

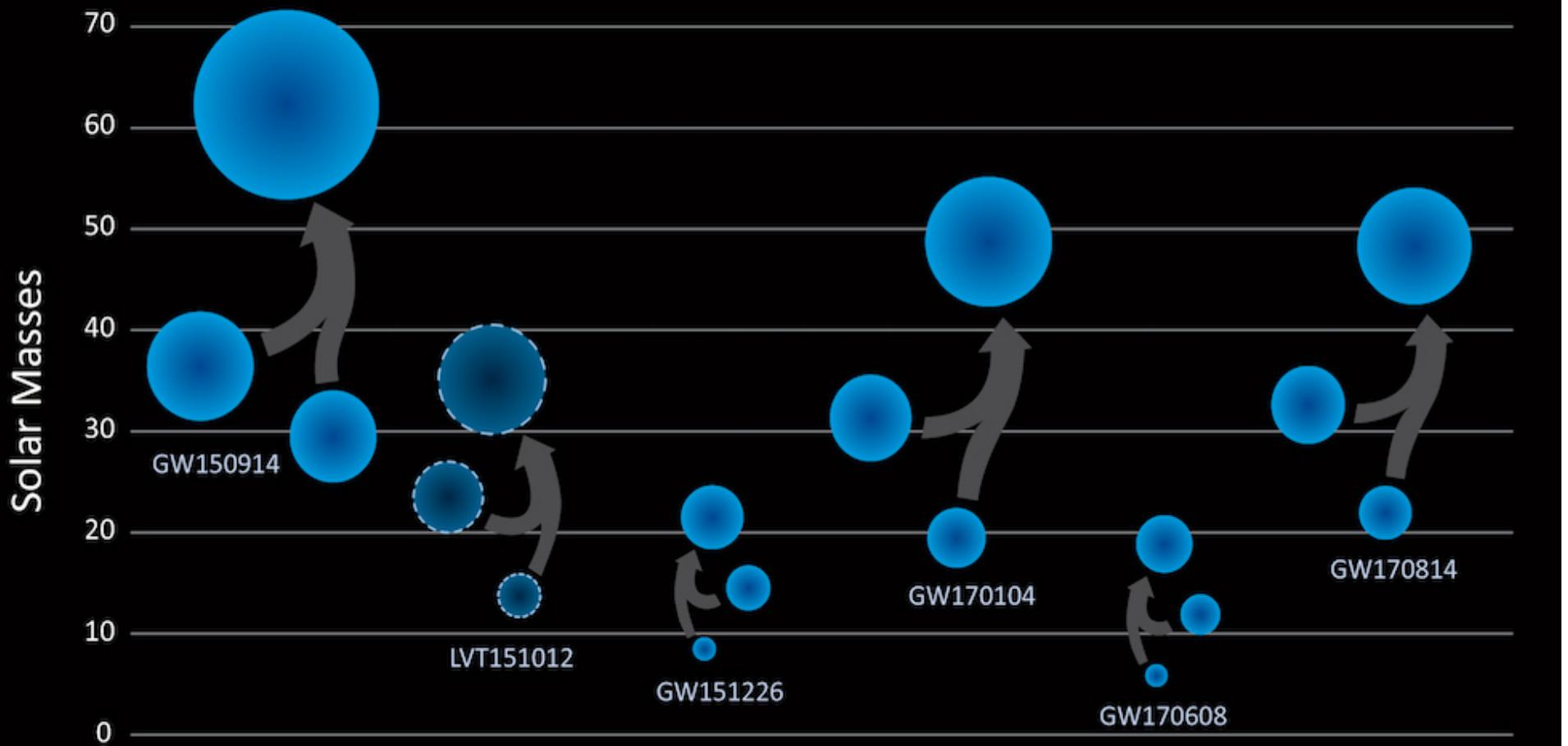
# LIGO Results/Surprises?

