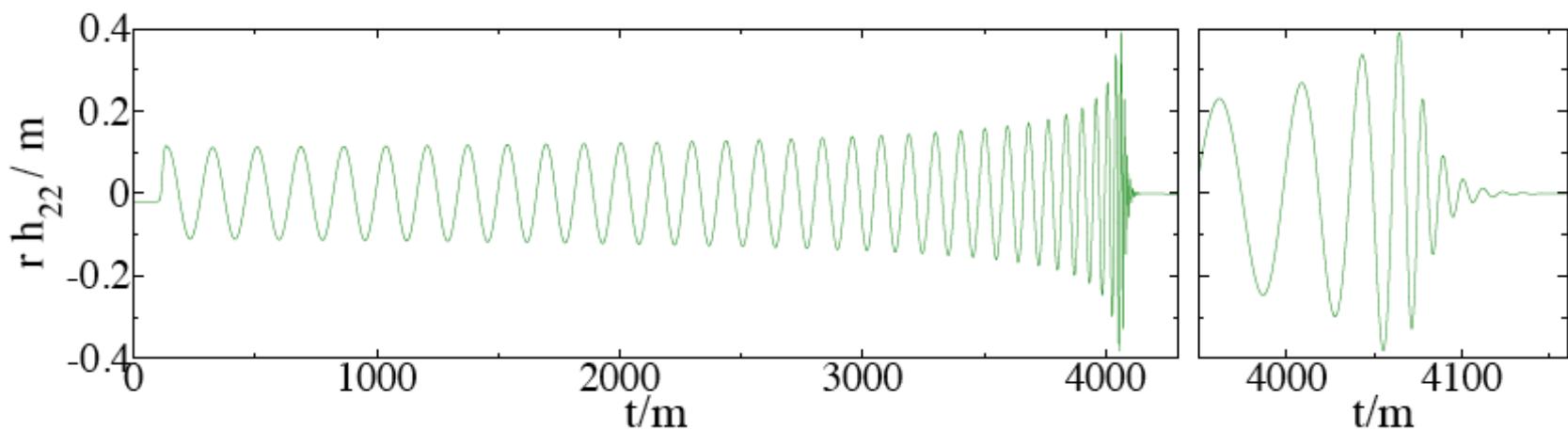
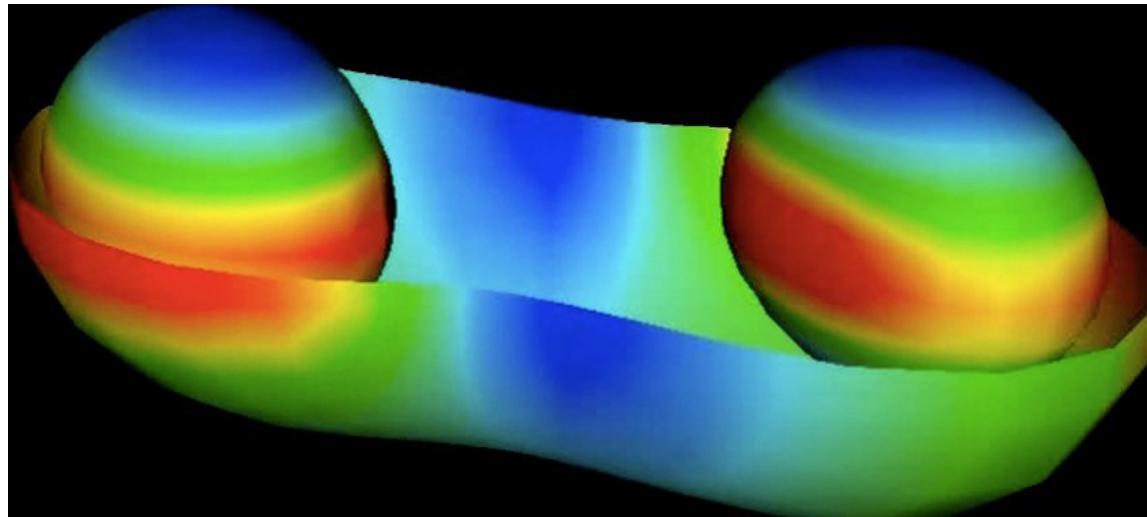


Masses in the Stellar Graveyard



GWTC-3: 90 merger events, with 2 NS/NS mergers, 3 NS/BH mergers

BH-BH Merger



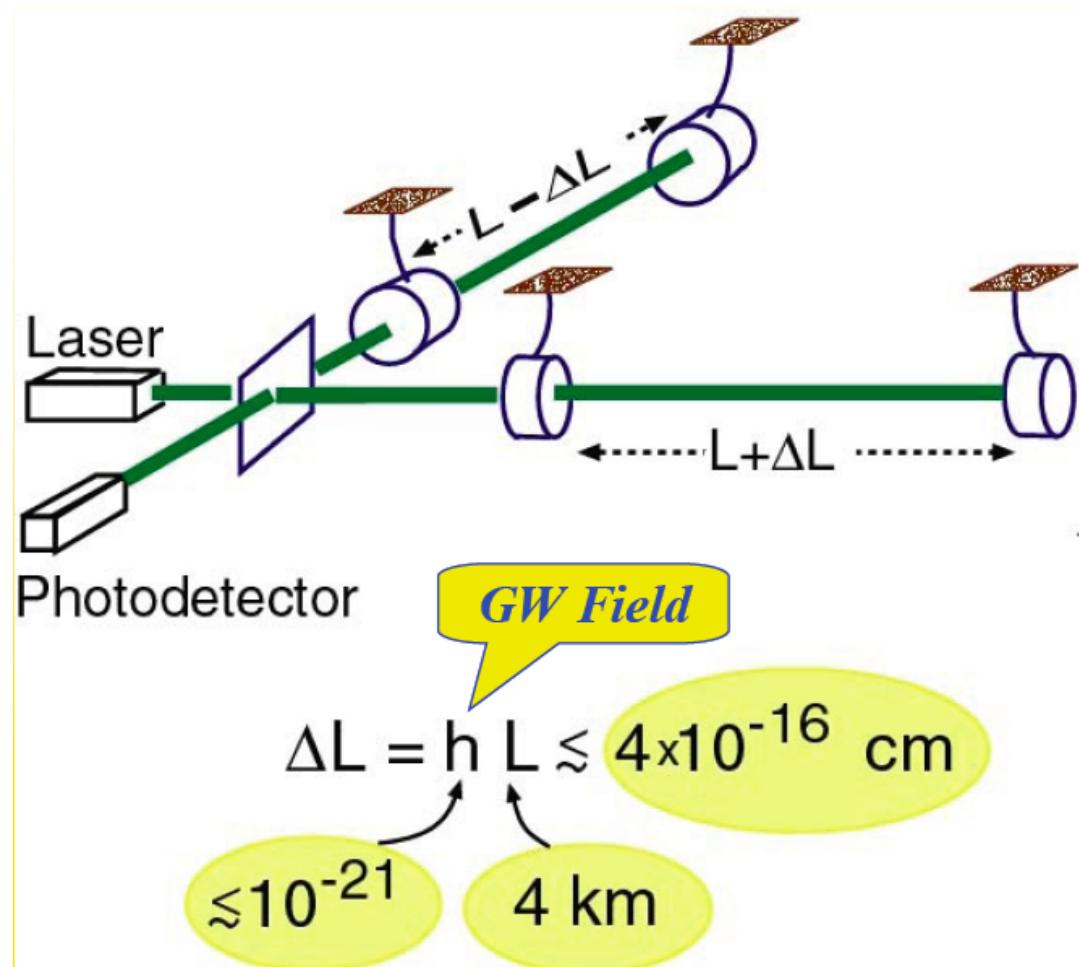
Cornell-Caltech collaboration

Gravitational Waves

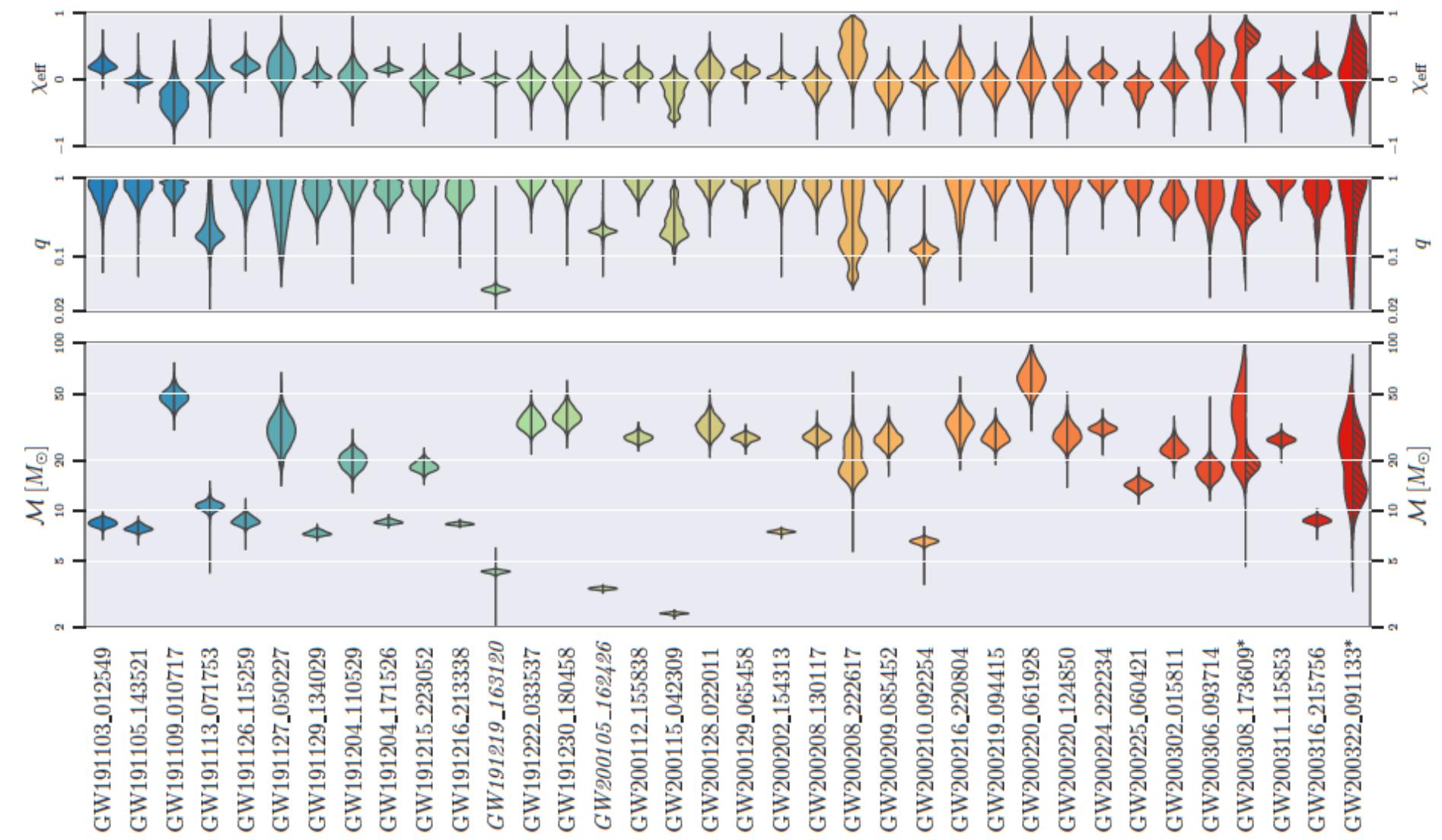
- Warpage of Spacetime
- Generated by time-dependent quadrupoles
- Detector response to passage of GWs:



Laser Gravitational Wave Interferometer



Kip Thorne



Gravitational waveform gives $M_1, M_2, \chi_{\text{eff}}$

$$\chi_{\text{eff}} \equiv \frac{m_1 \chi_1 + m_2 \chi_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

Formation of Merging BH Binaries

$$T_m \approx 10^{10} \text{ yrs} \left(\frac{60M_\odot}{m_1 + m_2} \right)^2 \left(\frac{15M_\odot}{\mu} \right) \left(\frac{a_0}{0.2 \text{ AU}} \right)^4 (1 - e_0^2)^{7/2}$$

Formation Channels of Merging BH Binaries

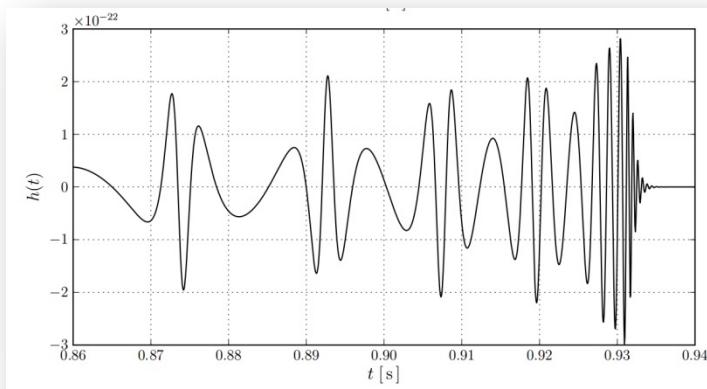
- Isolated Binary Evolution
- Dynamical Formation:
several flavors: star clusters, triples (multiples), AGN disks

How to distinguish different channels?

Rates (uncertain)?

Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band



Formation Channels of Merging BH Binaries

- Isolated Binary Evolution
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several flavors: star clusters, triples, AGN disks

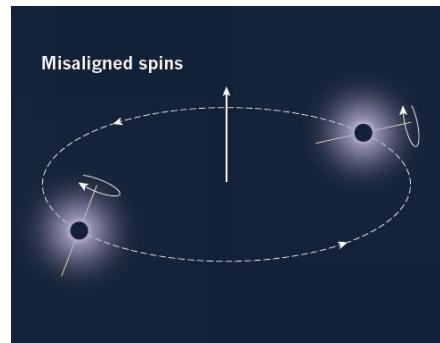
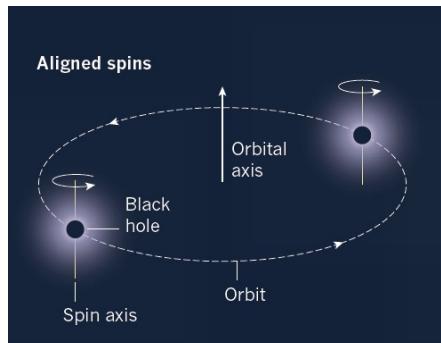
How to distinguish different channels?

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Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band

Spin-orbit misalignment



Formation Channels of Merging BH Binaries

- Isolated Binary Evolution
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several flavors: star clusters, triples, AGN disks

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Rates (uncertain)?

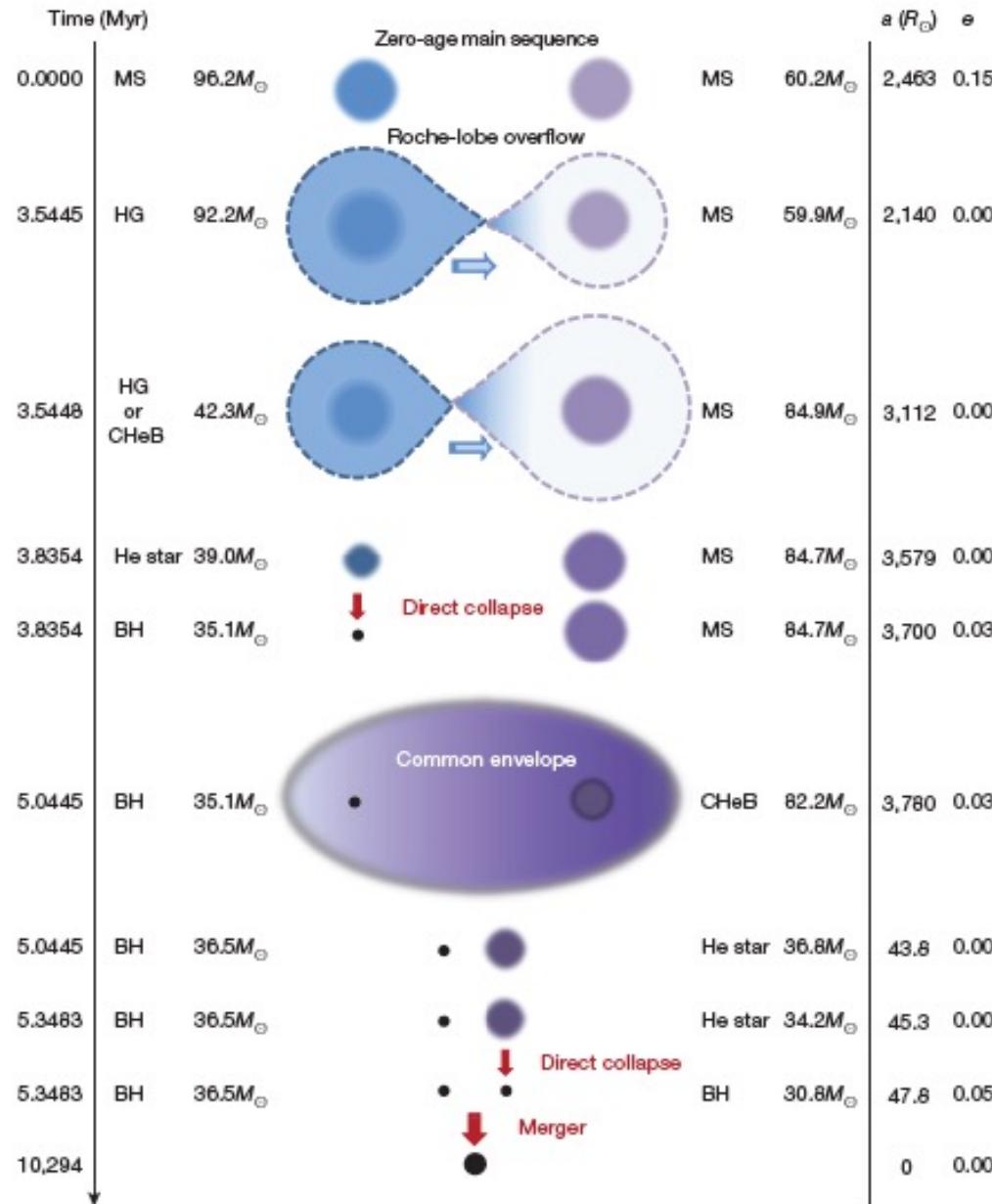
Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band

Spin-orbit misalignment

EM counterpart

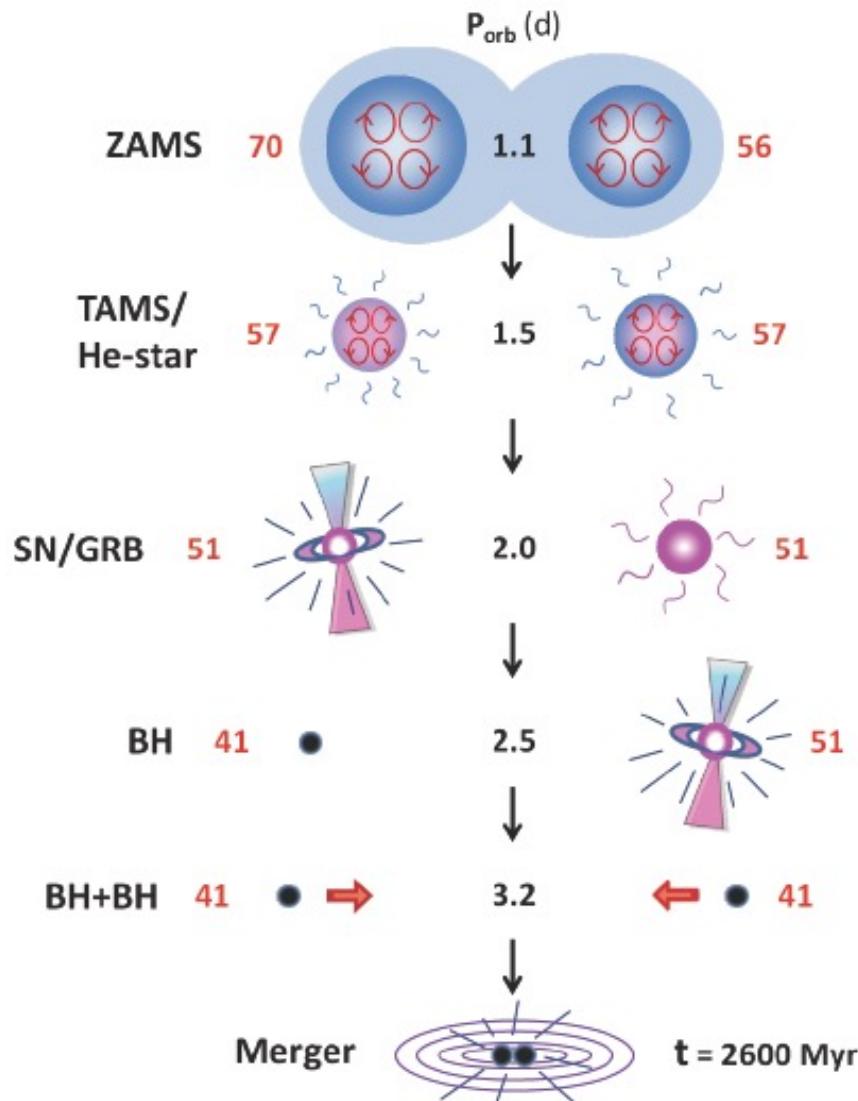
Isolated Binary Evolution Channel: Standard



many papers, uncertain physical ingredients (e.g. common envelope)

Produce
circular orbit at 10 Hz
mostly aligned spin-orbit

Isolated Binary Evolution Channel: Chemically homogeneous evolution



Produce
circular orbit at 10 Hz
mostly aligned spin-orbit

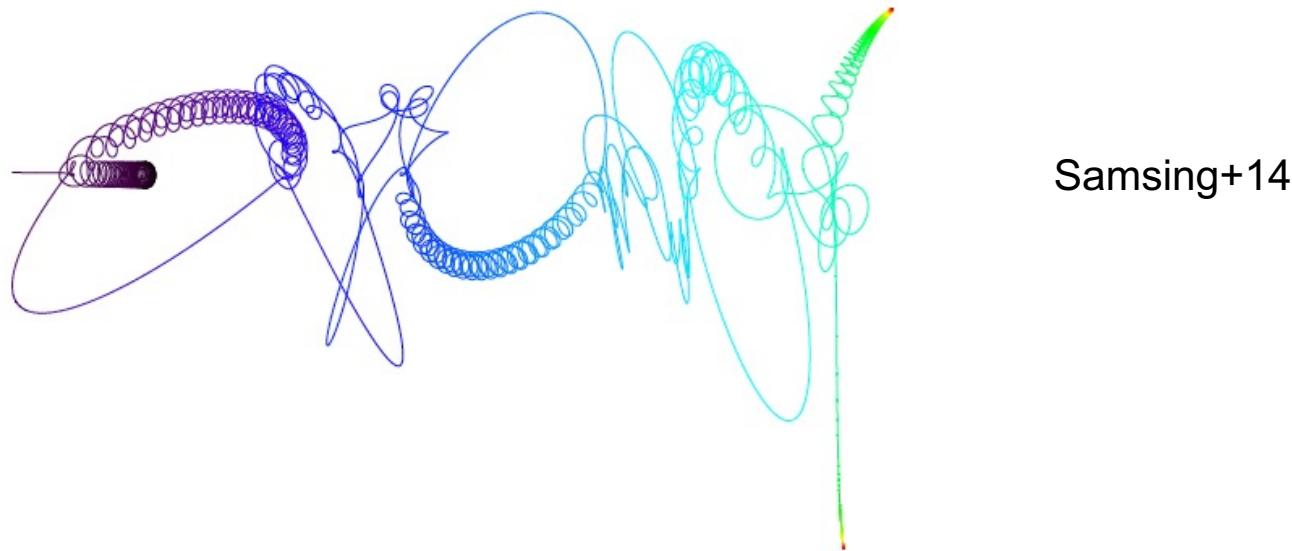
Dynamical Formation Channels

several flavors...

