# Tertiary-Induced BH Binary Mergers

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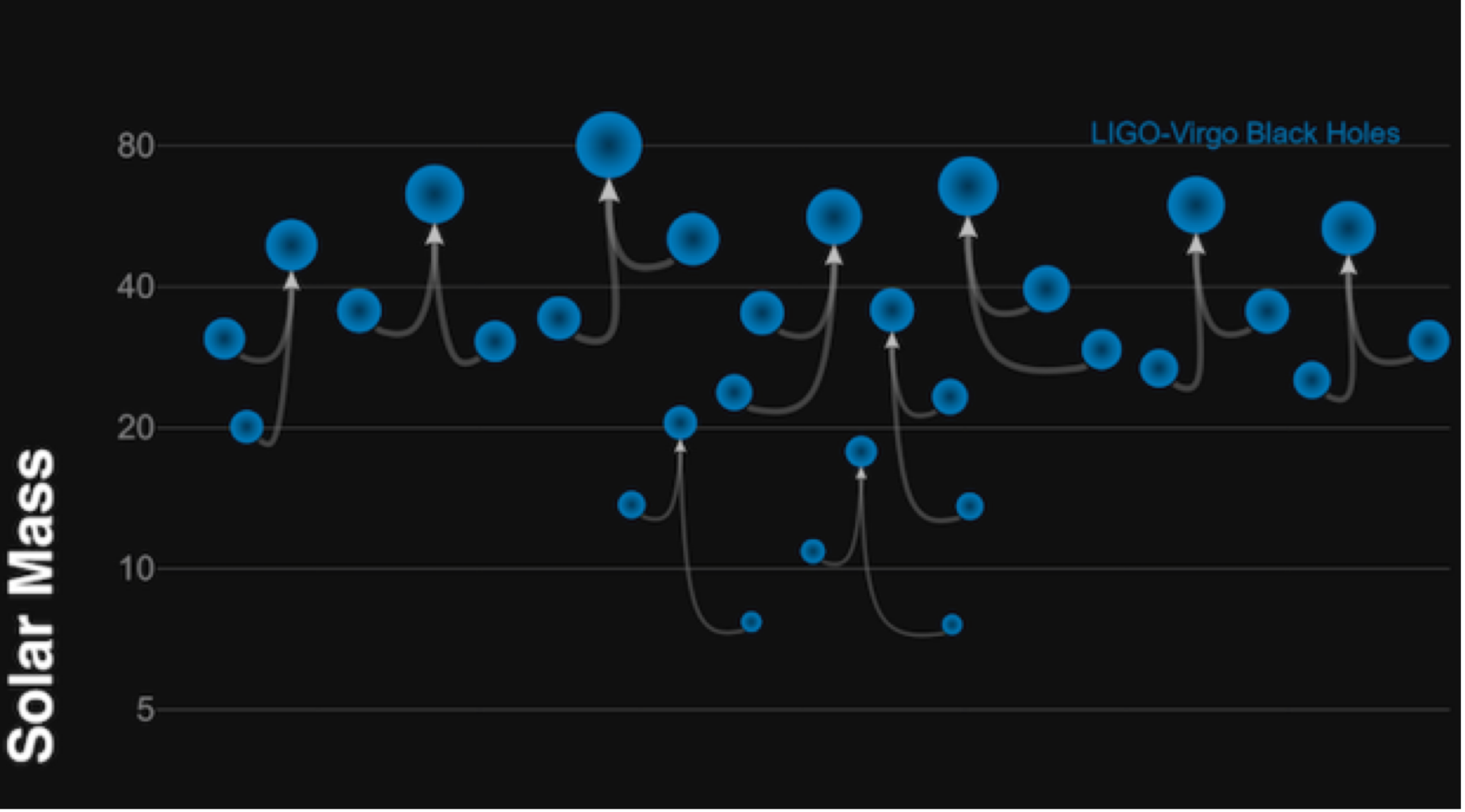
Cornell University

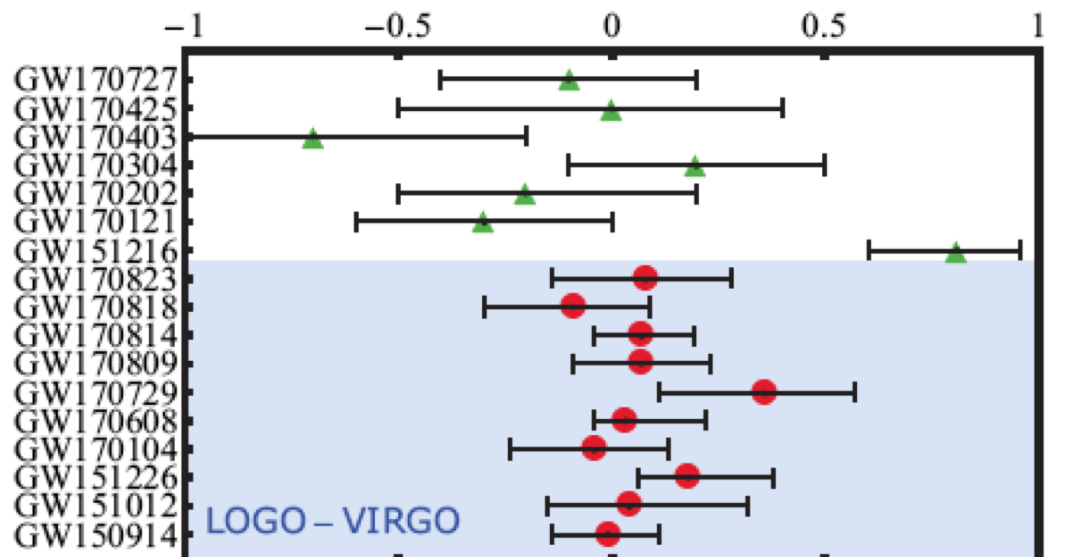
Caltech Tapir Seminar 3/6/2020




Gravitational waveform gives

$$M_1, M_2, \chi_{\rm eff}$$

$$\chi_{\rm eff} \equiv \frac{m_1 \boldsymbol{\chi}_1 + m_2 \boldsymbol{\chi}_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$



# Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation: several flavors (star clusters, triples…)

**How to distinguish different channels?**

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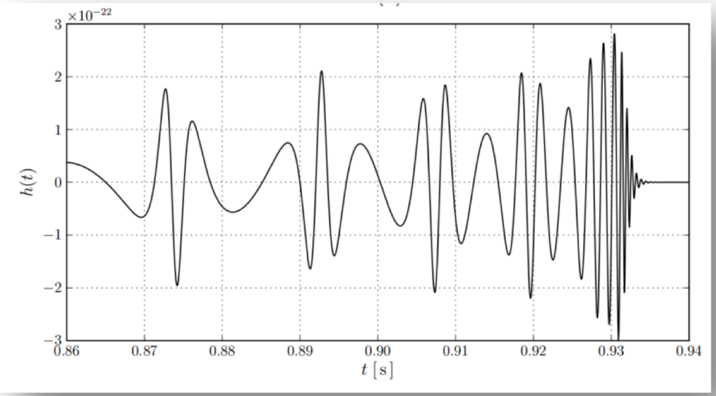
Residual eccentricity when enter LIGO band (10Hz)



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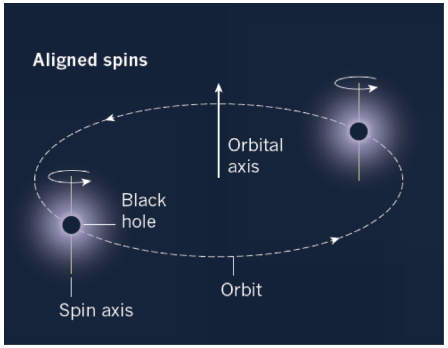
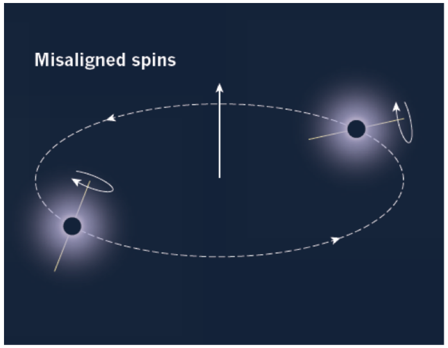
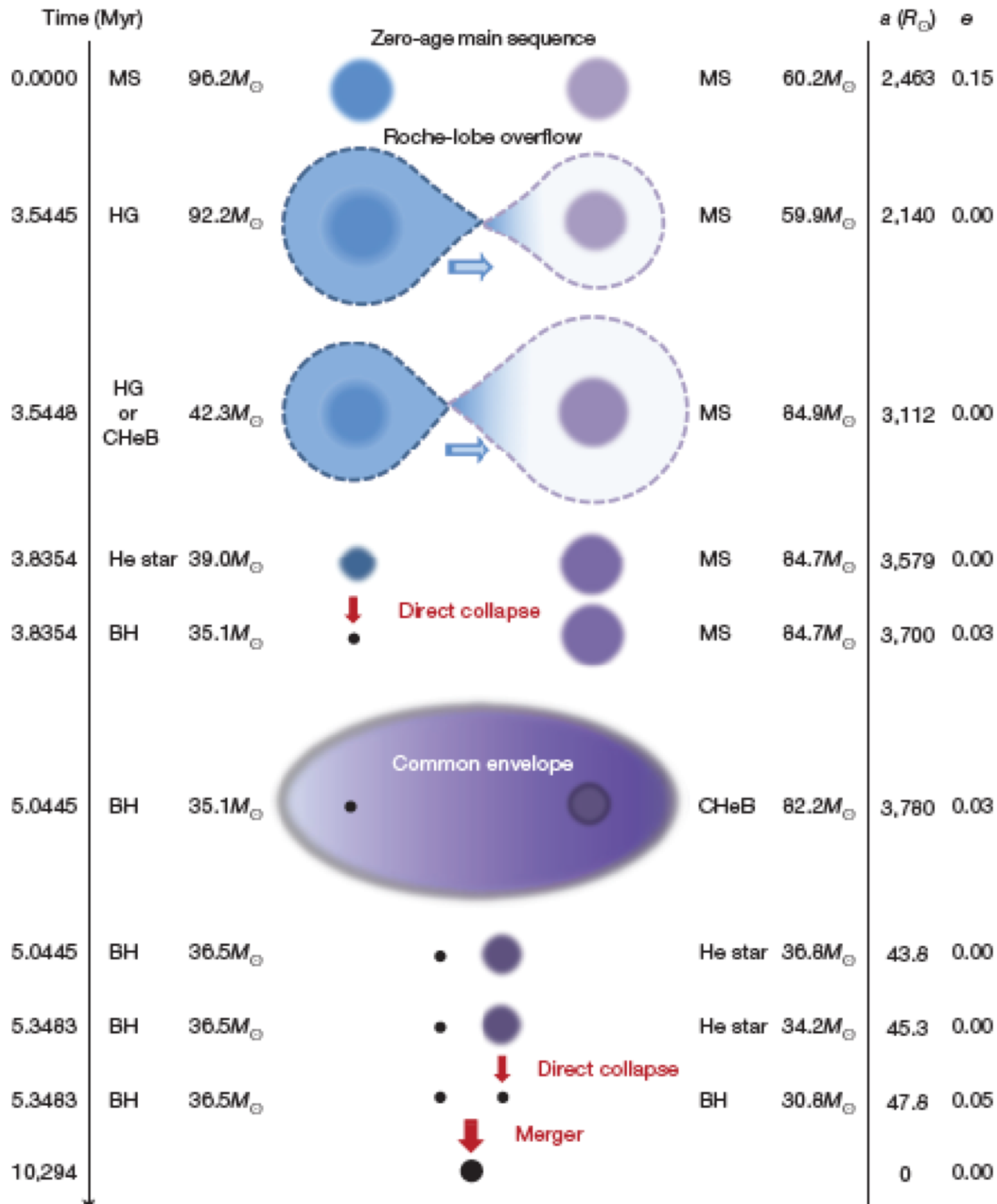
Spin-orbit misalignmemt