**Dong Lai**

Cornell University

“Exploding Universe” Workshop, TDLI, 5/28/2018

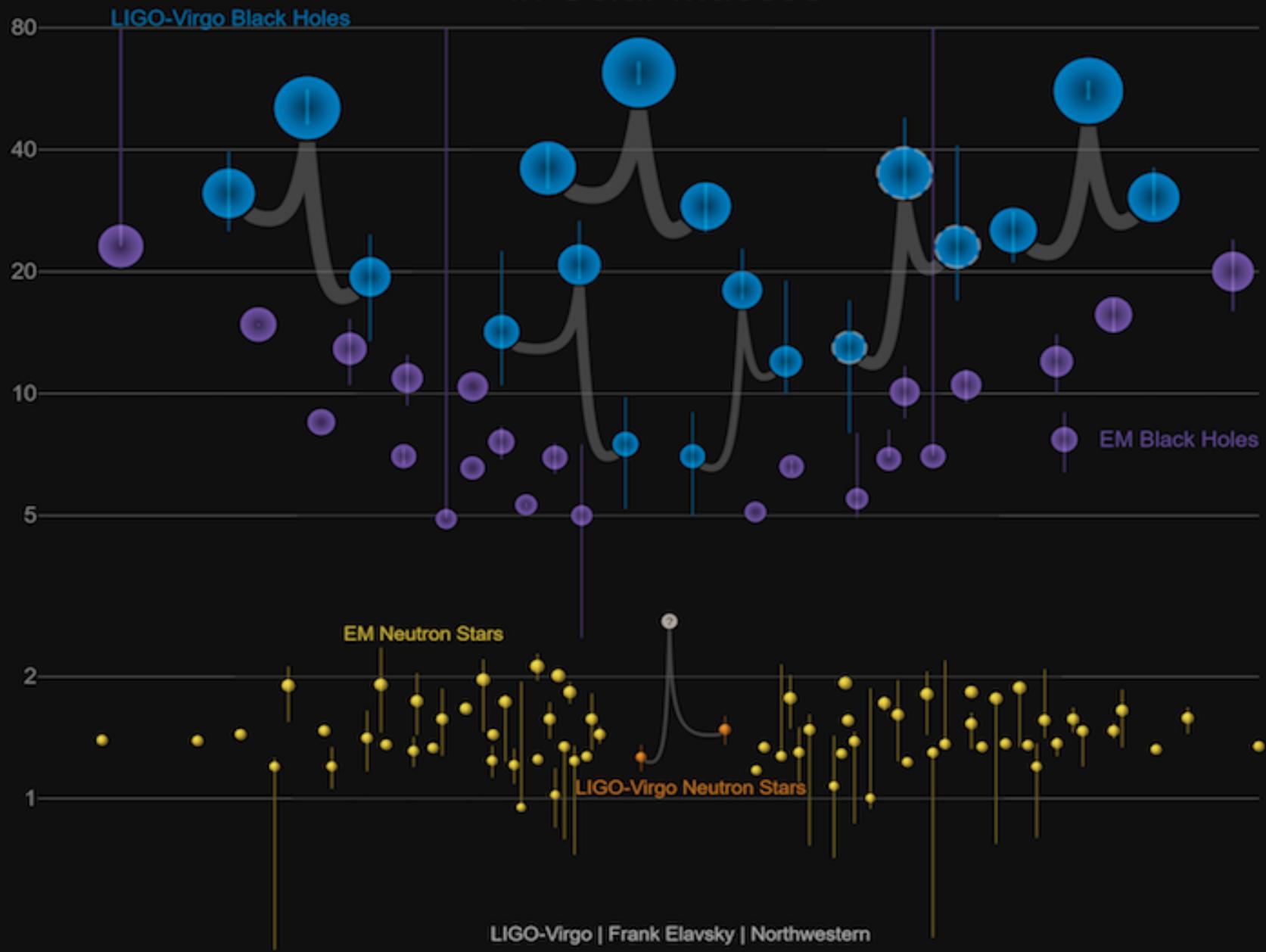
# Black Holes of Known Mass



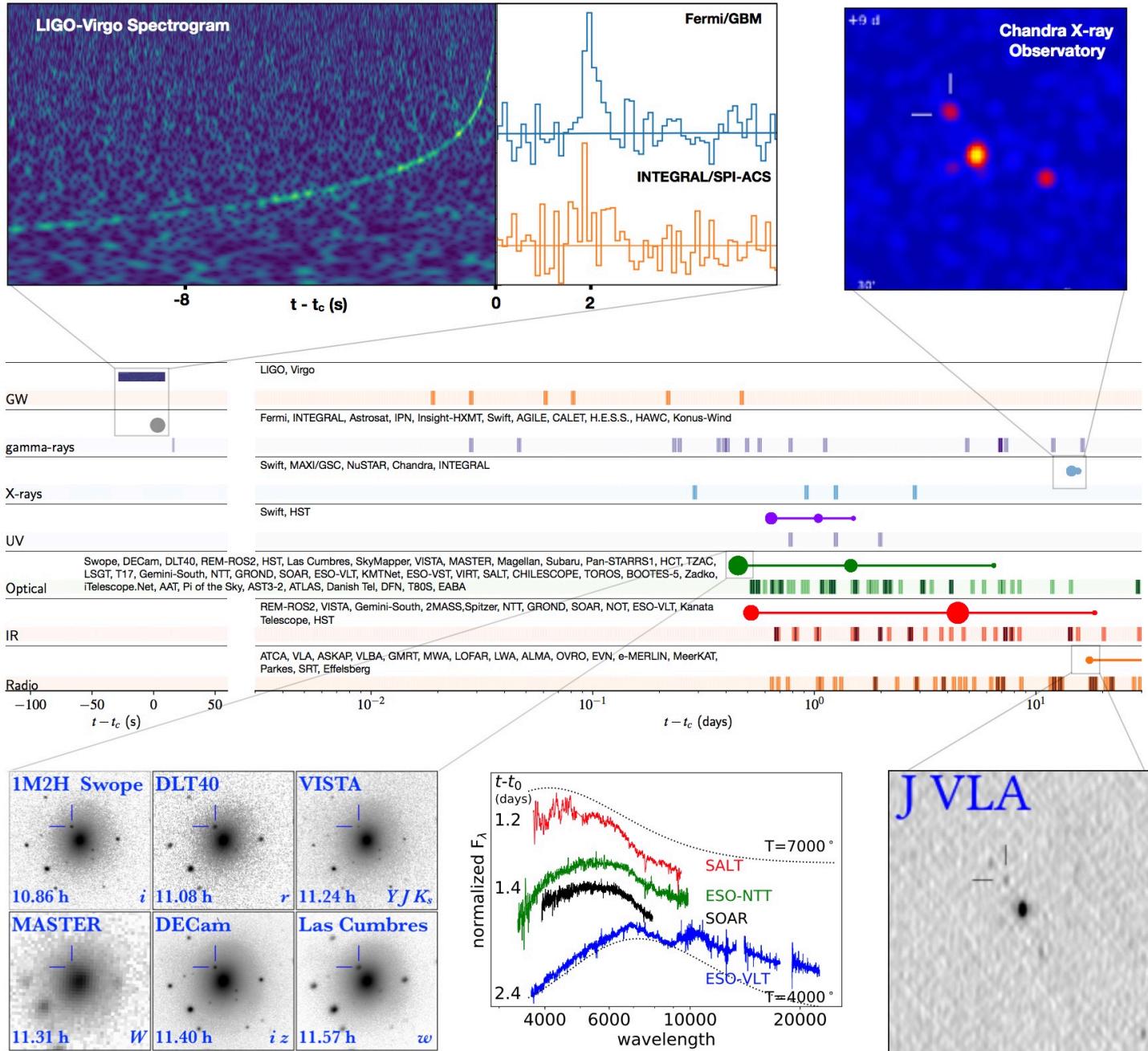
LIGO/VIRGO

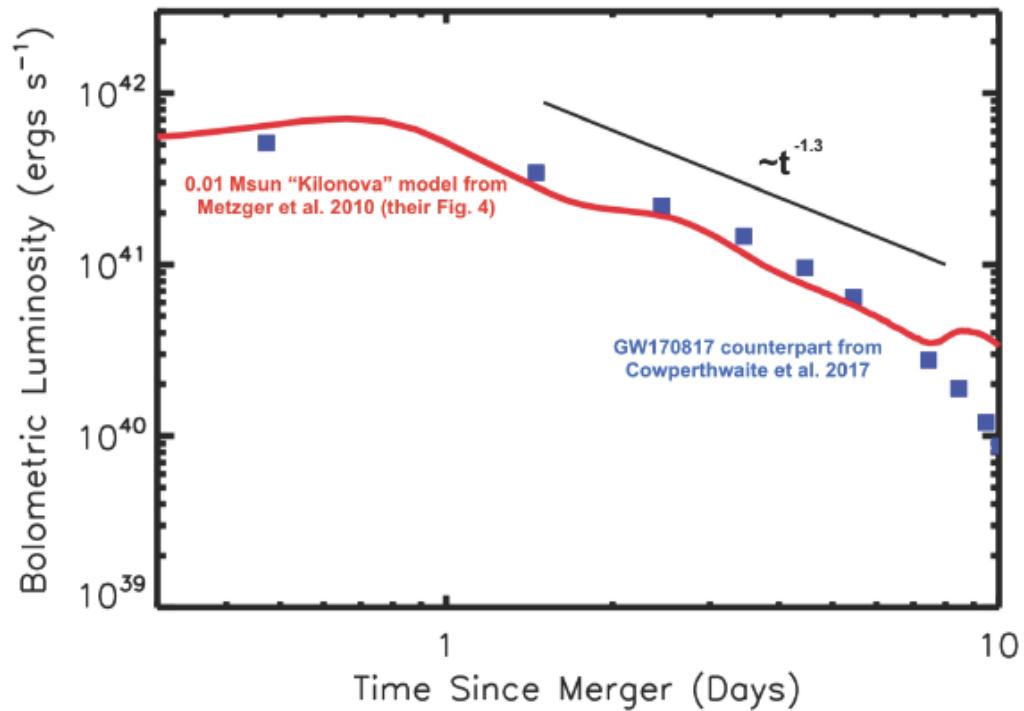
# Masses in the Stellar Graveyard

*in Solar Masses*

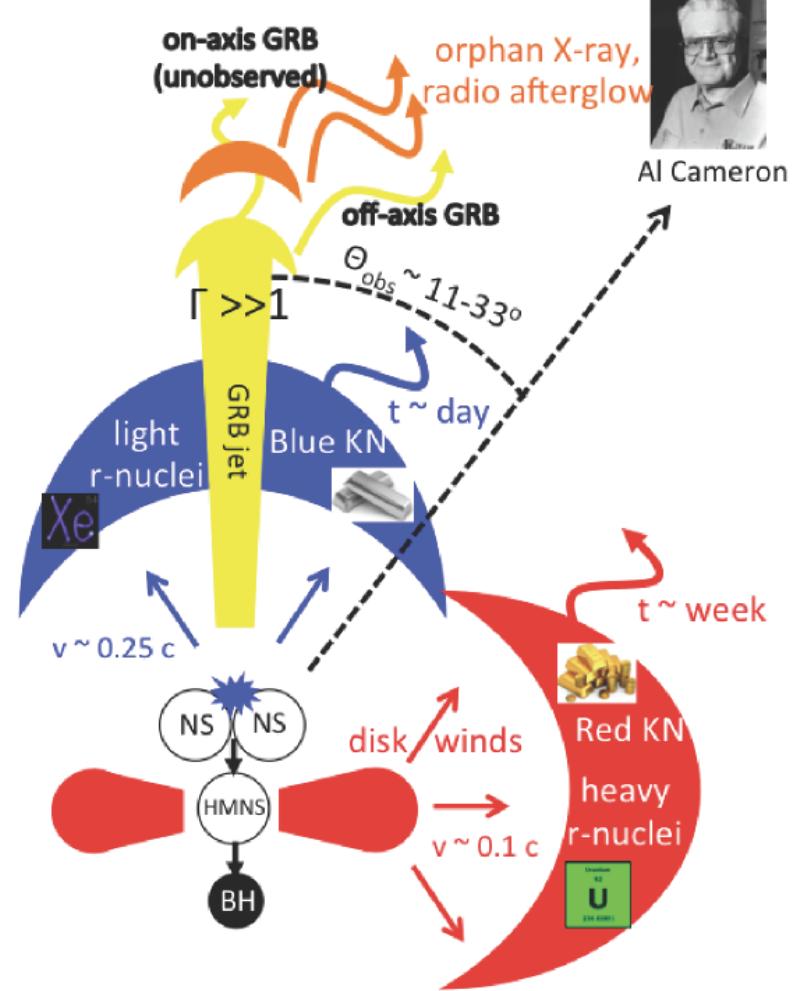


# GW170817 / AT2017gfo





Metzger 2017



# LIGO Surprises?

1. Tidal Resonances → NS EOS
2. Dynamical Formation of BH Binaries and Spin-Orbit Misalignments

**Dong Lai**

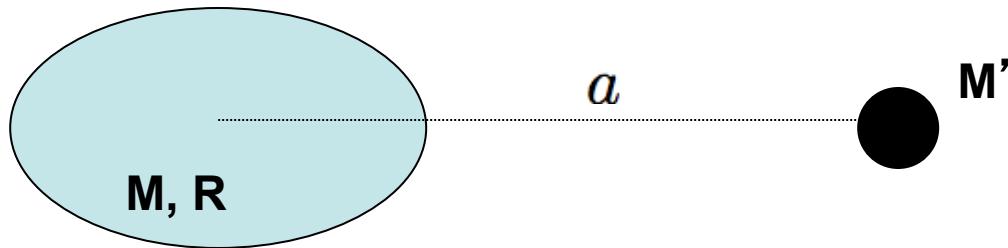
Cornell University

# **Tides in Merging NS Binaries**

**→ Probe NS EOS**

- Quasi-Equilibrium tides
- Resonant (Dynamical) tides

# “Quasi-Equilibrium” Tide (F-mode Distortion)



$$V = -\frac{MM'}{a} - \mathcal{O}\left(k_2 \frac{M'^2 R^5}{a^6}\right) \quad k_2 = \text{Love number}$$

$$d\Phi = d\Phi^{(0)} \left[ 1 - \mathcal{O}\left(\frac{k_2 M' R^5}{Ma^5}\right) \right] \quad (\text{GW Phase Shift})$$

→ **Important only at small separation (just prior to merger)**

(Kochanek; Bildsten & Cutler; Lai, Rasio & Shapiro, etc 1990s)

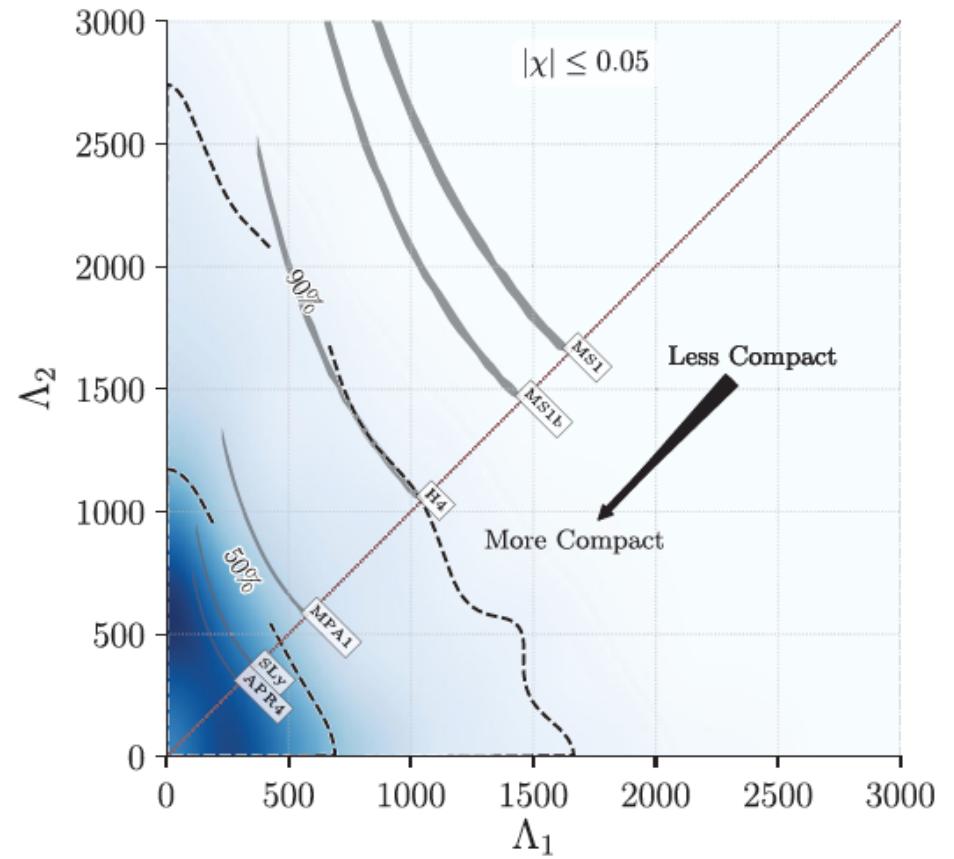
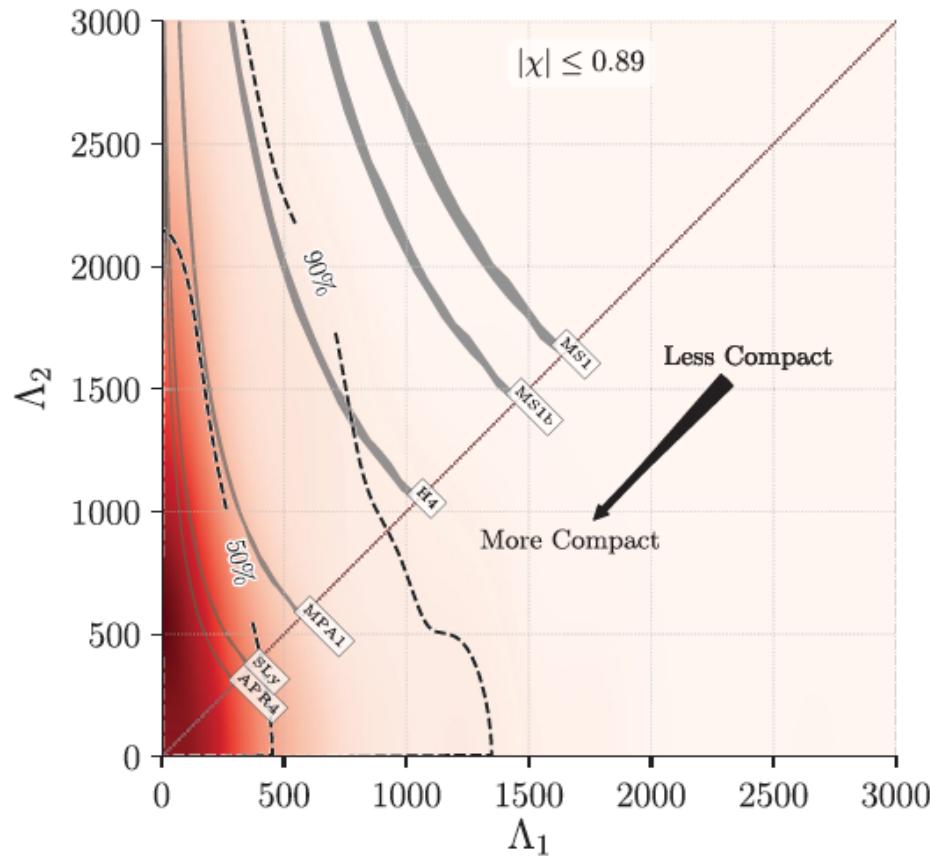
**Numerical GR Quasi-equilibrium NS binary sequence**

(Baumgarte, Shapiro, Teukolsky, Shibata, Meudon group, etc. 1990s--200x)

**Recent (semi-analytic) GR calculation of tidal effect**

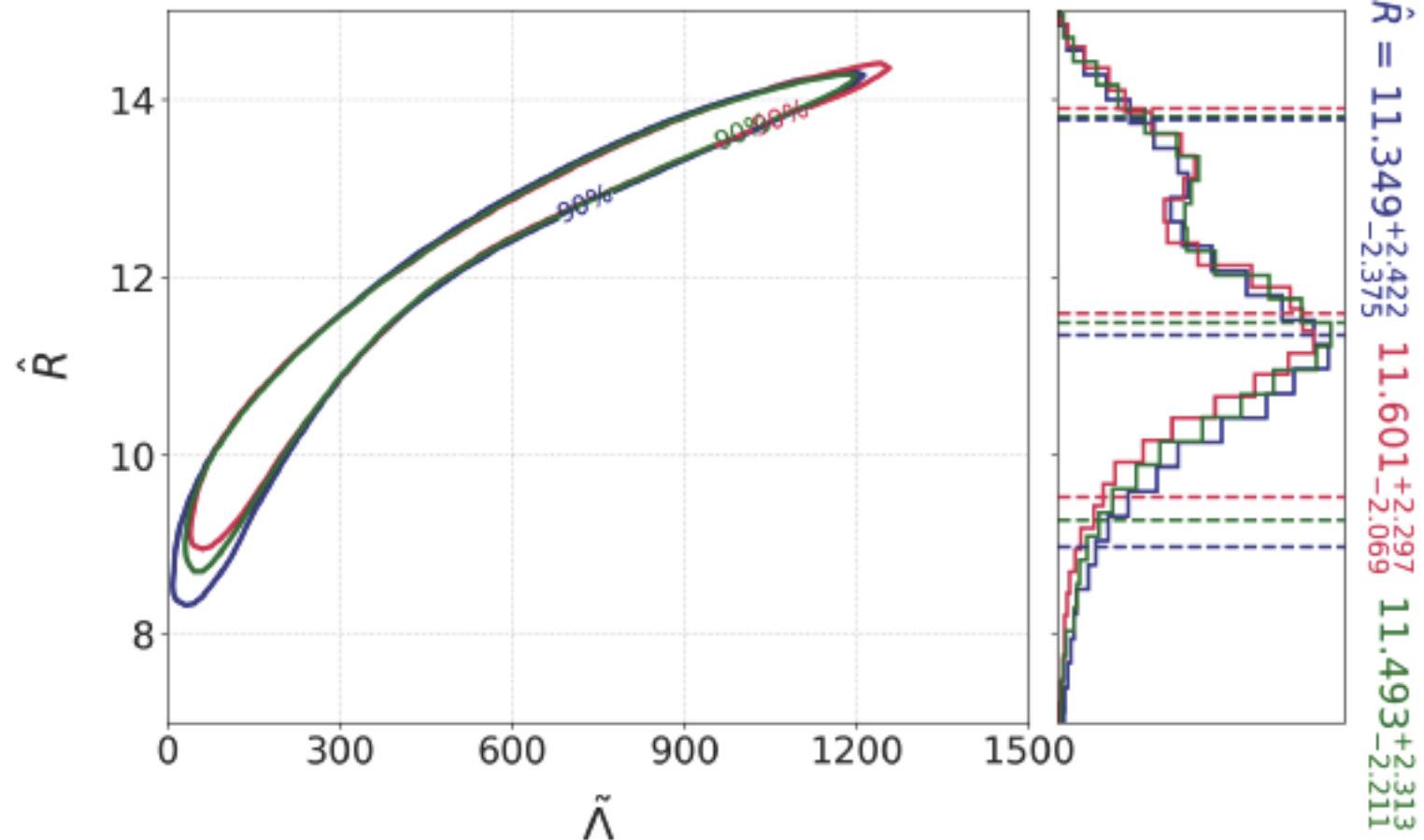
(Hinderer, Flanagan, Poisson, Damour, Penner, Andersson, Jones, etc., 2008+)

# GW170817:



$$\Lambda_2^{(i)} = \frac{2}{3} k_2^{(i)} \left[ \left( \frac{c^2}{G} \right) \left( \frac{R_i}{M_i} \right) \right]^5$$

# GW170817:



De, Finstad, Lattimer et al. 2018

# Resonant Tides: Excitations of Internal Modes

**NS has low-frequency oscillation modes:**

g-modes ( $\sim 100$  Hz)

inertial modes (incl. r-modes)

**Resonance:**  $\sigma_\alpha = m\Omega_{\text{orb}}$ ,  $m = 2, 3, \dots$

**Rosonant tidal excitations of NS modes during inspiral**

➔ transfer orbital energy to NS

➔ GW phase shift

# Resonant Excitations of NS Oscillations During Inspiral

$$\Delta\Phi \propto \frac{(Q_{\alpha,lm})^2}{\epsilon_\alpha \sigma_\alpha}, \quad Q_{\alpha,lm} = \int d^3x (\delta\rho_\alpha)^\star r^l Y_{lm}$$

## Non-rotating NS:

G-mode (Reisenegger & Goldreich 94; DL 94; Shibata 94)

## Rotating NS:

G-mode, F-mode, R-mode (Ho & DL 99)

Inertial modes (DL & Wu 06)

R-mode (excited by gravitomagnetic force; Racine & Flanagan 06)

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Inertial modes (DL & Wu 06)

R-mode (excited by gravitomagnetic force; Racine & Flanagan 06)

## Recent works:

- Superfluid NSs: Yu & Weinberg 2017
- Gravity-inertial modes, scalings (parameterized EOS): Xu & DL 2017



Wenrui Xu  
(Cornell → Princeton)

# G-modes

crustal density continuities

thermal stratification

stable composition stratification of core: symmetry energy, superfluidity

**Parameterize the uncertainties:**

$$P \propto \rho^\gamma, \quad \Gamma = (\partial \ln P / \partial \ln \rho)_{\text{ad}}$$

