

# Dynamical Formation of Merging Black-Hole Binaries

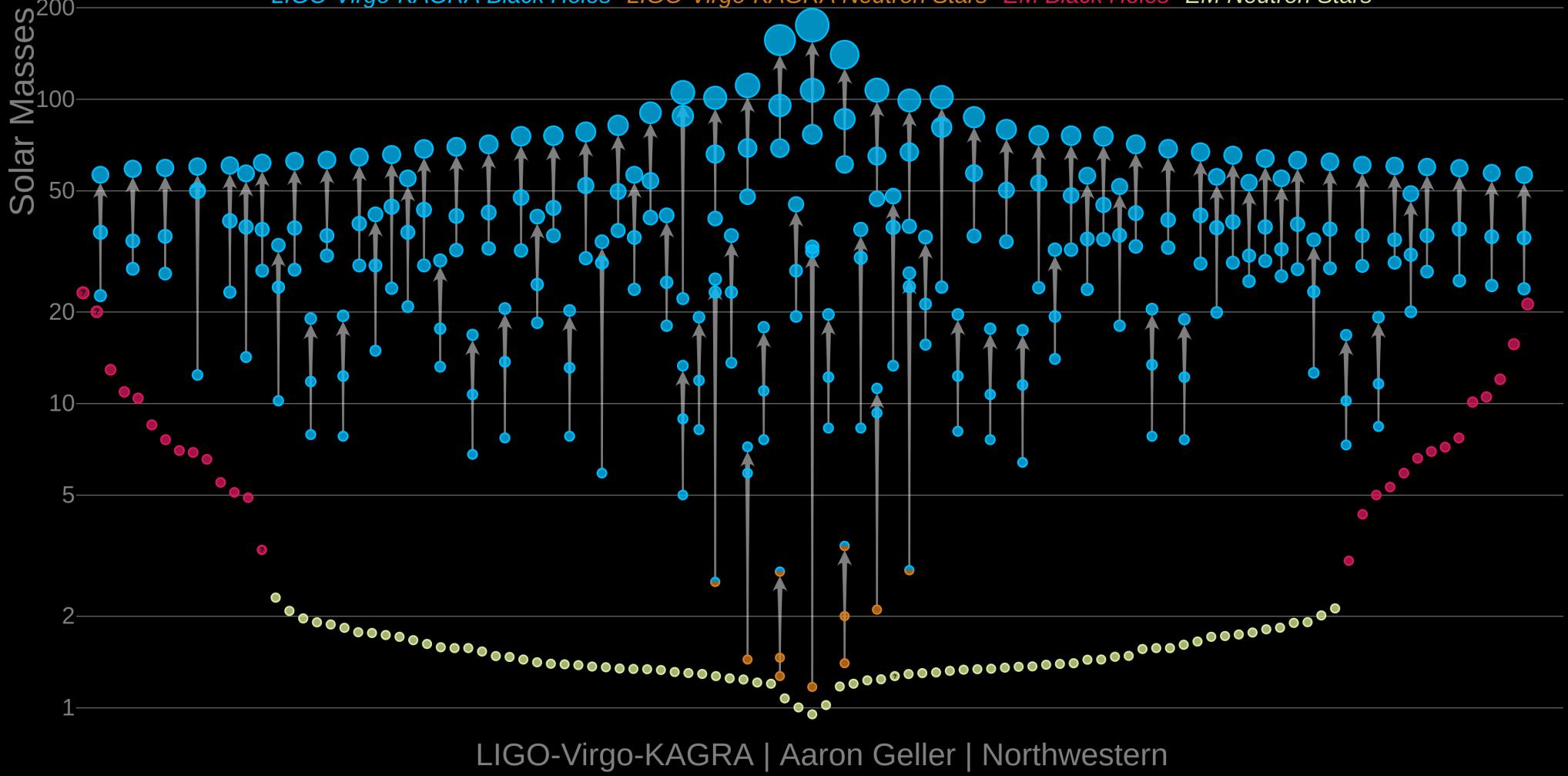
(a planetary dynamics talk disguised as a BH talk)

Dong Lai

Brouwer Award Lecture, 56<sup>th</sup> AAS-DDA annual meeting, Atlanta, 5/20/2025

# Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



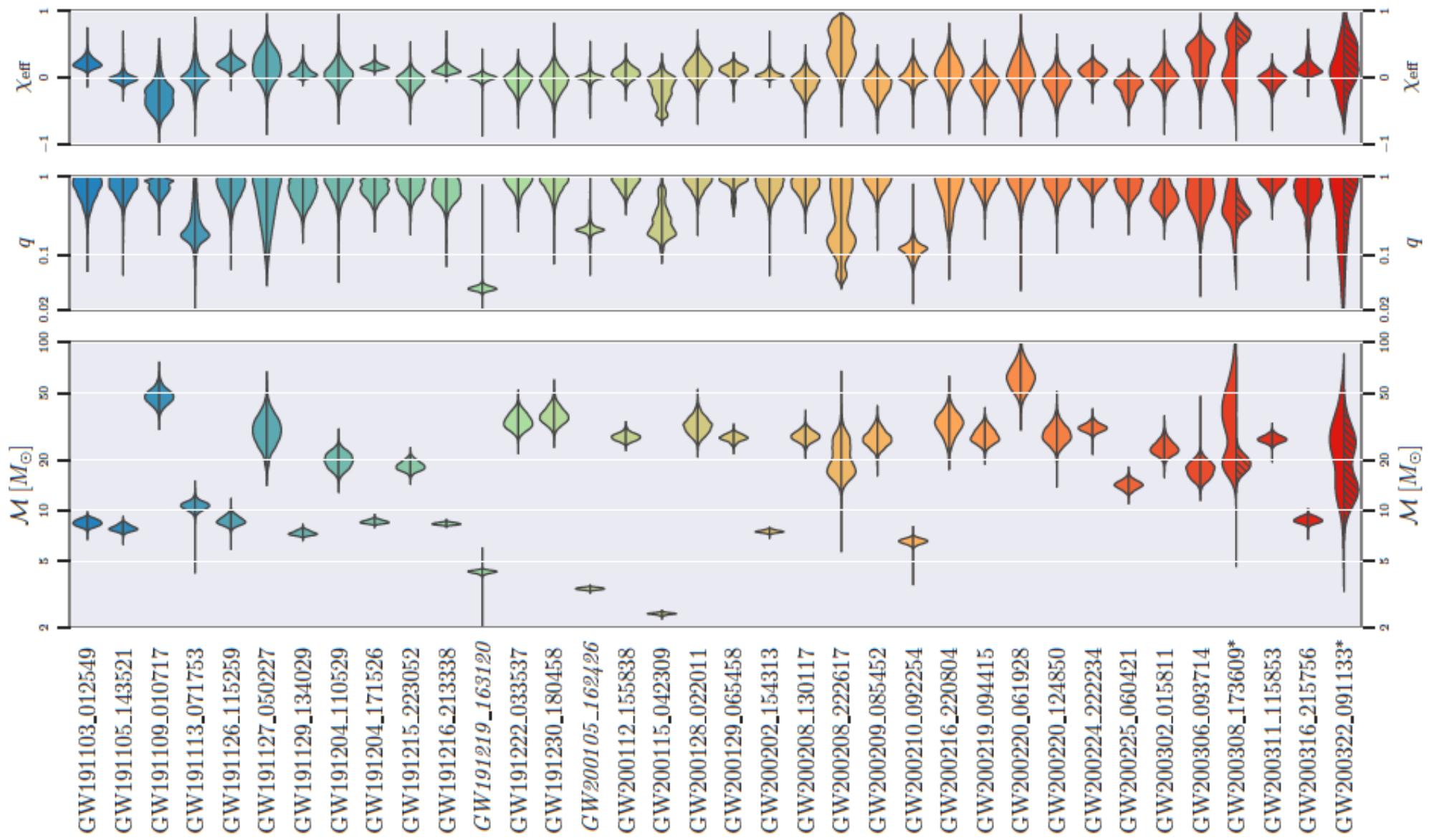
GWTC-3: 90 merger events, with 2 NS/NS mergers, 3 NS/BH mergers

## Observation Run 4 (O4)

S233750w*	10:03:07-03	1553	2545 ± 404	HES	$4.5 \cdot 10^{-10}$	0.85	0.0	0.009	0.035	0.007
S233750w*	12:03:07-03	2714	35760 ± 1155	HES	$3.4 \cdot 10^{-10}$	0.874	0.0	0.0	0.003	0.0
S233750w*	20:03:07-03	1227	3314 ± 805	HES	$4.3 \cdot 10^{-10}$	0.867	0.0	0.0	0.003	0.0041
S233750w*	20:03:07-03	7016	4647 ± 1056	HES	$7.0 \cdot 10^{-10}$	0.862	0.0	0.0	0.004	0.0041
S233750w*	20:03:07-03	3373	4647 ± 1056	HES	$7.0 \cdot 10^{-10}$	0.862	0.0	0.0	0.004	0.0041
S233750w*	20:03:07-03	2525	2556 ± 500	HES	$5.6 \cdot 10^{-10}$	0.845	0.0	0.0	0.009	0.001
S233750w*	20:03:07-03	2488	7	HES	$9.2 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233750w*	20:03:07-03	3664	4574 ± 1385	HES	$5.1 \cdot 10^{-10}$	-0.153	0.0	0.0	0.007	0.0078
S233751w*	03:11:01-01	4173	5242 ± 8219	HES	$4.9 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.004	0.004
S233751w*	03:11:01-01	4554	7	HES	$8.6 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233752w*	03:11:01-01	4239	7	HES	$4.2 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233753w*	03:11:01-01	1094	7	HES	$9.3 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233754w*	03:11:01-01	3038	7	HES	$9.3 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233755w*	03:11:01-01	2551	5258 ± 283	HES	$5.0 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0004	0.008	0.016
S233755w*	03:11:01-01	1117	1559 ± 435	HES	$5.5 \cdot 10^{-10}$	0.317	0.0	0.0	0.007	0.0
S233755w*	03:11:01-01	3415	4163 ± 205	HES	$5.0 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.00057	0.044	0.048
S233756w*	03:11:01-01	20240	27274	L	$5.0 \cdot 10^{-10}$	+0.0	0.0	+1.0	0.0	1.4e-2
S233756w*	03:11:01-01	4893	2719 ± 1445	HES	$5.0 \cdot 10^{-10}$	+0.0	0.0	+0.0	0.0	0.006
S233756w*	03:11:01-01	1963	1495 ± 448	HES	$3.4 \cdot 10^{-10}$	+0.0	0.0	+0.002	0.055	0.0093
S233757w*	03:11:01-01	310	1001 ± 242	HES	$5.2 \cdot 10^{-10}$	-0.015	0.0	-0.174	0.055	0.0022
S233758w*	03:11:01-01	35165	572 ± 1740	HES	$7.7 \cdot 10^{-10}$	-0.045	0.0	-0.002	0.057	0.015
S233759w*	03:11:01-01	4239	3984 ± 7110	HES	$9.4 \cdot 10^{-10}$	-0.042	0.0	-0.00019	0.049	0.0049
S233759w*	03:11:01-01	11677	7	HES	$2.7 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233759w*	03:11:01-01	3775	5442 ± 1852	HES	$3.0 \cdot 10^{-10}$	-0.120	0.0	0.0007	0.0	0.0008
S233760w*	03:11:01-01	9498	5072 ± 1020	HES	$7.1 \cdot 10^{-10}$	-0.264	0.0	0.0	0.0018	0.041
S233760w*	03:11:01-01	8119	1049 ± 8712	HES	$5.2 \cdot 10^{-10}$	-0.279	0.0	-0.18	0.0	2.4e-2
S233761w*	03:11:01-01	3106	1055 ± 1145	HES	$6.0 \cdot 10^{-10}$	-0.040	0.0	-0.001	0.043	0.003
S233761w*	03:11:01-01	35165	530 ± 1	L	$7.7 \cdot 10^{-10}$	-0.10	0.0	+1.0	0.0	7.6e-2
S233761w*	03:11:01-01	4516	437 ± 1025	HES	$6.6 \cdot 10^{-10}$	-0.105	0.0	-0.001	0.0	0.0007
S233762w*	03:11:01-01	1917	2962 ± 1467	HES	$4.4 \cdot 10^{-10}$	-0.081	0.0	-0.008	0.0	0.042
S233762w*	03:11:01-01	9304	5992 ± 8237	HES	$6.8 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.008	0.007
S233762w*	03:11:01-01	3114	5104 ± 8711	HES	$4.8 \cdot 10^{-10}$	-0.090	0.0	-0.009	0.042	0.002
S233763w*	03:11:01-01	2687	2687 ± 1000	HES	$5.8 \cdot 10^{-10}$	-0.080	0.0	-0.009	0.042	0.002
S233763w*	03:11:01-01	5298	4952 ± 1853	HES	$10^{-10}$	-0.0009	0.0	-1.0	0.0	2.1e-5
S233764w*	03:11:01-01	5095	7	HES	$5.0 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233764w*	03:11:01-01	1074	5203 ± 2157	HES	$2.4 \cdot 10^{-10}$	-0.059	0.0	-0.009	0.050	0.0024
S233765w*	03:11:01-01	5554	5554 ± 1000	HES	$5.5 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233765w*	03:11:01-01	5598	7	HES	$5.5 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233766w*	03:11:01-01	6435	7	HES	$5.2 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0
S233766w*	03:11:01-01	1977	3040 ± 1439	HES	$1.0 \cdot 10^{-10}$	-0.0009	0.0	-0.009	0.0041	0.017
S233766w*	03:11:01-01	30244	7	HES	$5.0 \cdot 10^{-10}$	+1.0 <sup>+0.0</sup>	0.0	0.0	0.0	0.0

S230507a	01:54:00-01: 01:54:25	1995	4020 ± 8476	HL	$4.0 \cdot 10^{-6}$	0.00	0.0	0.005	0.0073	0.013	
S230507w	02:08:00-02: 01:00:25	7627	16.151 ± 3154	HL	$4.0 \cdot 10^{-6}$	0.0	0.0	0.704	0.834	0.367	
S230508w	02:08:00-02: 01:00:25	7605	1595 ± 527	HL	$2.3 \cdot 10^{-6}$	0.005	0.0	0.0057	0.0074	0.006	
S230508w	02:08:00-02: 01:00:25	5658	4551 ± 1571	HL	$2.3 \cdot 10^{-6}$	0.0	0.0	0.0058	0.0073	0.017	
S230508w	02:08:00-02: 01:00:25	689	7	HL	$8.0 \cdot 10^{-7}$	$-1.0^{+0.07}_{-0.07}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	27756	1629 ± 1684	H	$2.0$	$0.0 \pm 0.1$	$0.0 \pm 0.0$	$-1.0$	0.0	1.0e-5	
S230510w	02:08:00-02: 01:00:25	5178	7	HL	$8.0 \cdot 10^{-7}$	$-1.0^{+0.08}_{-0.08}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	1537	2870 ± 827	HL	$2.0 \cdot 10^{-6}$	0.041	0.0	0.0	0.0053	0.0	0.0053
S230510w	02:08:00-02: 01:00:25	3381	7	HL	$8.0 \cdot 10^{-7}$	$-1.0^{+0.08}_{-0.08}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	259	7	HL	$4.0 \cdot 10^{-7}$	$-1.0^{+0.07}_{-0.07}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	756	1481 ± 460	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	0.00005	0.0	0.00005
S230510w	02:08:00-02: 01:00:25	3380	5139 ± 1225	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5
S230510w	02:08:00-02: 01:00:25	524	1124 ± 342	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5
S230510w	02:08:00-02: 01:00:25	1407 ± 444	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5	
S230510w	02:08:00-02: 01:00:25	4655	4651 ± 7310	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5
S230510w	02:08:00-02: 01:00:25	835	2750 ± 587	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5
S230510w	02:08:00-02: 01:00:25	4687	7	HL	$8.0 \cdot 10^{-7}$	$-1.0^{+0.07}_{-0.07}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	420	7	HL	$8.0 \cdot 10^{-7}$	$-1.0^{+0.07}_{-0.07}$	0.0	0.0	0.0	1	
S230510w	02:08:00-02: 01:00:25	1777	3462 ± 1044	HL	$1.0 \cdot 10^{-5}$	0.354	0.0	0.0	0.010	0.0	0.004
S230510w	02:08:00-02: 01:00:25	386	1576 ± 396	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	0.00000	0.0	0.00001
S230510w	02:08:00-02: 01:00:25	1999	3614 ± 8792	HL	$8.0 \cdot 10^{-6}$	0.215	0.0	0.0	-1.0	0.0	1.0e-5
S230510w	02:08:00-02: 01:00:25	2793	5125 ± 1859	HL	$7.0 \cdot 10^{-6}$	0.201	0.0	0.0	0.0004	0.0004	0.0005
S230510w	02:08:00-02: 01:00:25	1022	2867 ± 996	HL	$9.0 \cdot 10^{-6}$	0.207	0.0	0.0	0.0000	0.0004	0.0004
S230510w	02:08:00-02: 01:00:25	14402	1022 ± 3211	HL	$6.0 \cdot 10^{-7}$	$-1.0^{+0.14}_{-0.14}$	0.0	0.0	0.000	0.0004	0.0004
S230510w	02:08:00-02: 01:00:25	4743	7930 ± 2718	H	$7.0 \cdot 10^{-6}$	$-1.0^{+0.17}_{-0.17}$	0.0	0.0	0.007	0.000	0.002
S231005w	02:13:00-02: 03:00:25	5482	6417 ± 2346	HL	$2.0 \cdot 10^{-5}$	0.777	0.0	0.0	0.0000	0.0	0.0022
S231005w	02:13:00-02: 03:00:25	2497	3917 ± 1835	HL	$2.0 \cdot 10^{-5}$	0.381	0.0	0.0	0.0000	0.0004	0.0015
S231007w	02:13:00-02: 03:00:25	2663	3721 ± 1231	HL	$6.0 \cdot 10^{-7}$	$-1.0^{+0.14}_{-0.14}$	0.0	0.0	0.746	0.0004	0.264
S231007w	02:13:00-02: 03:00:25	9428	7	HL	$4.0 \cdot 10^{-7}$	$-1.0^{+0.15}_{-0.15}$	0.0	0.0	0.0	0.0	1
S231008w	02:13:00-02: 03:00:25	5887	5531 ± 1306	HL	$1.0 \cdot 10^{-5}$	0.568	0.0	0.0	0.0000	0.0	0.0004
S231008w	02:13:00-02: 03:00:25	1697	2857 ± 965	HL	$1.0 \cdot 10^{-5}$	0.563	0.0	0.0	0.0003	0.0	0.0003
S231017w	02:13:00-02: 03:00:25	10355	7	HL	$8.0 \cdot 10^{-6}$	$-1.0^{+0.16}_{-0.16}$	0.0	0.0	0.0	1	
S231019w	02:13:00-02: 03:00:25	1334	1168 ± 361	HL	$1.0 \cdot 10^{-5}$	0.357	0.0	0.0	0.0003	0.0004	0.0003
S231020w	02:13:00-02: 03:00:25	8889	2846 ± 704	HL	$3.0$	$0.0 \pm 0.1$	$0.0 \pm 0.0$	$0.0000$	0.0	0.00005	

## 100+ candidates



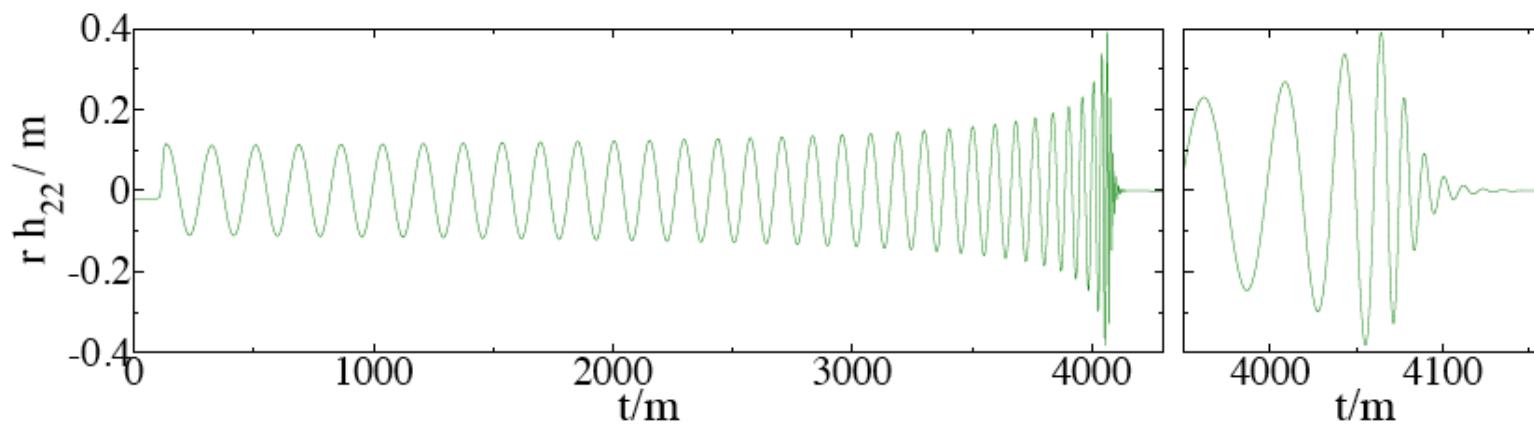
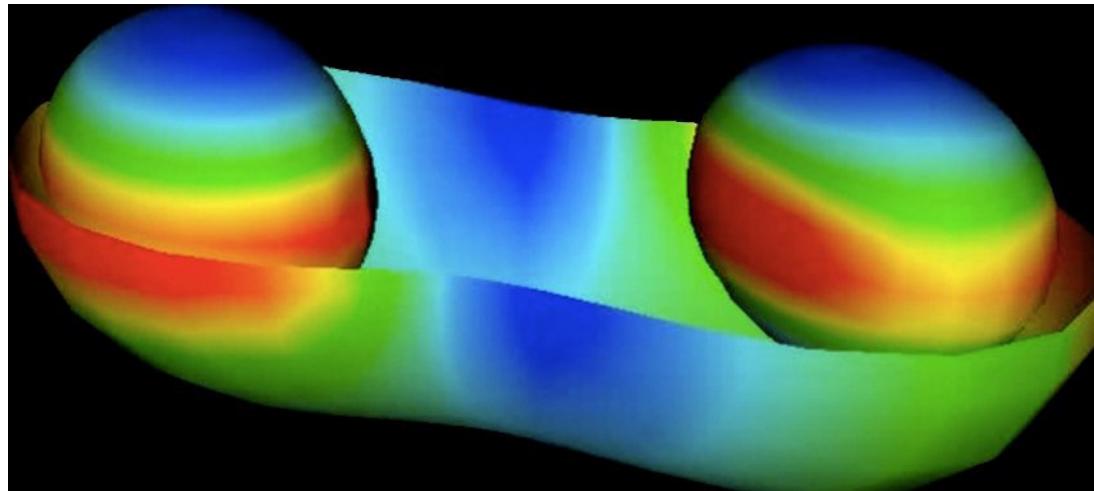
Gravitational waveform gives  $M_1, M_2, \chi_{\text{eff}}$

$$\chi_{\text{eff}} \equiv \frac{m_1 \chi_1 + m_2 \chi_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

# What to do with these GW detections?

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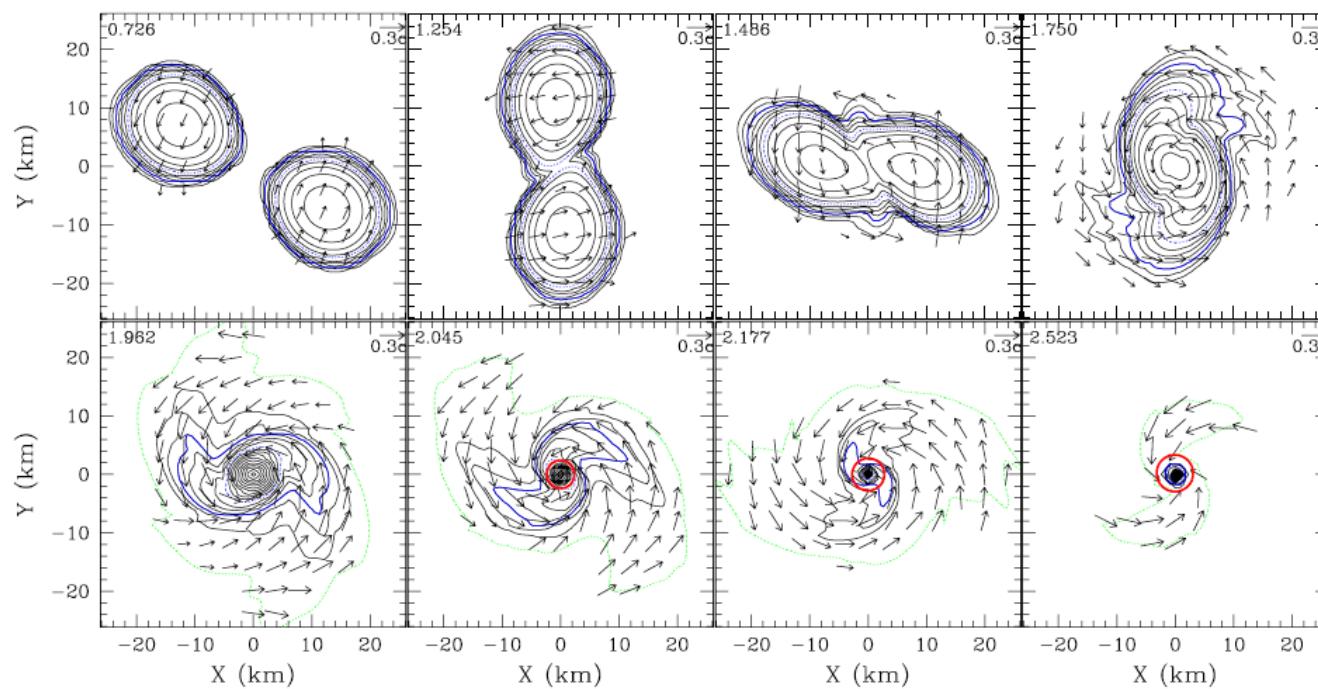
1. Test gravity theory in nonlinear regime:



Cornell-Caltech collaboration

# What to do with these GW detections?

1. Test gravity theory in nonlinear regime
2. Study dense nuclear matter



Shibata et al.