# Standard Binary Evolution Channel:

many papers, uncertain physical ingredients




**Produce**

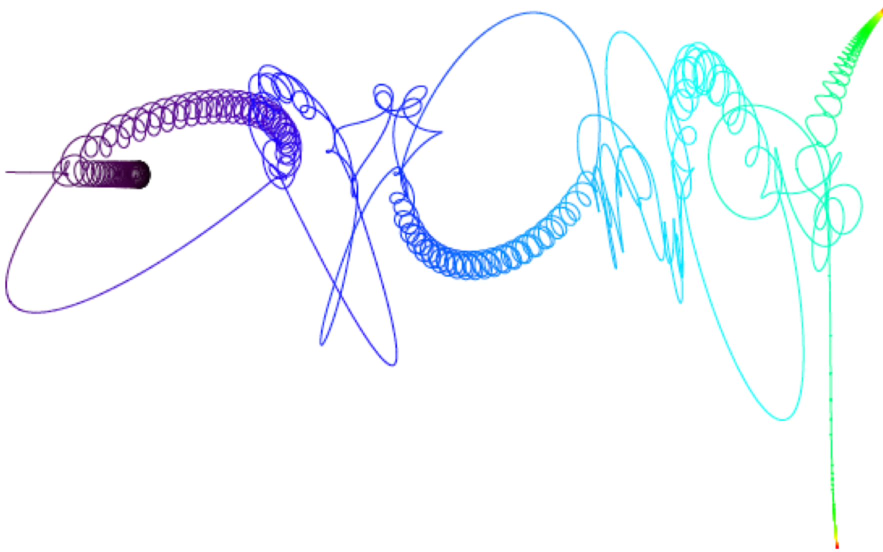
**circular orbit at 10 Hz**
**aligned spin-orbit**

Belczynski +16

# Dynamical Formation Channels

several flavors…

## 1. Dense clusters: binary-single scatterings ➔ tight binary



Samsing+14

**Produce mostly circular orbit when enter LIGO band (10 Hz) ??**

**Expect random spin-orbit orientations**

Portegies Zwart & McMillan 2000; Rodriguez et al.2015; Chatterjee et al.2017; Samsing et al. 2018; …

# Dynamical Formation Channels

several flavors…

1. **Dense clusters: binary-single scatterings → tight binary**

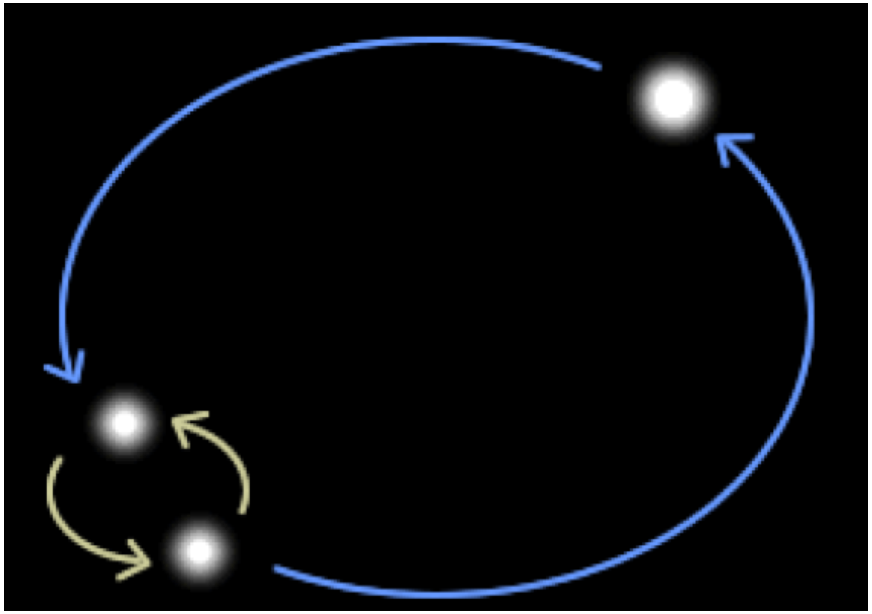
2. **Tertiary-Induced Mergers:**

   Mergers induced by (gentle) perturbations from tertiary companion

   stellar triples in galactic field, binary around SMBH

# Tertiary-Induced Binary Mergers

**merger window, residual eccentricity, spin-orbit misalignments**



Liu & DL 2017,2018, 2019
Liu, DL & Wang 2019a,b

Bin Liu (Cornell)

Previous/related works (in various contexts):

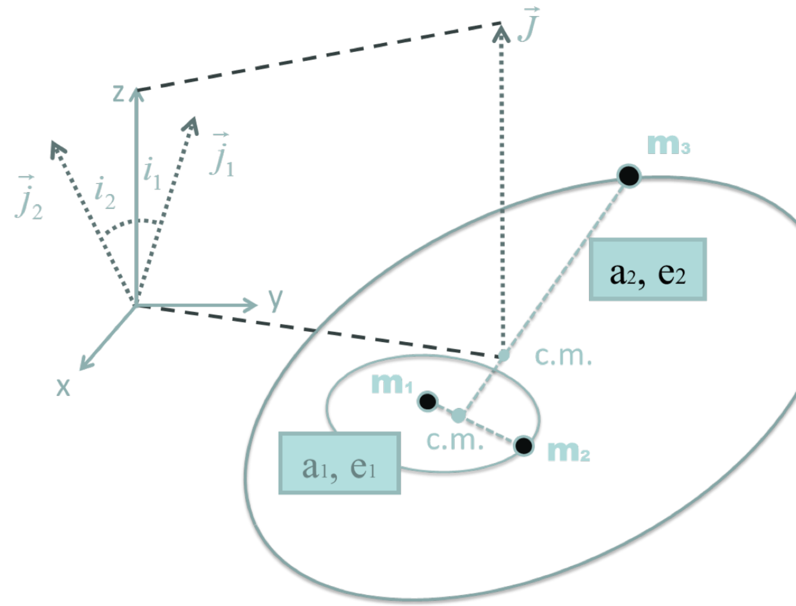
e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Thompson 2011; Antonini et al. 2012,2014,2017, Silsbee & Tremaine 2017; Petrovich & Antonini 2017...

# Lidov-Kozai oscillations

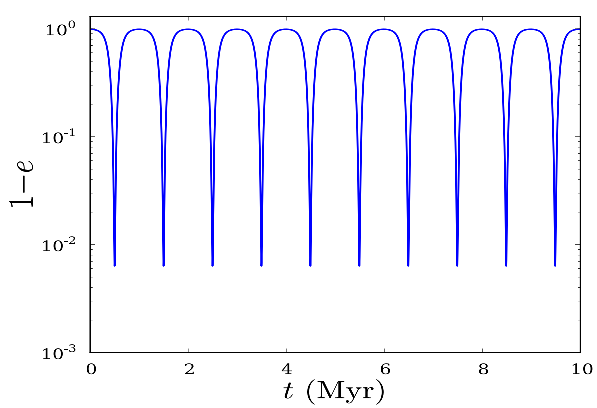
- **Inner binary + tertiary companion**

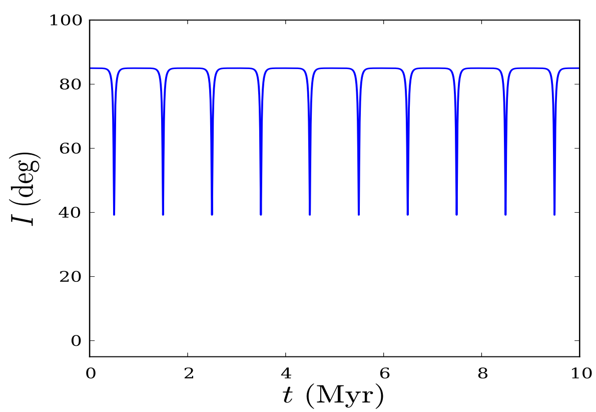
  --- Eccentricity and inclination oscillations induced if $i > 40°$

  --- If i large (85-90 degrees), get extremely large eccentricities (e > 0.99)