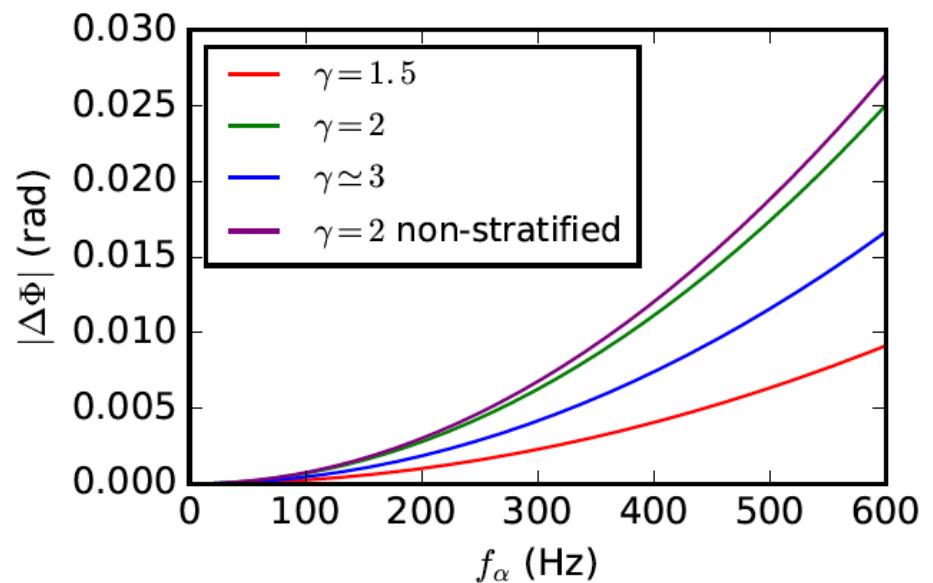
$$\omega_\alpha \propto (\Gamma - \gamma)^{1/2} M^{1/2} R^{-3/2}$$

$$Q_{\alpha,2m} \propto \Gamma - \gamma,$$

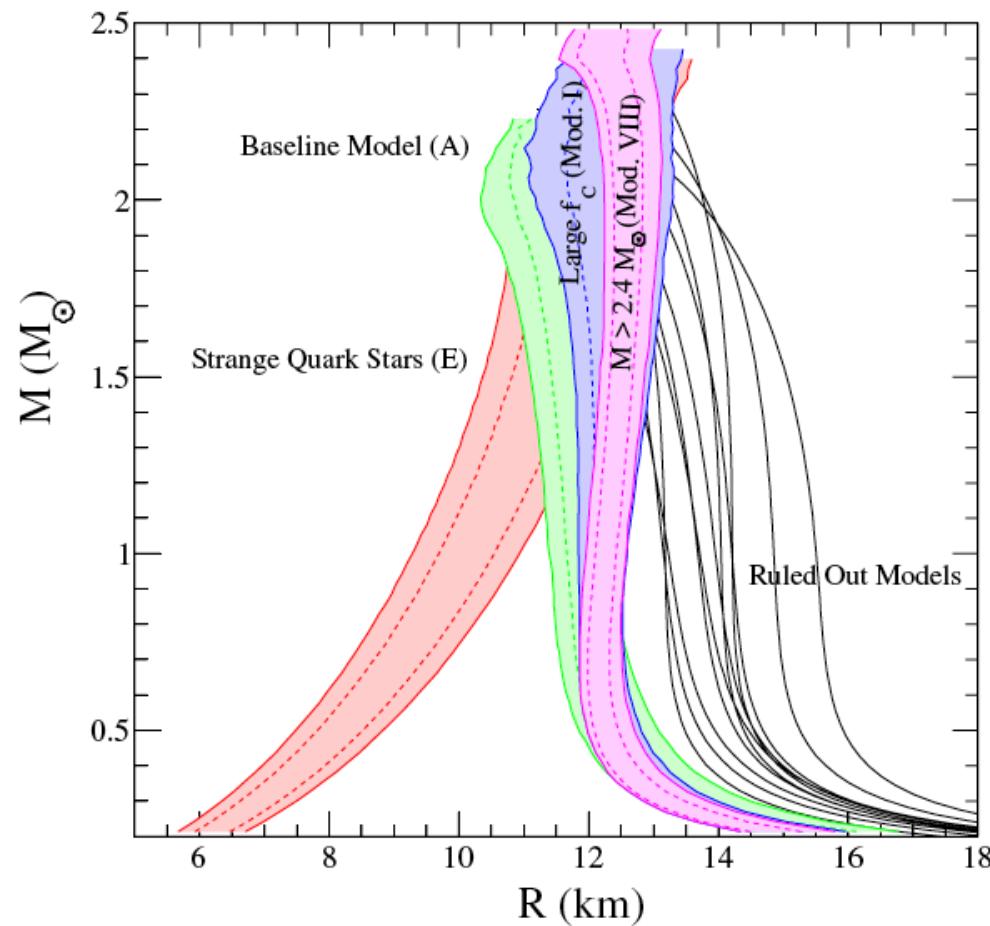
$$\begin{aligned} \Delta\Phi = & -0.060 \left( \frac{R_{10}^8}{M_{1.4}^6} \right) \frac{2}{q(1+q)} \left( \frac{f_\alpha}{100 \text{ Hz}} \right)^2 \\ & \times \left( \frac{\bar{f}_\alpha}{100 \text{ Hz}} \right)^{-4} \left( \frac{\bar{Q}_{\alpha,22}}{10^{-3}} \right)^2, \end{aligned}$$

Xu & DL 17

**Note:**  $M = 1.2M_\odot$ ,  $R = 13$  km NS  
 → x 21 Important !



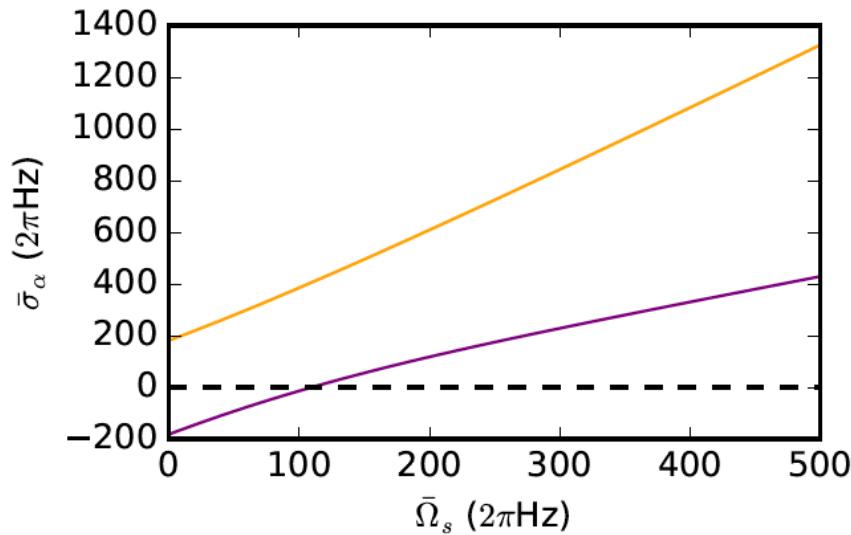
# NS Mass-Radius



Measured NS mass:  
1.17–2.0 Solar mass

Steiner et al 2013

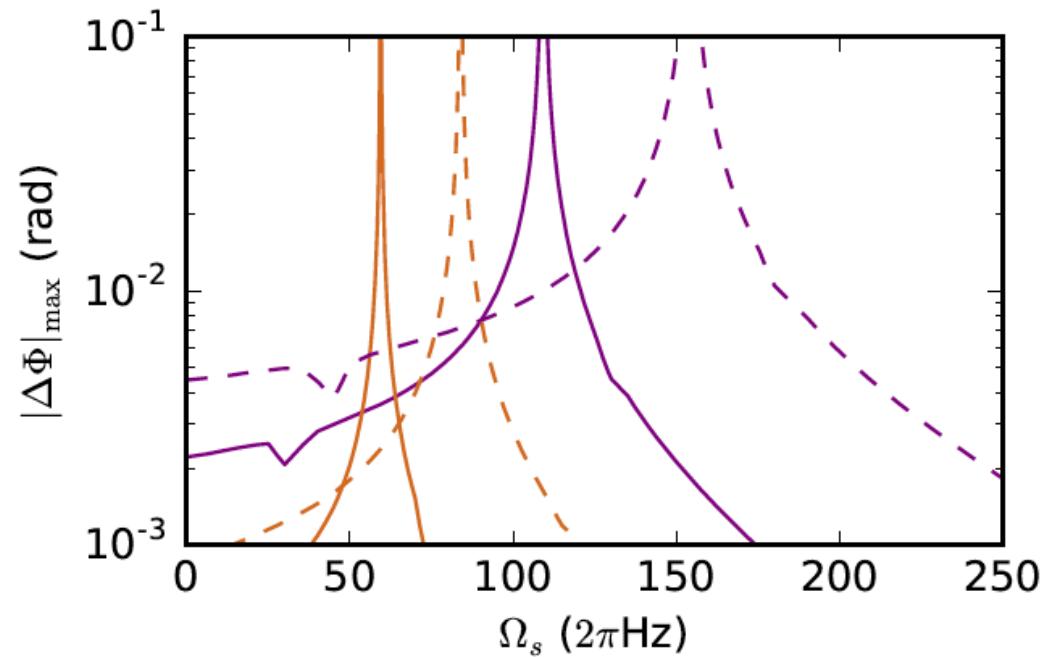
## With rotation: Inertial-Gravity Modes



Rotation can reduce (retrograde)  
g-mode  $|\sigma_\alpha|$ , thus increase  $|\Delta\Phi|$

$\gamma = 2$      $\Gamma - \gamma = 0.01$   
 $\gamma \approx 3$      $\Gamma - \gamma = 0.02$

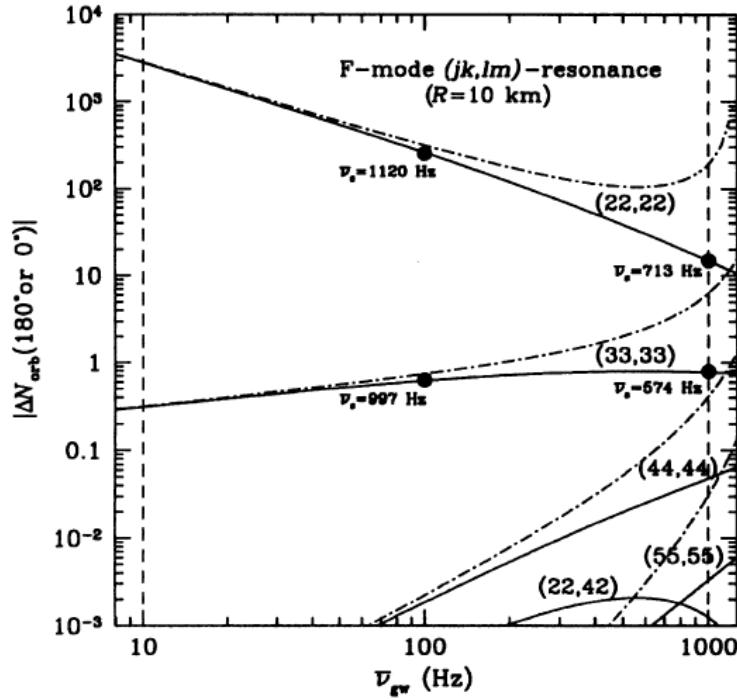
Xu & DL 2017



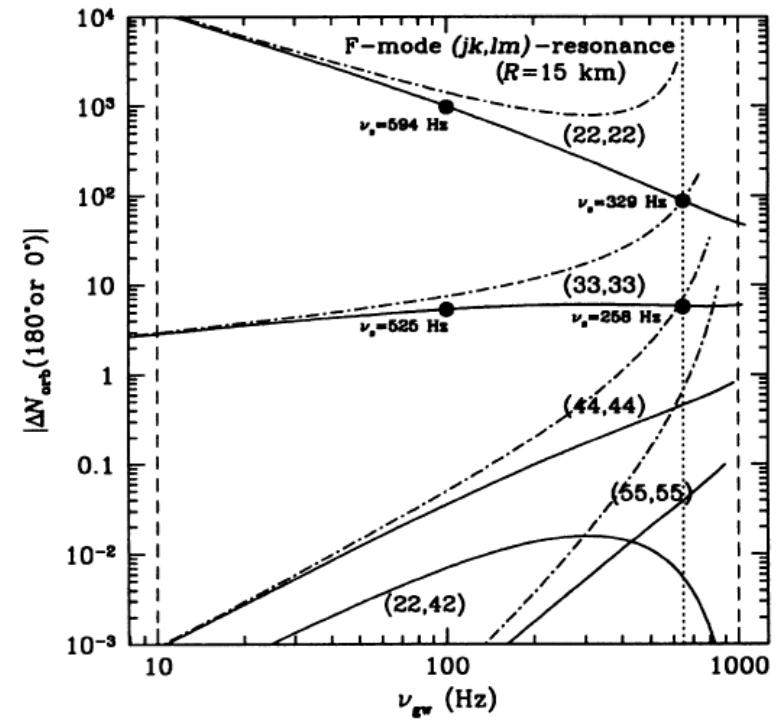
# Resonant excitation of F-modes?

Very strong ... possible with **very rapid rotations**

Wynn Ho & DL 1999



**Figure 2.** Orbital cycle change  $|(\Delta N_{\text{orb}})_{jk,lm}|$ , with  $M = 1.4 M_{\odot}$ ,  $R = 10 \text{ km}$ ,  $q = 1$ , as a function of the normalized gravitational wave frequency,  $\bar{\nu}_{\text{gw}} = \nu_{\text{gw}} M_{1.4}^{-1/2} R_{10}^{3/2} = (2/m) |\nu_{\alpha}| M_{1.4}^{-1/2} R_{10}^{3/2}$ , for the f-mode  $(jk,lm)$ -resonance. The labels in the figure give the values of  $(jk,lm)$ . The solid lines are for the  $\omega_{\alpha}^{(r)} > 0$  and  $\omega_{\alpha} > 0$  (spin-retrograde) stable modes with  $(\Delta N_{\text{orb}})_{jk,lm}(180^\circ) < 0$ . The dot-dashed lines are for the  $\omega_{\alpha}^{(r)} > 0$  and  $\omega_{\alpha} < 0$  (spin-prograde) unstable modes with  $(\Delta N_{\text{orb}})_{jk,lm}(0^\circ) > 0$ . Note



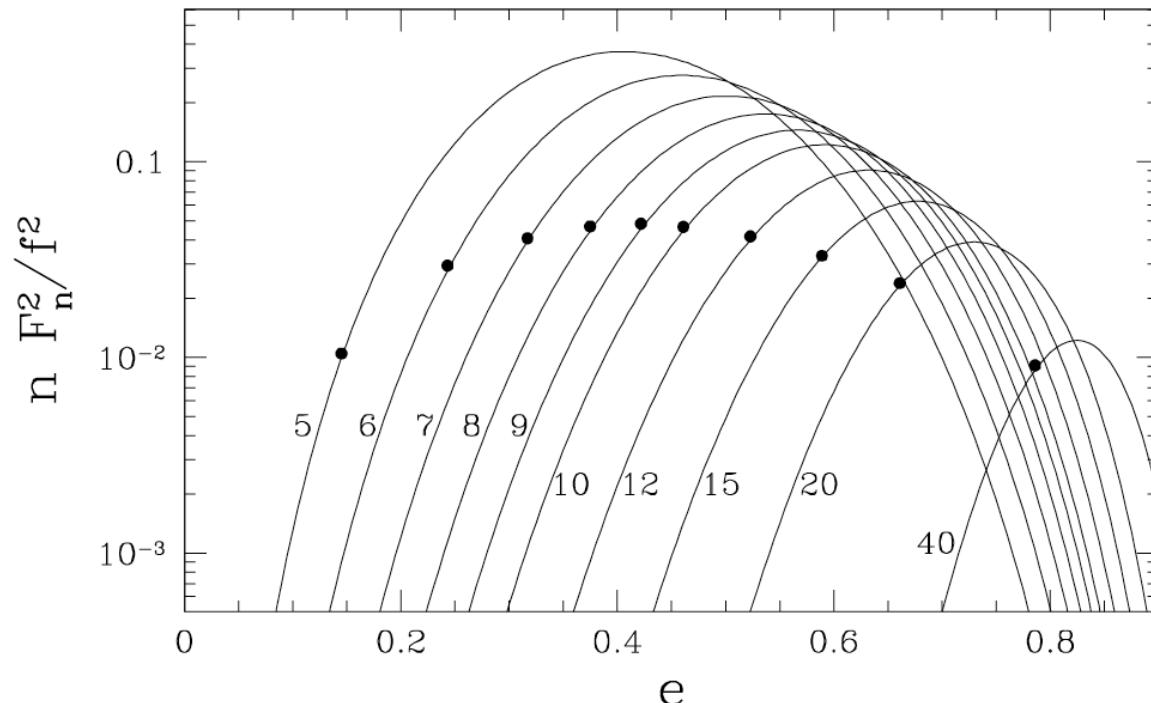
**Figure 4.** Orbital cycle change  $|(\Delta N_{\text{orb}})_{jk,lm}|$ , with  $M = 1.4 M_{\odot}$ ,  $R = 15 \text{ km}$ ,  $q = 1$ , as a function of the gravitational wave frequency,  $\nu_{\text{gw}} = (2/m) |\nu_{\alpha}|$ , for the f-mode  $(jk,lm)$ -resonance. The labels in the figure give the values of  $(jk,lm)$ . The solid lines are for the  $\omega_{\alpha}^{(r)} > 0$  and  $\omega_{\alpha} > 0$  (spin-retrograde) stable modes with  $(\Delta N_{\text{orb}})_{jk,lm}(180^\circ) < 0$ . The dot-dashed lines are for the  $\omega_{\alpha}^{(r)} > 0$  and  $\omega_{\alpha} < 0$  (spin-prograde) unstable modes with