# What to do with these GW detections?

1. Test gravity theory in nonlinear regime
2. Study dense nuclear matter

## Inspiral phase:

Tidal/spin deformation (quasi-static tide)      well-understood (since 1990s): many papers...  
Resonant tide (excitation of g-mode etc) ?      e.g. Lai 1994; Yu et al 2024...

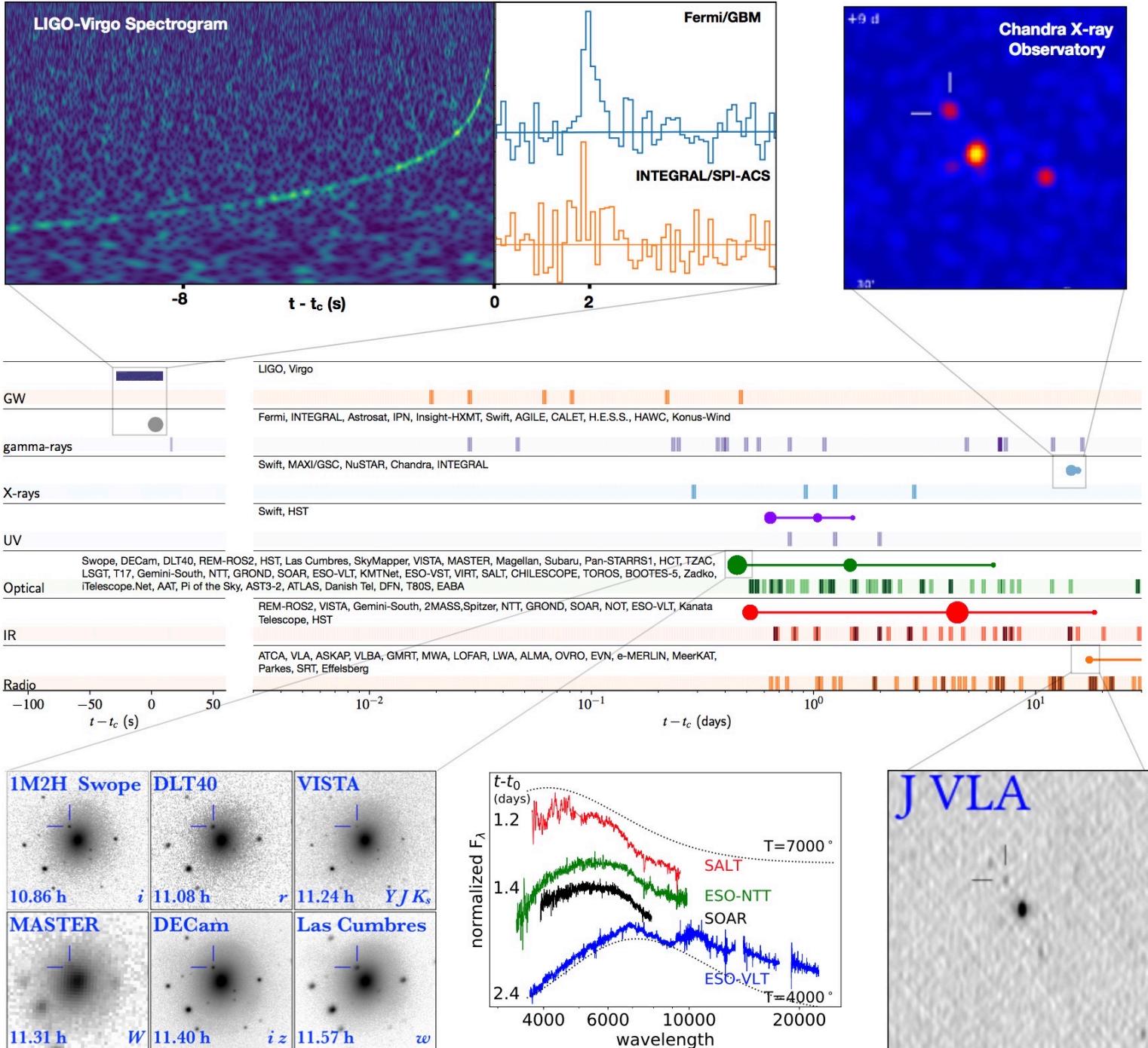
## Merger waveform:

Remnant NS (hyper-massive or supermassive) or BH ?

## What to do with these GW detections?

1. Test gravity theory in nonlinear regime
2. Study dense nuclear matter
3. Nucleosynthesis & EM counterparts

# LIGO's first NS Binaries: GW170817 / AT2017gfo



# What to do with these GW detections?

1. Test gravity theory in nonlinear regime
2. Study dense nuclear matter
3. Nucleosynthesis & EM counterparts
4. Astrophysics of BH binary formation

# Formation of Merging BH Binaries

$$T_m \approx 10^{10} \text{ yrs} \left( \frac{60M_\odot}{m_1 + m_2} \right)^2 \left( \frac{15M_\odot}{\mu} \right) \left( \frac{a_0}{0.2 \text{ AU}} \right)^4 \left( 1 - e_0^2 \right)^{7/2}$$

Final AU problem...

# Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

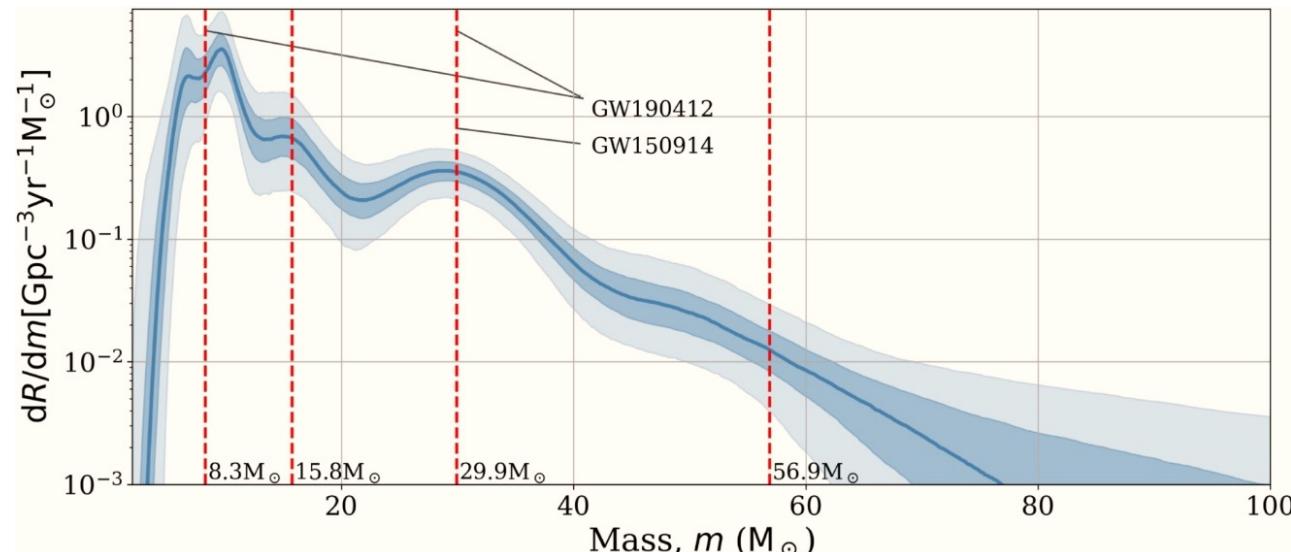
-- Dynamical Formation:

several flavors: star clusters, triples (multiples), AGN disks

## How to distinguish different channels?

Rates (uncertain)?

Masses and mass ratio



# Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation:

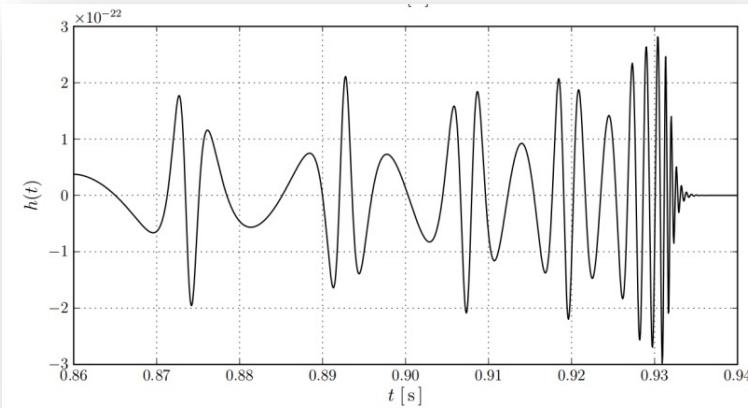
several flavors: star clusters, triples (multiples), **AGN disks**

## How to distinguish different channels?

Rates (uncertain)?

Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band



# Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation:

several flavors: star clusters, triples, AGN disks

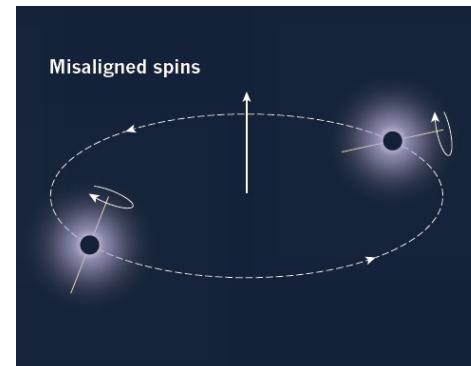
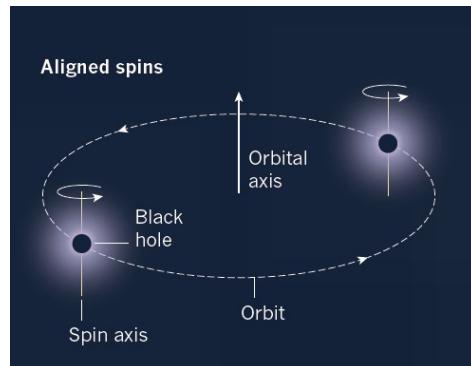
## How to distinguish different channels?

Rates (uncertain)?

Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band

Spin-orbit misalignment



$$\chi_{\text{eff}} \equiv \frac{m_1 \chi_1 + m_2 \chi_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

# Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation:

several flavors: star clusters, triples, AGN disks

## How to distinguish different channels?

Rates (uncertain)?

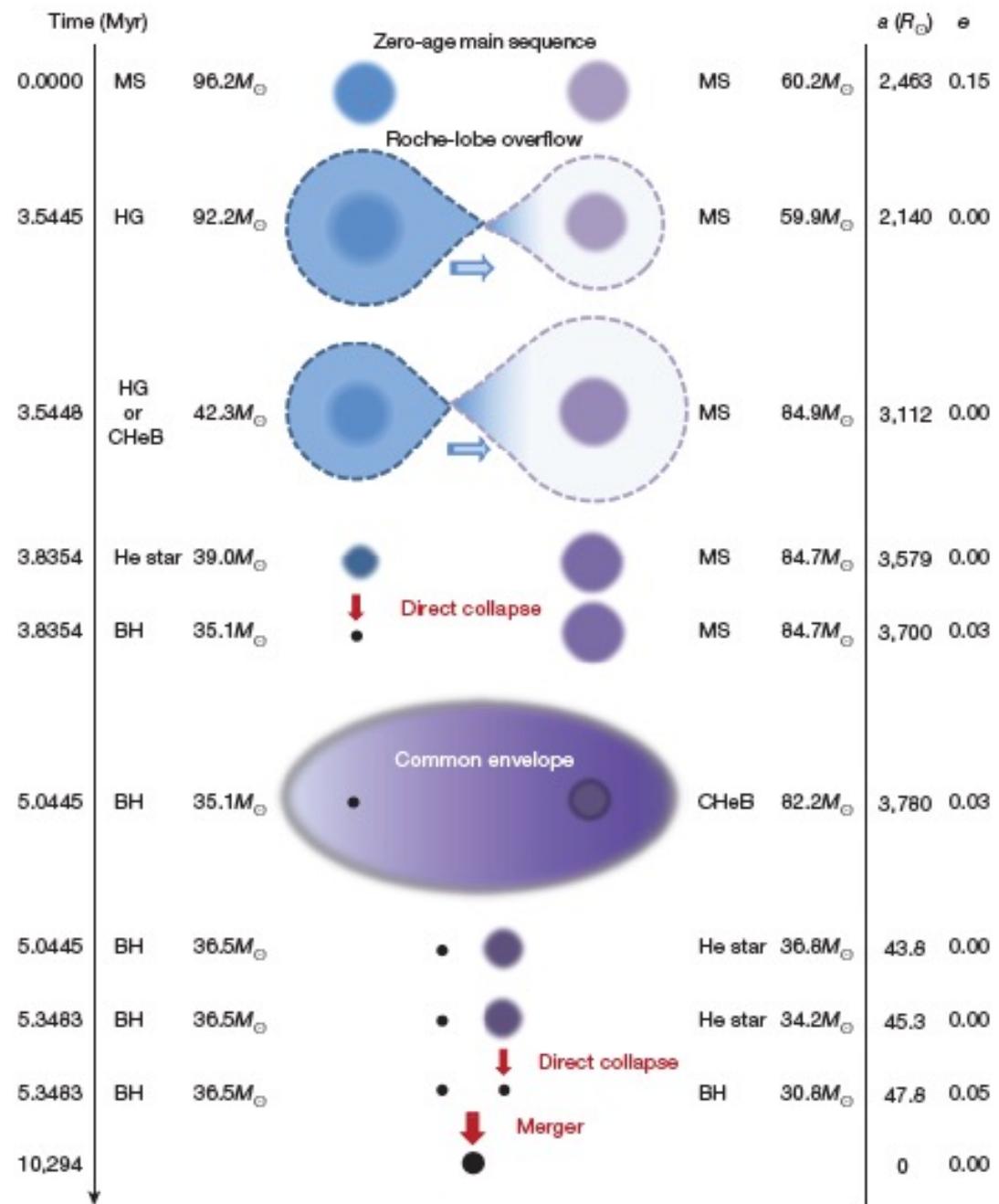
Masses and mass ratio

Residual eccentricity when enter LIGO band (10Hz) or lower-f band

Spin-orbit misalignment

EM counterpart

# Isolated Binary Evolution Channel: Standard



many papers, uncertain physical ingredients  
(e.g. common envelope;  
Gaia BH1,2,3 ??)

Produce  
circular orbit at 10 Hz  
mostly aligned spin-orbit

Belczynski +16

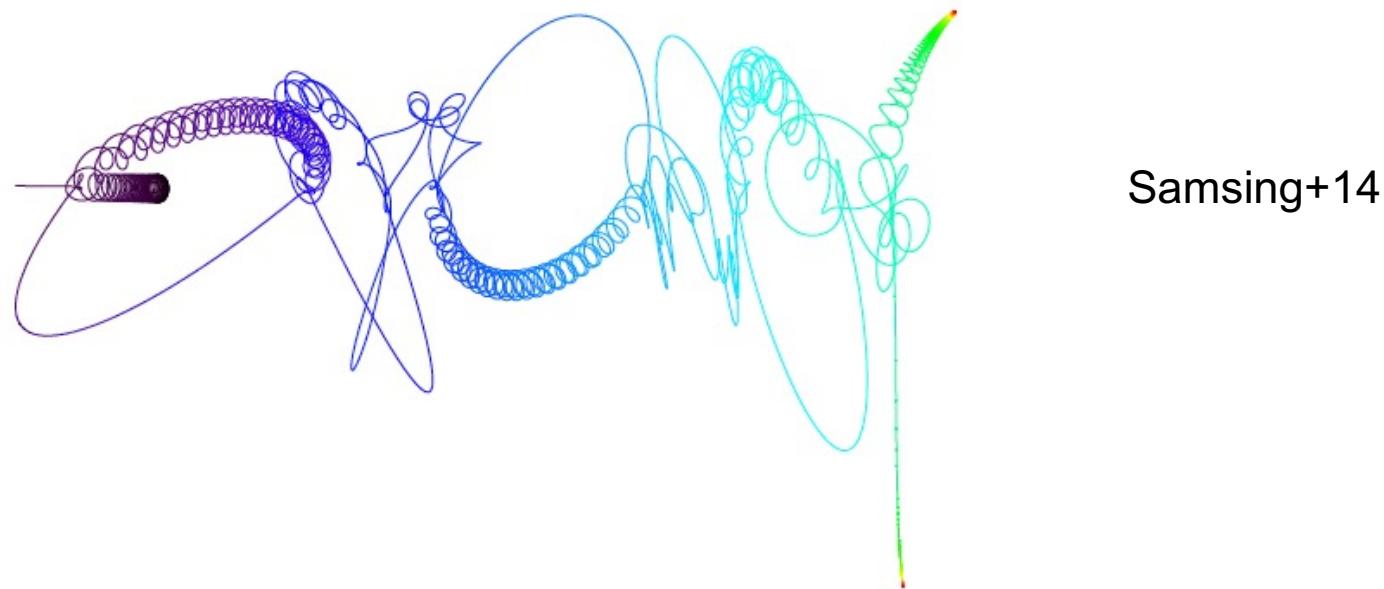
# Dynamical Formation Channels

several flavors...

# Dynamical Formation Channels

several flavors...

## 1. Dense clusters: binary-single scatterings → tight binary



Enough BHs in clusters? Kicks? GCs or Nuclear Star Clusters?

**Produce mostly circular orbit when enter LIGO band (10 Hz) ??  
Expect random spin-orbit orientations**

# Dynamical Formation Channels

several flavors...

**1. Dense clusters: binary-single scatterings → tight binary**

**2. Tertiary-Induced Mergers:**

Mergers induced by (gentle) perturbations from tertiary companion  
(via Lidov-Kozai or other secular resonance effects)

stellar triples in galactic field, binary around SMBH

# Lidov-Kozai Effect

Can perturbation from the Moon make Earth's satellites fall?