Dynamical Formation Channels

several flavors...

1. Dense clusters: binary-single scatterings → tight binary



Enough BHs in clusters? Kicks? GCs or Nuclear Star Clusters?

**Produce mostly circular orbit when enter LIGO band (10 Hz) ??
Expect random spin-orbit orientations**

Dynamical Formation Channels

several flavors...

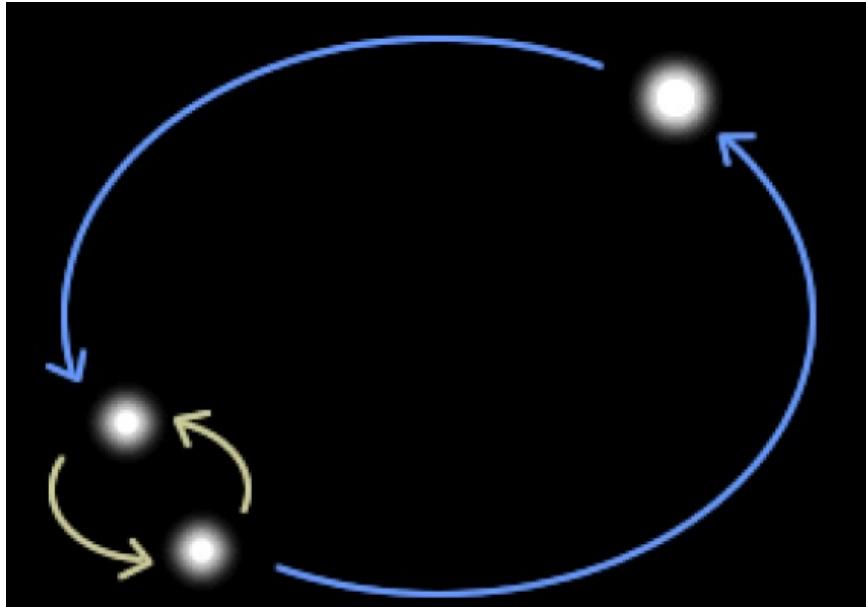
1. Dense clusters: binary-single scatterings → tight binary

2. Tertiary-Induced Mergers:

Mergers induced by (gentle) perturbations from tertiary companion
stellar triples in galactic field, binary around SMBH

Tertiary-Induced Binary Mergers

merger window, mass ratio, GR effects, spin-orbit misalignments



Bin Liu
(Cornell→Niels Bohr Inst)



Yubo Su
(Cornell, Ph.D.22
→ Princeton)

Liu & DL 2017-2022
Su et al.2021a,b

Previous/related works (in various contexts):

e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003;
Thompson 2011; Antonini et al. 2012,2014,2017,
Silsbee & Tremaine 2017; Petrovich & Antonini 2017...

Lidov-Kozai Effect

Can perturbation from the Moon make Earth's satellites fall?



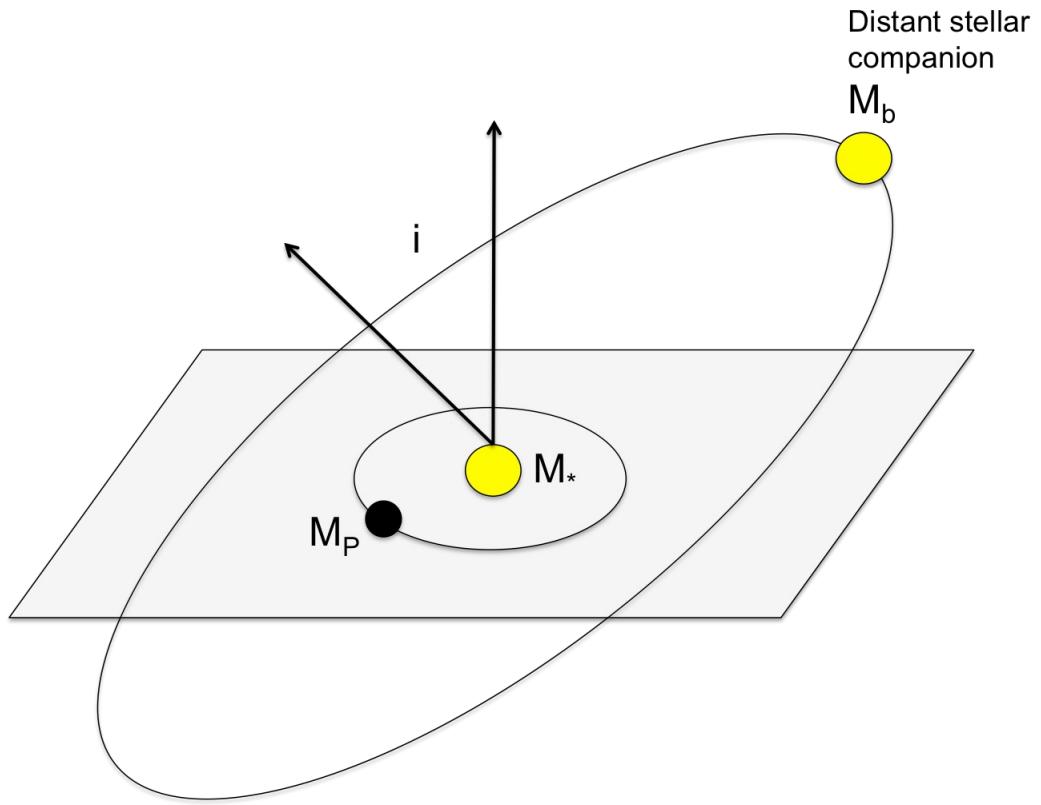
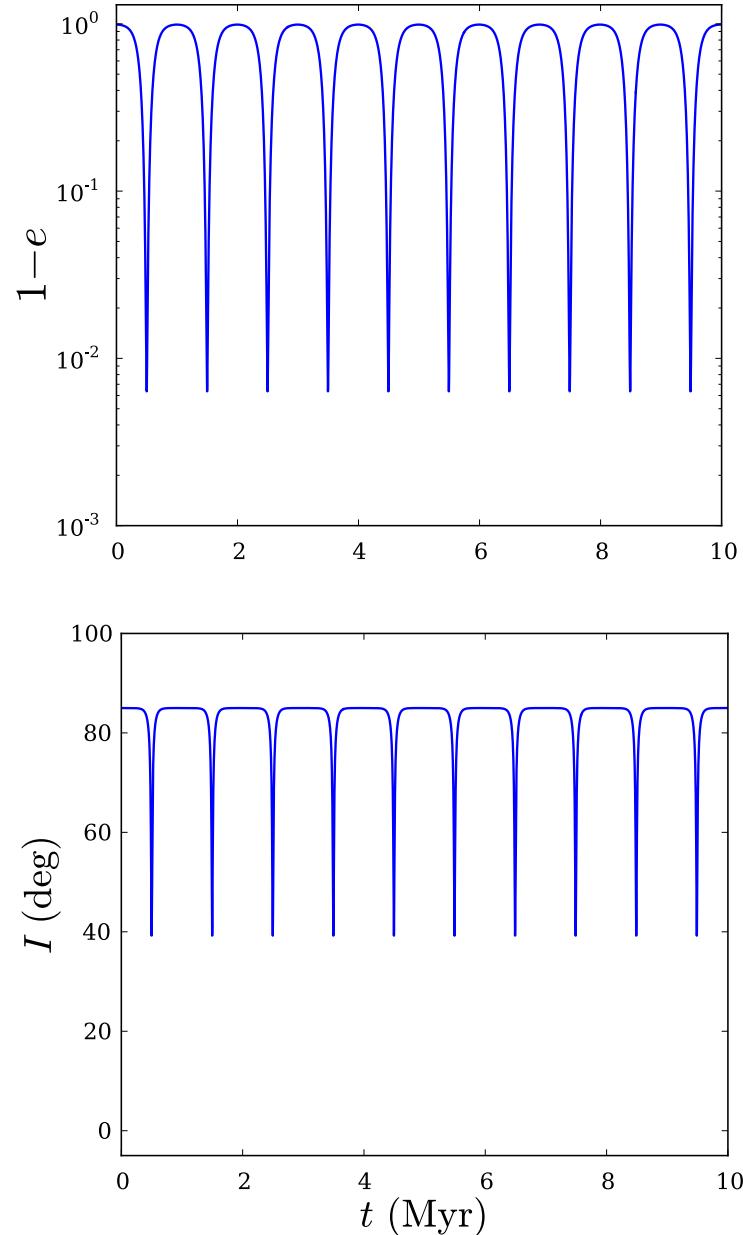
Planet. Space Sci., 1962, Vol. 9, pp. 719 to 759. Pergamon Press Ltd. Printed in Northern Ireland

THE EVOLUTION OF ORBITS OF ARTIFICIAL SATELLITES OF PLANETS UNDER THE ACTION OF GRAVITATIONAL PERTURBATIONS OF EXTERNAL BODIES

M. L. LIDOV

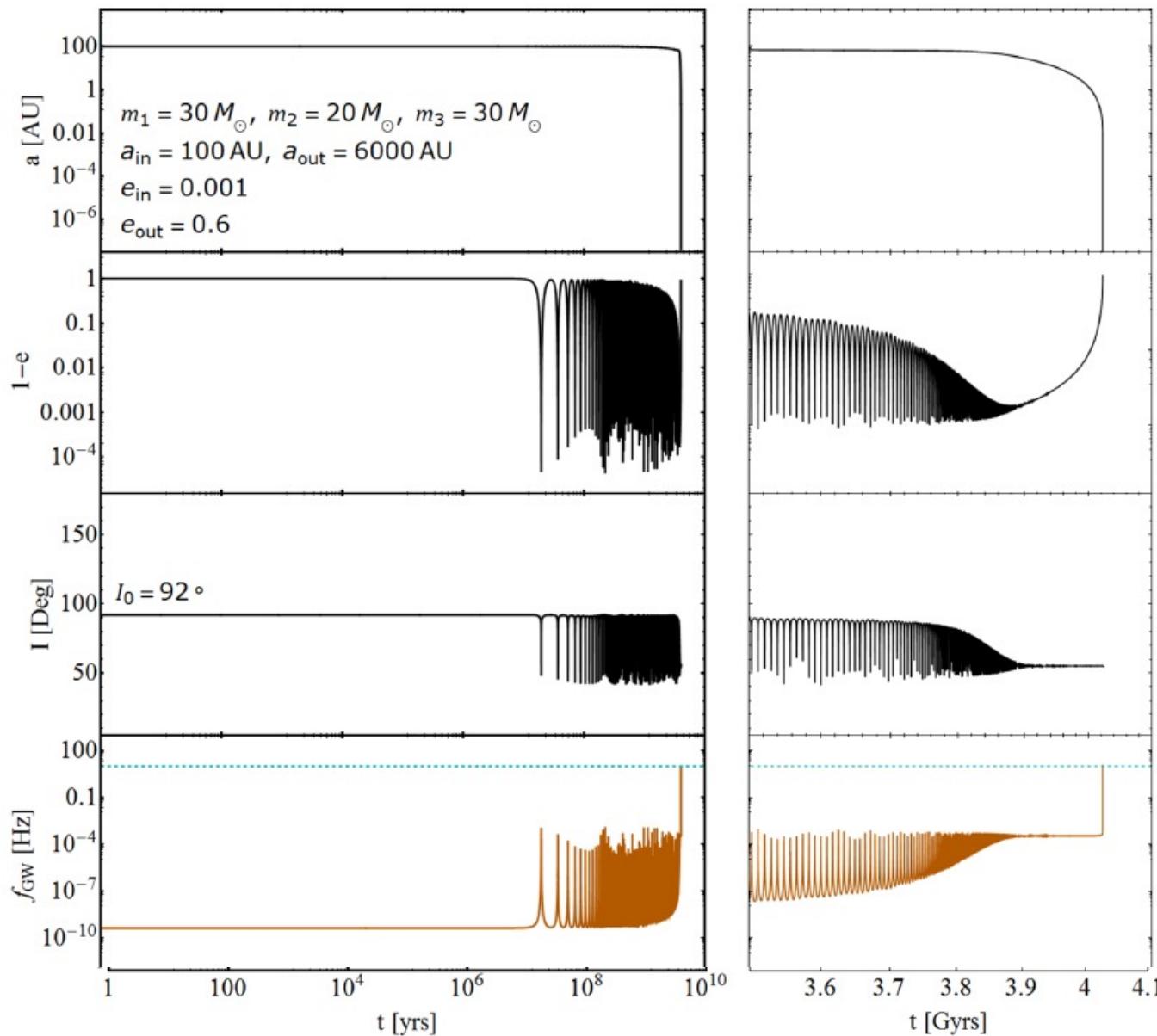
Translated by H. F. Cleaves from *Iskusstvennye Sputniki Zemli*, No. 8, p. 5, 1961.

Lidov-Kozai Effect



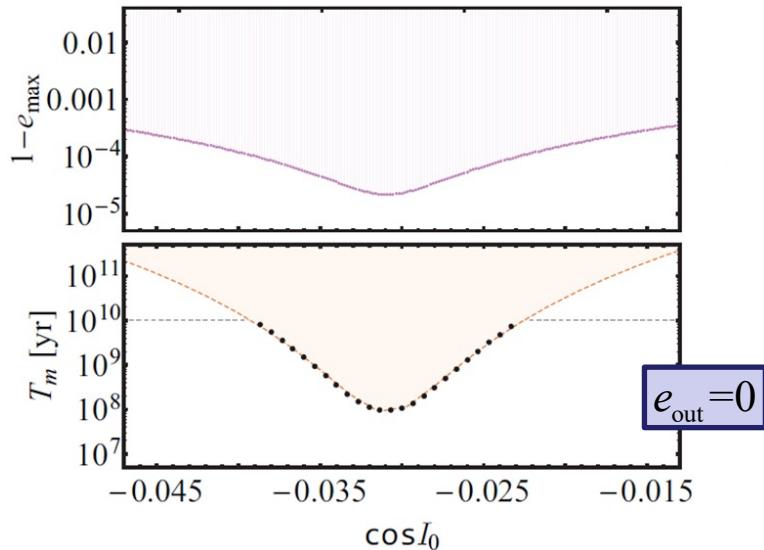
- Eccentricity and inclination oscillations induced if $i > 40$ degrees.
- If i large (85-90 degrees), get extremely large eccentricities ($e > 0.99$)

LK oscillation + Gravitation Radiation



Inclination window for merger

Fixed inner binary: $m_1=30M_{\odot}$, $m_2=20M_{\odot}$, $a_{\text{in},0}=100\text{AU}$



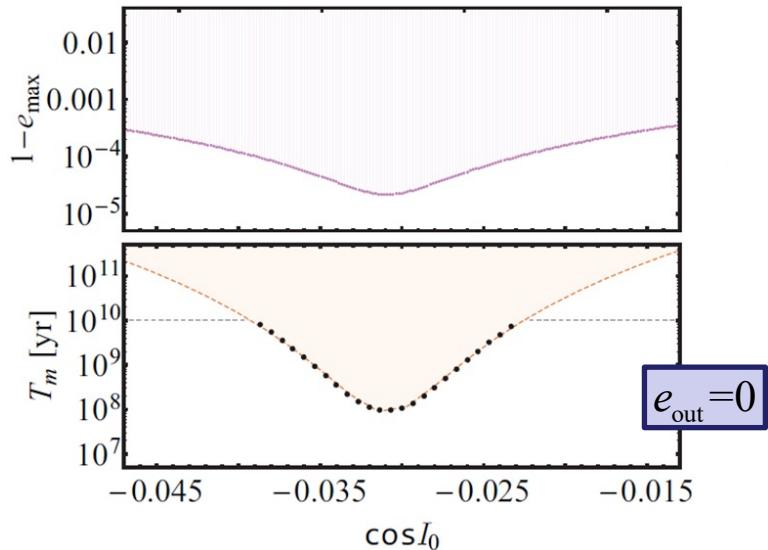
Fixed quadrupole strength $\frac{m_3}{a_{\text{out,eff}}^3}$

Quadrupole LK:
 e_{max} vs I_0 analytic:
LK driving compete with
GR apsidal precession

Merger window (almost)
analytic

Inclination window for merger

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{\text{in},0}=100\text{AU}$



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Quadrupole LK:
 e_{max} vs I_0 analytic:
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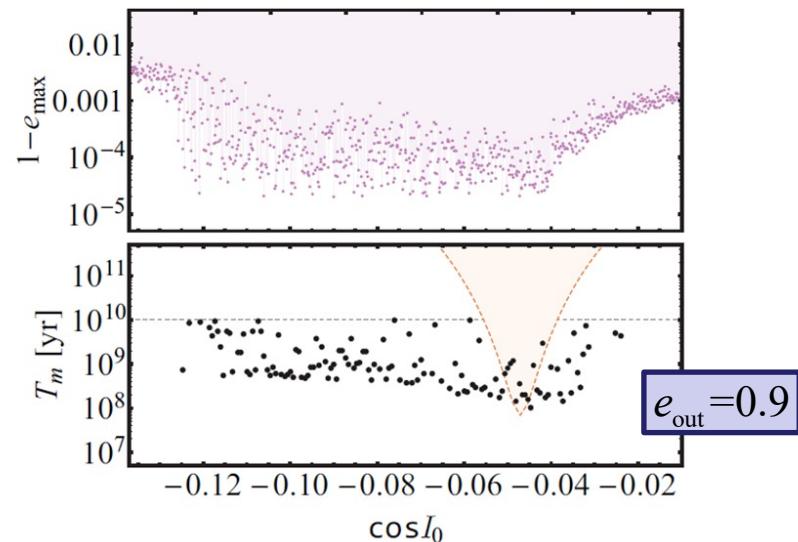
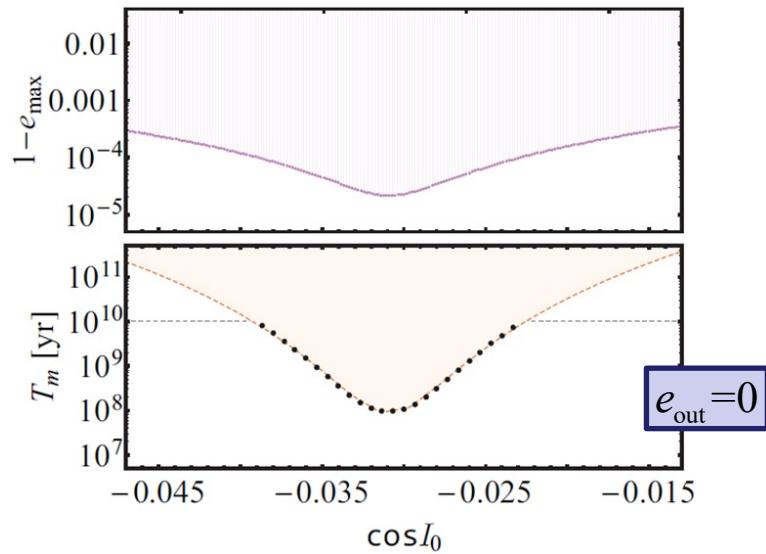
Merger window (almost)
analytic

Octupole effect makes orbital evolution “chaotic”:

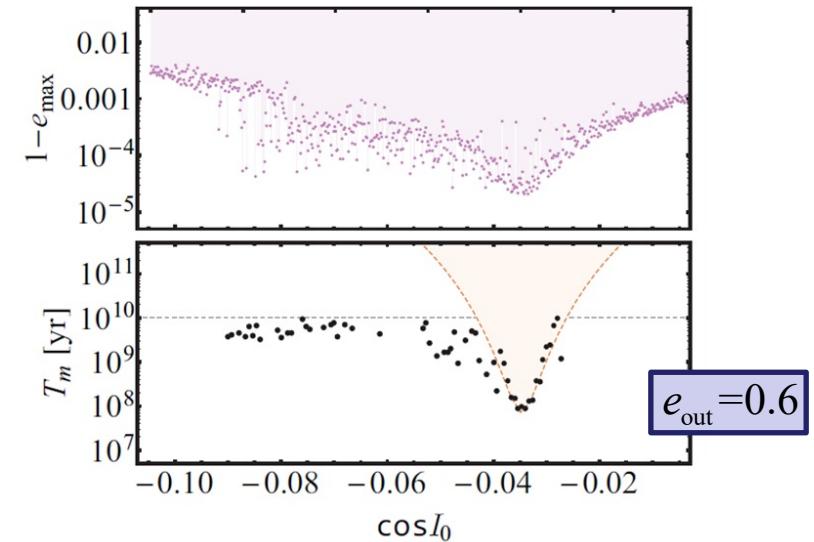
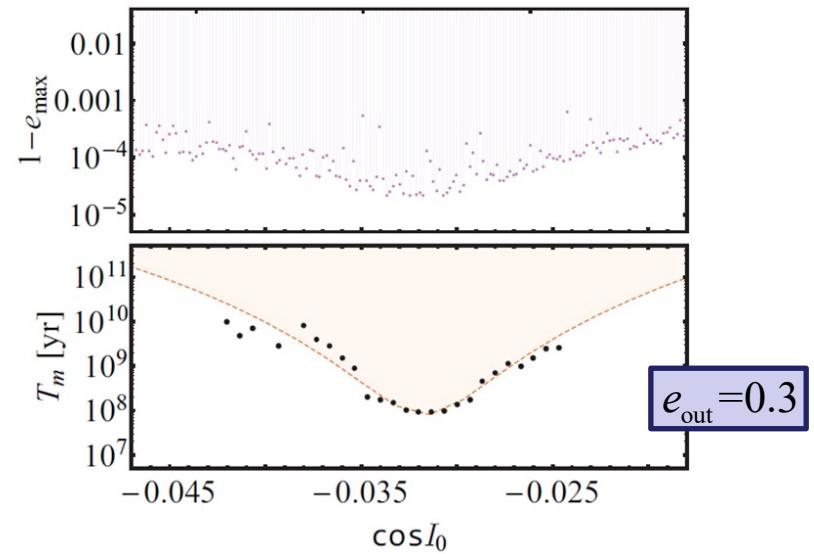
$$\epsilon_{\text{oct}} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\text{out}}} \frac{e_{\text{out}}}{1 - e_{\text{out}}^2}$$

Inclination window for merger

Fixed inner binary: $m_1=30M_{\odot}$, $m_2=20M_{\odot}$, $a_{\text{in},0}=100\text{AU}$