# Resonant excitation of F-modes?

## Eccentric Mergers

Resonance occurs:  $\sigma_\alpha = n\Omega_{\text{orb}}$  (even for  $l = m = 2$  tides)

$$\Delta\Phi = -\frac{151}{q(1+q)} \left(\frac{R}{5M}\right)^5 \left(\frac{\hat{Q}_\alpha}{\hat{\sigma}_\alpha}\right)^2 \left(\frac{nF_n^2(e)}{f(e)}\right)$$



DL & Chernoff 2018

# Summary I:

Resonant tidal excitation of modes occurs at low frequency (< 100 Hz).  
If strong, would provide a unique probe of EOS:

- g-modes: Important for low-M, large-R NSs
- rotation enhances resonance.
- f-mode resonance: Important for very rapid rotation

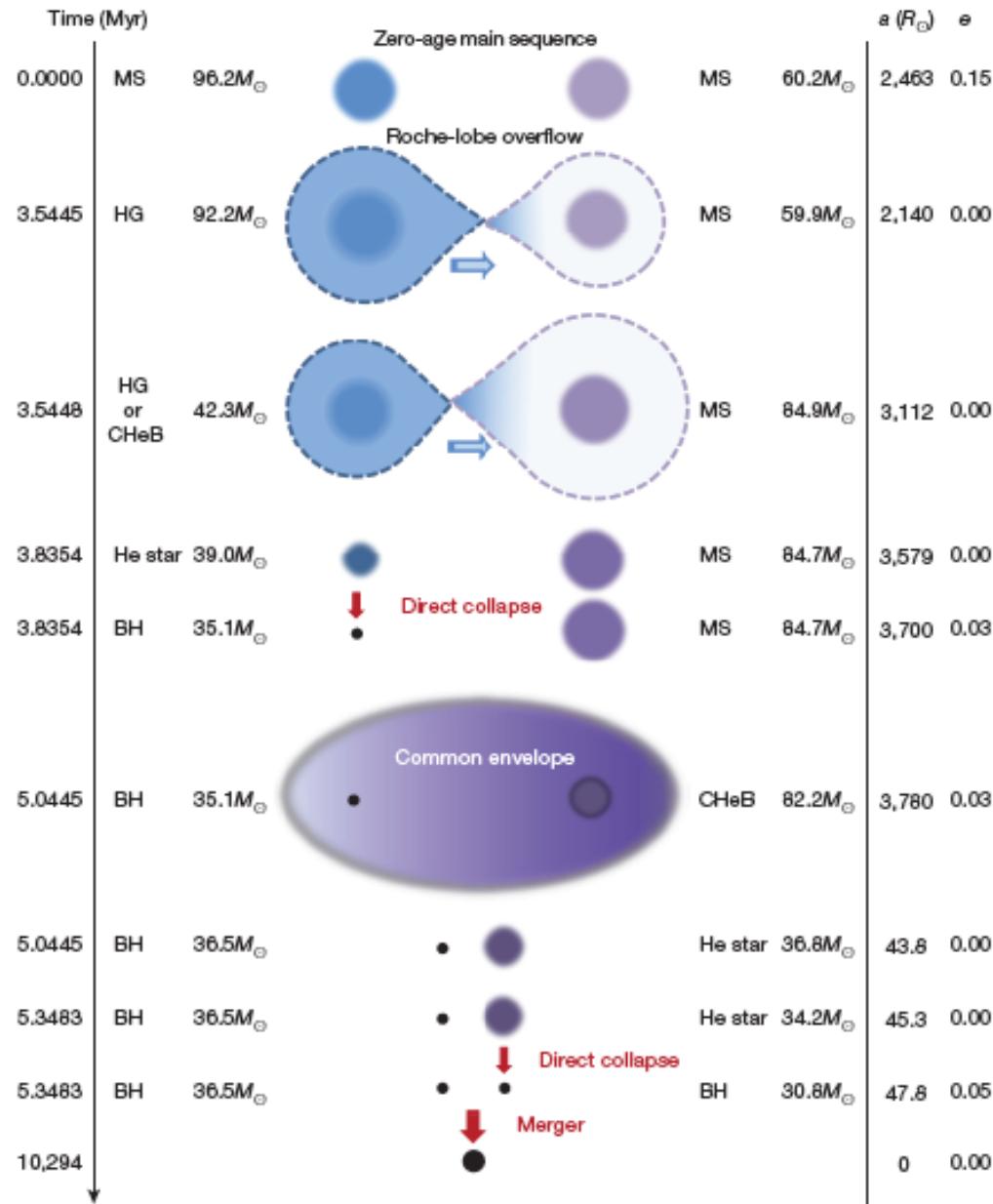
**Note:** Resonant tides are certainly important for WD binaries  
(or WD-NS, WD-BH): LISA band  
Also for 1-10 Hz band (Cosmic Explorer, Einstein Telescope)

# Formation Channels of Black Hole Binaries

A large and important topics...

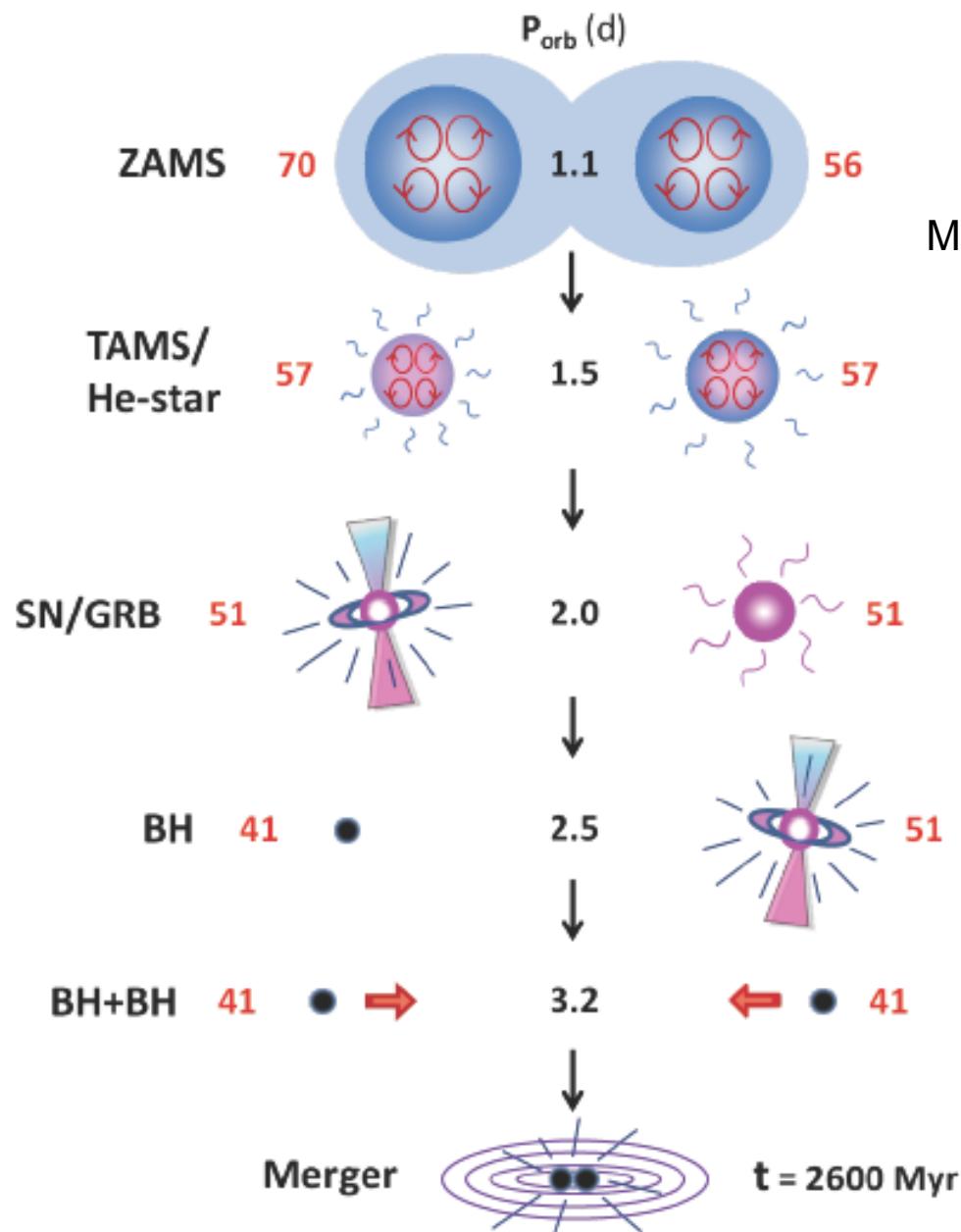
LIGO O3 may have ~1 BH binary events per week !

# Standard Binary Evolution Channel:

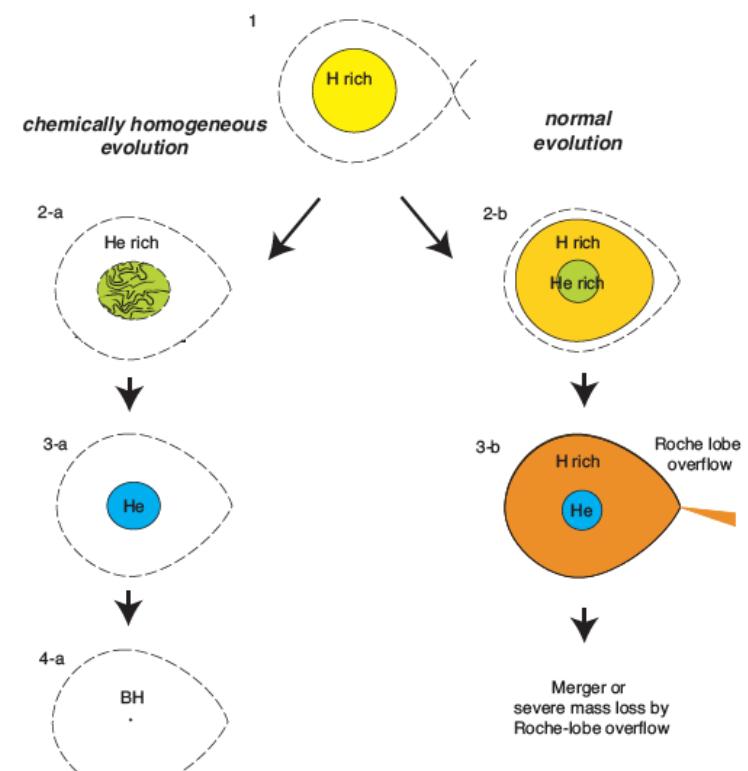


Belczynski +16

# Chemically Homogeneous Evolution Channel



Marchant, Langer et al 2016

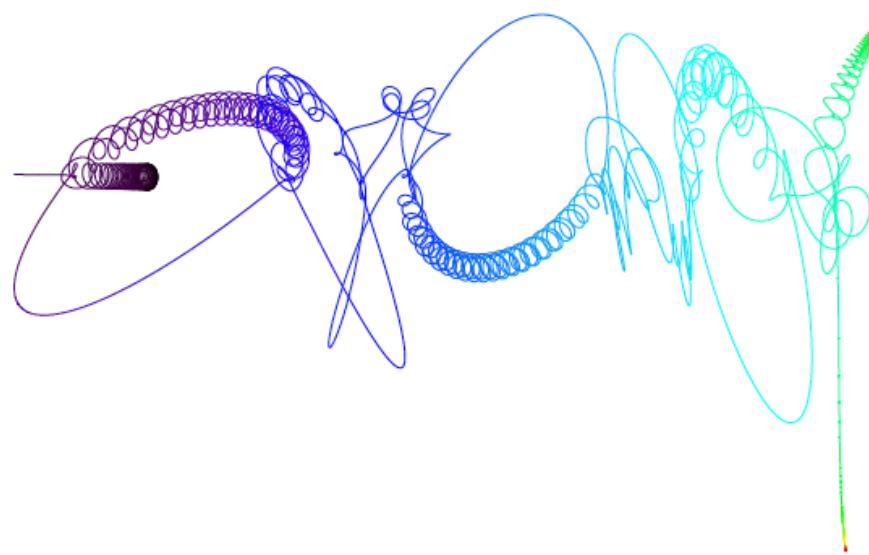


Mandel & de Mink 2016

# Dynamical Formation Channels

many flavors, many papers ...

-- Binary + single scatterings dense clusters → Tight binary



Samsing+14

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many flavors, many papers ...

- Binary + single scatterings dense clusters → Tight binary
- Binary mergers induced by external companion (SMBH, stellar triple)

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- Binary mergers induced by external companion (SMBH, stellar triple)

## → Eccentric mergers in LIGO band

e.g., Silsbee & Tremaine (2017) claim a few % of no-so hierarchical triples that lead to merger will enter LIGO band with  $e>0.999$

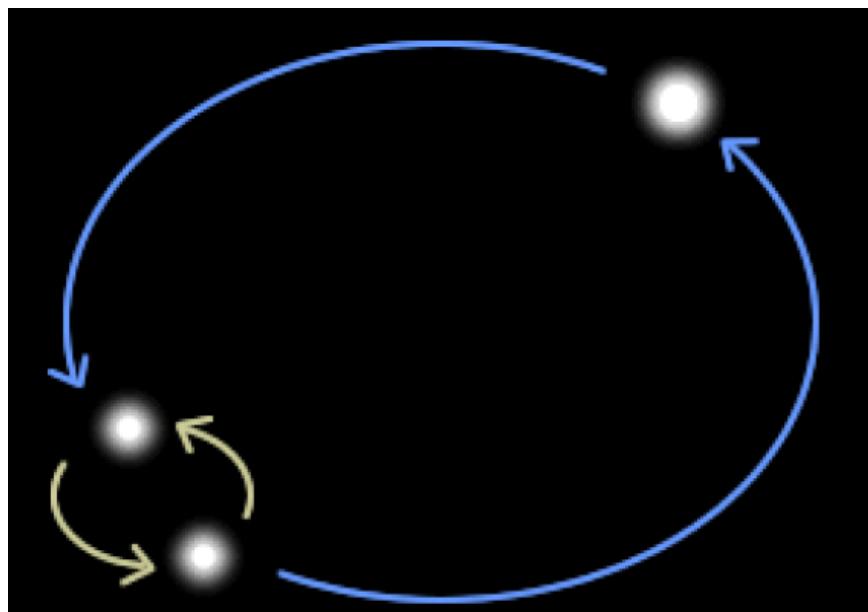
Eccentric mergers in LIGO band are probably rare...

