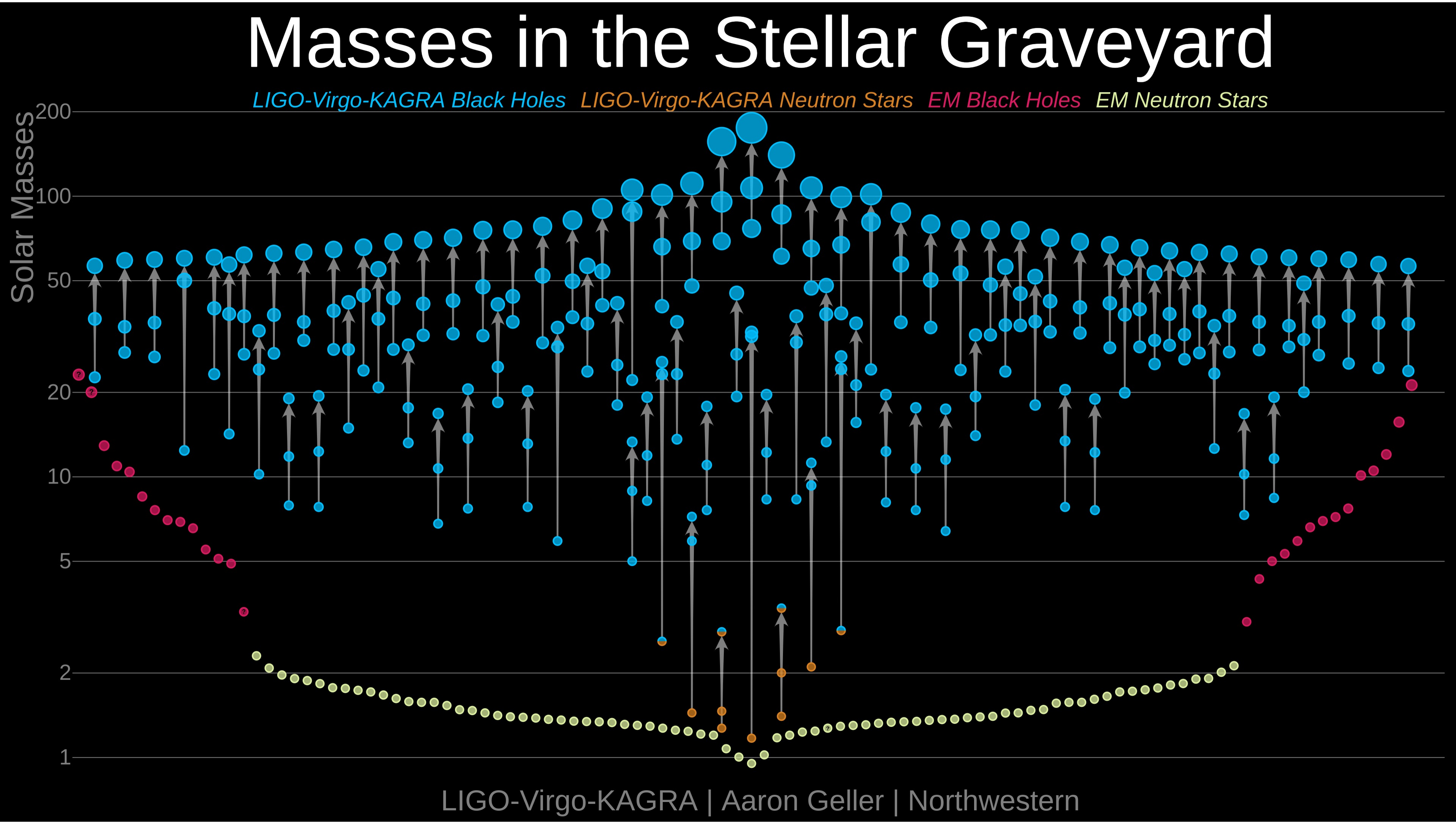
Dynamical Formation of Merging Compact Binaries

合并黑洞双星的动力学形成

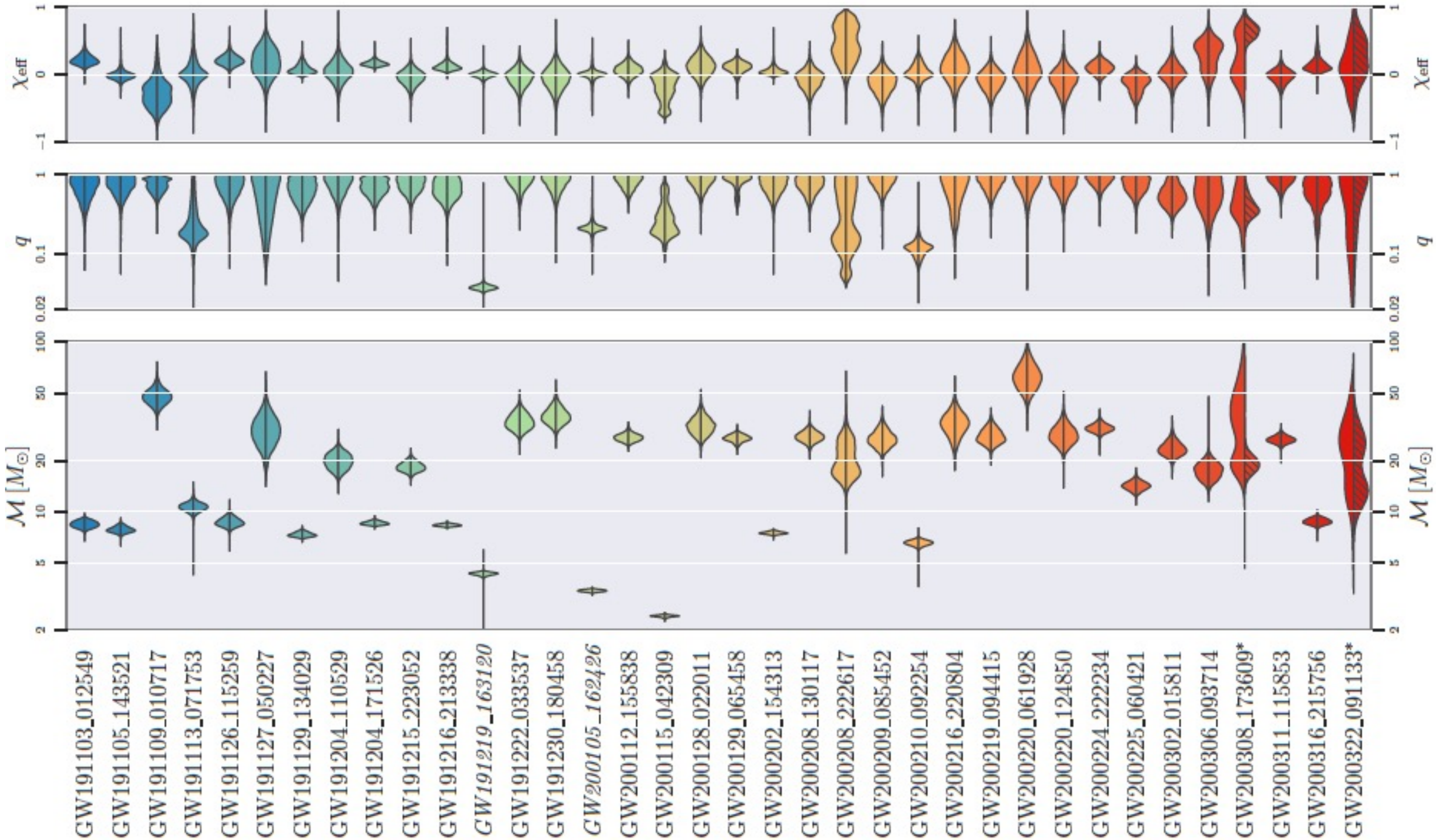
Dong Lai (赖东)

Cornell University

Sun Yat-Sen University, Jan. 6, 2022



GWTC-3 (11/2021): 90 merger events, with 2 NS/NS mergers, 3 NS/BH mergers



Gravitational waveform gives $M_1, M_2, \chi_{\rm eff}$

$$\chi_{\rm eff} \equiv \frac{m_1 \boldsymbol{\chi}_1 + m_2 \boldsymbol{\chi}_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

Formation of Merging BH Binaries

$$T_\mathrm{m} \approx\ 10^{10}\mathrm{yrs}\left(\frac{60M_\odot}{m_1+m_2}\right)^2\left(\frac{15M_\odot}{\mu}\right)\left(\frac{\mathbf{a_0}}{0.2\mathrm{AU}}\right)^2\left(\mathbf{1}-\mathbf{e_0}^{\mathbf{2}}\right)^{7/2}$$

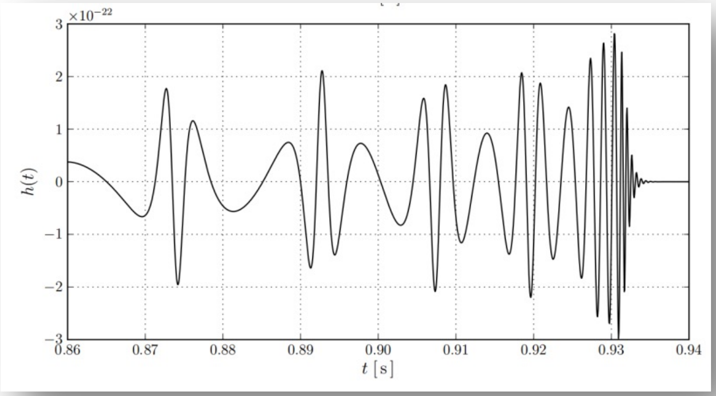
Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation:
several flavors: star clusters, triples (multiples), AGN disks

How to distinguish different channels?

Rates (uncertain)?
Masses and mass ratio
Residual eccentricity when enter LIGO band (10Hz) or lower-f band



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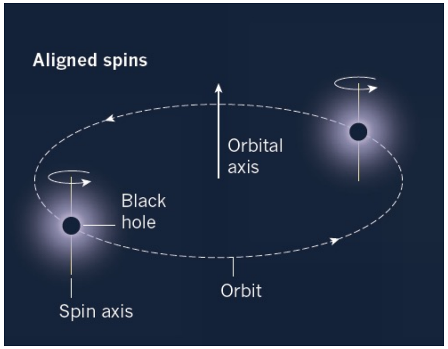
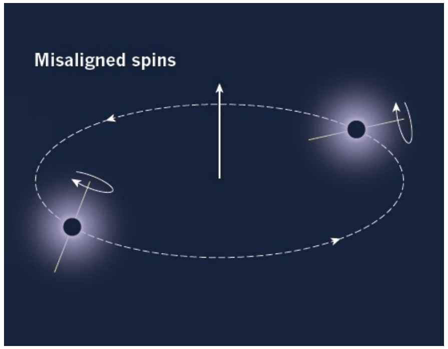
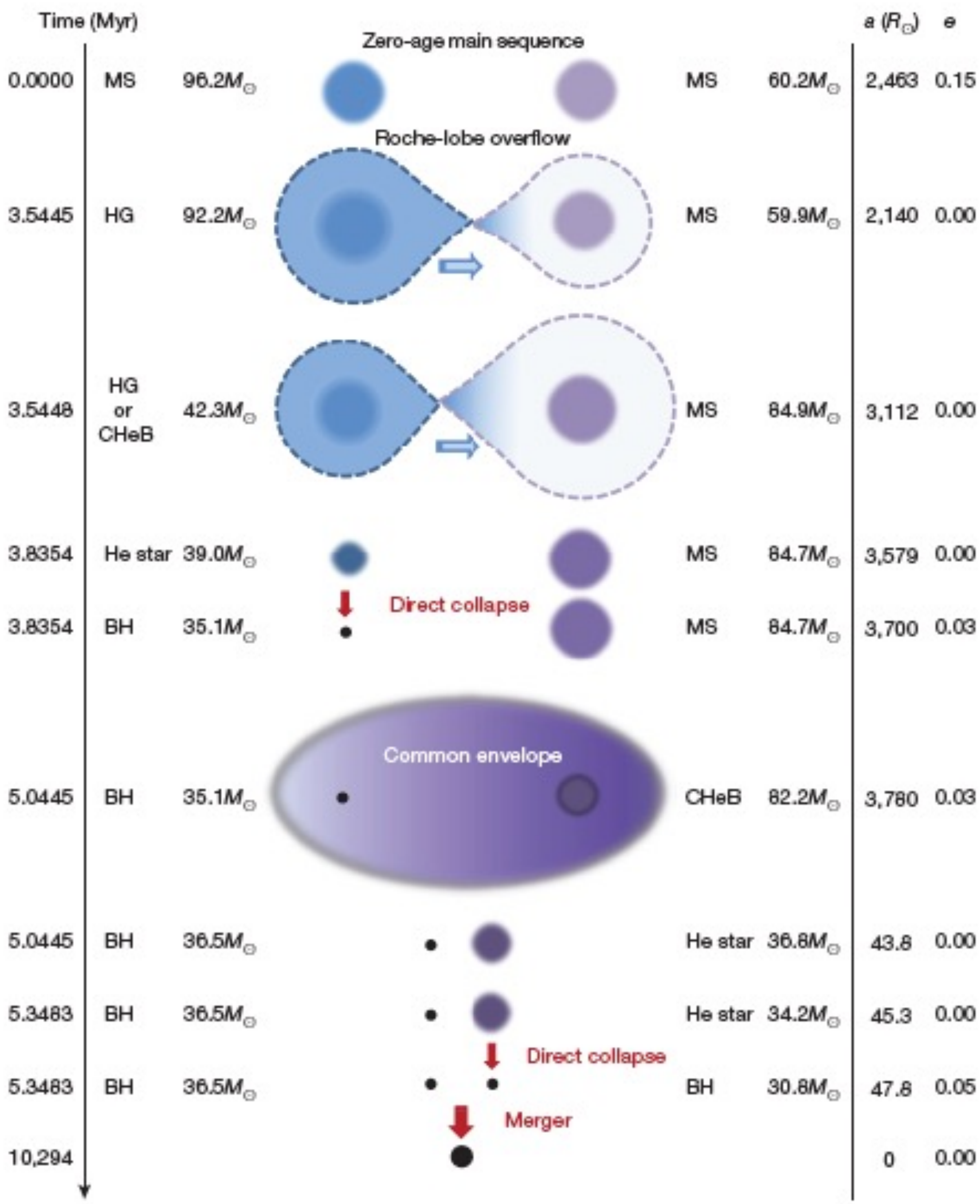
Residual eccentricity when enter LIGO band (10Hz) or lower-f band
Spin-orbit misalignment





Isolated Binary Evolution Channel: Standard



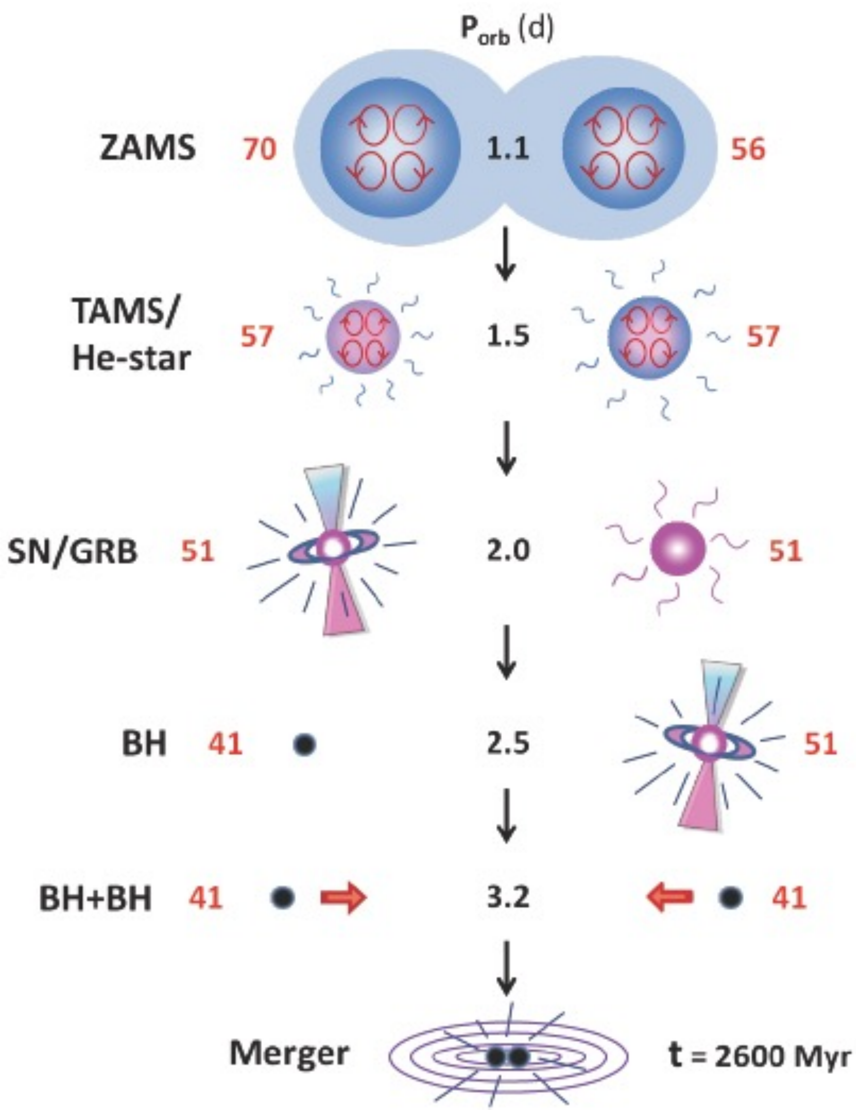
many papers, uncertain physical ingredients (e.g. common envelope)

Produce

circular orbit at 10 Hz
mostly aligned spin-orbit

Belczynski +16

Isolated Binary Evolution Channel:
Chemically homogeneous evolution



Produce

circular orbit at 10 Hz

mostly aligned spin-orbit

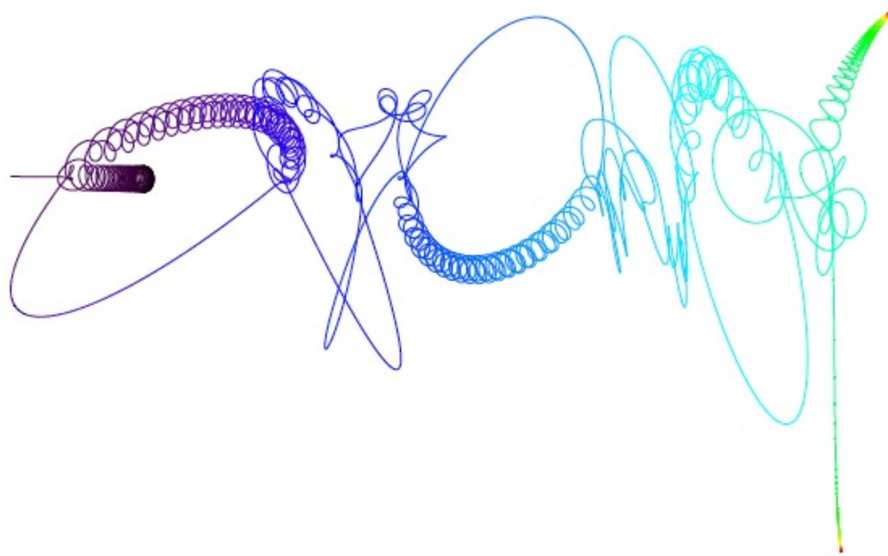
Merchant, Langer et al. 2016

Mandel & de Mink 2016

Dynamical Formation Channels

several flavors…

1. Dense clusters: binary-single scatterings ➔ tight binary



Samsing+14

Enough BHs in clusters? Kicks? GCs or Nuclear Star Clusters?