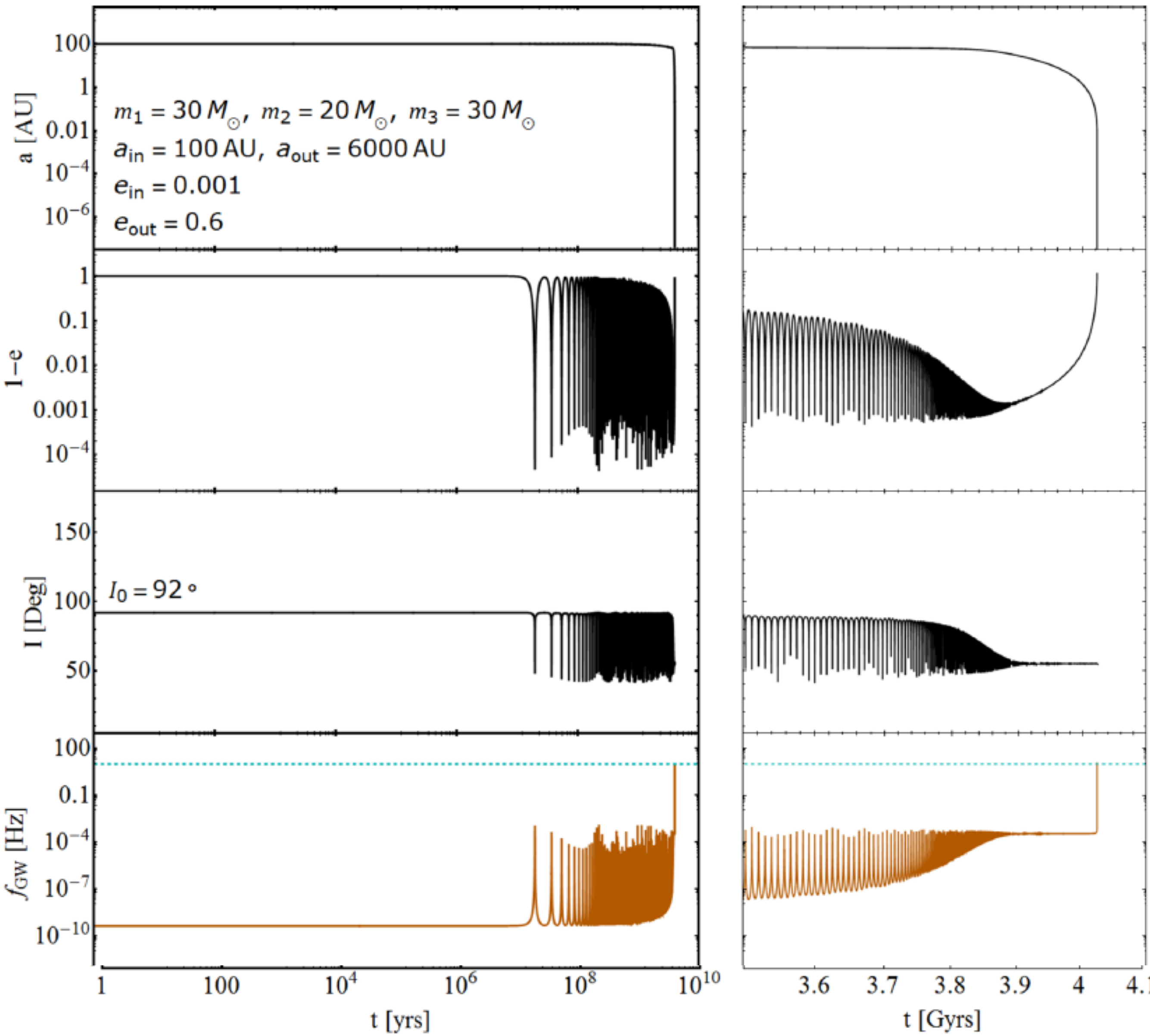



- **Orbital (secular) evolution**



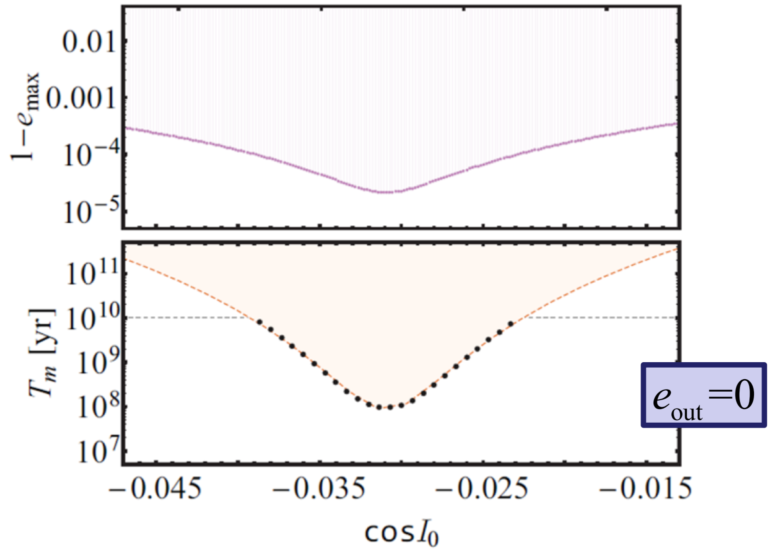
# LK oscillation + Gravitation Radiation



# Inclination window for merger

**Fixed inner binary:** $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100$AU

Fixed $m_3/a_{\rm out}^3$ value




Quadrupole LK:
$e_{max}$ vs $I_0$ analytic:
LK driving compete with GR apsidal precession

Merger window (almost) analytic

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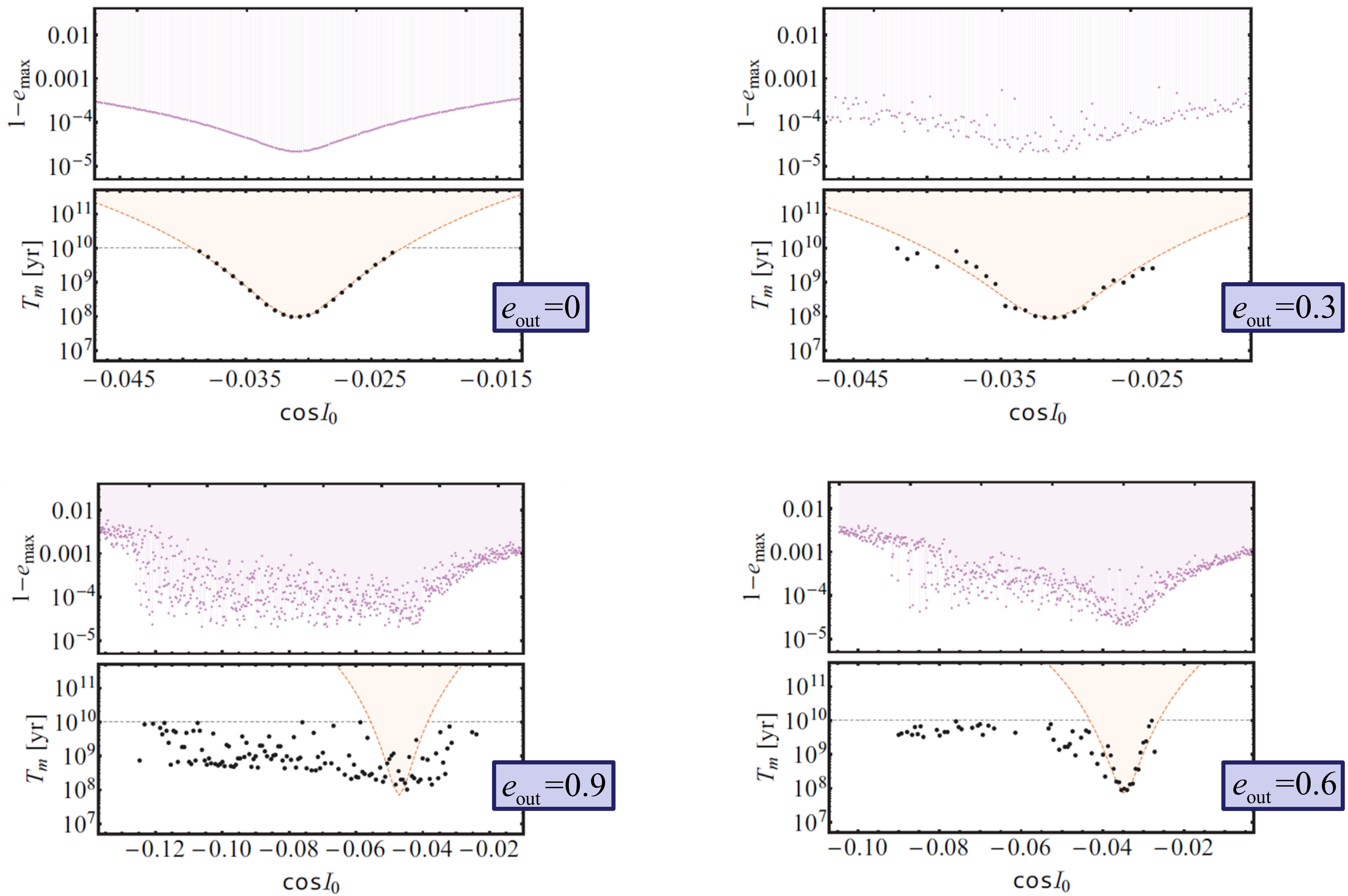

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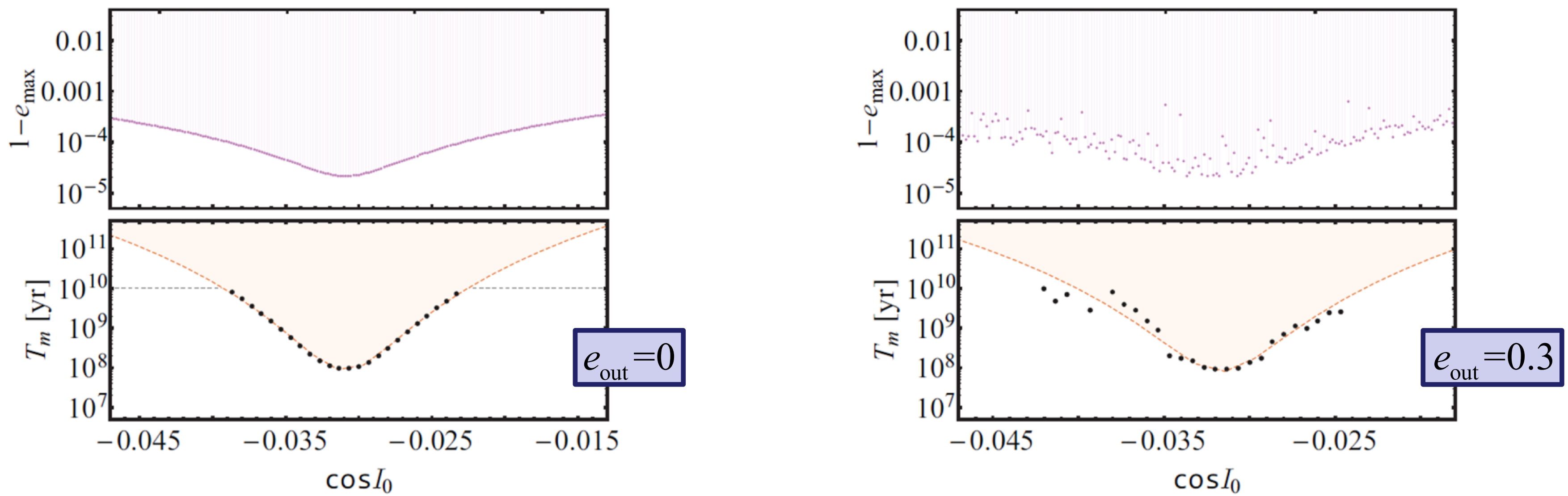
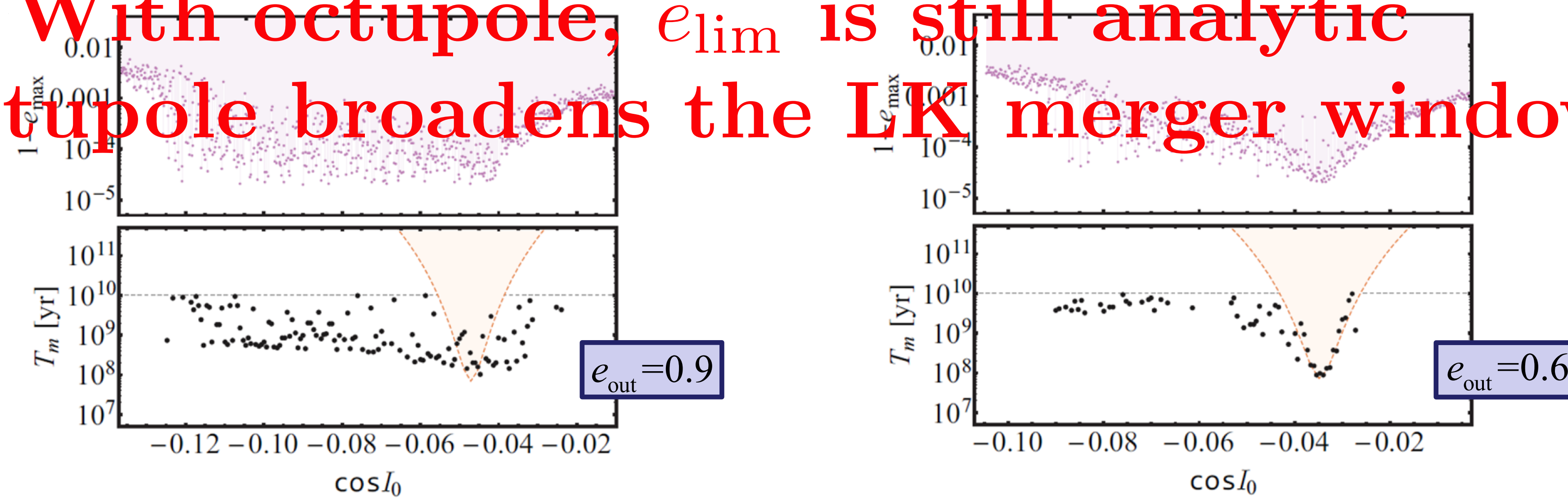
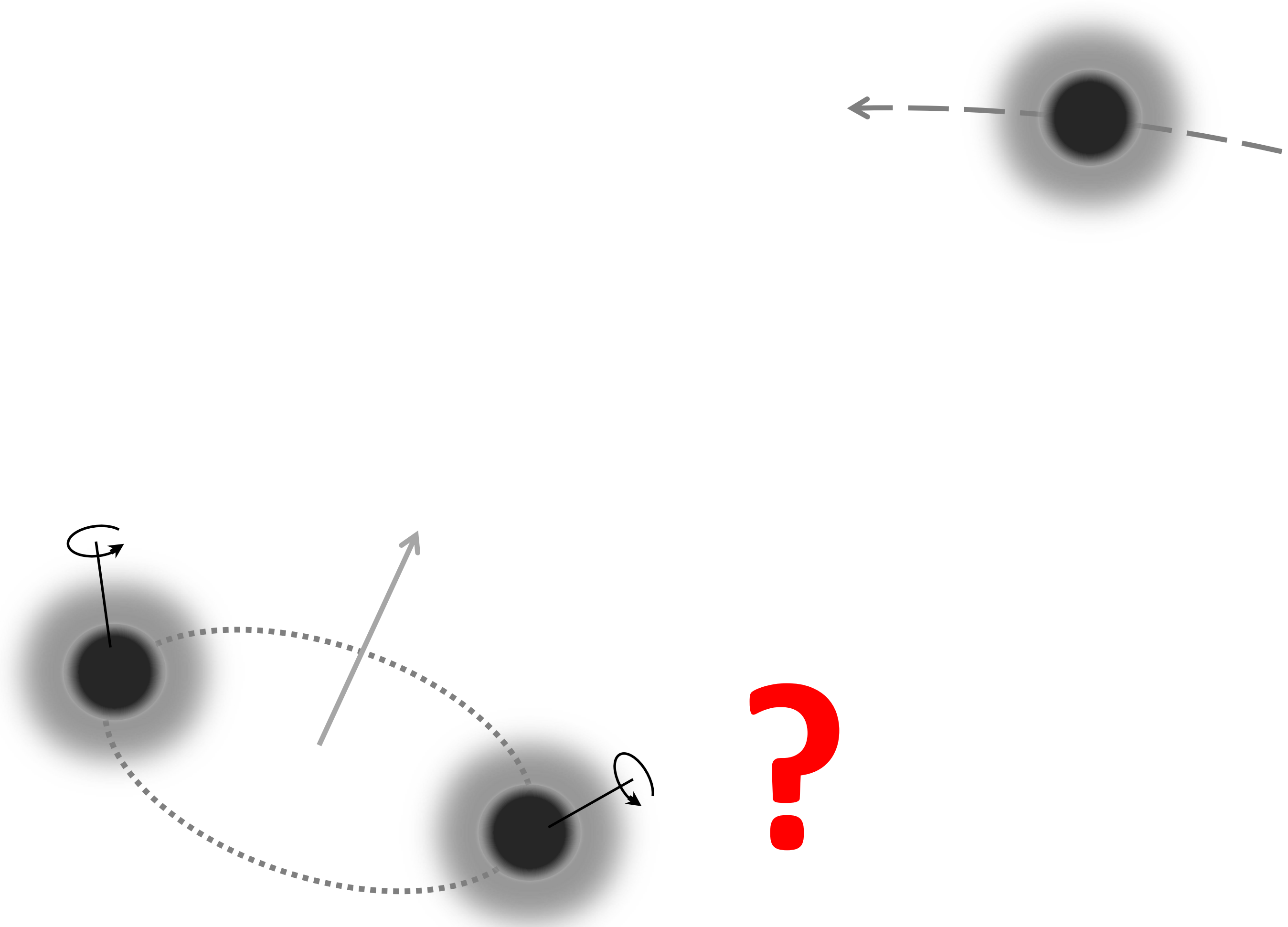