But “**high-e mergers**” induced/enhanced by tertiary companion  
(binary goes through high-e phase but has circularized when entering LIGO band) may not be rare (Competitive with normal binary evolution channel?)

#### **Previous works:**

e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003;  
Thompson 2011; Antonini et al. 2014,2017,  
Silsbee & Tremaine 2017...

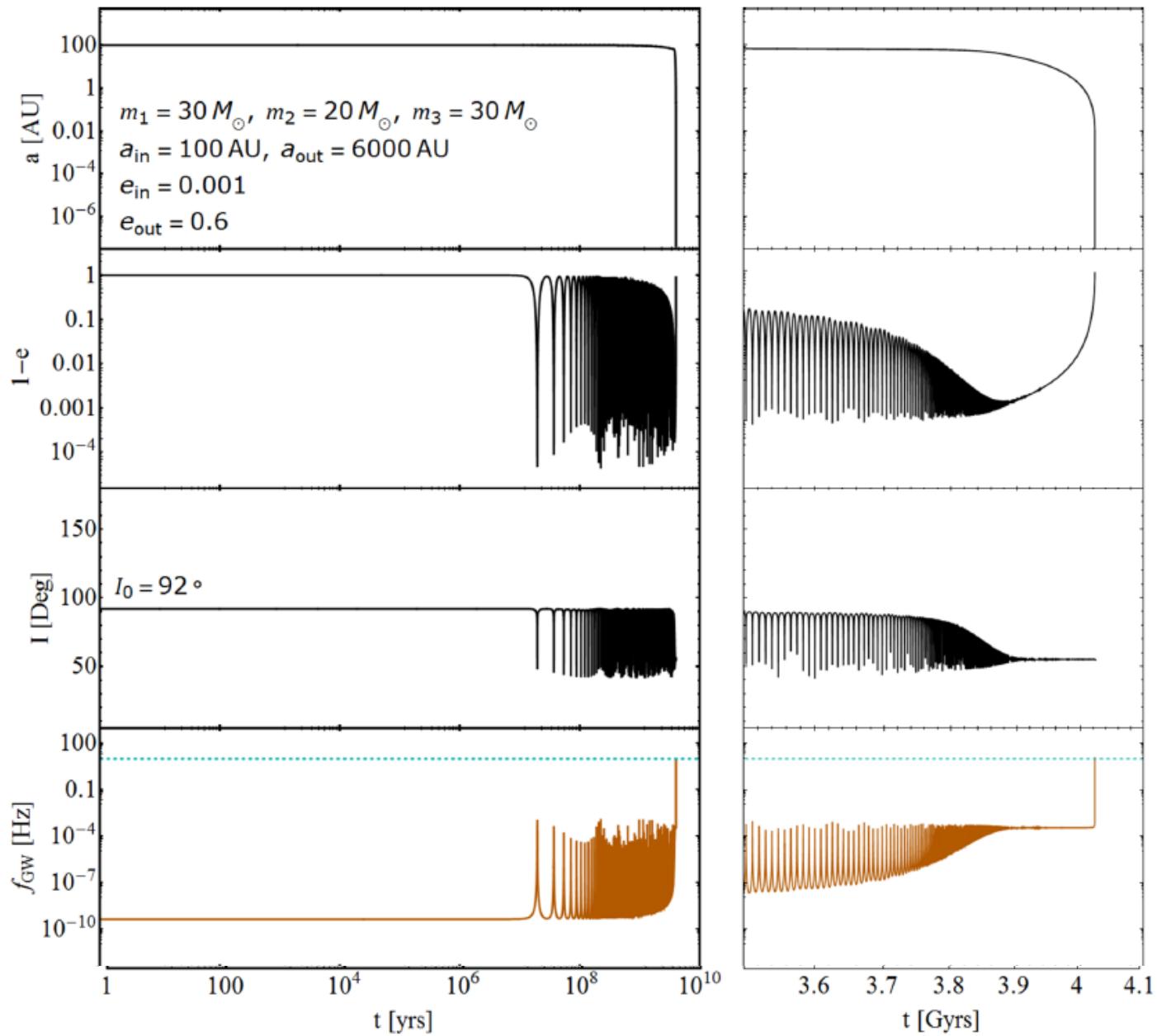
# Binary Mergers Induced by Tertiary Companions and Spin-Orbit Misalignments



B.Liu & DL 2017,2018

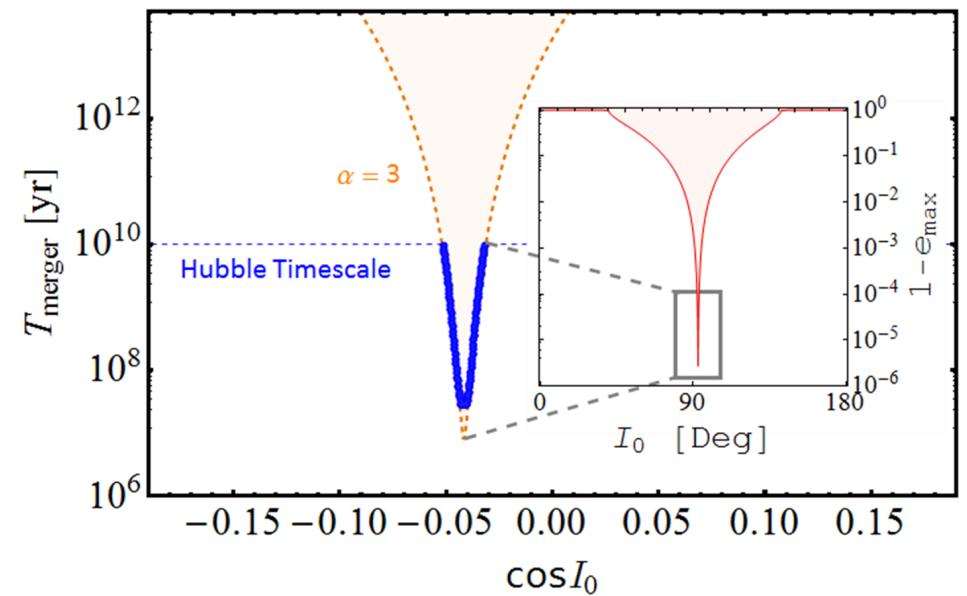
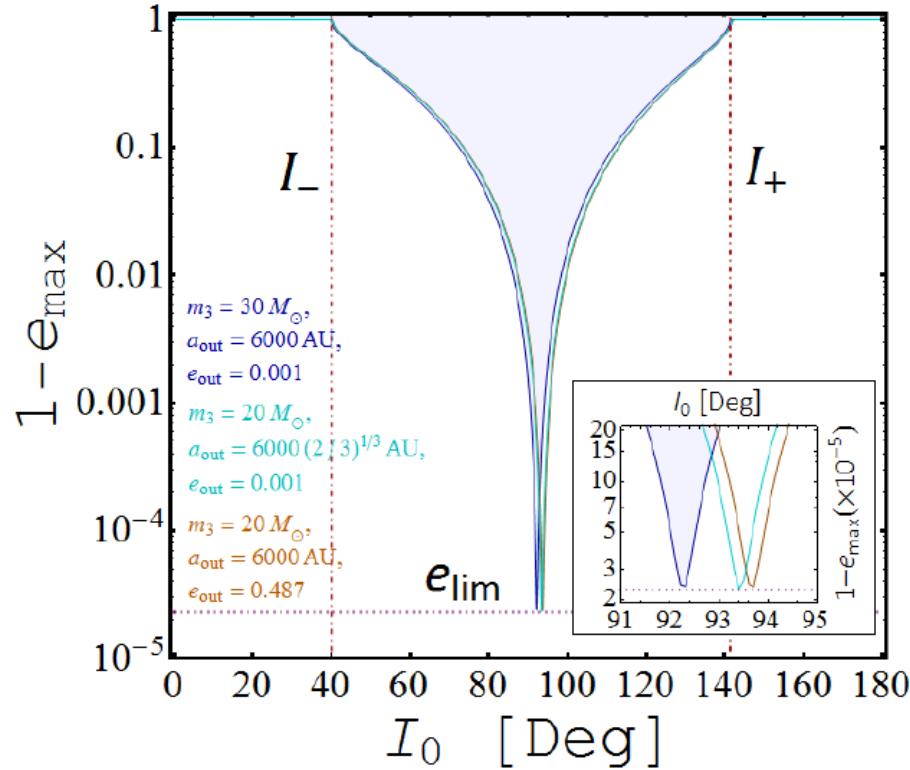


Bin Liu (SHAO, Cornell)



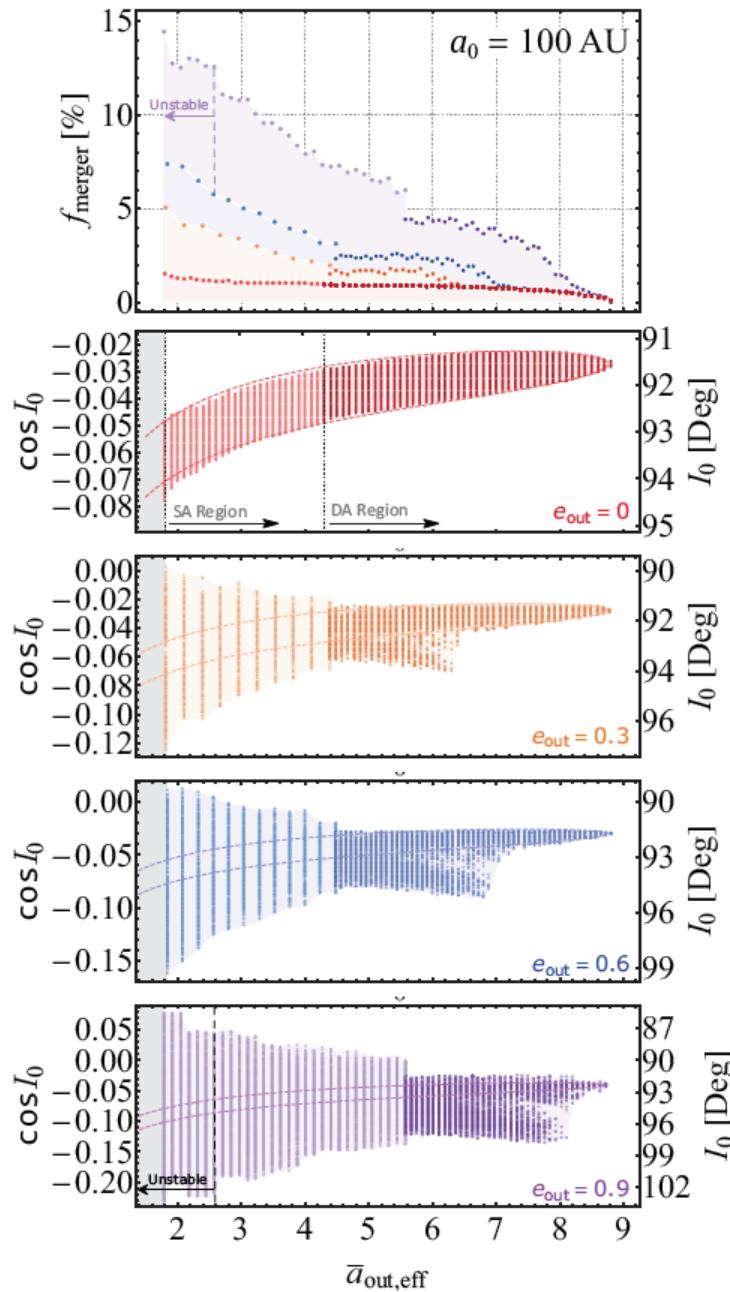
Lidov-Kozai oscillations

# Inclination Window for Merger



Many old/new semi-analytic expressions are summarized/derived in  
B.Liu & DL 2018, arXiv1805.03202

# Merger Window and Merger Fraction



$$\bar{a}_{\text{out,eff}} = \left( \frac{a_{\text{out}} \sqrt{1 - e_{\text{out}}^2}}{1000 \text{AU}} \right) \left( \frac{m_3}{30 M_{\odot}} \right)^{-1/3}$$

# Spin Puzzle ?

$$\chi_{\text{eff}} = \frac{m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$$