Produce mostly circular orbit when enter LIGO band (10 Hz) ??

Expect random spin-orbit orientations

Portegies Zwart & McMillan 2000; Rodriguez et al.2015; Chatterjee et al.2017; Samsing et al. 2018; …

Dynamical Formation Channels

several flavors…

1. **Dense clusters: binary-single scatterings ➔ tight binary**

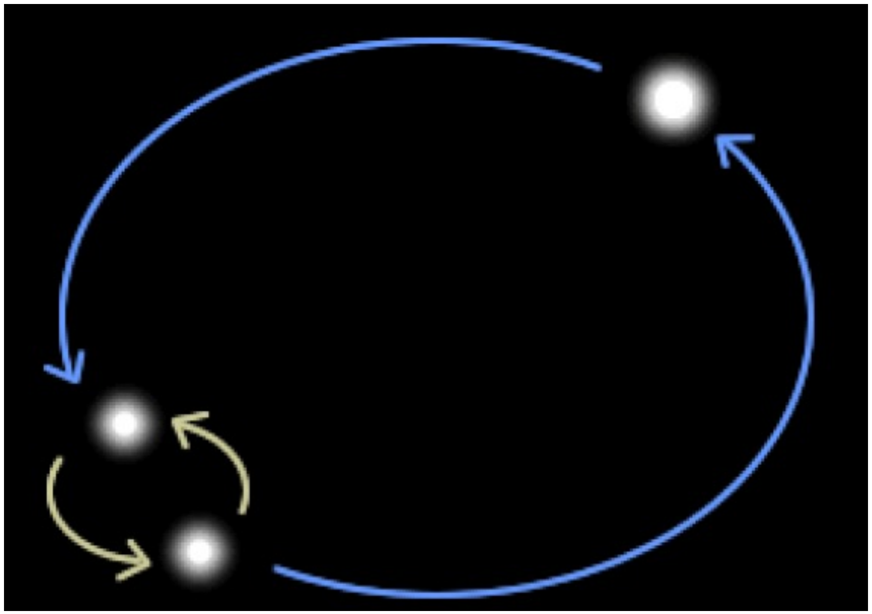
2. **Tertiary-Induced Mergers:**

   Mergers induced by (gentle) perturbations from tertiary companion

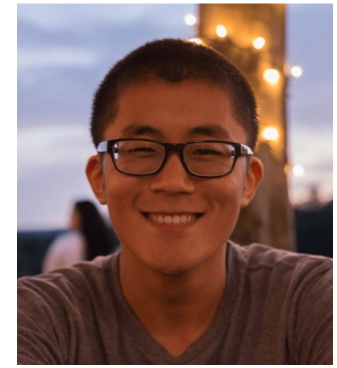
   stellar triples in galactic field, binary around SMBH

Tertiary-Induced Binary Mergers

merger window, mass ratio, GR effects, spin-orbit misalignments



Bin Liu
(Cornell→Niels Bohr Inst)



Yubo Su
(Cornell, Ph.D.22)

Liu & DL 2017,2018,2019,2020,2021
Liu, DL & Wang 2019a,b
Su et al.2021a,b

Previous/related works (in various contexts):

e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Thompson 2011; Antonini et al. 2012,2014,2017, Silsbee & Tremaine 2017; Petrovich & Antonini 2017...

Lidov-Kozai Effect

Can perturbation from the Moon make Earth's satellites fall?

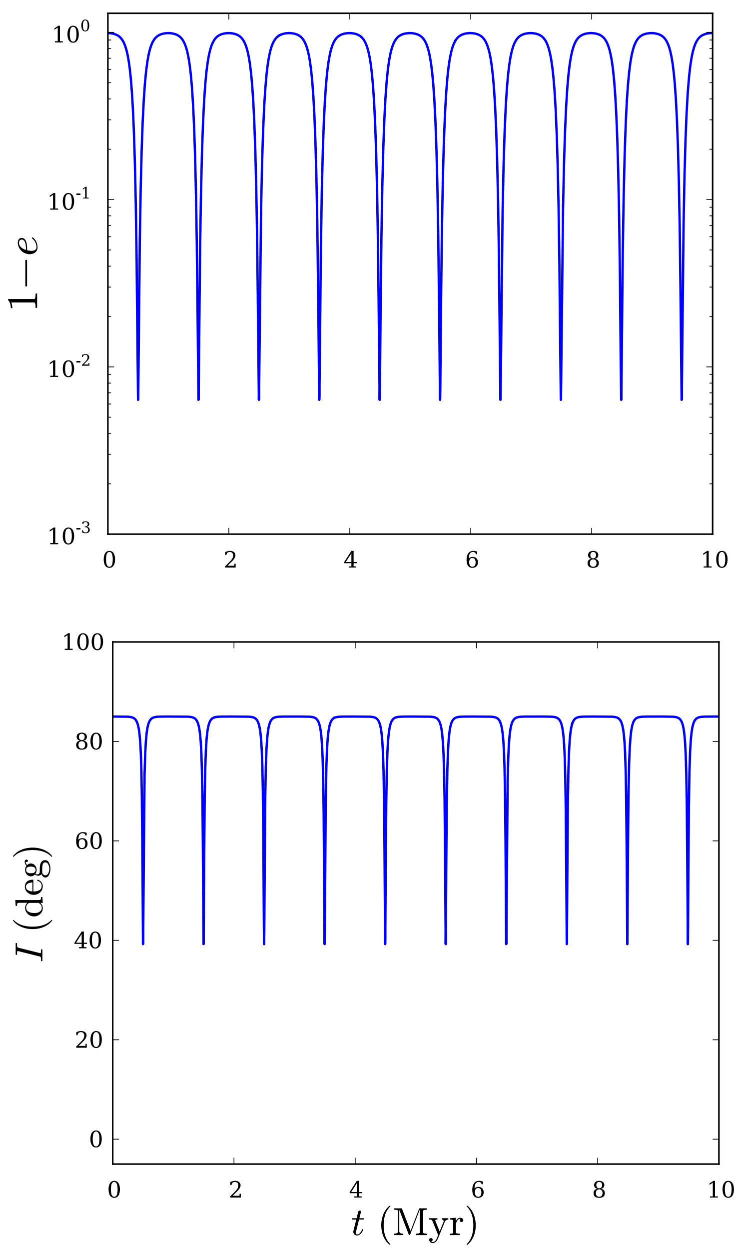
Planet. Space Sci., 1962, Vol. 9, pp. 719 to 759. Pergamon Press Ltd. Printed in Northern Ireland

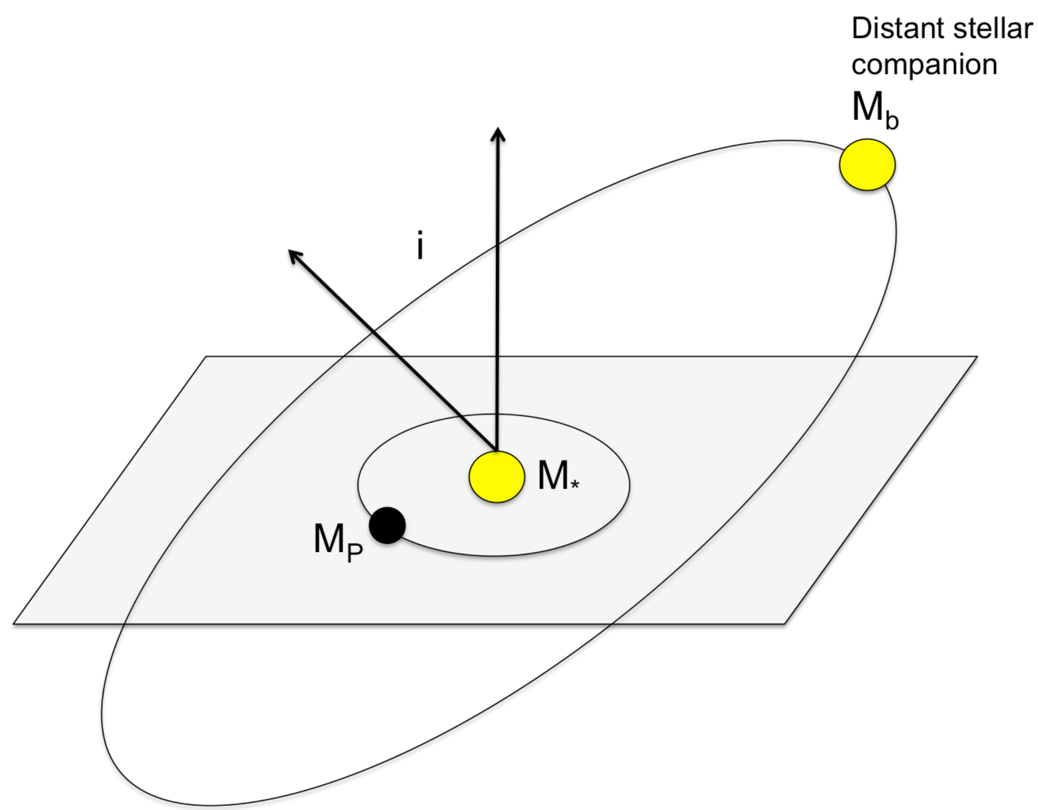
THE EVOLUTION OF ORBITS OF ARTIFICIAL SATELLITES OF PLANETS UNDER THE ACTION OF GRAVITATIONAL PERTURBATIONS OF EXTERNAL BODIES

M. L. LIDOV

Translated by H. F. Cleaves from *Iskusstvennye Sputniki Zemli*, No. 8, p. 5, 1961.

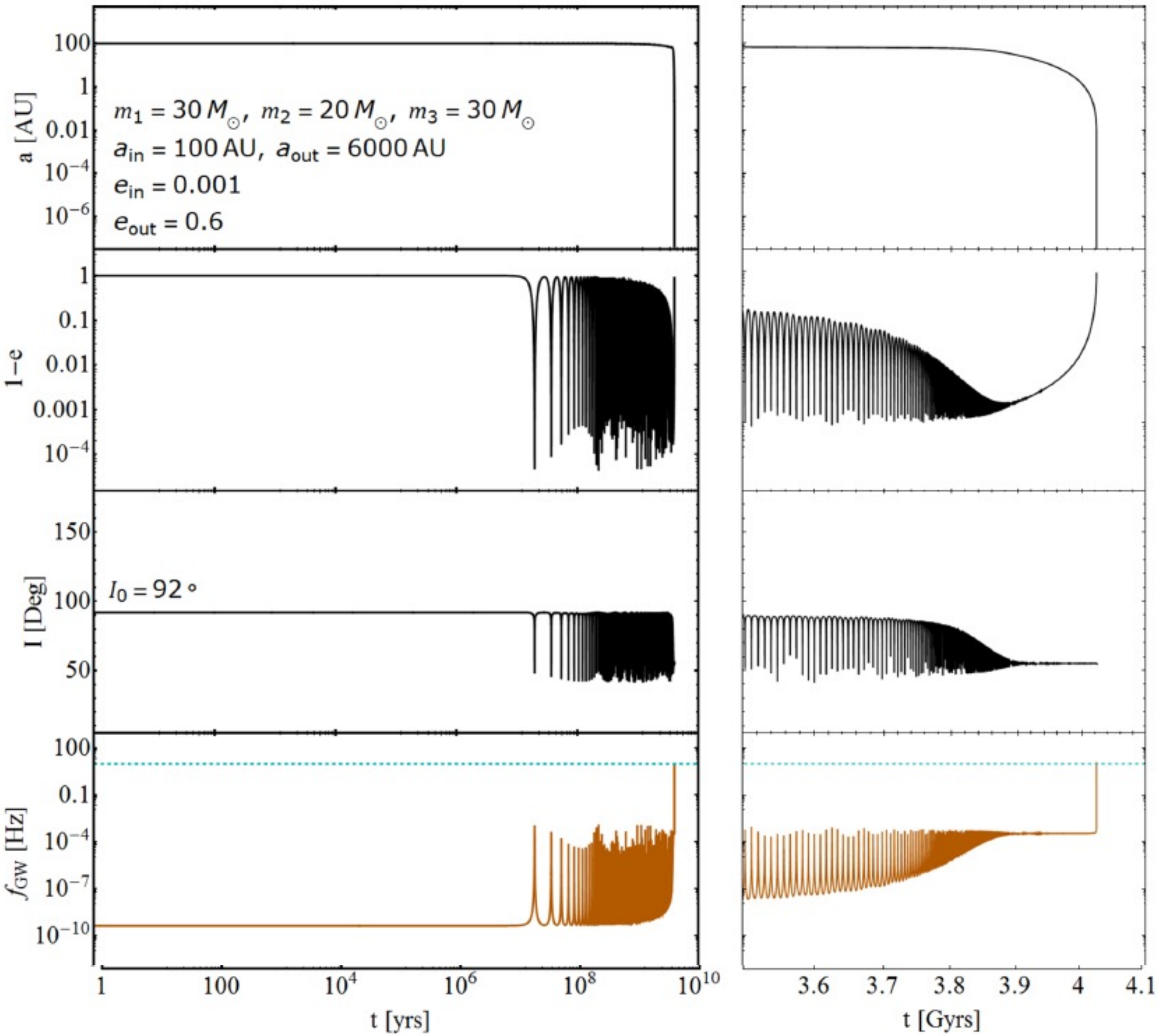
Lidov-Kozai Effect





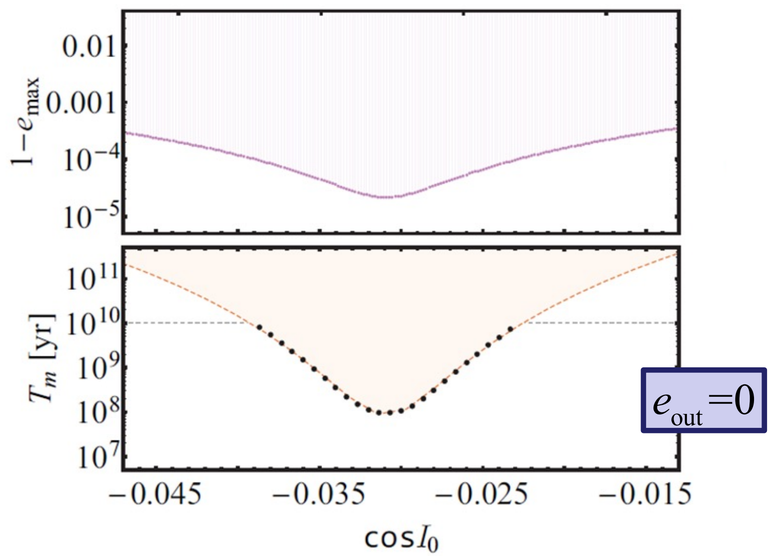
- Eccentricity and inclination oscillations induced if i > 40 degrees.
- If i large (85-90 degrees), get extremely large eccentricities (e > 0.99)

LK oscillation + Gravitation Radiation



Inclination window for merger

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100AU$



Fixed quadrupole strength $\frac{m_3}{a^3_{\rm out,eff}}$

Quadrupole LK:
e_{max} vs I_0 analytic:
LK driving compete with GR apsidal precession

Merger window (almost) analytic

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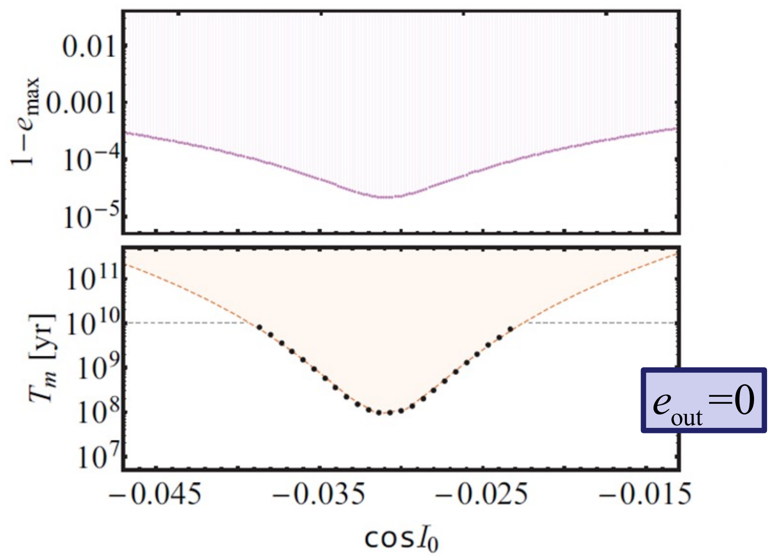


Fixed quadrupole strength $\frac{m_3}{a^3_{\rm out,eff}}$

Quadrupole LK:
e_{max} vs I_0 analytic:
LK driving compete with GR apsidal precession

Merger window (almost) analytic

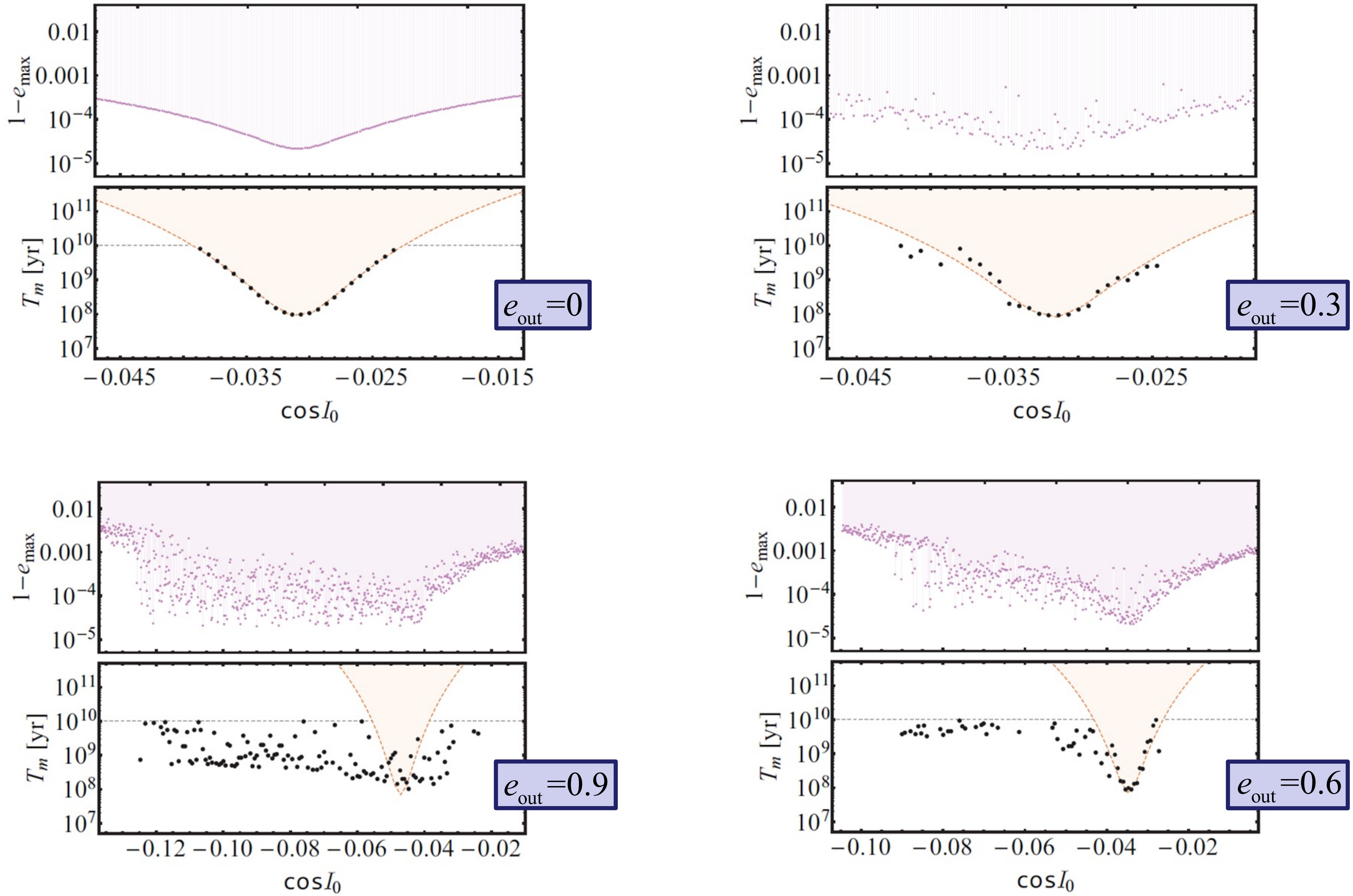
Octupole effect makes orbital evolution "chaotic":

$$\epsilon_{\rm oct} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\rm out}} \frac{e_{\rm out}}{1 - e^2_{\rm out}}$$