**With octupole, $e_{\rm lim}$ is still analytic**

**Octupole broadens the LK merger window**



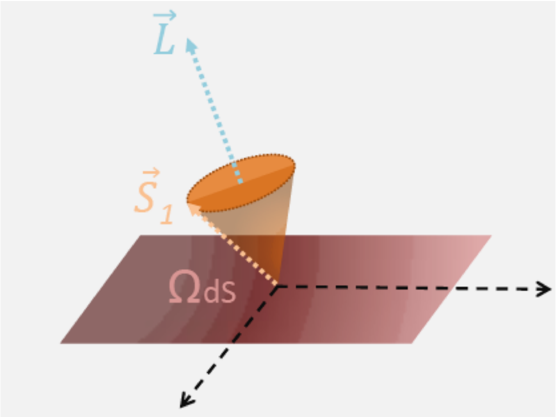
# What about the BH Spin?



# Spin-Orbit Coupling

**The de Sitter precession of spin around the angular momentum axis of the binary**

$$\frac{d\hat{S}_1}{dt} = \Omega_{dS} \hat{L} \times \hat{S}_1 \qquad \Omega_{dS} = \frac{3Gn\left(m_2 + \mu/3\right)}{2c^2 a\left(1-e^2\right)}$$

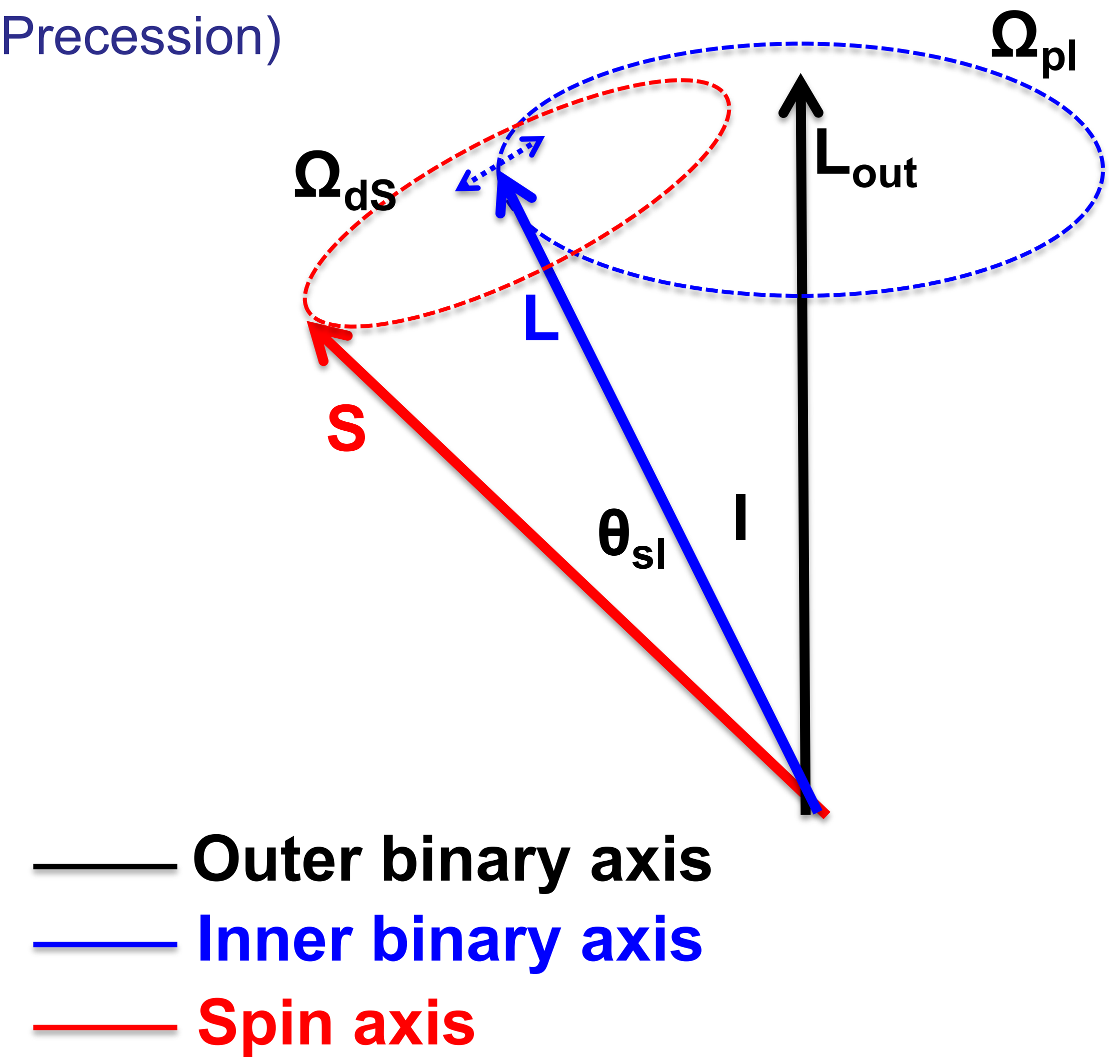


# BH spin dynamics during LK oscillations

$$\frac{d\hat{\mathbf{S}}_1}{dt} = \Omega_{\rm dS}\hat{\mathbf{L}} \times \hat{\mathbf{S}}_1$$ (de Sitter Precession)

But **L** precesses and nutates during LK oscillations

$$\Omega_{\rm pl} \simeq \frac{3(1+e^2)}{t_{\rm LK}\sqrt{1-e^2}}|\sin 2I|$$



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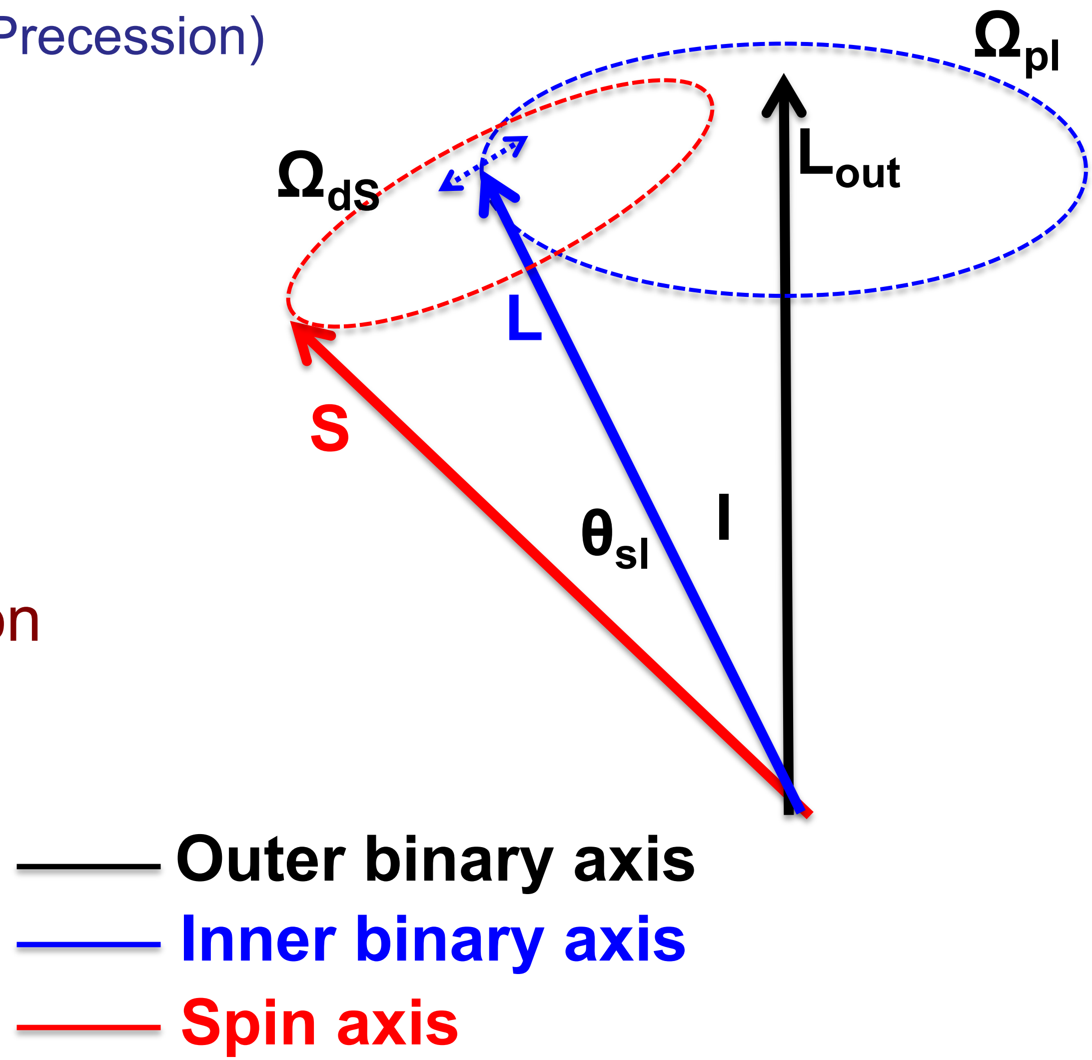
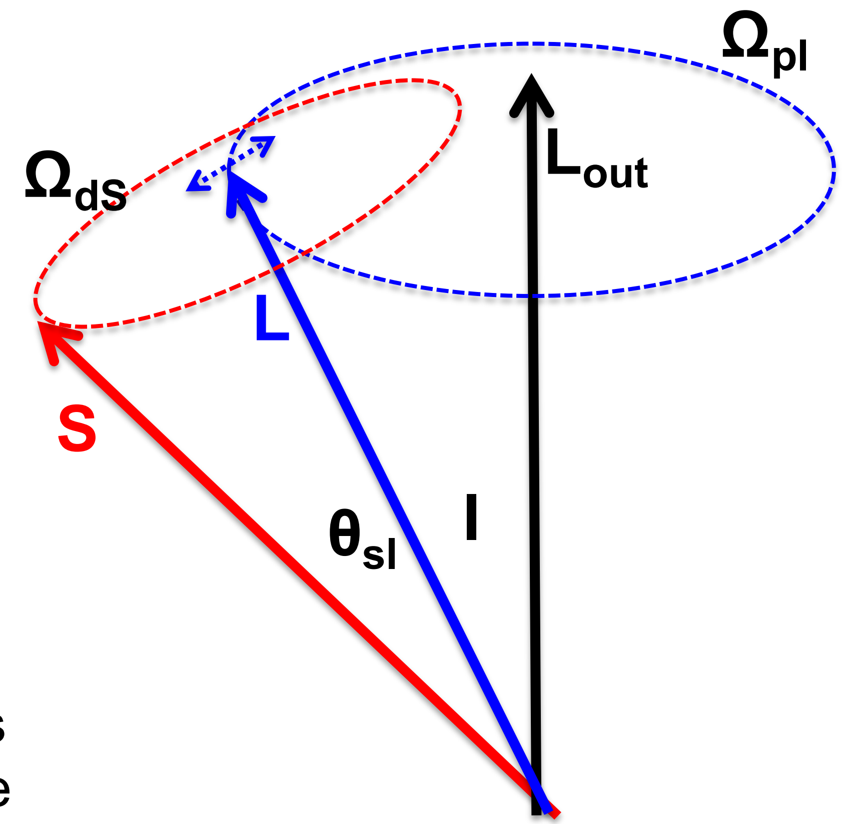
Spin dynamics depends on