GW150914:  $-0.06^{+0.14}_{-0.14}$

GW151226:  $0.21^{+0.2}_{-0.1}$

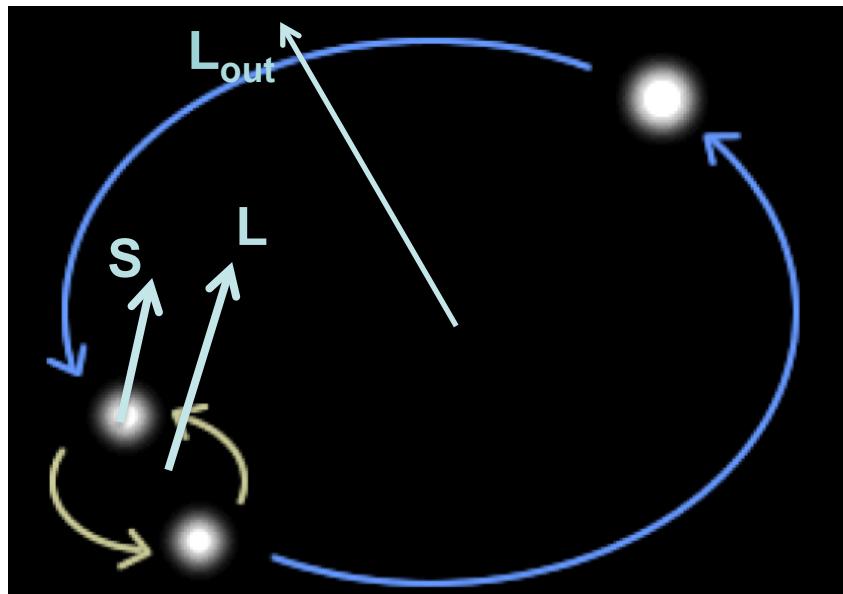
GW170104:  $-0.12^{+0.21}_{-0.3}$

GW170608:  $0.07^{+0.23}_{-0.09}$

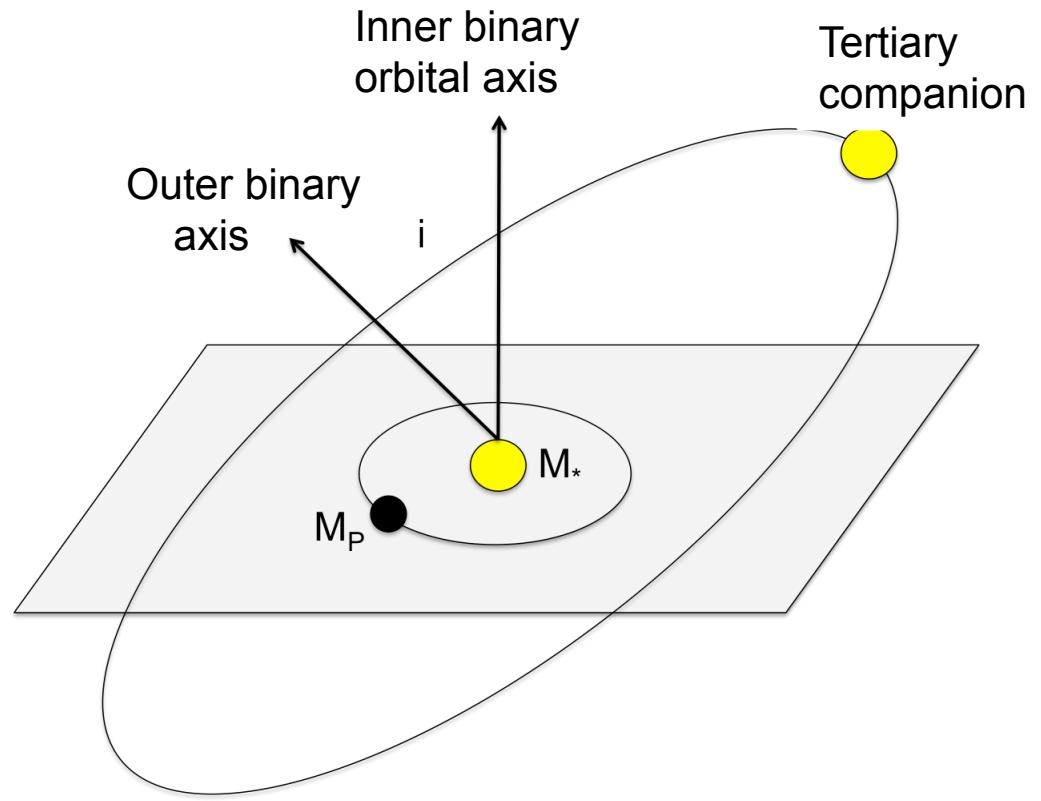
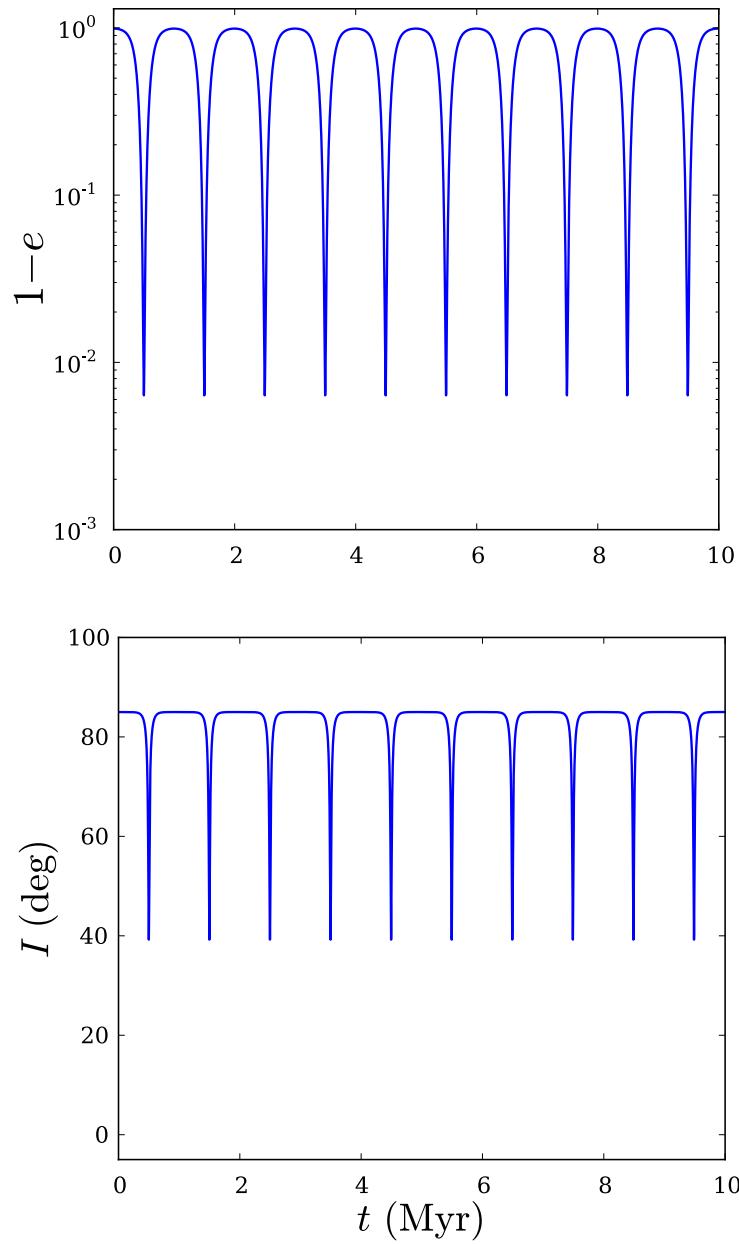
GW170814:  $0.06^{+0.12}_{-0.12}$

# Spin-Orbit Misalignments in Merging Binaries with Tertiary Companions

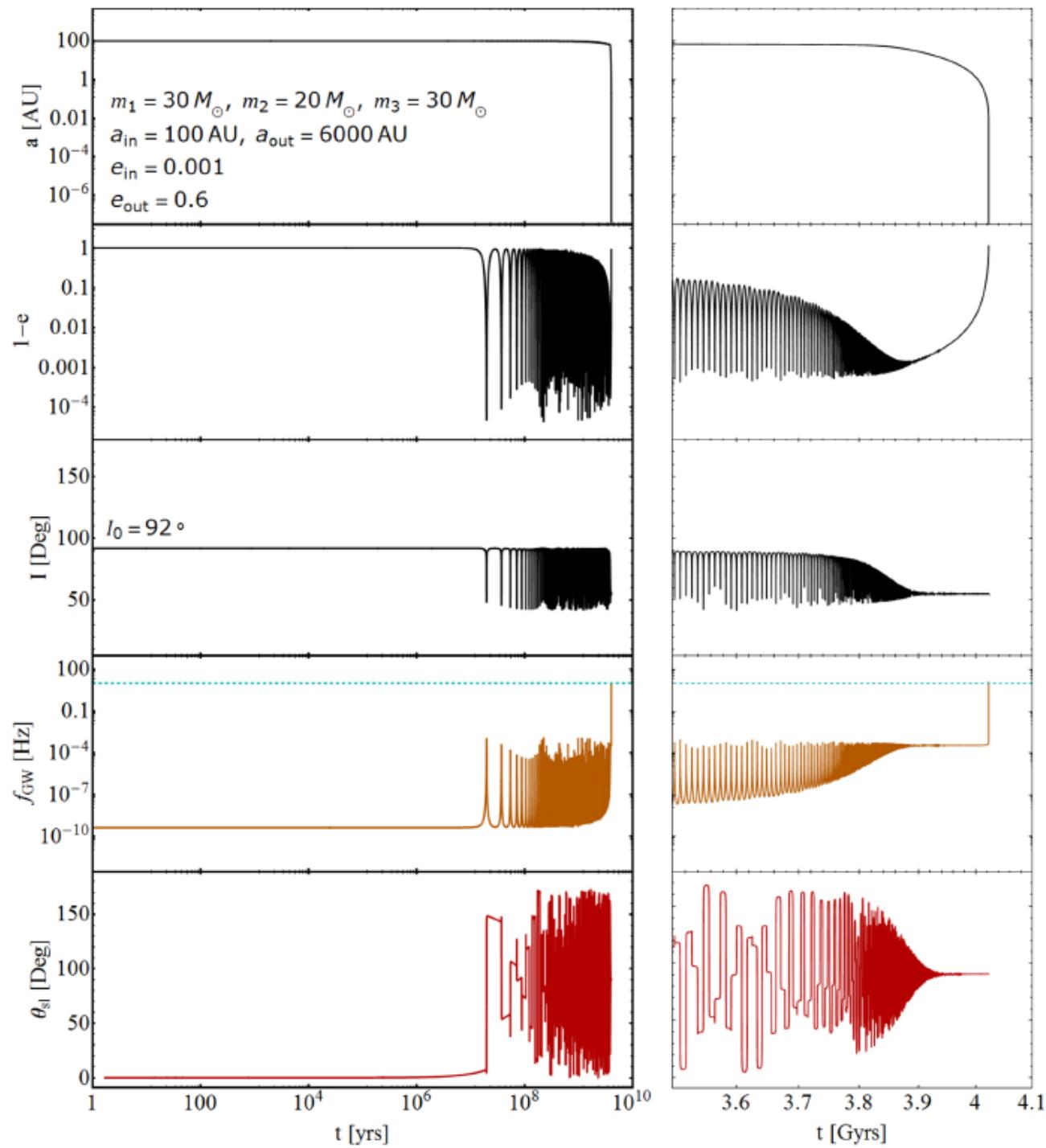
Bin Liu & DL 2017, 2018



# Lidov-Kozai Oscillations



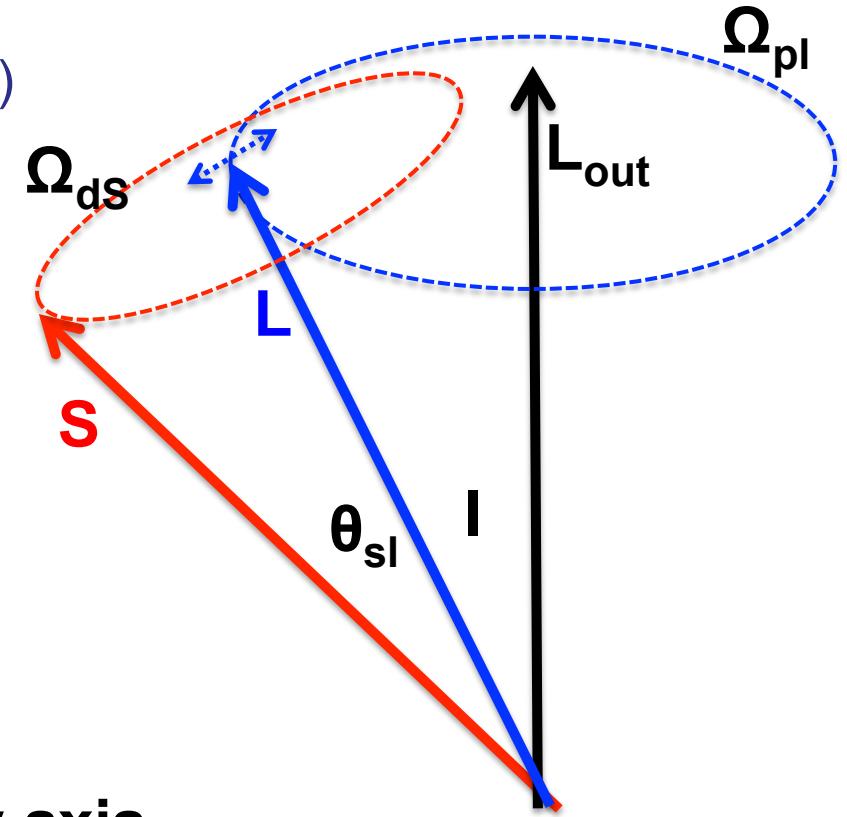
- Eccentricity and inclination oscillations induced if  $I > 40$  degrees.
- If  $I$  large (85-90 degrees), get extremely large eccentricities ( $e > 0.99$ )



# BH Spin Dynamics

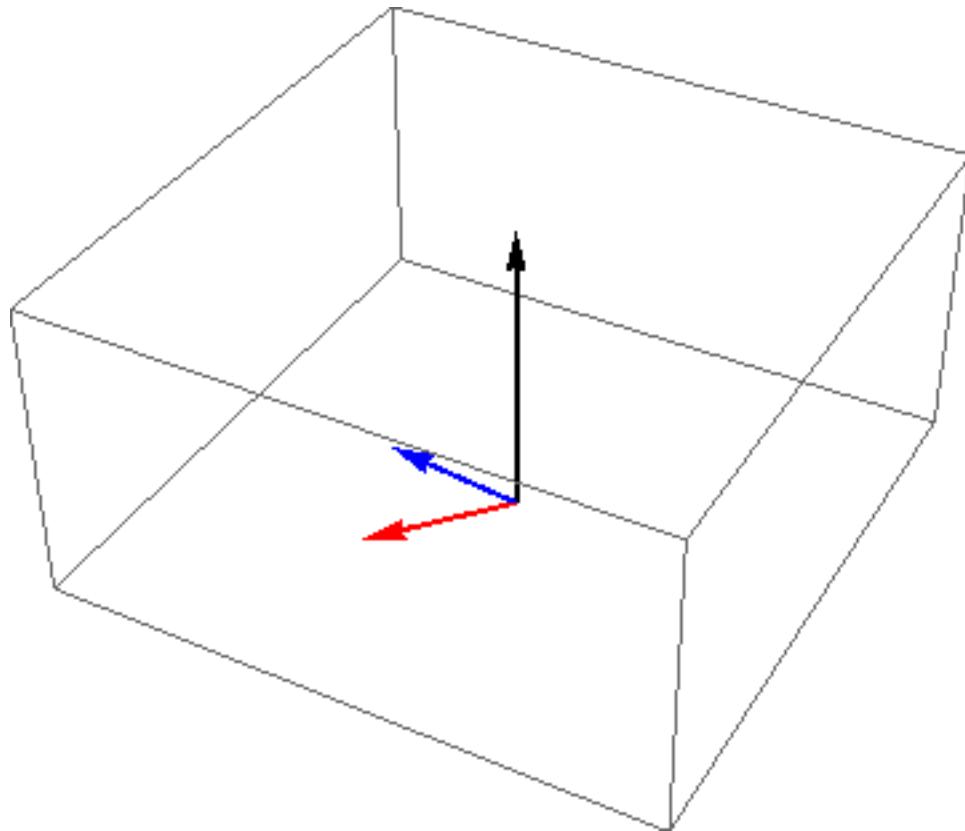
$$\frac{d\hat{\mathbf{S}}_1}{dt} = \Omega_{\text{dS}} \hat{\mathbf{L}} \times \hat{\mathbf{S}}_1 \quad (\text{de Sitter Precession})$$

$$\Omega_{\text{dS}} = \frac{3Gn(m_2 + \mu/3)}{2c^2a(1 - e^2)}$$



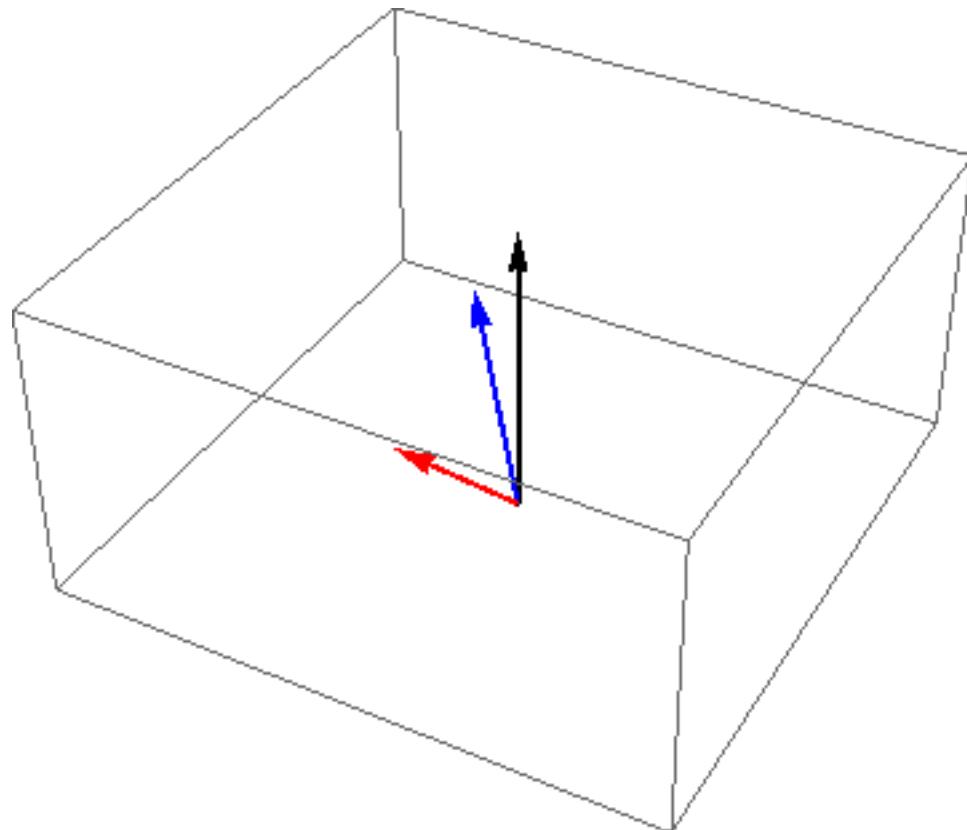
- Outer binary axis
- Inner binary axis
- Spin axis

