# NS Seismology and Merging Binaries

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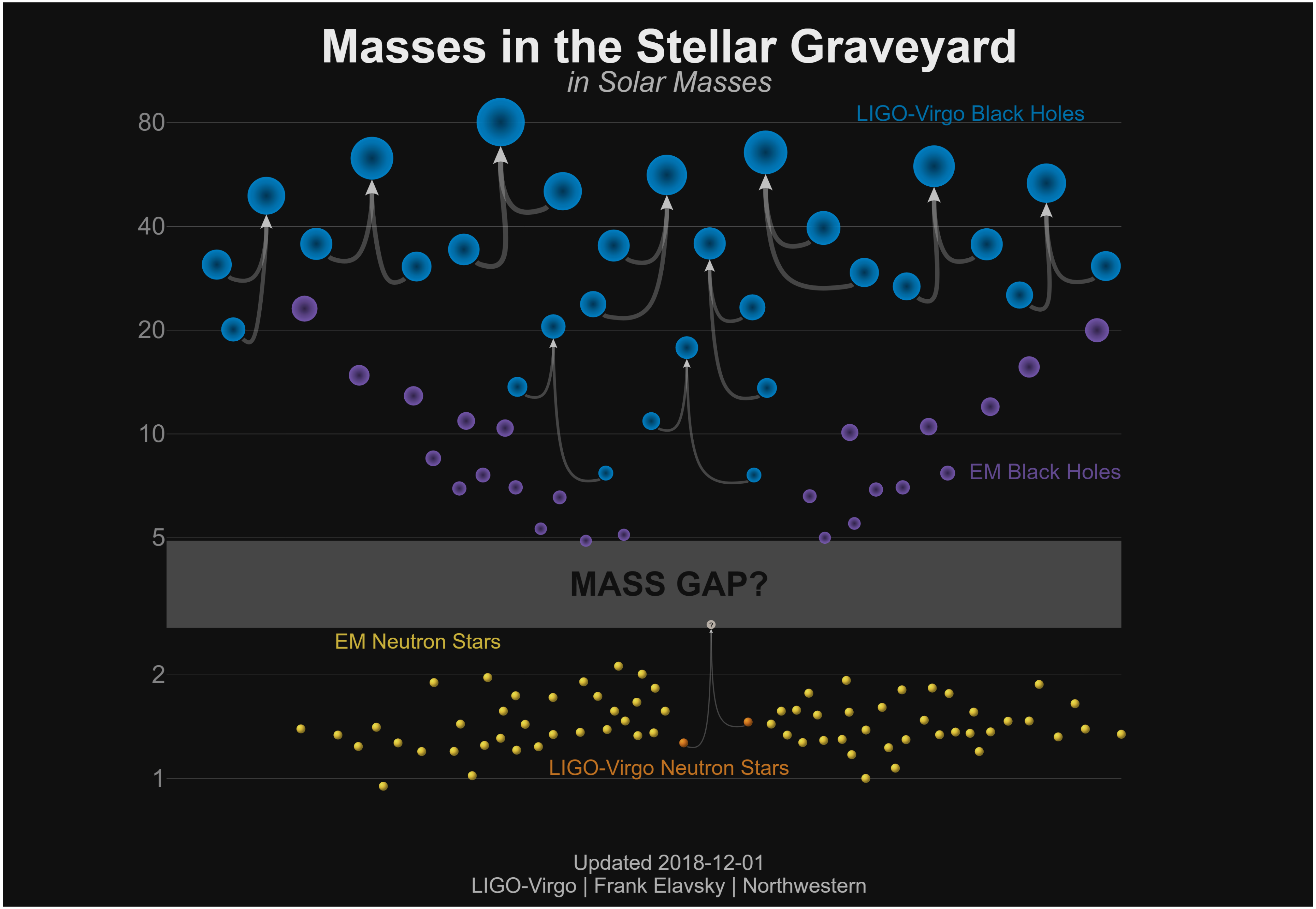
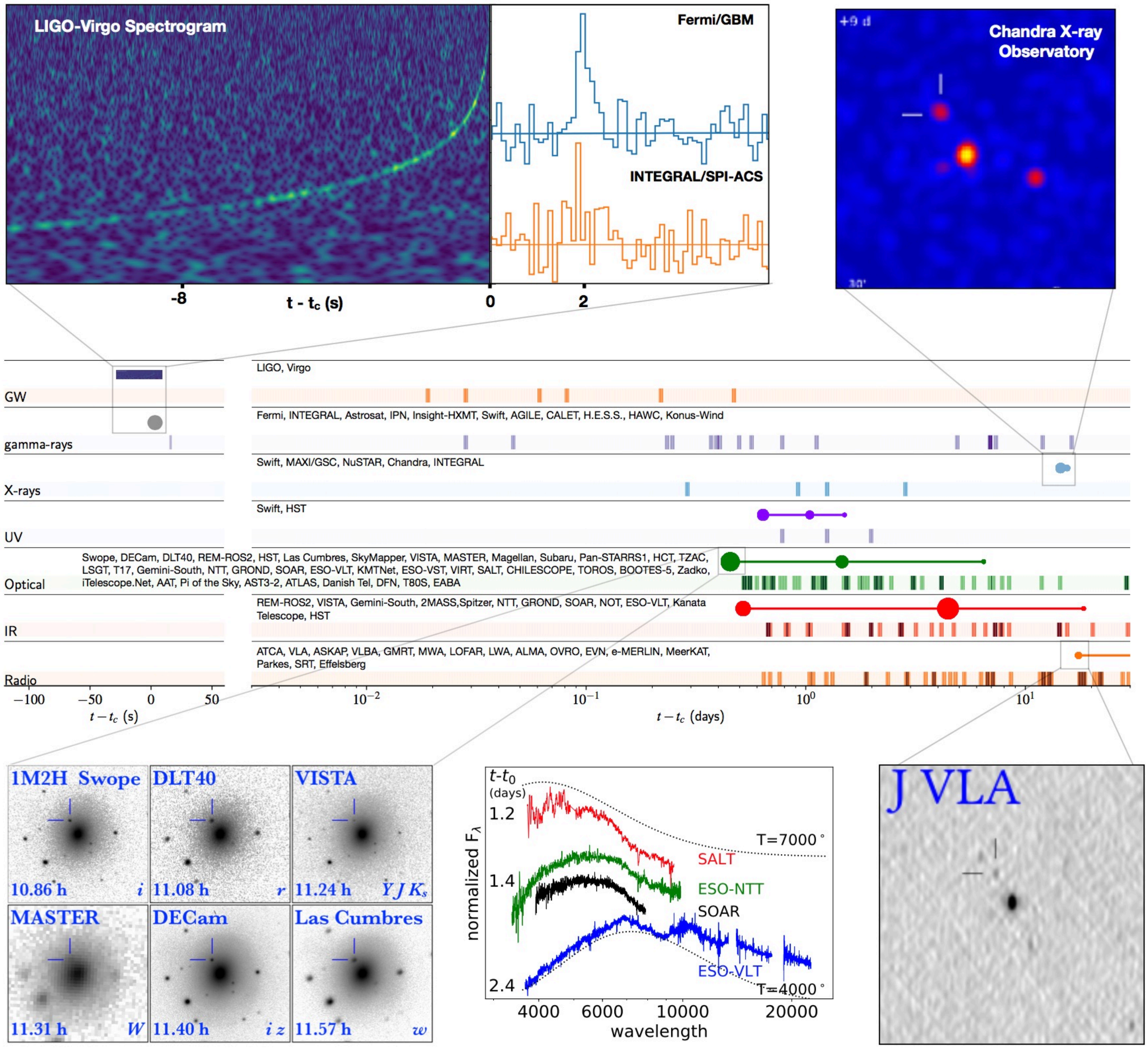


