Dynamical Tides in Binaries

I. Merging White Dwarf Binaries

II. Kepler KOI-54

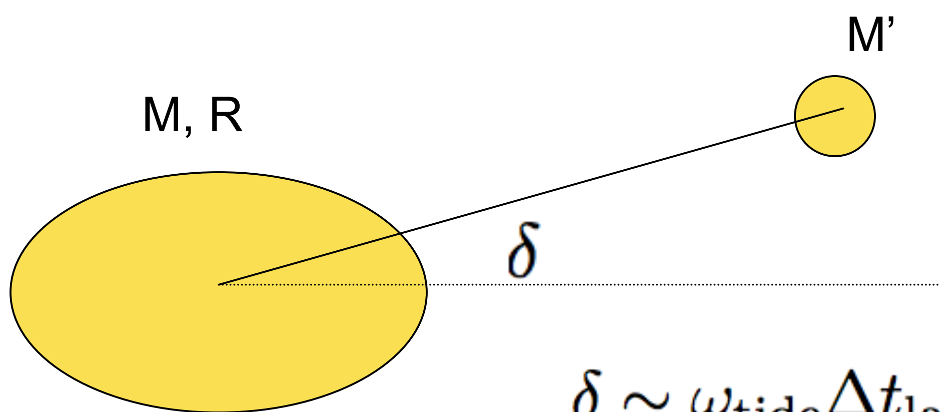
III. Hot Jupiter Systems

Dong Lai

Cornell University

4/5/2012, IAS, Princeton

Equilibrium Tide



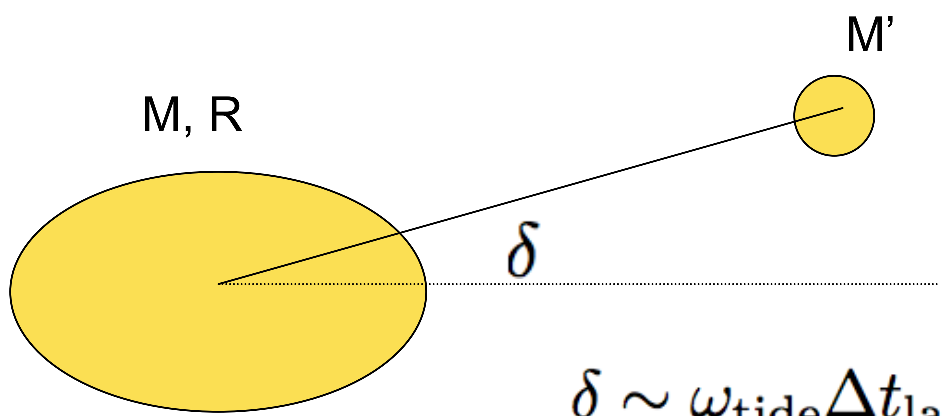
$$\delta \sim \omega_{\rm tide} \Delta t_{\rm lag} \sim 1/Q$$

$$\omega_{\rm tide} = 2(\Omega_{\rm orb} - \Omega_s)$$

$$\mathrm{Torque} \sim G \left(\frac{M'}{a^3} \right)^2 R^5 \, \delta$$

$$\dot{E}_{\rm tide} = \mathrm{Torque} \cdot \Omega$$

Equilibrium Tide



$$\delta \sim \omega_{\rm tide}\Delta t_{\rm lag} \sim 1/Q$$

$$\omega_{\rm tide} = 2(\Omega_{\rm orb} - \Omega_s)$$

$$\text{Torque} \sim G\left(\frac{M'}{a^3}\right)^2 R^5\,\delta$$

$$\dot{E}_{\rm tide} = \text{Torque}\cdot\Omega$$

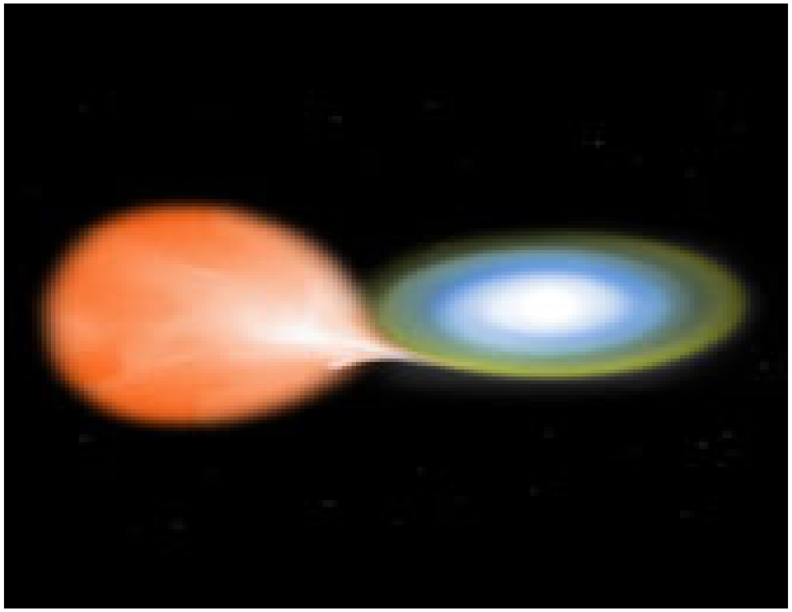
Problems:

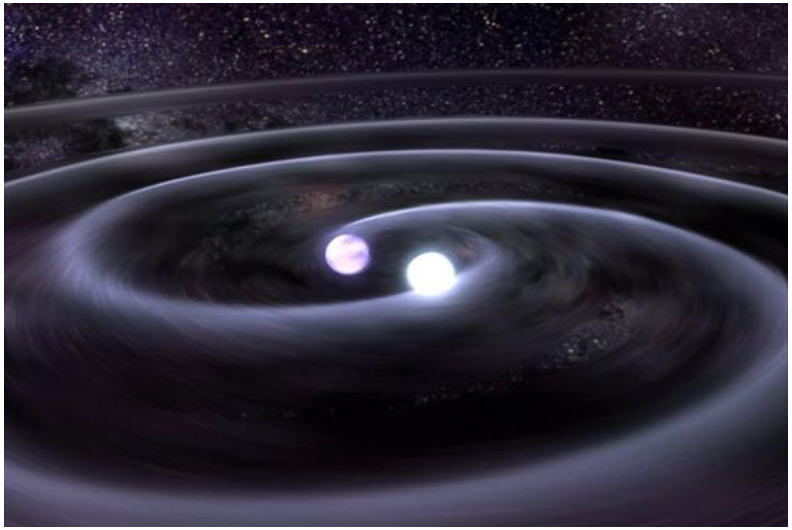
-- Parameterized theory

-- The physics of tidal dissipation is more complex:
Excitation/damping of internal waves/modes (Dynamical Tides)

-- For some applications, the parameterization is misleading

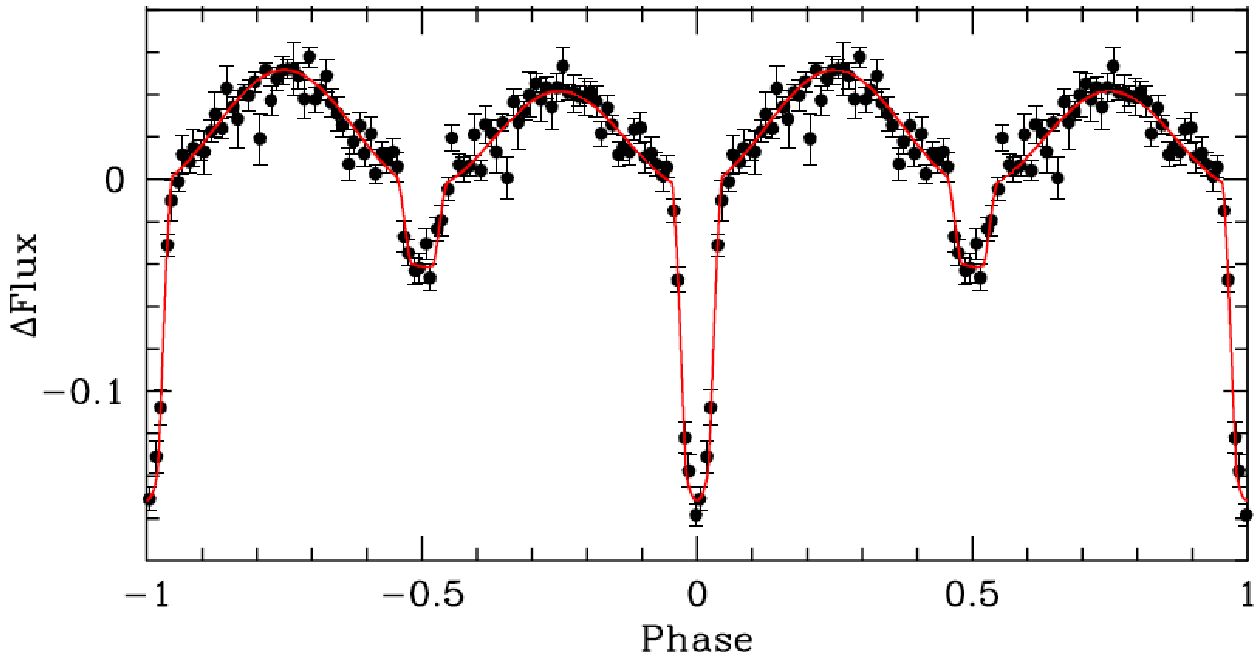
Compact White Dwarf Binaries





-- May lead to various outcomes: SN Ia, transients, AICs, etc
(SN Ia: single vs double-WDs ? Sub-Chandra Mass?**)**
-- Gravitational waves (eLISA-NGO)

12 min orbital period double WD eclipsing binary



Primary & secondary eclipses
Ellipsoidal (tidal) distortion
Doppler boosting

Brown et al. 2011

-- will merge in 0.9 Myr
-- large GW strain ==> LISA
-- orbital decay measurable from eclipse timing

Dynamical Tides in Compact WD Binaries

Jim Fuller & DL 2011,12

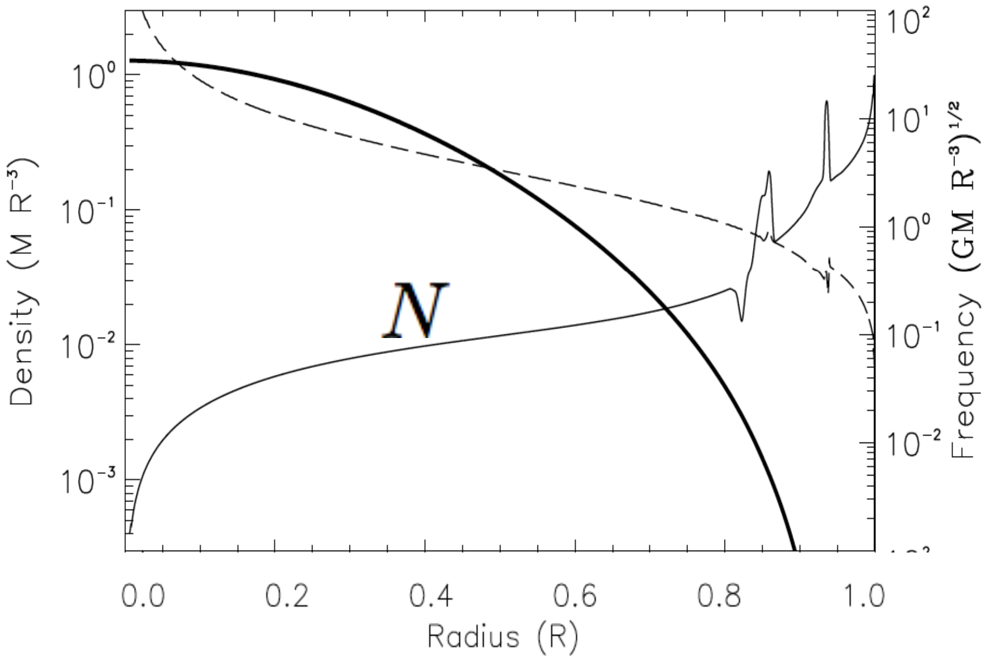
Issues:

-- Spin-orbit synchronization?