Inclination window for merger

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100AU$

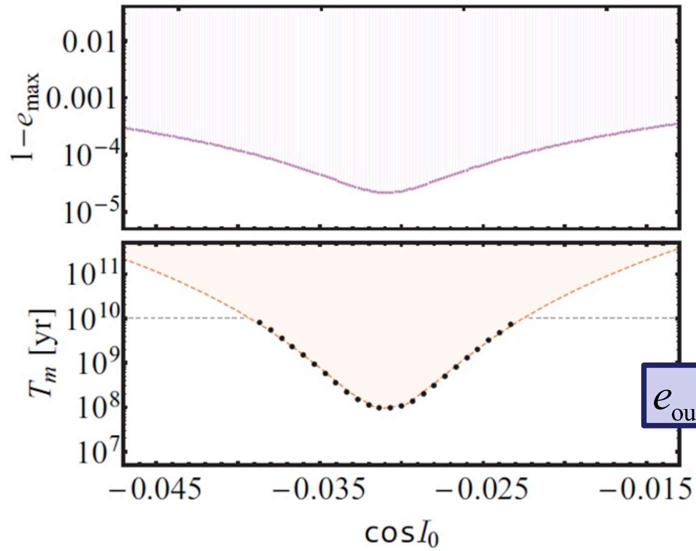
Fixed quadrupole strength $\frac{m_3}{a^3_{\mathrm{out,eff}}}$

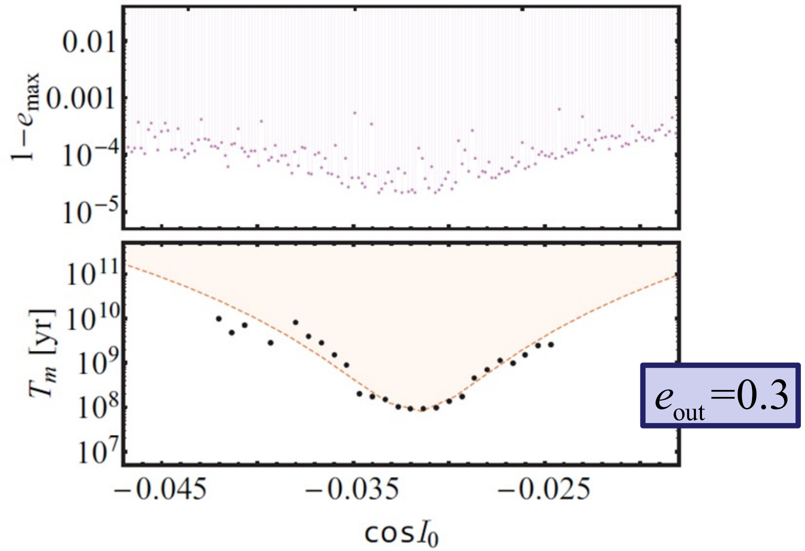


Inclination window for merger

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100AU$

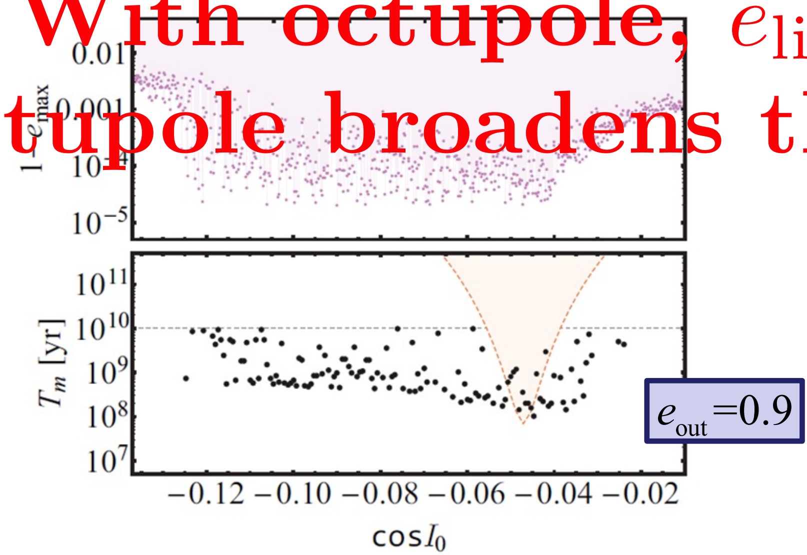
Fixed $m_3/a_{\rm out}^3$ value

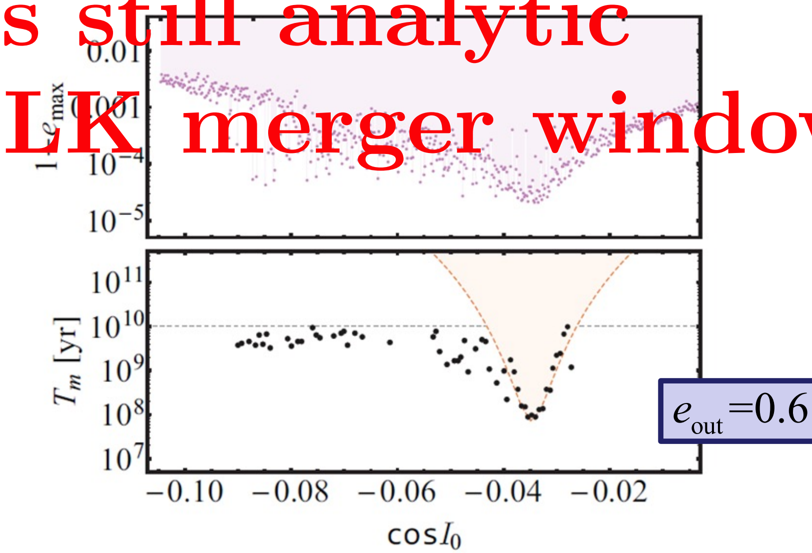




With octupole, $e_{\rm lim}$ is still analytic

Octupole broadens the LK merger window

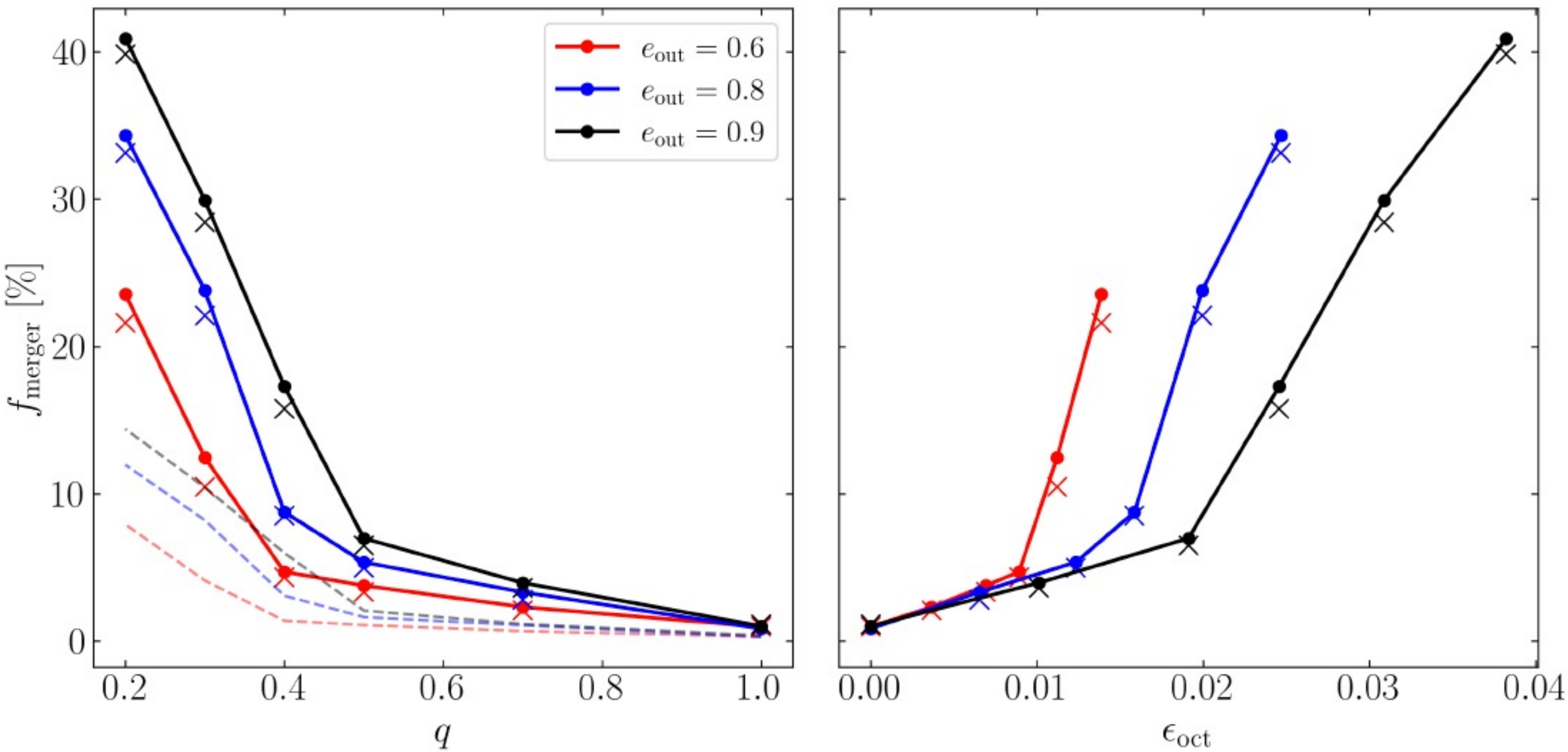




Octupole effect depends on

$$\epsilon_{\rm oct} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\rm out}} \frac{e_{\rm out}}{1 - e_{\rm out}^2}$$

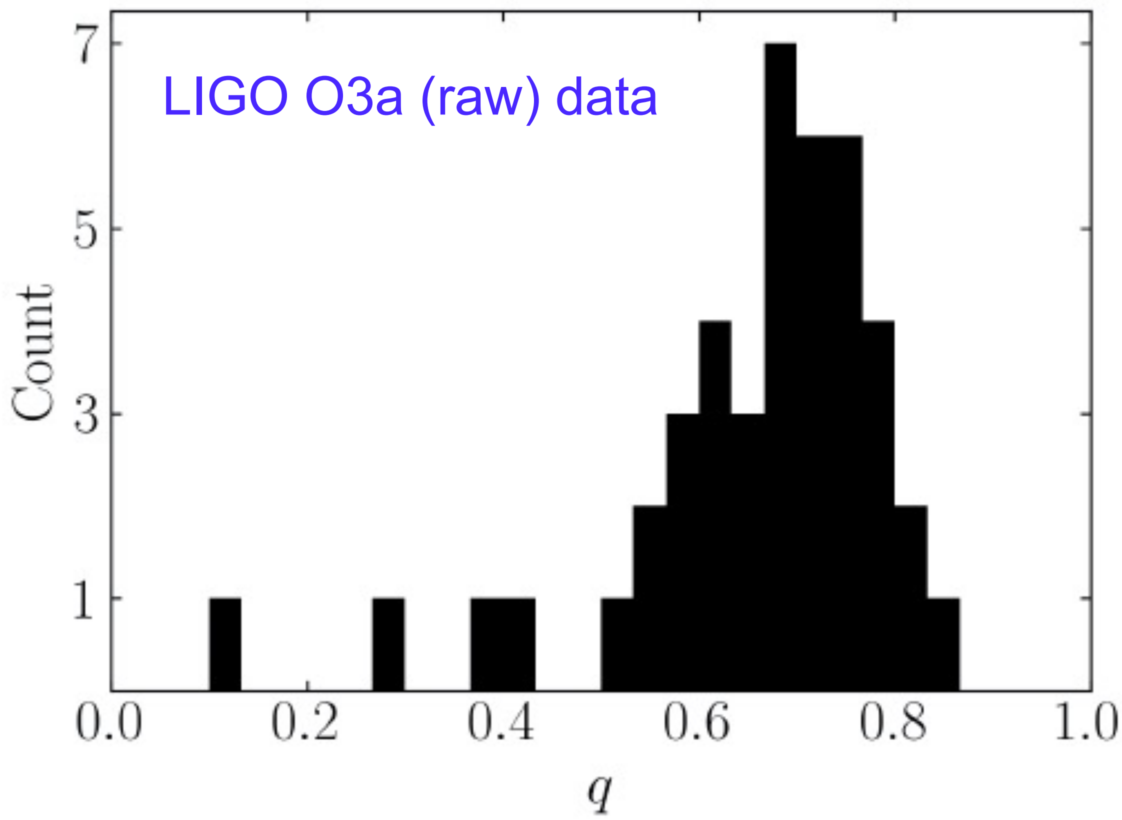
➔ Large $e_{\rm out}$ and/or small $q = m_2/m_1$ increases merger window



a = 100 AU, $a_{out,eff}$ = 3600 AU, $m_{12} = 50M_\odot$, $m_3 = 30M_\odot$

Su et al. 2021

Is this compatible with the observed q-distribution of merging BH binaries?



-- Depends on the initial q-distribution of BH binaries.

-- Maybe incompatible with data if octupole effect is strong…

Octupole effect depends on

$$\epsilon_{\mathrm{oct}} = \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{a_{\mathrm{out}}} \frac{e_{\mathrm{out}}}{1 - e_{\mathrm{out}}^2}$$

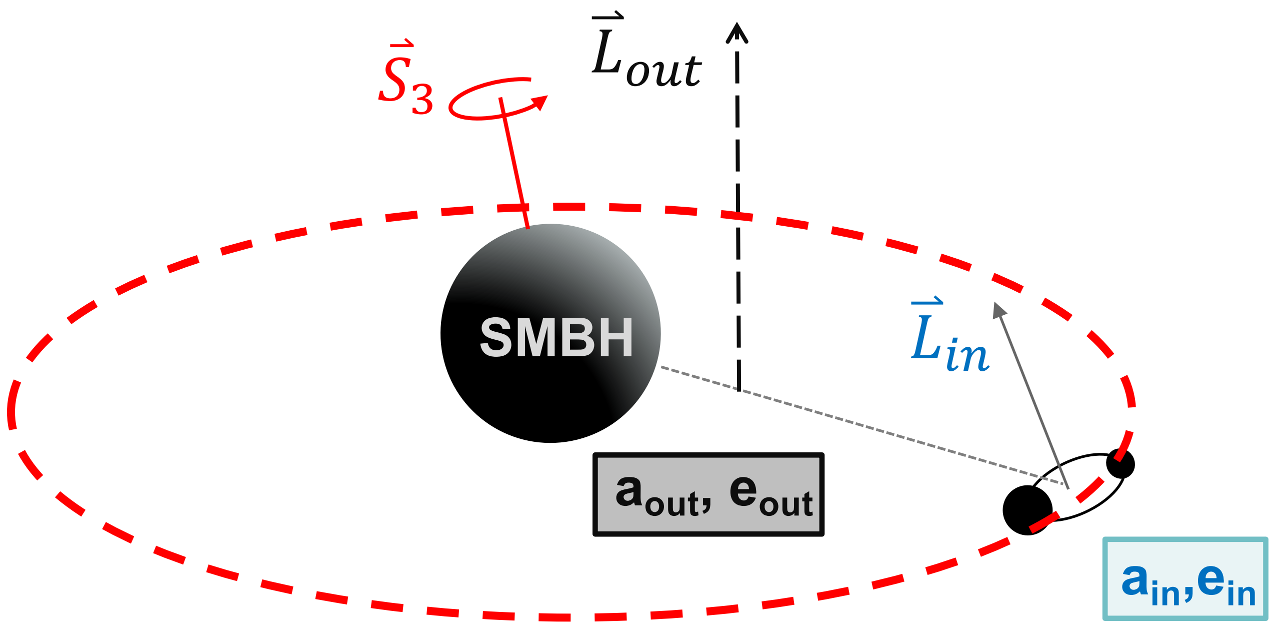
If the tertiary is a (super)massive BH, then $a_{\mathrm{out}} \gg a$

➔ Small octupole effect

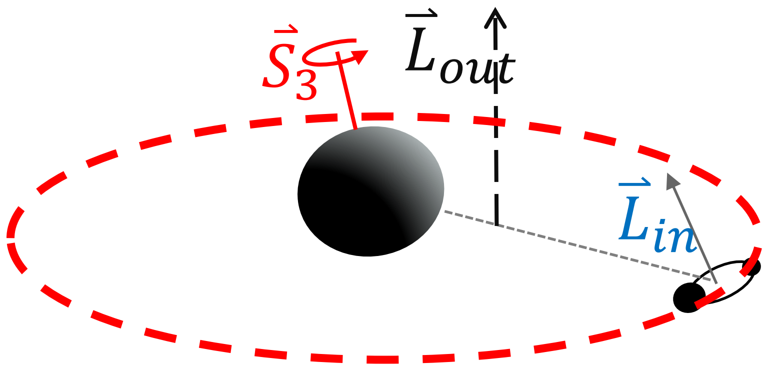
➔ Weak dependence of merger fraction on mass ratio

What happens if the tertiary is a Supermassive BH ?