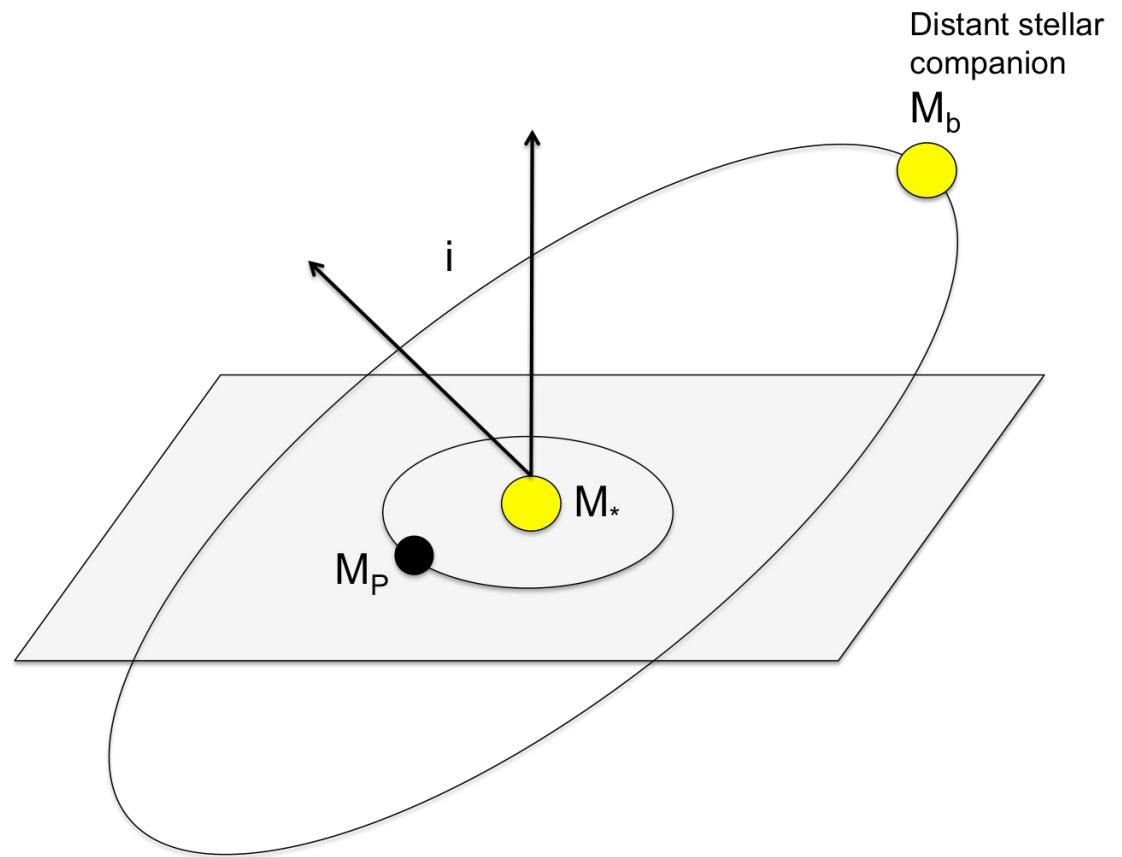
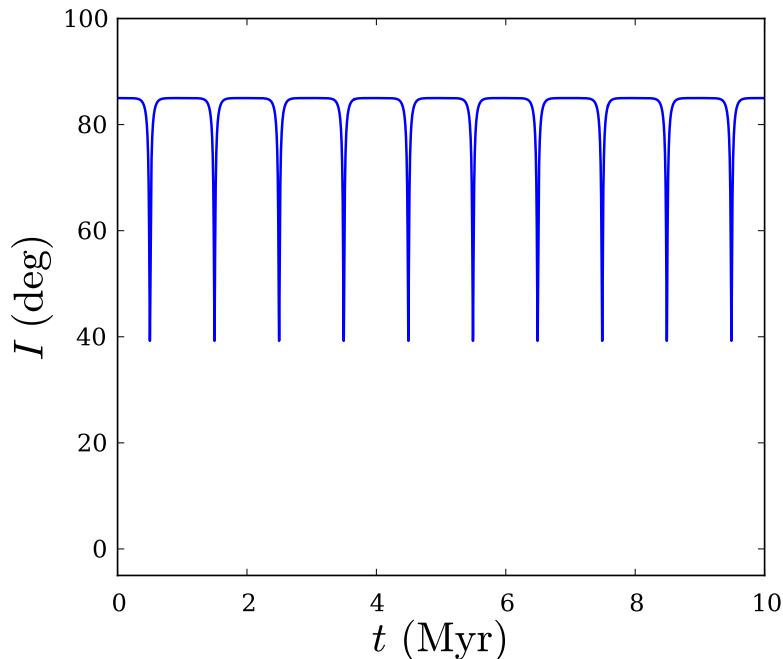
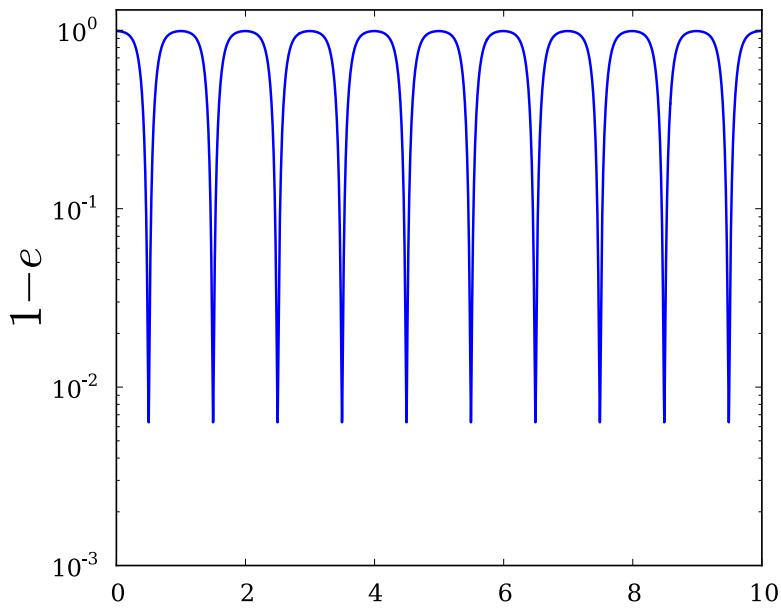
Planet. Space Sci., 1962, Vol. 9, pp. 719 to 759. Pergamon Press Ltd. Printed in Northern Ireland

## THE EVOLUTION OF ORBITS OF ARTIFICIAL SATELLITES OF PLANETS UNDER THE ACTION OF GRAVITATIONAL PERTURBATIONS OF EXTERNAL BODIES

M. L. LIDOV

Translated by H. F. Cleaves from *Iskusstvennye Sputniki Zemli*, No. 8, p. 5, 1961.

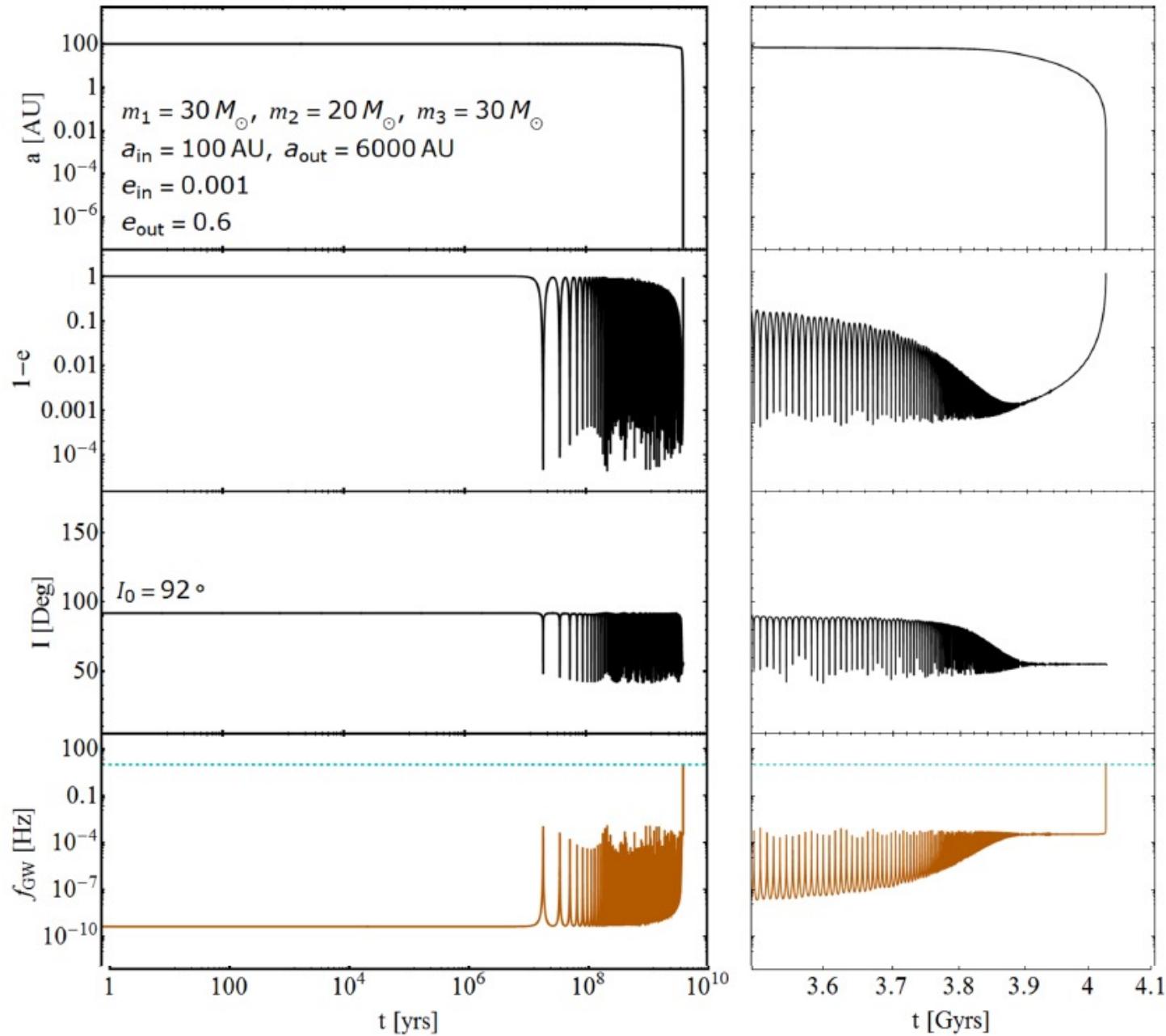
# Lidov-Kozai Effect



- Eccentricity and inclination oscillations induced if  $i > 40$  degrees.
- If  $i$  large (depending on octupole strength), get extremely large eccentricities ( $e > 0.99$ )

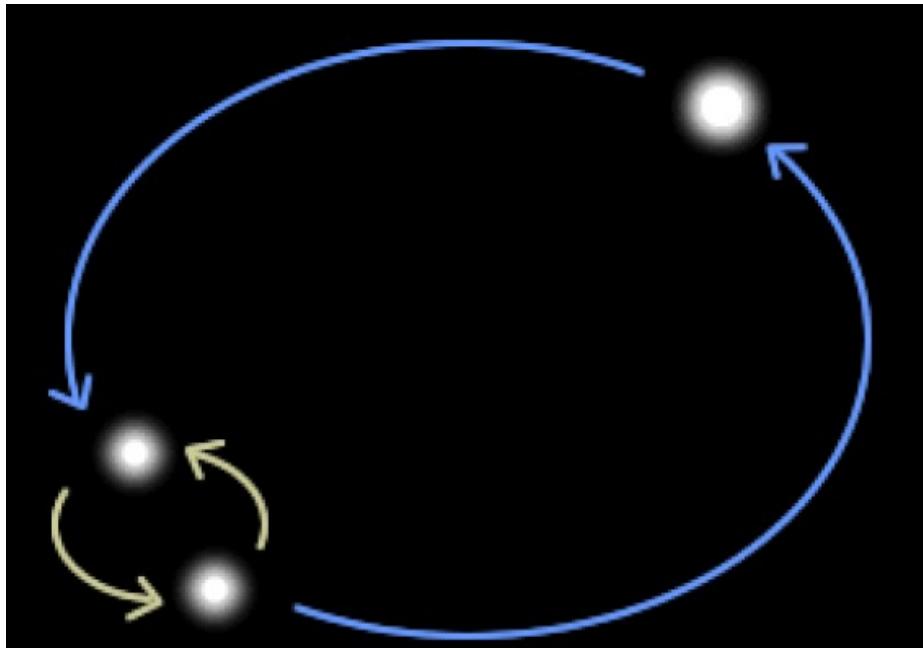
See Naoz (2016) review

# LK oscillation + Gravitation Radiation



# Tertiary-Induced Binary Mergers

merger window, mass ratio, GR effects, spin-orbit misalignments



Bin Liu  
(Cornell → Niels Bohr Inst  
→ Zhejiang U.)



Yubo Su  
(Cornell, Ph.D.22  
→ Princeton)

Liu & DL 2017-2023  
Su et al. 2021a,b; Su+2024

Previous/related works (in various contexts):

e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003;  
Thompson 2011; Antonini et al. 2012,2014,2017,  
Silsbee & Tremaine 2017; Petrovich & Antonini 2017....

Note: Related/general effects of galactic tidal potential  
Hamilton & Rafikov 2019-2021 (a la Heisler-Tremaine)

# Summary of Tertiary-Induced Mergers

Some general “predictions”:

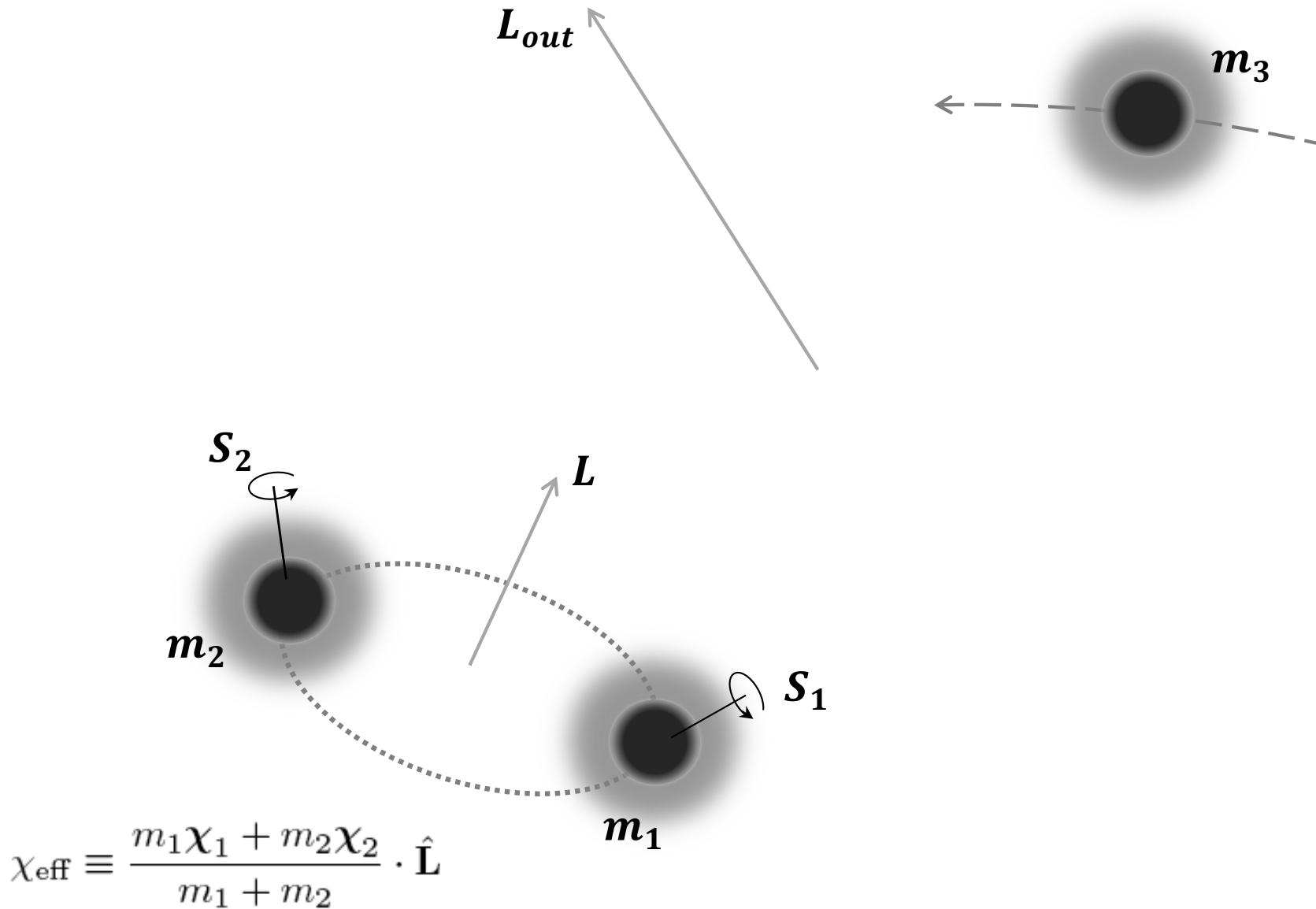
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SMBH spin enhances the LK merger fraction;  
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Synergy between galactic tide and stellar flybys (Winter-Granic+2024)

# Summary of Tertiary-Induced Mergers

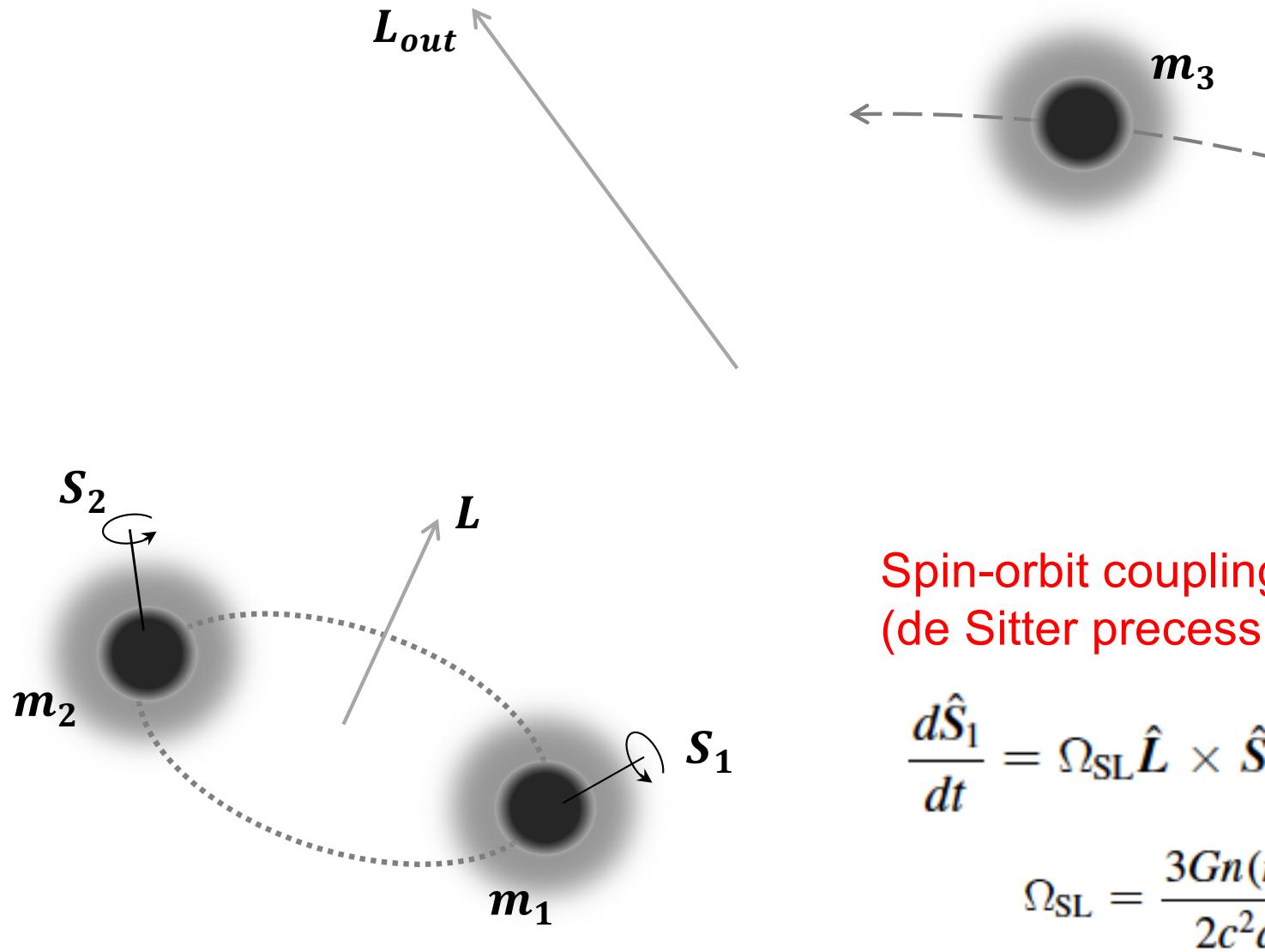
Some general “predictions”:

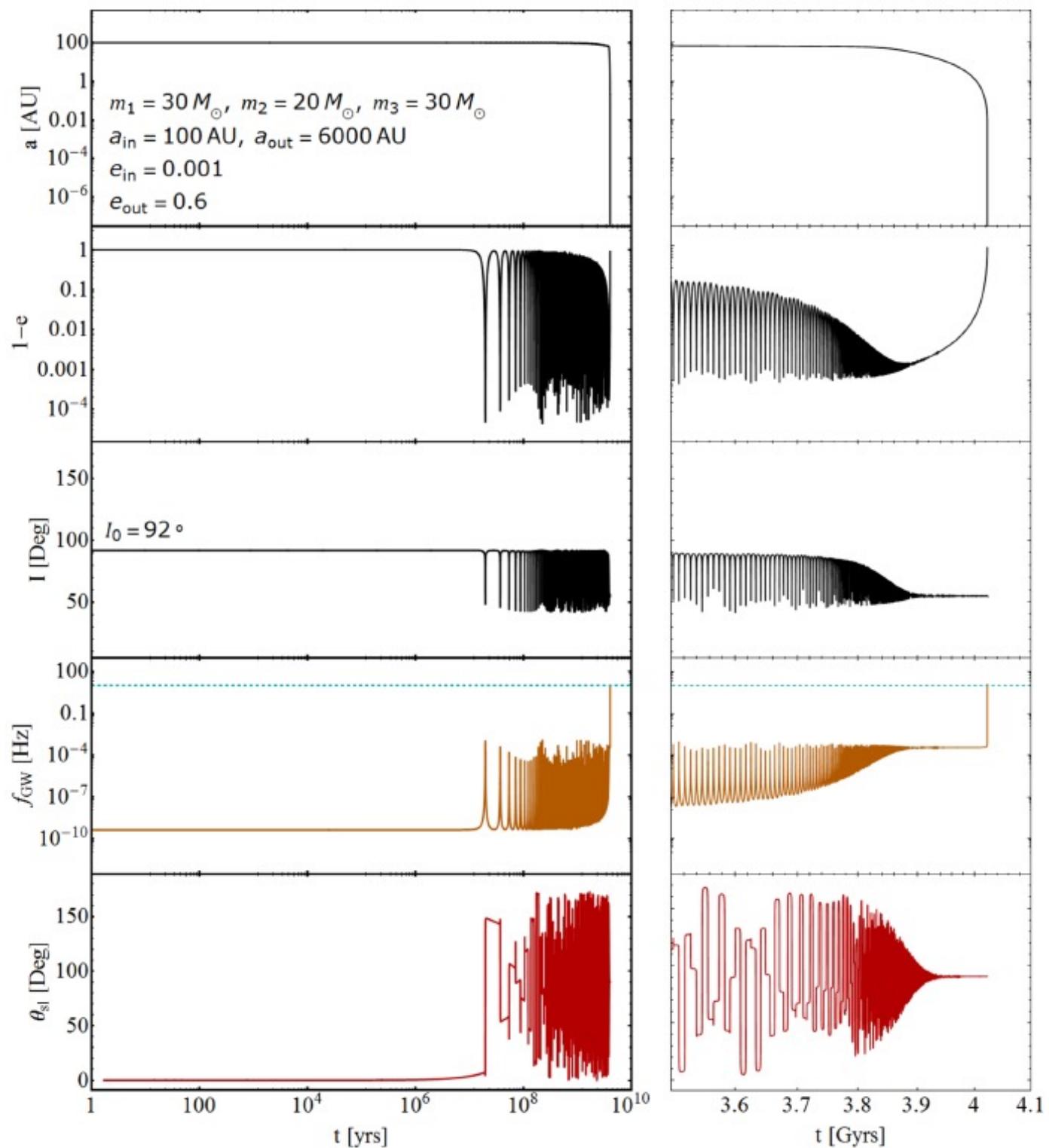
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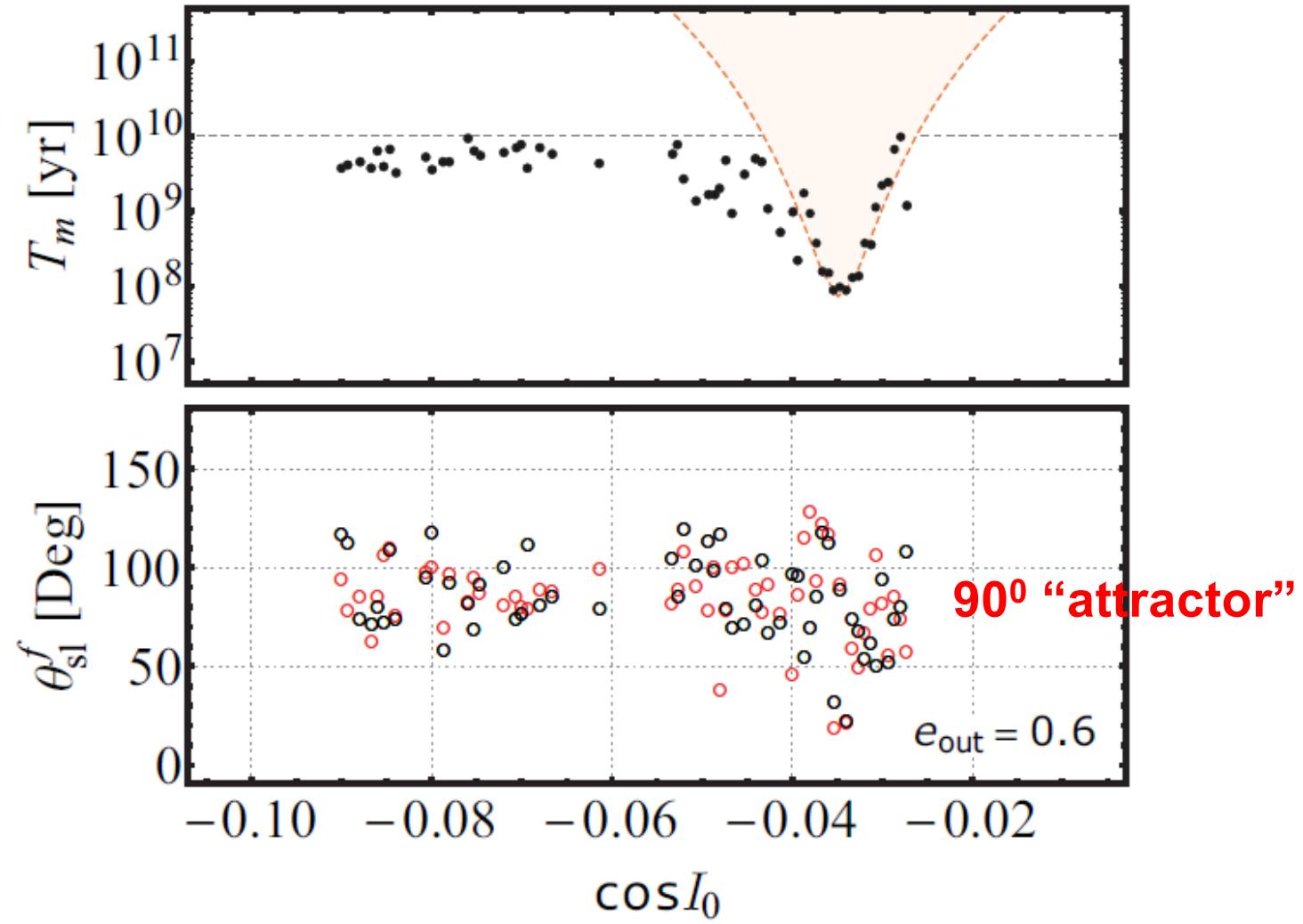
# How does BH spin axis evolve ?