If  $|\Omega_{ds}| \gg |\Omega_{pl}|$ : S follows L adiabatically



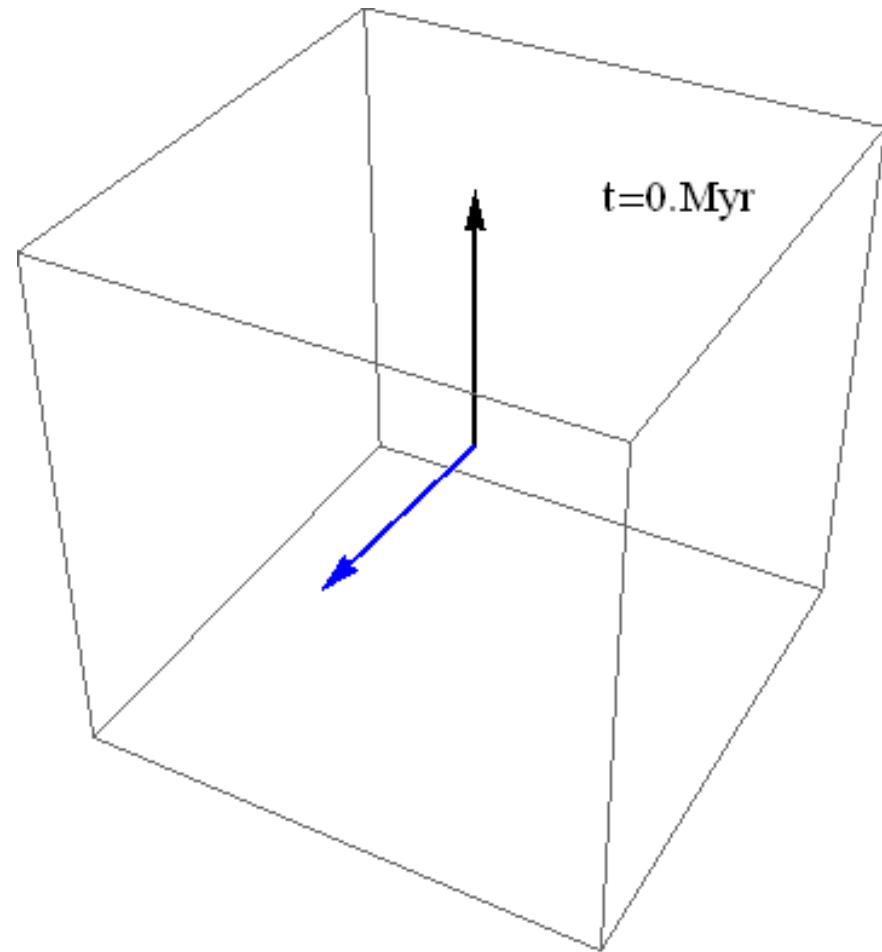
- Outer binary axis
- Inner binary axis
- Spin axis

If  $|\Omega_{ds}| \ll |\Omega_{pl}|$ : Non-adiabatic



- Outer binary axis
- Inner binary axis
- Spin axis

If  $|\Omega_{ds}| \sim |\Omega_{pl}|$ :

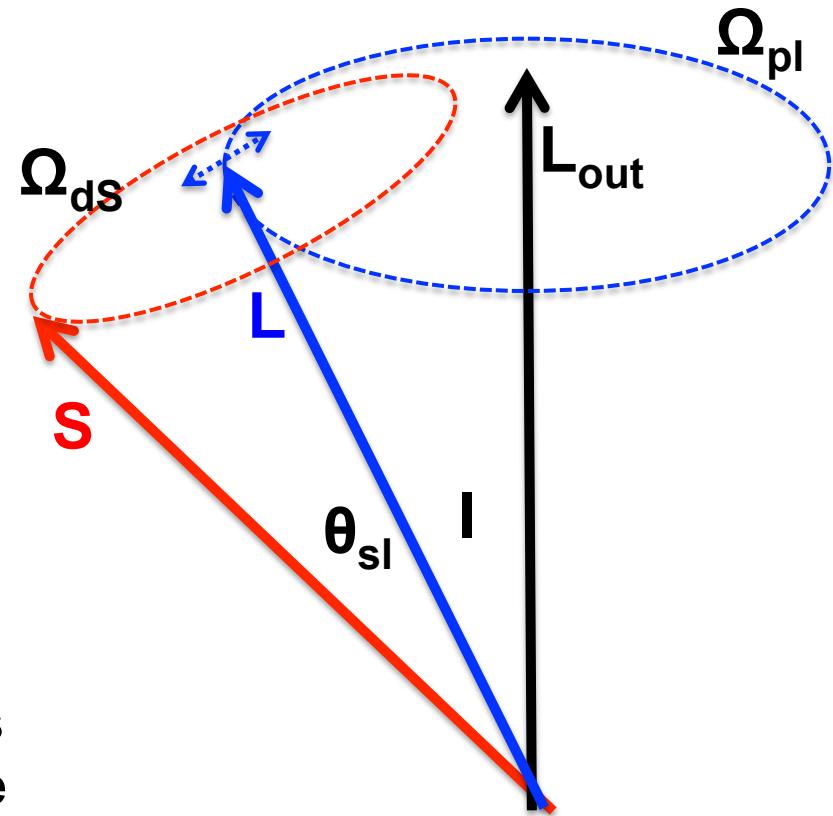


- Outer binary axis
- Inner binary axis
- Spin axis

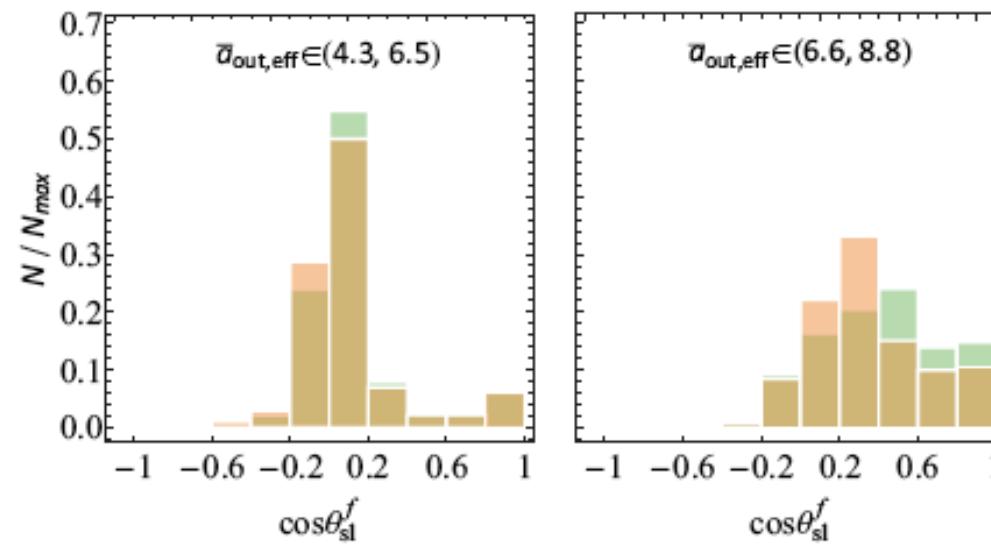
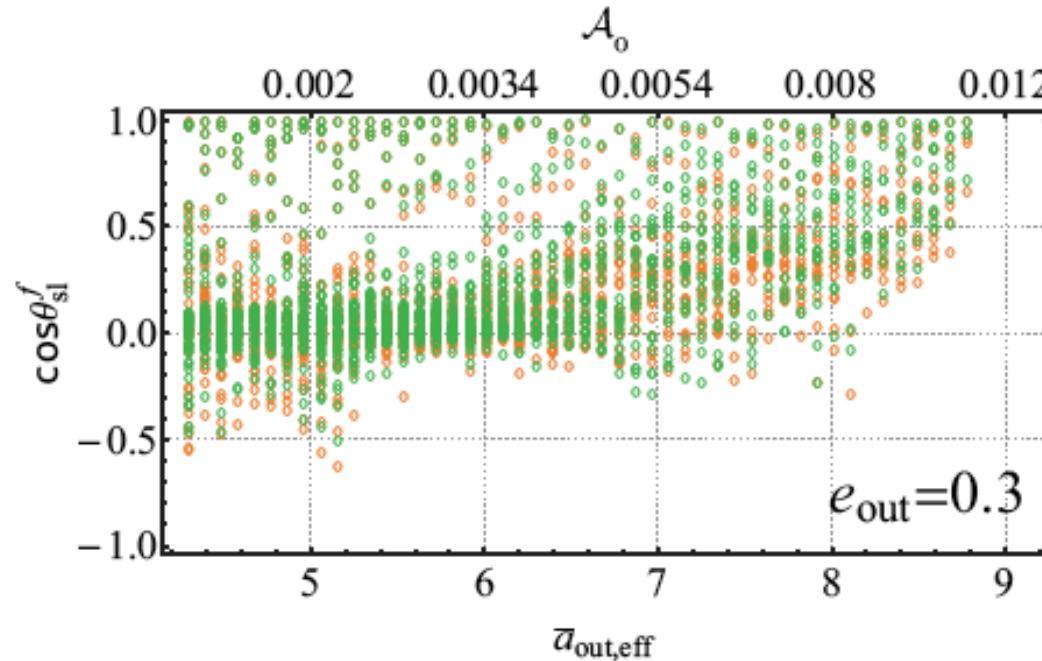
# BH Spin Evolution

$$\Omega_{dS}/\Omega_{pl}$$

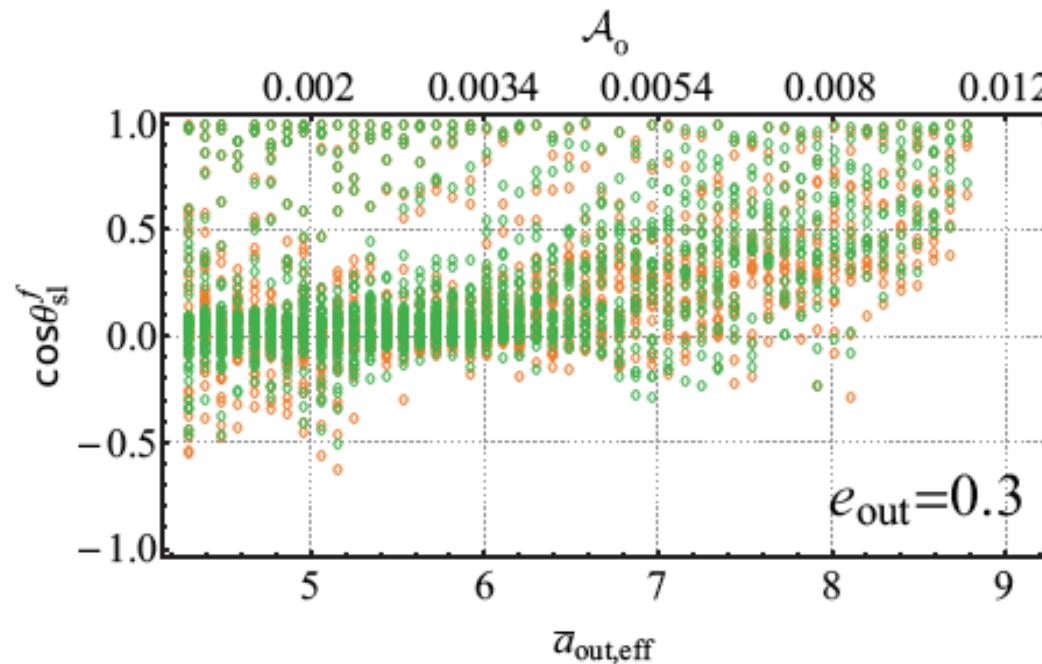
changes from  $\ll 1$  (non-adiabatic)  
to  $\gg 1$  (adiabatic) as the orbit decays  
→ Final spin-orbit misalignment angle



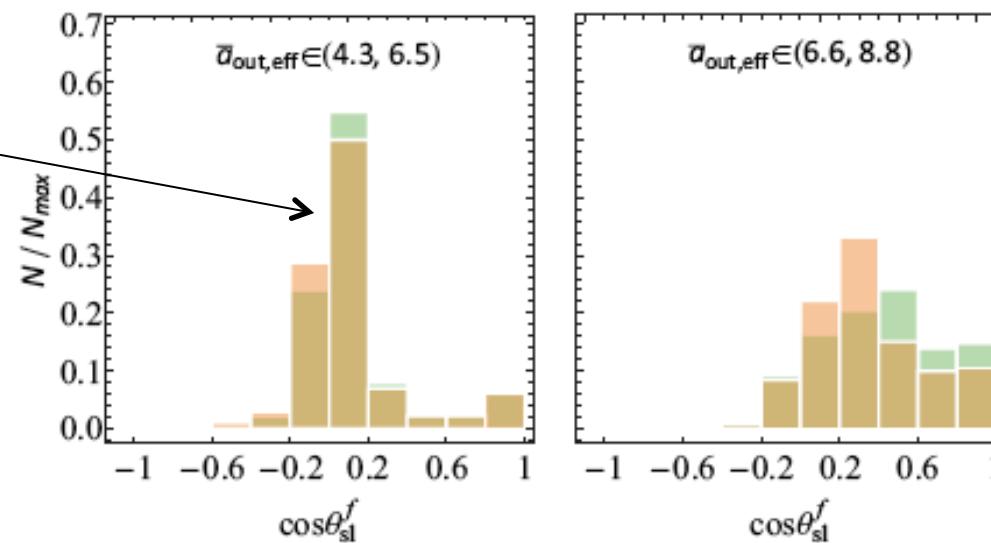
## Final Spin-Orbit Misalignment Distribution (starting from alignment)