Relativistic Effects induced by the SMBH



Three leading-order GR effects (recognizing L_{in} behaves like a "spin")



Effect I: ***de-Sitter-like Precession of L_{in} around L_{out}***

--- 1.5 PN

$$\begin{cases} \left.\dfrac{d\mathbf{L}}{dt}\right|_{\mathrm{L_{in}L_{out}}} = \Omega^{(\mathrm{GR})}_{\mathrm{L_{in}L_{out}}}\hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{L}, \\ \left.\dfrac{d\mathbf{e}}{dt}\right|_{\mathrm{L_{in}L_{out}}} = \Omega^{(\mathrm{GR})}_{\mathrm{L_{in}L_{out}}}\hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}, \end{cases}$$

Effect II: ***Precession of L_{out} around S_3***

--- 1.5 PN

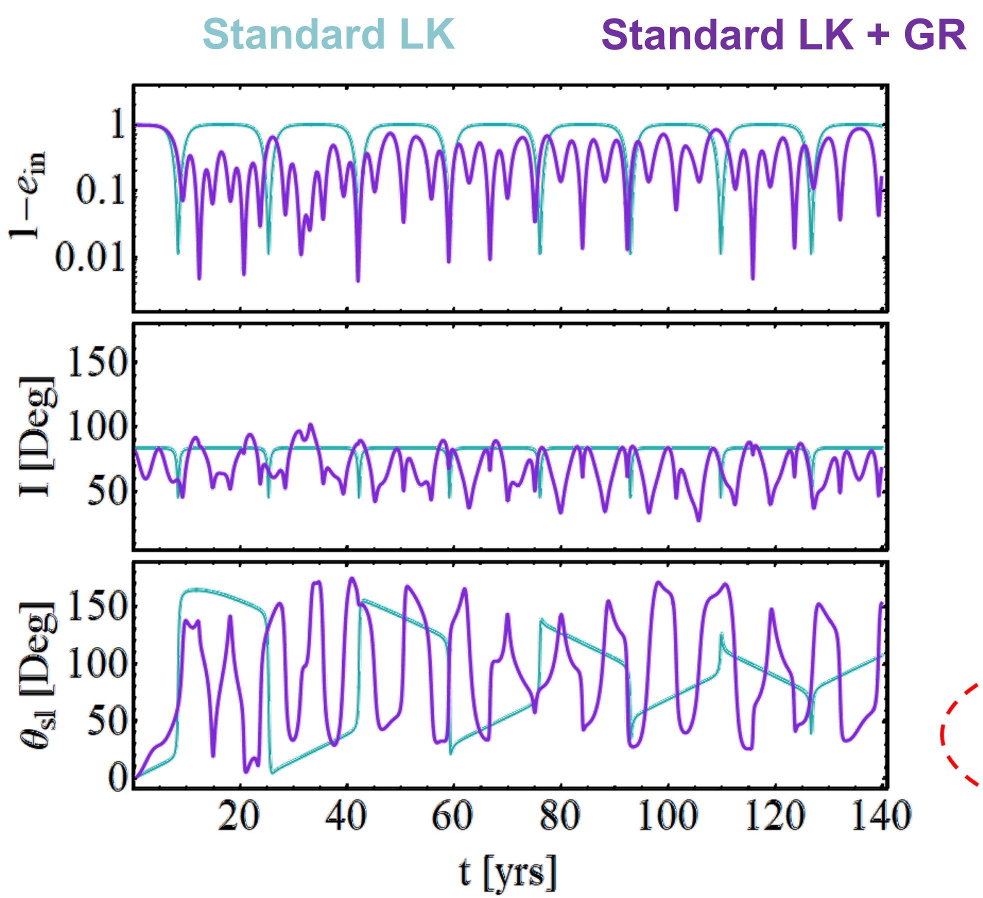
$$\begin{cases} \left.\dfrac{d\mathbf{L}_{\mathrm{out}}}{dt}\right|_{\mathrm{L_{out}S_3}} = \Omega_{\mathrm{L_{out}S_3}}\hat{\mathbf{S}}_3 \times \mathbf{L}_{\mathrm{out}}, \\ \left.\dfrac{d\mathbf{e}_{\mathrm{out}}}{dt}\right|_{\mathrm{L_{out}S_3}} = \Omega_{\mathrm{L_{out}S_3}}\hat{\mathbf{S}}_3 \times \mathbf{e}_{\mathrm{out}} \\ \qquad -3\Omega_{\mathrm{L_{out}S_3}}(\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3)\hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}_{\mathrm{out}} \end{cases}$$

Effect III: ***Lense-Thirring Precession of L_{in} around S_3***

--- 2 PN

$$\begin{cases} \left.\dfrac{d\mathbf{L}}{dt}\right|_{\mathrm{L_{in}S_3}} = \Omega_{\mathrm{L_{in}S_3}}\hat{\mathbf{S}}_3 \times \mathbf{L} \\ \qquad -3\Omega_{\mathrm{L_{in}S_3}}(\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3)\hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{L}, \\ \left.\dfrac{d\mathbf{e}}{dt}\right|_{\mathrm{L_{in}S_3}} = \Omega_{\mathrm{L_{in}S_3}}\hat{\mathbf{S}}_3 \times \mathbf{e} \\ \qquad -3\Omega_{\mathrm{L_{in}S_3}}(\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3)\hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}. \end{cases}$$

Evolution Example



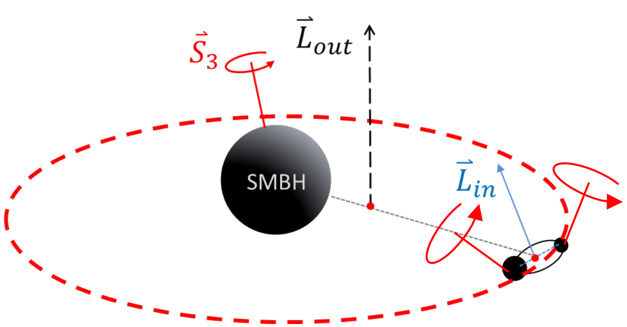
System Parameters:

$(m_1, m_2, m_3) = (30, 20, 2.3 \times 10^9)\ M_\odot$

$(a_{in,0}, a_{out}) = (0.1, 500)$ AU

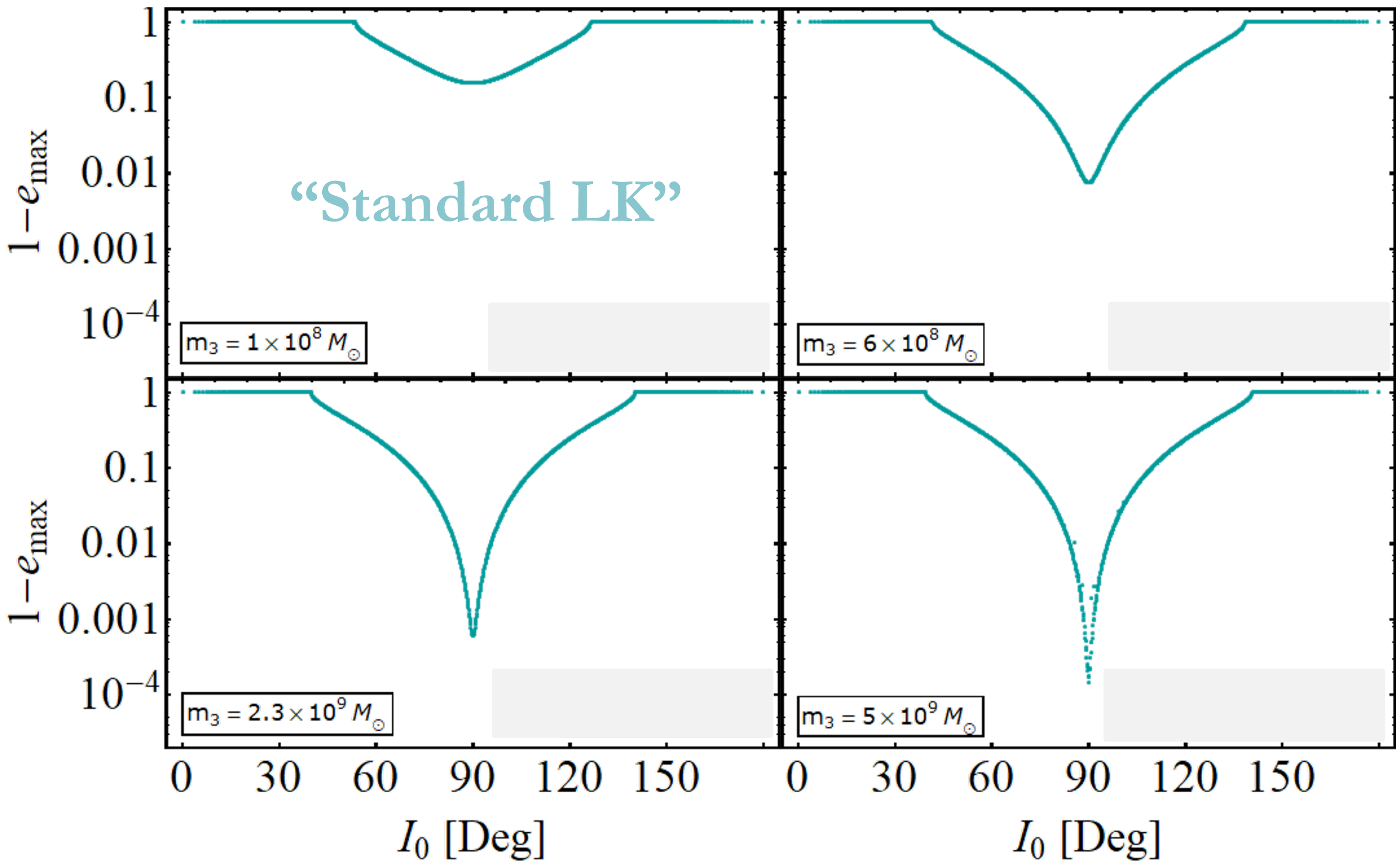
$(e_{in,0}, e_{out,0}) = 0.001$

$I_0 = 84^\circ$



Eccentricity Excitation

$(m_1, m_2) = (30, 20)\ M_\odot$; $(a_{in,0}, a_{out}) = (0.1, 500)$ AU ; Circular Orbits



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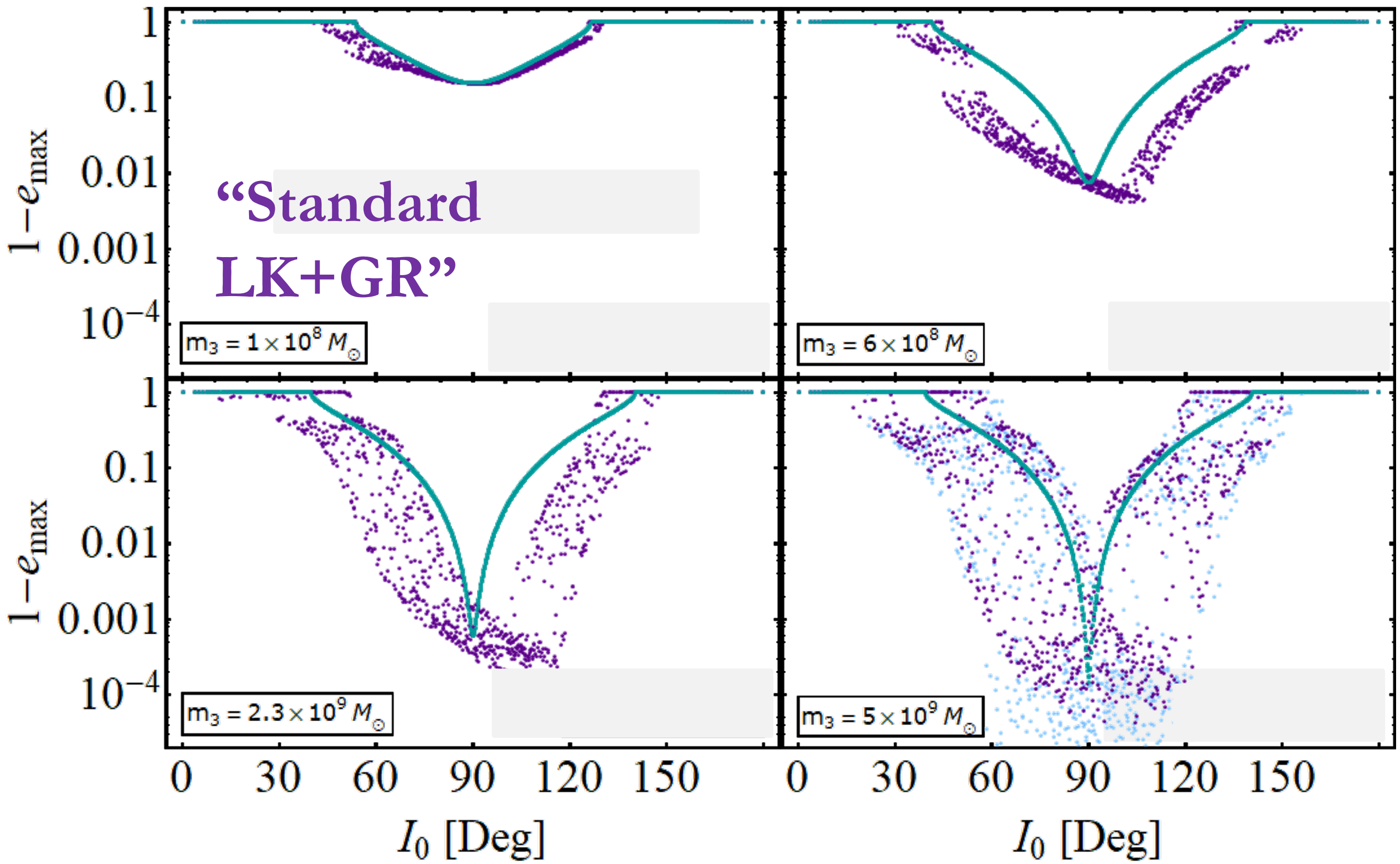
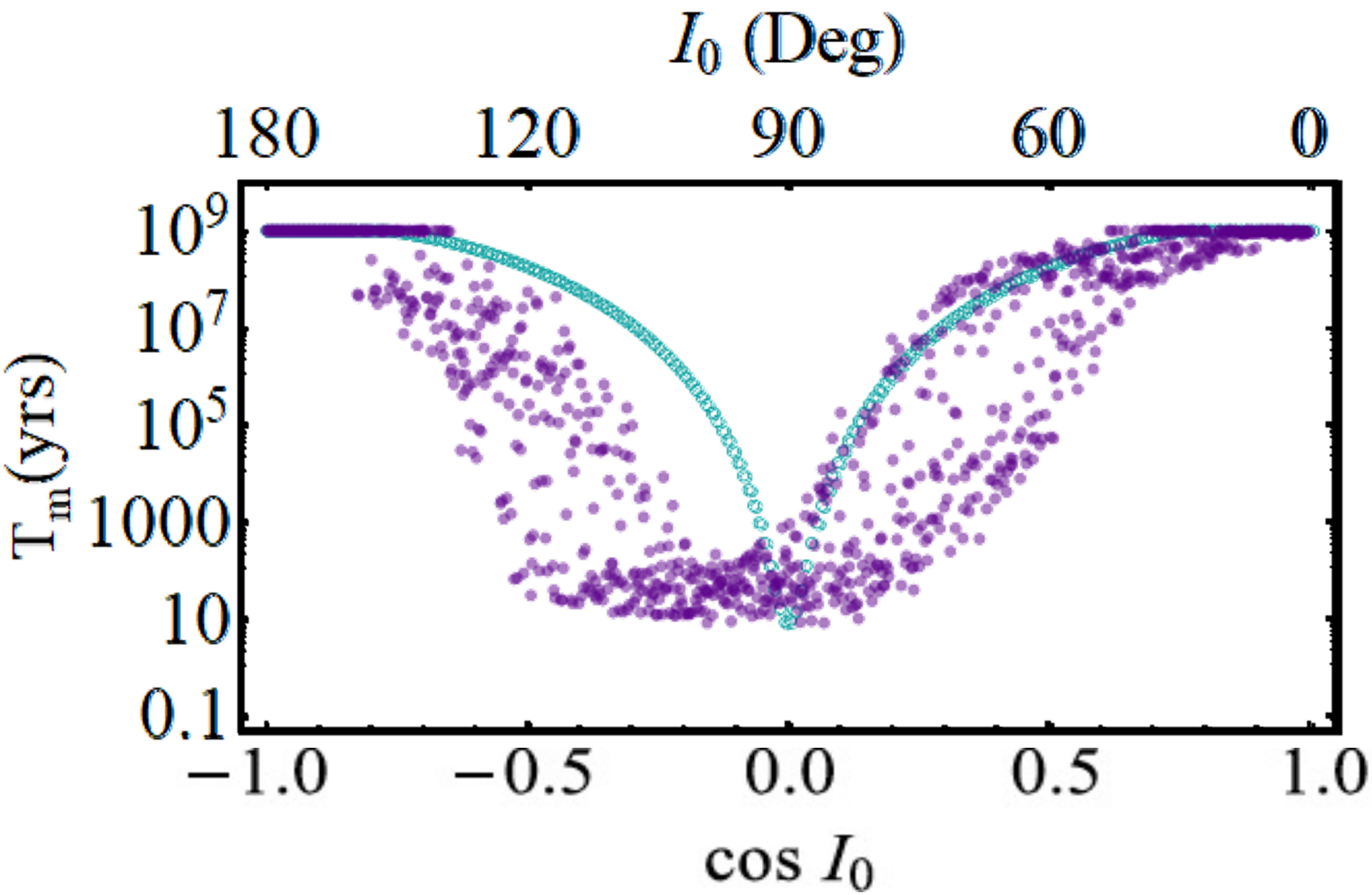


GR effects broaden merger window



"Standard LK+GW"

"Standard LK+GR+GW"

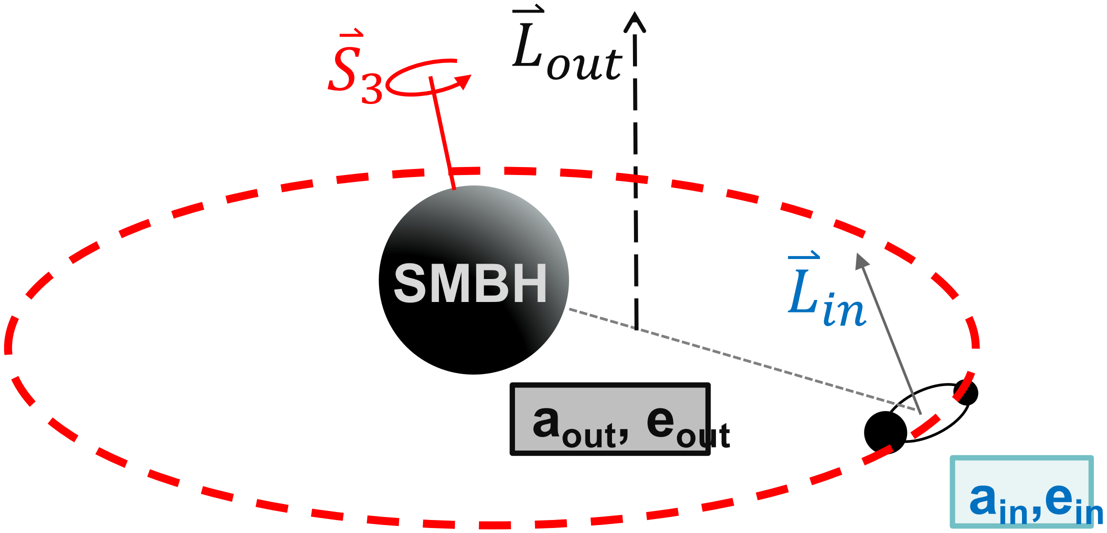
$(m_1, m_2, m_3) = (30, 20, 2.3\times10^9)\ M_\odot$

$(a_{in,0}, a_{out}) = (0.1, 500)$ AU

GR effects broaden Merger window: Why?