# How does BH spin axis evolve ?







Liu & Lai 2018

# BH spin dynamics during LK oscillations

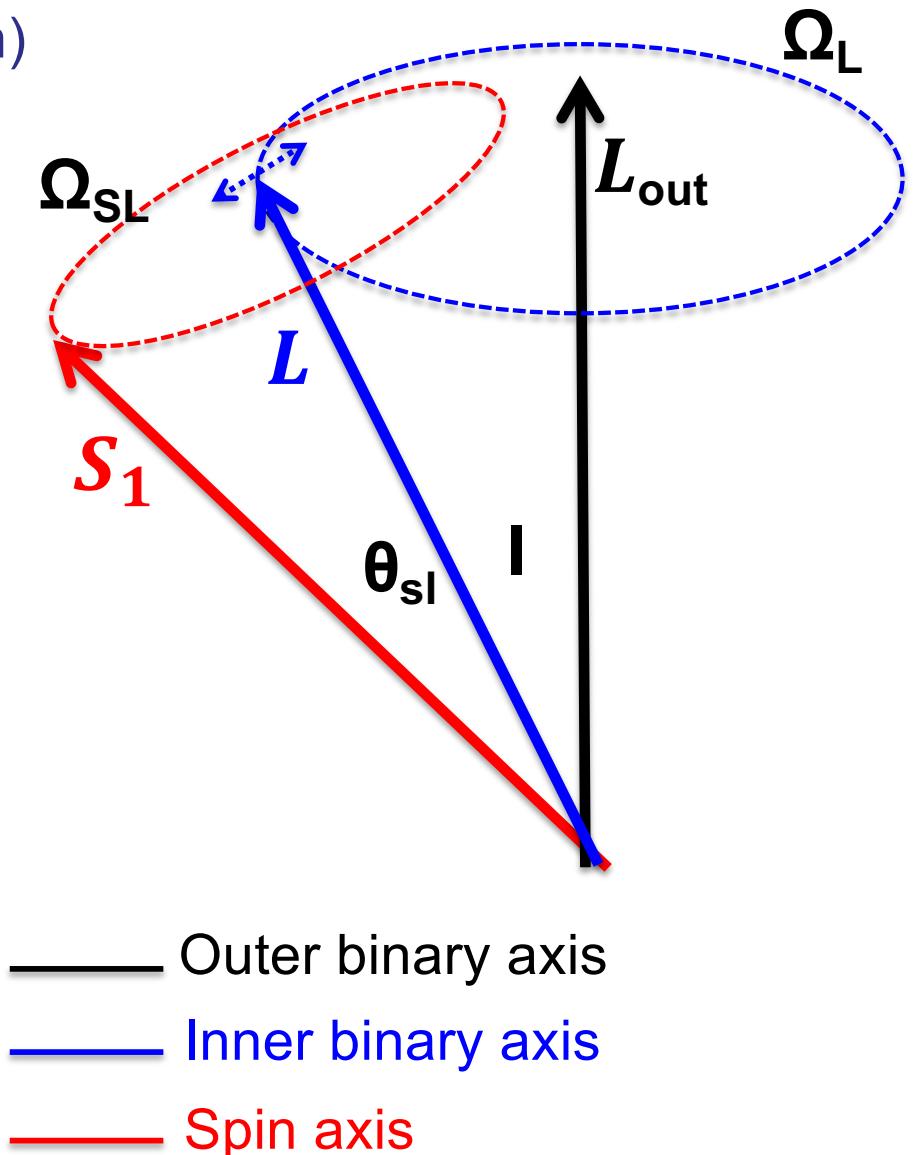
$$\frac{d\hat{S}_1}{dt} = \Omega_{SL}\hat{L} \times \hat{S}_1, \text{ (de Sitter Precession)}$$

$$\Omega_{SL} = \frac{3Gn(m_2 + \mu/3)}{2c^2a(1 - e^2)} \propto a^{-5/2}$$

But  $L$  precesses and nutates around  $L_{out}$

$$\Omega_L \sim \frac{1}{t_{LK}} \times f(e, I)$$

$$\frac{1}{t_{LK}} = n \frac{m_3}{m_{12}} \left( \frac{a}{a_{out}} \right)^3 \propto a^{3/2}$$



In the frame co-rotating with  $\mathbf{L}$

$$\left( \frac{d\hat{\mathbf{S}}_1}{dt} \right)_{\text{rot}} = \boldsymbol{\Omega}_e \times \hat{\mathbf{S}}_1$$
$$\boldsymbol{\Omega}_e \equiv \Omega_L \hat{\mathbf{L}}_{\text{out}} + \Omega_{SL} \hat{\mathbf{L}}$$

$\Omega_L$  and  $\Omega_{SL}$  both vary ( $a, e, I$ )

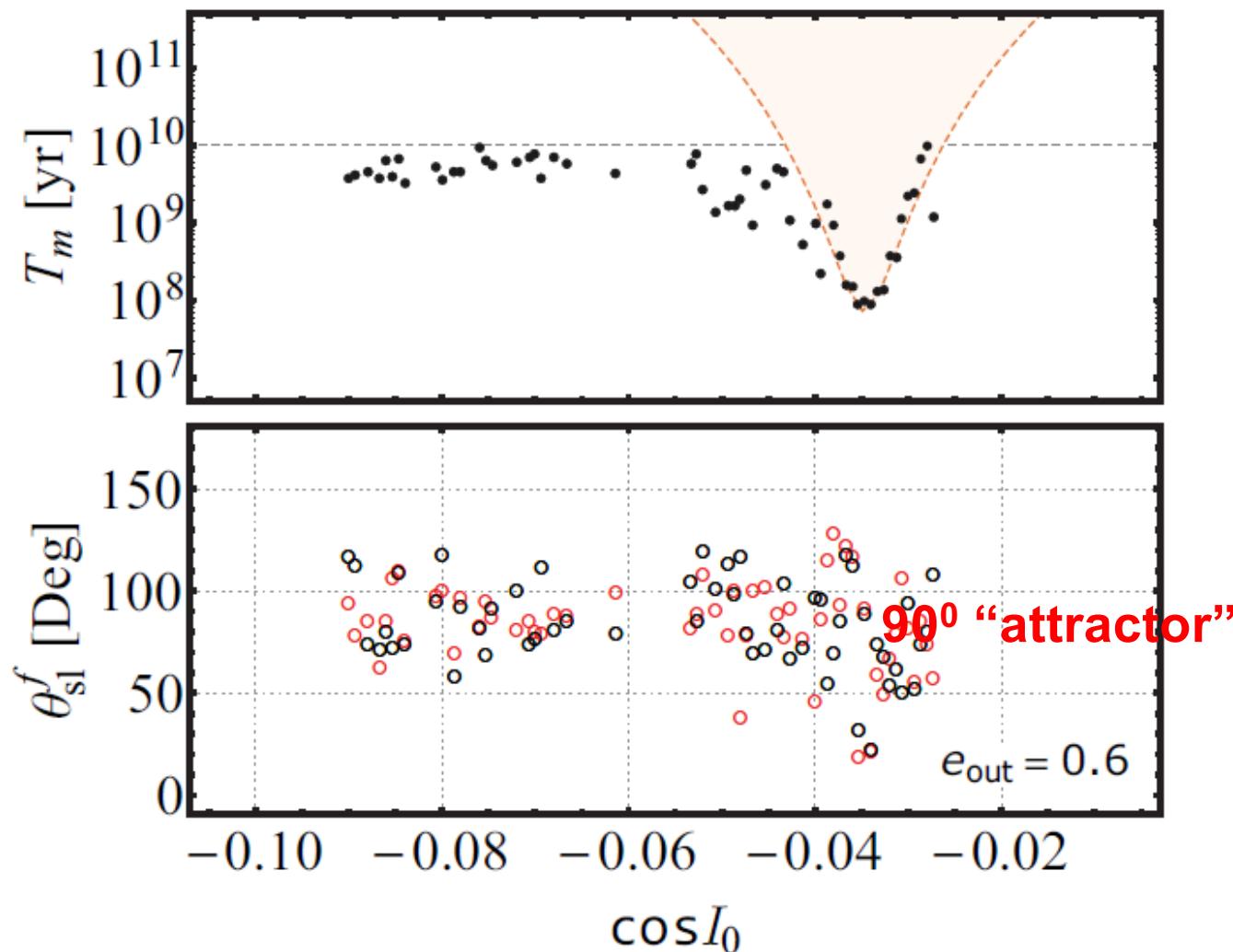
Over orbital decay timescale ( $\gg t_{LK}$ ), LK oscillation of  $e$  and  $I$  can “average” out...

$$\bar{\Omega}_{SL} \propto a^{-5/2}, \quad \bar{\Omega}_L \propto a^{3/2}$$

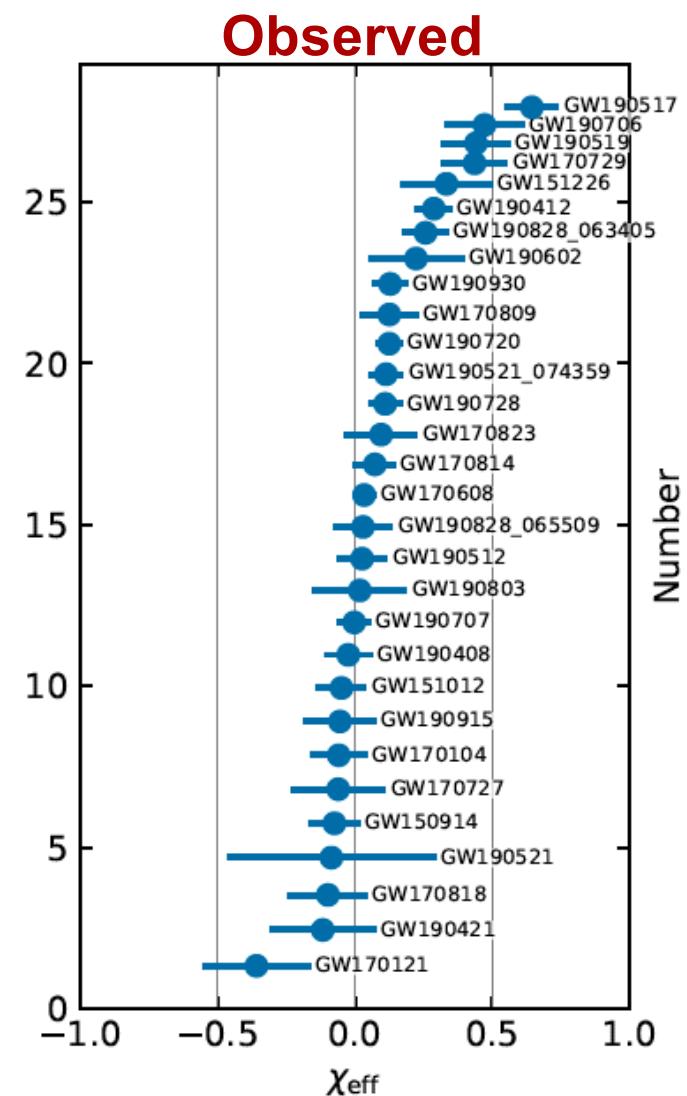
- $\mathbf{S}_1$  processes rapidly around  $\boldsymbol{\Omega}_e$ , which is slowly varying
- $\theta_{Se}$  (the angle between  $\mathbf{S}_1$  and  $\boldsymbol{\Omega}_e$ ) is adiabatic invariant

Initial (large  $a$ ):       $\bar{\Omega}_L \gg \bar{\Omega}_{SL} \implies \boldsymbol{\Omega}_e \propto \hat{\mathbf{L}}_{\text{out}}$

Final (small  $a$ ):       $\bar{\Omega}_L \ll \bar{\Omega}_{SL} \implies \boldsymbol{\Omega}_e \propto \hat{\mathbf{L}}$

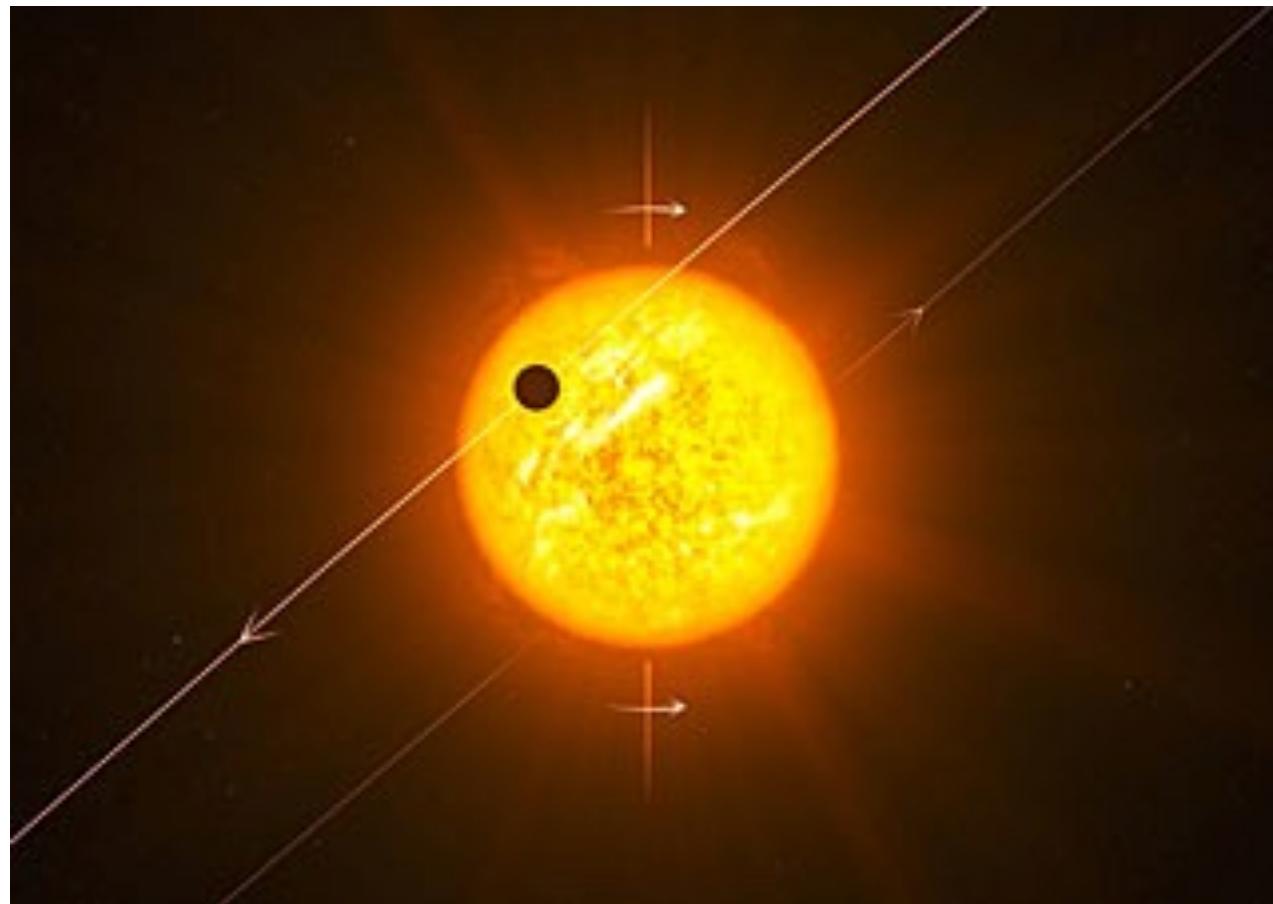


Liu & Lai 2018



$$\chi_{\text{eff}} \equiv \frac{m_1 \chi_1 + m_2 \chi_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

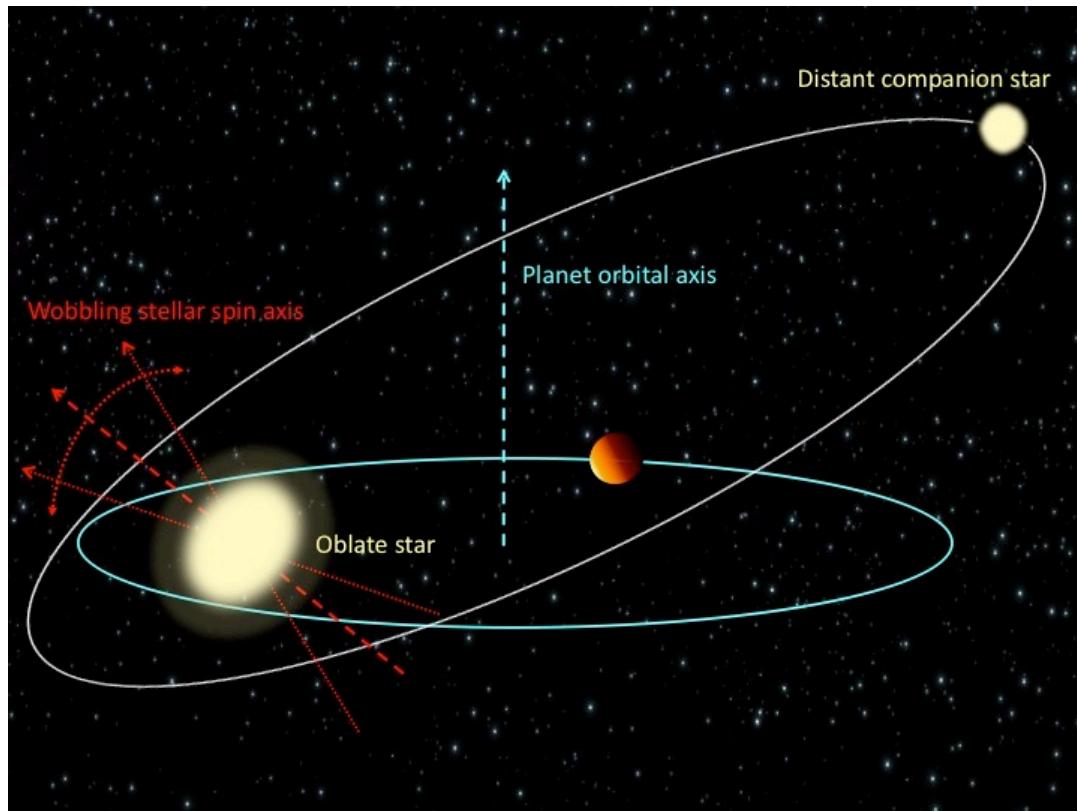
## Digression: **Stellar Obliquity (S-L misalignment) of Hot Jupiter Systems**



## One of the ways of forming Hot Jupiters: Tertiary-induced high-e migration

- Planet forms at  $\sim$  a few AU
  - Interaction with companion (other star or planets) pumps planet into high-e orbit (e.g. Lidov-Kozai)
  - Tidal dissipation in planet during high-e phases causes orbital decay
- ➔ Combined effects can result in planets in  $\sim$  few days orbit

# What happens to stellar spin axis as the planet undergoes Lidov-Kozai Oscillations ?



Star rotates → oblate  
→ **S** precesses around **L**

$$\Omega_{\text{SL}} \propto -\frac{\Omega_* m_p}{a^3(1-e^2)^{3/2}} \cos \theta_{\text{sl}}$$