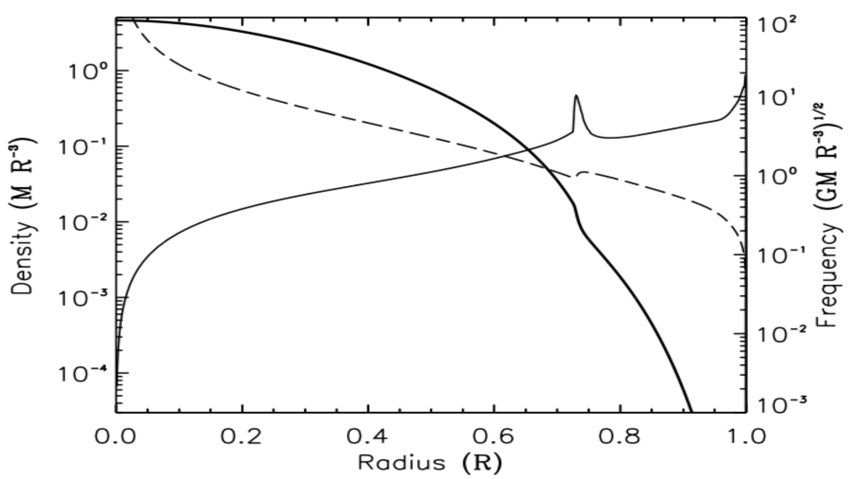
-- Tidal dissipation and heating?

-- Effect on orbital decay rate? (e.g. eLISA-NGO)

White Dwarf Propagation Diagram



$0.6 M_\odot,\ 8720\,\mathrm{K}$



$0.3 M_\odot,\ 12000\,\mathrm{K}$

Resonant Tidal Excitation of G-modes

As the orbit decays, resonance occurs when

$$\omega = 2(\Omega_{\rm orb} - \Omega_s) = \omega_\alpha$$

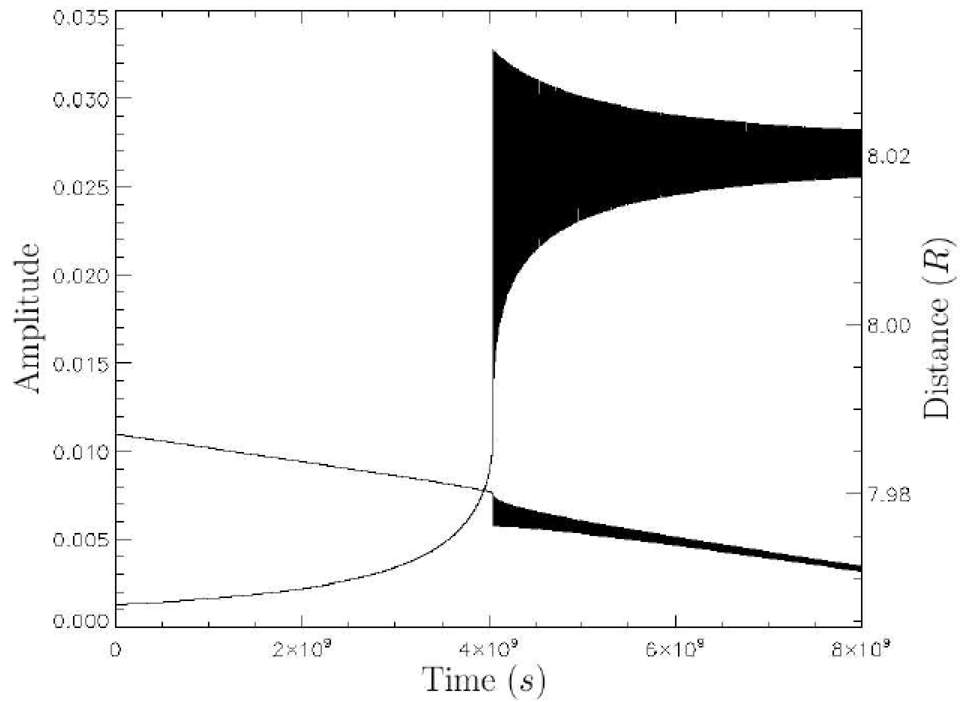
Calculation: mode amplitude evolution + Orbital evolution

Resonant Tidal Excitation of G-modes

As the orbit decays, resonance occurs when

$$\omega = 2(\Omega_{\rm orb} - \Omega_s) = \omega_\alpha$$

Calculation: mode amplitude evolution + Orbital evolution



Result: Surface displacement is ~ R

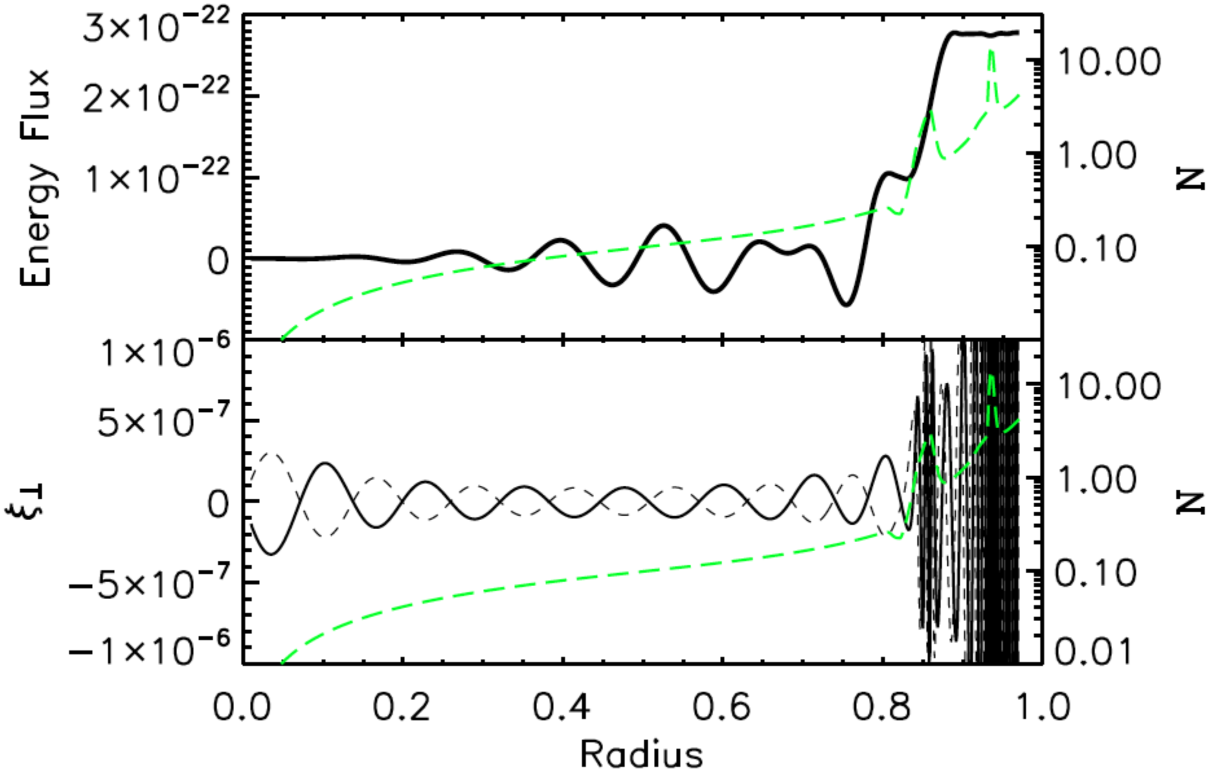
==> Dissipation==> No standing wave

“Continuous” Excitation of Gravity Waves

Waves are excited in the interior/envelope, propagate outwards and dissipate near surface

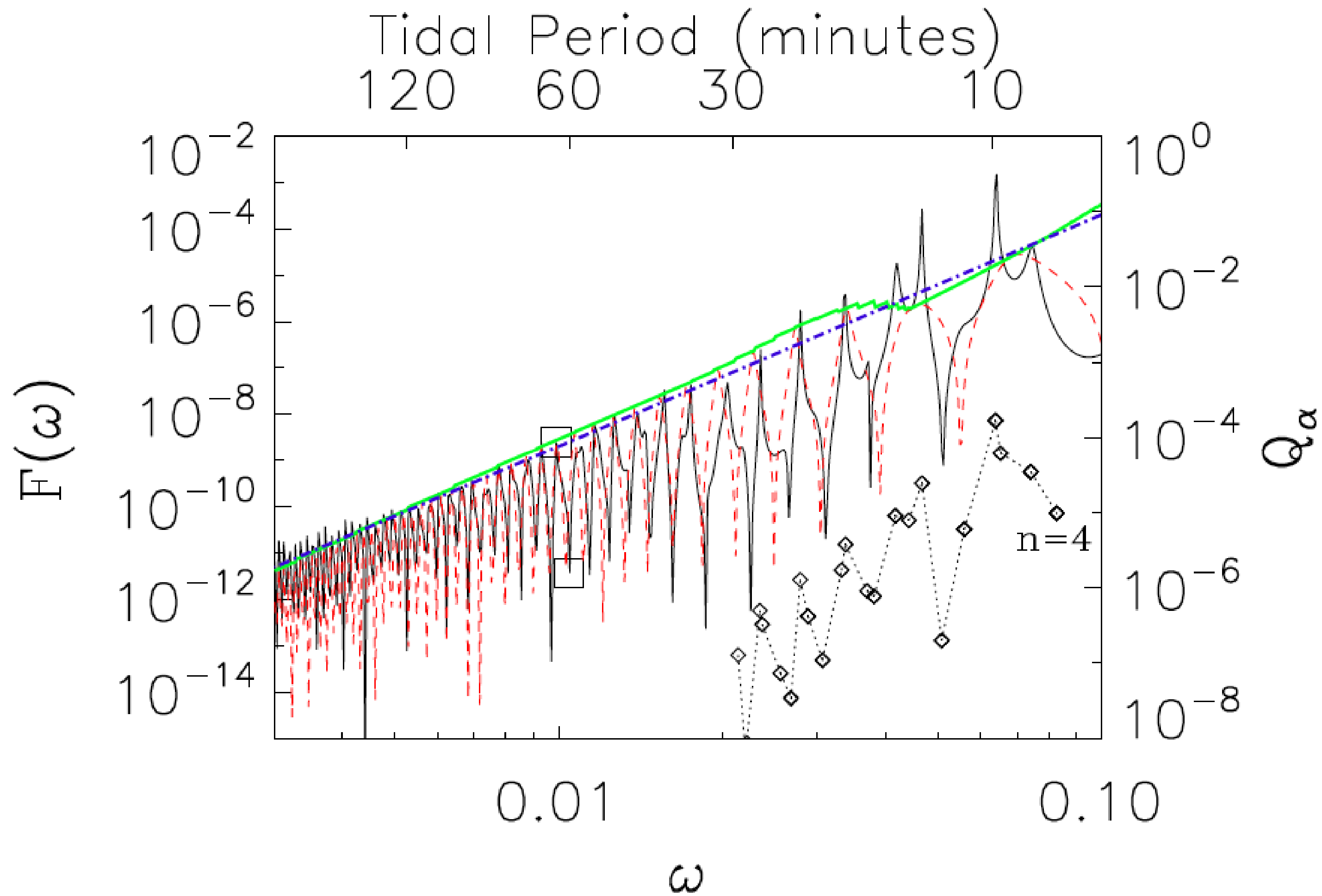
“Continuous” Excitation of Gravity Waves

Waves are excited in the interior/envelope, propagate outwards and dissipate near surface