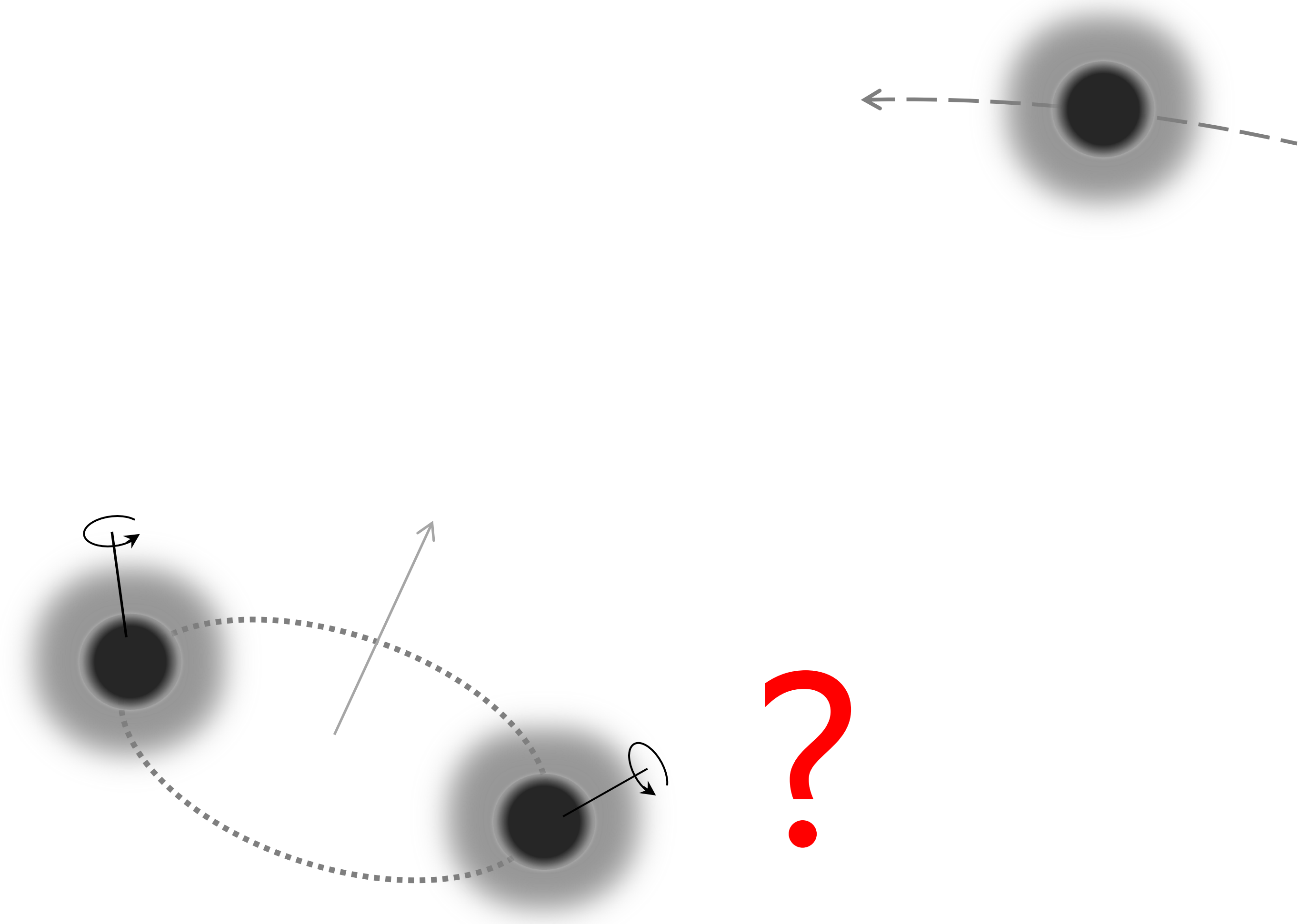
Inclination Resonance

$$\gamma \equiv \frac{\Omega_{\mathrm{L_{in}L_{out}}}}{\Omega_{\mathrm{L_{out}S_3}}} = \frac{\Omega^{(\mathrm{N})}_{\mathrm{L_{in}L_{out}}} + \Omega^{(\mathrm{GR})}_{\mathrm{L_{in}L_{out}}}}{\Omega_{\mathrm{L_{out}S_3}}}$$



Mutual inclination is excited when the two precession rates are equal

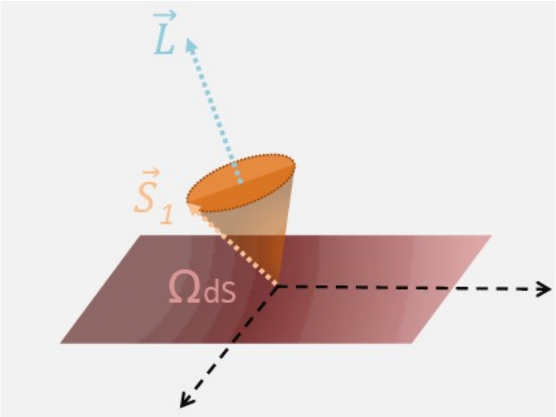
What about the BH Spin?



Spin-Orbit Coupling

The de Sitter precession of spin around the angular momentum axis of the binary

$$\frac{d\hat{S}_1}{dt} = \Omega_{dS} \hat{L} \times \hat{S}_1 \qquad \Omega_{dS} = \frac{3Gn\left(m_2 + \mu/3\right)}{2c^2 a\left(1-e^2\right)}$$

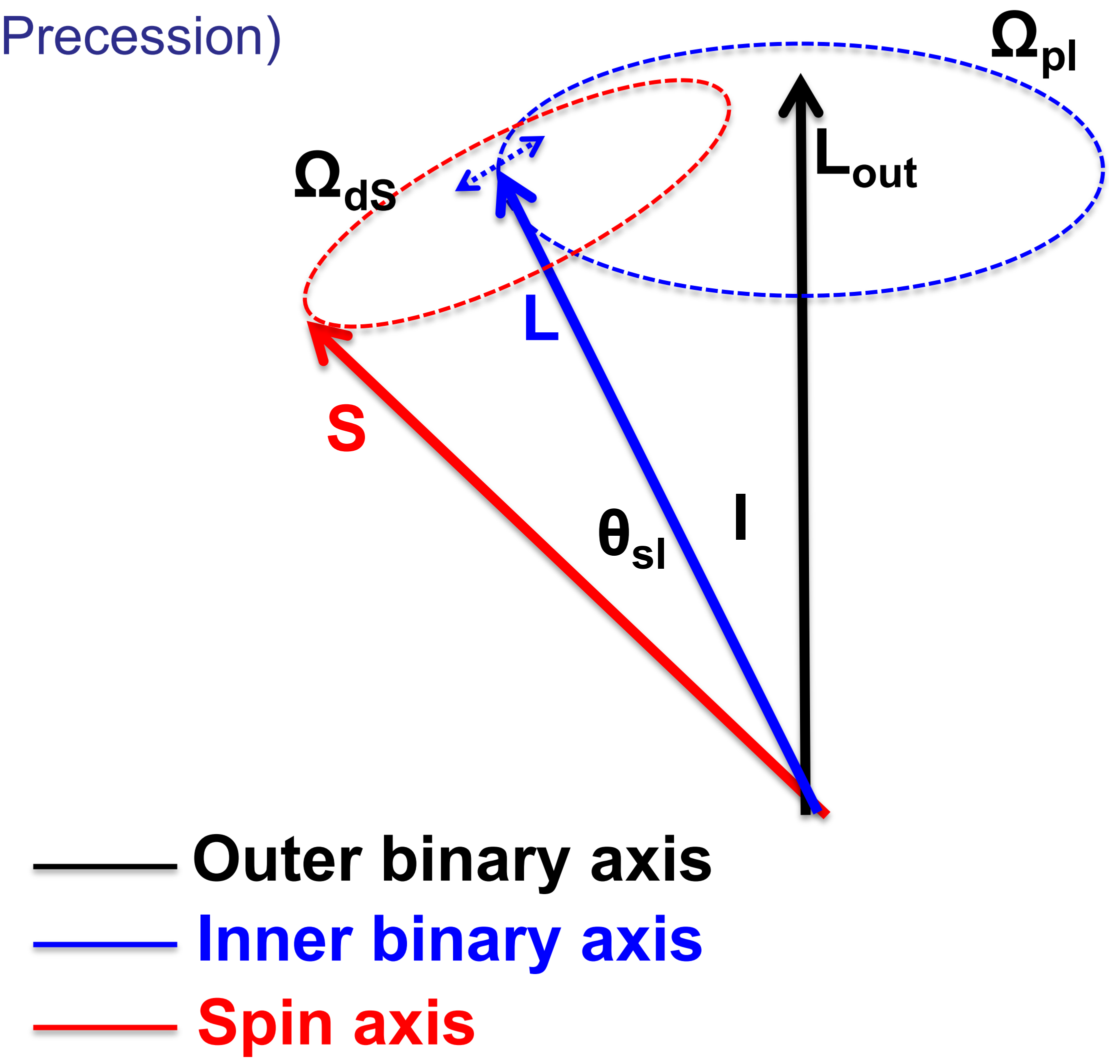


BH spin dynamics during LK oscillations

$$\frac{d\hat{\mathbf{S}}_1}{dt} = \Omega_{\rm dS}\hat{\mathbf{L}} \times \hat{\mathbf{S}}_1$$ (de Sitter Precession)

But **L** precesses and nutates during LK oscillations

$$\Omega_{\rm pl} \simeq \frac{3(1+e^2)}{t_{\rm LK}\sqrt{1-e^2}}|\sin 2I|$$



BH spin dynamics during LK oscillations

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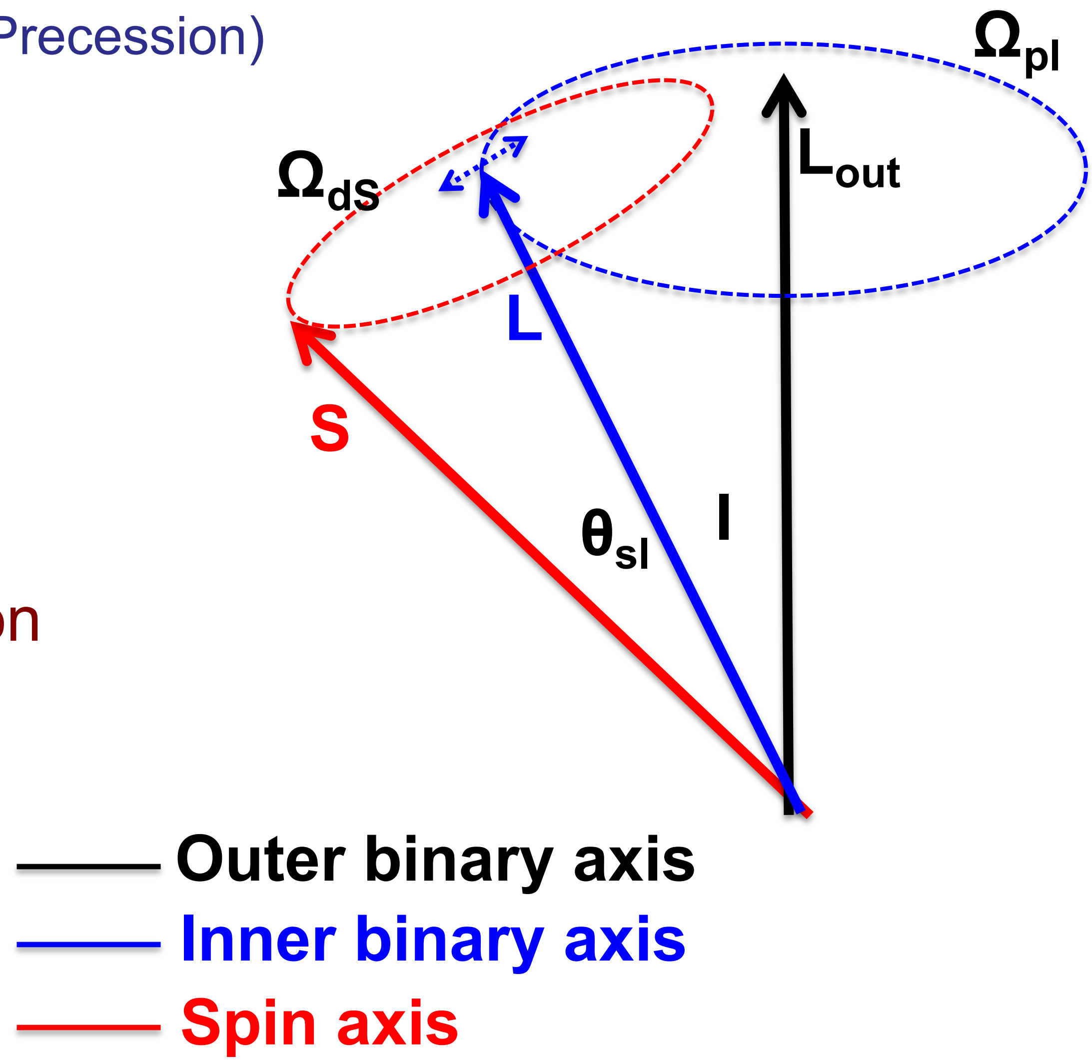
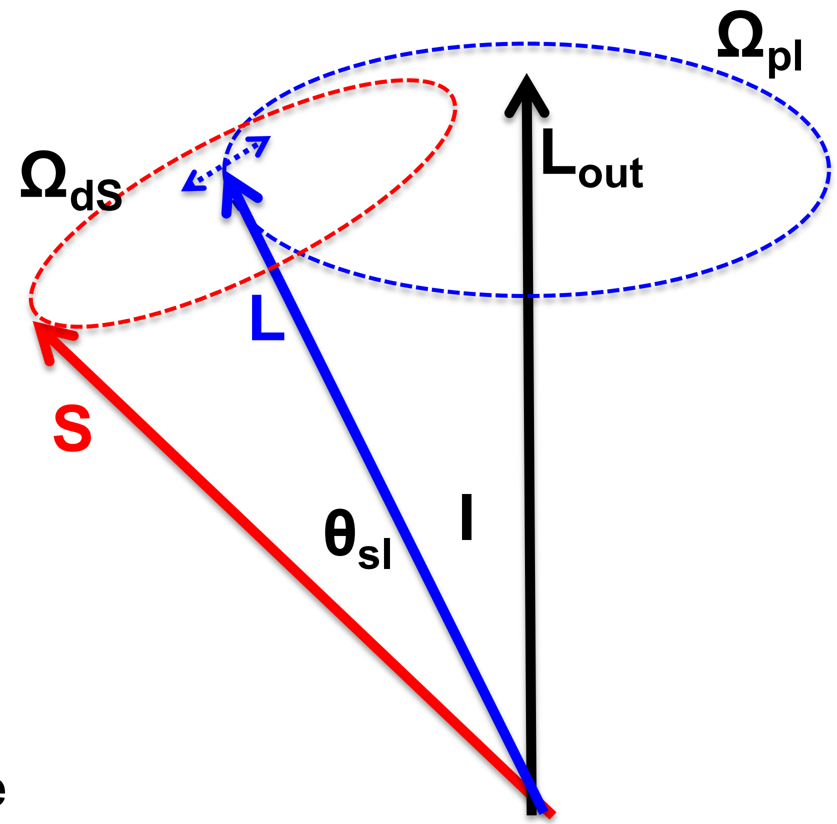
Spin dynamics depends on

$$\Omega_{\rm dS} \ \text{vs} \ \Omega_{\rm pl}$$

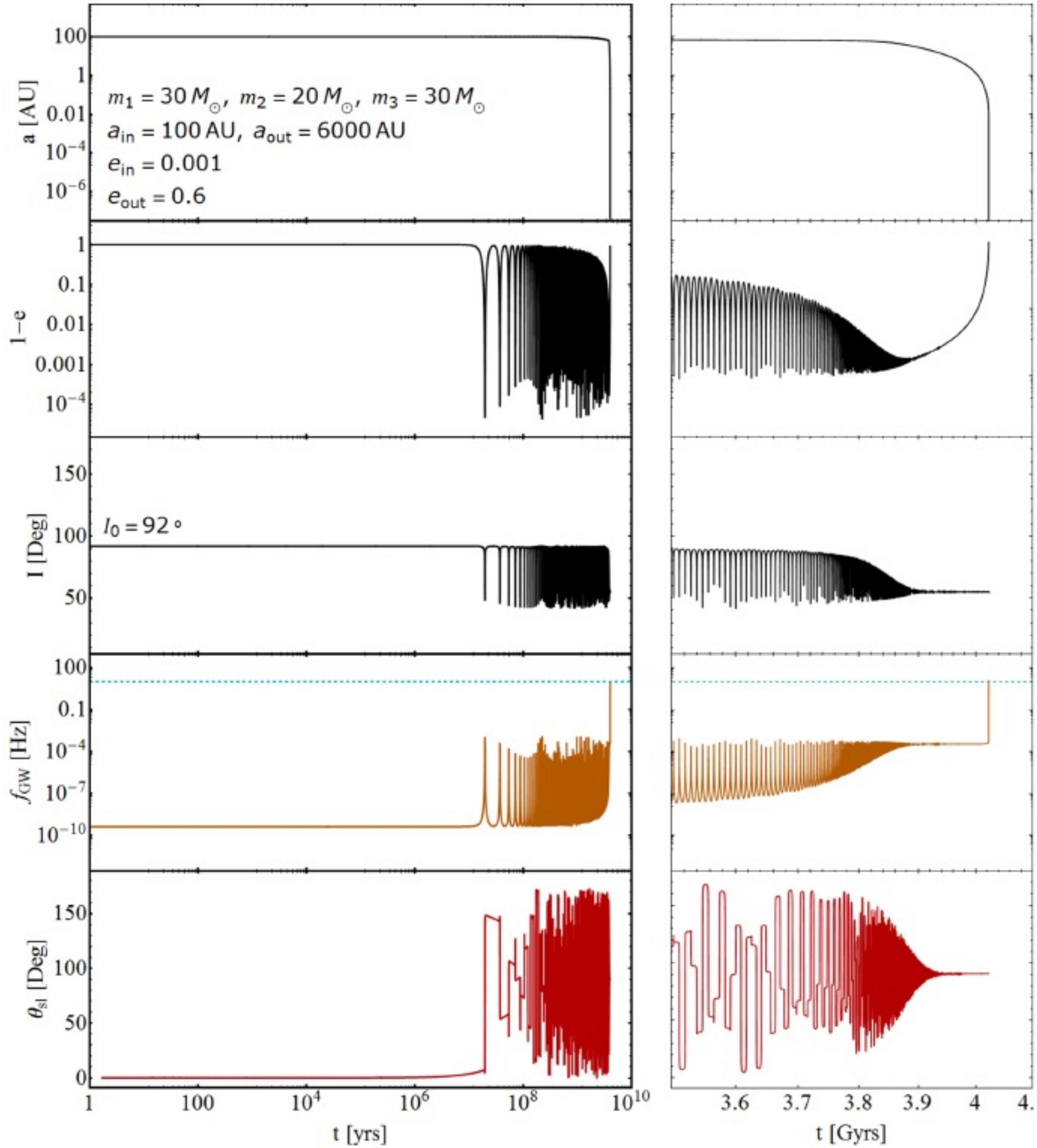


BH spin evolution in LK-induced orbital decay



$$\Omega_{\rm dS}/\Omega_{\rm pl}$$

changes from << 1 (non-adiabatic)
to >> 1 (adiabatic) as the orbit decays
➔ Final spin-orbit misalignment angle