# Stellar spin dynamics during LK oscillations

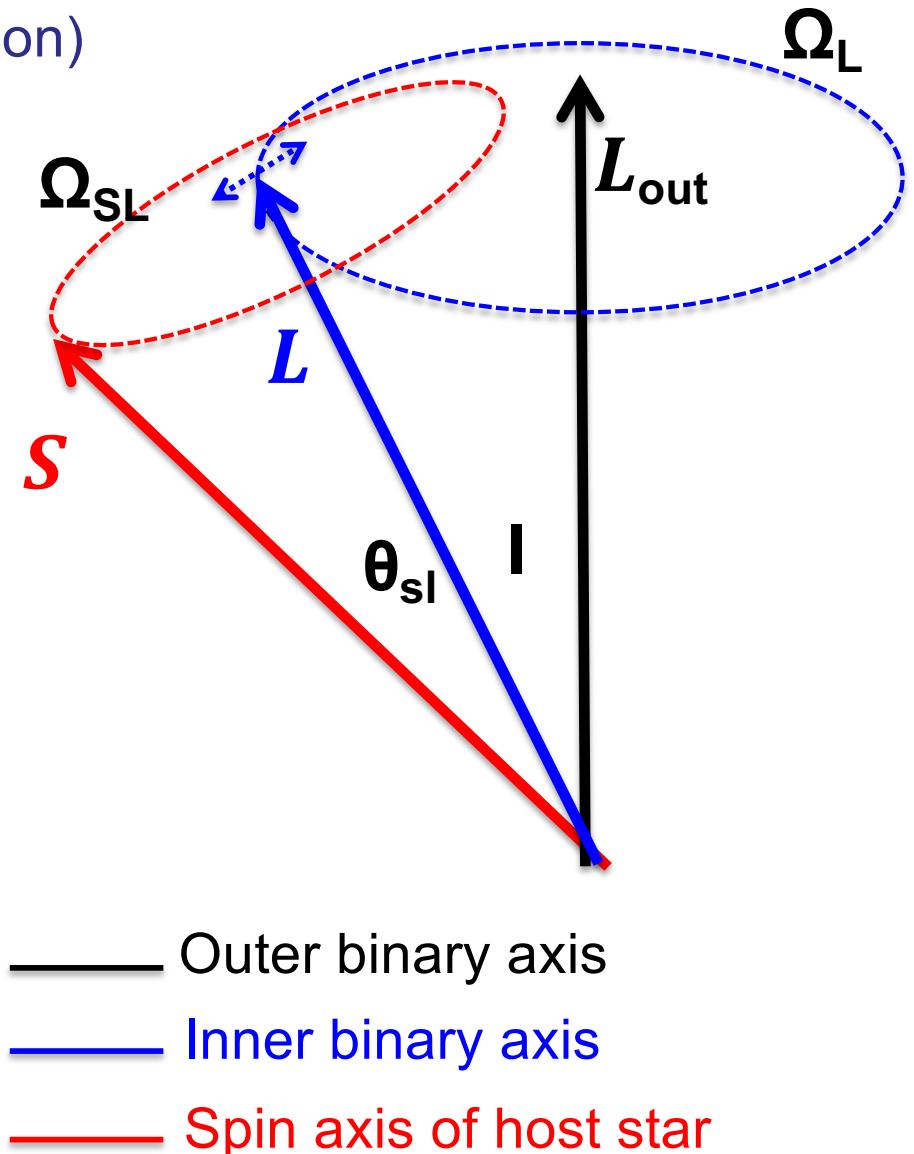
$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{SL}\hat{\mathbf{L}} \times \hat{\mathbf{S}} \quad (\text{Newtonian Precession})$$

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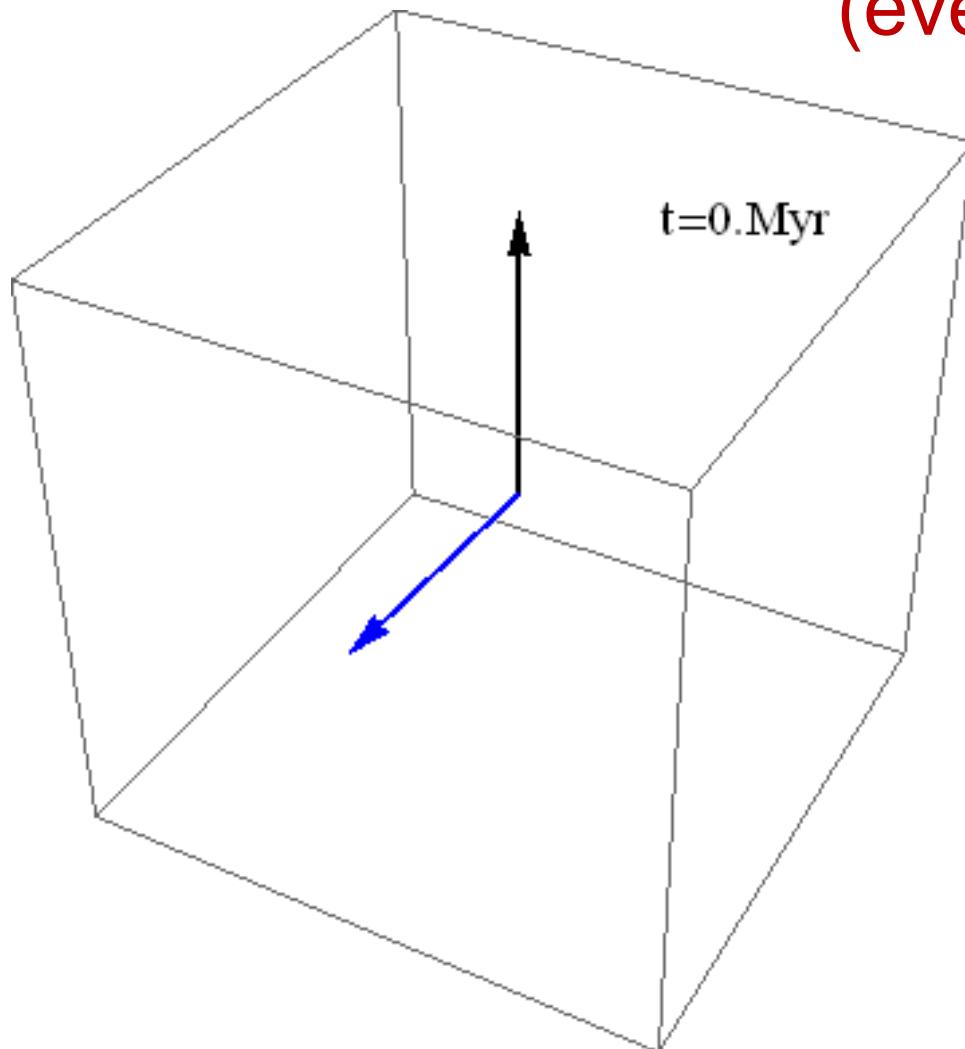
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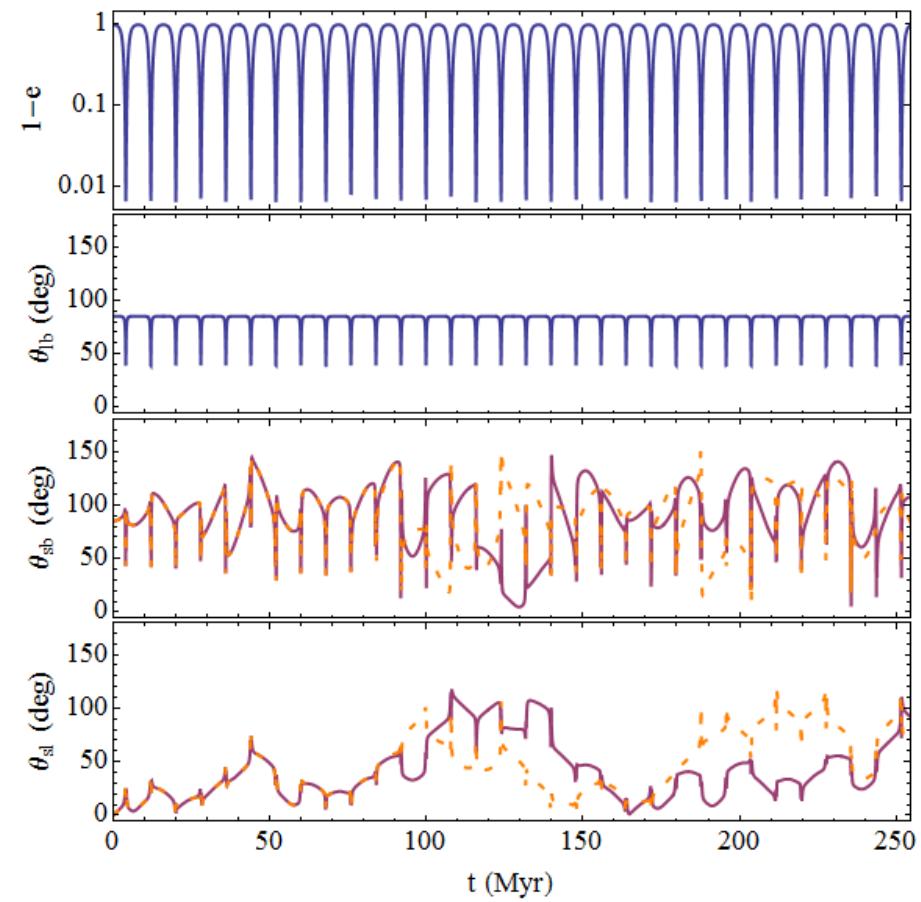
$$\frac{1}{t_{LK}} = n \frac{m_3}{m_{12}} \left( \frac{a}{a_{out}} \right)^3 \propto a^{3/2}$$



# Spin evolution can be chaotic (even when the orbit is regular)



- Outer binary axis
- Planet orbital axis
- Stellar spin axis



Storch, Anderson & Lai 2014  
Anderson, Storch & Lai 2016

## BH spin:

$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{SL}\hat{\mathbf{L}} \times \hat{\mathbf{S}} \quad (\text{de Sitter Precession})$$

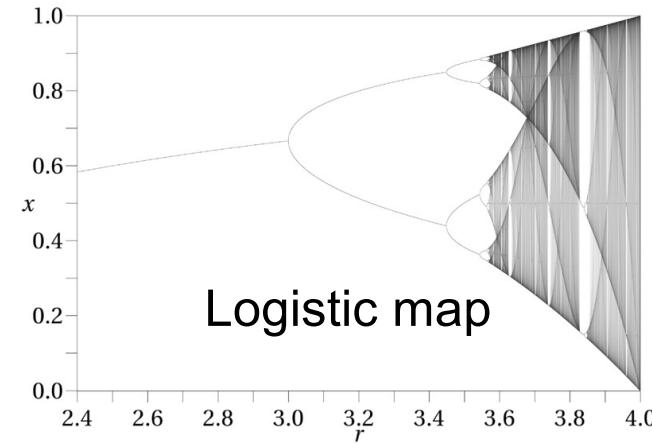
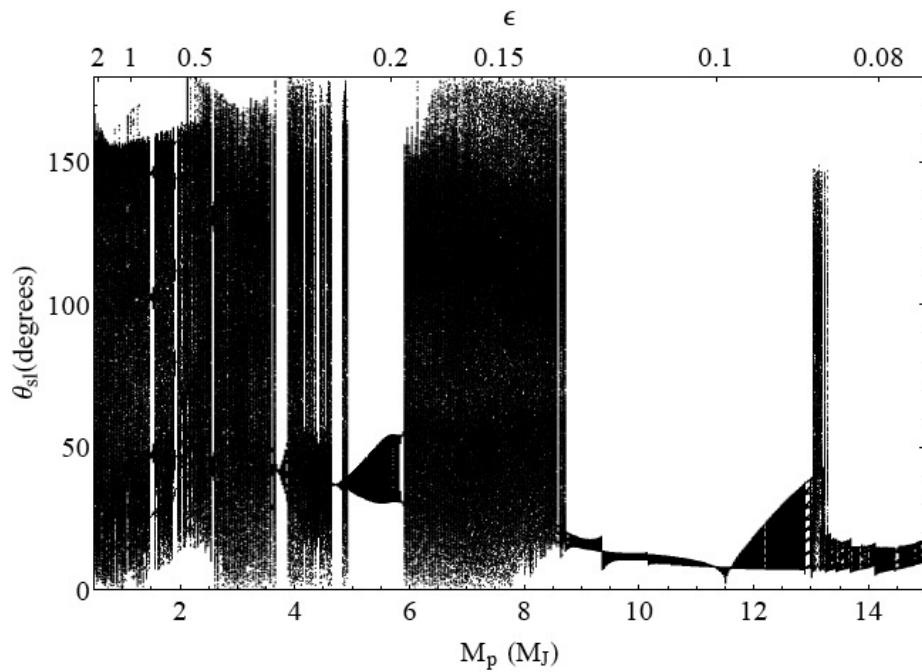
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## Stellar Spin:

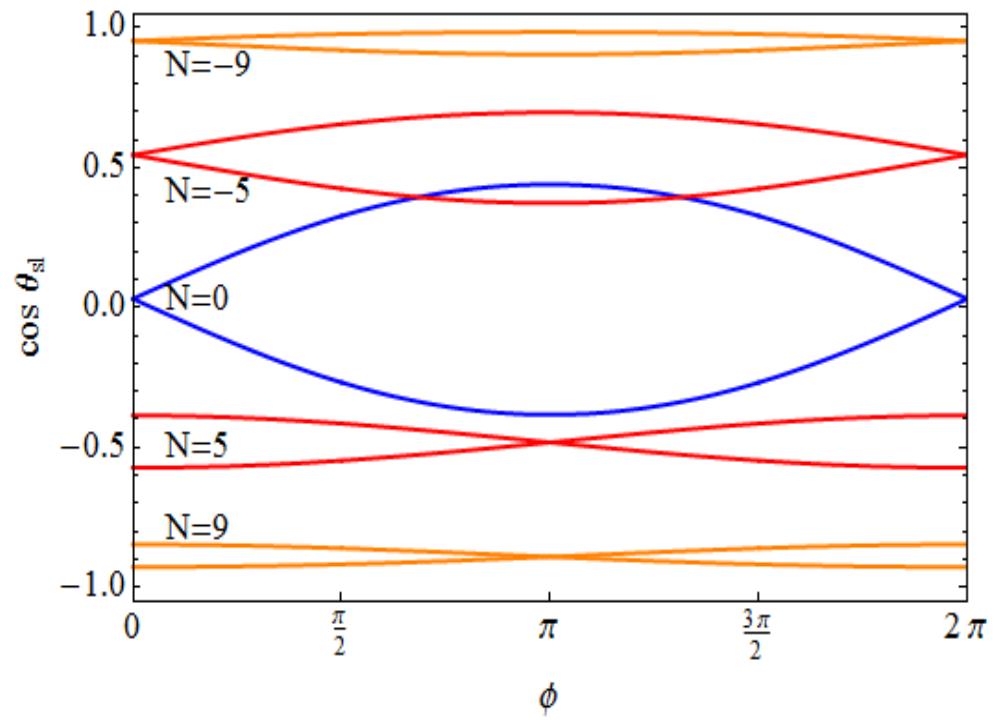
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$$\Omega_{SL} \propto -\frac{\Omega_\star m_p}{a^3(1 - e^2)^{3/2}} \cos \theta_{sl}$$

# Theory of Spin Chaos during Lidov-Kozai



Logistic map



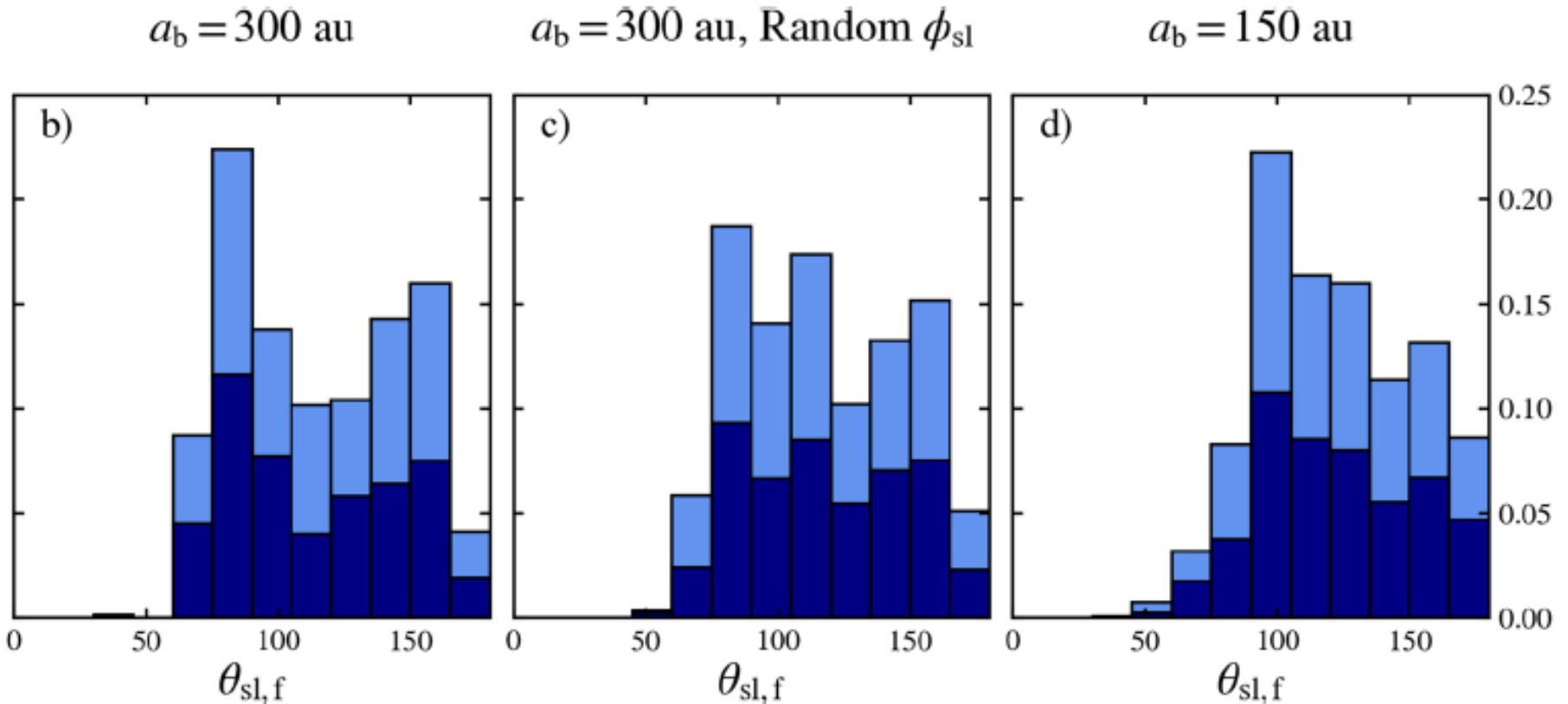
Chaos arises from overlapping resonances (Chirikov criterion)

Storch & Lai 2015, 2017

# Population synthesis of Lidov-Kozai high-e migration

## “Predicted” S-L angle Distribution

### (including star-disk-binary interaction prior to LK phase)



Vick, Su & Lai (2023)



Michelle Vick  
(Cornell PhD 20→Brown)



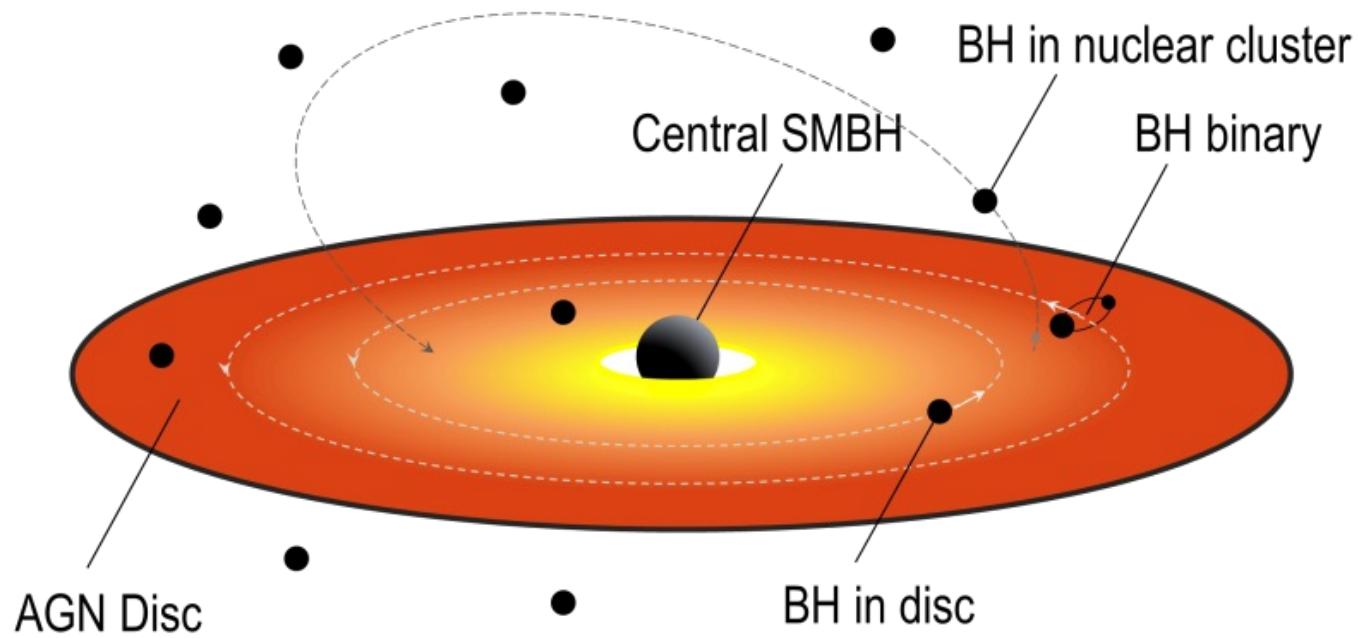
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# Dynamical Formation Channel:

## 3. Binary BH Mergers in AGN (Active Galactic Nuclei) disks



# Empirical evidence for massive stars (-> stellar mass BHs) in AGN disks

Galactic Center:

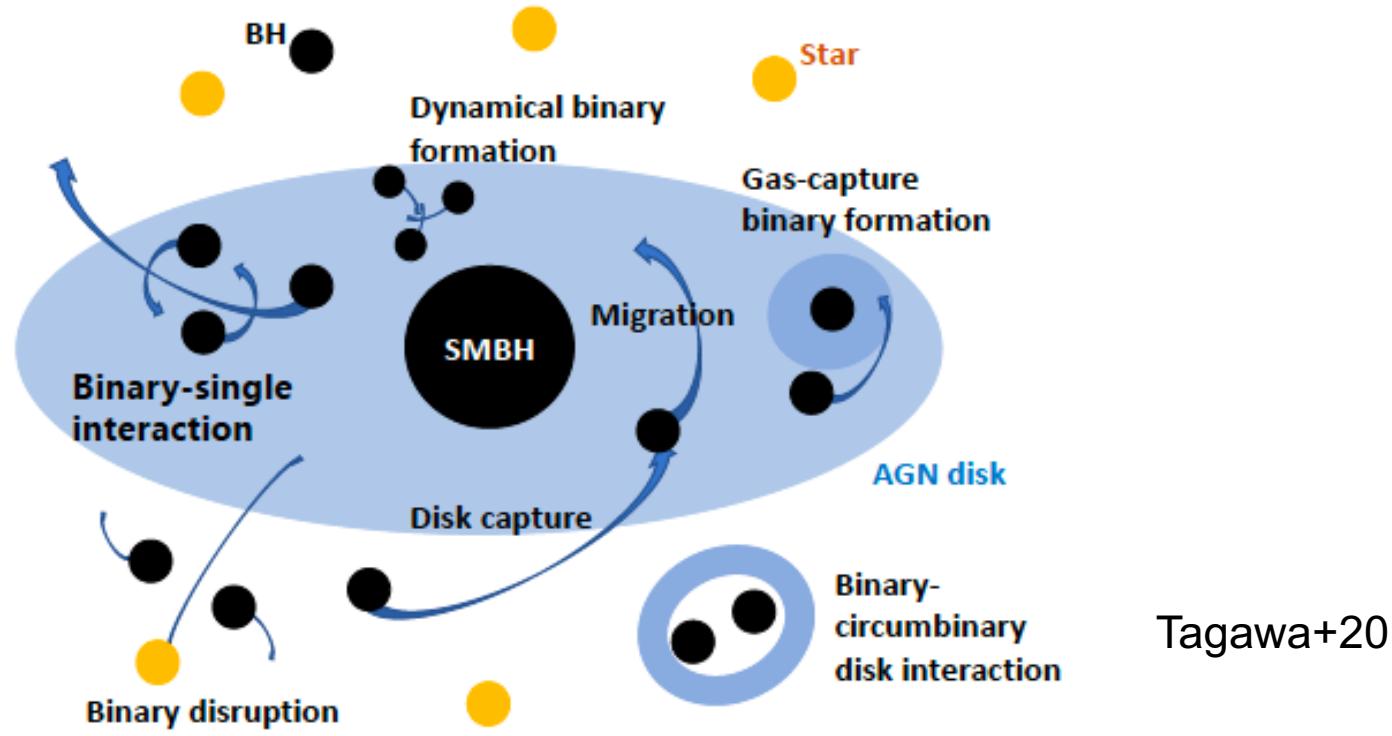
0.04-0.5 pc: OB/WR stars in disk

Broad emission lines of AGN

=> metal rich (independent of redshift)

# Dynamical Formation Channel:

## 3. Binary BH Mergers in AGN (Active Galactic Nuclei) disks



Several merger scenarios...