$$\Omega_{\rm dS} \text{ vs } \Omega_{\rm pl}$$

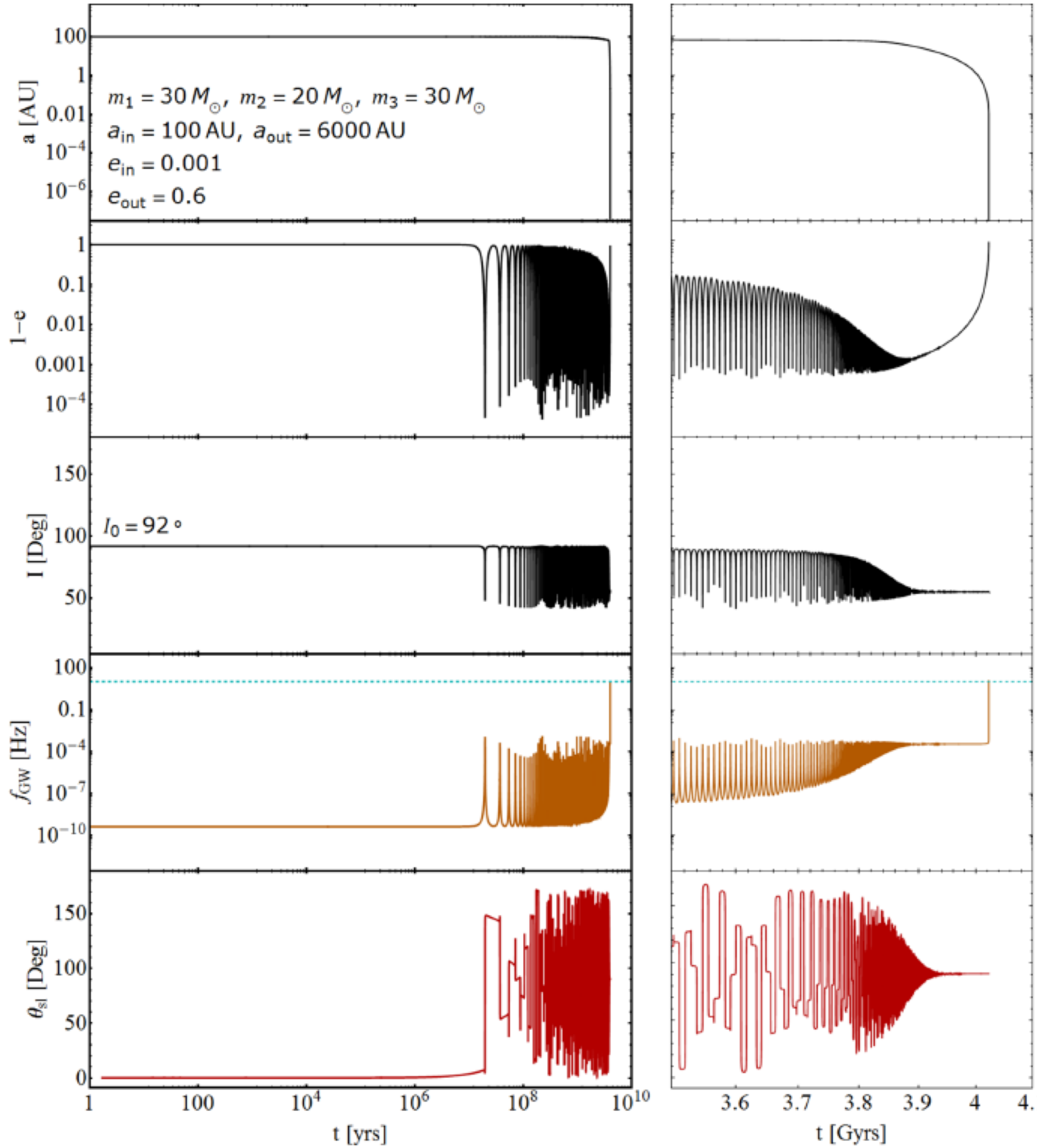


# BH spin evolution in LK-induced orbital decay




$$\Omega_{\rm dS}/\Omega_{\rm pl}$$

changes from << 1 (non-adiabatic)
to >> 1 (adiabatic) as the orbit decays
➔ Final spin-orbit misalignment angle

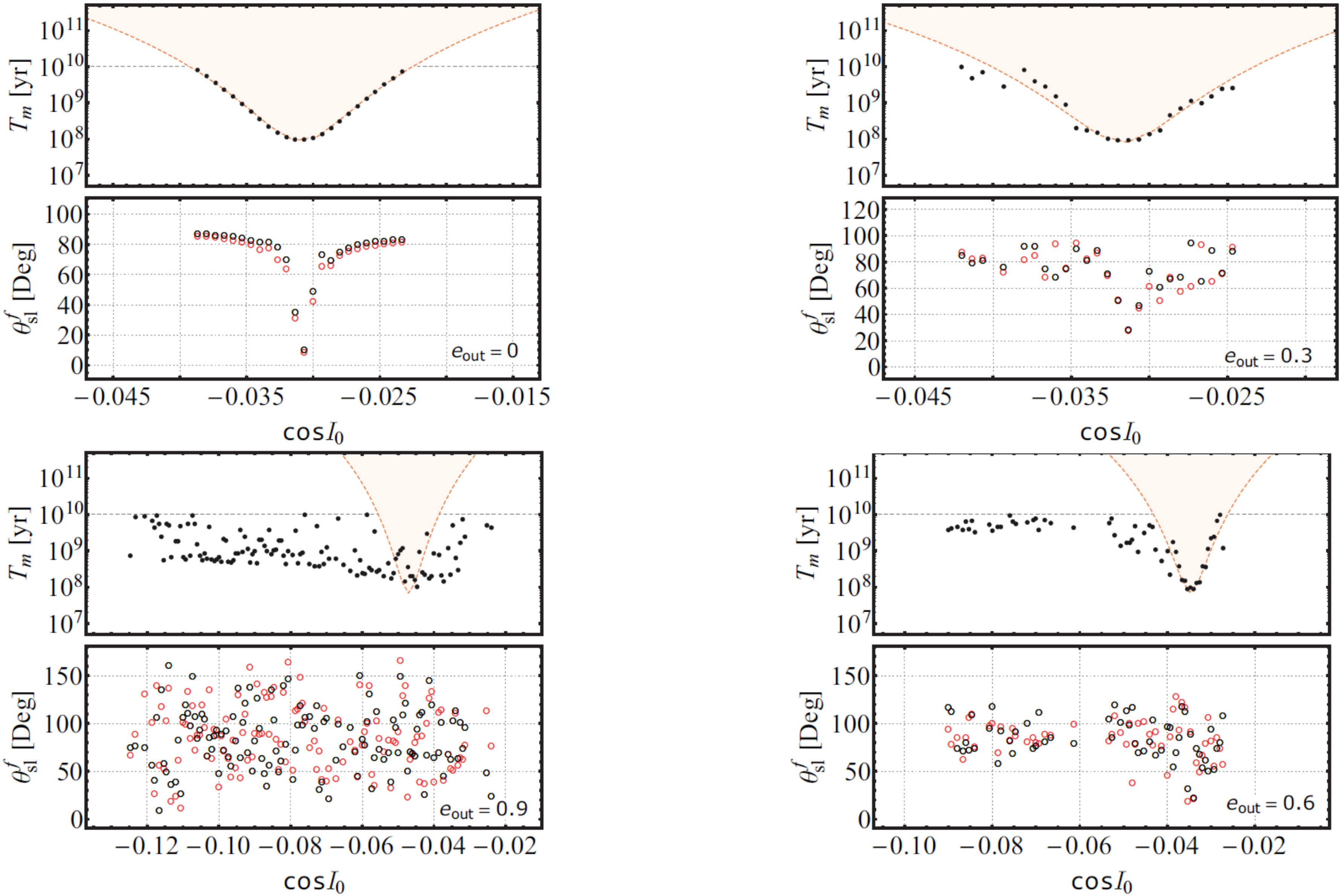


$m_1 = 30\,M_\odot,\ m_2 = 20\,M_\odot,\ m_3 = 30\,M_\odot$
$a_{\rm in} = 100\,{\rm AU},\ a_{\rm out} = 6000\,{\rm AU}$
$e_{\rm in} = 0.001$
$e_{\rm out} = 0.6$
$I_0 = 92^\circ$
a [AU]
1−e
I [Deg]
$f_{\rm GW}$ [Hz]
$\theta_{\rm sl}$ [Deg]
t [yrs]
t [Gyrs]