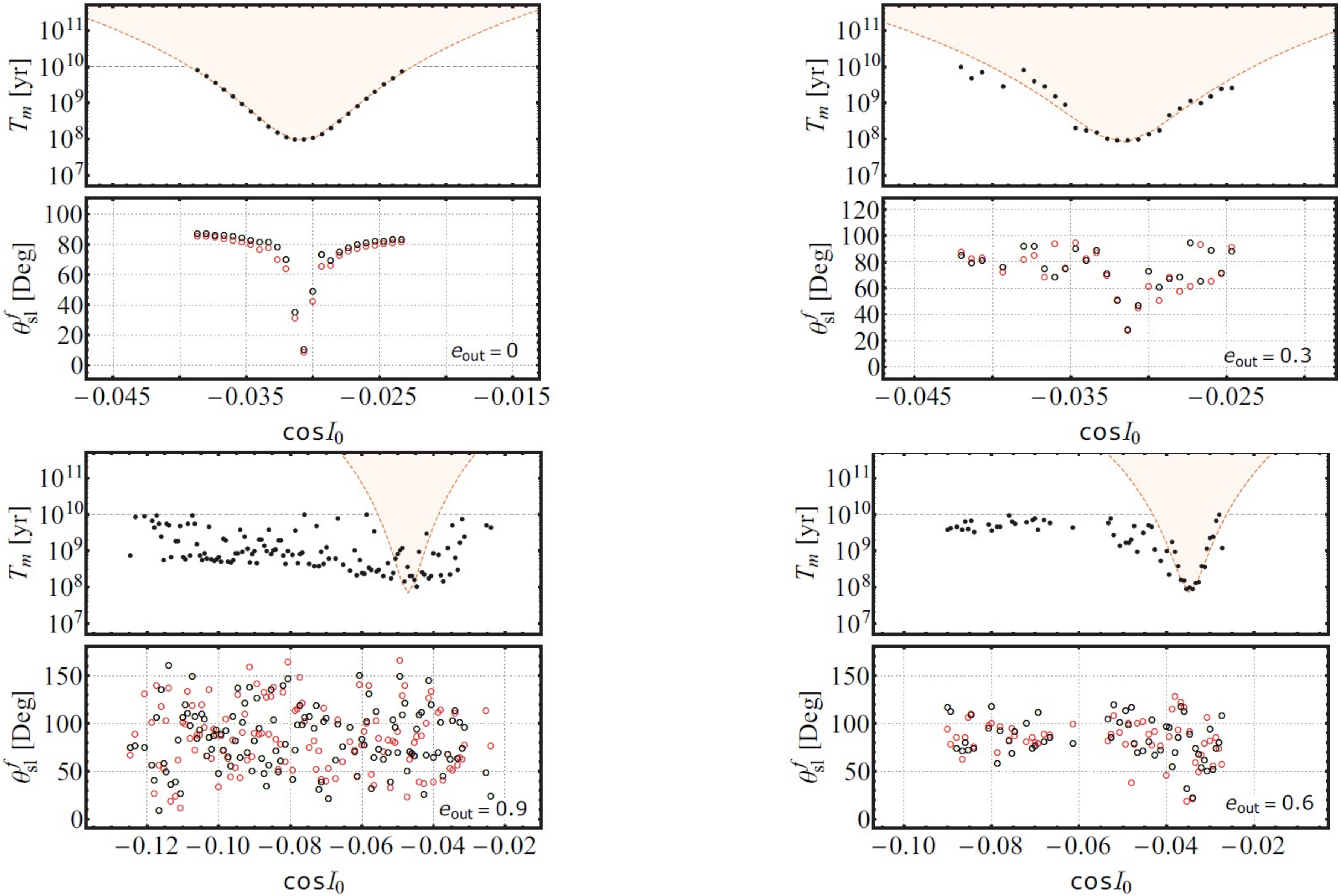


$m_1 = 30\,M_\odot,\ m_2 = 20\,M_\odot,\ m_3 = 30\,M_\odot$
$a_{\rm in} = 100\,\mathrm{AU},\ a_{\rm out} = 6000\,\mathrm{AU}$
$e_{\rm in} = 0.001$
$e_{\rm out} = 0.6$
$I_0 = 92^\circ$
a [AU]
1−e
I [Deg]
$f_{\rm GW}$ [Hz]
$\theta_{\rm sl}$ [Deg]
t [yrs]
t [Gyrs]

Merger Window and Final Spin-Orbit Misalignments

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100AU$

Fixed $m_3/a_{\rm out}^3$ value



Merger Window and Final Spin-Orbit Misalignments

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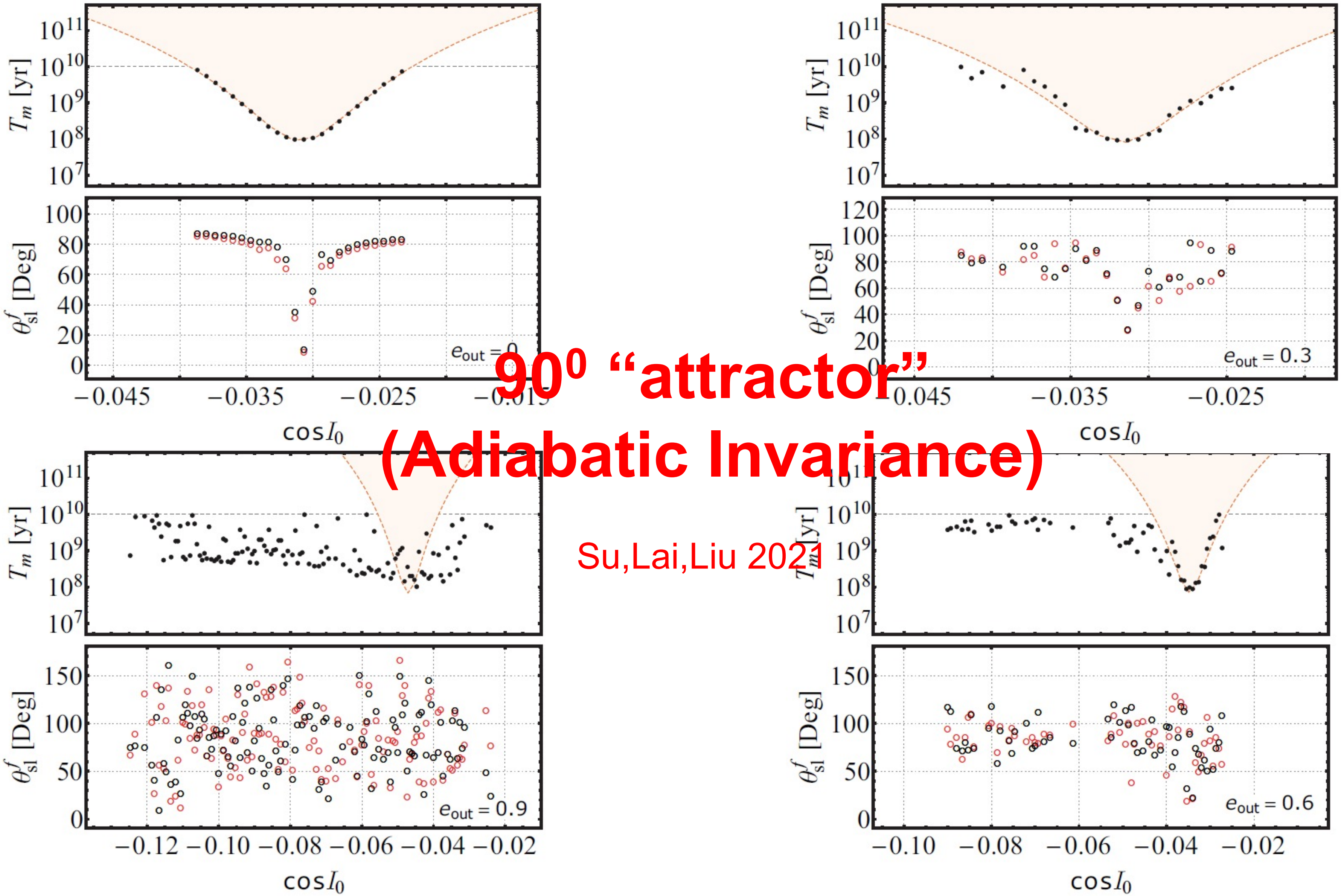
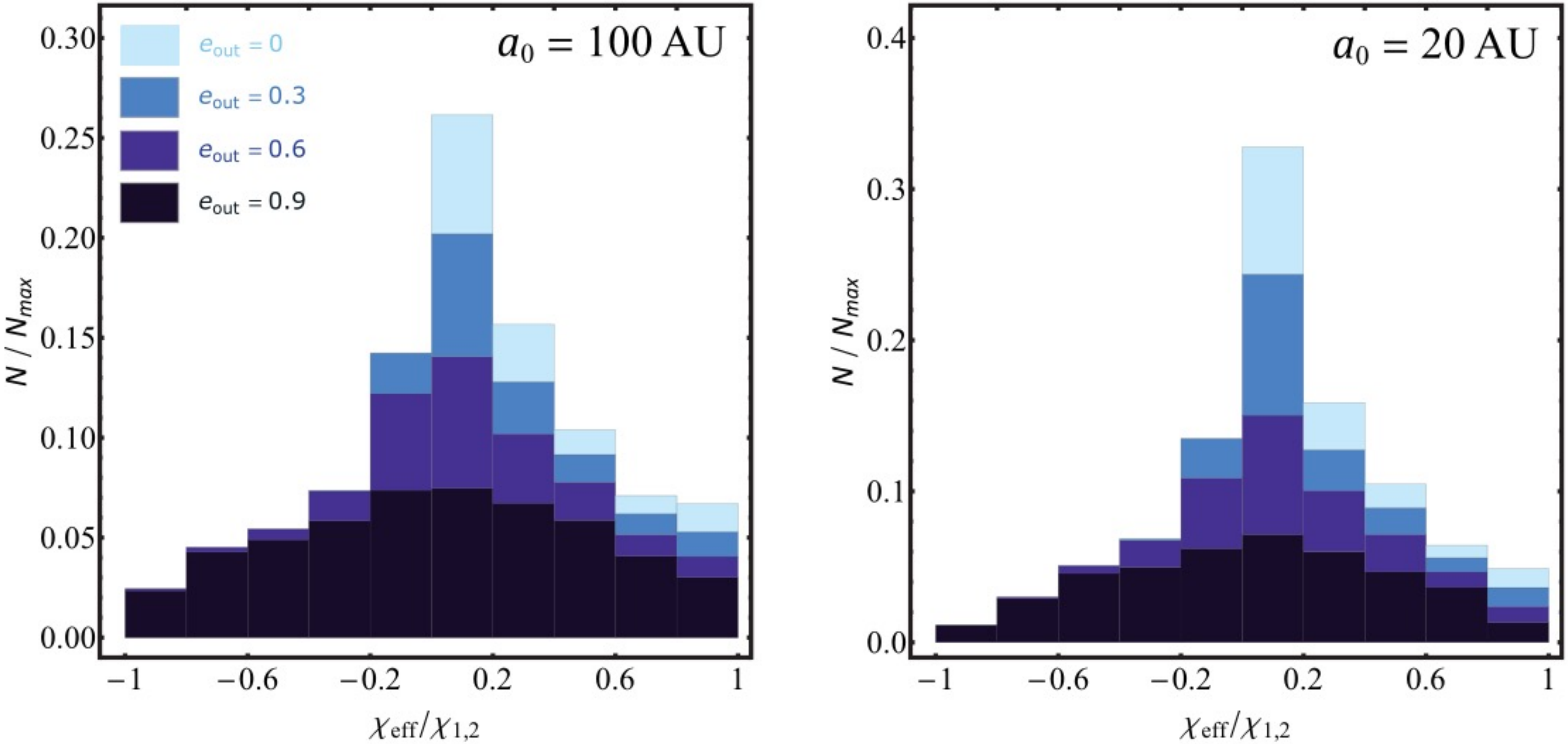


90^0 "attractor" (Adiabatic Invariance)

Su,Lai,Liu 2021

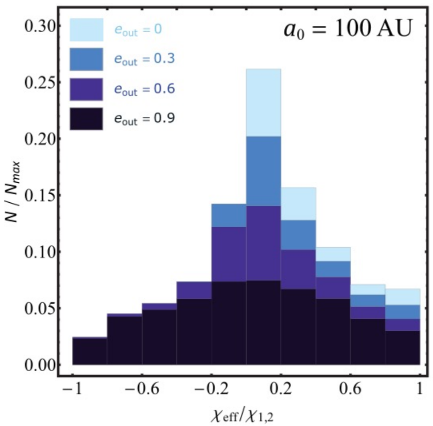
Effective Spin Distribution

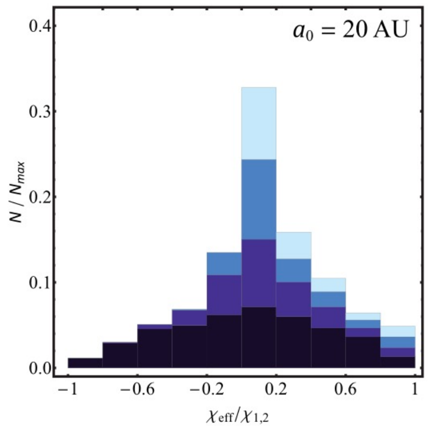
Effective spin parameter $\chi_{\rm eff} = \dfrac{m_1\chi_1 \cos\theta_{s_1l} + m_2\chi_2 \cos\theta_{s_2l}}{m_{12}}$



Effective Spin Distribution

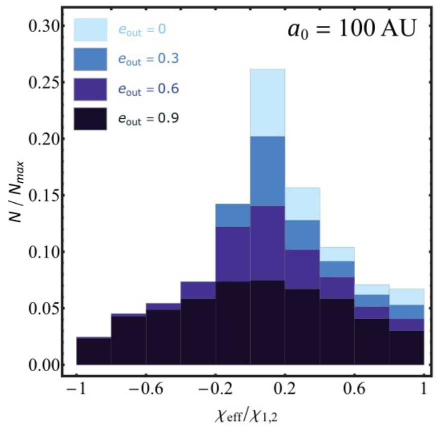
For "reasonable" initial binary/triple parameters (e_0=0, distant companions)

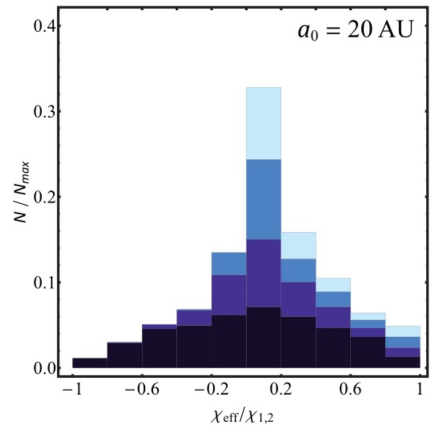




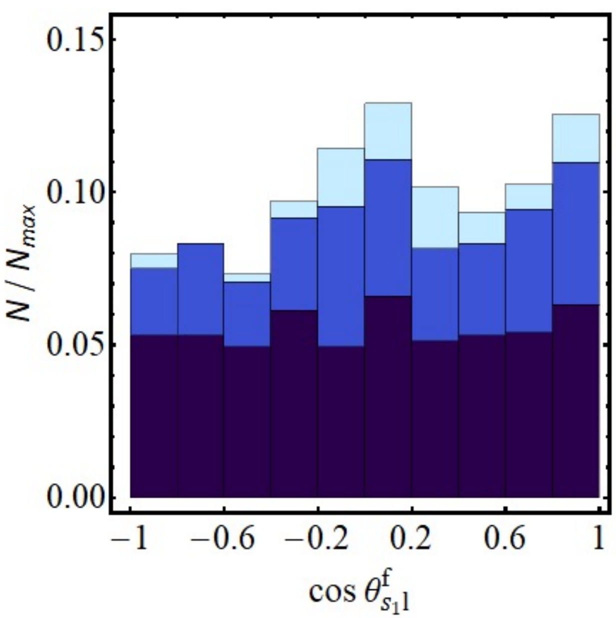
Effective Spin Distribution

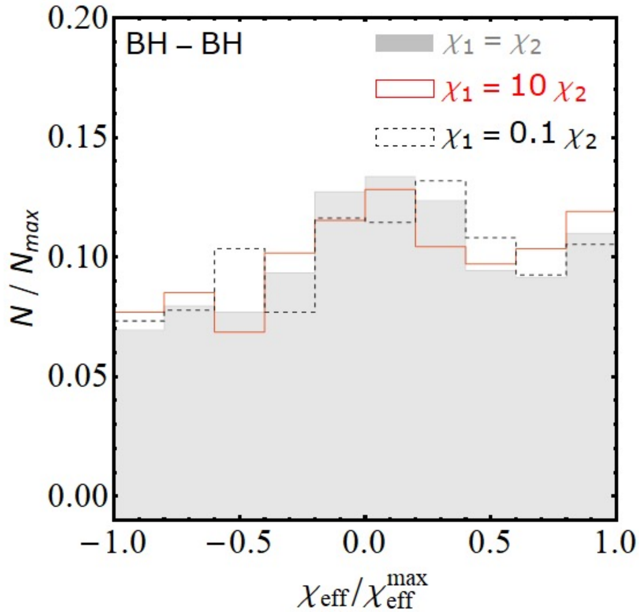
For "reasonable" initial binary/triple parameters (e_0=0, distant companions)





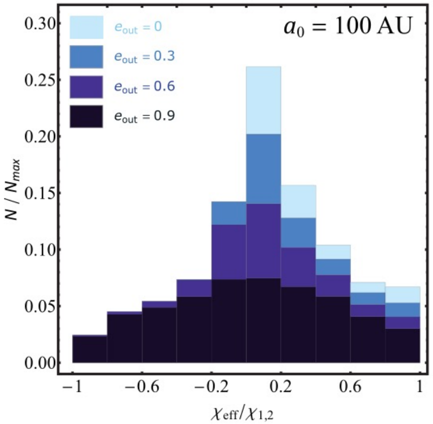
Consider ALL possible parameters

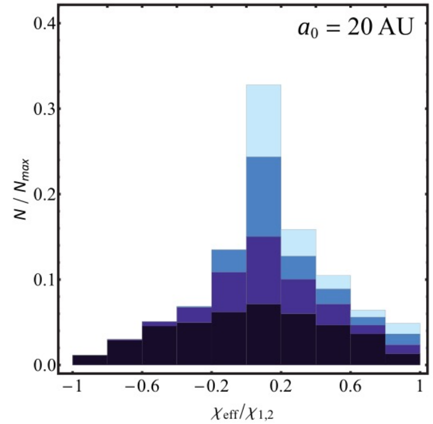


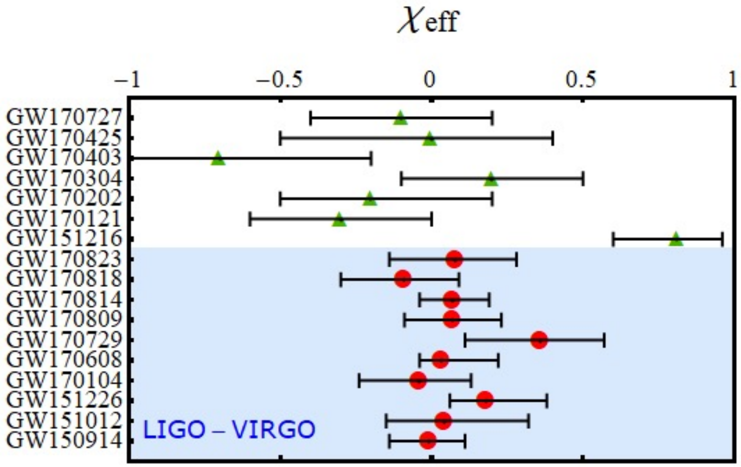


Effective Spin Distribution

For “reasonable” initial binary/triple parameters (e_0=0, distant companions)