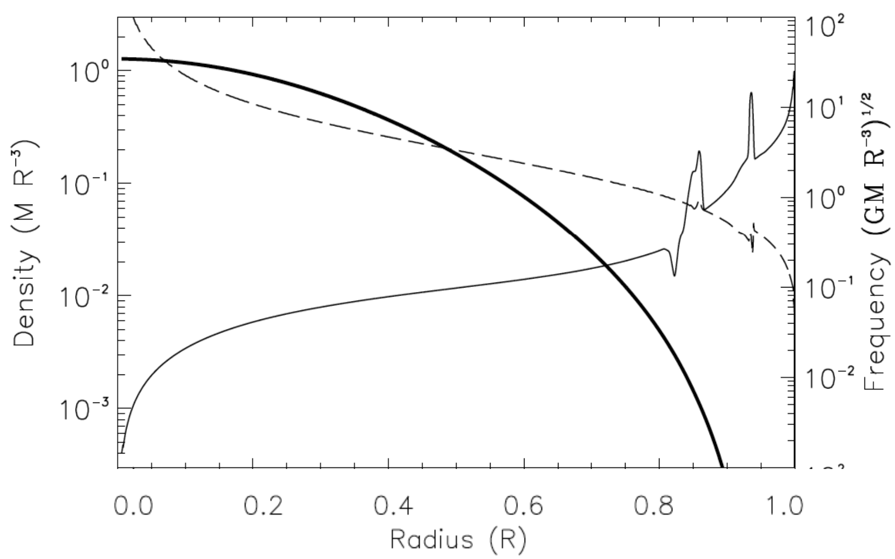


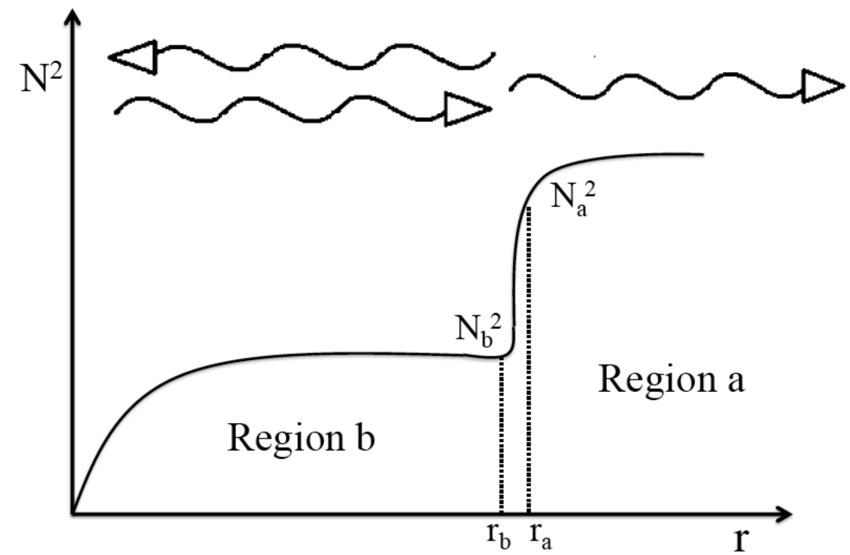
$M = 0.6 M_\odot, \quad \omega = 0.01$

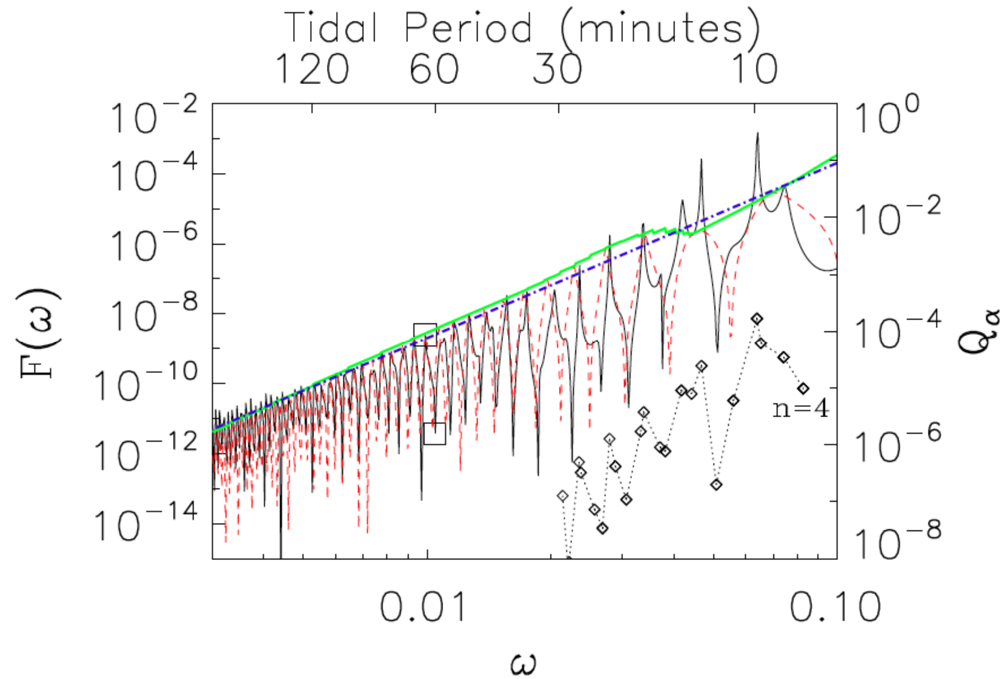
$$\text{Torque} = G\left(\frac{M'}{a^3}\right)^2 R^5\ F(\omega)$$



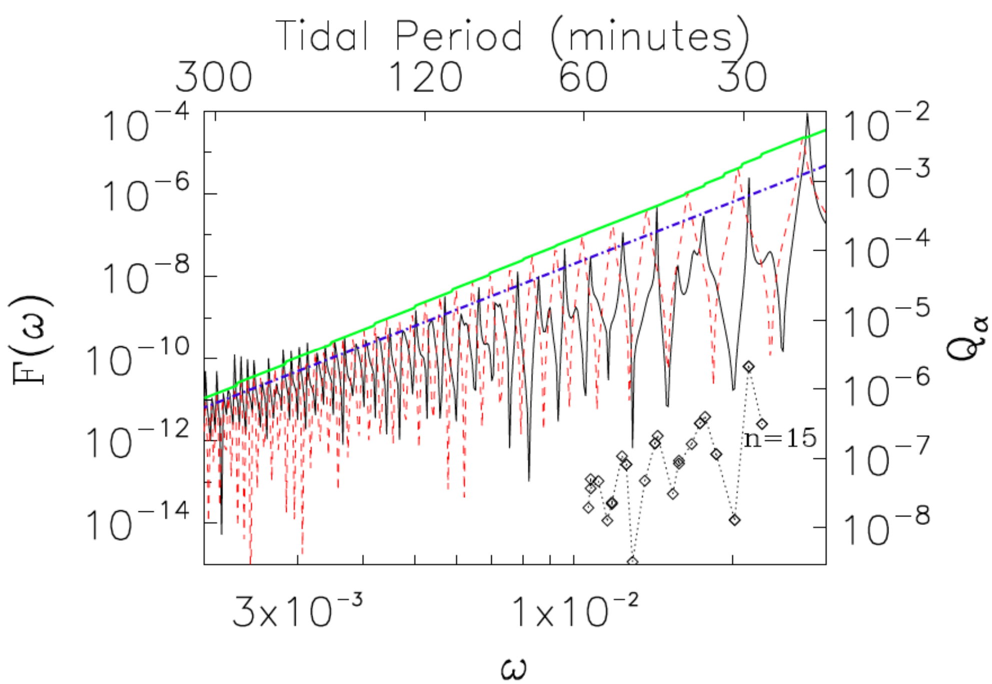
Why is $F(\omega)$ not smooth ?