Previous works: Bellovary+16, Bartos+16, Stone+17, McKernan+18, Secunda+18, Yang+19, Tagawa+20...

## **AGN channel merger scenarios:** (leading to different “predictions”)

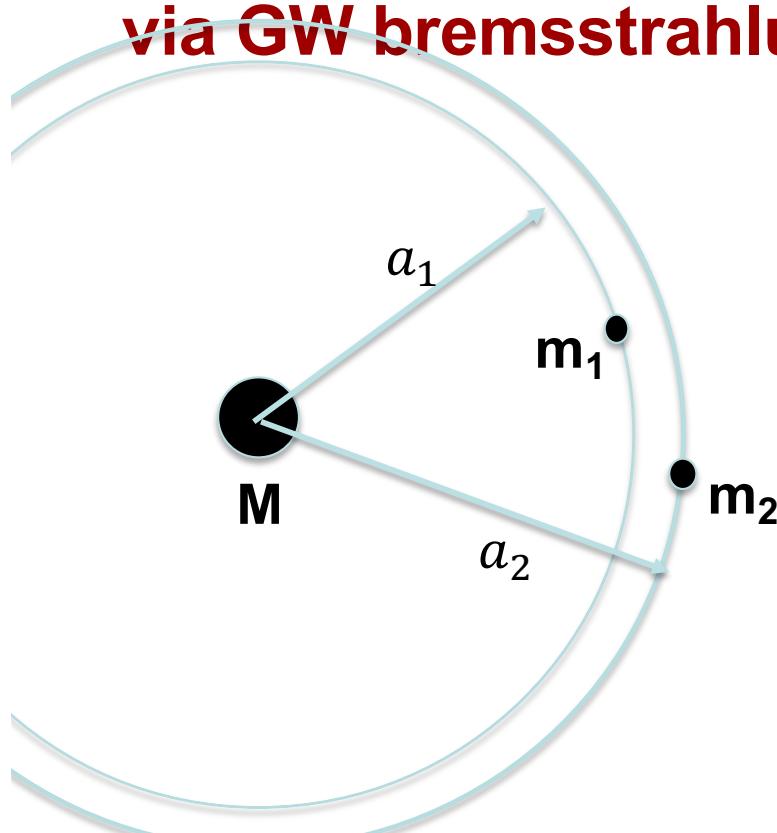
- Gas-free (essentially) mergers
  - AGN disk helps to bring BHs to a plane...
- Gas-assisted mergers

# Gas-free single-single “collision” via GW bremsstrahlung



Li, Lai & Rodet 2022  
Also: Rom, Sari & Lai 2024

Jiaru Li, Cornell Ph.D.23  
→ Northwestern



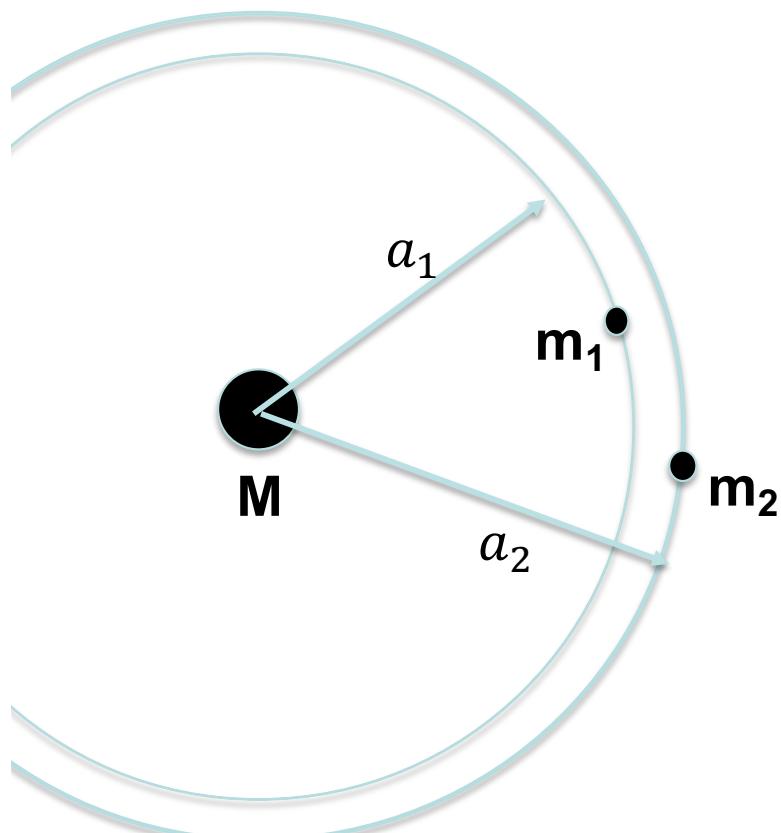
Two BHs ( $m_1, m_2$ ) on closely-packed, nearly circular, nearly-coplanar orbits around a SMBH (M) (e.g. brought together by migration in AGN disks)

When  $a_2 - a_1 \lesssim 3.46 R_H$

$$R_H = a_1 \left( \frac{m_{12}}{3M} \right)^{1/3}, \quad m_{12} = m_1 + m_2$$

orbits are dynamically unstable.  
**What happens to the two BHs?**

## Two planets in unstable orbits around a star:



Three outcomes:

1. Ejection of lower-mass planet
2. Planet-planet collision
3. Injection into the the near vicinity of star

## Two BHs in unstable orbits around a SMBH:

Since  $M/m_{12} \sim 10^6 \gg 1$

Ejection and injection are not possible  
(takes many orbits > Hubble time)

Since  $\frac{GMm_{1,2}}{a} \gg \frac{Gm_1m_2}{a}, \frac{Gm_1m_2}{R_H}$

→ The two BHs undergo “chaotic” motion, experience recurring closer encounters (separation  $< R_H$ )

**For VERY close encounter:**

$$\text{GW emission} \quad \Delta E_{\text{GW}} \sim \frac{\mu^2 m_{12}^{5/2}}{r_{\text{rel}}^{7/2}} \gtrsim \frac{G m_1 m_2}{R_{\text{H}}}$$

$$\xrightarrow{} \frac{r_{\text{rel}}}{R_{\text{H}}} \lesssim 10^{-4} \left( \frac{4\mu}{m_{12}} \right)^{2/7} \left( \frac{10^6 m_{12}}{M} \right)^{10/21} \left( \frac{a_1}{100M} \right)^{-5/7}$$

Capture radius for forming “permanent” binary  
due to GW bremsstrahlung

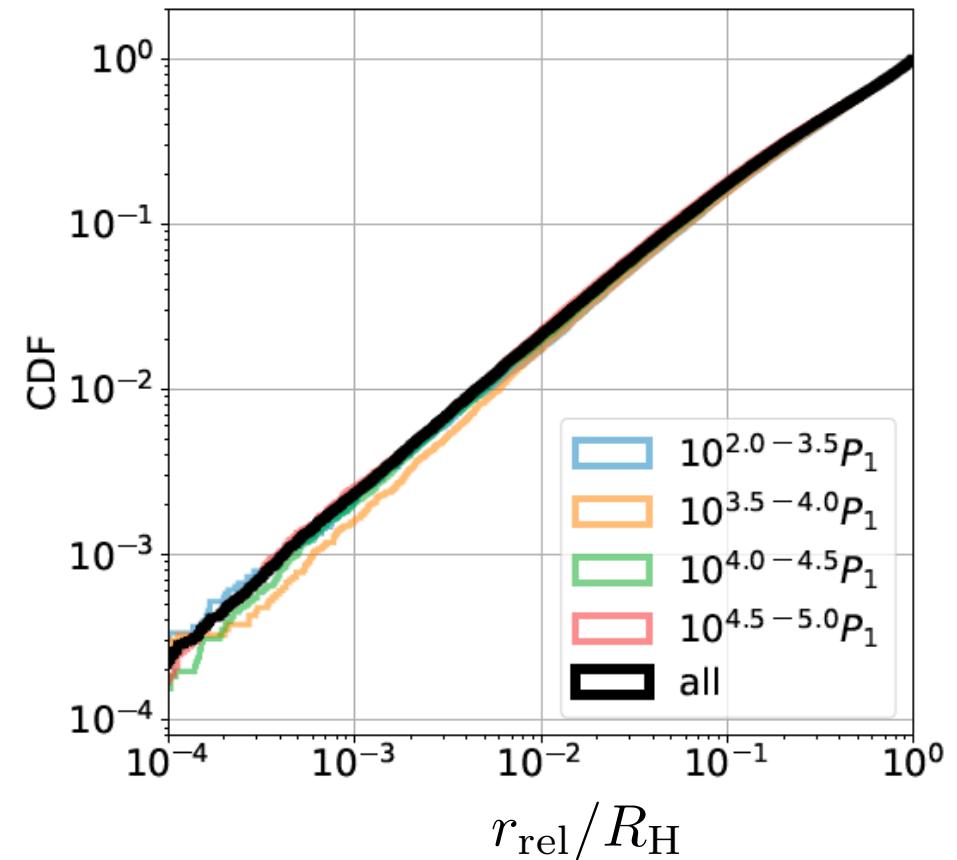
# How likely/often does this happen? ("loss cone" problem...)

Probability of very close encounters (with separation  $< r_{\text{rel}} \ll R_{\text{H}}$ )

$$P(< r_{\text{rel}}) \simeq \frac{r_{\text{rel}}}{R_{\text{H}}}$$

$$\leftrightarrow \frac{dP}{d l_{\text{rel}}} \propto l_{\text{rel}}$$

$$l_{\text{rel}} \simeq \sqrt{2m_{12}r_{\text{rel}}}$$



Two BHs get captured into a (very eccentric) binary via GW bremsstrahlung

$$f_{\text{cap}} \simeq (1.4 \text{ Hz}) \left( \frac{4\mu}{m_{12}} \right)^{-3/7} \left( \frac{M}{10^8 M_\odot} \right)^{-2/7} \left( \frac{m_{12}}{100 M_\odot} \right)^{-5/7} \left( \frac{a_1}{100 M} \right)^{-3/7}$$

Once captured, it will take a few orbital period to merge  
it enters LIGO band with  $e \gtrsim 0.1$

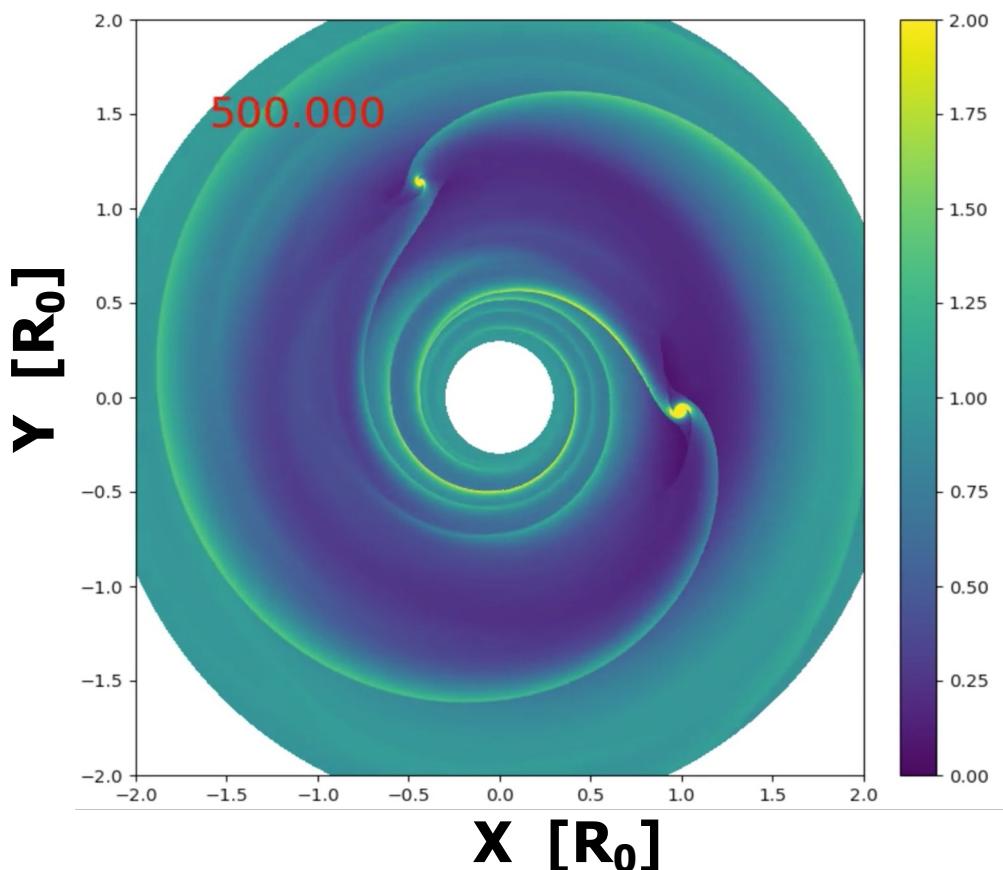
This mechanism always produces eccentric mergers

Li, Lai & Rodet 2022  
Rom, Sari & Lai 2024

# Gas Effects: Formation of BH binaries

## hydrodynamics simulations

Jiaru Li ... Hui Li's LANL group... 2023,2025



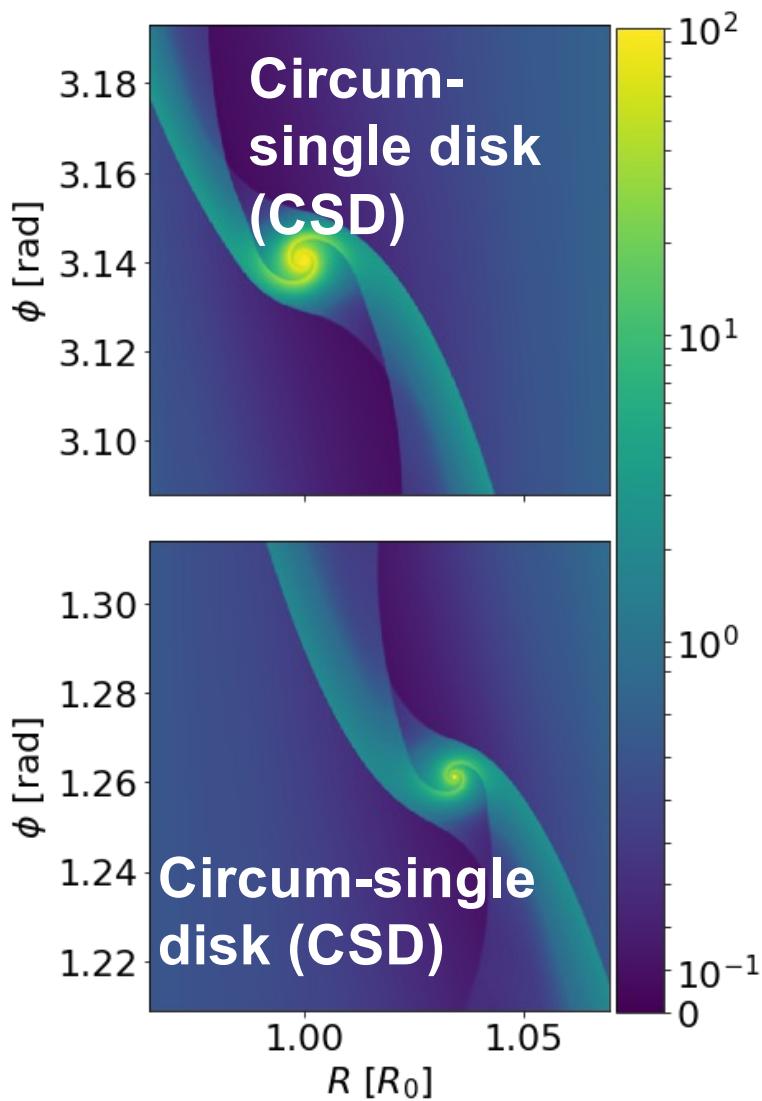
- Initial condition:

$$a_2 - a_1 = 2R_H$$

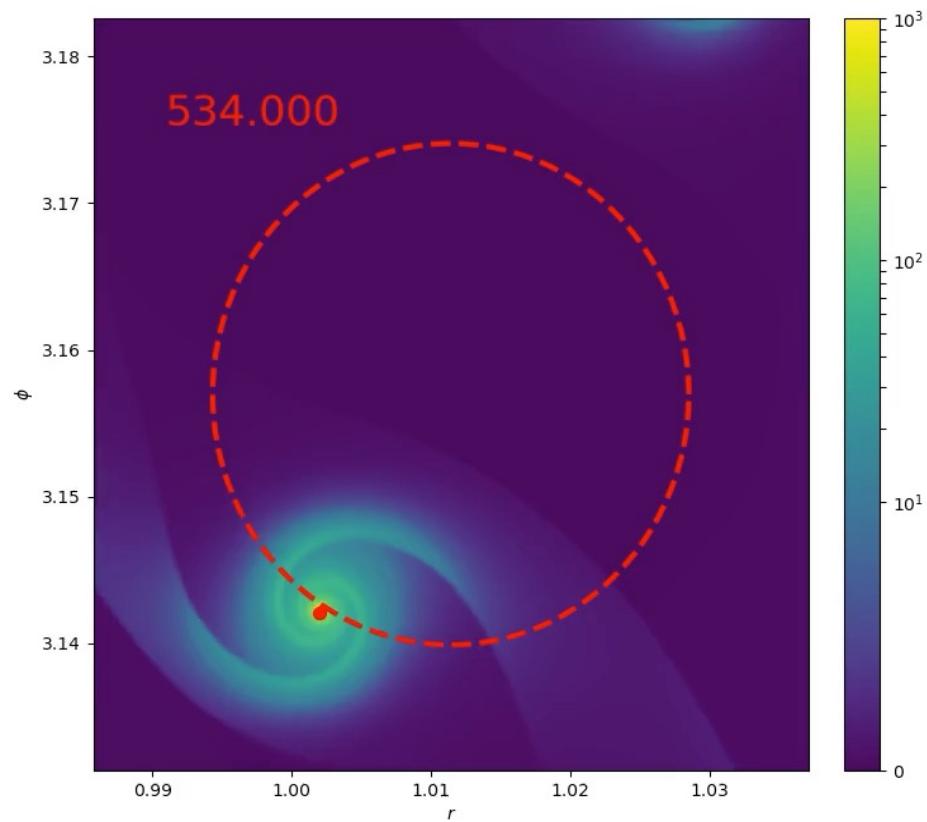
- Simulation setup:

- $M_{\text{SMBH}} = 1$ ,  $m_1 = 10^{-5}$ ,  $m_2 = 5 \times 10^{-6}$
- Thin disk  $H/R = 0.01$ , low viscosity  $\alpha = 0.01$ .
- Isothermal disk.
- High resolution with  $50 \rightarrow 100$  grid cells per  $R_H$ , where  $R_H = 0.017R_0$

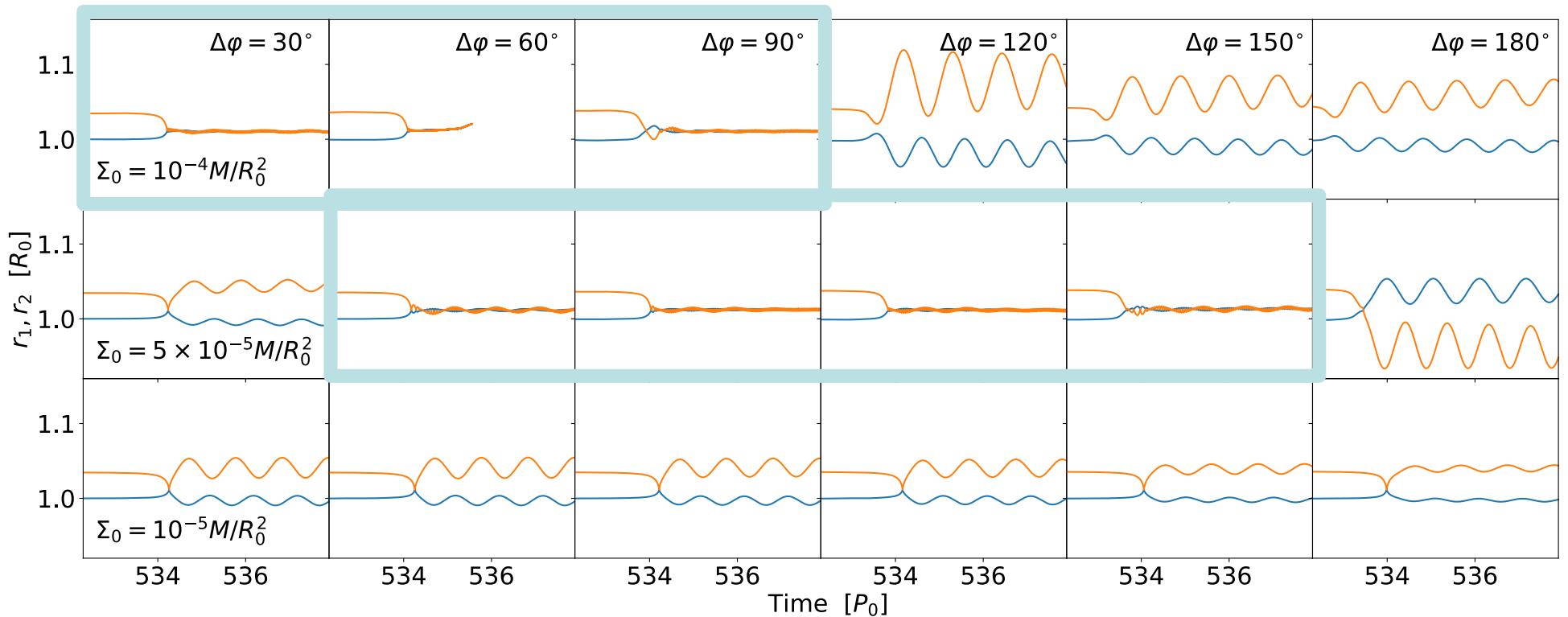
See also Rowan et al. 2023,24;  
Whitehead et al 2024