# Merger Window and Final Spin-Orbit Misalignments

**Fixed inner binary:** $m_1=30M_\odot$ , $m_2=20M_\odot$, $a_{in,0}$=100AU

Fixed $m_3/a_{\rm out}^3$ value



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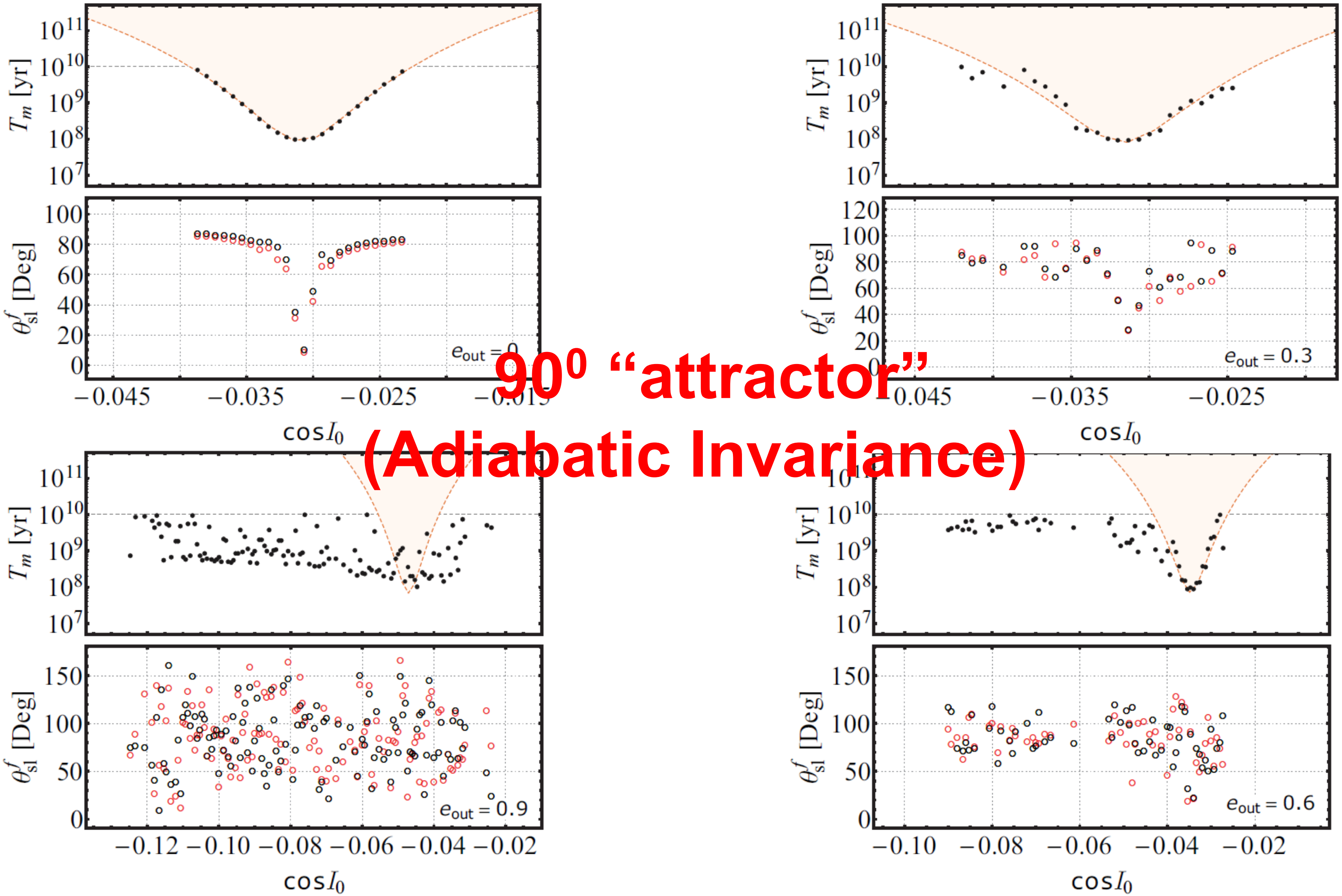
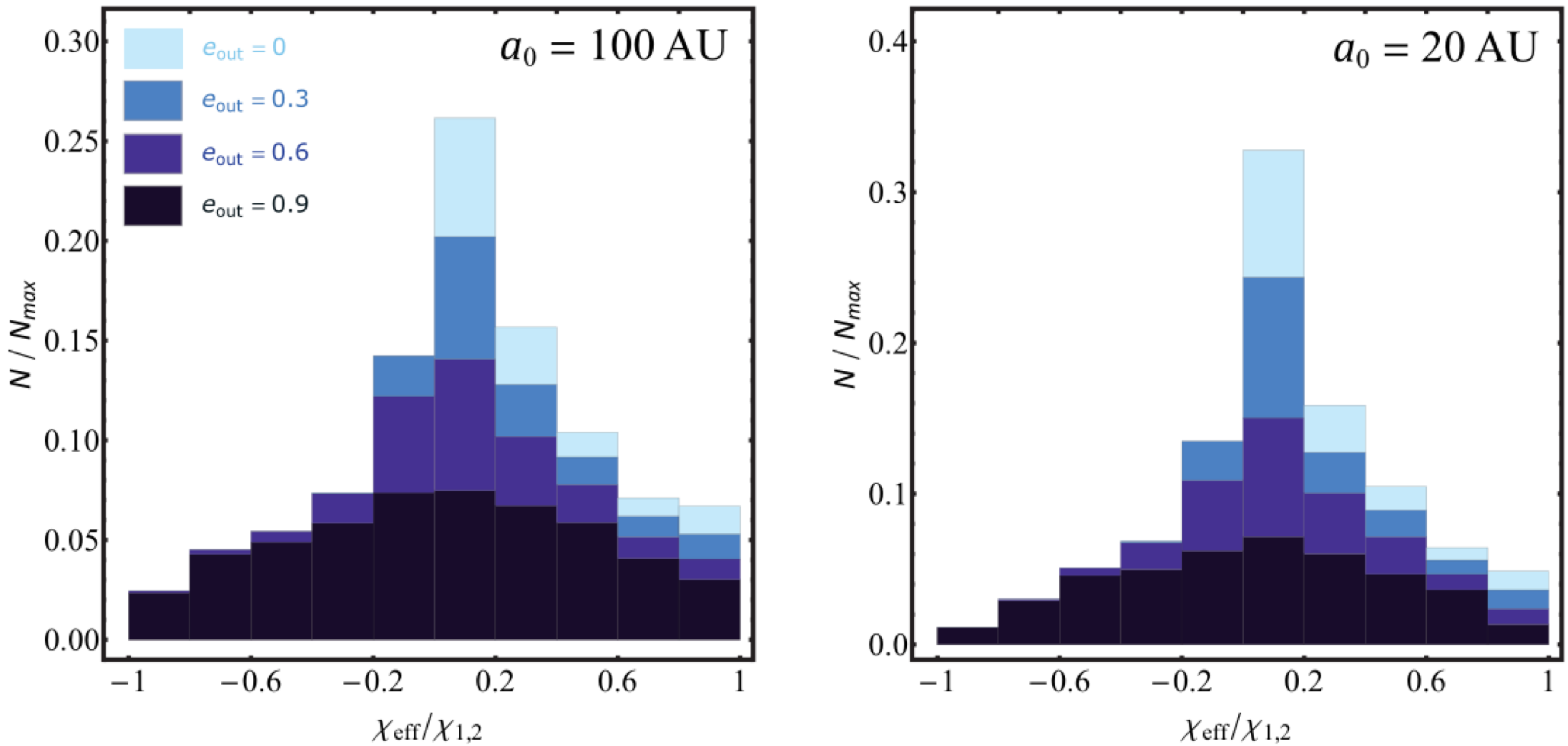



**$90^0$ "attractor"**

**(Adiabatic Invariance)**

# Effective Spin Distribution

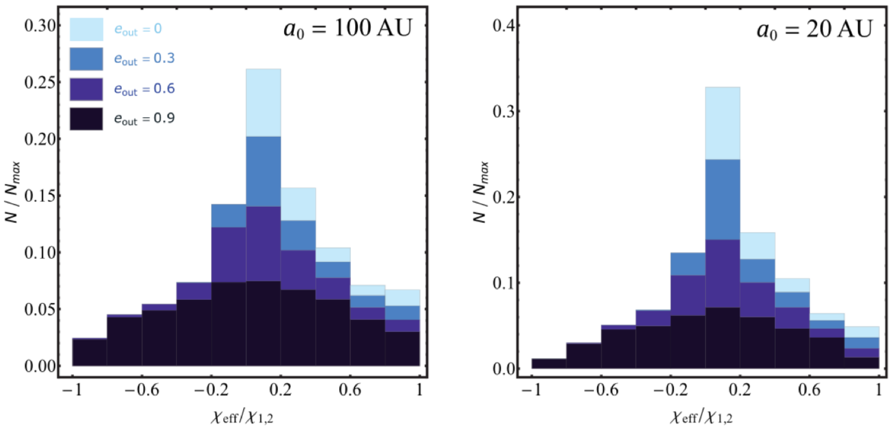
Effective spin parameter $\chi_{\rm eff} = \dfrac{m_1\chi_1 \cos\theta_{\rm s_1 l} + m_2\chi_2 \cos\theta_{\rm s_2 l}}{m_{12}}$




**A unique signature of LK-induced mergers**

# Effective Spin Distribution

For "reasonable" initial binary/triple parameters ($e_0$=0, distant companions)



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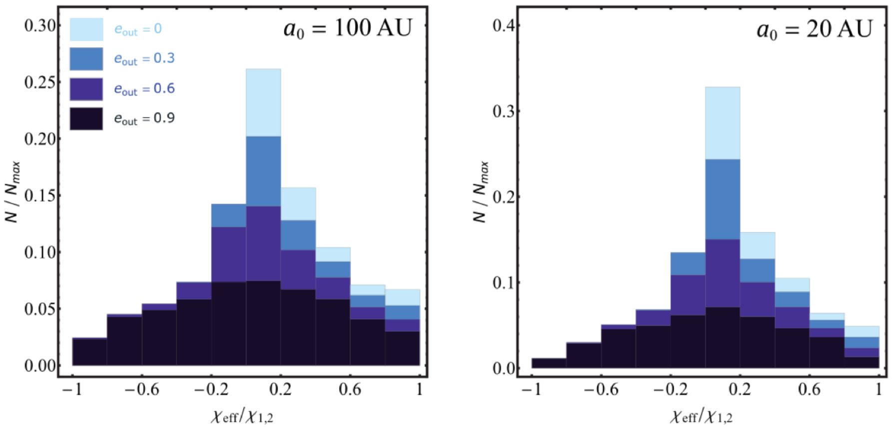
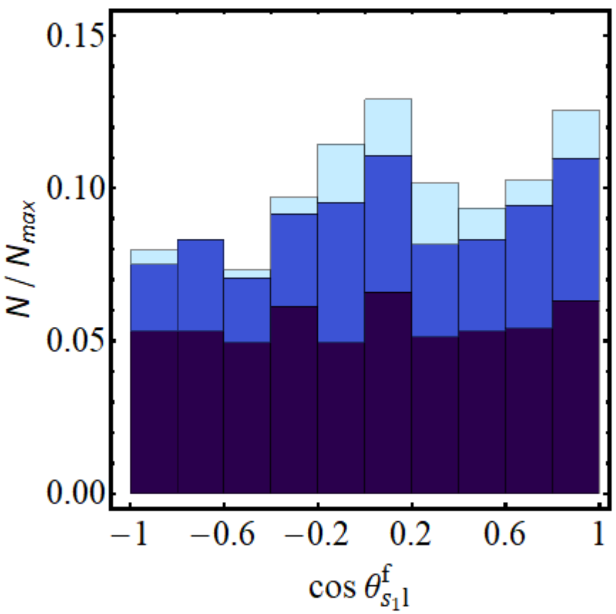
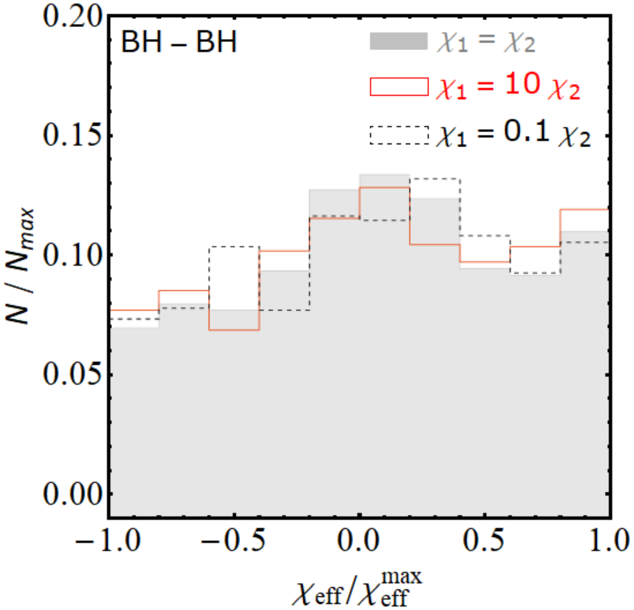