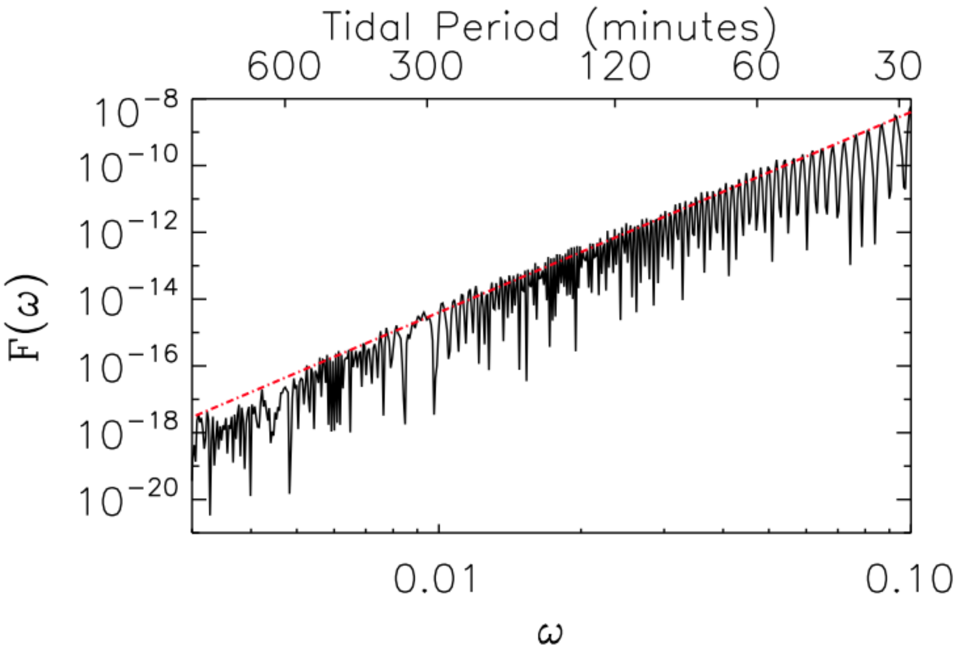




$M = 0.6 M_\odot,\ T = 8720$ K

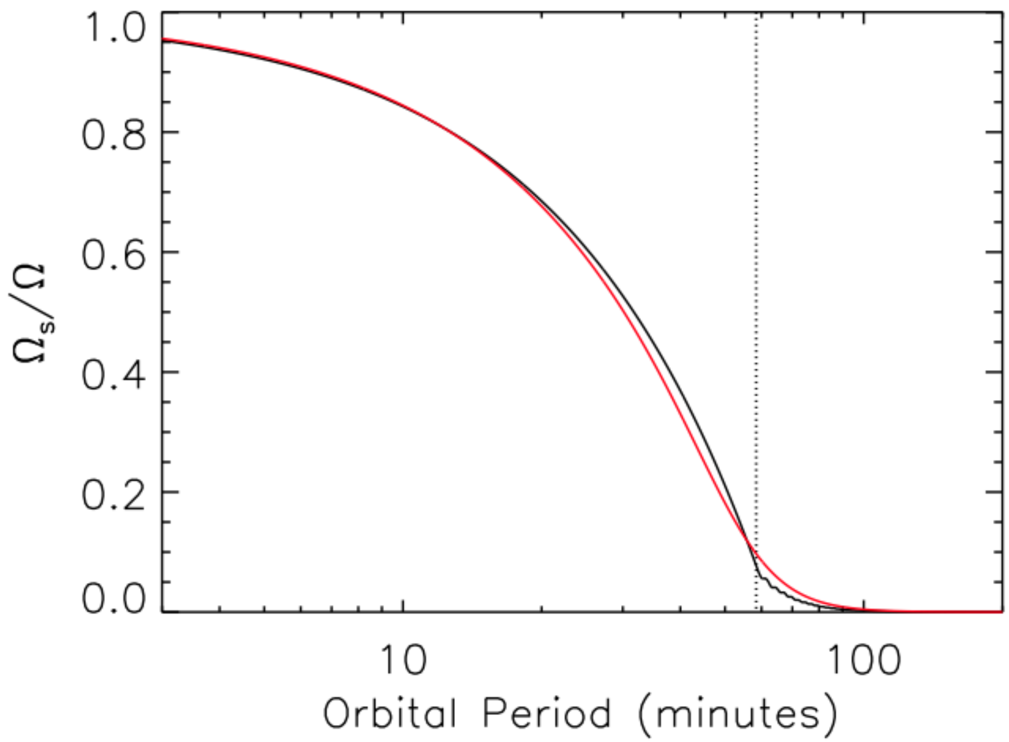


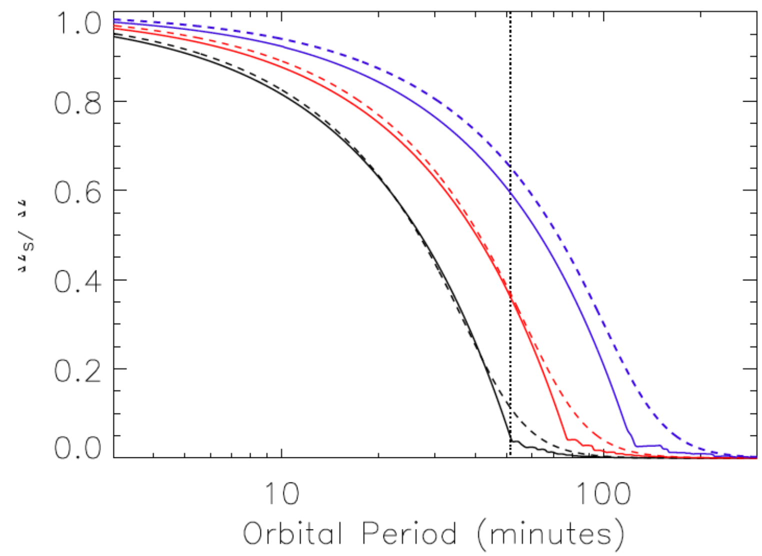
$M = 0.6 M_\odot,\ T = 5080$ K



$M = 0.3M_{\odot},\ T = 12000$ K

Spin-Orbit Synchronization



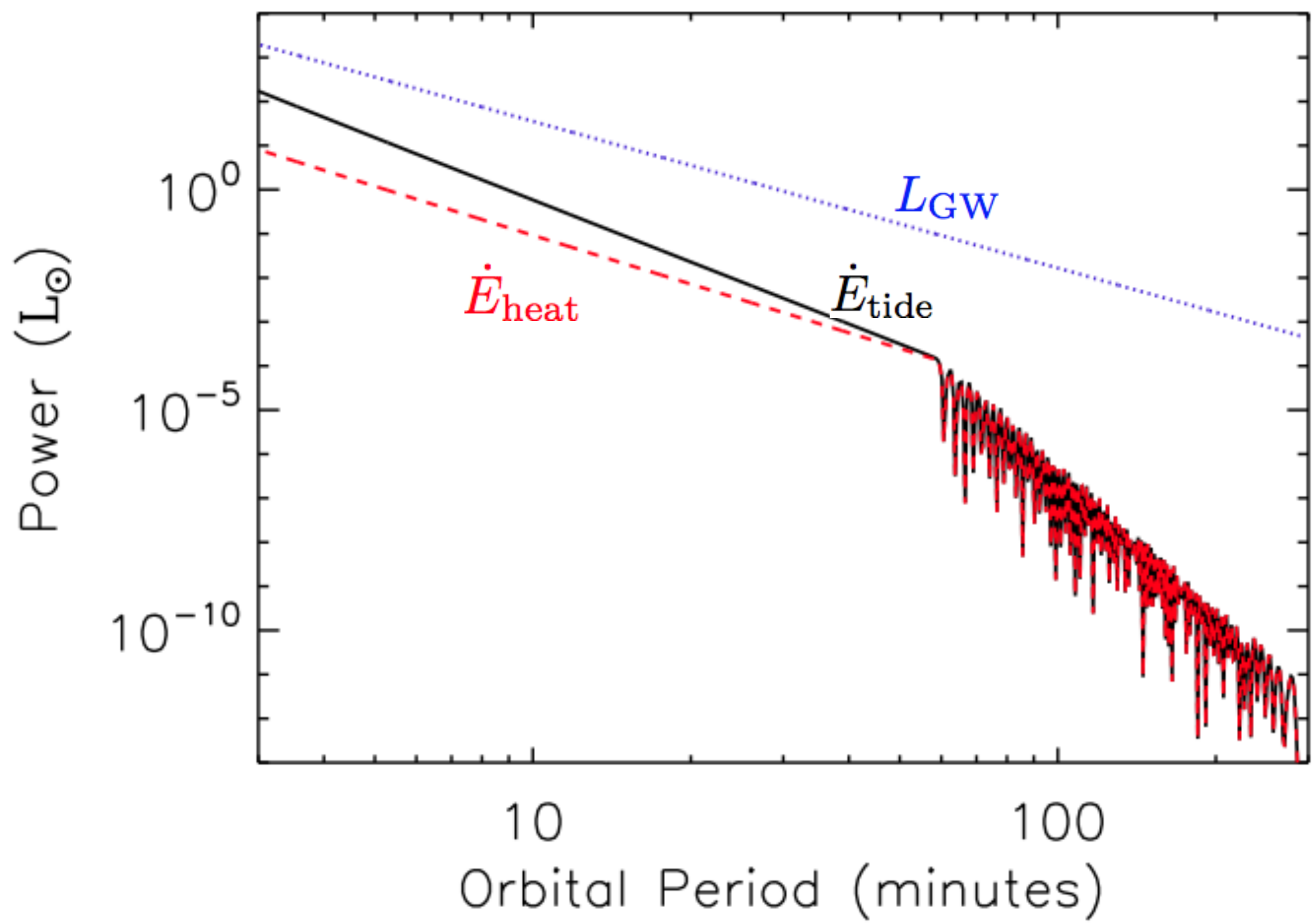


Critical orbital Ω_c : $\quad \dot{\Omega}_s = \dfrac{\text{Torque}}{I} \simeq \dot{\Omega}_{\rm orb} = \dfrac{3\Omega_{\rm orb}}{2t_{\rm GW}}$

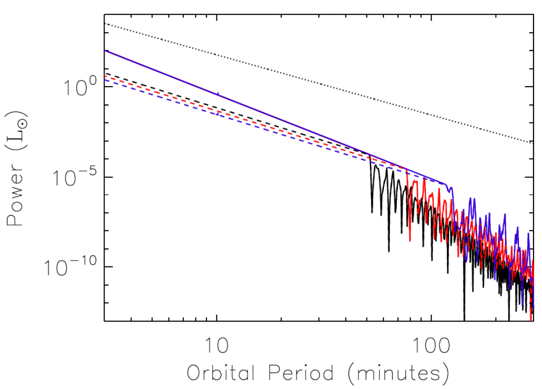
For $\Omega_{\rm orb} > \Omega_c$: $\quad \dot{\Omega}_s > \dot{\Omega}_{\rm orb}$

$$\dot{\Omega}_s - \dot{\Omega}_{\rm orb} \ll \dot{\Omega}_{\rm orb} \Longrightarrow \dot{E}_{\rm tide} = \Omega_{\rm orb} T \simeq \frac{3I\Omega_{\rm orb}^2}{2t_{\rm GW}}$$

Tidal Heating Rate



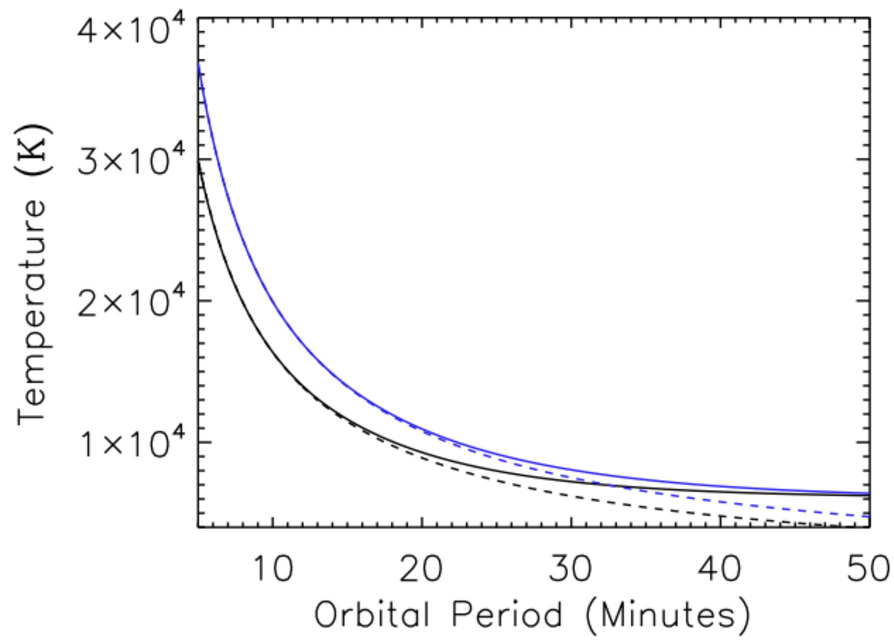
$$\dot{E}_{\rm heat} = \dot{E}_{\rm tide}\left(1 - \frac{\Omega_s}{\Omega_{\rm orb}}\right)$$



Consequences of Tidal Heating

Depend on where the heat is deposited …

If deposited in shallow layer:
- thermal time short
- ==> change T_{eff}



If deposited in deeper layer:
- thermal time longer than orbital
- ==> ? (nuclear flash?)

Stay tuned…

Summary: Tides in White Dwarf Binaries

-- Dynamical tides: Continuous excitation of gravity waves, outgoing…

-- Spin synchronized prior to merger (but not completely)

-- Tidal heating important…

Dynamical Tides in Eccentric Binaries: Tidally Excited Oscillations in KEPLER KOI-54

Jim Fuller & DL 2012 (arXiv:1107.4594)

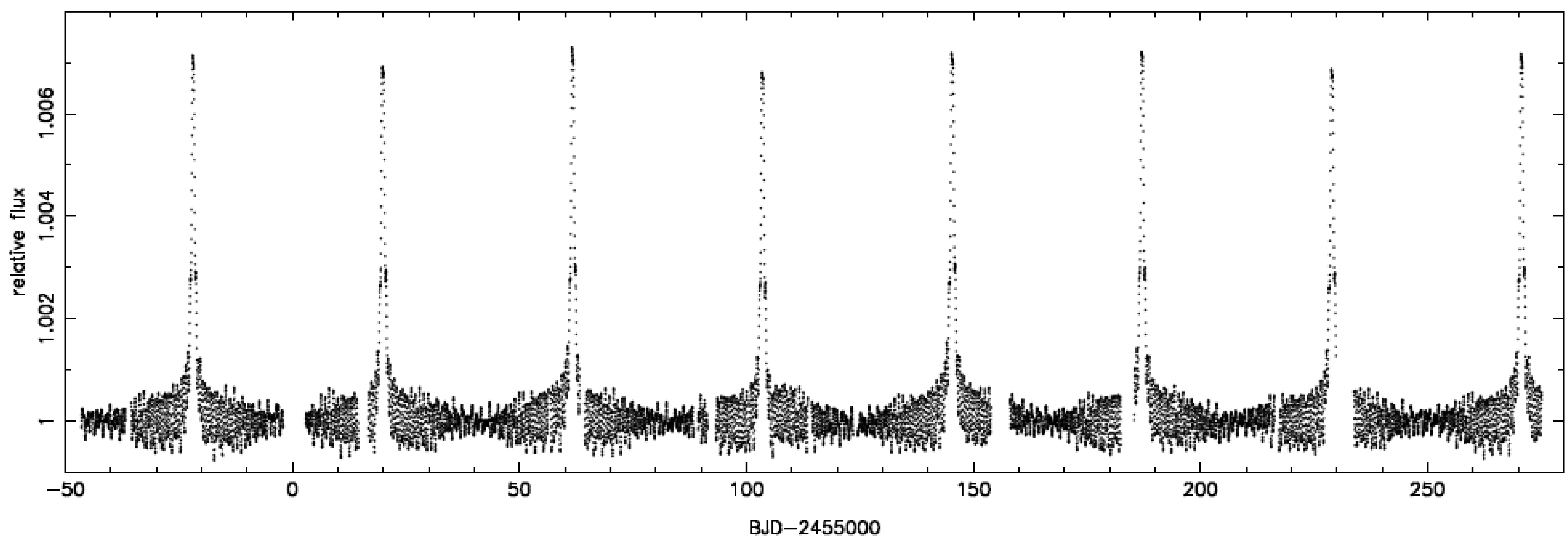
Other references:

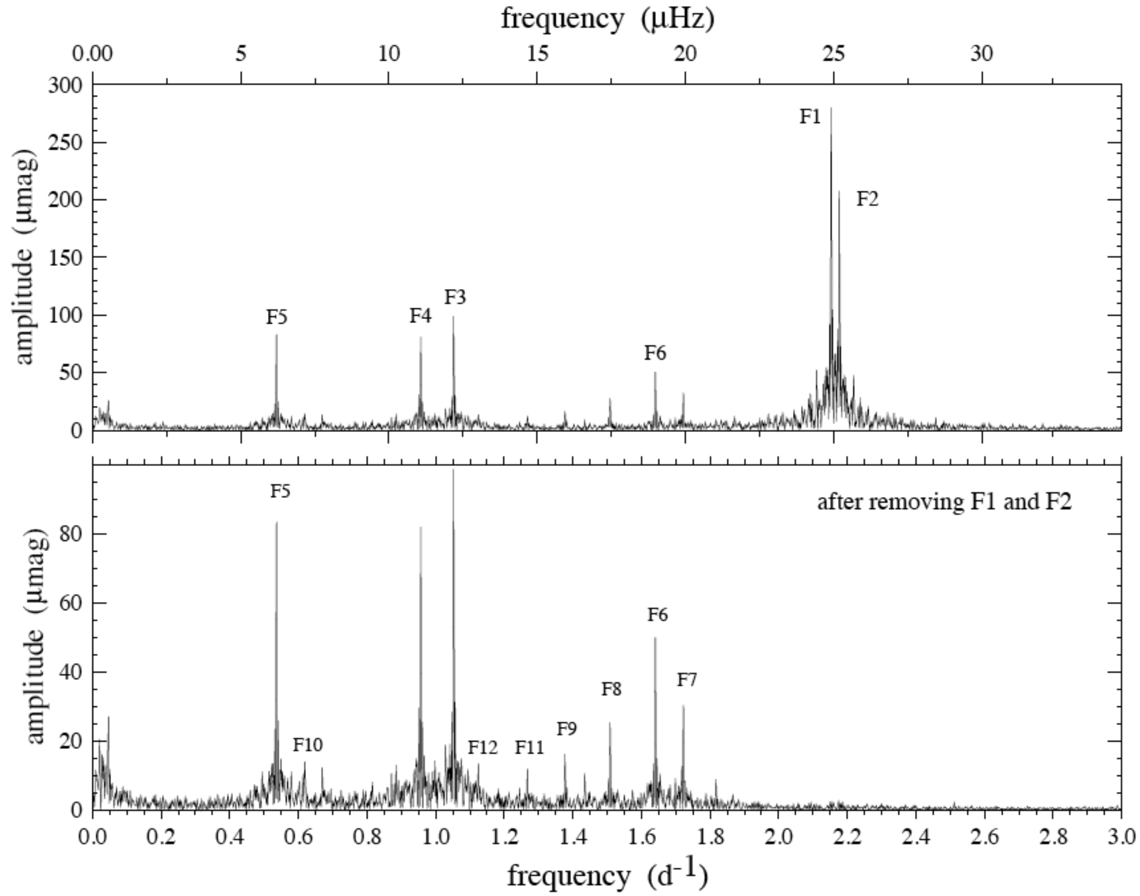
-- W. Welsh et al (Kepler team) 2011 (arXiv:1102.1730) (Discovery)

-- Burkart, Quataert, Arras & Weinberg, arXiv:1108.3832

KOI-54a,b Binary

A-type stars: 2.32, 2.38 M_{sun}
P=42 days, e=0.834, face-on (5.5 deg)
--> At periastron: $a_p = 6.5R, \quad f_p = 20 f_{\rm orb}$





30 pulsations (21 are integer $\times f_{\rm orb}$)

$22.42 f_{\rm orb} \rightarrow 91 f_{\rm orb}$

Welsh et al 2011

amplitude (μmag)	f/f_{orbit}
297.7	90.00
229.4	91.00
97.2	44.00
82.9	40.00
82.9	22.42
49.3	68.58
30.2	72.00
17.3	63.07
15.9	57.58
14.6	28.00
13.6	53.00
13.4	46.99
12.5	39.00
11.6	59.99
11.5	37.00
11.4	71.00
11.1	25.85
9.8	75.99
9.3	35.84
9.1	27.00
8.4	42.99
8.3	45.01
8.1	63.09
6.9	35.99
6.8	60.42
6.4	52.00
6.3	42.13
5.9	33.00
5.8	29.00
5.7	48.00

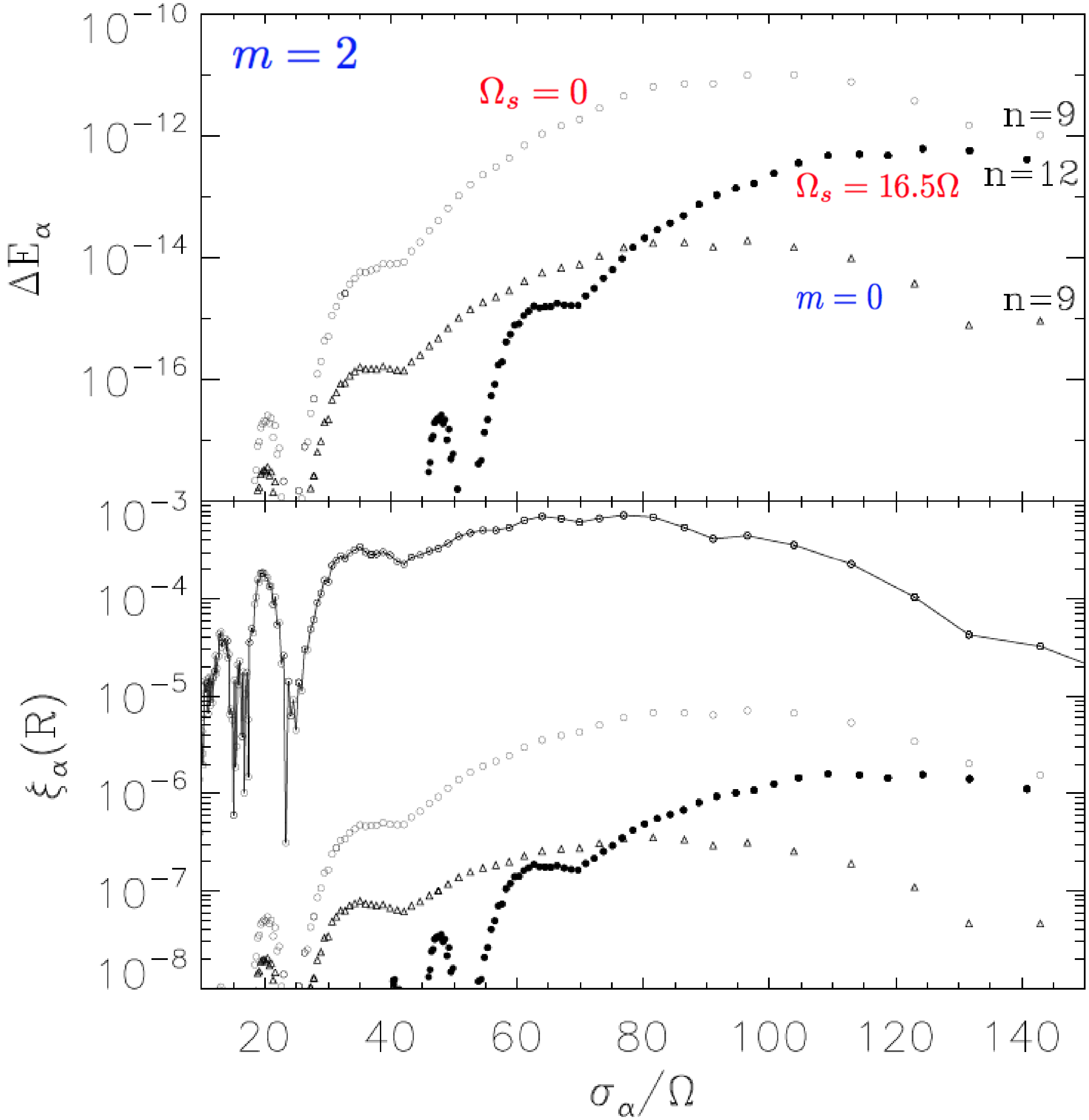
Tidally Forced Oscillations

Decompose tidal potential into orbital harmonics $N\Omega$ ==>

$$\boldsymbol{\xi}(\mathbf{r}_i,t) = \sum_{N=-\infty}^{\infty}\sum_{\alpha}\frac{GM'W_{lm}Q_\alpha}{2\varepsilon_\alpha a^{l+1}}\frac{F_{Nm}\boldsymbol{\xi}_\alpha(\mathbf{r}_i)}{(\sigma_\alpha - N\Omega) - i\gamma_\alpha}e^{-iN\Omega t}$$

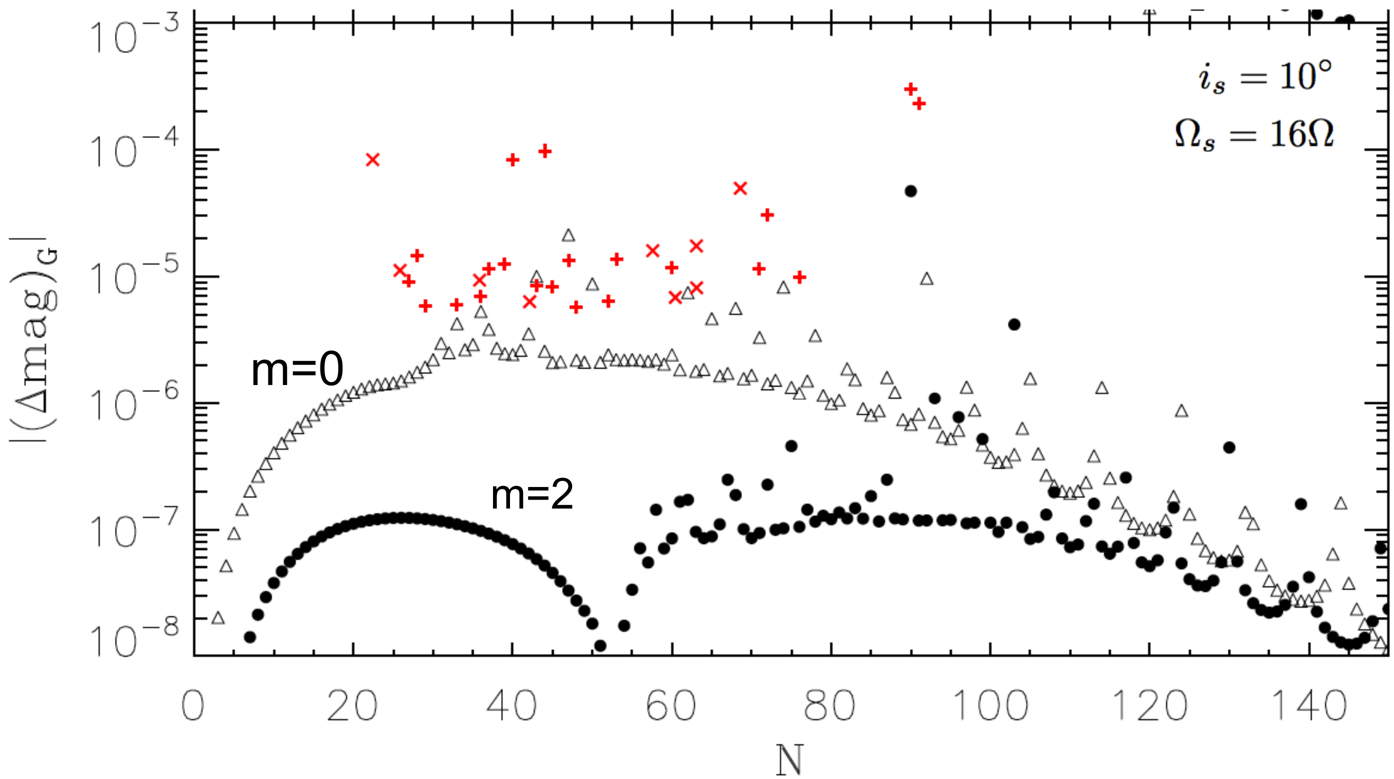
$$= \sum_{N=1}^{\infty}\sum_{\alpha'}\frac{GM'W_{lm}Q_\alpha}{2\varepsilon_\alpha a^{l+1}}\boldsymbol{\xi}_\alpha(\mathbf{r}_i)$$