## Formation of a binary



**Outcomes:** depend on gas density,  
initial  $a_2 - a_1 = KR_{\text{H}}$  and orbital phase

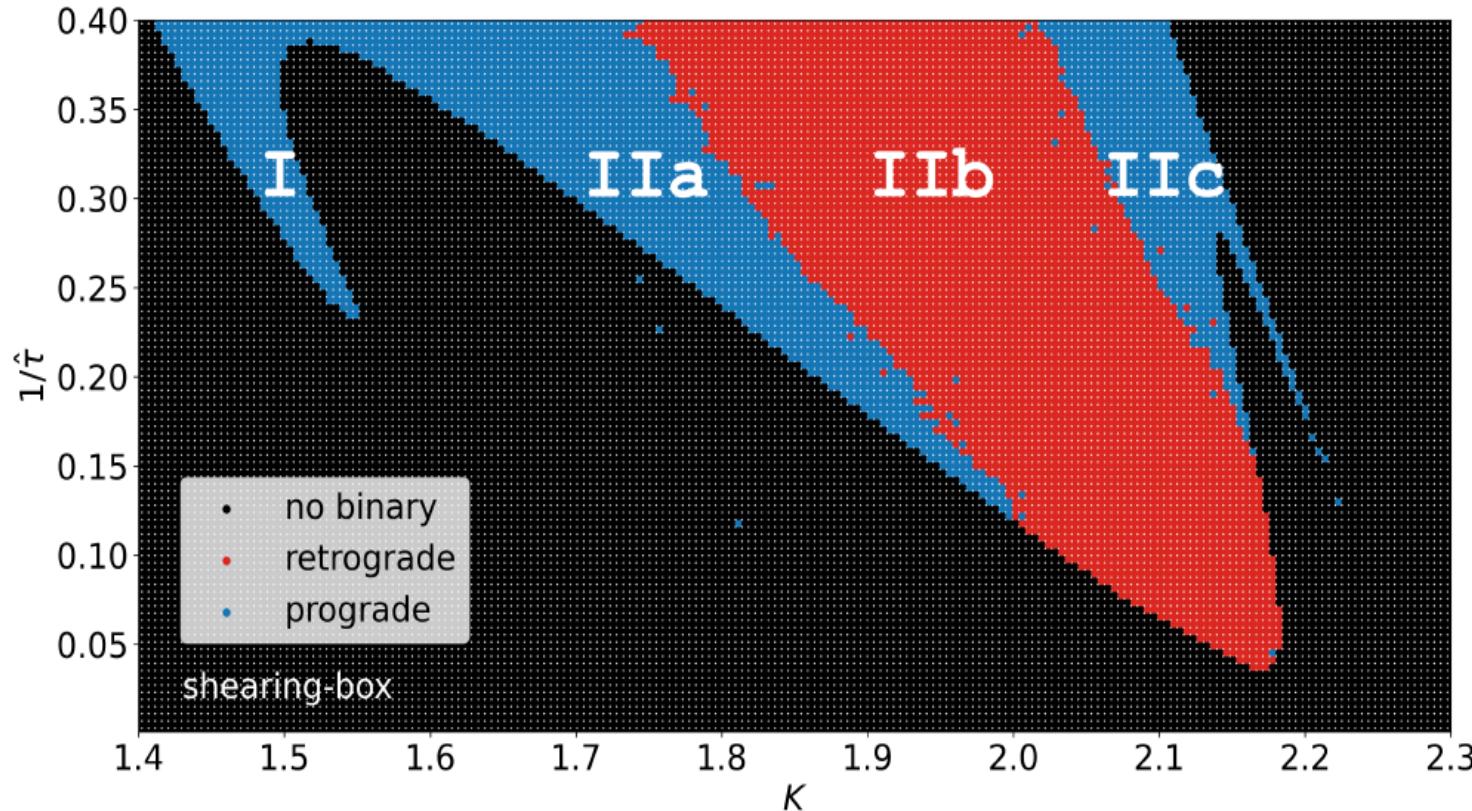


## Gas-assisted binary formation: Toy model calculation

Gas drag on each BH:  $F_{\text{drag}} = -m \frac{\mathbf{v} - \mathbf{v}_K}{\tau}$

# Gas-assisted binary formation: Toy model calculation

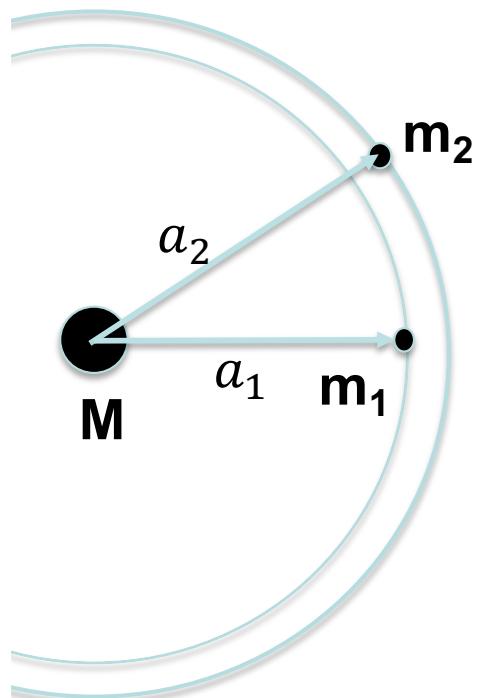
$$\text{Gas drag on each BH: } \mathbf{F}_{\text{drag}} = -m \frac{\mathbf{v} - \mathbf{v}_K}{\tau}$$



Qian, Jiaru Li & DL 2024

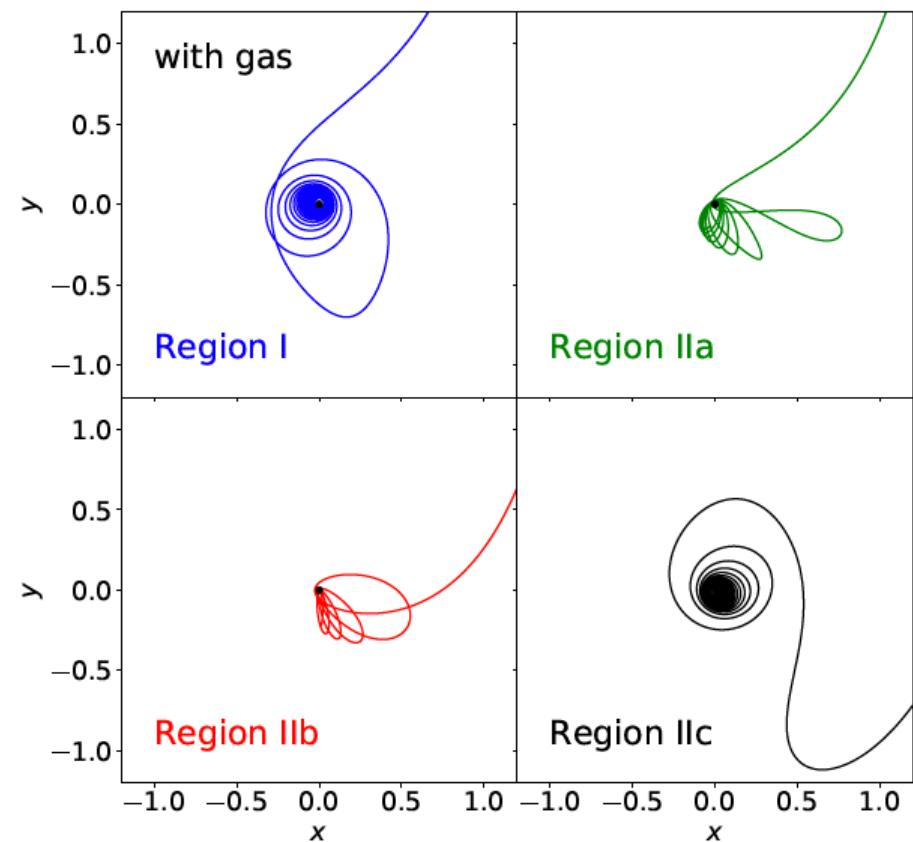
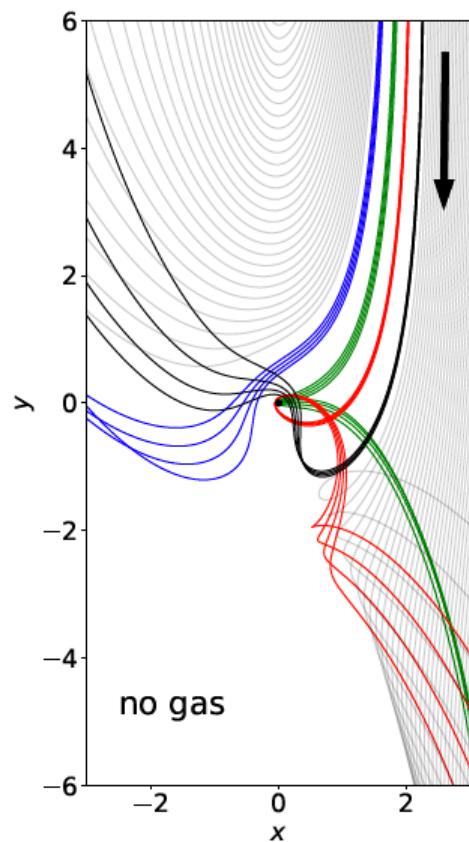
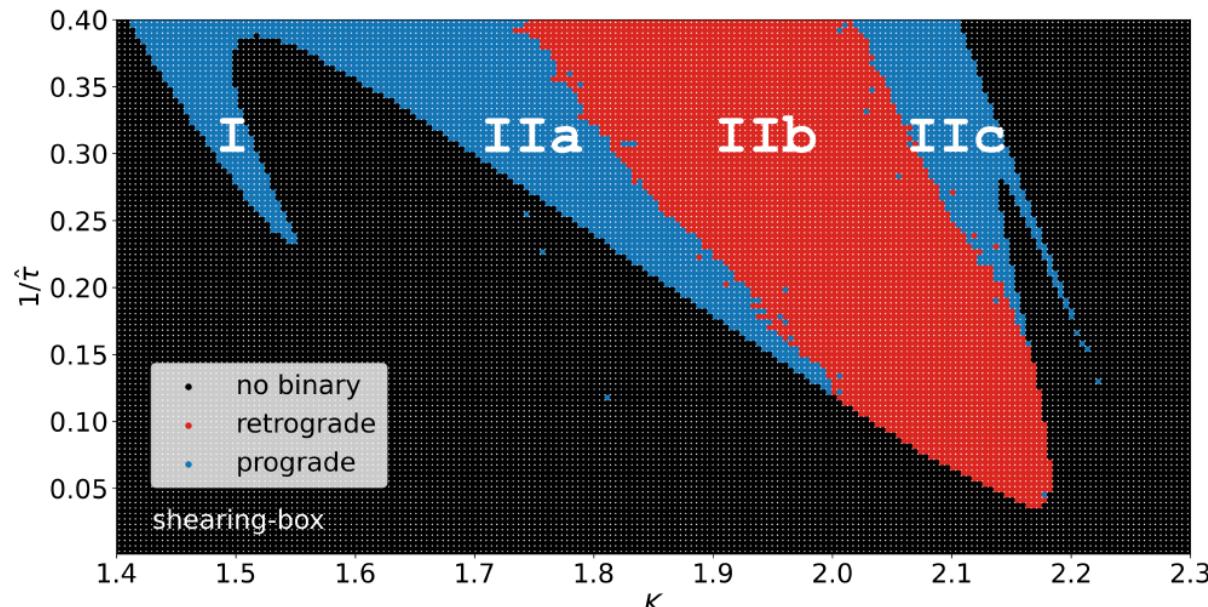
Kecheng (Stephon) Qian  
(Cornell '24 -> UC Berkeley)

See also  
Tjarda Boekholt et al. 2023  
Mark Dodici & Tremaine 2024



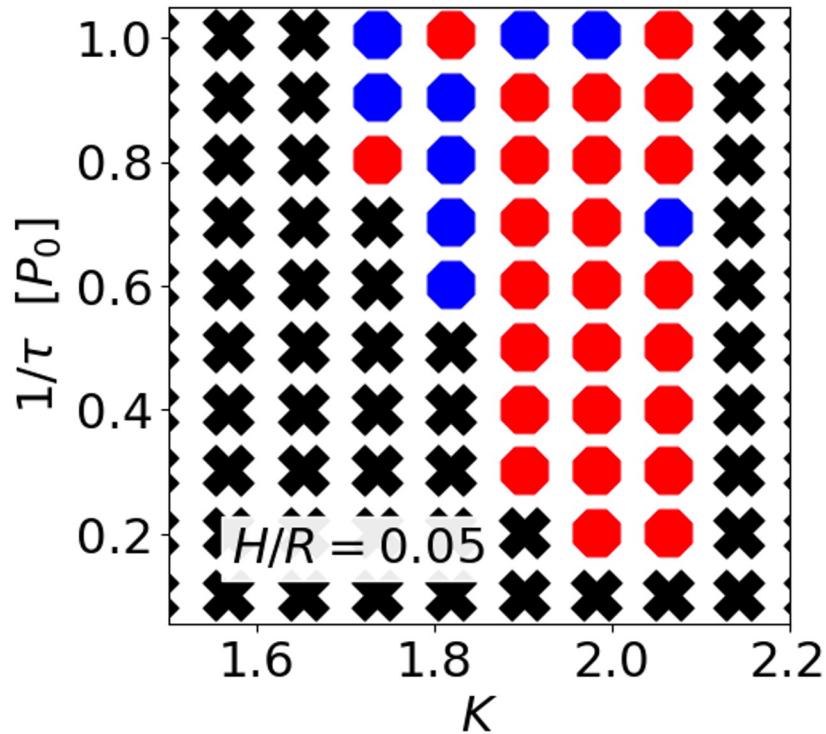
$$a_2 - a_1 = KR_H$$

See Petit & Helon 1986  
“Satellite Encounters”

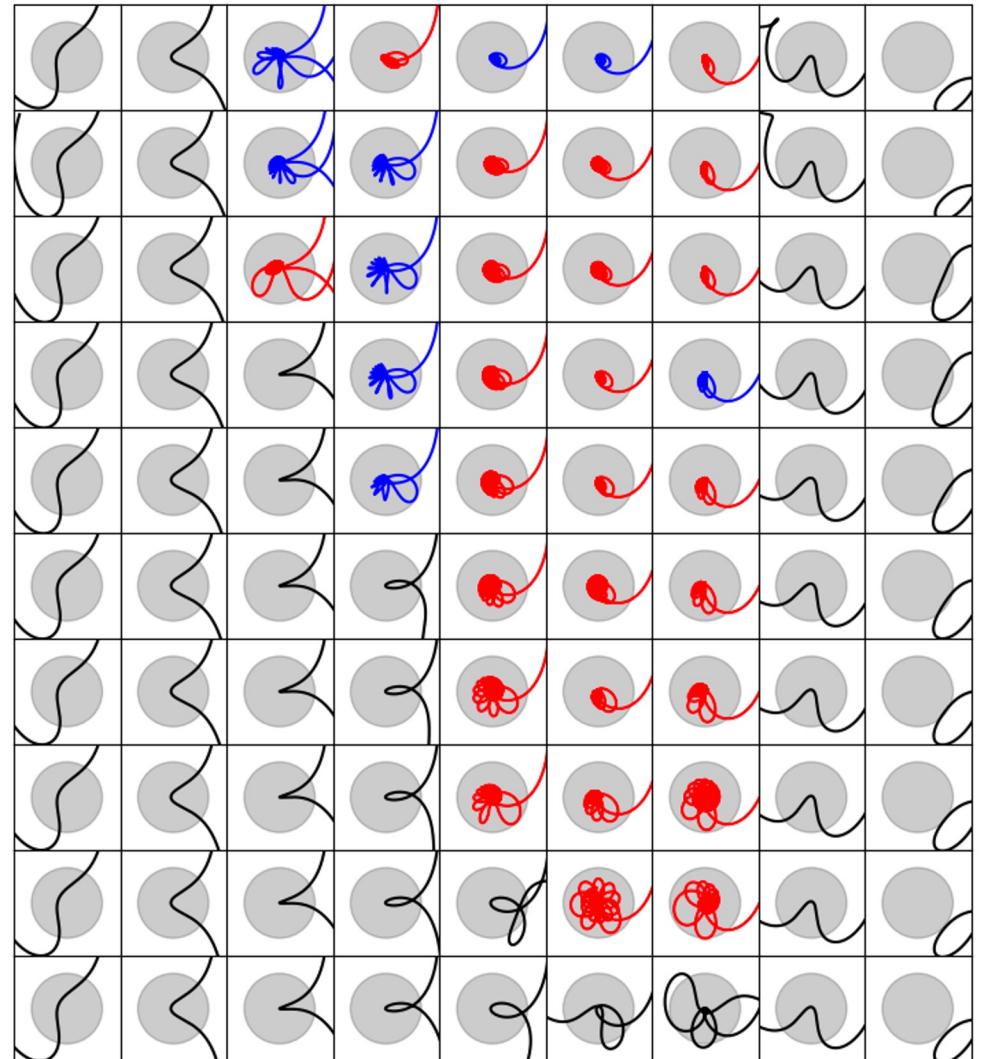


# 2D Hydro simulation survey:

Jiaru Li et al, 2025 (in prep)



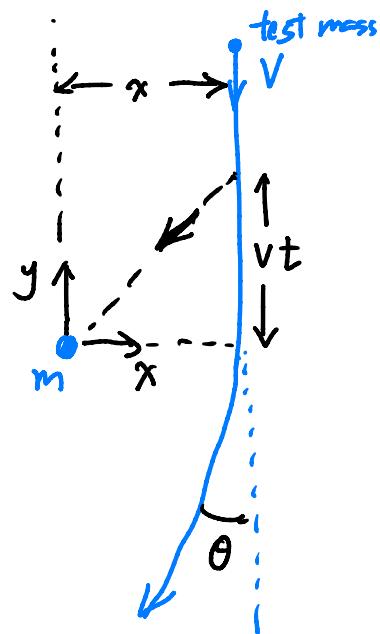
Relative trajectories of the BHs  $\Rightarrow$



Capture happens only for  $1.7 \lesssim K \lesssim 2.1$       Why?  $\rightarrow$

## Digression:

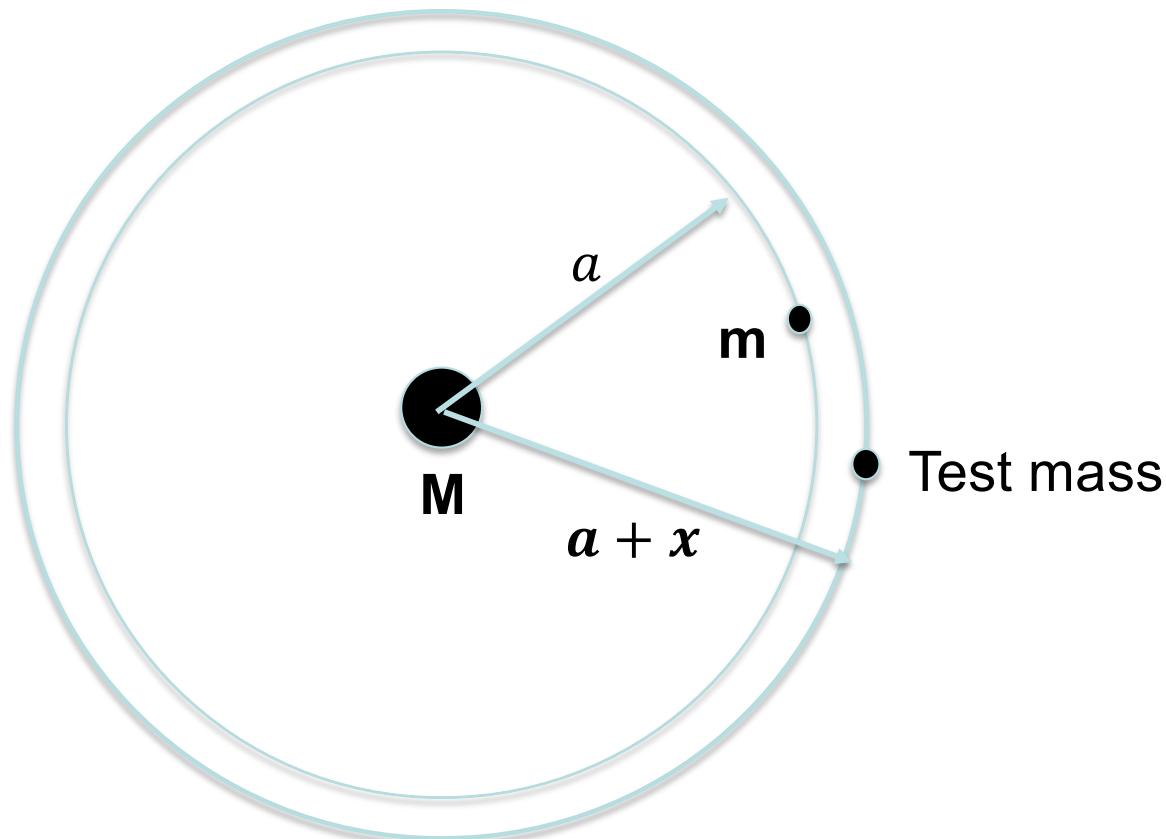
### Two-body encounter



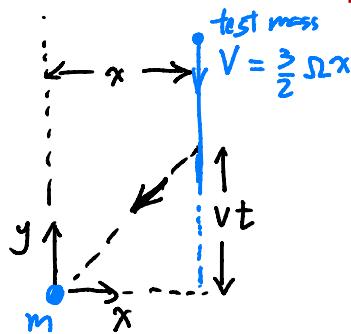
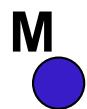
Impulse approximation

$$\Delta P_x \approx - \frac{Gm}{v} \int_{-\infty}^{+\infty} \frac{x dt}{(x^2 + v^2 t^2)^{3/2}}$$
$$= - \frac{2 Gm}{x v}$$

$$\text{Deflection angle } \theta \approx \frac{2 Gm}{x v^2}$$



# Two-body encounter in the presence of M



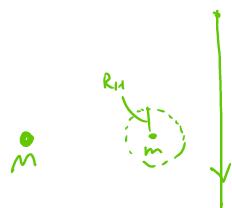
$\Delta J$  = loss of angular momentum by the test mass due to  $m$ 's gravity from  $\infty$  to the point of the closest approach

Impulse approximation

$$\Delta J = \int_{-\infty}^0 dt \alpha |F_y|$$

$$= \frac{Gma}{x^2} \frac{2}{3\pi}$$

Depending on  $x$ , there are 3 outcomes



\* If  $\Delta J(x) < \sqrt{GM(a+x)} - \sqrt{GM(a+R_H)} \approx \sqrt{Gma} \frac{x-R_H}{za}$