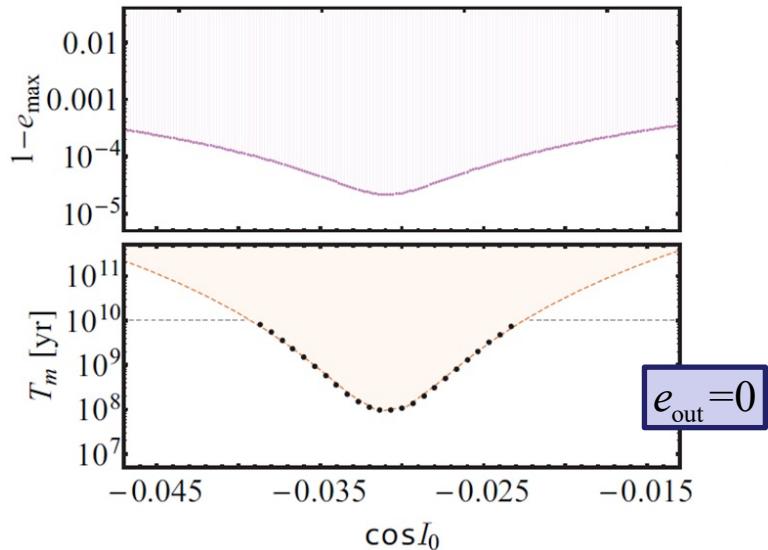


Fixed quadrupole strength $\frac{m_3}{a_{\text{out,eff}}^3}$

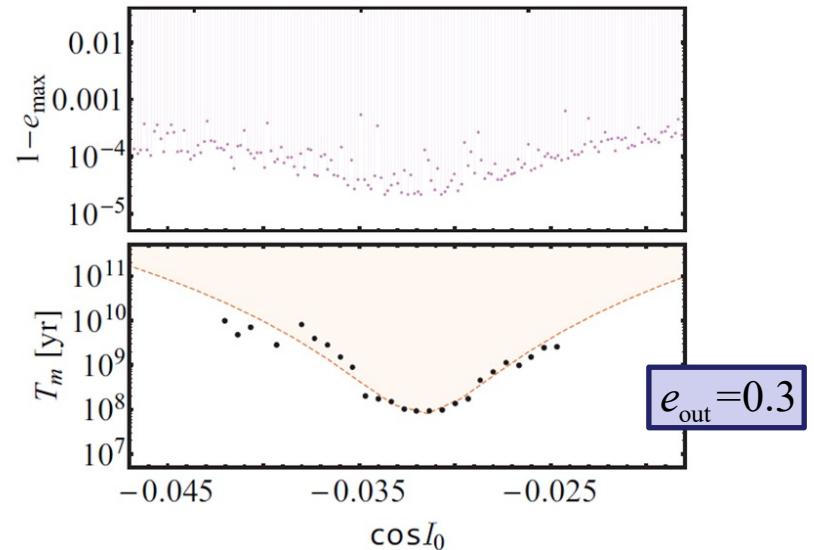


Inclination window for merger

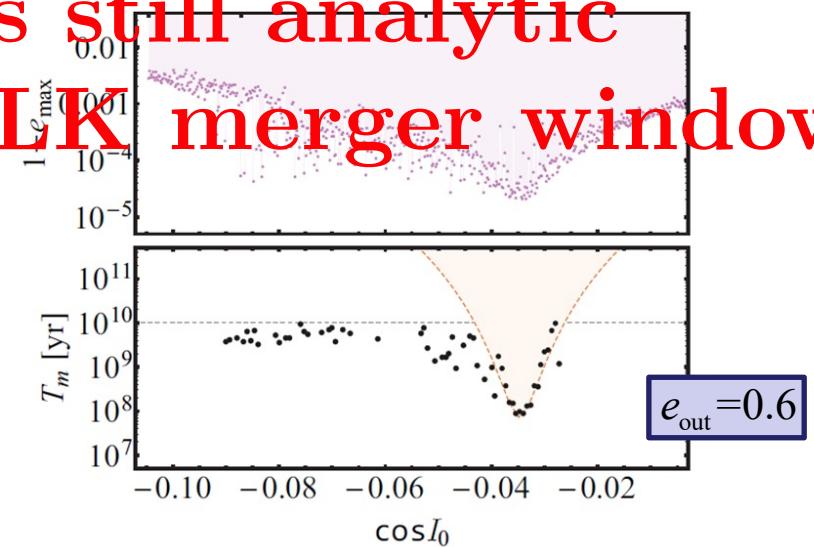
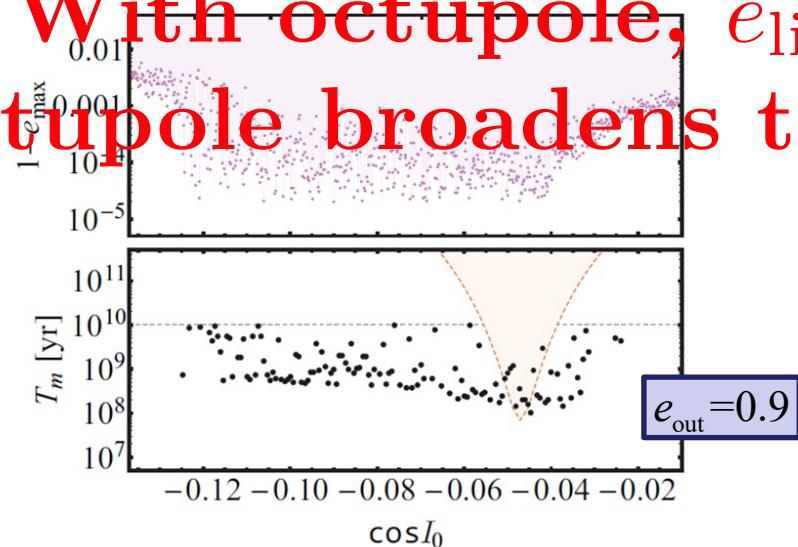
Fixed inner binary: $m_1=30M_{\odot}$, $m_2=20M_{\odot}$, $a_{\text{in},0}=100\text{AU}$



Fixed m_3/a_{out}^3 value



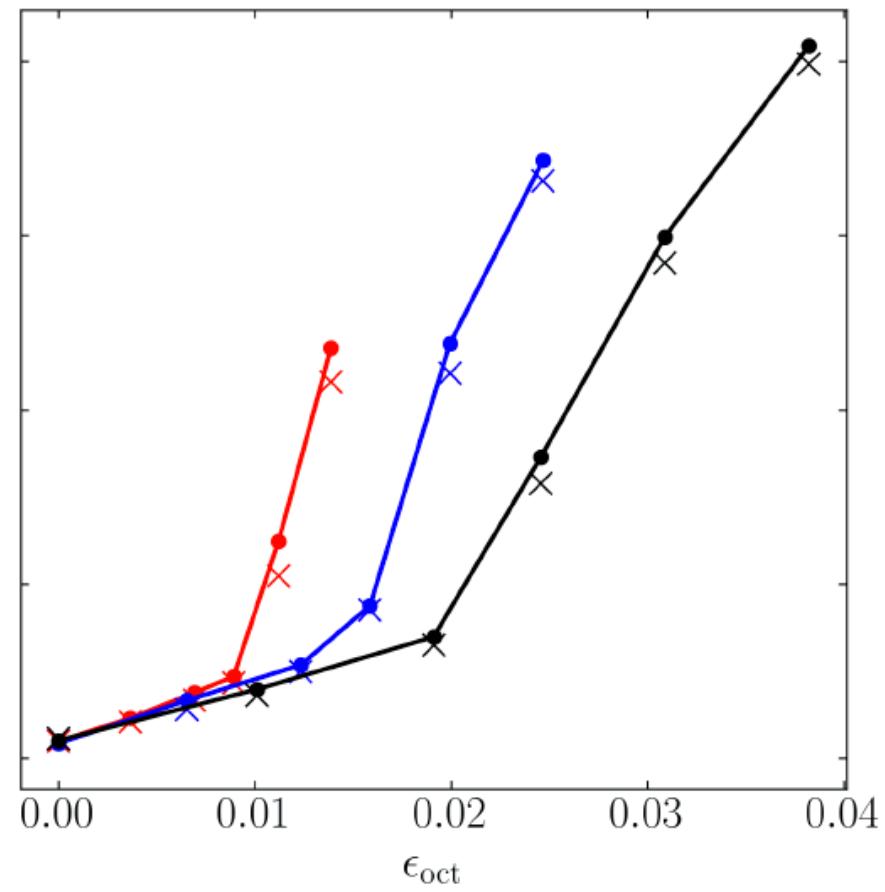
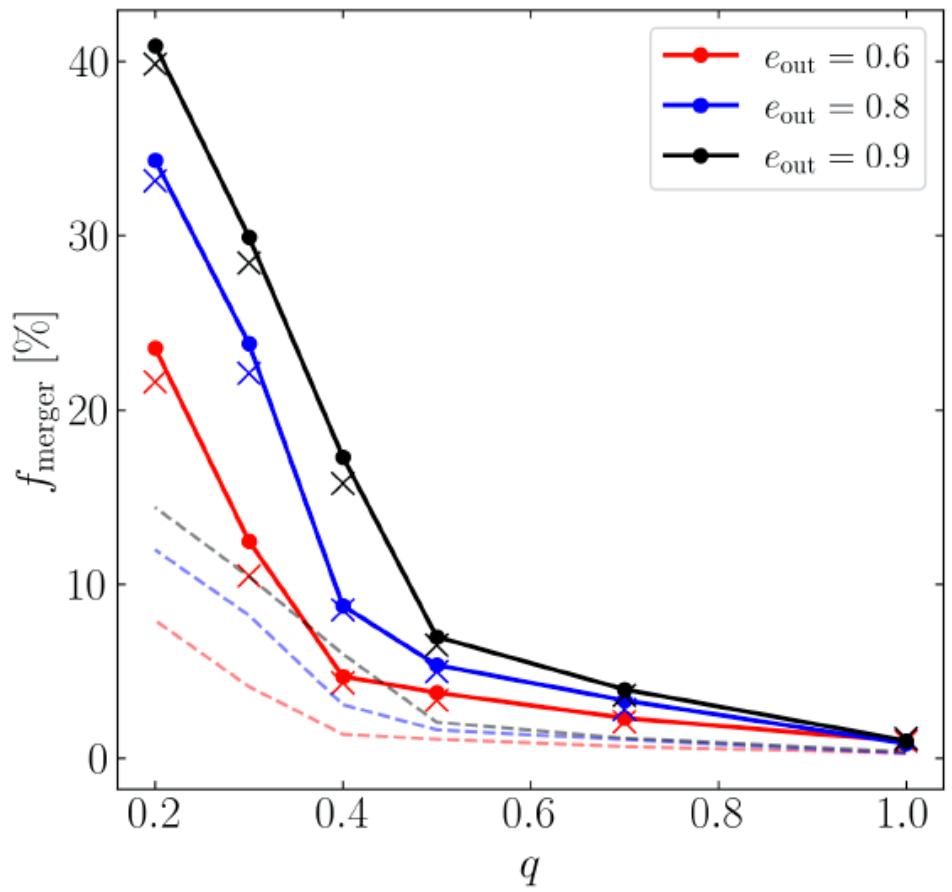
**With octupole, e_{lim} is still analytic
Octupole broadens the LK merger window**



Octupole effect depends on

$$\epsilon_{\text{oct}} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\text{out}}} \frac{e_{\text{out}}}{1 - e_{\text{out}}^2}$$

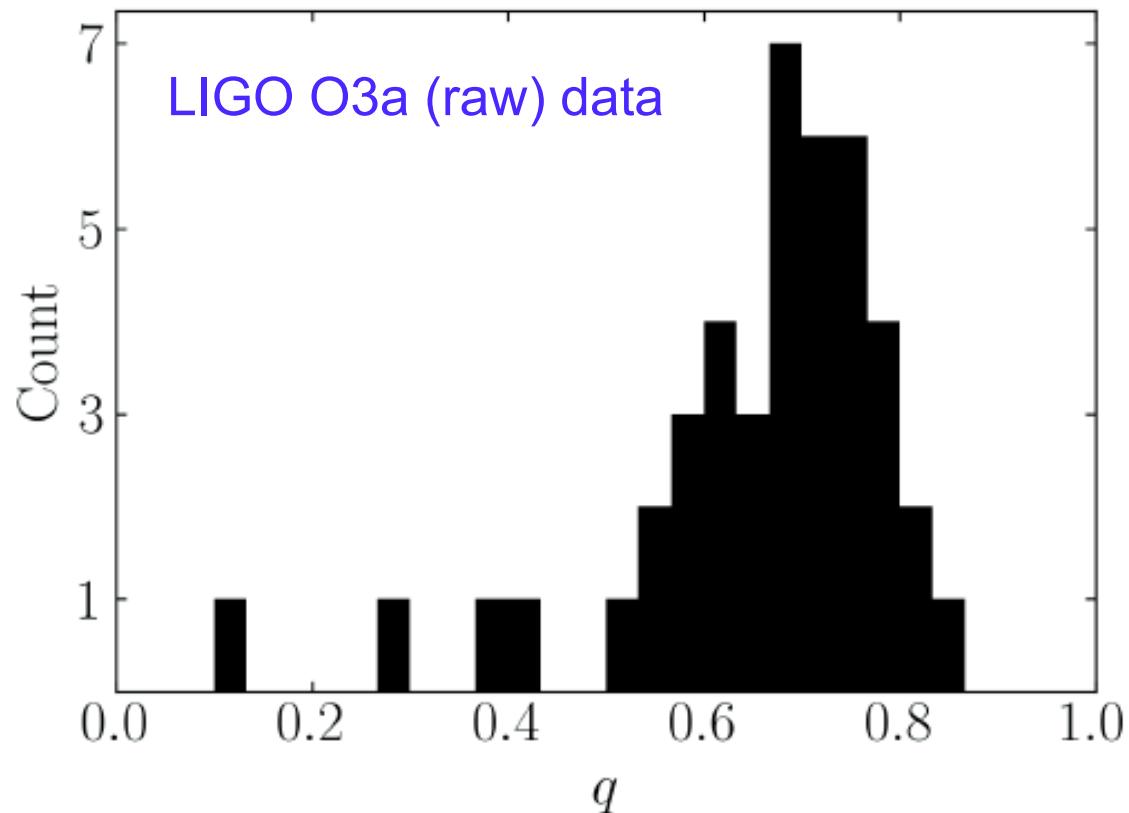
- Large e_{out} and/or small $q = m_2/m_1$ increases merger window



$a = 100 \text{ AU}$, $a_{\text{out,eff}} = 3600 \text{ AU}$, $m_{12} = 50M_{\odot}$, $m_3 = 30M_{\odot}$

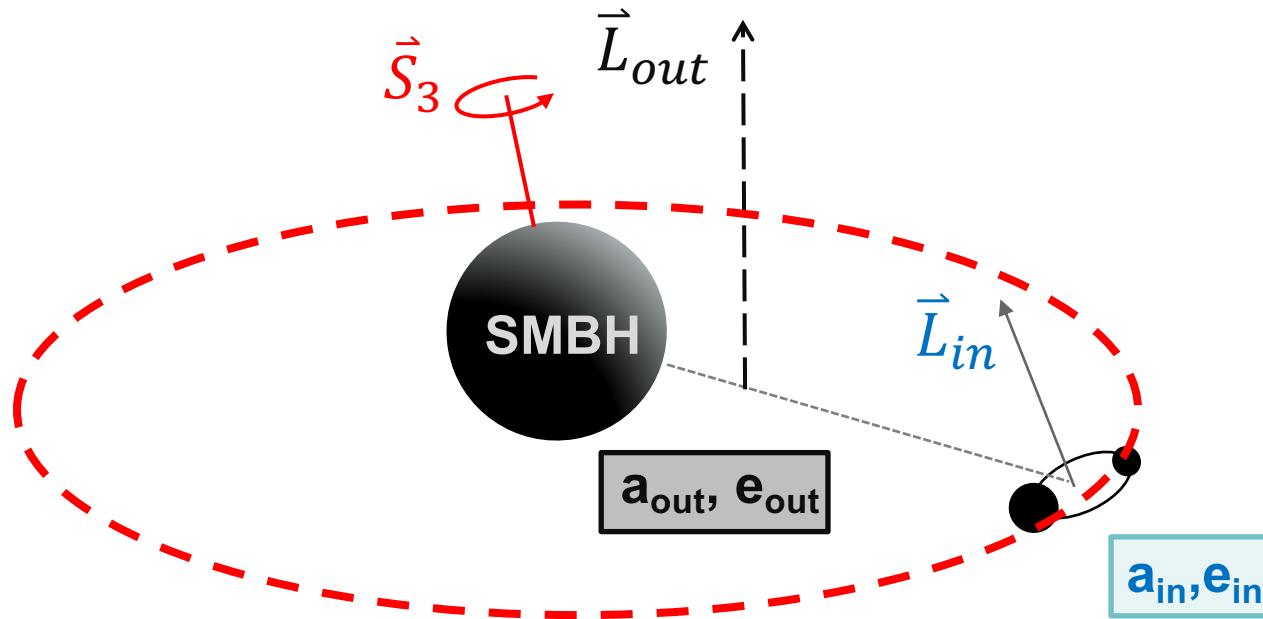
Su et al. 2021

Is this compatible with the observed q-distribution
of merging BH binaries?



- Depends on the initial q -distribution of BH binaries.
- Maybe incompatible with data if octupole effect is strong...

What happens if the tertiary is a Supermassive BH ?

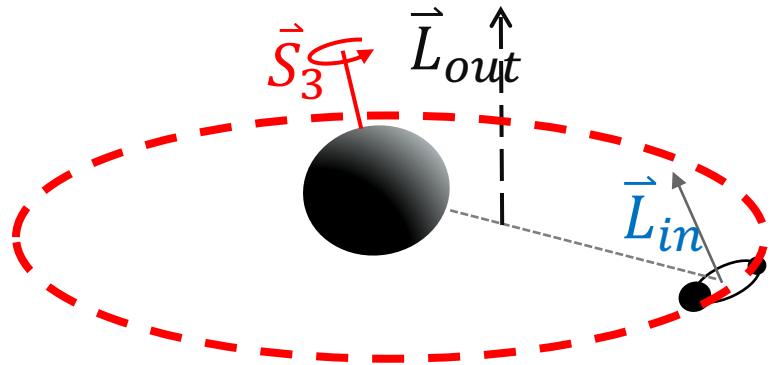


Since $a_{\text{out}} \gg a$, octupole effect is negligible

$$\epsilon_{\text{oct}} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\text{out}}} \frac{e_{\text{out}}}{1 - e_{\text{out}}^2}$$

Relativistic Effects induced by the SMBH

Three leading-order GR effects (recognizing L_{in} behaves like a “spin”)



Effect I: de-Sitter-like Precession of L_{in} around L_{out}

--- 1.5 PN

$$\left\{ \begin{array}{l} \frac{d\mathbf{L}}{dt} \Big|_{L_{in}L_{out}} = \Omega_{L_{in}L_{out}}^{(GR)} \hat{\mathbf{L}}_{out} \times \mathbf{L}, \\ \frac{d\mathbf{e}}{dt} \Big|_{L_{in}L_{out}} = \Omega_{L_{in}L_{out}}^{(GR)} \hat{\mathbf{L}}_{out} \times \mathbf{e}, \end{array} \right.$$

Effect II: Precession of L_{out} around S_3

--- 1.5 PN

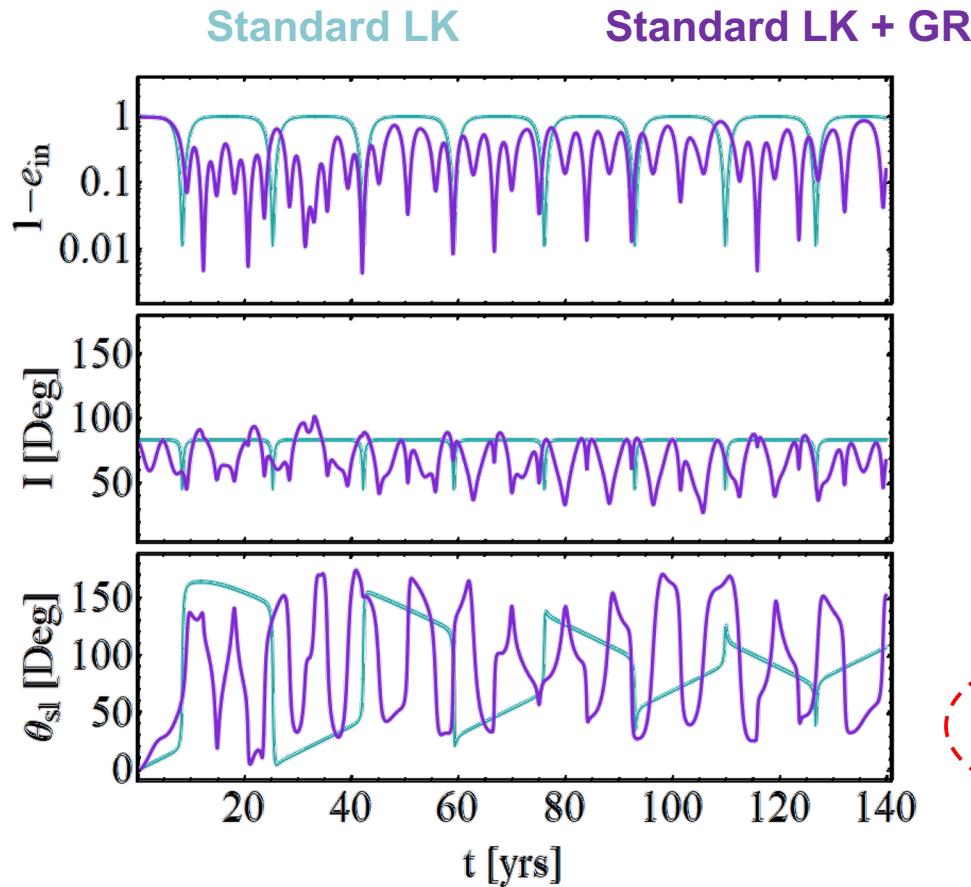
$$\left\{ \begin{array}{l} \frac{d\mathbf{L}_{out}}{dt} \Big|_{L_{out}S_3} = \Omega_{L_{out}S_3} \hat{\mathbf{S}}_3 \times \mathbf{L}_{out}, \\ \frac{d\mathbf{e}_{out}}{dt} \Big|_{L_{out}S_3} = \Omega_{L_{out}S_3} \hat{\mathbf{S}}_3 \times \mathbf{e}_{out} \\ \quad - 3\Omega_{L_{out}S_3} (\hat{\mathbf{L}}_{out} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{out} \times \mathbf{e}_{out} \end{array} \right.$$

Effect III: Lense-Thirring Precession of L_{in} around S_3

--- 2 PN

$$\left\{ \begin{array}{l} \frac{d\mathbf{L}}{dt} \Big|_{L_{in}S_3} = \Omega_{L_{in}S_3} \hat{\mathbf{S}}_3 \times \mathbf{L} \\ \quad - 3\Omega_{L_{in}S_3} (\hat{\mathbf{L}}_{out} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{out} \times \mathbf{L}, \\ \frac{d\mathbf{e}}{dt} \Big|_{L_{in}S_3} = \Omega_{L_{in}S_3} \hat{\mathbf{S}}_3 \times \mathbf{e} \\ \quad - 3\Omega_{L_{in}S_3} (\hat{\mathbf{L}}_{out} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{out} \times \mathbf{e}. \end{array} \right.$$