$$\times\left[\frac{F_{Nm}e^{-iN\Omega t}}{(\sigma_\alpha - N\Omega) - i\gamma_\alpha} + \frac{F_{-Nm}e^{iN\Omega t}}{(\sigma_\alpha + N\Omega) - i\gamma_\alpha}\right] + c.c. \quad (23)$$



$m = 2$
$\Omega_s = 0$
n=9
n=12
$\Omega_s = 16.5\Omega$
$m = 0$
n=9
ΔE_α
$\xi_\alpha(R)$
σ_α/Ω

Flux Variations



Most of the observed flux variations are explained by m=0 modes (more visible for near face-on orientation)

Exception: 90,91 harmonics, which require very close resonances $(N\Omega = \omega_\alpha)$

Why N=90,91 ?

The probability of seeing high-amplitude modes

Consider mode near resonance $\omega_\alpha = (N + \epsilon)\Omega$

- By chance

$$P_{|\epsilon|<\epsilon_0} \simeq 2\epsilon_0$$

likely for N=20-80 ($\epsilon_0 \sim 0.1$)

- If mode dominates tidal energy transfer

$$P_{|\epsilon|<\epsilon_0} = \frac{\Delta t_{\rm res}}{\Delta t_{\rm nonres}} \sim \frac{8\pi^2}{3}\epsilon_0^3$$

unlikely for N=90,91 $(\text{require } \epsilon_0 < 0.01)$

Resonance Locking

- Tidal excitation of modes ==> Orbitdal decay, spinup of star, change mode frequency

$$\omega_\alpha = \omega_\alpha^{(0)} + mB_\alpha\Omega_s$$

- At resonance, $\dfrac{\omega_\alpha}{\Omega} = N$

- Mode can stay in resonance if $\dfrac{d}{dt}\left(\dfrac{\omega_\alpha}{\Omega}\right) = 0$ or $\left(\dfrac{\dot\omega_\alpha}{\omega_\alpha}\right)_{\rm tide} = \left(\dfrac{\dot\Omega}{\Omega}\right)_{\rm tide}$

==> $N_c = m\left(\dfrac{B_\alpha \mu a^2}{3I}\right)^{1/2} \simeq 130 - 145$

$$\left(\frac{\dot\Omega}{\Omega}\right)_{\rm tide} = \left(\frac{N}{N_c}\right)^2 \left(\frac{\dot\omega_\alpha}{\omega_\alpha}\right)_{\rm tide}$$

Resonance Locking (continued)

Including intrinsic stellar spin-down torque:

$$\dot{\Omega}_s = (\dot{\Omega}_s)_{\rm tide} + (\dot{\Omega}_s)_{\rm sd}$$

===>

$$\frac{\dot{\omega}_\alpha}{\omega_\alpha} = \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide} + \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm sd}$$

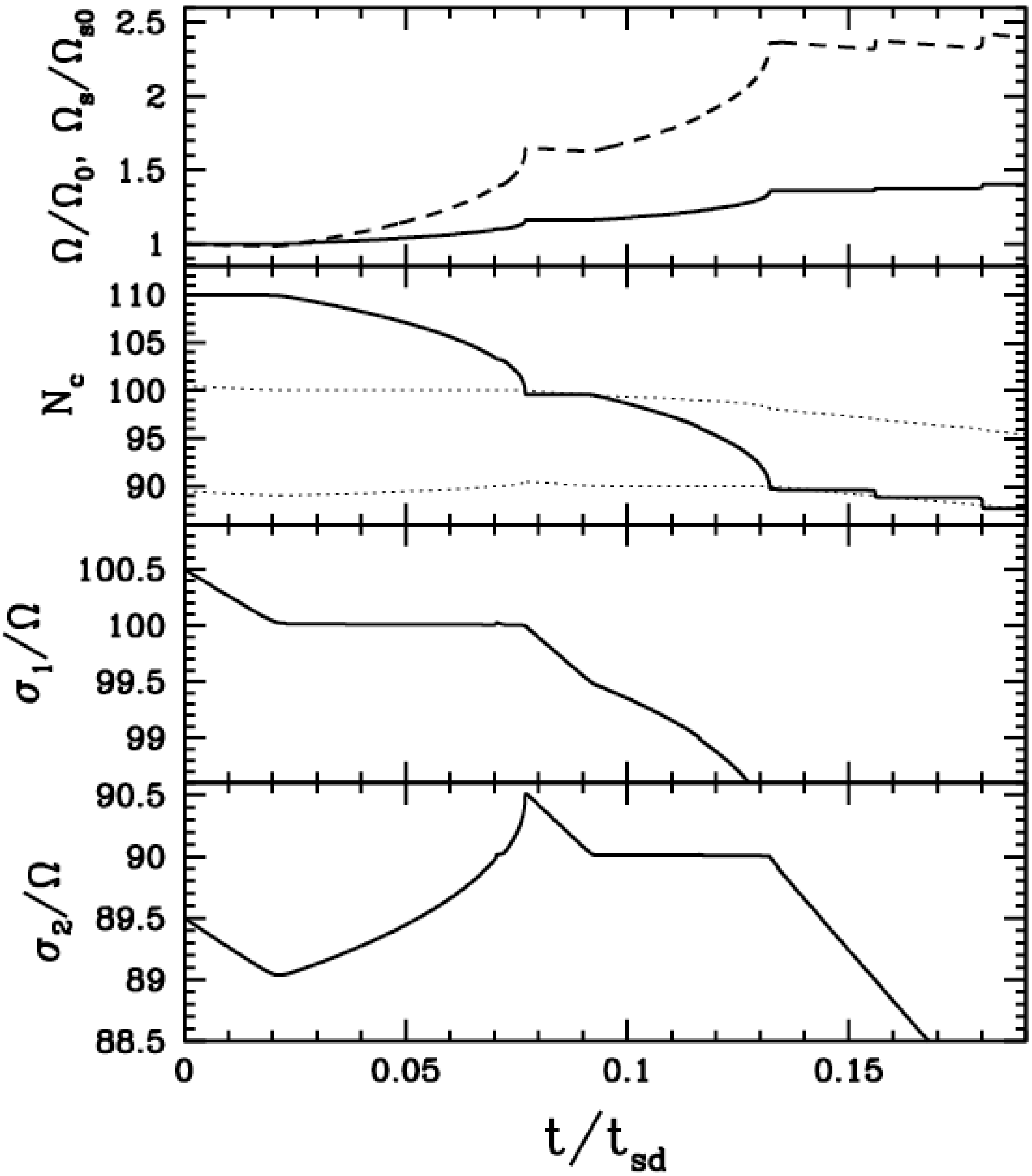
$$\frac{\dot{\Omega}}{\Omega} = \left(\frac{\dot{\Omega}}{\Omega}\right)_{\rm tide} = \left(\frac{N}{N_c}\right)^2 \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide}$$

===> **Mode can lock into resonance if** $N < N_c$

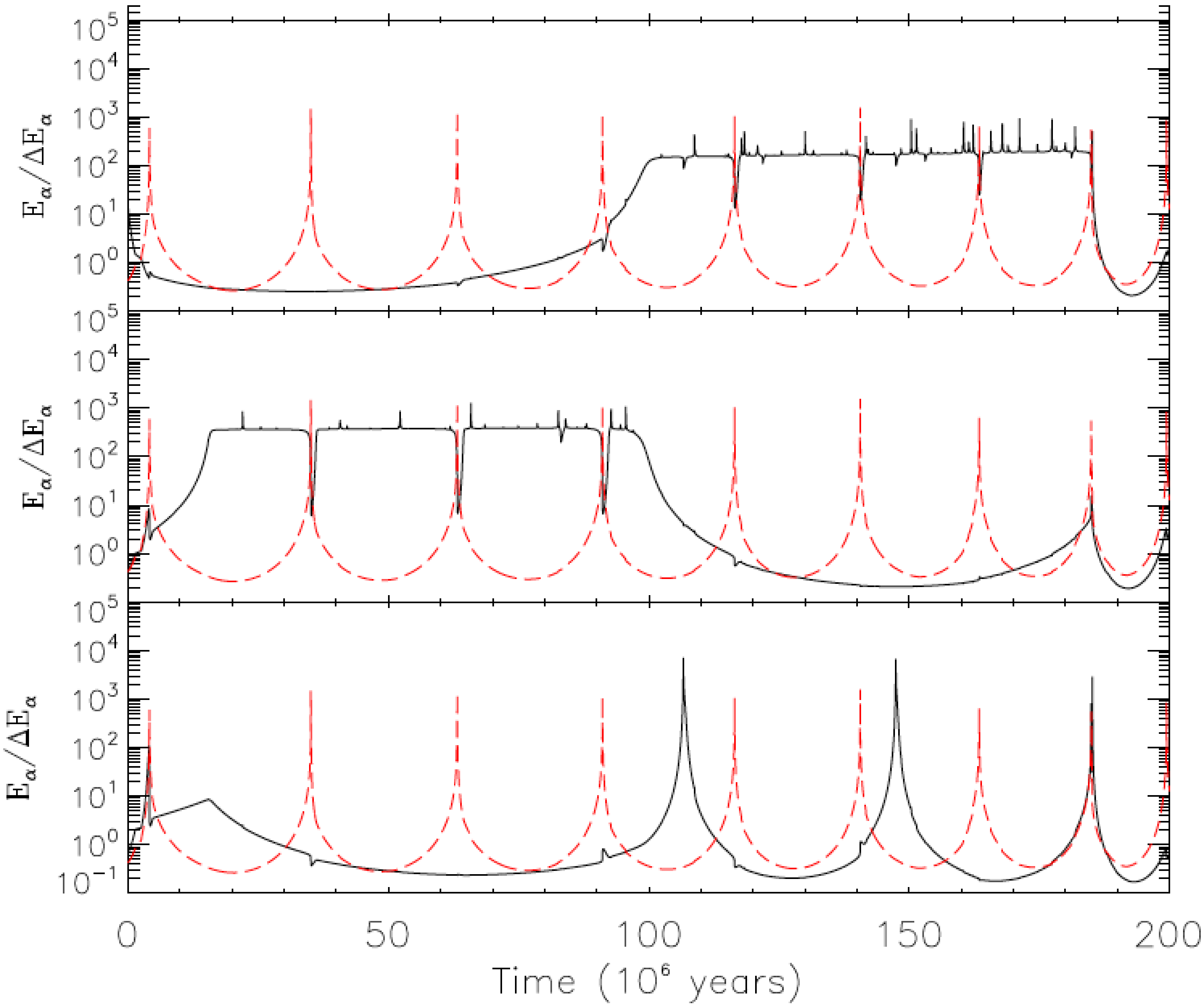
$$\frac{\omega_\alpha}{\Omega} < N_c$$

Resonance Locking: Numerical Examples

Coupled evolution of orbit, spin and mode amplitudes…



Ω/Ω_0, Ω_s/Ω_{s0}
2.5
2
1.5
1
N_c
110
105
100
95
90
σ_1/Ω
100.5
100
99.5
99
σ_2/Ω
90.5
90
89.5
89
88.5
0
0.05
0.1
0.15
t/t_{sd}