Masses in the Stellar Graveyard
in Solar Masses
LIGO-Virgo Black Holes
EM Black Holes
MASS GAP?
EM Neutron Stars
LIGO-Virgo Neutron Stars
80
40
20
10
5
2
1
Updated 2018-12-01
LIGO-Virgo | Frank Elavsky | Northwestern

# GW170817 / AT2017gfo



# Probe Nuclear EOS from NS Binary Mergers

**Fate of the remnant of NS/NS**

maximum TOV mass

**Final merger GW of NS/NS or NS/BH**

characteristic cut-off frequency ~$(GM/R^3)^{1/2}$

**Tidal effects on GW during Inspiral**

## Tidal Seismology in Merging NS Binaries
## ➔ Probe NS EOS

-- Quasi-Equilibrium tides

f-mode, p-modes

-- Resonant (Dynamical) tides

g-modes, r-modes, mixed modes

# Nonradial Oscillation Modes of NS

**Acoustic modes (p-modes):**

Stationary sound waves, depends on sound speed $c_s = (\partial P/\partial\rho)^{1/2}$

f-mode: global acoustic mode (no radial node), $\sigma \sim \sqrt{G\bar{\rho}} \sim 2\ \mathrm{kHz}$

**G-modes (gravity modes = buoyancy modes):**

Arise from stable stratification:

Density jumps in crust: interface modes

Core composition gradient: $x=n_p/n$ increases with density

$$N = g\left[\left(\frac{\partial\rho}{\partial x}\right)_P \left(\frac{dx}{dP}\right)\right]^{1/2}$$

~100 Hz or less

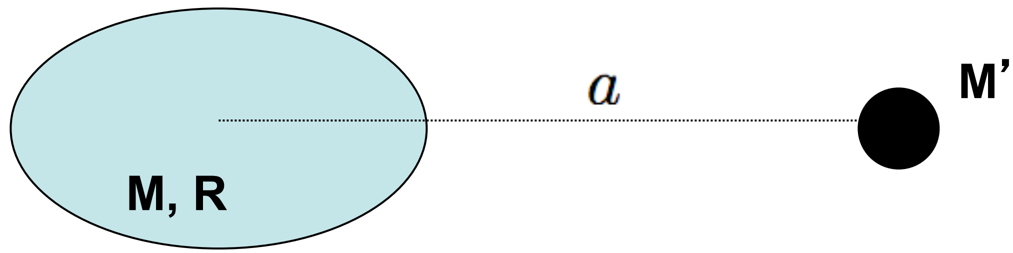
depends on symmetry energy

depends on superfluidity···

**R-modes (inertial modes, Rossby modes):**

arise from Coriolis force $\sigma \sim \Omega_s$

# "Quasi-Equilibrium" Tide (F-mode Distortion)




Tidally induced quadrupole $\sim k_2 \dfrac{M' R^5}{a^3}$ $\qquad k_2 = \text{Love number}$

$$V = -\frac{MM'}{a} - \mathcal{O}\left(k_2 \frac{M'^2 R^5}{a^6}\right)$$

$$\Rightarrow d\Phi = d\Phi^{(0)}\left[1 - \mathcal{O}\left(\frac{k_2 R^5}{a^5}\right)\right]$$

**(GW Phase Shift)**

➔ Important only at small separation (just prior to merger)

(Kochenek; Bildsten & Cutler; Lai, Rasio & Shaipro, etc 1990s)