## Consider ALL possible parameters



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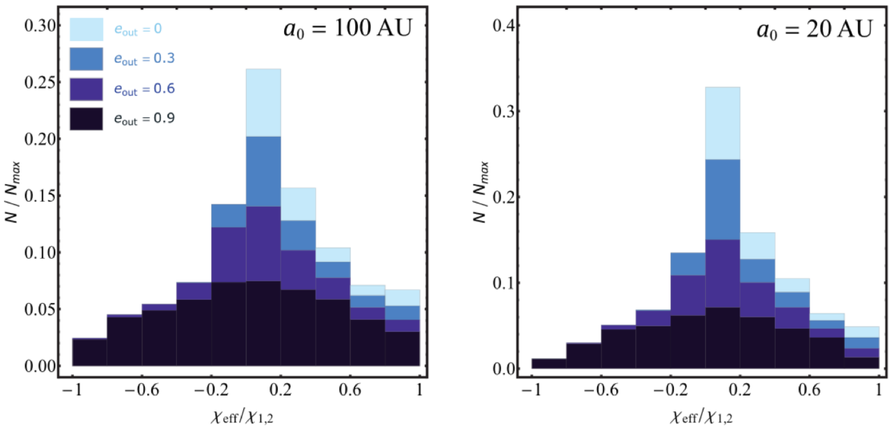


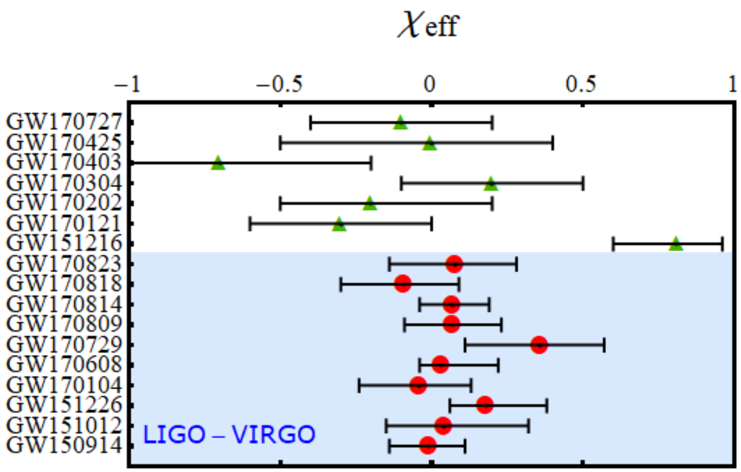


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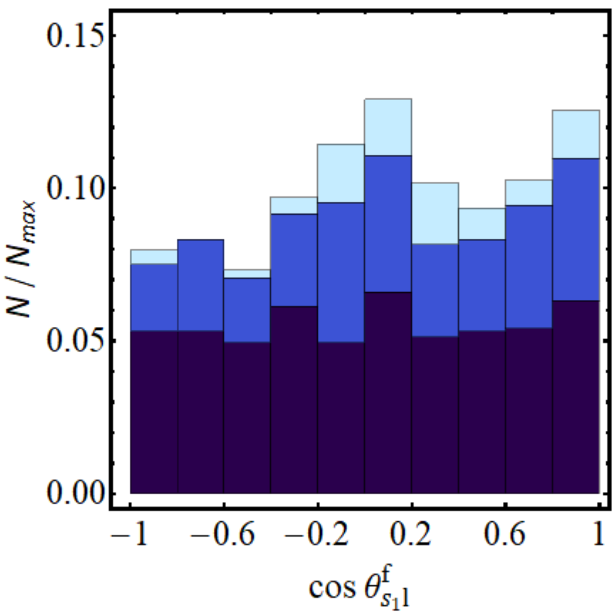
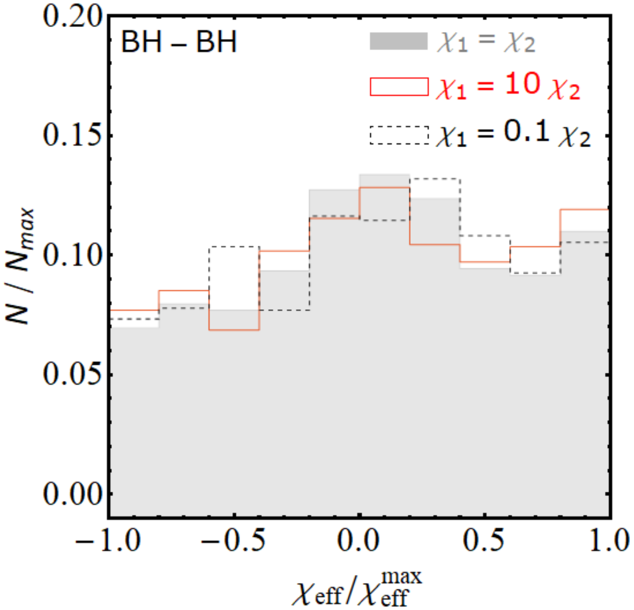
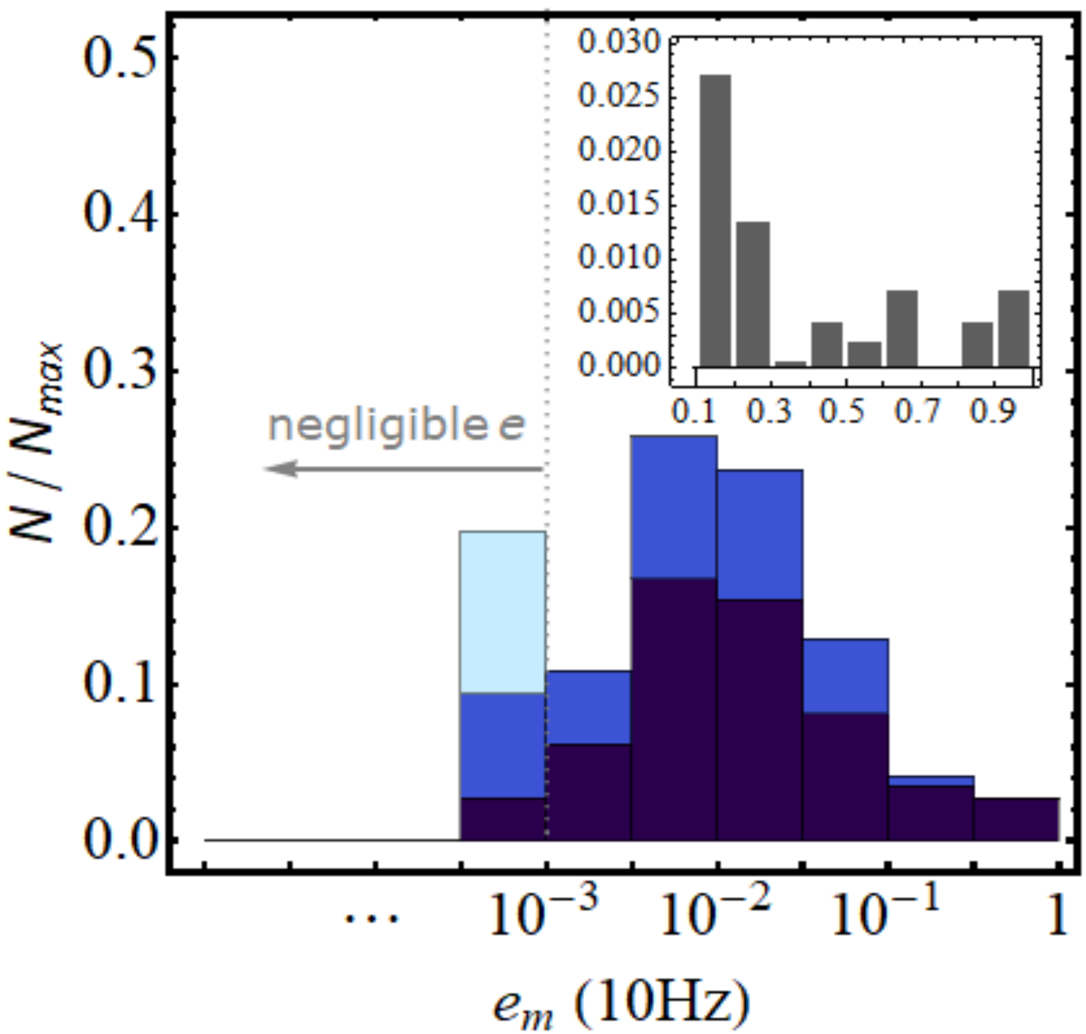