Evolution Example



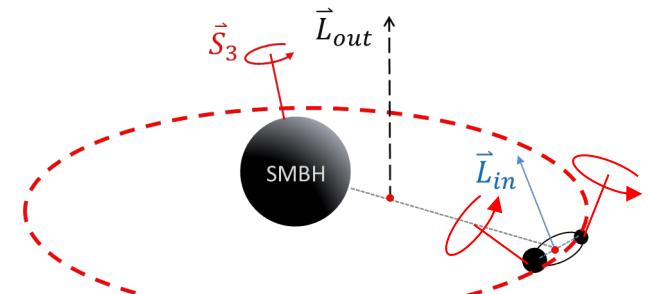
System Parameters:

$$(m_1, m_2, m_3) = (30, 20, 2.3 \times 10^9) M_\odot$$

$$(a_{\text{in},0}, a_{\text{out}}) = (0.1, 500) \text{ AU}$$

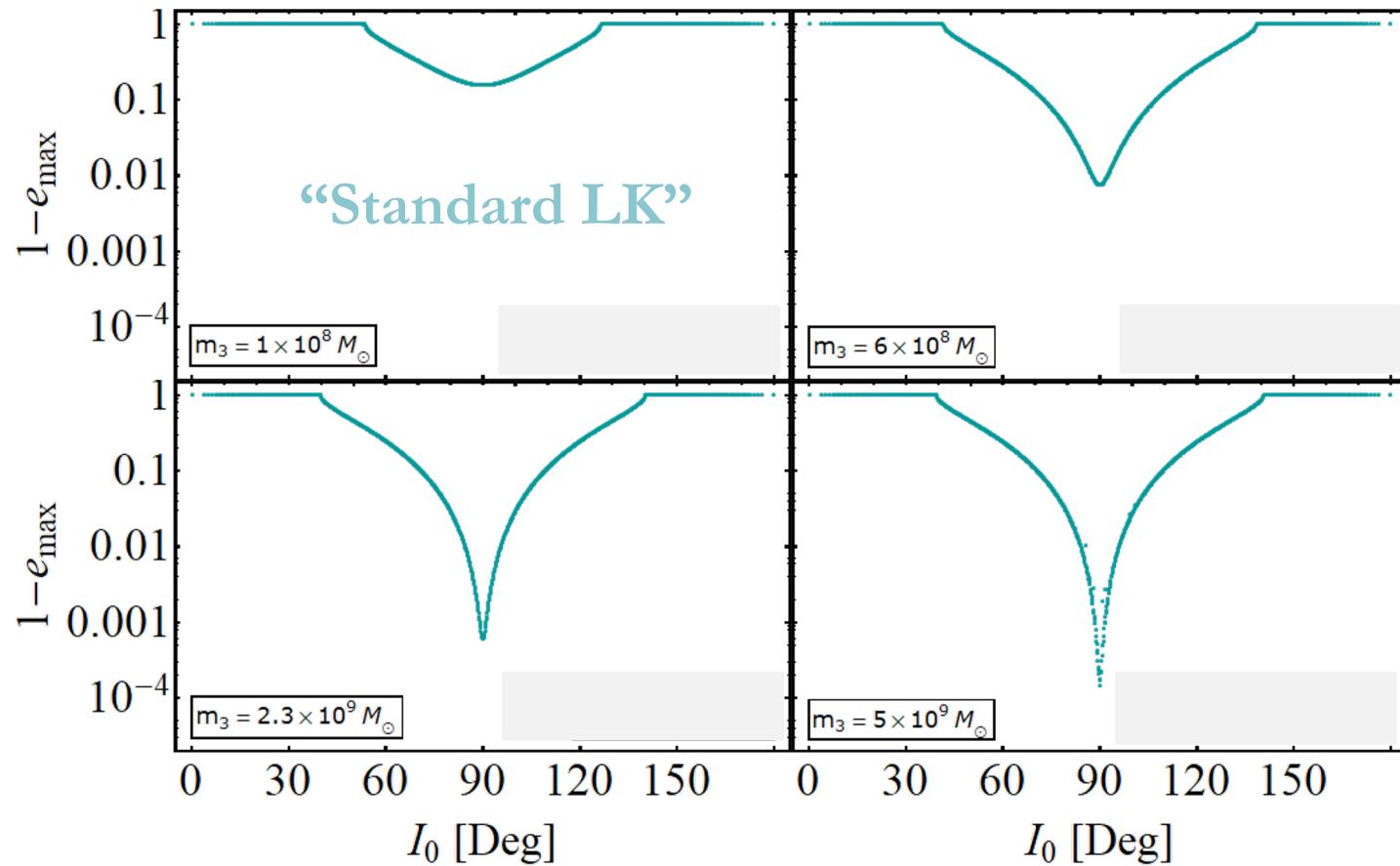
$$(e_{\text{in},0}, e_{\text{out},0}) = 0.001$$

$$I_0 = 84^\circ$$



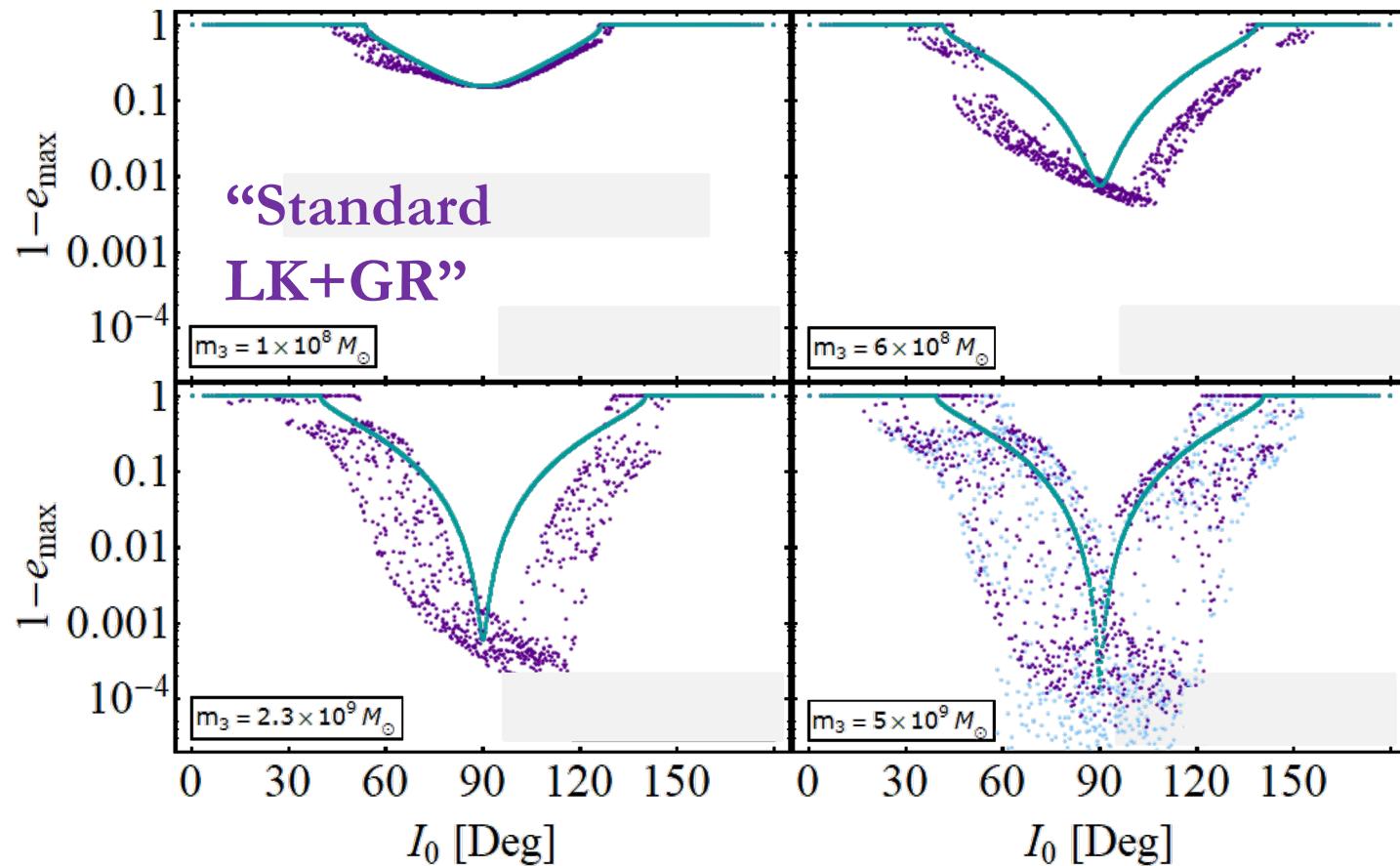
Eccentricity Excitation

$(m_1, m_2) = (30, 20) M_\odot$; $(a_{in,0}, a_{out}) = (0.1, 500)$ AU; Circular Orbits



Eccentricity Excitation

$(m_1, m_2) = (30, 20) M_\odot$; $(a_{\text{in},0}, a_{\text{out}}) = (0.1, 500)$ AU; Circular Orbits

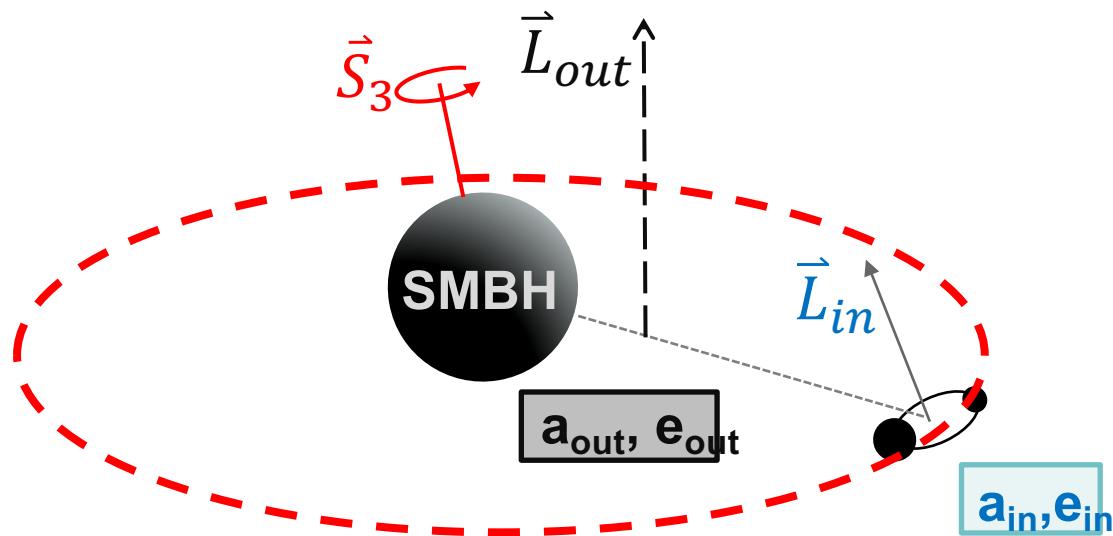


==> GR effects broaden the merger window

GR effects broaden Merger window: Why?

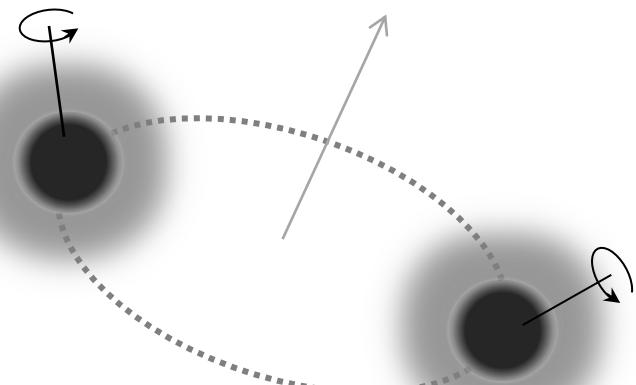
Inclination Resonance

$$\gamma \equiv \frac{\Omega_{\text{LinLout}}}{\Omega_{\text{LoutS}_3}} = \frac{\Omega_{\text{LinLout}}^{(\text{N})} + \Omega_{\text{LinLout}}^{(\text{GR})}}{\Omega_{\text{LoutS}_3}}$$

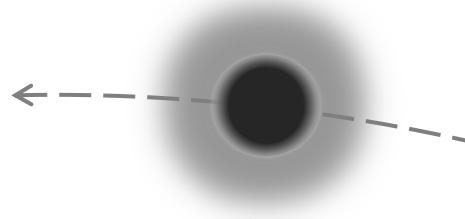


Mutual inclination is excited when the two precession rates are equal

What about the BH Spins of merging BHs?



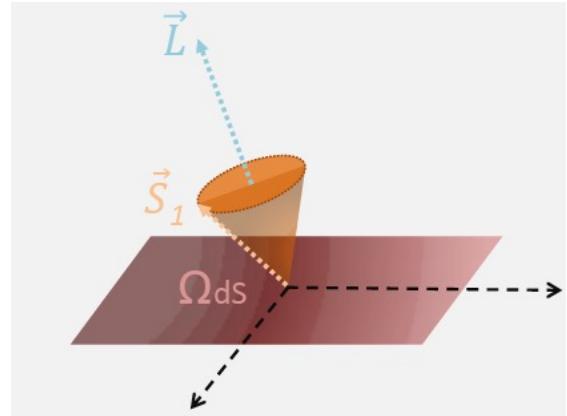
?



Spin-Orbit Coupling

The de Sitter precession of spin around the angular momentum axis of the binary

$$\frac{d\hat{S}_1}{dt} = \Omega_{ds} \hat{L} \times \hat{S}_1 \quad \Omega_{ds} = \frac{3Gn(m_2 + \mu/3)}{2c^2 a(1-e^2)}$$

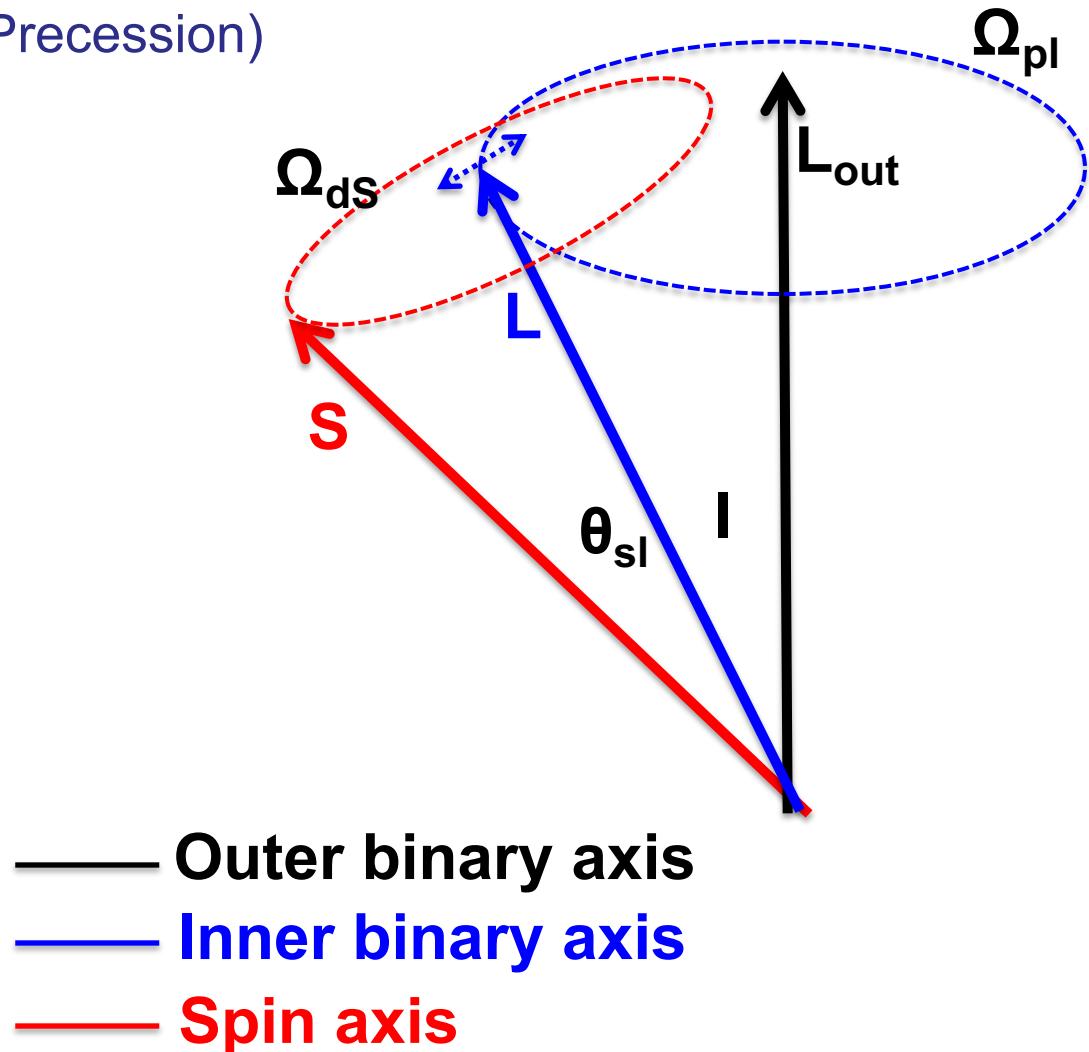


BH spin dynamics during LK oscillations

$$\frac{d\hat{\mathbf{S}}_1}{dt} = \Omega_{\text{dS}} \hat{\mathbf{L}} \times \hat{\mathbf{S}}_1 \quad (\text{de Sitter Precession})$$

But \mathbf{L} precesses and nutates during LK oscillations

$$\Omega_{\text{pl}} \simeq \frac{3(1+e^2)}{t_{\text{LK}} \sqrt{1-e^2}} |\sin 2I|$$



BH spin dynamics during LK oscillations

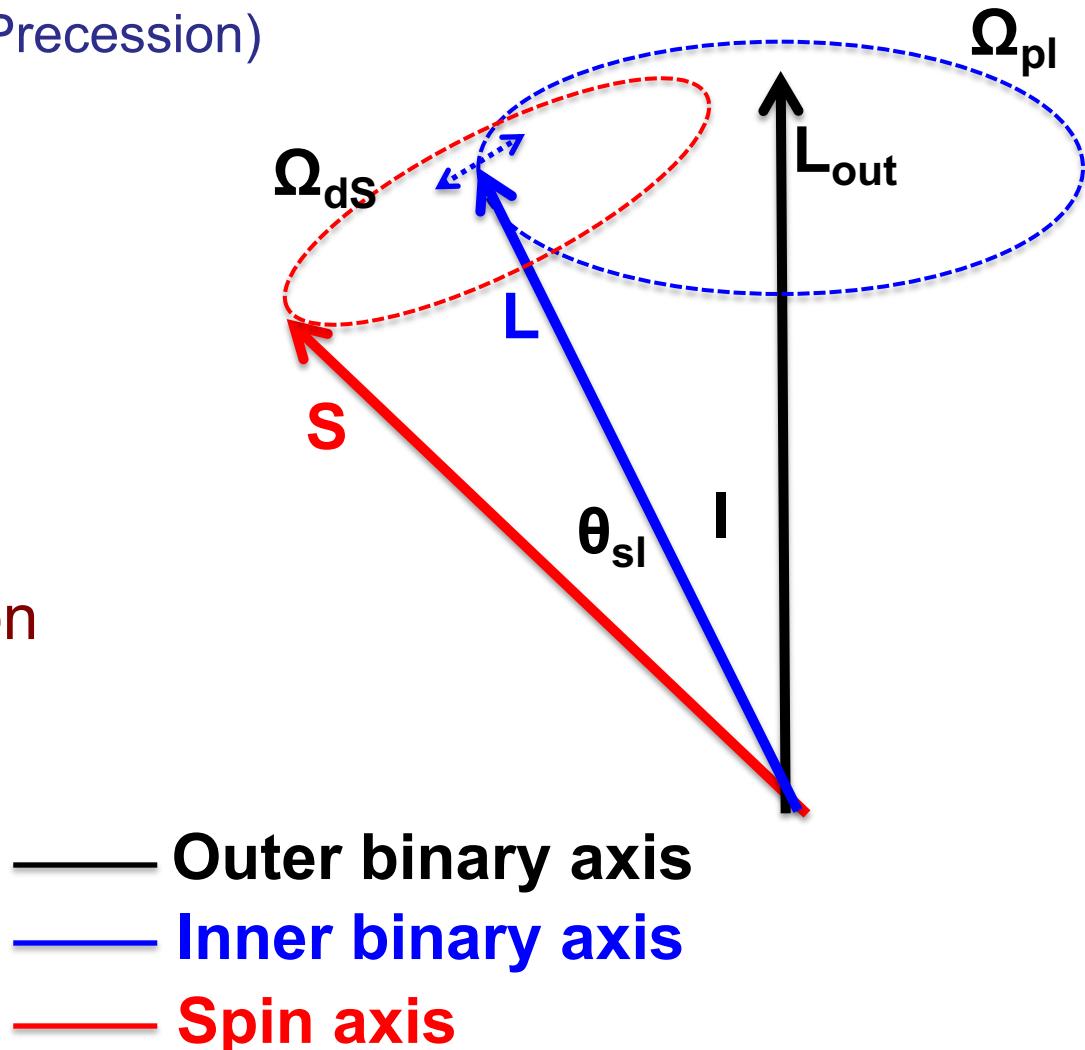
$$\frac{d\hat{\mathbf{S}}_1}{dt} = \Omega_{\text{dS}} \hat{\mathbf{L}} \times \hat{\mathbf{S}}_1 \quad (\text{de Sitter Precession})$$

But \mathbf{L} precesses and nutates during LK oscillations

$$\Omega_{\text{pl}} \simeq \frac{3(1+e^2)}{t_{\text{LK}} \sqrt{1-e^2}} |\sin 2I|$$