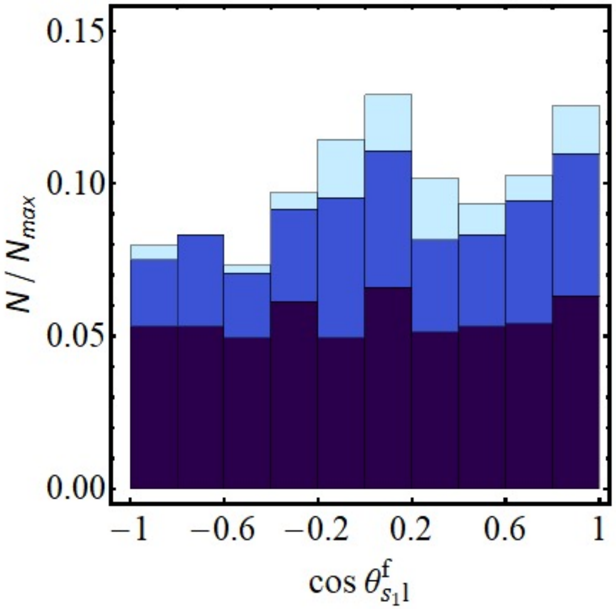


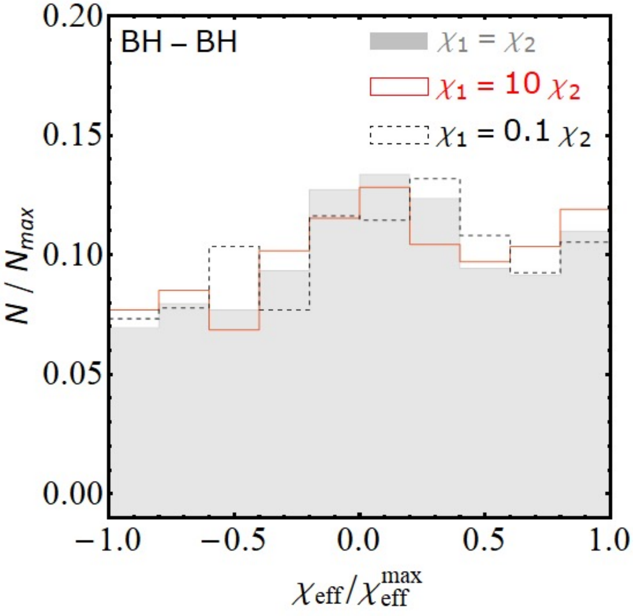




Observed

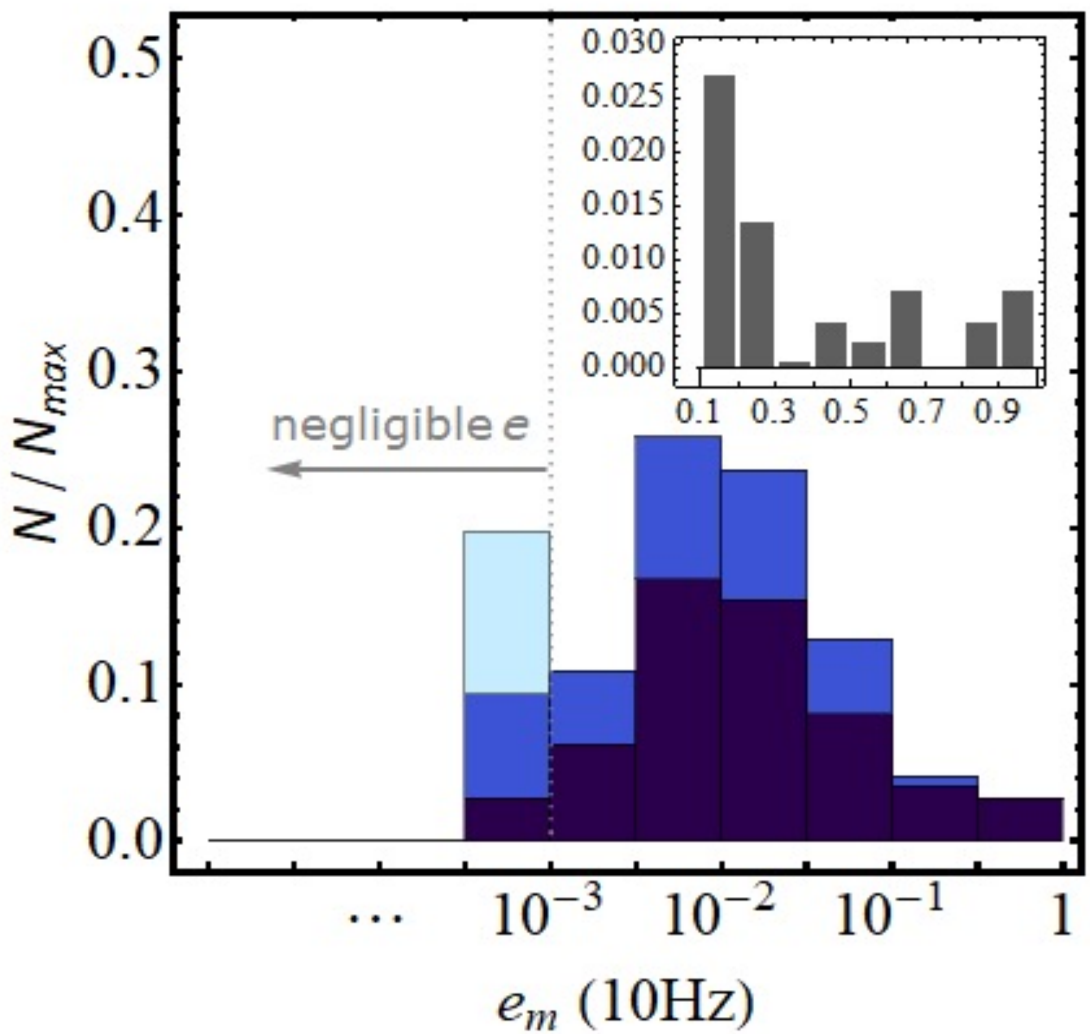
Consider ALL possible parameters





Residual Eccentricity (at 10 Hz)

BH-BH mergers



10% have $e_m > 0.1$

1% have $e_m > 0.9$

LISA/Taiji/Tianqin
would be very useful

Hierarchical Black-Hole Mergers in Multiple Systems

LIGO/VIRGO O3 Events

Parameters	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2
GW190412	30	8	0.43	
GW190814	23.2	2.6	0.07	
GW190521	85	66	0.69	0.73

- **Exceptional Features**

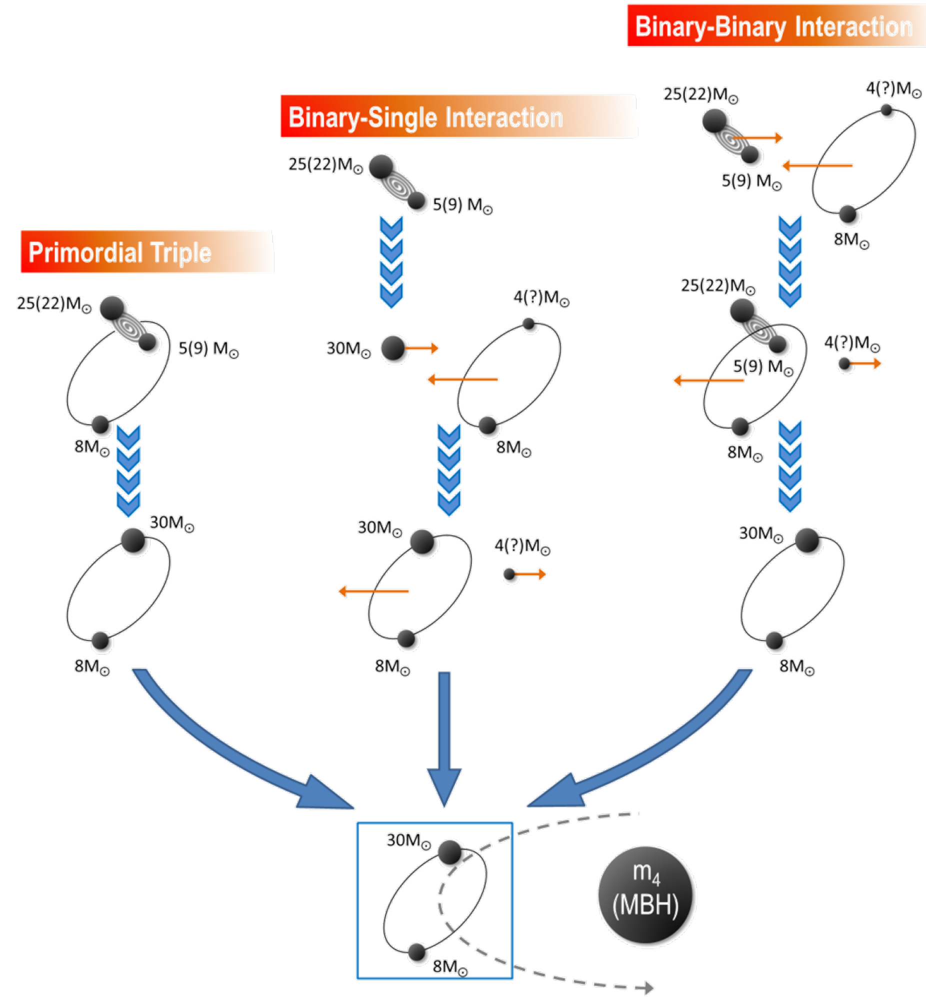
  can be naturally explained if one or both components are the **remnants of the previous mergers**!

Event

Parameters	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2
GW190412	30	8	0.43	

BH BH

Formation of GW190412-like binaries



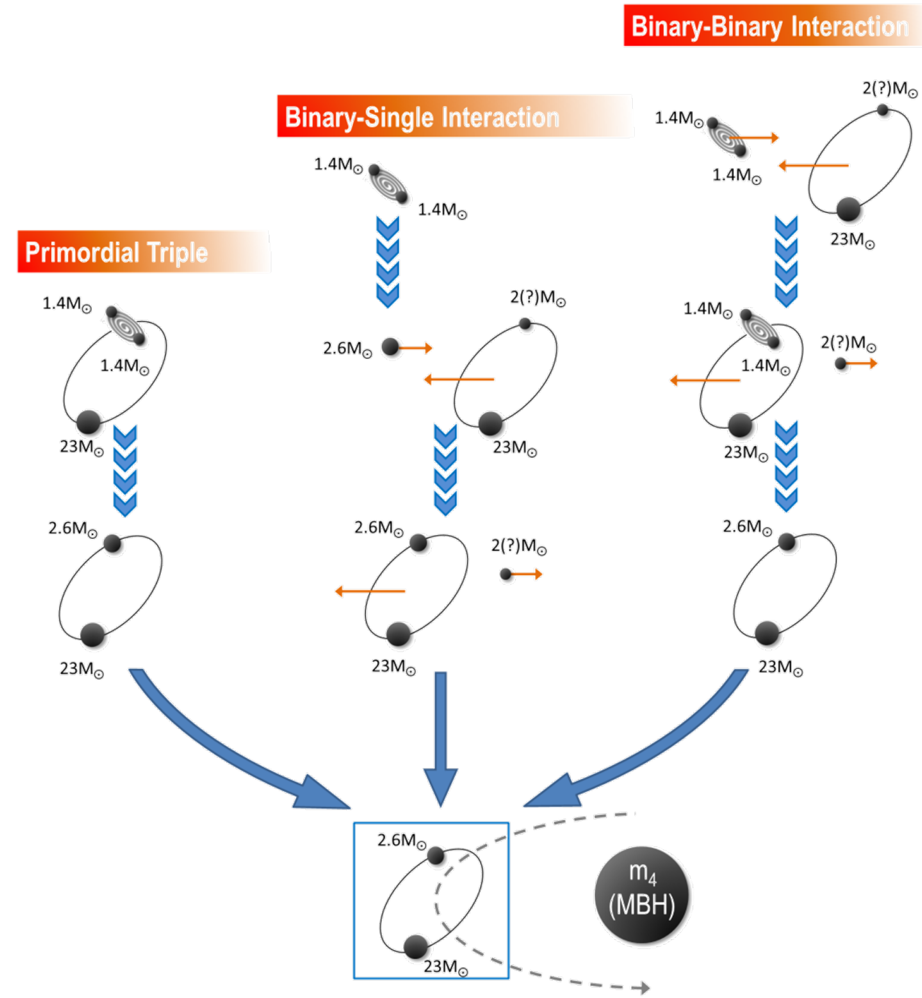
Liu & Lai MNRAS 2021

Event

Parameters	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2
GW190814	23.2	2.6	0.07	

NS NS

Formation of GW190814-like binaries



Liu & Lai MNRAS 2021

BH

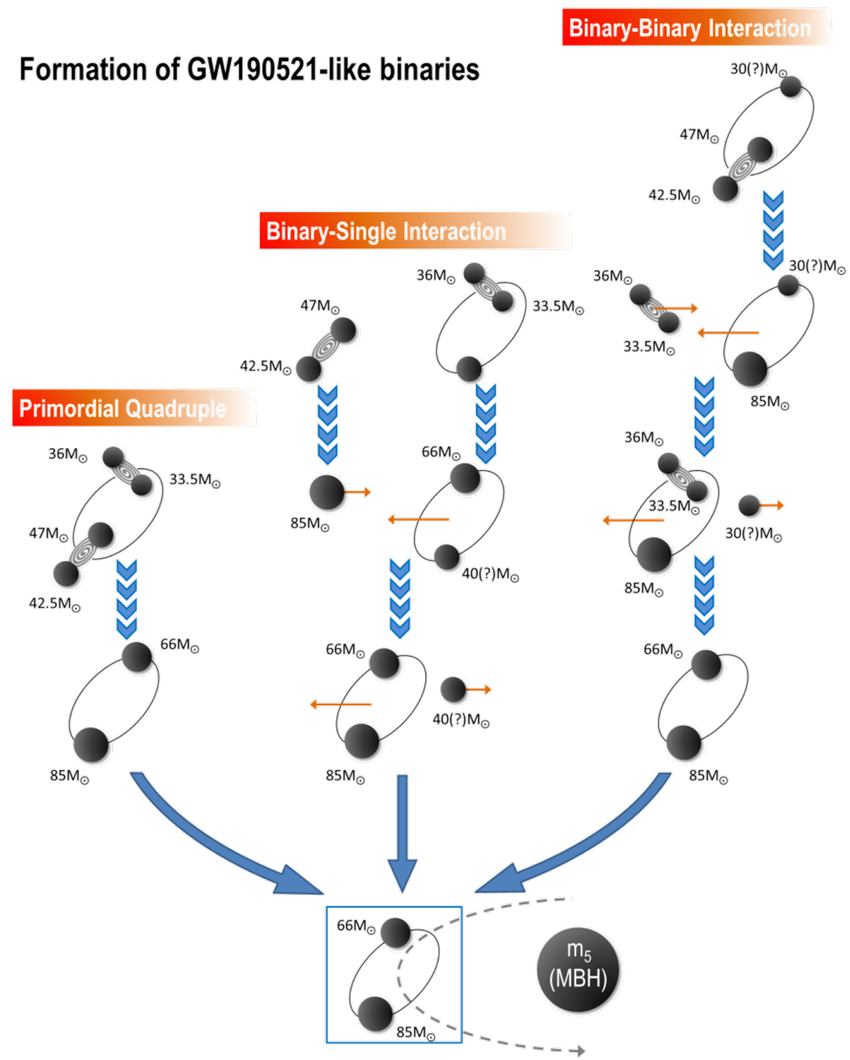
BH

Event

Parameters	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2
GW190521	85	66	0.69	0.73

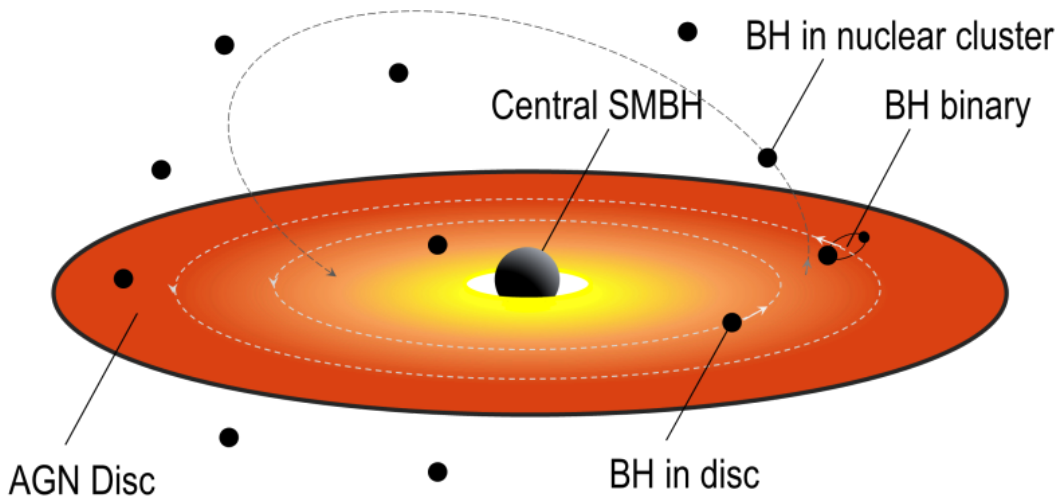
BH

BH



Liu & Lai MNRAS 2021

Binary BH Mergers in AGN disks



Bellovary+16, Bartos+16, Stone+17, McKernan+18, Secunda+18, Yang+19, Tagawa+20, etc

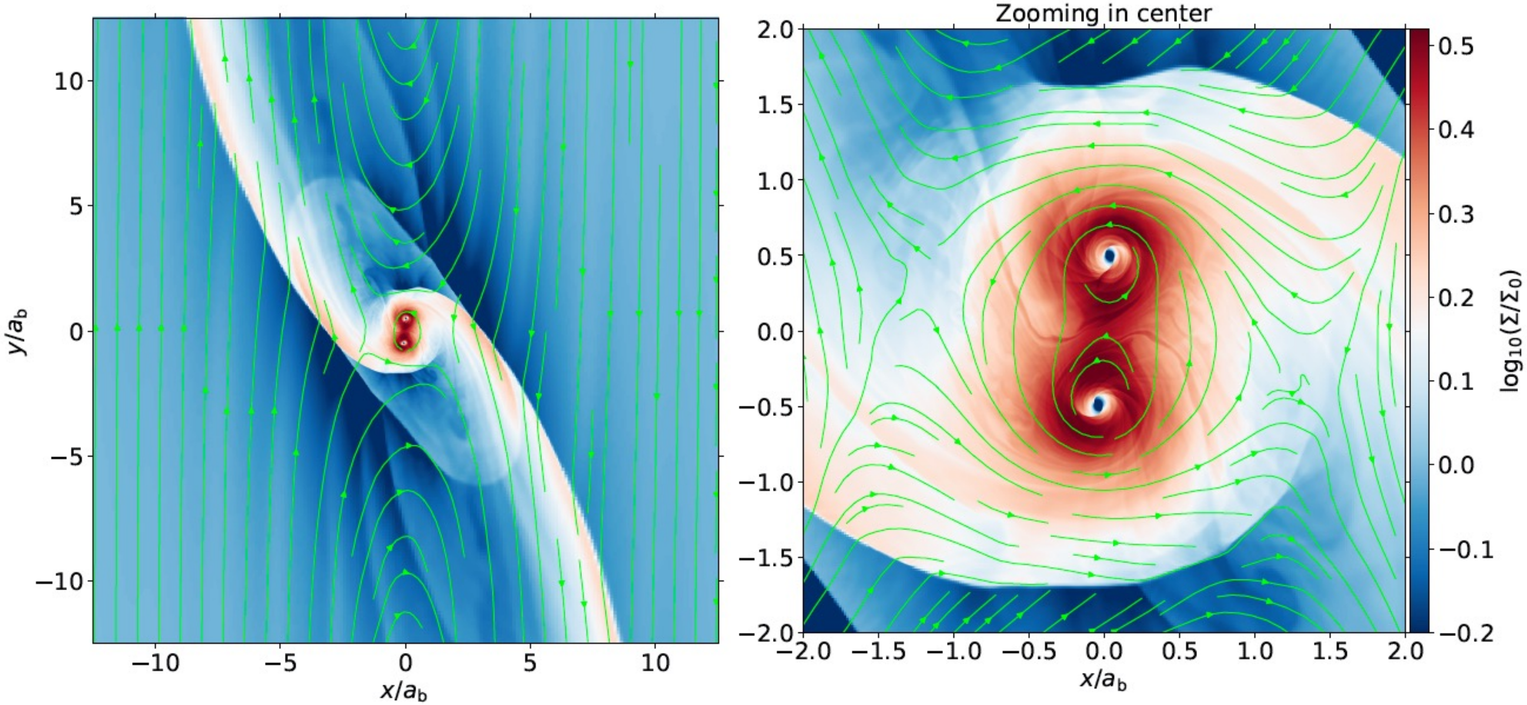
Simulations of binary in AGN disks

Baruteau, Cuadra & Lin 2011: Orbit decays.