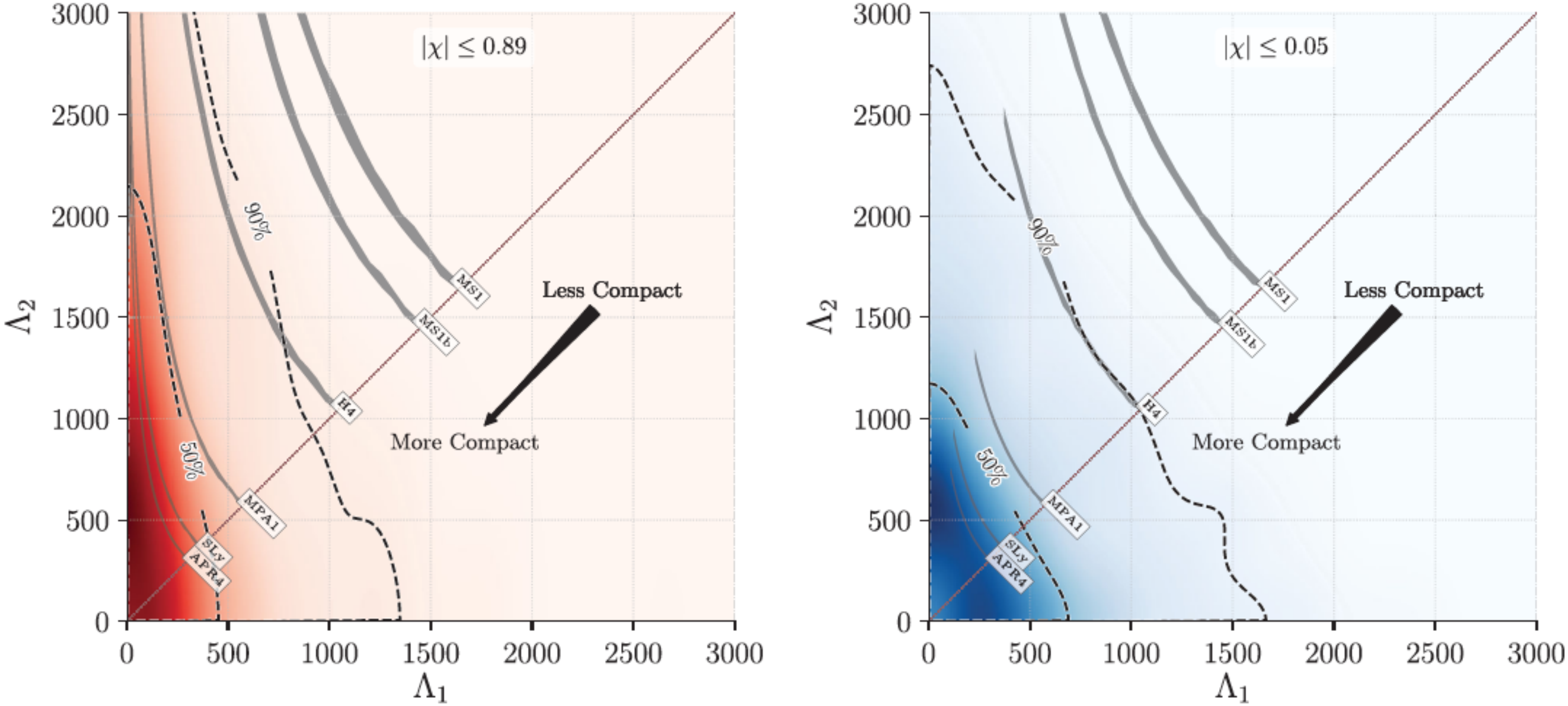
Numerical GR Quasi-equilibrium NS binary sequence

(Baumgarte, Shapiro, Teukolsky, Shibata, Meudon group, etc. 1990s--200x)

Recent (semi-analytic) GR calculation of tidal effect

(Hinderer, Flanagan, Poisson, Damour, Penner, Andersson, Jones, etc., 2008+)

# GW170817:




$$\Lambda_2^{(i)} = \frac{2}{3} k_2^{(i)} \left[ \left( \frac{c^2}{G} \right) \left( \frac{R_i}{M_i} \right) \right]^5$$

## Quasi-Equilibrium Tide = F-mode $(l = 2)$

$$\ddot{\xi} + \sigma^2 \xi \sim e^{i2\Omega t}$$

$$\Rightarrow \quad \xi \sim \frac{e^{i2\Omega t}}{\sigma^2 - (2\Omega)^2}$$

The usual tidal distortion calculation assumes $\sigma_{\mathrm{f}} \gg 2\Omega$

**Dynamical correction to "equilibrium tide":**

$$d\Phi = d\Phi^{(0)} \left[ 1 - \mathcal{O}\left( \frac{k_2 R^5}{a^5} \right) \frac{1}{1 - 4\Omega^2/\sigma_{\mathrm{f}}^2} \right]$$

# Resonant Tides:
## Excitations of Internal Modes

**NS has low-frequency oscillation modes:**

g-modes (~100 Hz)
r-modes

$$\ddot{\xi} + \sigma_\alpha^2 \xi \sim e^{im\Omega t}$$

**Resonance:** $\sigma_\alpha = m\Omega_{\rm orb}, \quad m = 2, 3, \cdots$

**Rosonant tidal excitations of NS modes during inspiral**

➔ transfer orbital energy to NS
➔ GW phase shift

# Resonant Excitations of NS Oscillations During Inspiral

$$\Delta\Phi \propto \frac{(Q_{\alpha,lm})^2}{\epsilon_\alpha \sigma_\alpha}, \quad Q_{\alpha,lm} = \int d^3x\,(\delta\rho_\alpha)^\star\, r^l Y_{lm}$$

**Non-rotating NS:**

G-mode (Reisenegger & Goldreich 94; DL 94; Shibata 94)

**Rotating NS:**

G-mode, F-mode, R-mode (Ho & DL 99)

Inertial modes (DL & Wu 06)

R-mode (excited by gravitomagnetic force; Racine & Flanagan 06)

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## Recent works:

- Superfluid NSs: Yu & Weinberg 2017
- Gravity-inertial modes, scalings (parameterized EOS): Xu & DL 2017

Wenrui Xu
(Cornell → Princeton)

# G-modes

crustal density continuities

stable composition stratification of core: symmetry energy, superfluidity

**Parameterize the uncertainties:**

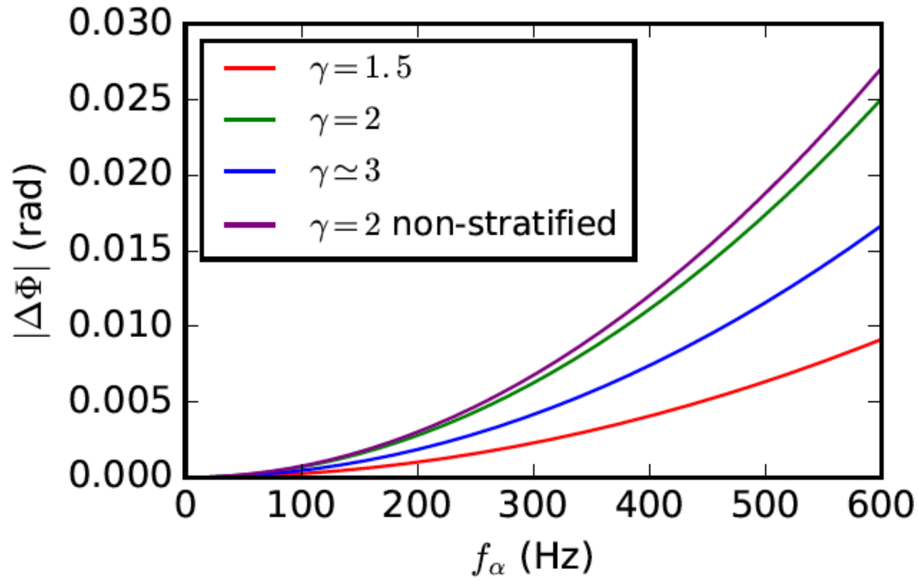
$P \propto \rho^{\gamma}, \quad \Gamma = (\partial \ln P / \partial \ln \rho)_{\rm ad}$