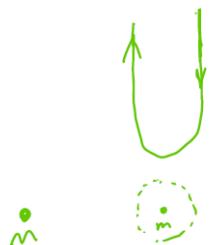
$$\Leftrightarrow x > 2R_H$$

the particle loses little AM and will always stay outside  $R_H$

\* If  $\Delta J(x) > \sqrt{GM(a+x)} - \sqrt{Gma} \approx \sqrt{Gma} \frac{x}{za}$

$$\Leftrightarrow x \approx 1.6 R_H$$

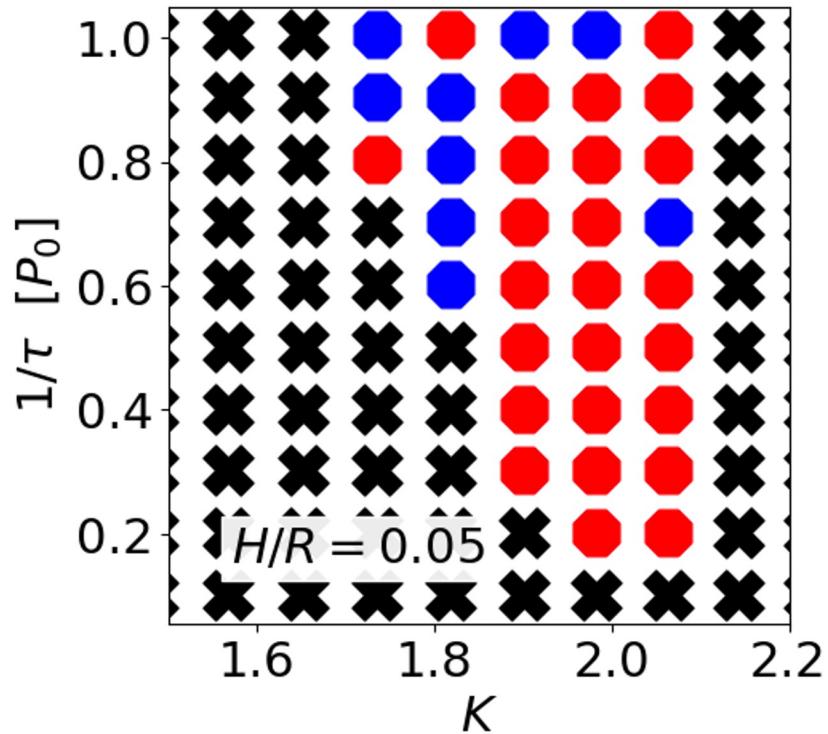
the particle loses enough AM to be pushed into the  $x < 0$  region  
 $\Rightarrow$  horseshoe orbit



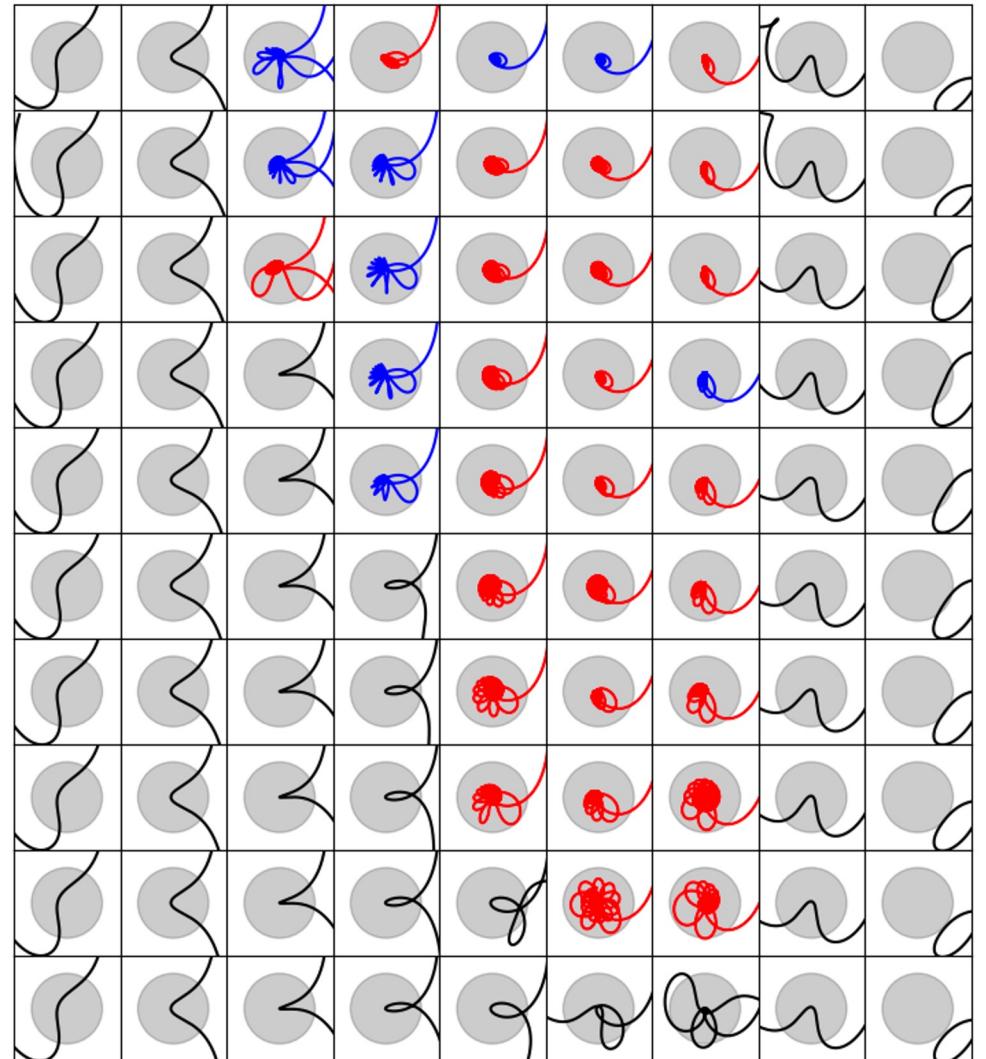
\* If  $1.6 R_H \leq x \leq 2R_H$ , the particle collides with Hill sphere.

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Jiaru Li et al, 2025 (in prep)

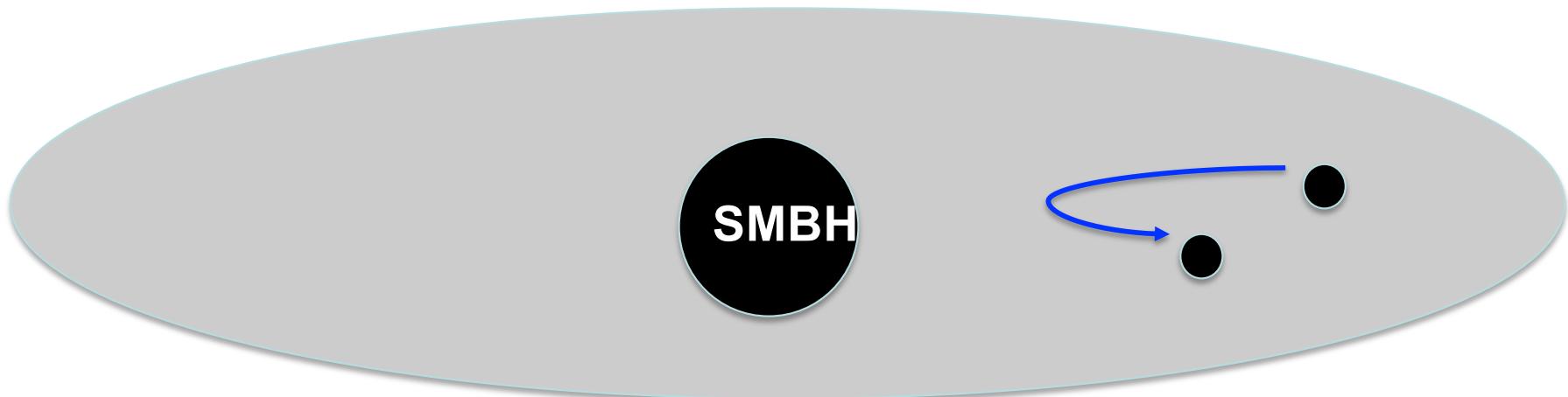


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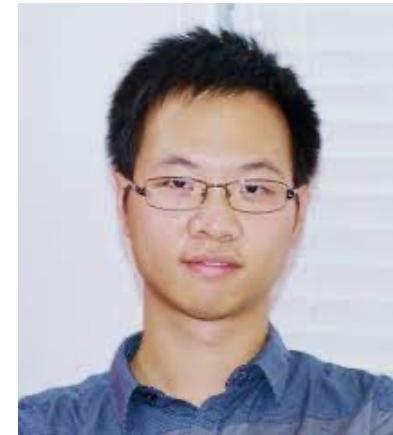
# Gas effects: Embedded Binary in AGN Disk



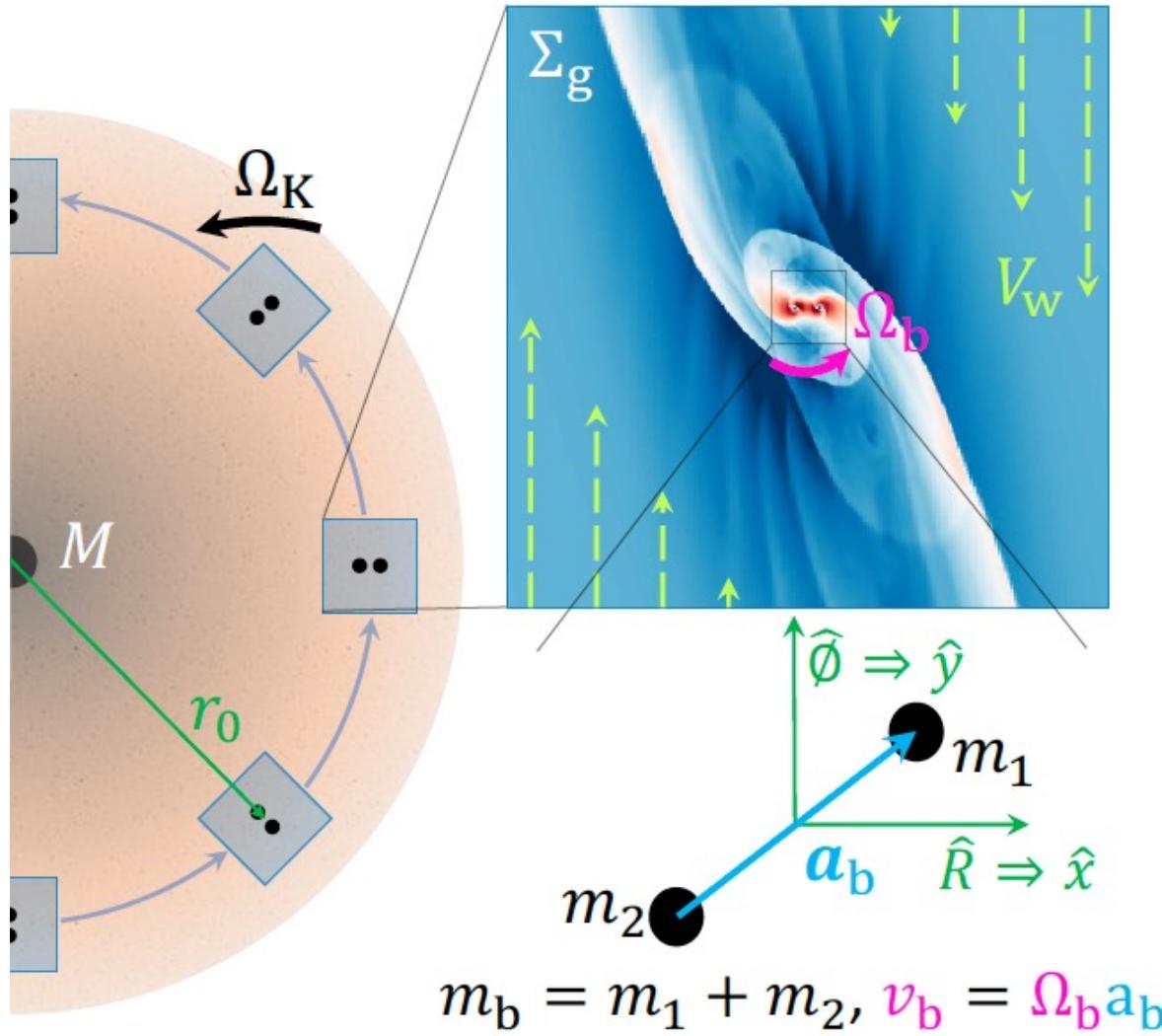
Baruteau et al. 2011  
Y.Li,... H.Li... 2021  
Dempsey, H.Li... 2023  
R.Li & DL 2022,23,24

# Local simulations of binary in disk

R.Li & Lai 2022,2023,2024

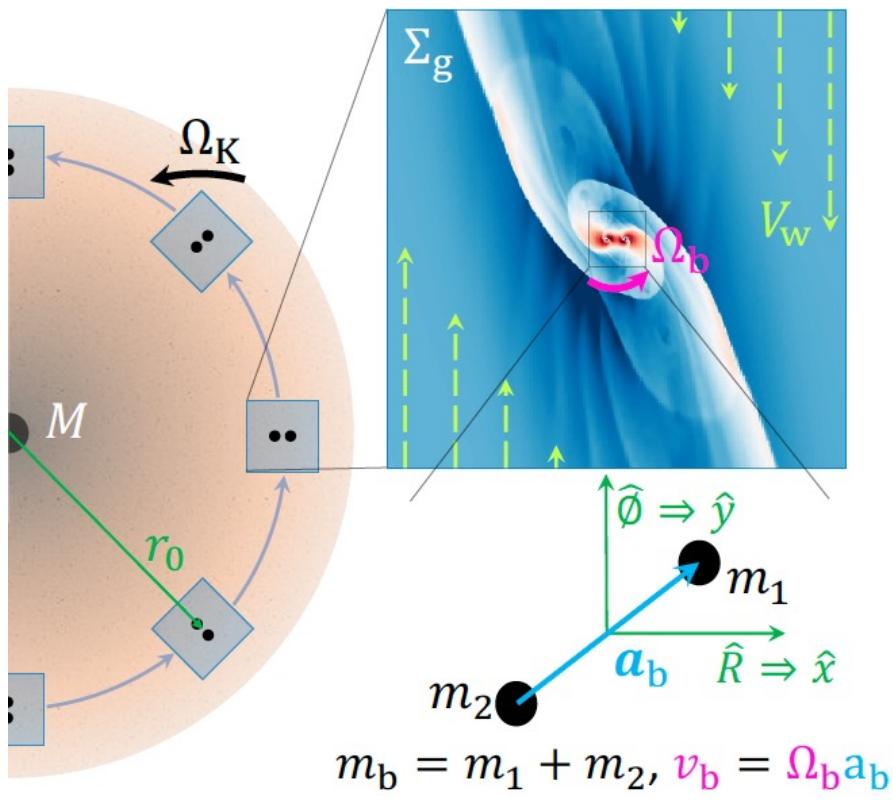


Dr. Rixin Li  
(Cornell -> Berkeley)



Local shearing box  
(not "local wind tunnel box"  
used by Kaaz et al. 2021)

ATHENA  
Mesh refinement  
Resolution:  $a_b \sim 250$  cells  
zero softening in gravity



## Length scales of the problem:

$$a_b, \quad R_B \sim \frac{Gm_b}{c_\infty^2}, \quad R_H \sim r_0 \left( \frac{m_b}{M} \right)^{1/3}, \quad H$$

## Velocity scales of the problem:

$$v_b, \quad c_\infty, \quad V_{\text{shear}}$$

## → Dimensionless ratios:

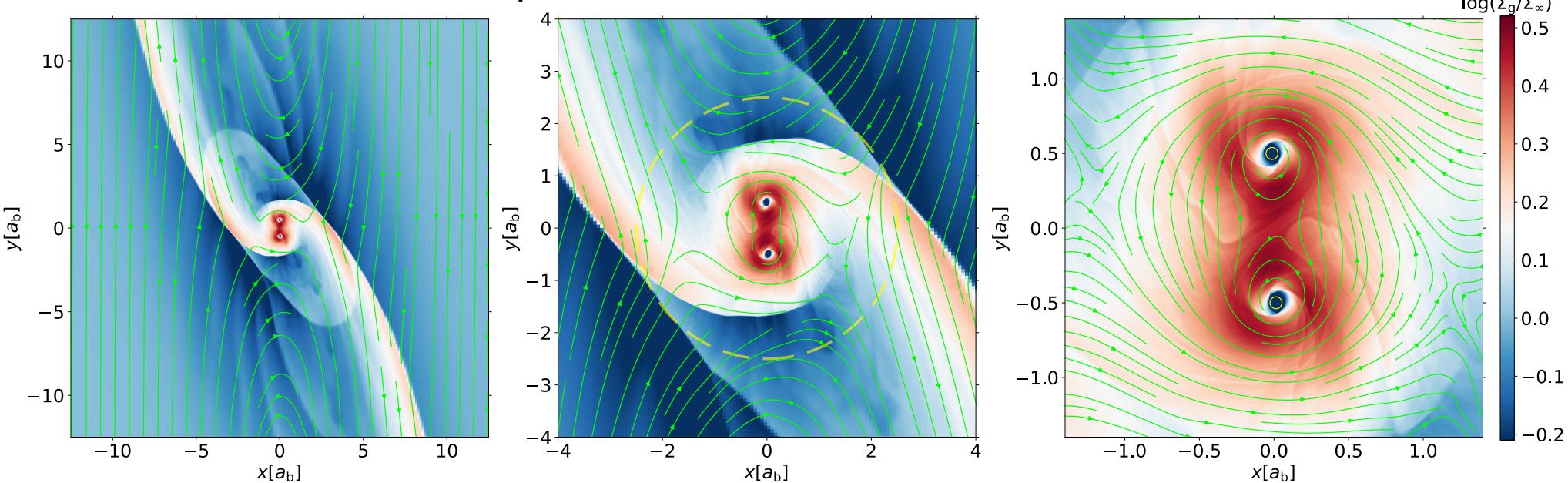
$$\frac{q}{h^3}, \quad \frac{R_H}{a_b} \equiv \lambda$$

where  $q = m_b/M, h = H/r_0$

$m_2/m_1, \quad e_b, \quad \text{EOS}$  (e.g.  $\gamma$  law)

# Example of flow structure

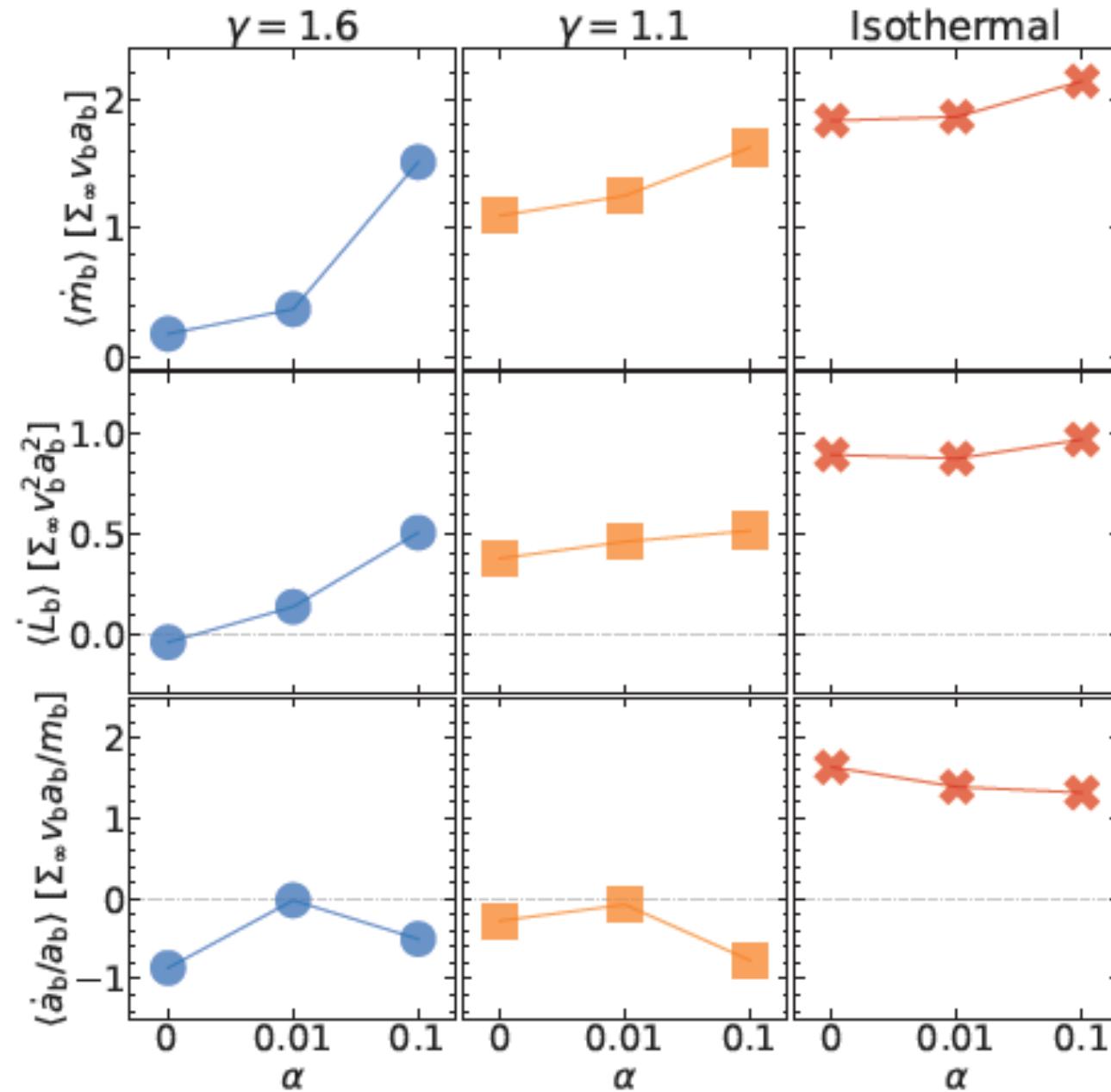
Pairs of bow shocks, spiral shocks



BH = absorbing sphere: sink radius:  $r_{\text{sink}} = 0.04 a_b \simeq 10$  cells  $\rightarrow \dot{m}_b$

Force on each BH: from gravity + accretion + pressure

$\rightarrow$  Torque on binary, energy transfer rate  $\rightarrow \dot{a}_b, \dot{e}_b$



Rixin Li & DL 2024

$$\frac{m_1}{m_2} = 1, \quad e_b = 0, \quad \frac{R_H}{H} = \left(\frac{q}{h^3}\right)^{1/3} = 3^{1/3}, \quad \frac{R_H}{a_b} = 5 \quad \text{prograde}$$

# Embedded binaries in gas disks

## Prograde binaries:

Orbital decay or expansion? Depends on gas thermodynamics, viscosity, size of accretor...

## Retrograde binaries:

Always orbital decay

In general, orbital evolution of the binary is much faster than migration of the center of mass of binary in AGN disk.

# Take-Home Messages

## Formation Channels of Merging BH Binaries:

Isolated binary evolution: "standard model"

Dynamical formation channels:

1. Dense star clusters
2. Tertiary-induced mergers
3. BH Mergers in AGN disks

All can contribute: Rates, branching ratios uncertain... "smoking guns"?

More detections (aLIGO, ET), LISA useful for probing dynamical formation

## Planetary dynamics applied to new/different regimes

e.g.

- Spin-orbit coupling: GR vs Newtonian gives different spin obliquities
- BH-BH scatterings around SMBH vs planet-planet scatterings
  - Loss cone via GW bremsstrahlung
- Binary capture vs pebble accretion...

# International Conference on Exoplanets and Planet Formation (EPF)

Shanghai, China | December 8-12, 2025

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Dong Lai, Josh Winn, Yanqin Wu

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