Spin dynamics depends on

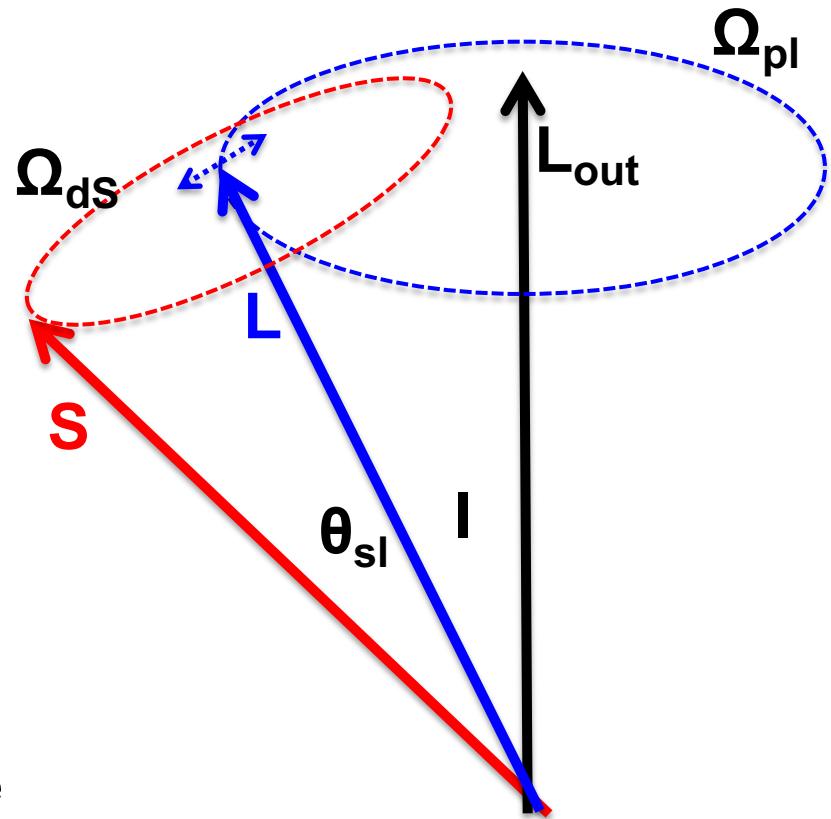
Ω_{dS} vs Ω_{pl}

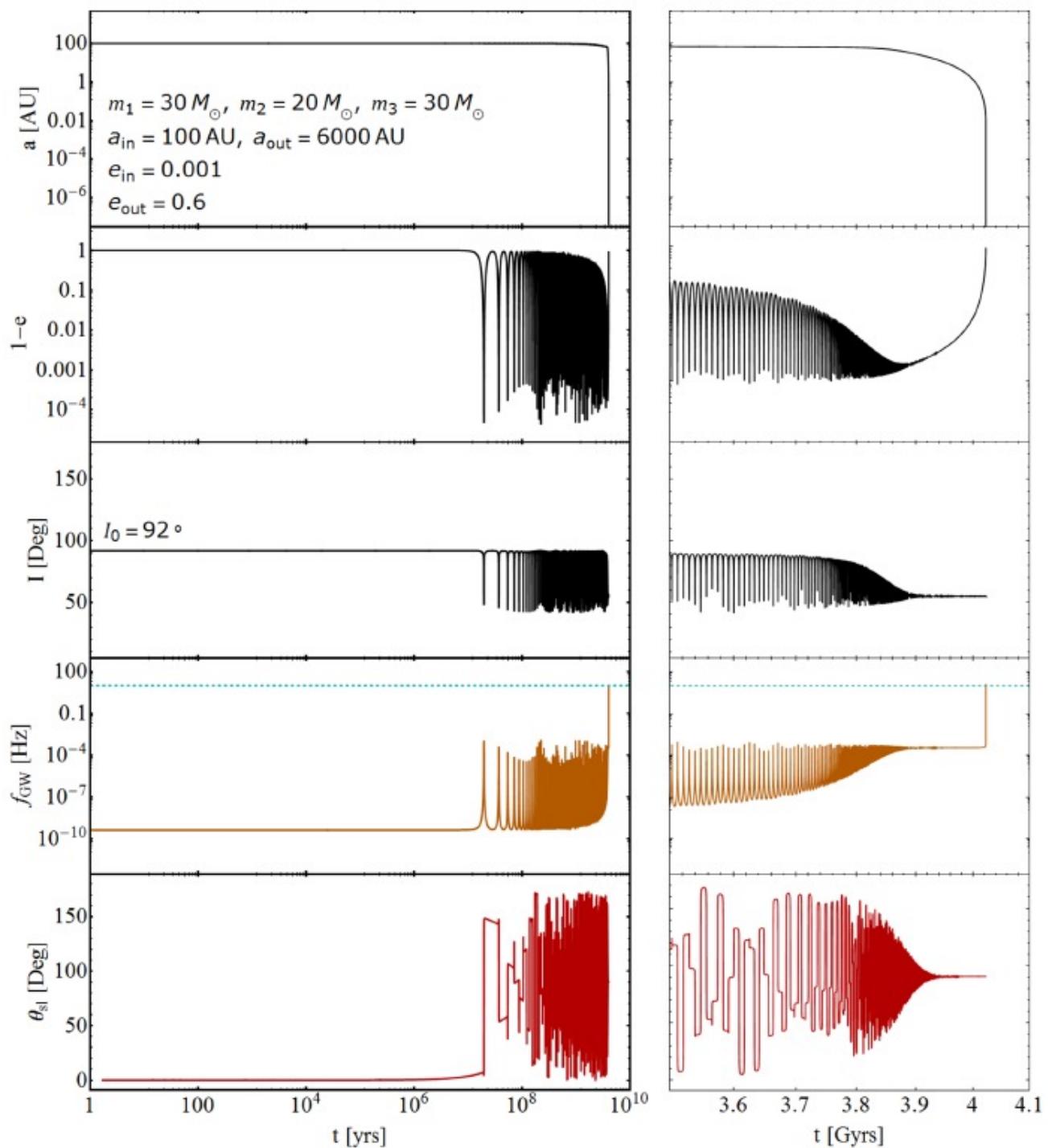


BH spin evolution in LK-induced orbital decay

$$\Omega_{dS}/\Omega_{pl}$$

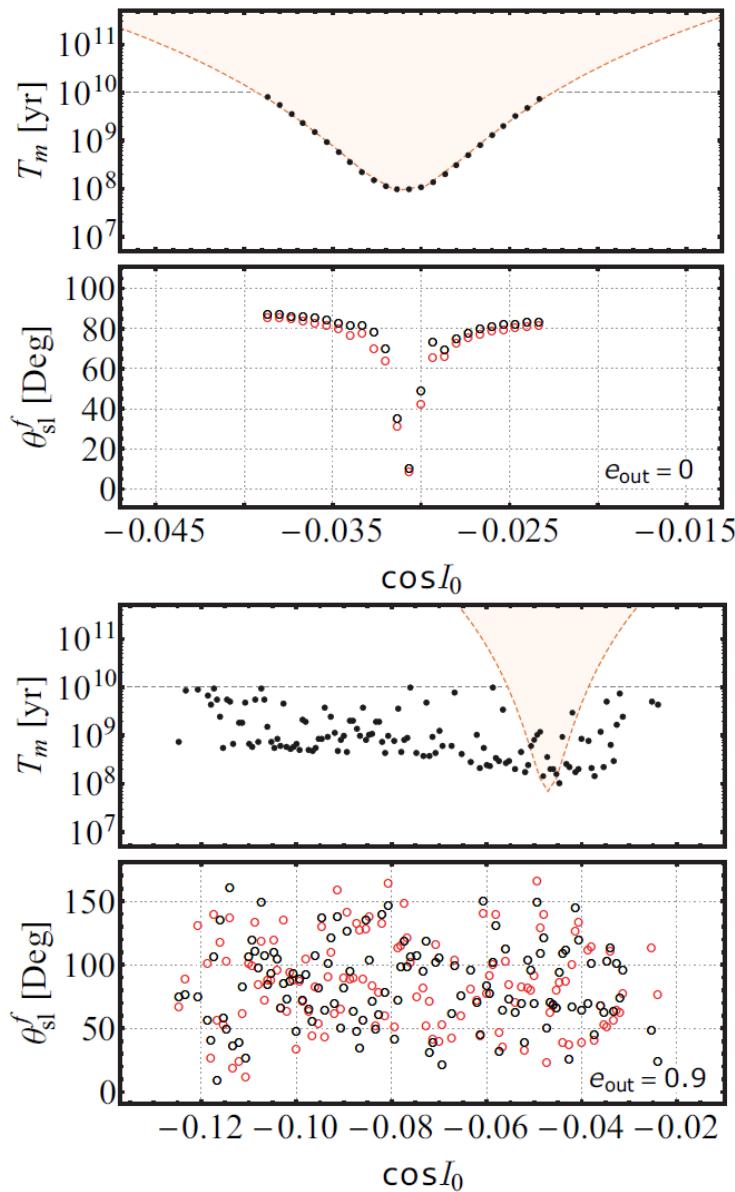
changes from $\ll 1$ (non-adiabatic)
to $\gg 1$ (adiabatic) as the orbit decays
→ Final spin-orbit misalignment angle



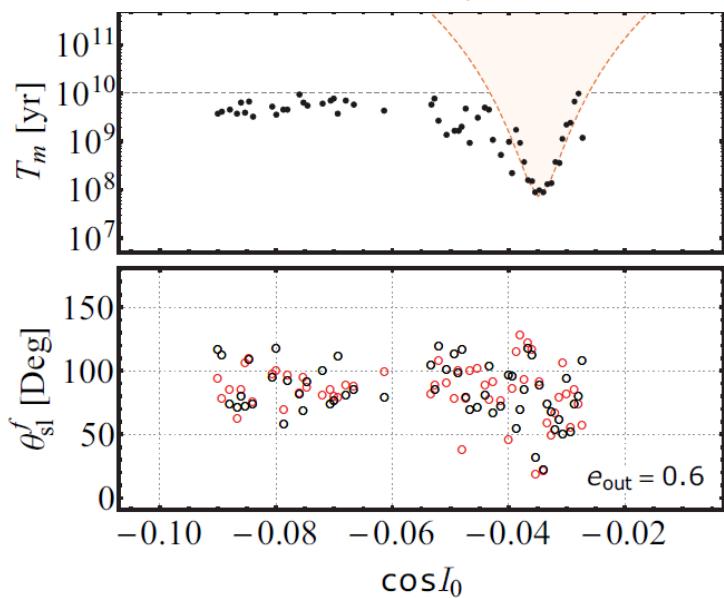
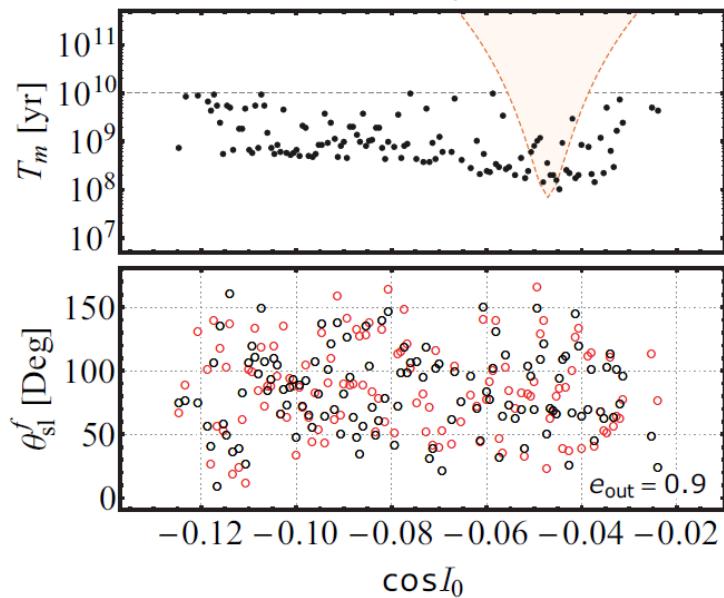
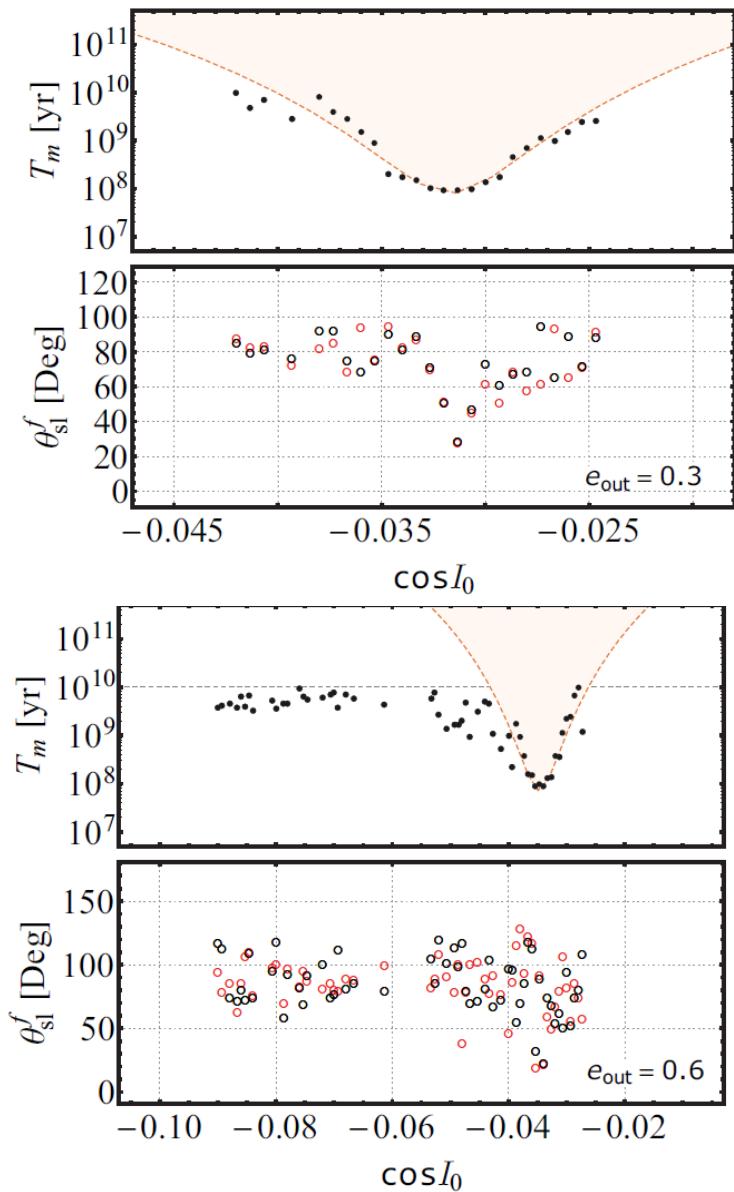


Merger Window and Final Spin-Orbit Misalignments

Fixed inner binary: $m_1=30M_{\odot}$, $m_2=20M_{\odot}$, $a_{\text{in},0}=100\text{AU}$

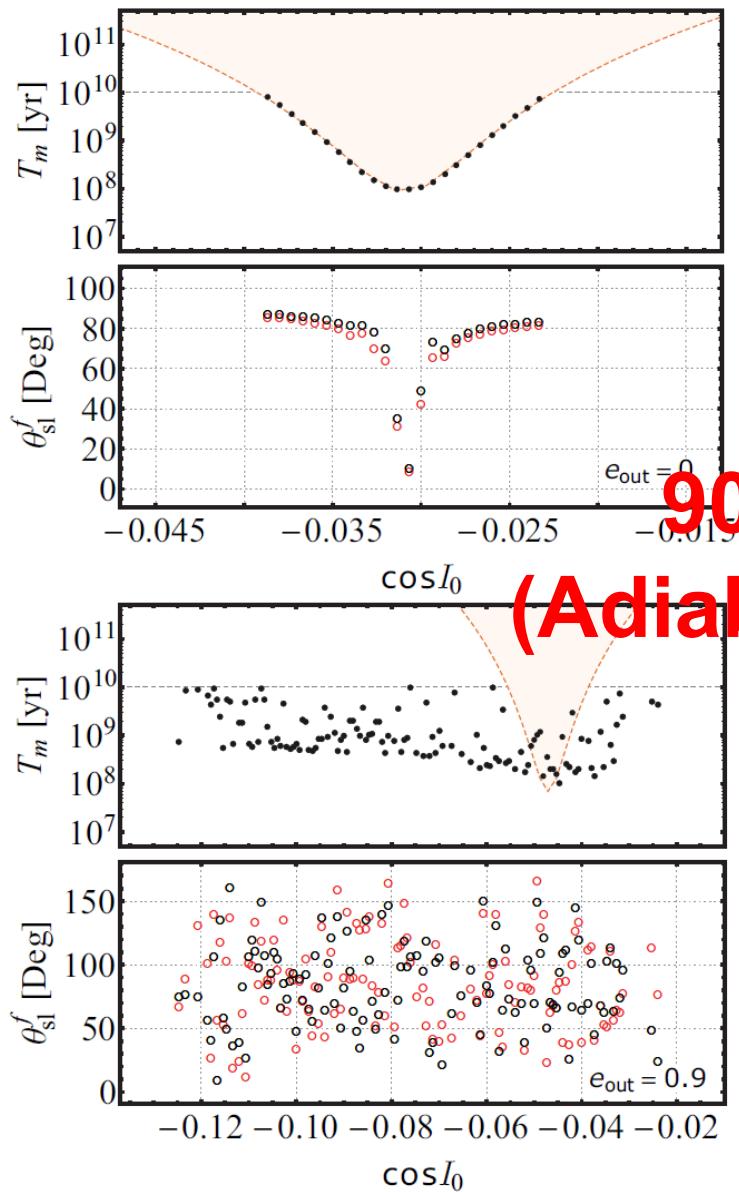


Fixed m_3/a_{out}^3 value

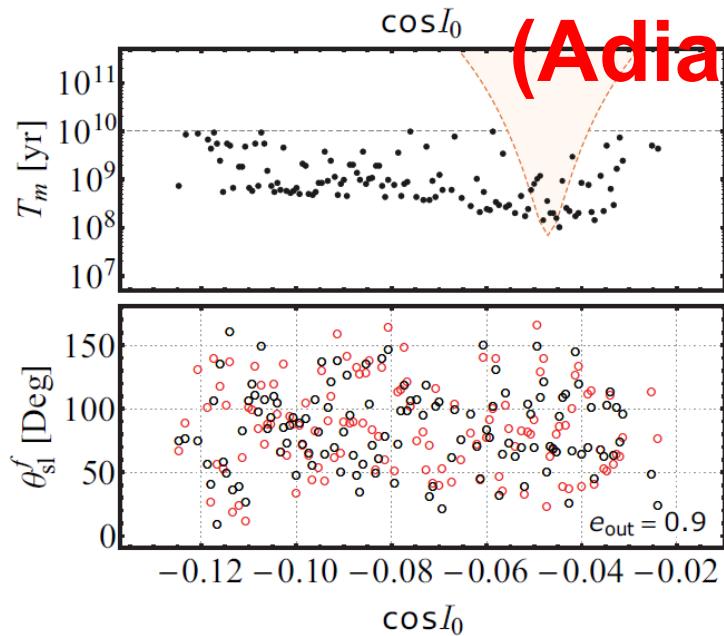
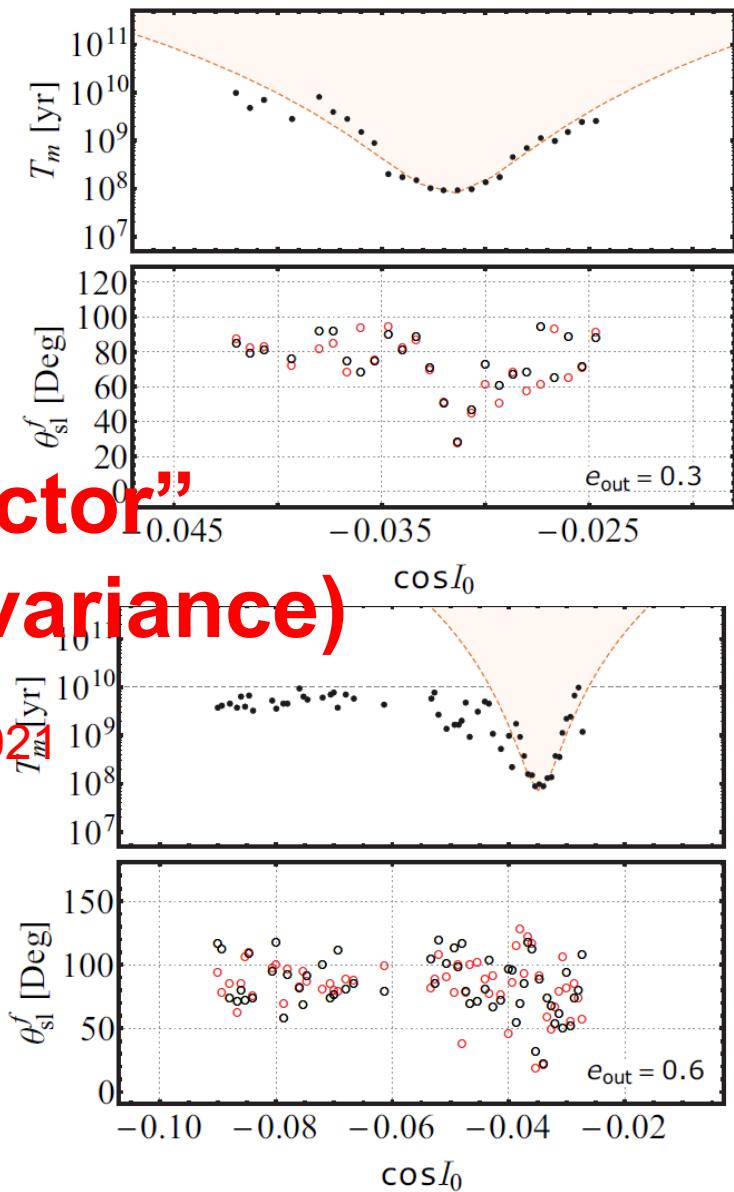


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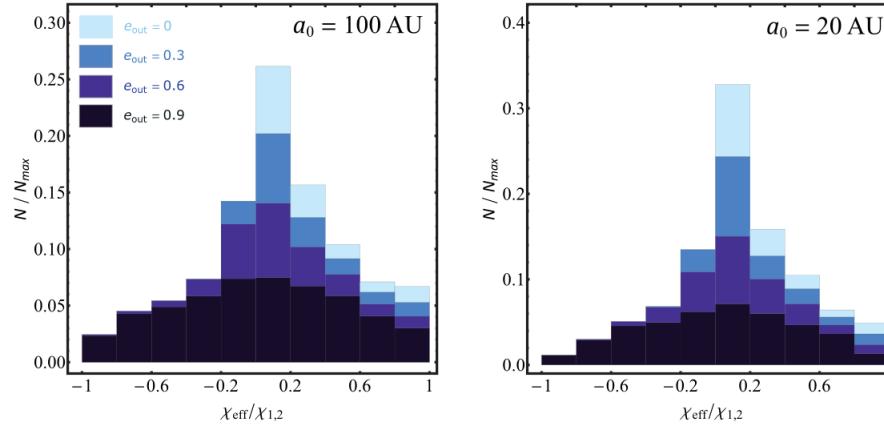


Su,Lai,Liu 2021

(Adiabatic Invariance)

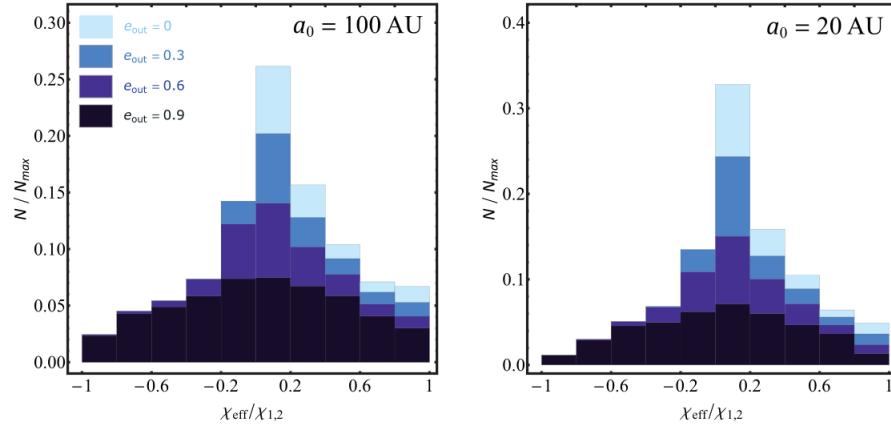
Effective Spin Distribution

For “some” initial binary/triple parameters ($e_0=0$, distant companions)