$$\omega_\alpha \propto (\Gamma - \gamma)^{1/2} M^{1/2} R^{-3/2}$$

$$Q_{\alpha,2m} \propto \Gamma - \gamma,$$

$$\Delta\Phi = -0.060 \left(\frac{R_{10}^8}{M_{1.4}^6}\right) \frac{2}{q(1+q)} \left(\frac{f_\alpha}{100\,\mathrm{Hz}}\right)^2 \times \left(\frac{\bar{f}_\alpha}{100\,\mathrm{Hz}}\right)^{-4} \left(\frac{\bar{Q}_{\alpha,22}}{10^{-3}}\right)^2,$$

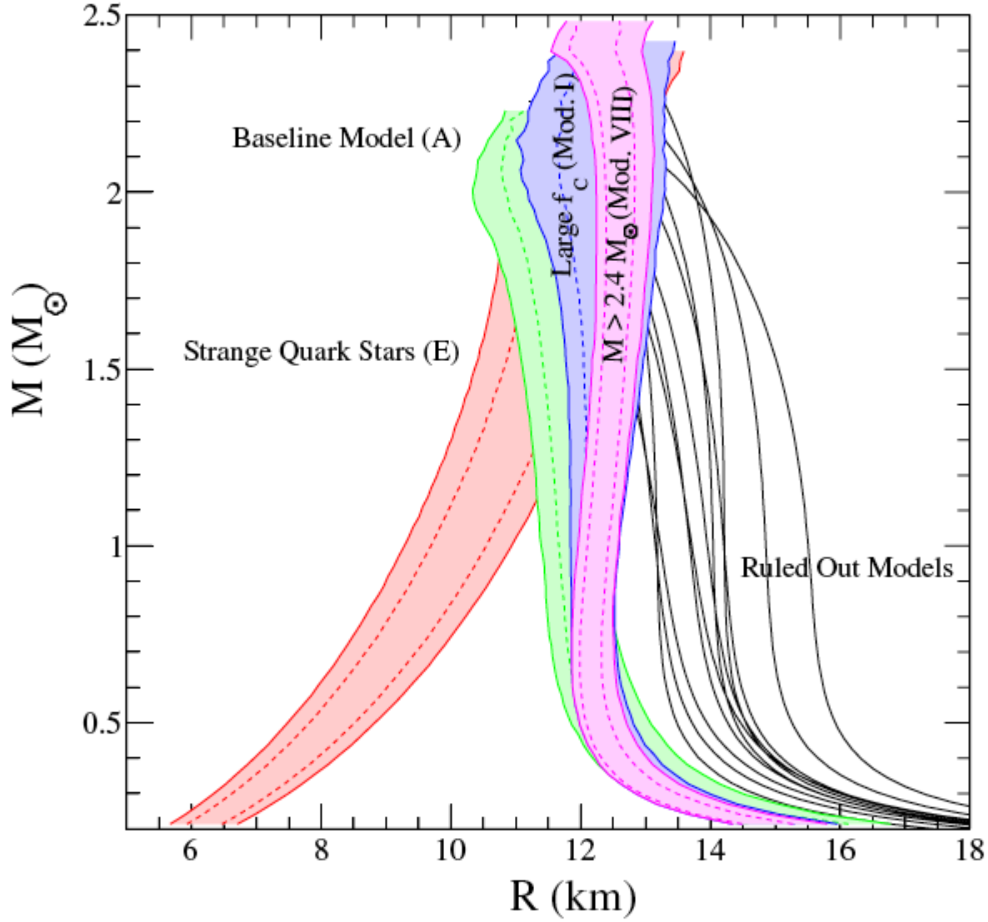
Xu & DL 17




**Note:** $M = 1.2 M_\odot,\ R = 13$ km NS

➔ x 21 Important !

# NS Mass-Radius




Measured NS mass:
1.17–2.0 Solar mass

Steiner et al 2013

# G-mode excitation during binary inspiral

$$\Delta\Phi = -0.060 \left(\frac{R_{10}^{8}}{M_{1.4}^{6}}\right) \frac{2}{q(1+q)} \left(\frac{f_\alpha}{100\,\mathrm{Hz}}\right)^2 \times \left(\frac{\bar{f}_\alpha}{100\,\mathrm{Hz}}\right)^{-4} \left(\frac{\bar{Q}_{\alpha,22}}{10^{-3}}\right)^2,$$

It is a coincidence that $|\Delta\Phi| \lesssim 1$... Unlike equilibrium tide

For white dwarf binaries, $|\Delta\Phi| \gg 1$ (LISA), lead to significant tidal heating

# Challenge to nuclear physicists:

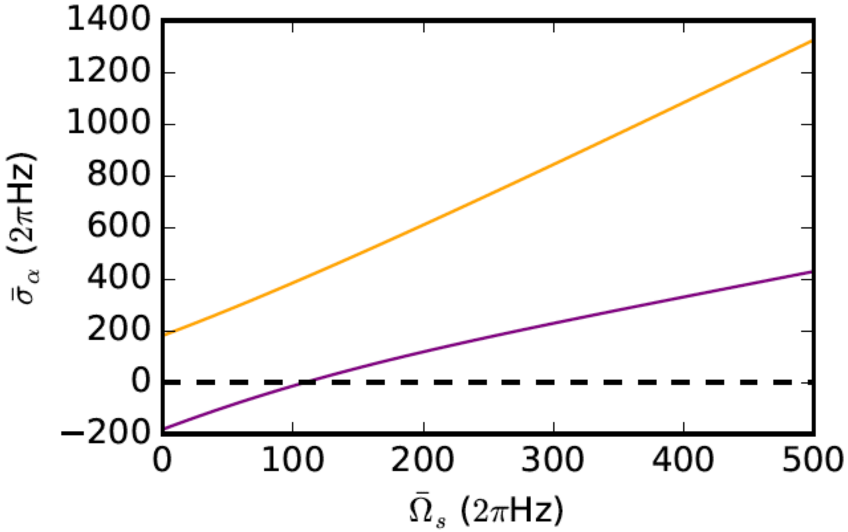
Survey $E_{\mathrm{sym}}(n)$ (constrained by exps/obs), calculate g-modes and $\Delta\Phi$