$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
50
100
150
200
Time (10^6 years)

Resonance Locking in Both Stars

- Locking in one star:

$$N_c = m\left(\frac{B_\alpha \mu a^2}{3I}\right)^{1/2} \simeq 130 - 145$$

- Similar modes are locked simultaneously in both stars

$$N_c = 92 - 102$$

- Explain the observed N=90,91 harmonics

Non-Linear Mode Coupling

- 9 oscillations detected at non-integer multiples of orbital frequencies

- Could be produced by nonlinear coupling to daughter modes

$$\omega_p = \omega_{d1} + \omega_{d2}$$

- In KOI-54,

$$\frac{\omega_2}{\Omega} = 91.00 \qquad \frac{\omega_5}{\Omega} = 22.42 \qquad \frac{\omega_6}{\Omega} = 68.58$$

- Other non-integer modes likely due to nonlinear coupling in which one of the daughter modes is invisible

amplitude (μmag)	f/f_{orbit}
297.7	90.00
229.4	91.00
97.2	44.00
82.9	40.00
82.9	22.42
49.3	68.58
30.2	72.00
17.3	63.07
15.9	57.58
14.6	28.00
13.6	53.00
13.4	46.99
12.5	39.00
11.6	59.99
11.5	37.00
11.4	71.00
11.1	25.85
9.8	75.99
9.3	35.84
9.1	27.00
8.4	42.99
8.3	45.01
8.1	63.09
6.9	35.99
6.8	60.42
6.4	52.00
6.3	42.13
5.9	33.00
5.8	29.00
5.7	48.00

Summary: Lessons from KOI-54

- Direct detection of tidally excited oscillations in eccentric binary
  ==> Dynamical tides at work

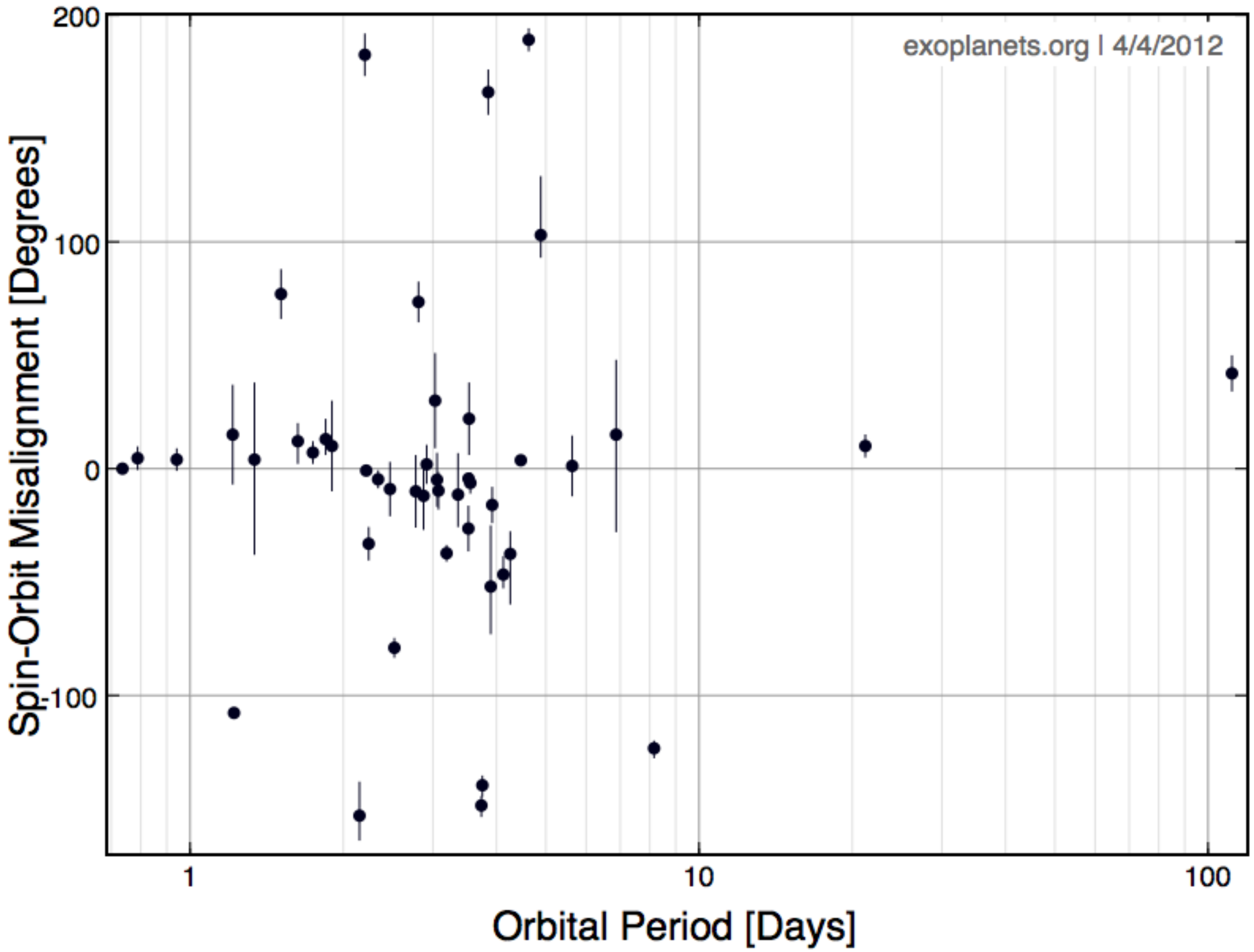
- Resonance locking

- First direct evidence of nonlinear mode coupling

- More such systems …

Tides in Planet-Hosting Stars:
Spin-Orbit Misalignment and Survival of Hot Jupiters

DL 2012 (arXiv:1109.4703)



4/2012: 43 systems measured, 22 misaligned

S^*-L_p misalignment in Exoplanetary Systems
==> The Importance of few-body interactions

1. Kozai + Tide migration by a distant companion star/planet

   (e.g., Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al. 11; Katz et al. 11)

2a. Planet-planet scatterings

   (e.g., Chatterjee et al. 08; Juric & Tremaine 08; Nagasawa et al 08)

2b. Planet-Planet Secular Chaos (“Internal Kozai”) + Tide

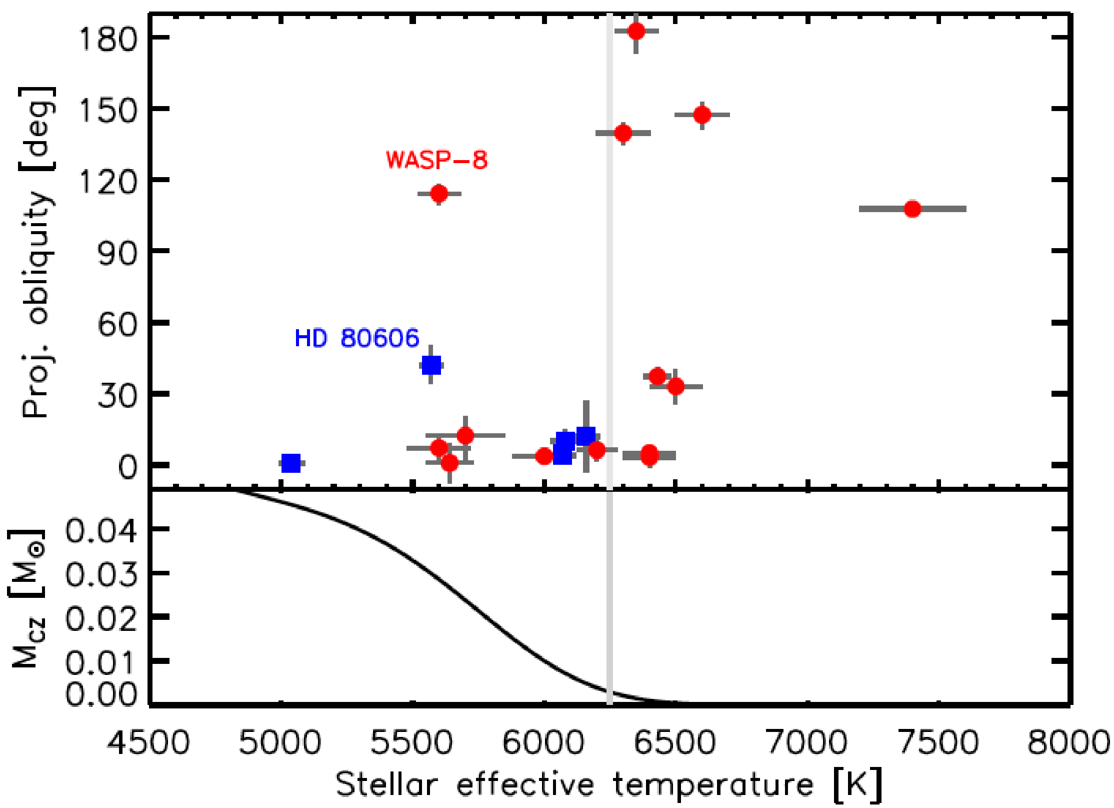
   (Wu & Lithwick 11; see Nagasawa et al. 08)

Misaligned protostar - protoplanetary disk ? (e.g. solar system)

(DL, Foucart & Lin ‘11; Bate et al. 10)

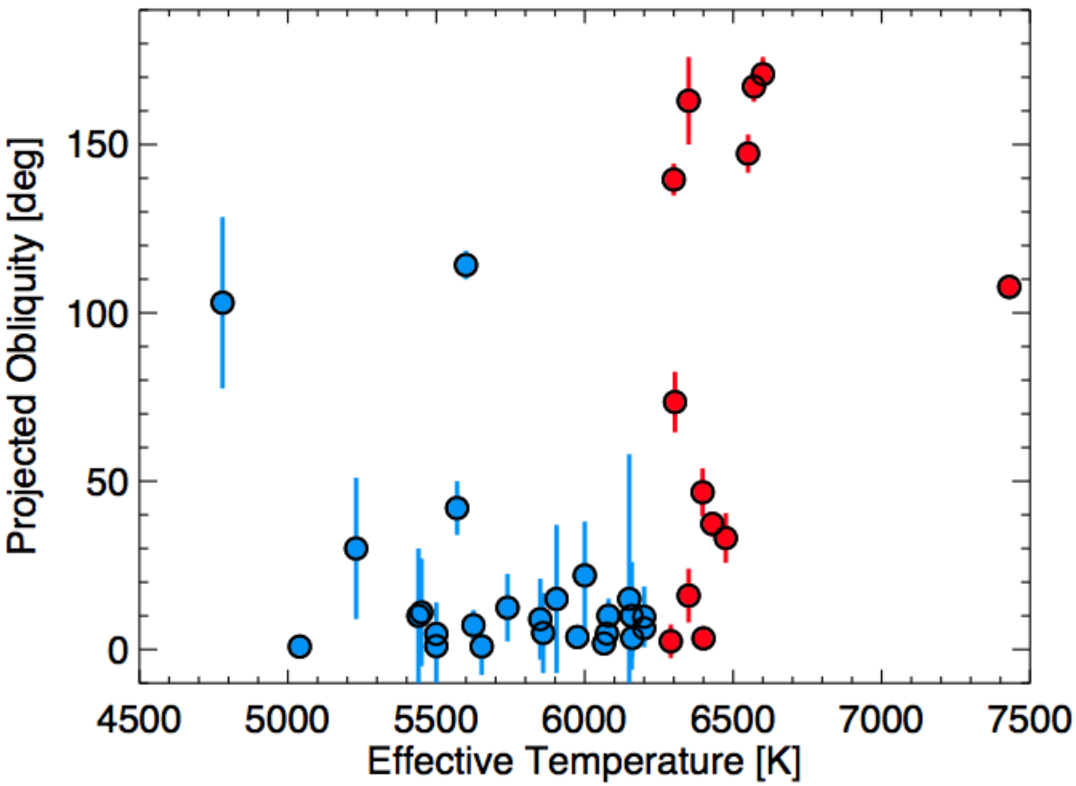
Correlation: Misalignment -- Stellar Temperature/Mass