Observed so far

# Residual Eccentricity (at 10 Hz)

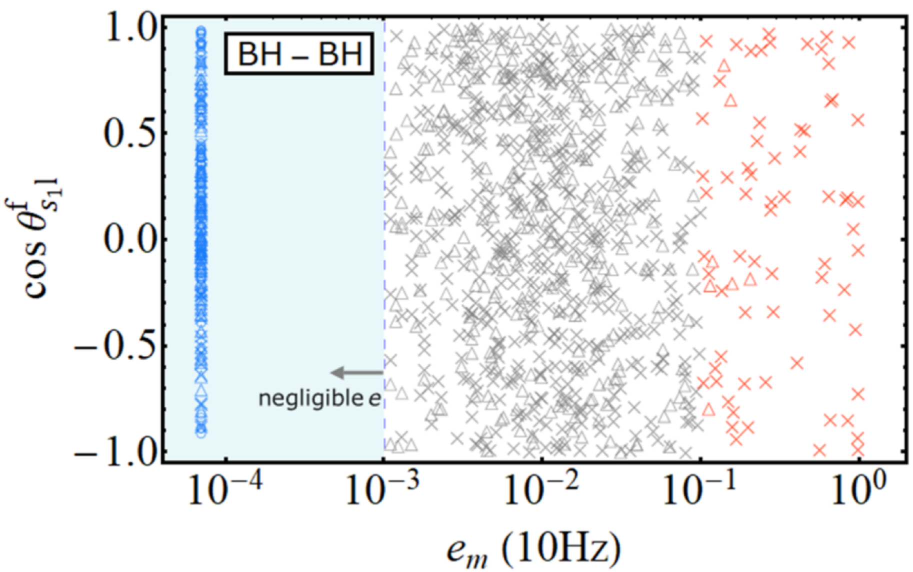
BH-BH mergers

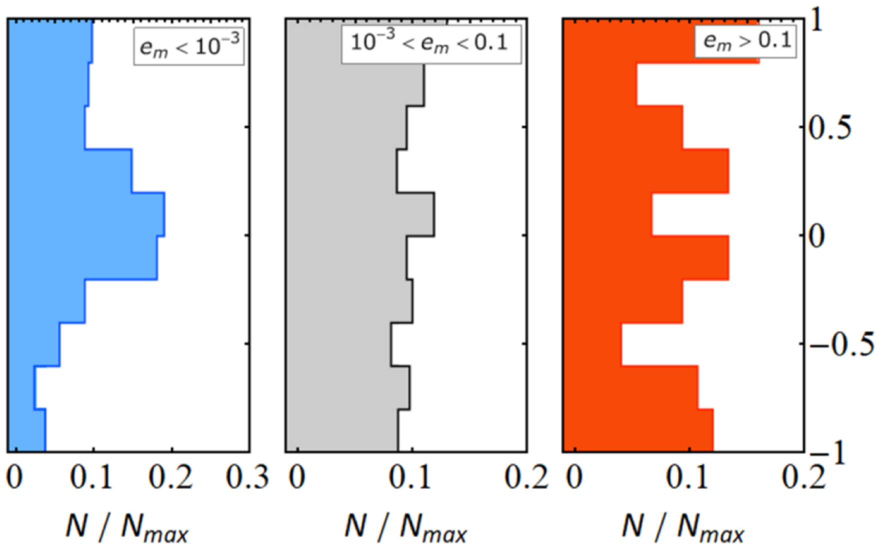



$10\%$ have $e_m > 0.1$

$1\%$ have $e_m > 0.9$

# Residual eccentricity vs Spin-orbit Mislaignment




Circular Mergers ($e_m < 10^{-3}$) prefer $\theta^{f}_{\rm sl} \sim 90°$

More eccentric Mergers has random $\theta^{f}_{\rm sl}$

# What happens if the tertiary is a Supermassive BH ?

--- Relativistic Effects induced by the SMBH

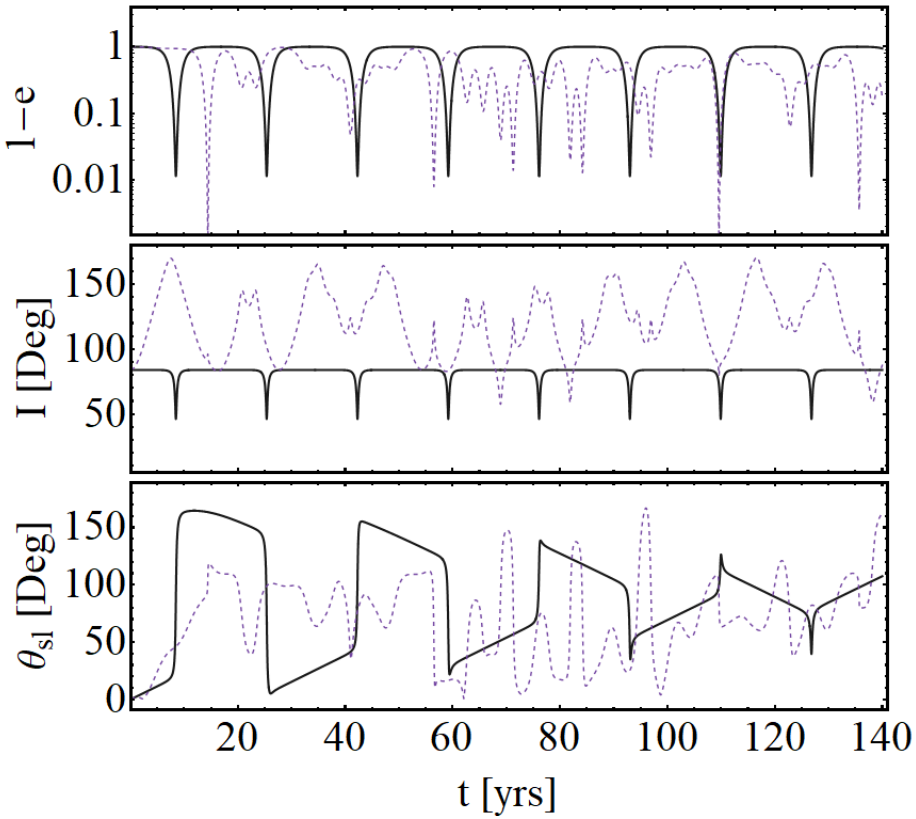
# *1. Lense-Thirring Precession of $L_{out}$ around $S_3$*

## --- introduced by the spin of the SMBH

$$\left.\frac{d\mathbf{L}_{\rm out}}{dt}\right|_{\rm L_{out}S_3} = \Omega_{\rm L_{out}S_3}\hat{\mathbf{S}}_3 \times \mathbf{L}_{\rm out},$$

$$\left.\frac{d\mathbf{e}_{\rm out}}{dt}\right|_{\rm L_{out}S_3} = \Omega_{\rm L_{out}S_3}\hat{\mathbf{S}}_3 \times \mathbf{e}_{\rm out} - 3\Omega_{\rm L_{out}S_3}(\hat{\mathbf{L}}_{\rm out}\cdot\hat{\mathbf{S}}_3)\hat{\mathbf{L}}_{\rm out} \times \mathbf{e}_{\rm out}$$

$$\Omega_{\rm L_{out}S_3} = \frac{GS_3(4+3m_{12}/m_3)}{2c^2 a_{\rm out}^3(1-e_{\rm out}^2)^{3/2}}.$$



## *2. de-Sitter-like Precession of $L_{in}$ around $L_{out}$*

--- modifies the eccentricity growth indirectly

$$\left.\frac{d\mathbf{L}_{\rm in}}{dt}\right|_{\rm L_{in}L_{out}} = \Omega_{\rm L_{in}L_{out}} \hat{\mathbf{L}}_{\rm out} \times \mathbf{L}_{\rm in}$$

$$\Omega_{\rm L_{in}L_{out}} \equiv \Omega^{\rm (N)}_{\rm L_{in}L_{out}} + \Omega^{\rm (GR)}_{\rm L_{in}L_{out}}$$

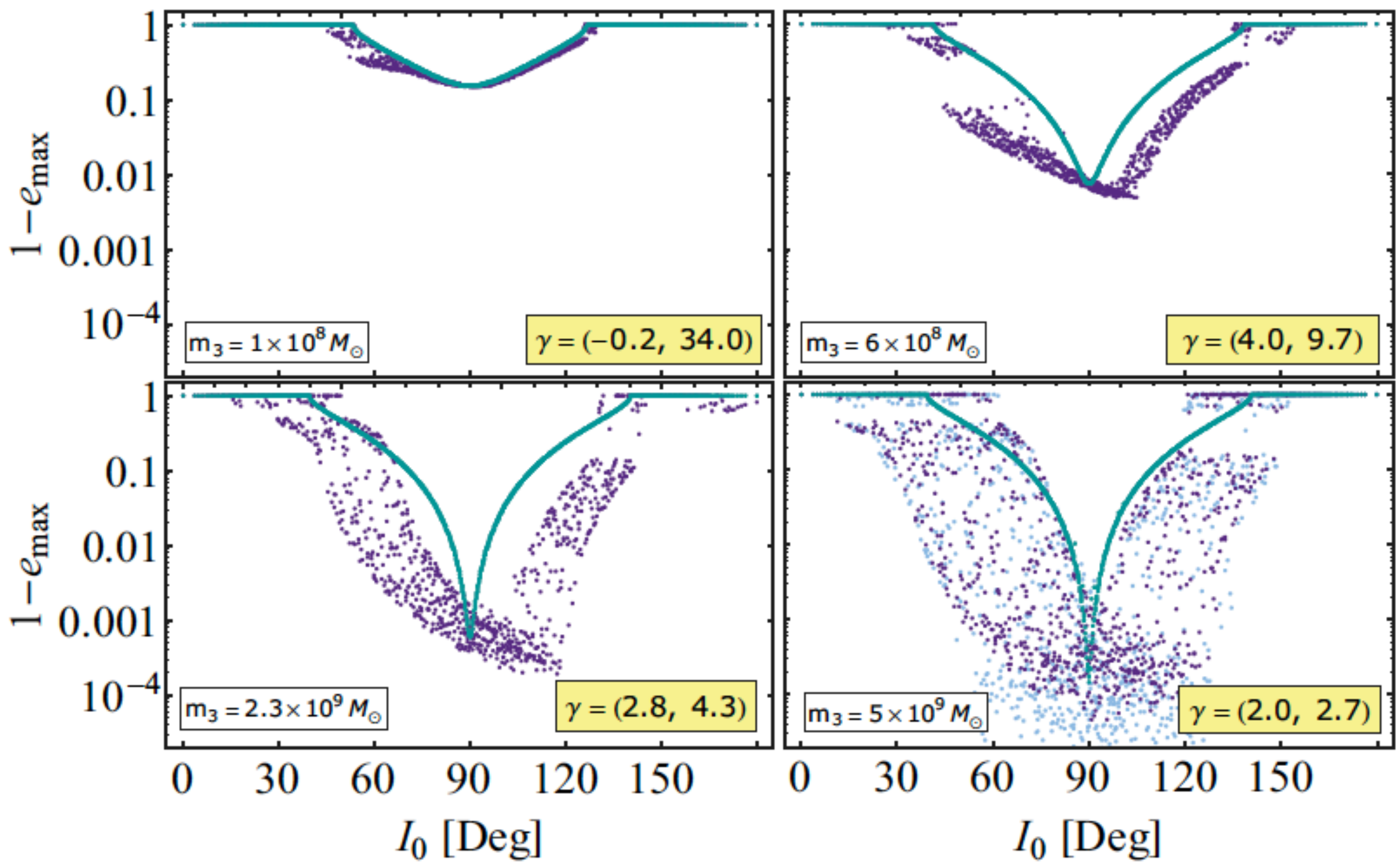
$$\Omega^{\rm (N)}_{\rm L_{in}L_{out}} = -\frac{3}{4}\Omega_{\rm LK}(\hat{\mathbf{L}}_{\rm out} \cdot \hat{\mathbf{L}}_{\rm in}) \qquad (\text{for } e_{\rm in} = 0).$$

$$\Omega^{\rm (GR)}_{\rm L_{in}L_{out}} = \frac{3}{2}\frac{G(m_3 + \mu_{\rm out}/3)n_{\rm out}}{c^2 a_{\rm out}(1 - e_{\rm out}^2)}$$

Cross terms in PN (Will 2014,18)

# Inclination Resonance

$$\gamma \equiv \frac{\Omega_{\mathrm{L_{in}L_{out}}}}{\Omega_{\mathrm{L_{out}S_3}}} = \frac{\Omega^{\mathrm{(N)}}_{\mathrm{L_{in}L_{out}}} + \Omega^{\mathrm{(GR)}}_{\mathrm{L_{in}L_{out}}}}{\Omega_{\mathrm{L_{out}S_3}}}$$



$1-e_{\rm max}$
$m_3 = 1\times10^8 M_\odot$
$\gamma = (-0.2,\ 34.0)$
$m_3 = 6\times10^8 M_\odot$
$\gamma = (4.0,\ 9.7)$
$m_3 = 2.3\times10^9 M_\odot$
$\gamma = (2.8,\ 4.3)$
$m_3 = 5\times10^9 M_\odot$
$\gamma = (2.0,\ 2.7)$
$I_0$ [Deg]

# Summary
# Formation Channels of Merging BH Binaries

## Standard isolated binary evolution channel:

uncertain physics (common envelope…)

➔ circular mergers ($e_m$=0)

aligned spin-orbit angle

## Dynamical formation channels:

"clean" physics, but "environmental" uncertainties

1. **Dense star clusters**

   ➔ mostly circular mergers ?

   expect random spin-orbit misalignments ?

2. **Tertiary-induced mergers**

   Perturbations from outer companion → Lidov-Kozai

   Spin-orbit coupling (de Sitter precession) important..

➔ 10% mergers have residual e>0.1 when entering LIGO band

Preference of $90^0$ spin-orbit misalignment, especially for circular mergers

Rates? All potentially compatible…

LISA useful for probing dynamical formation