# With rotation: pure r-modes (inertial modes)

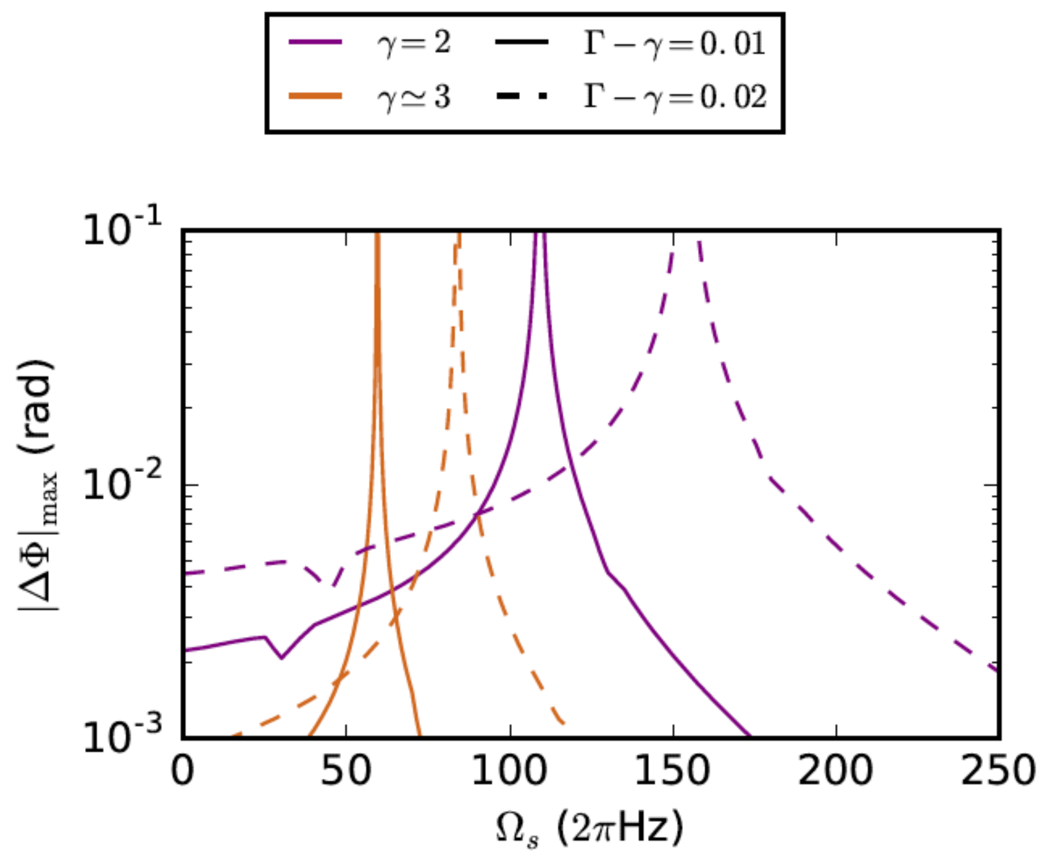
$$\Delta\Phi = \mp 0.0027 \left(\frac{R_{10}^8}{M_{1.4}^6}\right) \frac{2}{q(1+q)} \left(\frac{\epsilon_\alpha |\sigma_\alpha|}{\Omega_s^2}\right)^{-1} \times \left(\frac{Q_{\alpha,2m}}{0.02\hat{\Omega}_s^2}\right)^2 \left(\frac{f_s}{500\ \mathrm{Hz}}\right)^2 \left|\mathcal{D}_{m\pm 2}^{(2)}\right|^2,$$

Usually negligible (unless $R^8/M^6$ is large)⋯

# With rotation: Mixed modes (Inertial-Gravity Modes)




Rotation can reduce (retrograde) g-mode $|\sigma_\alpha|$, thus increase $|\Delta\Phi|$