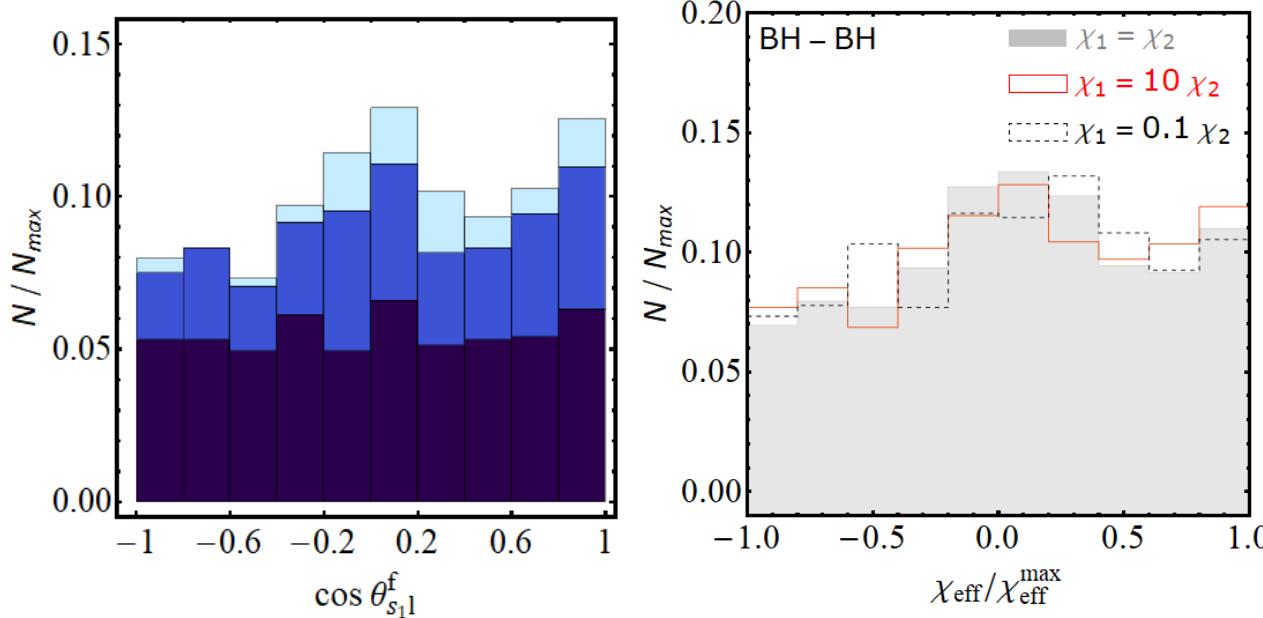


Effective Spin Distribution

For “some” initial binary/triple parameters ($e_0=0$, distant companions)

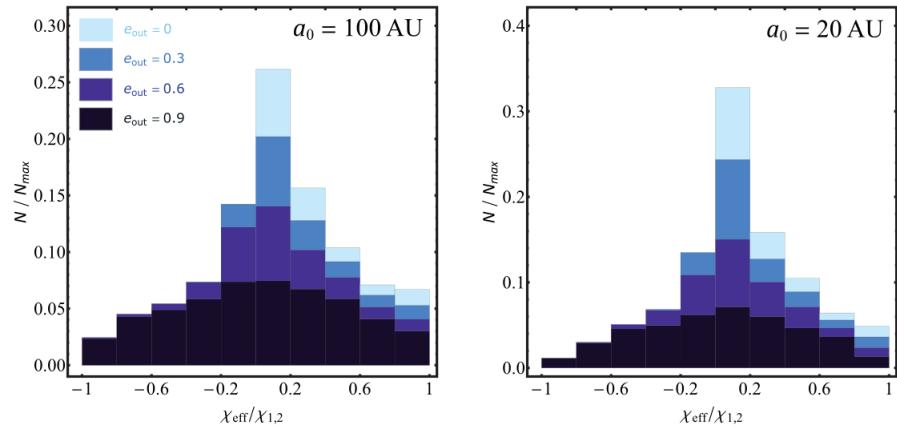


Consider ALL possible parameters

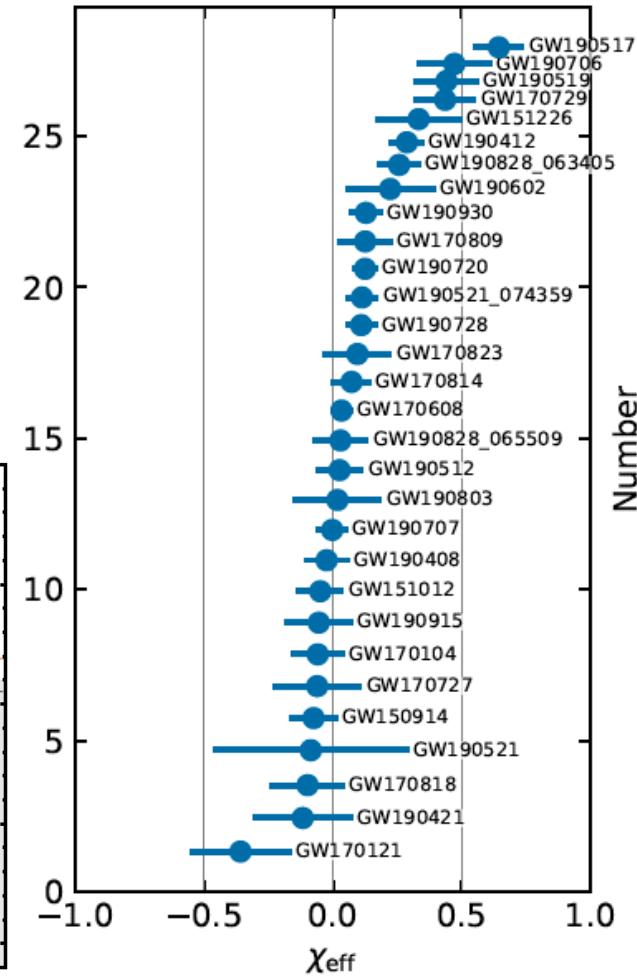


Effective Spin Distribution

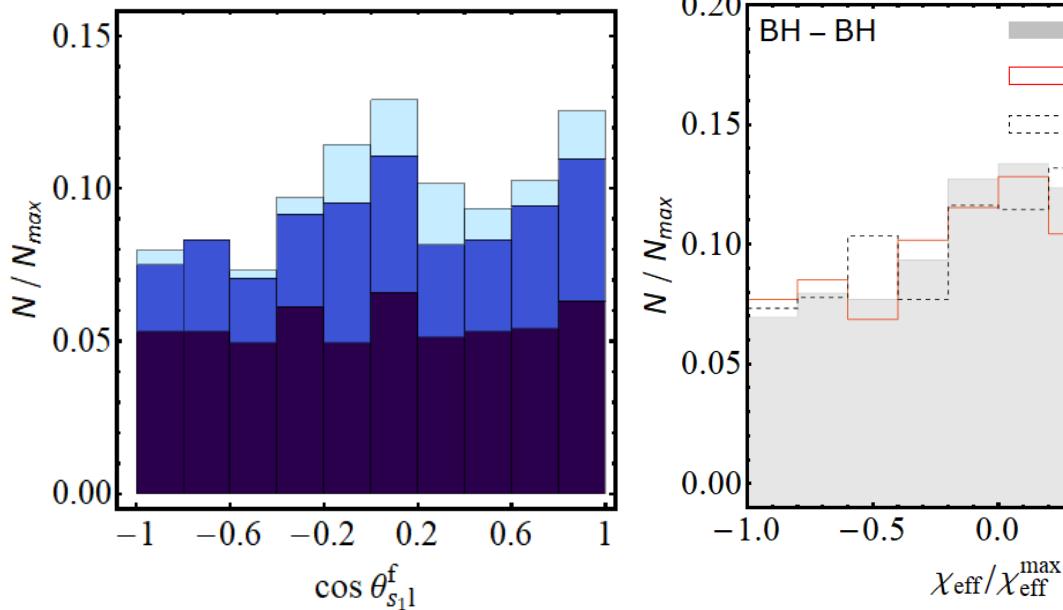
For “some” initial binary/triple parameters ($e_0=0$, distant companions)



Observed

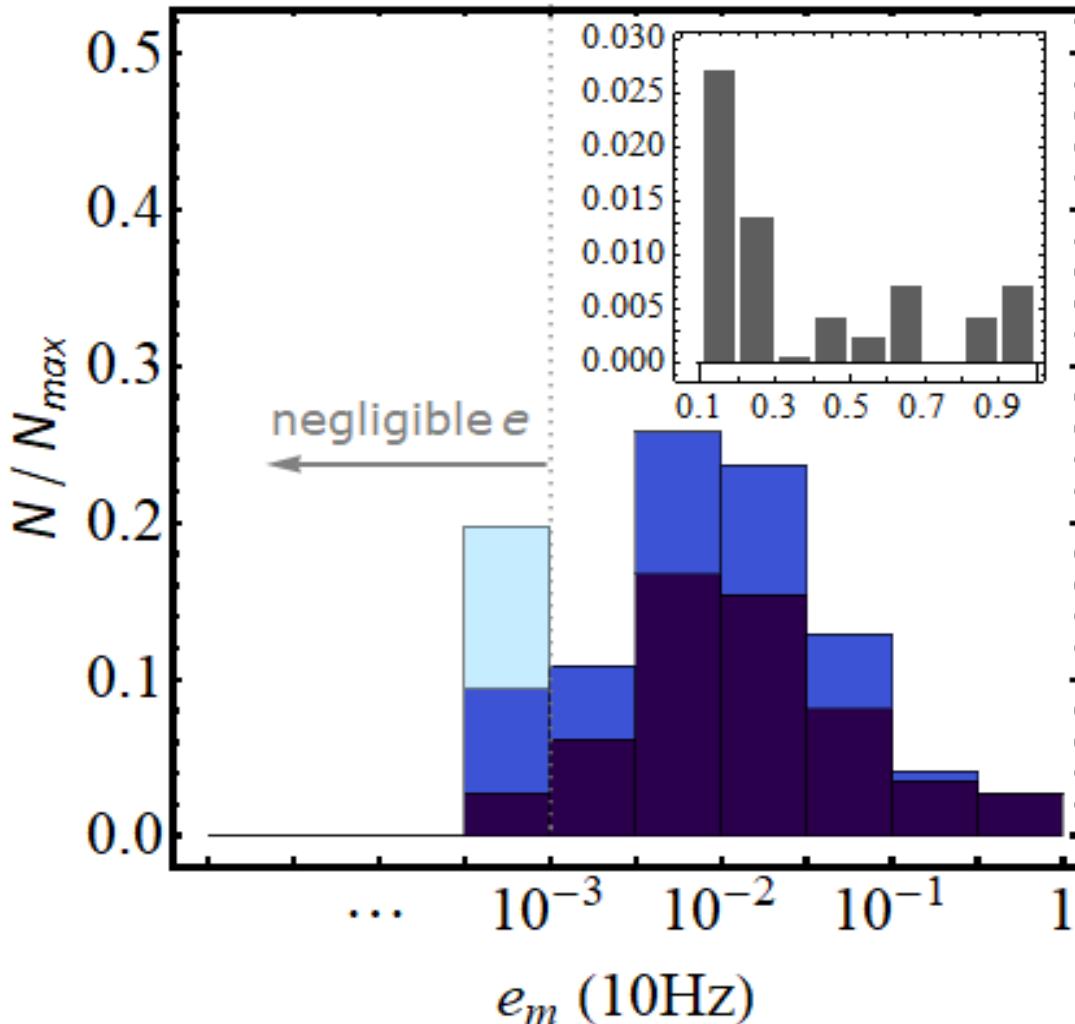


Consider ALL possible parameters



Residual Eccentricity (at 10 Hz)

BH-BH mergers



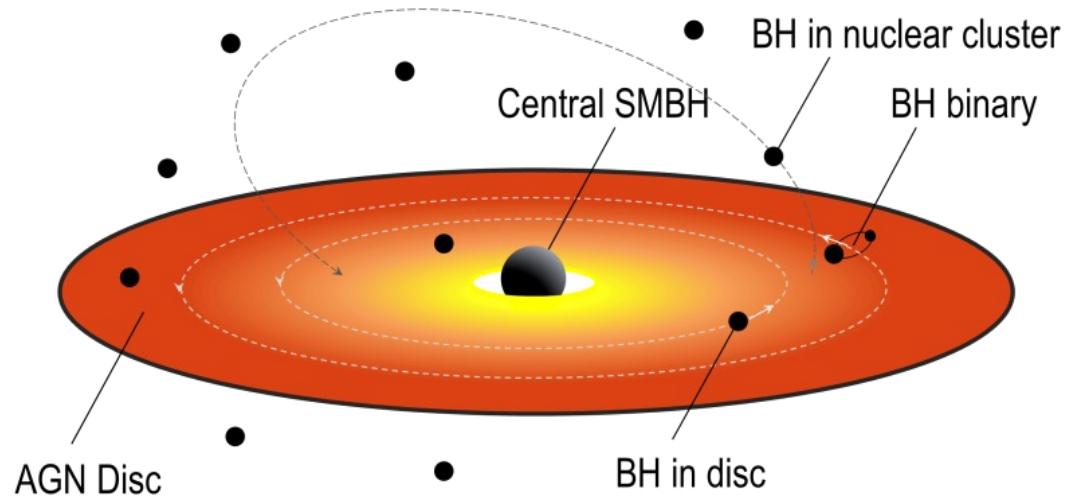
Very approximately

10% have $e_m > 0.1$

1% have $e_m > 0.9$

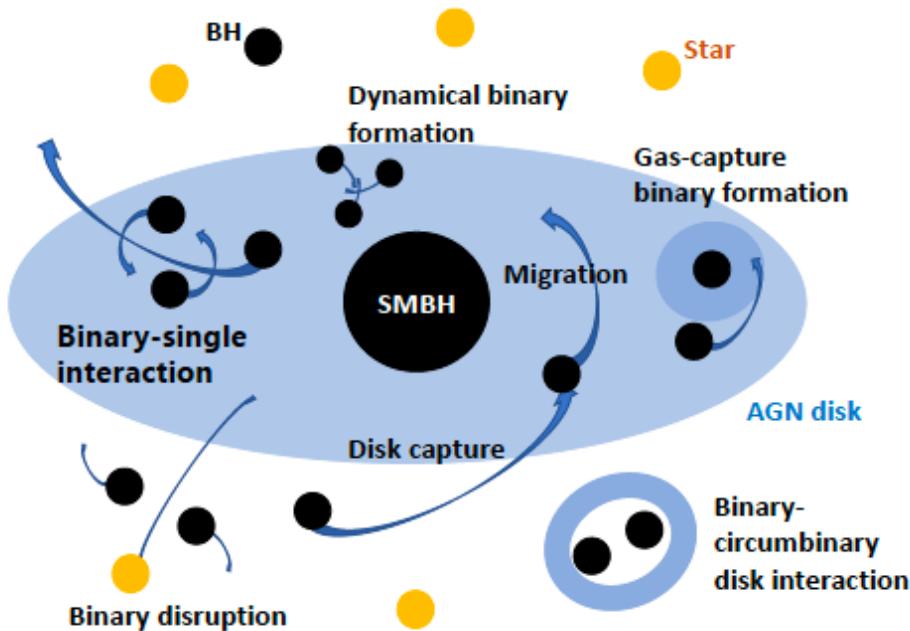
LISA/Taiji/Tianqin
would be very useful

Binary BH Mergers in AGN disks



Bellovary+16, Bartos+16, Stone+17, McKernan+18, Secunda+18, Yang+19, Tagawa+20, etc

Where do BH binaries in AGN disks come from?



1. Binaries form in disks via GI (\sim pc)
2. Binaries in nuclear clusters get captured in disks
3. Single BHs in AGN disks get captured in binaries

Tagawa, Haiman, Kocsis 2020

See also Bartos+17; Stone+17;

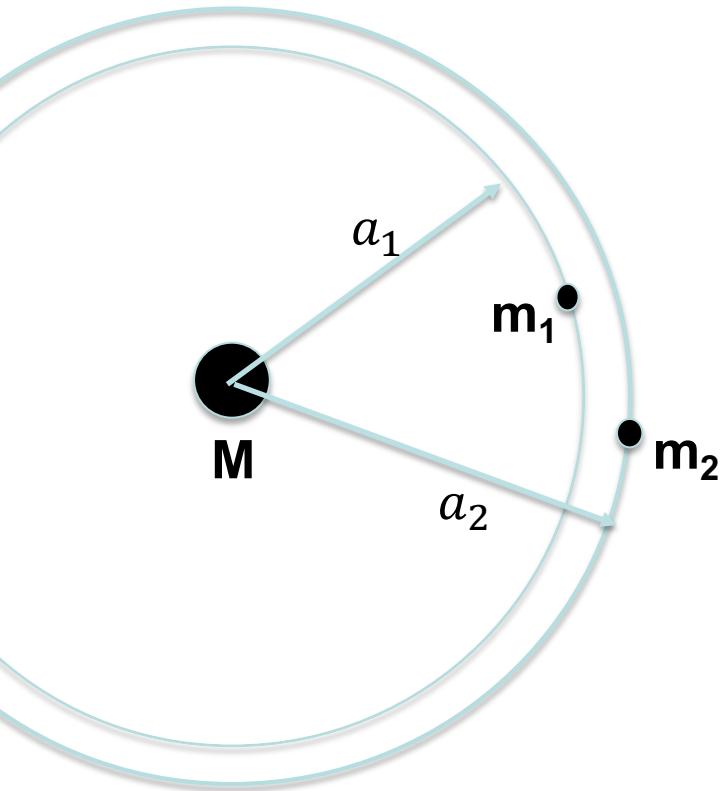
Long-Term Evolution of Tightly-Packed Stellar BHs in AGN Disks: Formation of Merging BH Binaries via Close Encounters

Jiaru Li, Lai, Rodet 2023



Jiaru Li
(Cornell Ph.D. 2023)

The Problem:



Two BHs (m_1, m_2) on closely-packed, nearly circular, nearly-coplanar orbits around a SMBH (M)
(e.g. brought together by migration in AGN disks)

When $a_2 - a_1 \lesssim 3.46 R_H$

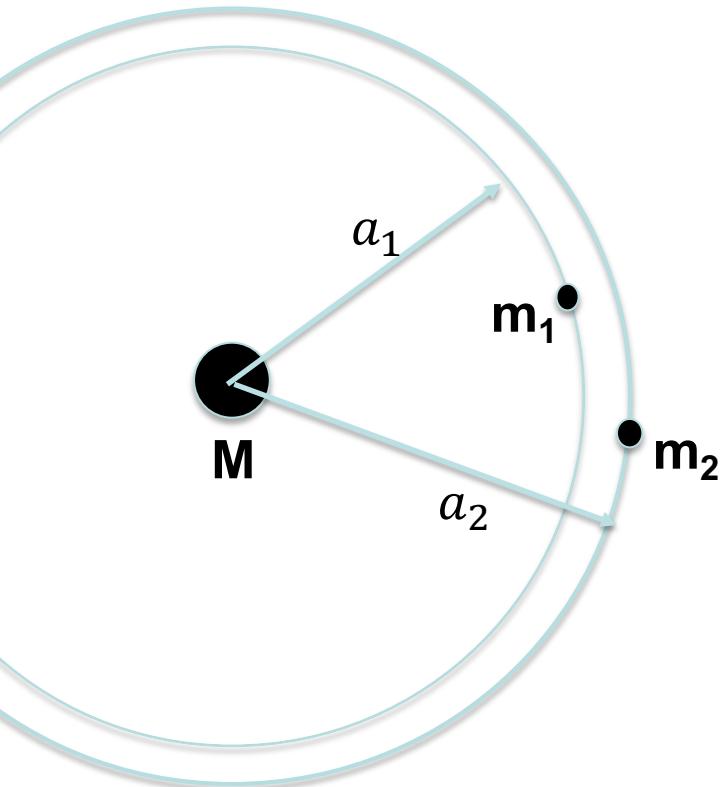
$$R_H = a_1 \left(\frac{m_{12}}{3M} \right)^{1/3}, \quad m_{12} = m_1 + m_2$$

orbits are dynamically unstable.

What happens to the two BHs?

Neglect gas effect for now...

Two planets in unstable orbits around a star:



Three outcomes:

1. Ejection of lower-mass planet
2. Planet-planet collision
3. Injection into the the near vicinity of star

Two BHs in unstable orbits around a SMBH:

Since $M/m_{12} \sim 10^6 \gg 1$

Ejection and injection are not possible
(takes many orbits > Hubble time)

Since $\frac{GMm_{1,2}}{a} \gg \frac{Gm_1m_2}{a}, \frac{Gm_1m_2}{R_H}$

→ The two BHs undergo “chaotic” motion, experience recurring closer encounters (separation $< R_H$)

For VERY close encounter:

$$\text{GW emission} \quad \Delta E_{\text{GW}} \sim \frac{\mu^2 m_{12}^{5/2}}{r_{\text{rel}}^{7/2}} \gtrsim \frac{Gm_1m_2}{R_{\text{H}}}$$


$$\frac{r_{\text{rel}}}{R_{\text{H}}} \lesssim 10^{-4} \left(\frac{4\mu}{m_{12}} \right)^{2/7} \left(\frac{10^6 m_{12}}{M} \right)^{10/21} \left(\frac{a_1}{100M} \right)^{-5/7}$$

Capture radius for forming “permanent” binary
due to GW bremsstrahlung

What is the capture rate?

For a typical “SMBH + 2 BHs” system (in unstable orbits), what is the cumulative capture rate to form real bound binary?