Y.Li et al (LANL) 2021: Orbit expands (if gravitational softening is small enough…)

Simulations of binary in AGN disks

Dr. Rixin Li
(Cornell)

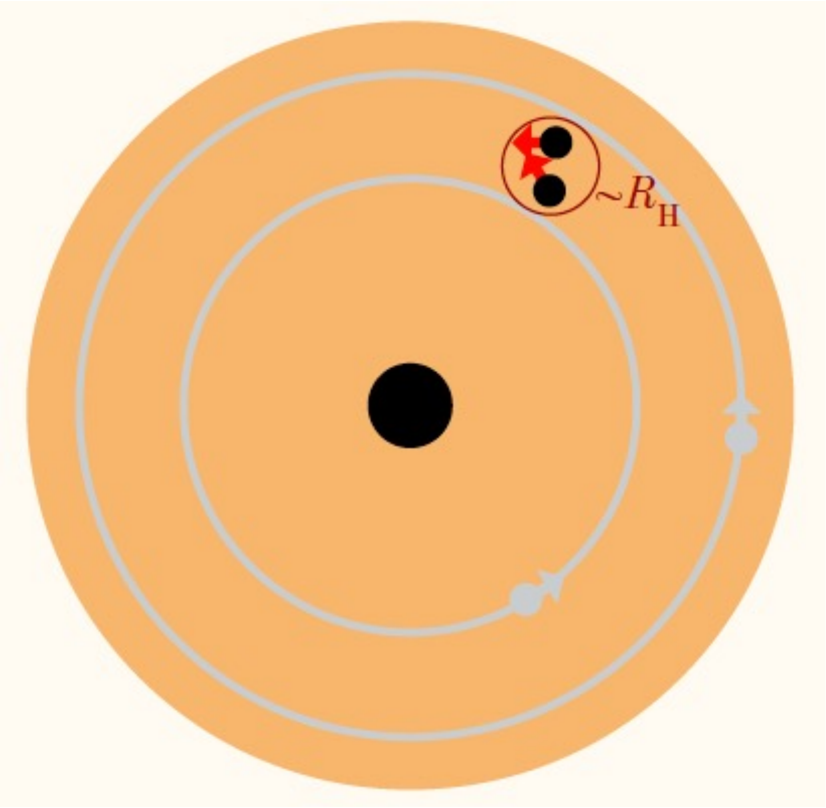


$$\frac{\dot{a}_b}{a_b} \sim -\frac{\dot{M}_b}{M_b}, \qquad \dot{M}_b \sim \text{Bondi rate}$$

Coefficients depend on various dimensionless parameters (e.g., a_b/R_H, Mach number, etc)

R.Li & DL 2021, in prep

Can/How two BHs in “migration trap” merge?



Two BHs in unstable orbits around SMBH

➔ “Chaoic” orbits

BHs can merge due to “gravitational bremsstrahlung”

2D “loss cone” for relative orbit

Jiaru Li, Lai, Rodet 2021 in prep

Summary
Formation Channels of Merging BH Binaries

Standard isolated binary evolution channel:

uncertain physics (common envelope…)

➔ circular mergers (e_m=0), mostly aligned spin-orbit angle

Dynamical formation channels:

"clean" physics, but "environmental" uncertainties

1. Dense star clusters

➔ mostly circular mergers ? expect random spin-orbit misalignments ?

2. Tertiary-induced mergers

Perturbations from outer companion → Lidov-Kozai
Octupole effect → mass ratio dependence
Binary mergers around SMBH: GR effects important
Spin-orbit misalignment

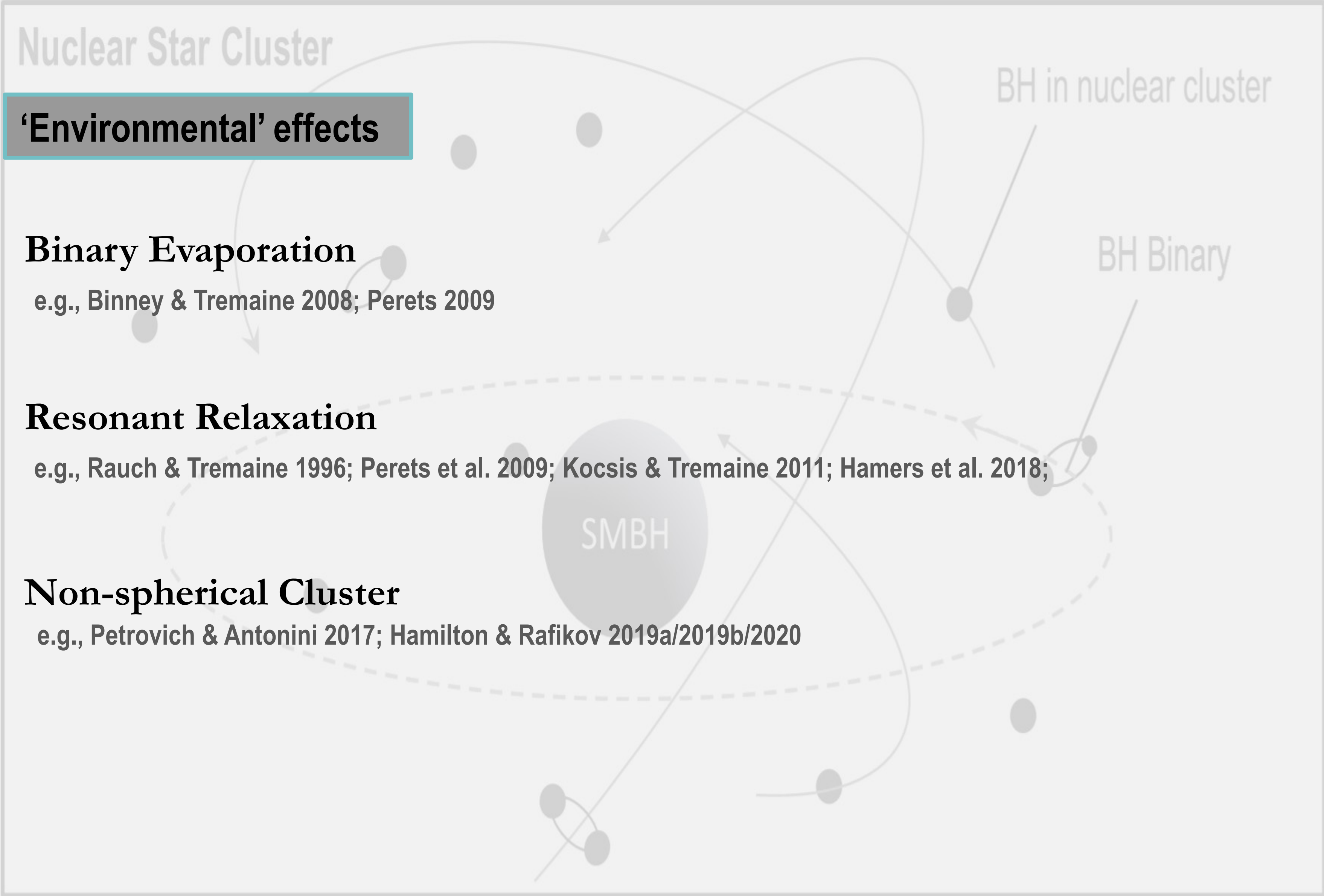
➔ ~10% mergers have residual e>0.1 when entering LIGO band
Preference of 90^0 spin-orbit misalignment, especially for circular mergers

3. BH Mergers in AGN disks

gas assisted mergers?

Rates? All potentially can play a role
LISA/Taiji/Tianqin useful for probing dynamical formation

Binary Dynamics Near a Supermassive BH

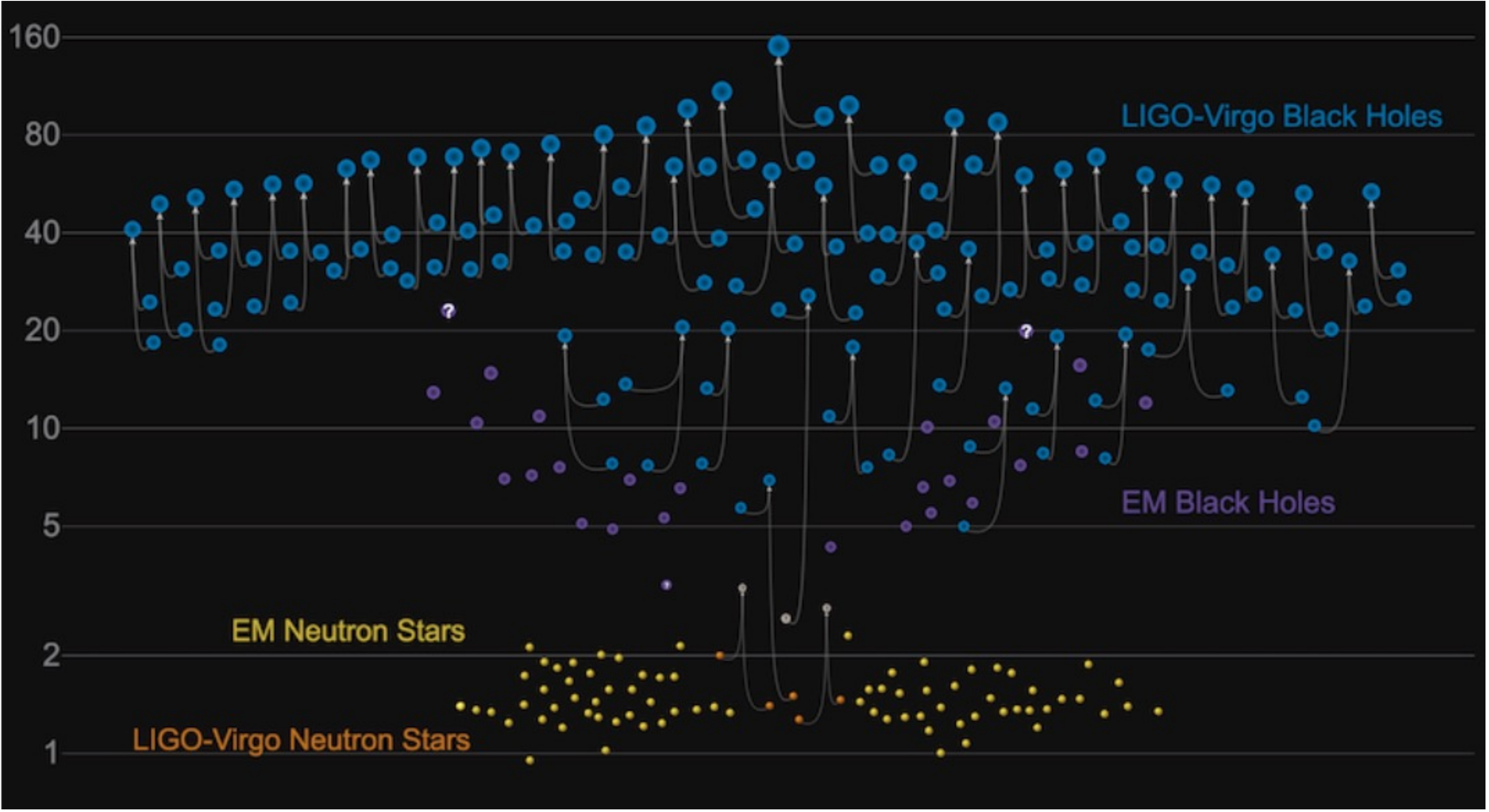


‘Environmental’ effects

- **Binary Evaporation**
  e.g., Binney & Tremaine 2008; Perets 2009

- **Resonant Relaxation**
  e.g., Rauch & Tremaine 1996; Perets et al. 2009; Kocsis & Tremaine 2011; Hamers et al. 2018;

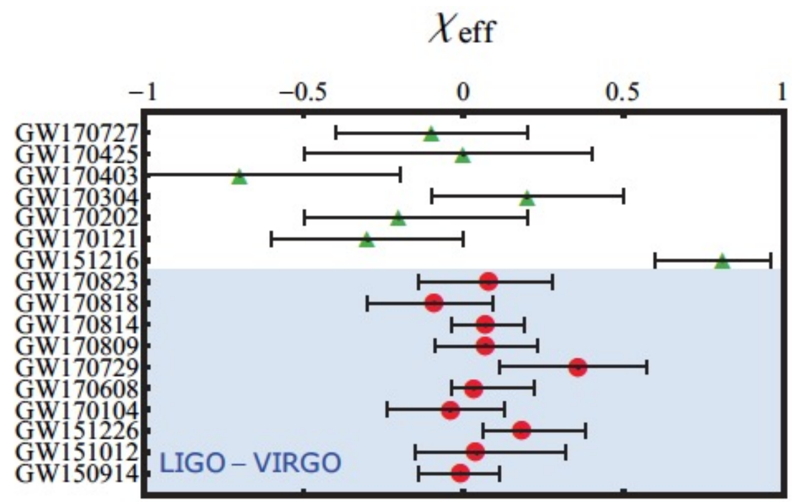
- **Non-spherical Cluster**
  e.g., Petrovich & Antonini 2017; Hamilton & Rafikov 2019a/2019b/2020



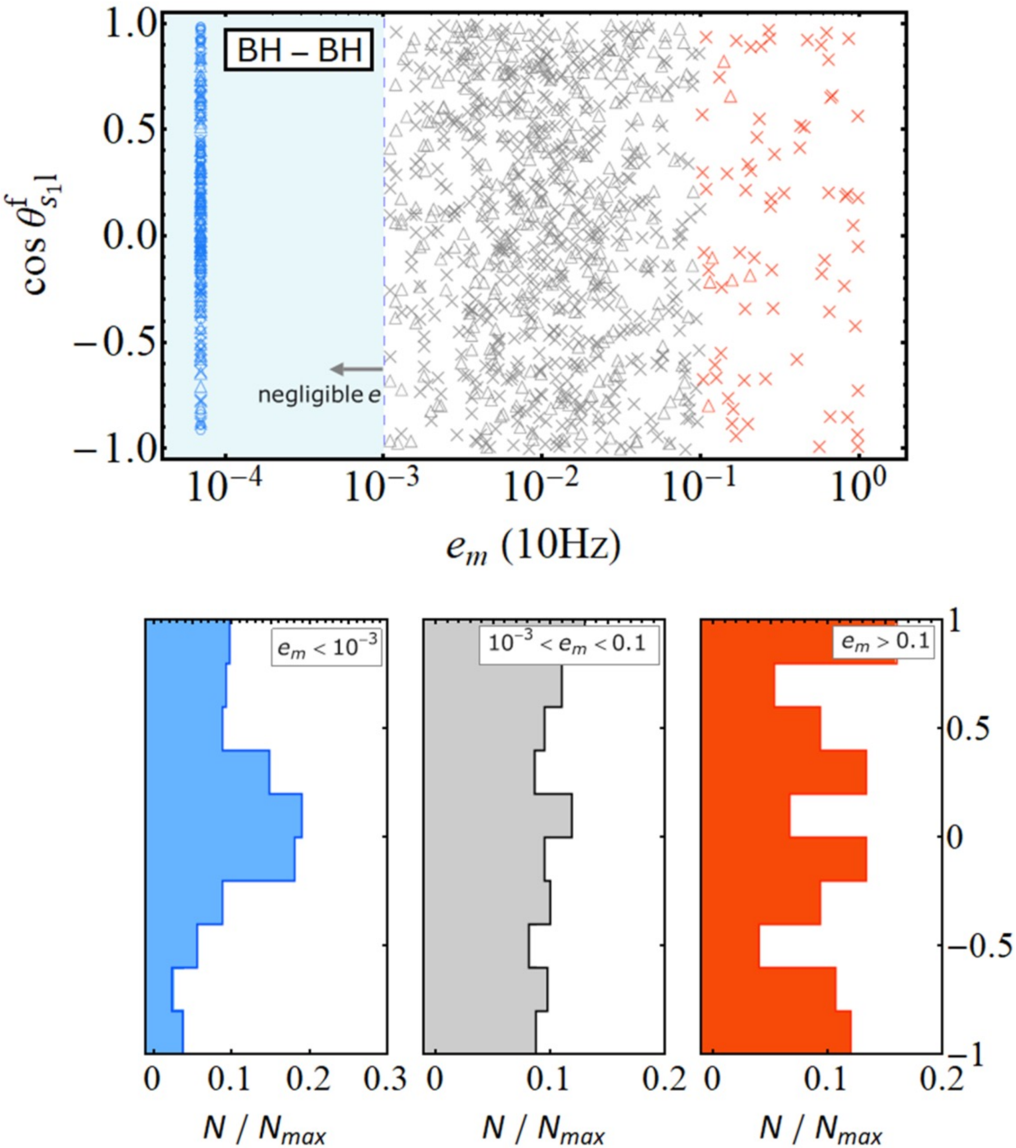
Gravitational waveform gives

$$M_1, M_2, \chi_{\rm eff}$$

$$\chi_{\rm eff} \equiv \frac{m_1\boldsymbol{\chi}_1 + m_2\boldsymbol{\chi}_2}{m_1+m_2}\cdot\hat{\mathbf{L}}$$



Residual eccentricity vs Spin-orbit Mislaignment



Circular Mergers ($e_m < 10^{-3}$)
prefer $\theta^{f}_{sl} \sim 90°$

More eccentric Mergers
has random θ^{f}_{sl}