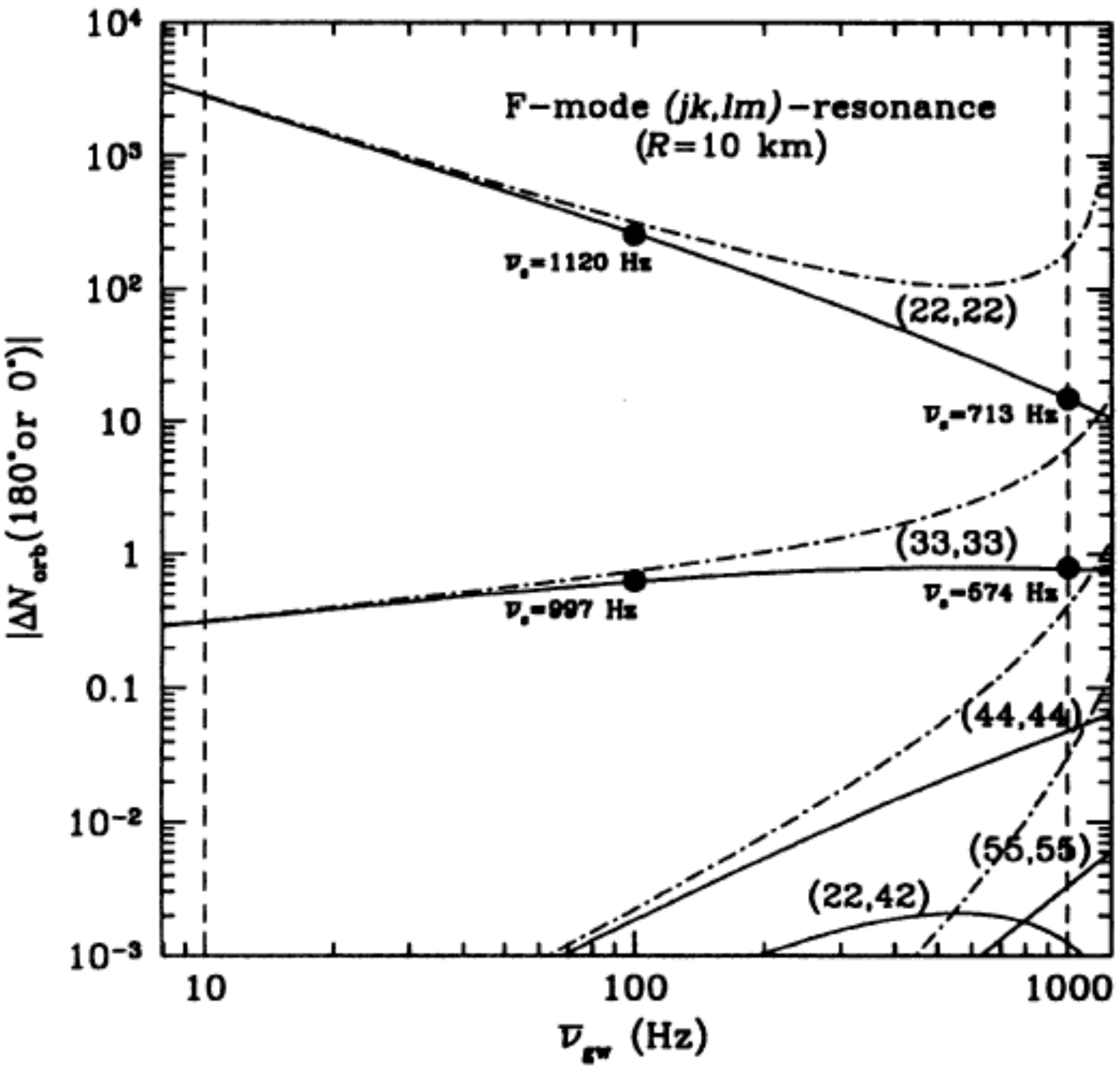



Xu & DL 2017

# Resonant excitation of F-modes?

Very strong … possible with **very rapid rotations**

Wynn Ho & DL 1999

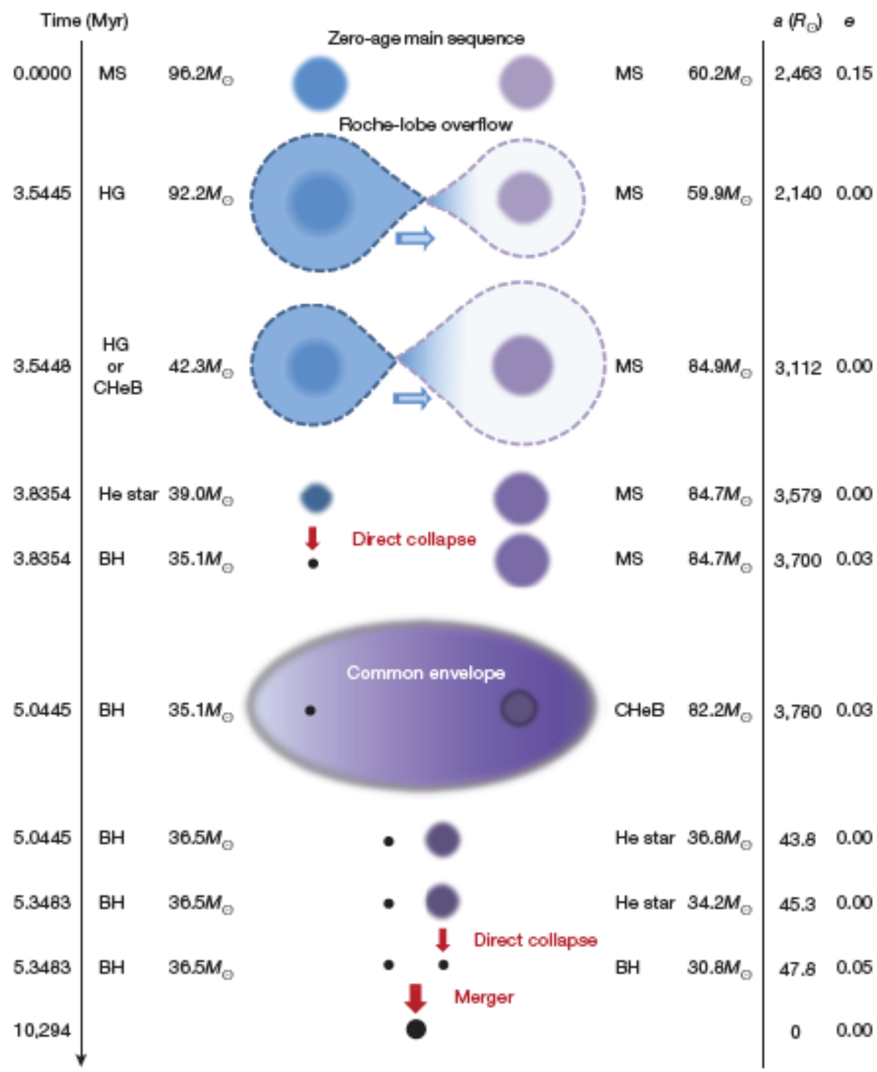



**Physics allows it.**
**Does Nature provide it?**

So far: Circular mergers

**What about eccentric mergers?**

# Standard Binary Evolution Channel ➔ Circular merger

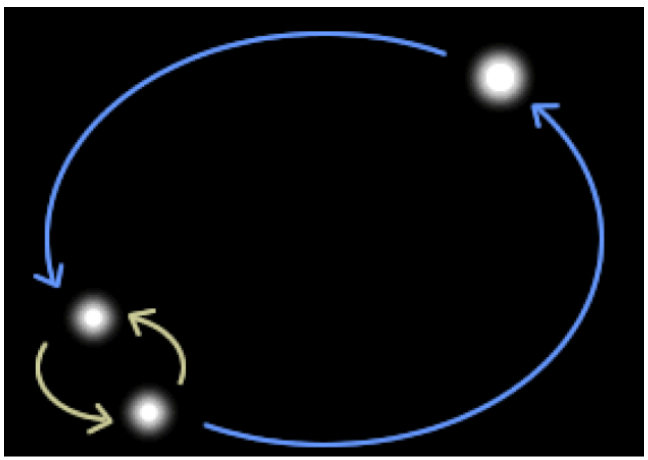


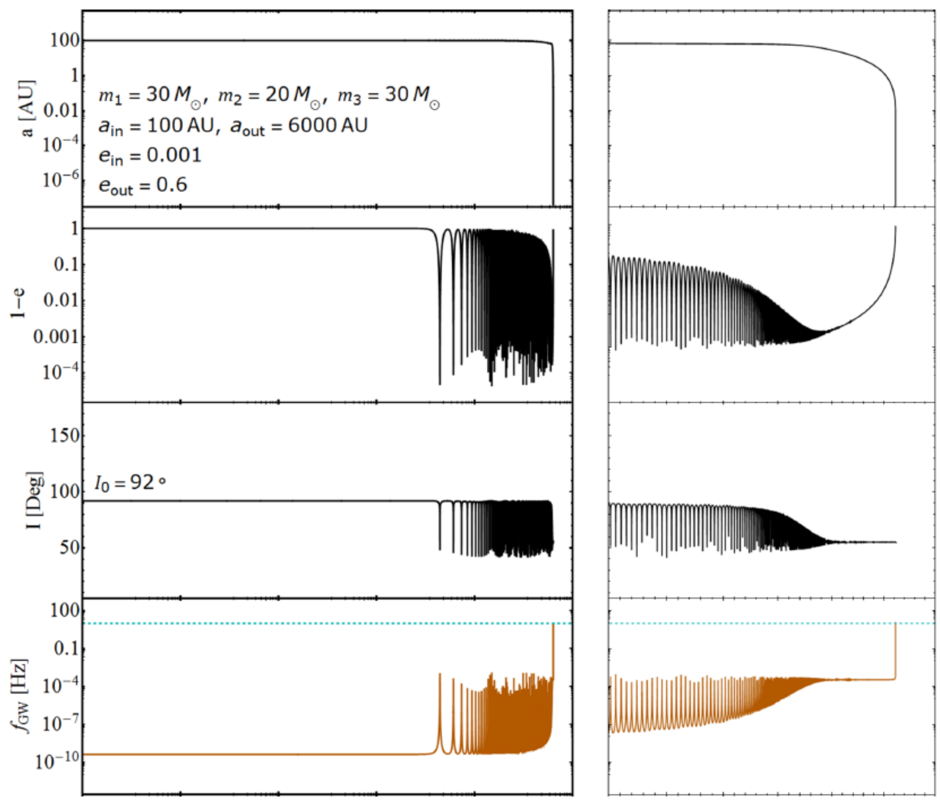

Belczynski +16

# **Possibility of eccentric mergers:**
Dynamical formation merging compact binaries