(1) Close encounter (CE0) rate

$$(2) P(< r_{\text{cap}}) \simeq \frac{r_{\text{cap}}}{R_{\text{H}}}$$


$$\langle N_{\text{cap}}(t) \rangle \simeq 6 \times 10^{-5} \left(\frac{t}{P_1} \right)^{0.52} \left(\frac{r_{\text{cap}}}{10^{-4} R_{\text{H}}} \right)$$

It takes $10^8 P_1$ (on average) for two BHs to capture into bound merging binary

Captured BH binary as GW source

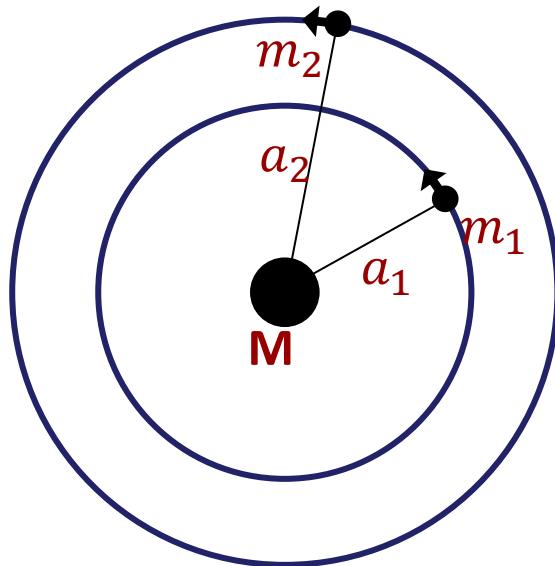
$$f_{\text{cap}} \simeq (1.4 \text{ Hz}) \left(\frac{4\mu}{m_{12}} \right)^{-3/7} \left(\frac{M}{10^8 M_\odot} \right)^{-2/7} \left(\frac{m_{12}}{100 M_\odot} \right)^{-5/7} \left(\frac{a_1}{100 M} \right)^{-3/7}$$

Once capture, it will take a few orbits to merge
it enters LIGO band with $e \gtrsim 0.5$

**This mechanism always produce very eccentric mergers
(not typical/most LIGO events)**

Formation of BH binaries: Gas effects

hydrodynamics simulations

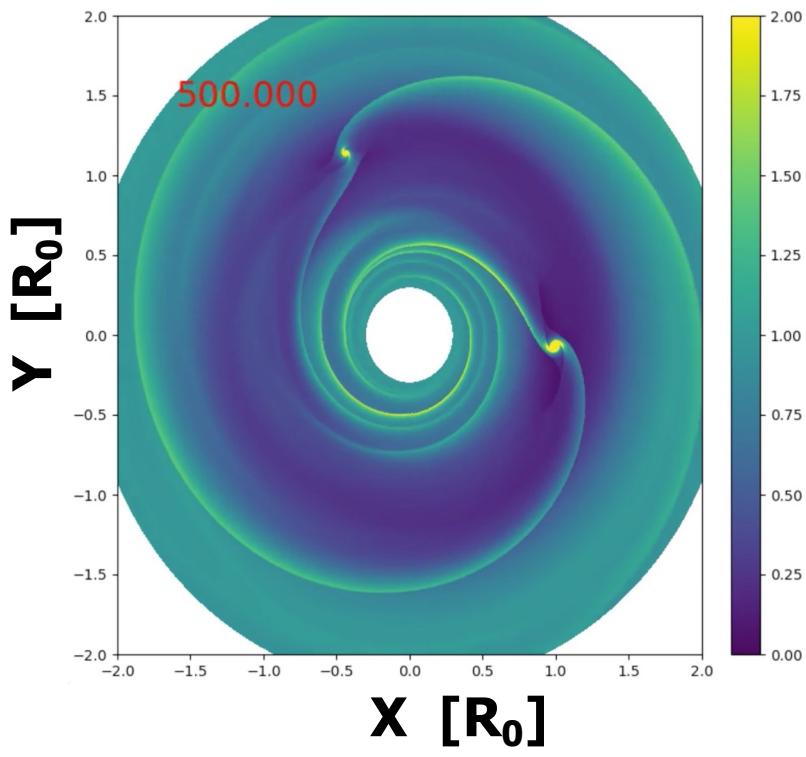


- Initial condition:

$$a_2 - a_1 = 2R_H$$

Formation of BH binaries: Gas effects

hydrodynamics simulations

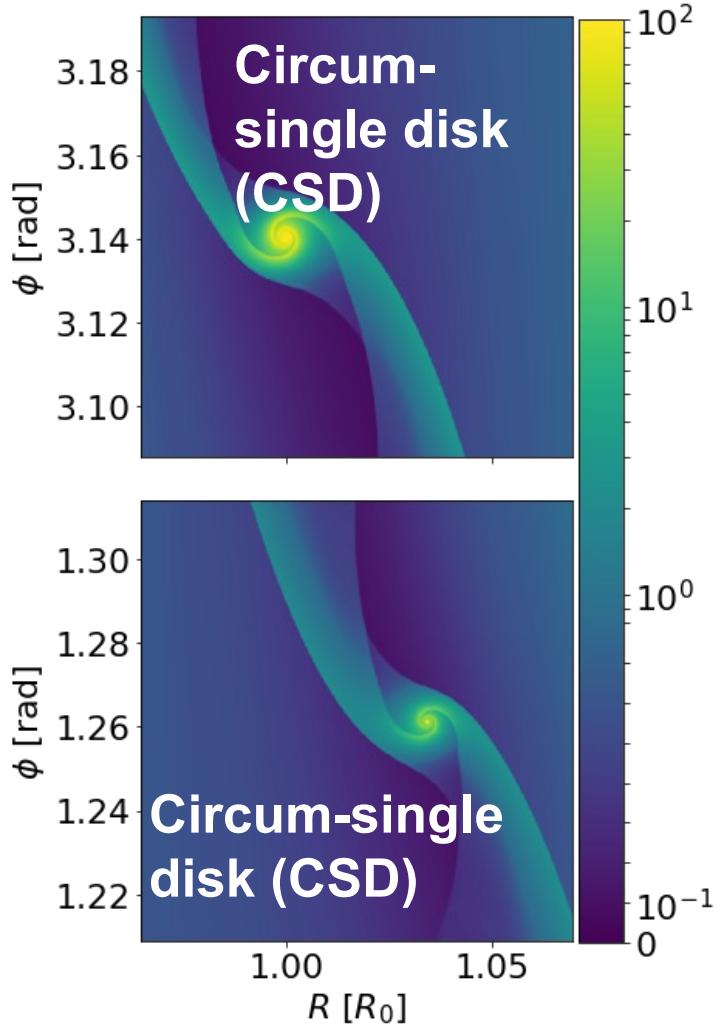


- Initial condition:

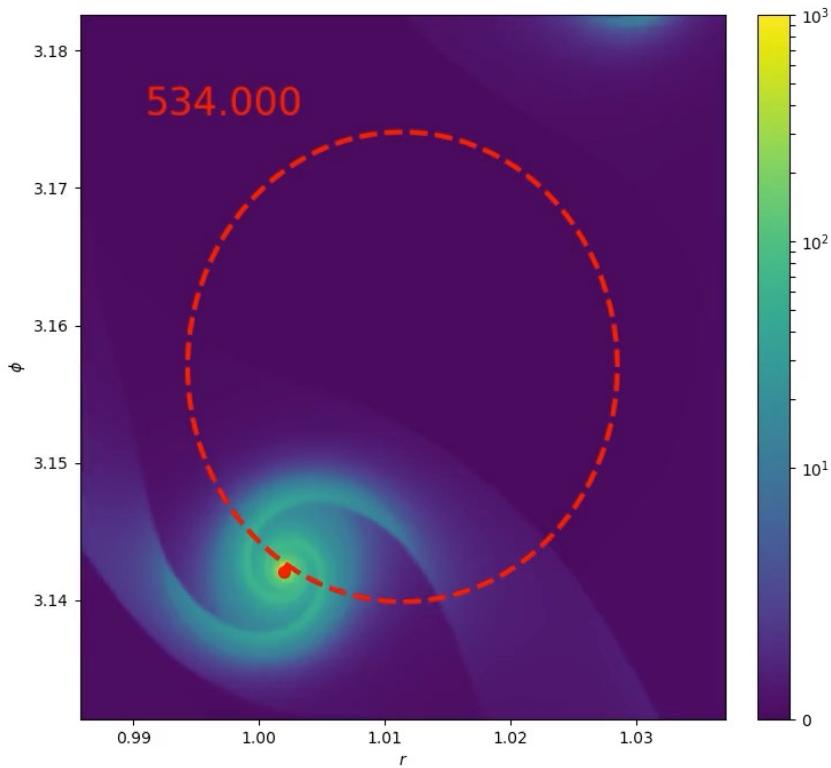
$$a_2 - a_1 = 2R_H$$

- Simulation setup:

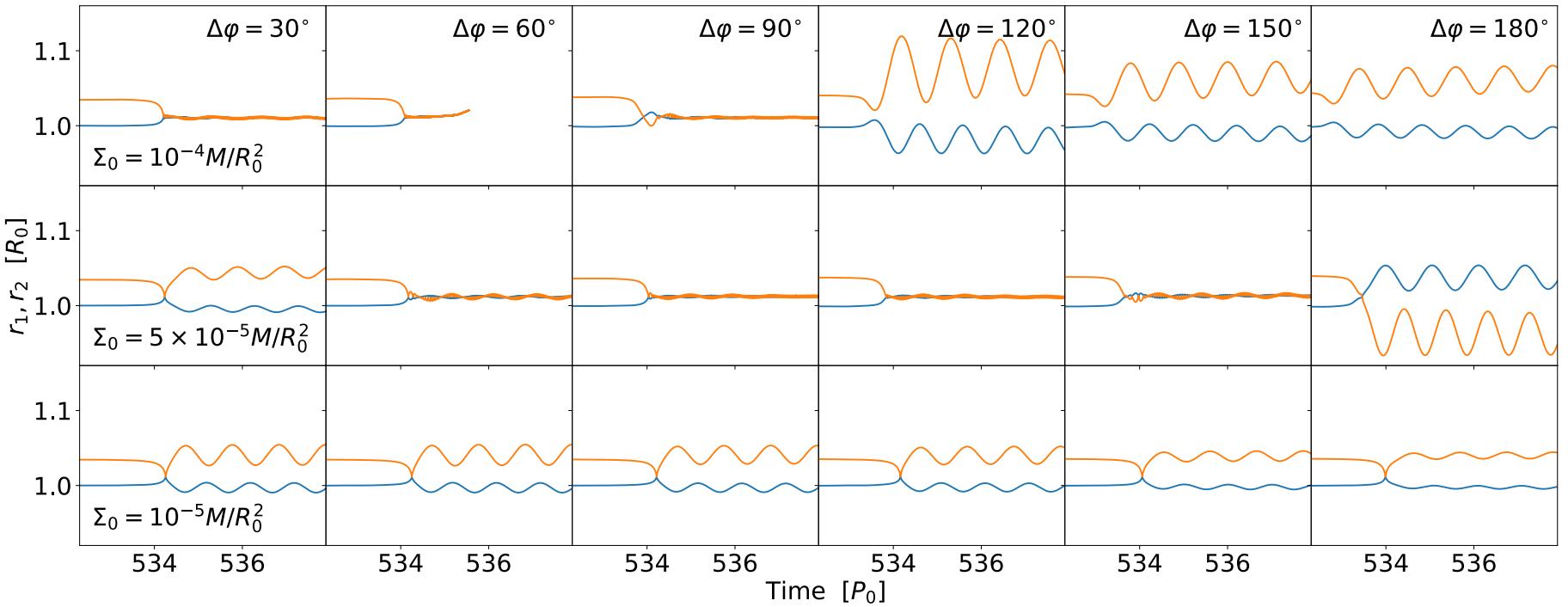
- $M_{\text{SMBH}} = 1$, $m_1 = 10^{-5}$, $m_2 = 5 \times 10^{-6}$
- Thin disk $H/R = 0.01$, low viscosity $\alpha = 0.01$.
- Isothermal disk.
- High resolution with $50 \rightarrow 100$ grid cells per R_H , where $R_H = 0.017R_0$



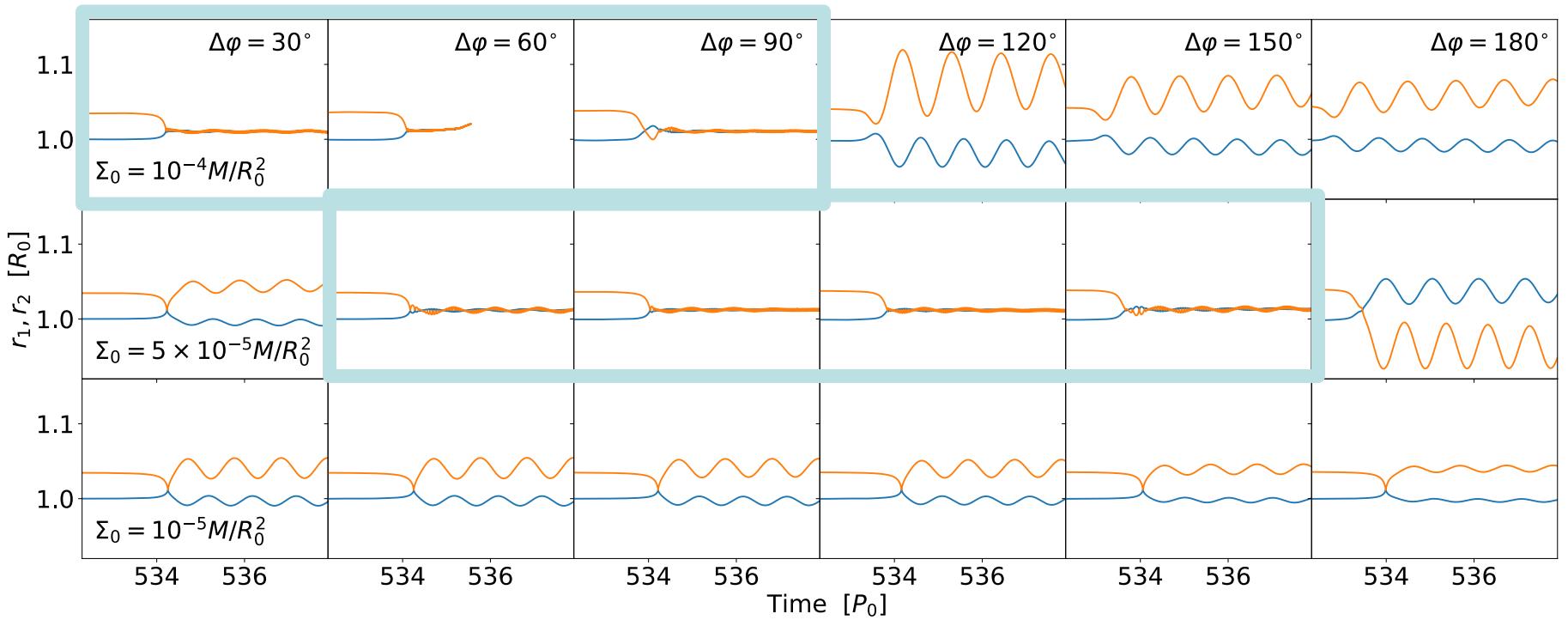
Formation of a binary



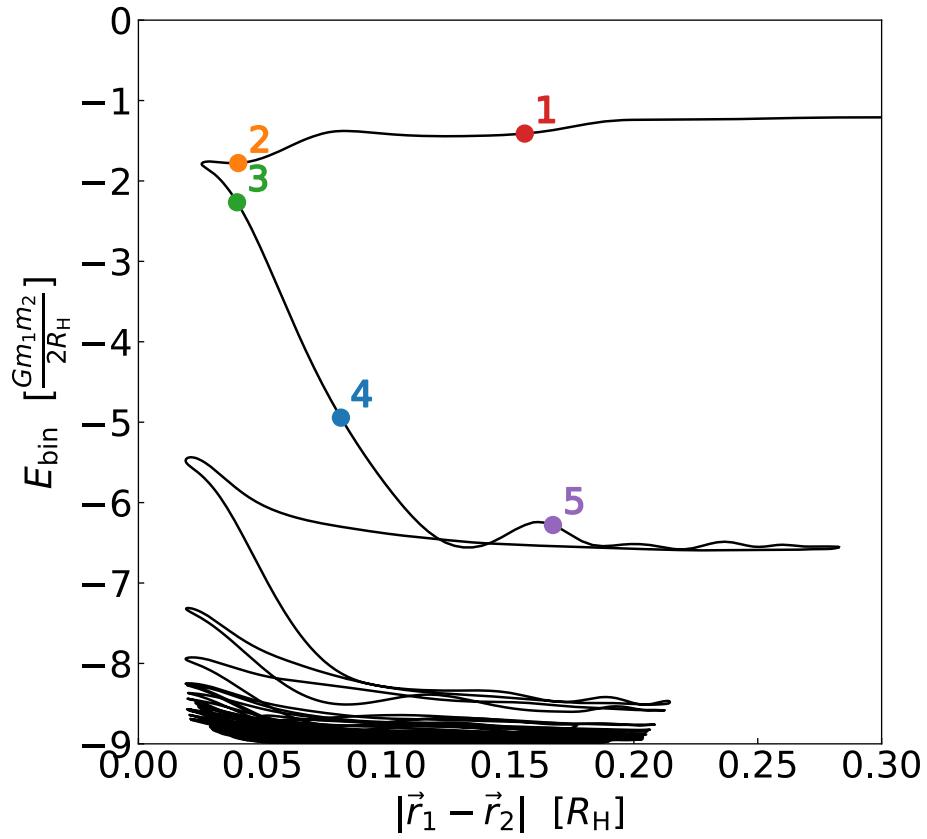
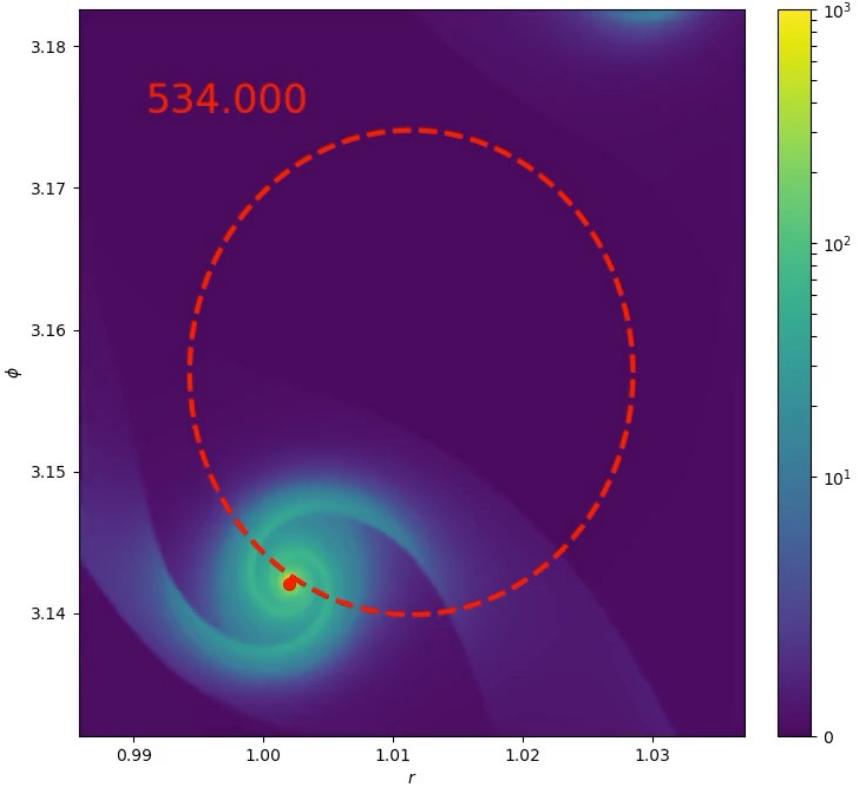
Simulation outcomes



Simulation outcomes

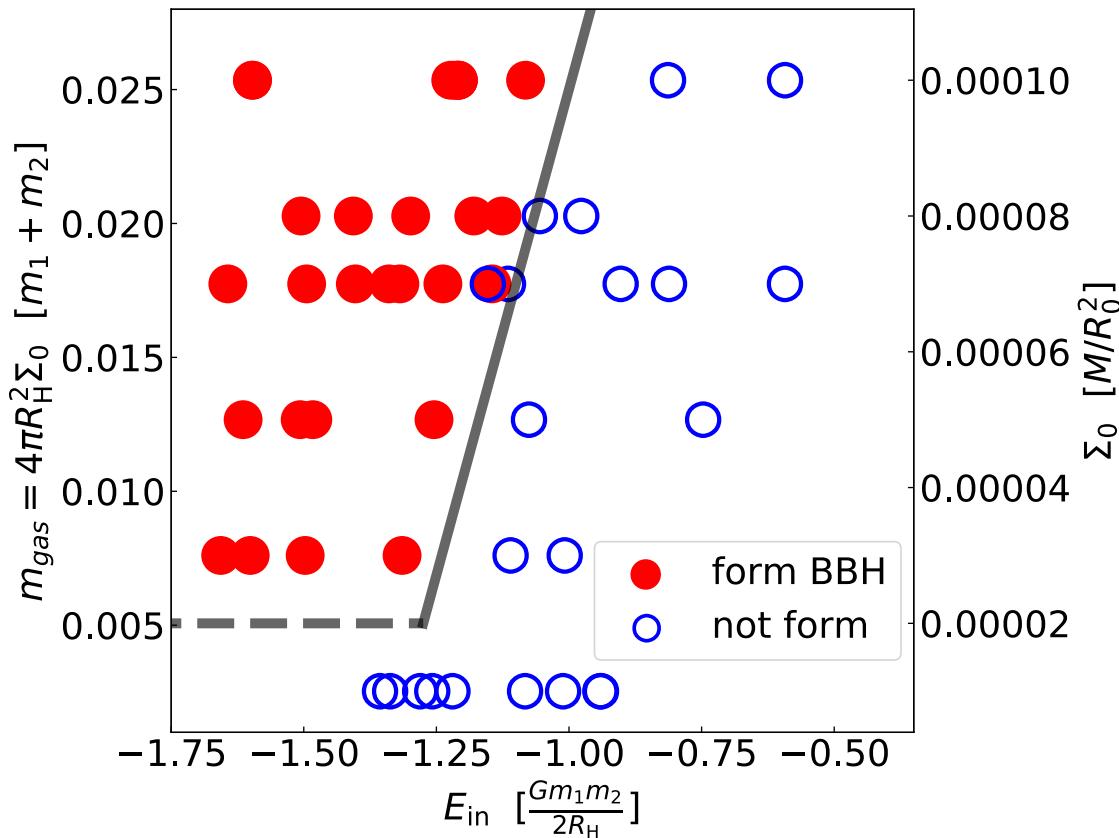


Analysis: formation mechanism -- a departure drag



$$E_{\text{bin}} = \frac{1}{2} \mu v_{\text{rel}}^2 - \frac{G m_1 m_2}{r_{\text{rel}}}$$

Analysis: criteria for binary formation



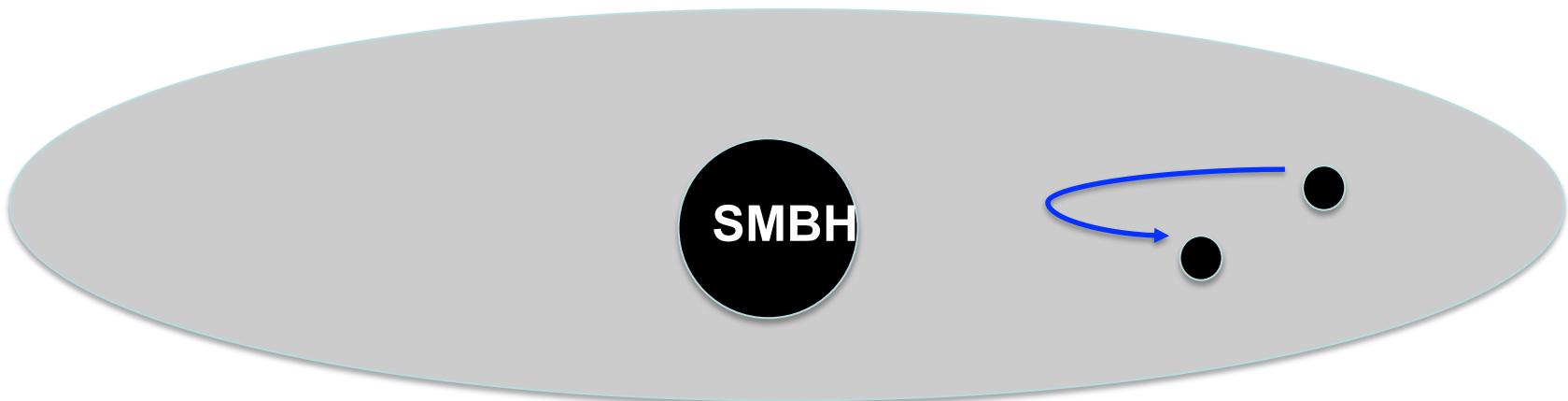
Forming binary requires:

- sufficiently large gas mass
- sufficiently small initial binary energy

$$E_{\text{in}} := \frac{1}{2}\mu v_{\text{rel}}^2 - \frac{Gm_1m_2}{r_{\text{rel}}} \quad \text{when } r_{\text{rel}} = 0.3R_H$$

Hydrodynamical interaction of Binary in ANG Disk

Assume a pre-existing binary

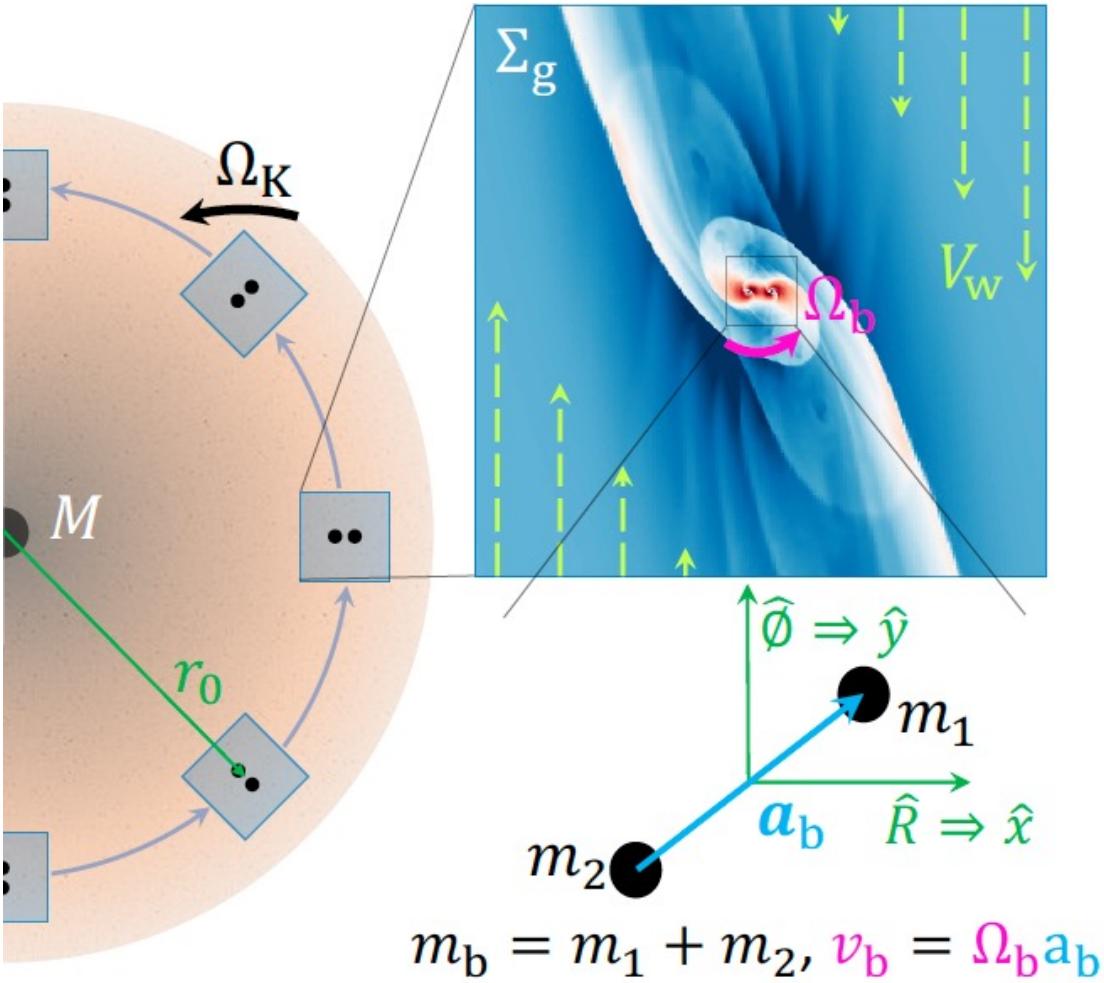


Local simulations of binary in disk

R.Li & Lai 2022,2023



Dr. Rixin Li
(Cornell)

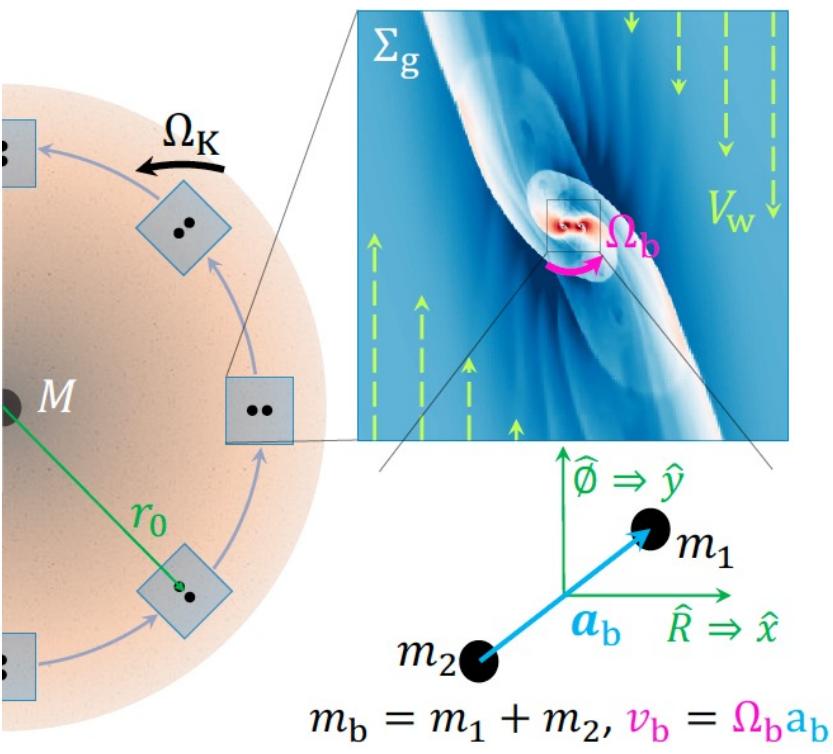


Local shearing box
(not "local wind tunnel box"
used by Kaaz et al. 2021)

ATHENA
Mesh refinement
Resolution: $a_b \sim 250$ cells
zero softening in gravity

Length scales of the problem:

$$a_b, \quad R_B \sim \frac{Gm_b}{c_\infty^2}, \quad R_H \sim r_0 \left(\frac{m_b}{M} \right)^{1/3}, \quad H$$



Velocity scales of the problem:

$$v_b, \quad c_\infty, \quad V_{\text{shear}}$$

→ Dimensionless ratios:

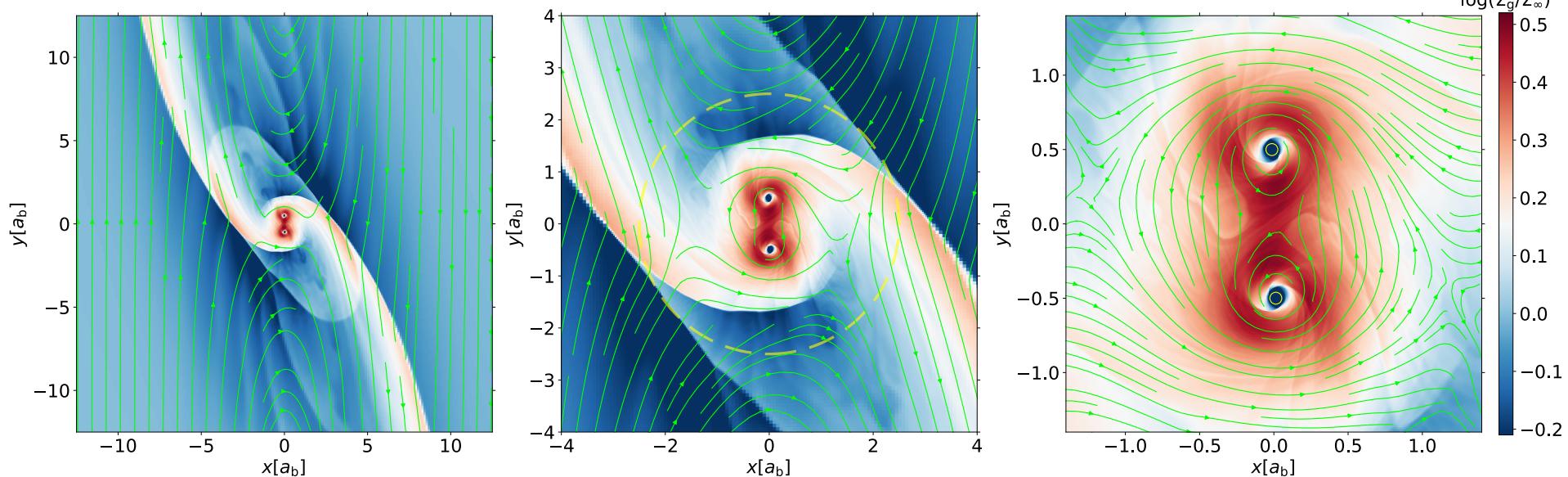
$$\frac{q}{h^3}, \quad \frac{R_H}{a_b} \equiv \lambda$$

where $q = m_b/M$, $h = H/r_0$

$m_2/m_1, e_b, \text{EOS}$ (e.g. γ law)

Example of flow structure

Pairs of bow shocks, spiral shocks



BH = absorbing sphere: sink radius: $r_{\text{sink}} = 0.04 a_b \simeq 10$ cells $\rightarrow \dot{m}_b$

Force on each BH: from gravity + accretion + pressure

\rightarrow Torque on binary, energy transfer rate $\rightarrow \dot{a}_b, \dot{e}_b$

Some “Typical” Results:

$\langle \dot{m}_b \rangle$: can be << Bondi-Hoyle-Lyttleton rate (even including shear)
depends on sink radius (for non-viscous flow)

$$\ell_0 = \frac{\langle \dot{L}_b \rangle}{\langle \dot{m}_b \rangle} = -0.23 v_b a_b$$
$$m_2/m_1 = 1, \quad e_b = 0$$
$$q/h^3 \sim 1, \quad \lambda = 5, \quad \gamma = 5/3$$

$$\frac{\langle \dot{a}_b \rangle}{a_b} = -4.81 \frac{\langle \dot{m}_b \rangle}{m_b}$$

Orbital evolution is always accompanied by accretion

Summary

Formation Channels of Merging BH Binaries

Standard isolated binary evolution channel:

uncertain physics (common envelope...)

→ circular mergers ($e_m=0$), mostly aligned spin-orbit angle

Dynamical formation channels:

“clean” physics, but “environmental” uncertainties

1. Dense star clusters

→ mostly circular mergers ? expect random spin-orbit misalignments ?

2. Tertiary-induced mergers

Perturbations from outer companion → Lidov-Kozai

Octupole effect → mass ratio dependence

Binary mergers around SMBH: GR effects important

Spin-orbit misalignment

→ ~10% mergers have residual $e>0.1$ when entering LIGO band

Preference of 90° spin-orbit misalignment, especially for circular mergers

3. BH Mergers in AGN disks

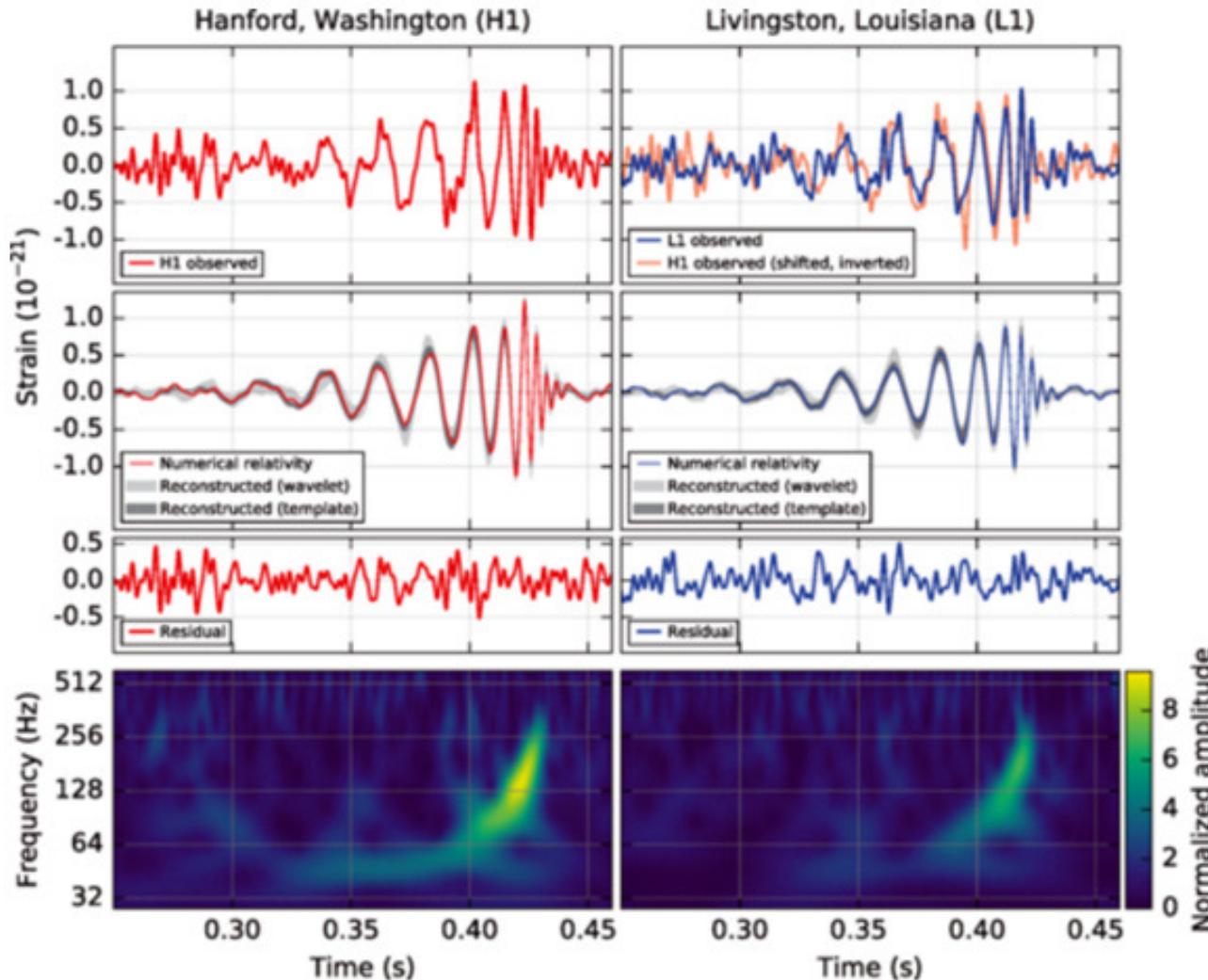
GW capture → very eccentric mergers

gas assisted mergers?

Rates? All potentially can play a role

LISA/Taiji/Tianqin useful for probing dynamical formation

LIGO's First Merging Binary Black Holes: GW150914



Binary Dynamics Near a Supermassive BH

Nuclear Star Cluster

'Environmental' effects

- **Binary Evaporation**

e.g., Binney & Tremaine 2008; Perets 2009

- **Resonant Relaxation**

e.g., Rauch & Tremaine 1996; Perets et al. 2009; Kocsis & Tremaine 2011; Hamers et al. 2018;

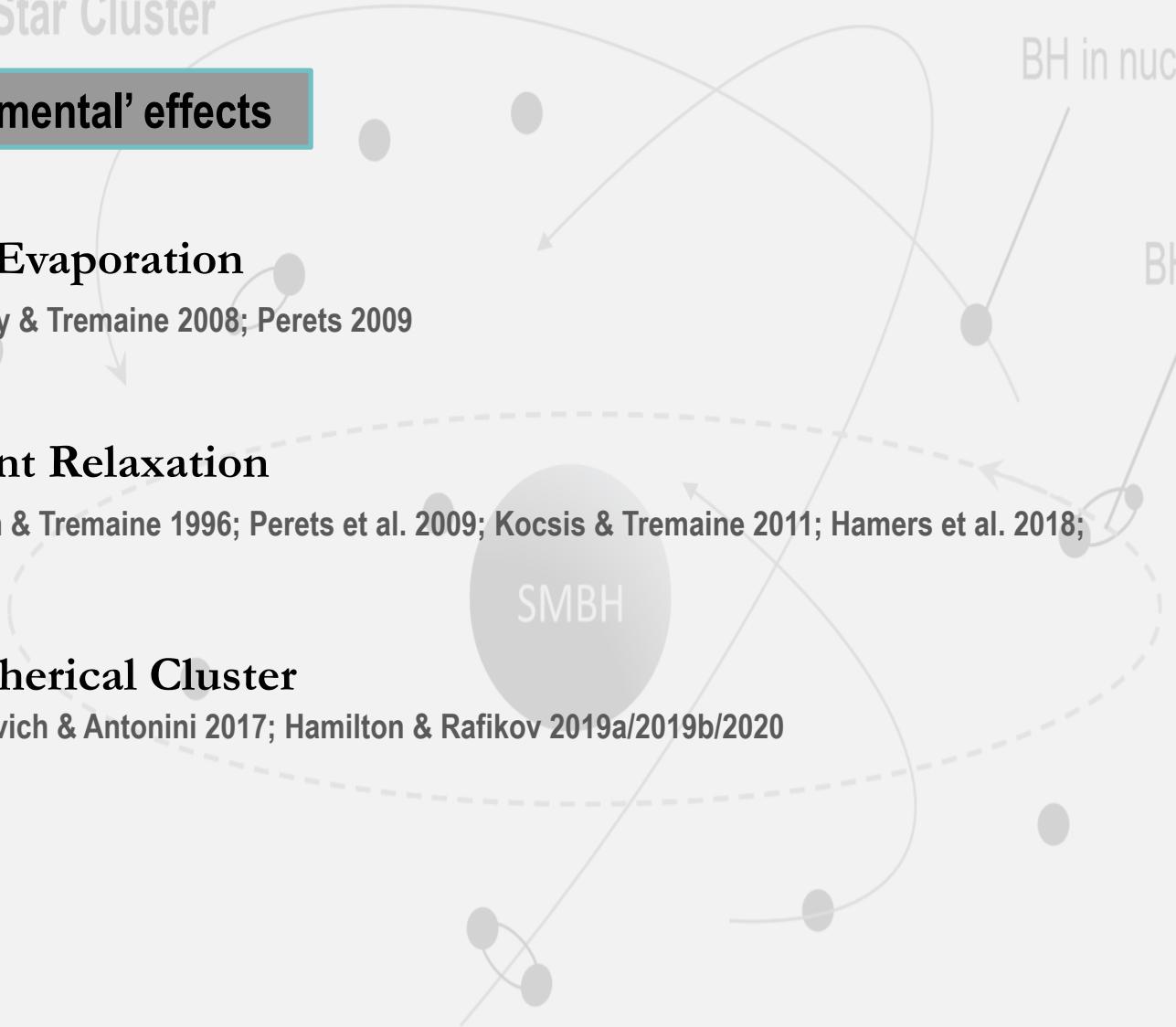
- **Non-spherical Cluster**

e.g., Petrovich & Antonini 2017; Hamilton & Rafikov 2019a/2019b/2020

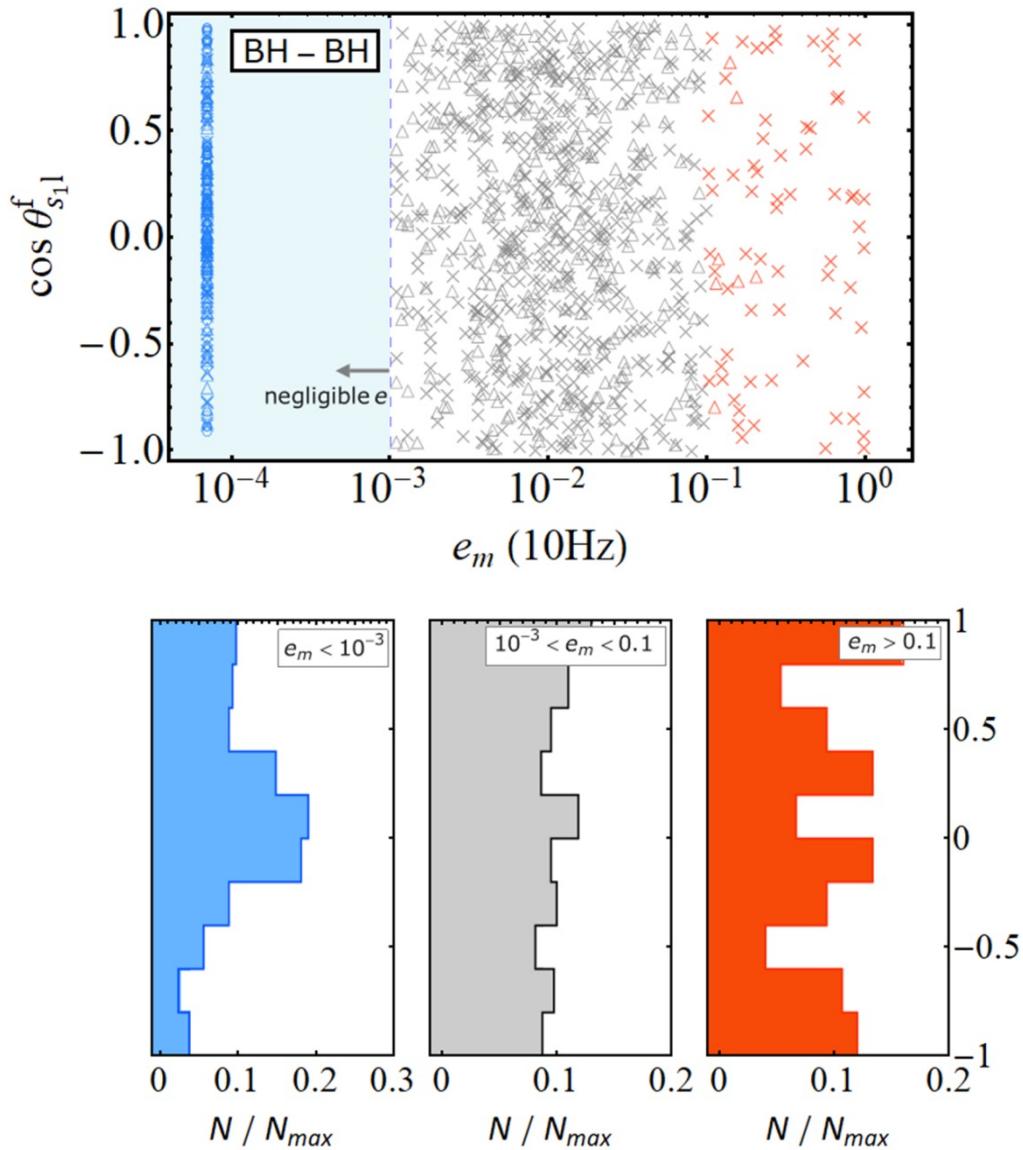
BH in nuclear cluster

BH Binary

SMBH



Residual eccentricity vs Spin-orbit Misalignment



Circular Mergers ($e_m < 10^{-3}$) prefer $\theta_{sl}^f \sim 90^\circ$

More eccentric Mergers has random θ_{sl}^f