Winn et al. 2010; Schlaufman 2010

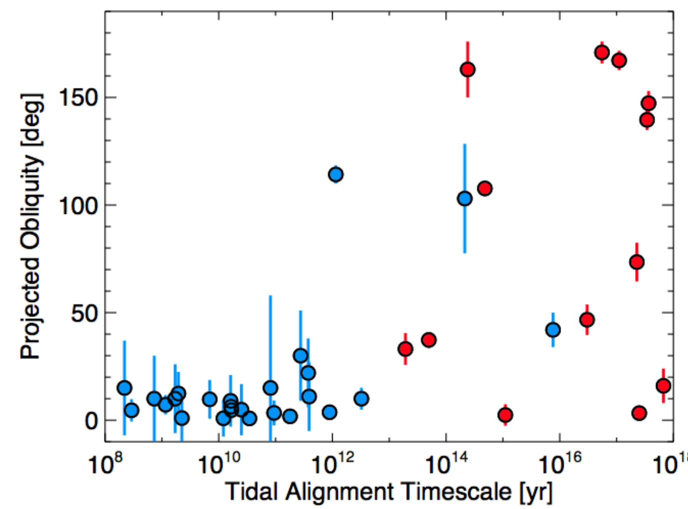


Correlation: Misalignment -- Stellar Temperature/Mass

Winn et al. 2010; Schlaufman 2010



From Josh Winn (MIT), 2011.9



Correlation: Misalignment -- Stellar Age

Triaud 2011

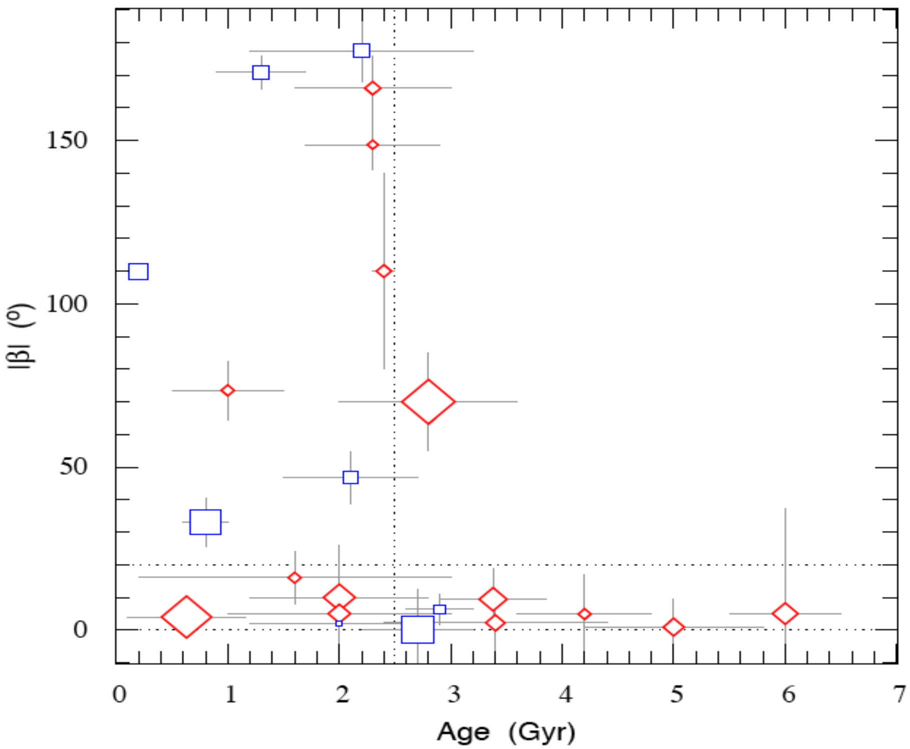


Fig. 2. Secure, absolute values of β against stellar age (in Gyr), for stars with $M_\star \geq 1.2\,M_\odot$. Size of the symbols scales with planet mass. In blue squares, stars with $M_\star \geq 1.3\,M_\odot$; in red diamonds $1.3 > M_\star \geq 1.2\,M_\odot$. Horizontal dotted line show where aligned systems are. Vertical dotted line shows the age at which where misaligned planets start to disappear.

Problem with Equilibrium Tide (with the parameterization…)

$$t_{\mathrm{decay}} \simeq 1.3 \left(\frac{Q'_\star}{10^7}\right) \left(\frac{M_\star}{10^3 M_p}\right) \left(\frac{P_{\mathrm{orb}}}{1\,\mathrm{d}}\right)^{13/3} \mathrm{Gyr}$$

$$\frac{t_{\mathrm{align}}}{t_{\mathrm{decay}}} \simeq \frac{2S_\star}{L} \simeq 2 \left(\frac{M_\star}{10^3 M_p}\right) \left(\frac{10\,\mathrm{d}}{P_s}\right) \left(\frac{1\,\mathrm{d}}{P_{\mathrm{orb}}}\right)^{1/3}$$

Possible Solution:

Different Tidal Q's for Orbital Decay and Alignment ?

Tidal Forcing Frequency=?

For aligned system

$$\omega = 2(\Omega_{\rm orb} - \Omega_s)$$

For misaligned system

$$\omega = m'\Omega_{\rm orb} - m\Omega_s \qquad m, m' = 0, \pm 1, \pm 2$$

7 physically distinct components

==> Effective tidal evolution equations with 7 different Q's

Tidal Dissipation in Rotating Stars/Planets

Importance of inertial waves…

(Ogilvie & Lin ‘04,’07; Goodman & Lackner ‘09, etc)

Inertial Waves in Rotating Fluid

Dispersion relation (in rotating frame)

$$\omega = \pm 2\,\boldsymbol{\Omega}_s \cdot \hat{\mathbf{k}}$$

Can only be excited if tidal forcing frequency satisfies

$$|\omega| < 2\Omega_s$$

Stellar Tides in Hot Jupiter Systems

For aligned system:

$$\omega = 2(\Omega_{\rm orb} - \Omega_s) \gg \Omega_s$$

==> Cannot excite inertial waves

For misaligned system:

$$\omega = m'\Omega_{\rm orb} - m\Omega_s$$

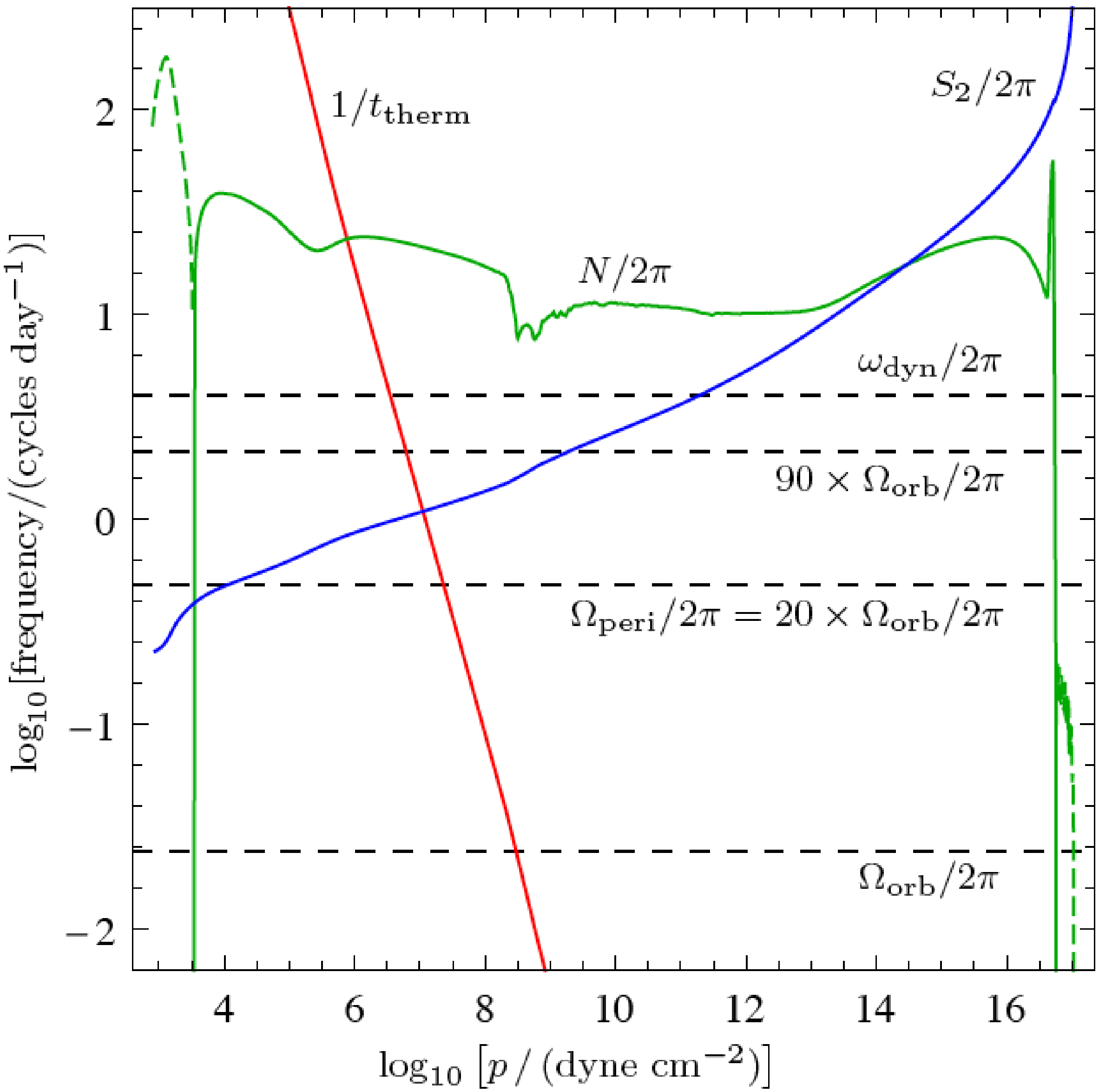
The m'=0, m=1 component has $\omega = -\Omega_s$

This component leads to alignment, but not orbital decay

Summary: Tidal Damping of Misalignment in Hot Jupiter Systems

-- Spin-orbit misalignment may be damped without orbital decay

-- Different Q's for different processes (Equilibrium tide parameterization misleading)


$1/t_{\rm therm}$
$S_2/2\pi$
$N/2\pi$
$\omega_{\rm dyn}/2\pi$
$90 \times \Omega_{\rm orb}/2\pi$
$\Omega_{\rm peri}/2\pi = 20 \times \Omega_{\rm orb}/2\pi$
$\Omega_{\rm orb}/2\pi$
$\log_{10}[\text{frequency}/(\text{cycles day}^{-1})]$
$\log_{10}[p/(\text{dyne cm}^{-2})]$
2
1
0
-1
-2
4
6
8
10
12
14
16