**-- Binary + single scatterings dense clusters**

**-- Binary mergers induced by external companion (SMBH, stellar triple)**

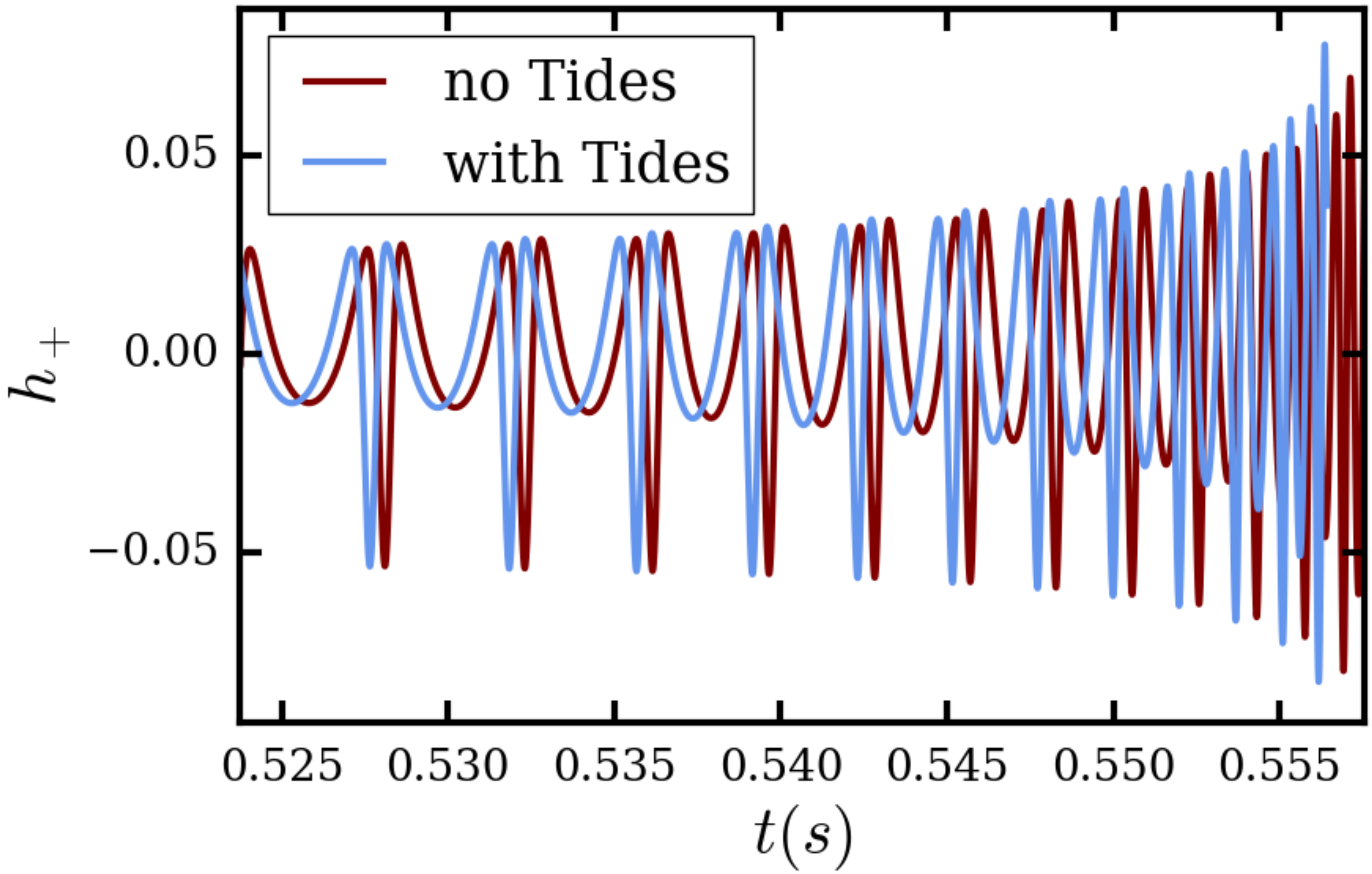





Bin Liu & DL 2017,18,19
also Silsbee & Tremaine 2017 etc

# Tidal effect in eccentric mergers

Michelle Vick & DL 2019




Initial $e$=0.9, $r_p$=5R

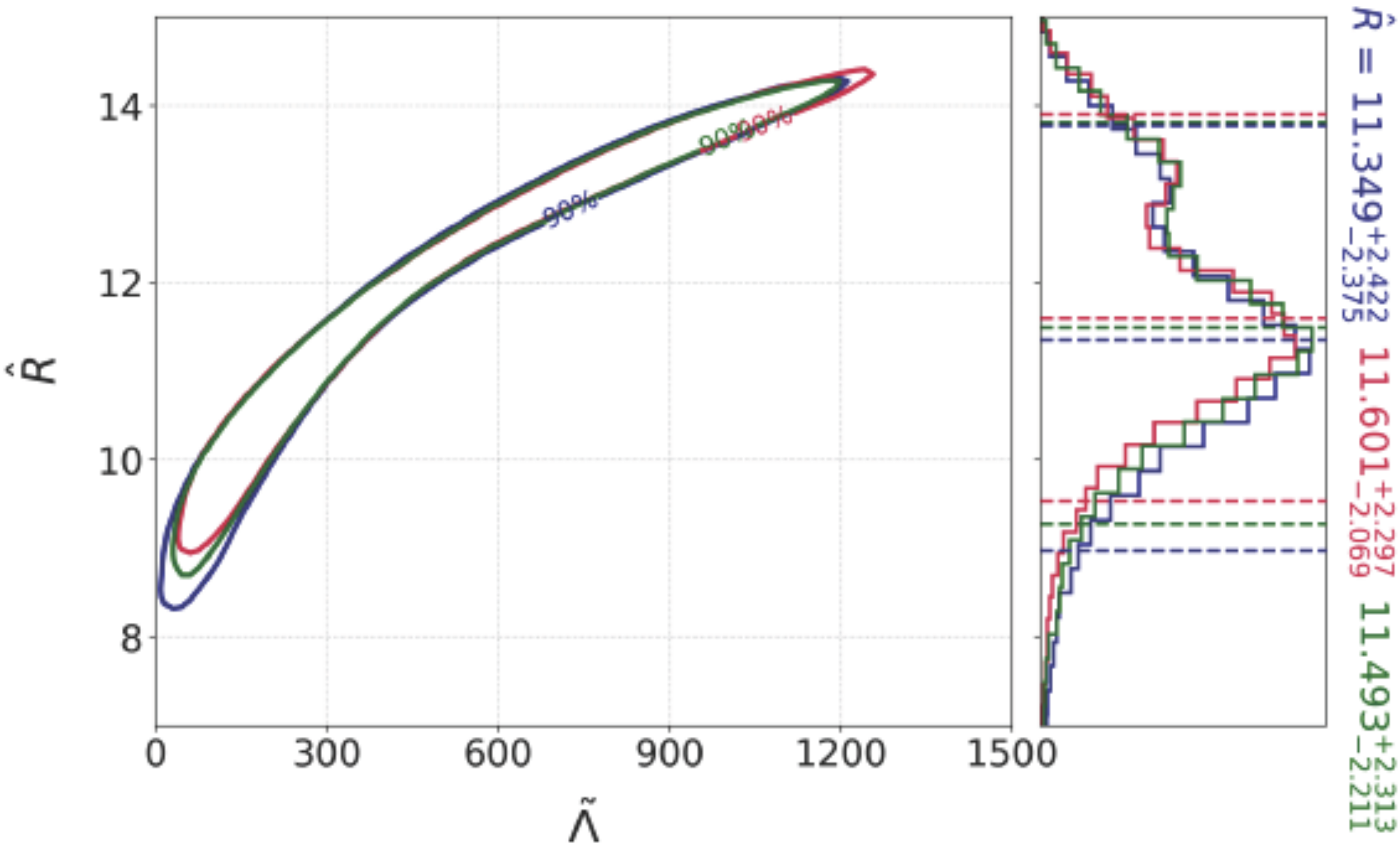
# Summary

- Tidal distortion (non-resonant f-mode) constrains R-M
- Resonant tidal excitation of modes occurs at low frequency (< 100 Hz).
  If strong, would provide a unique probe of EOS (e.g. symmetry energy)
  - -- g-modes: Important for low-M, large-R NSs
  - -- rotation enhances resonance.
  - -- f-mode resonance: Important for very rapid rotation
- Highly eccentric mergers ?

**Note**: Resonant tides are certainly important for WD binaries
(or WD-NS, WD-BH): LISA band
Also for 1-10 Hz band (Cosmic Explorer, Einstein Telescope)

# Thanks.

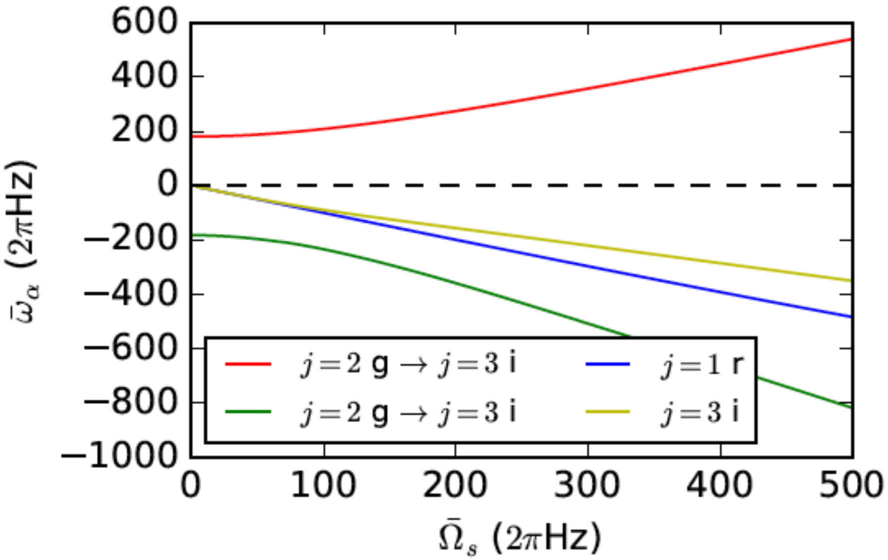
# GW170817:




De, Finstad, Lattimer et al. 2018

# With rotation: Many other inertial modes

## (modified by stratification)




Not as important…

Xu & DL 2017