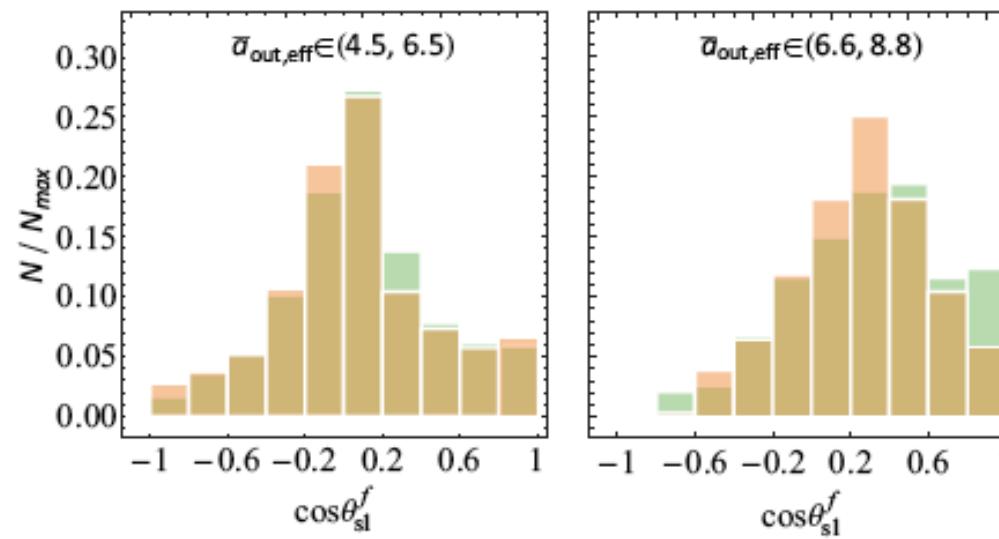
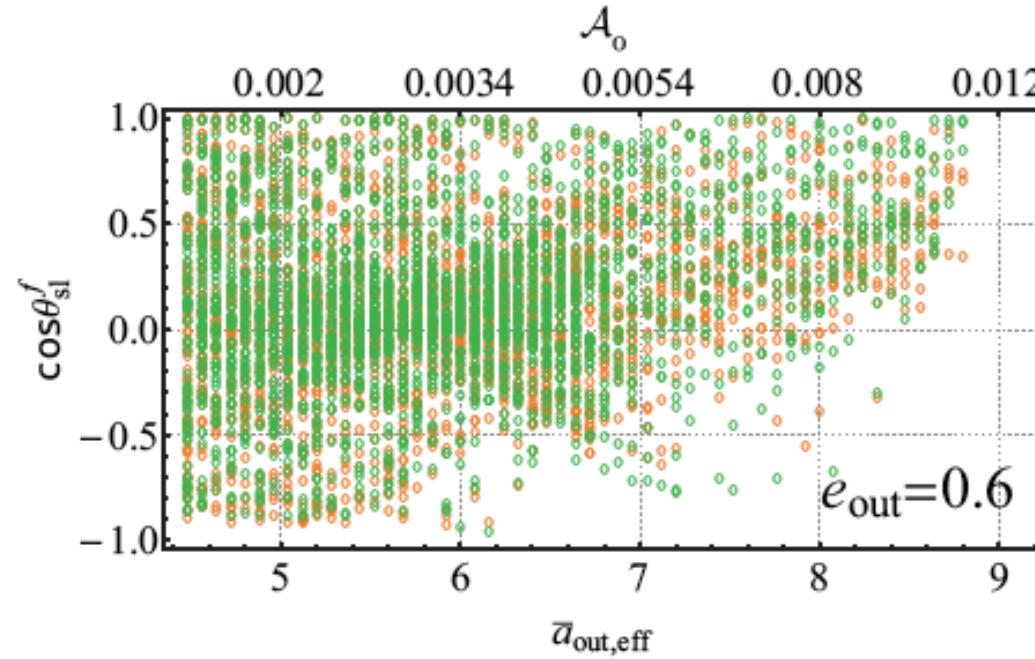
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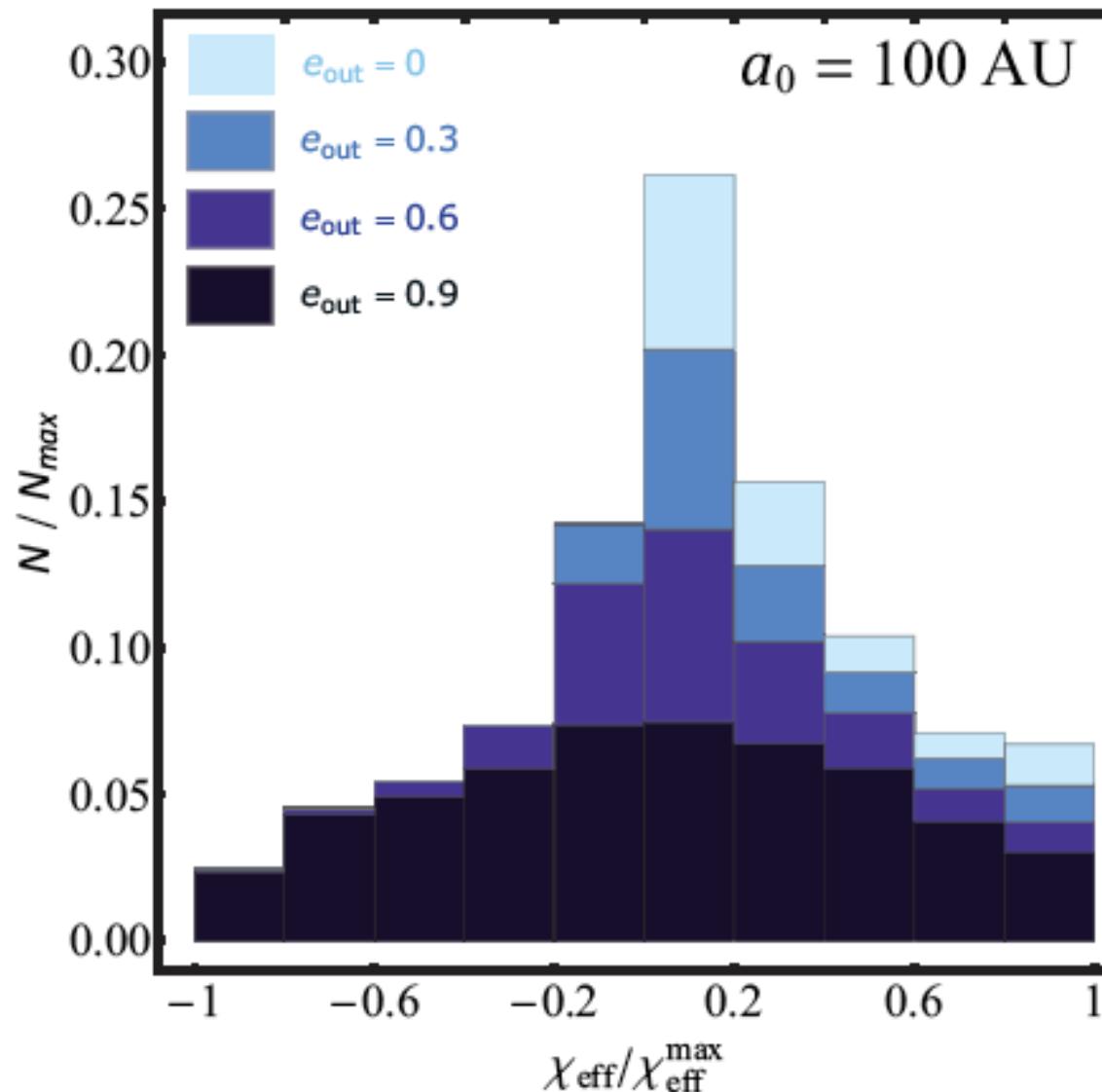
90 degree  
Attractor  
("adiabatic  
Invariance")



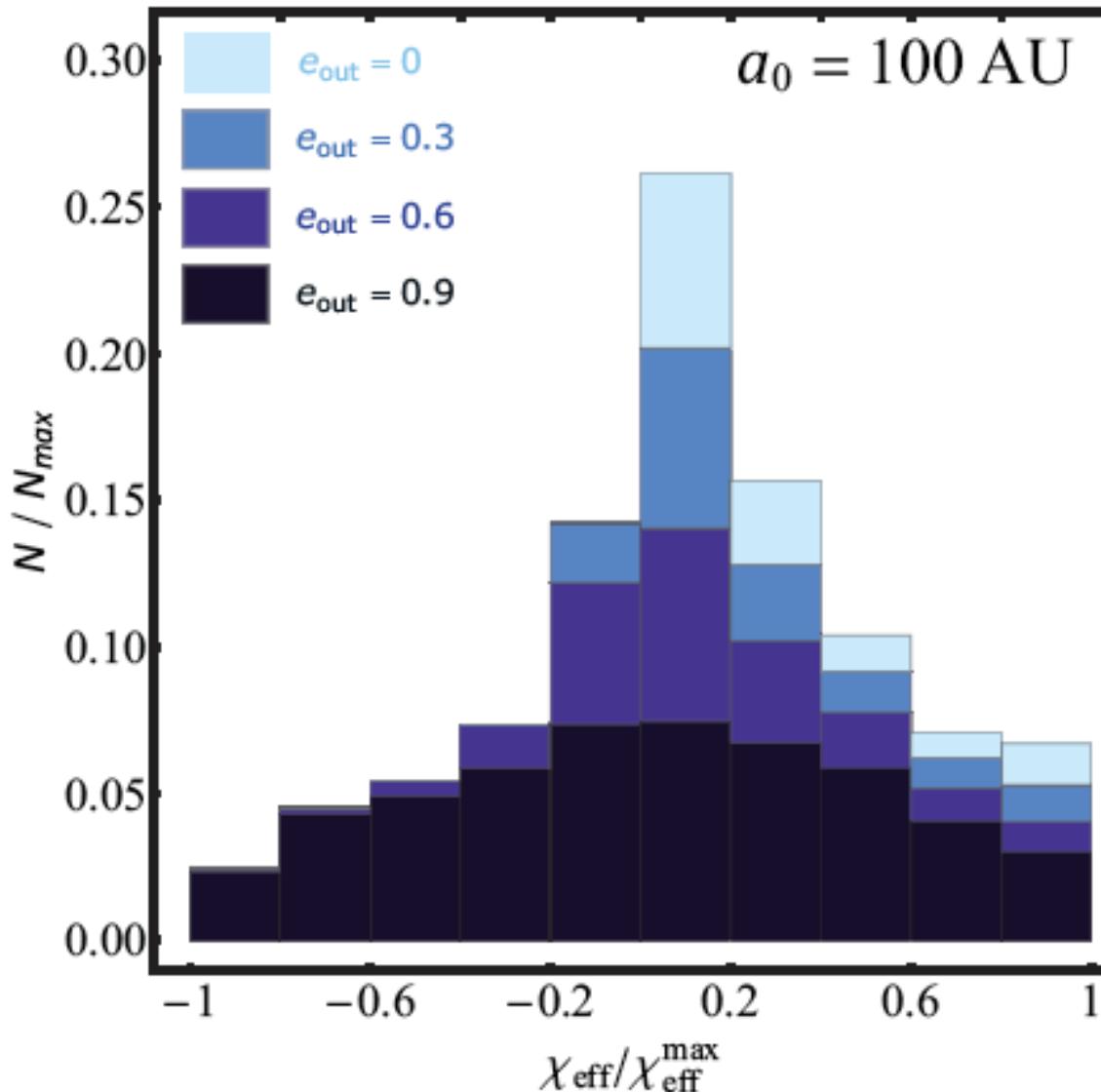
## Final Spin-Orbit Misalignment Distribution (starting from alignment)



Distribution of  $\chi_{\text{eff}} = \frac{m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$



Distribution of  $\chi_{\text{eff}} = \frac{m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2}{m_1 + m_2} \cdot \hat{\mathbf{L}}$



A unique signature  
of LK-induced  
mergers

# Summary

**Resonant tidal excitation of modes (<100 Hz) would probe EOS:**

Probably not important unless Nature is kind:

- g-modes: small M, large R
- rotation enhances resonance.
- f-mode resonance: very rapid rotation, eccentric merger.

**Dynamically Induced BH mergers may be common: many flavors...**

**Binary mergers induced by tertiary companions in triples**

may compete with standard channel ??

**Non-trivial spin-orbit misalignments in binary mergers with external companions**

**Thanks.**

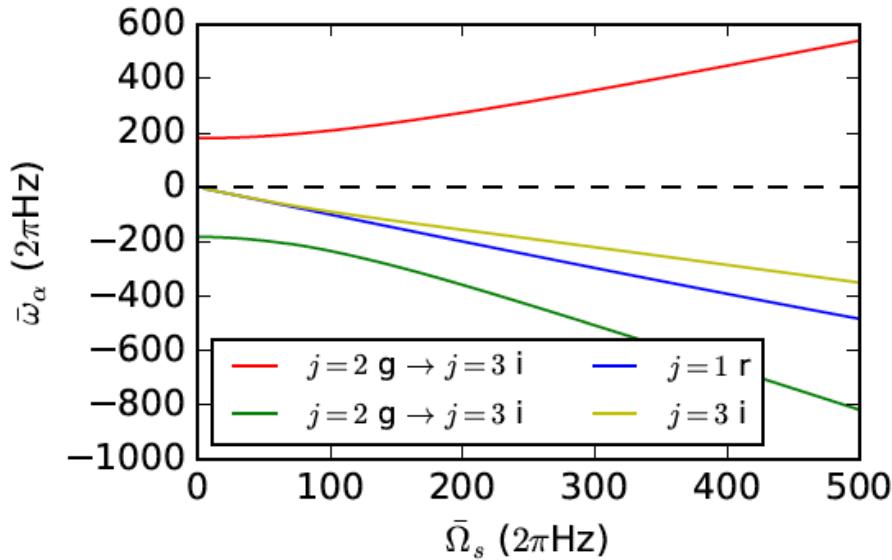
## “Quasi-Equilibrium” Tide (F-mode Distortion)

“dynamical correction” to equilibrium tide:

$$d\Phi = d\Phi^{(0)} \left[ 1 - \mathcal{O} \left( \frac{k_2 M' R^5}{M a^5} \right) \frac{1}{1 - 4\Omega^2/\omega_f^2} \right]$$

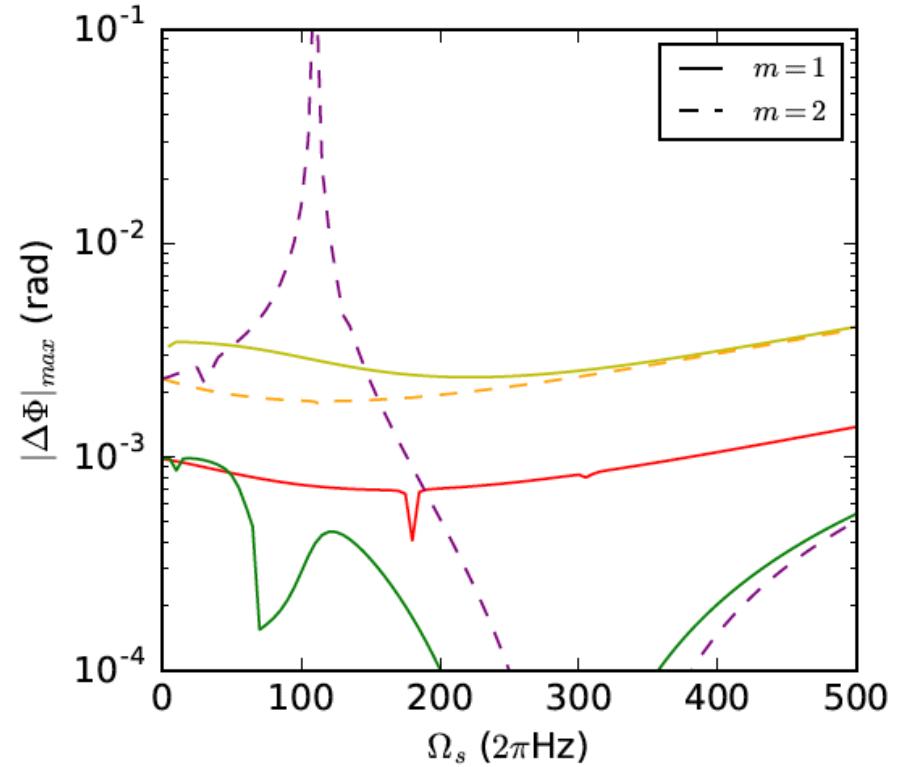
$$\text{where } \omega_{\text{tide}} = 2(\Omega - \Omega_s) \simeq 2\Omega$$

## With rotation: Many other inertial modes (modified by stratification)



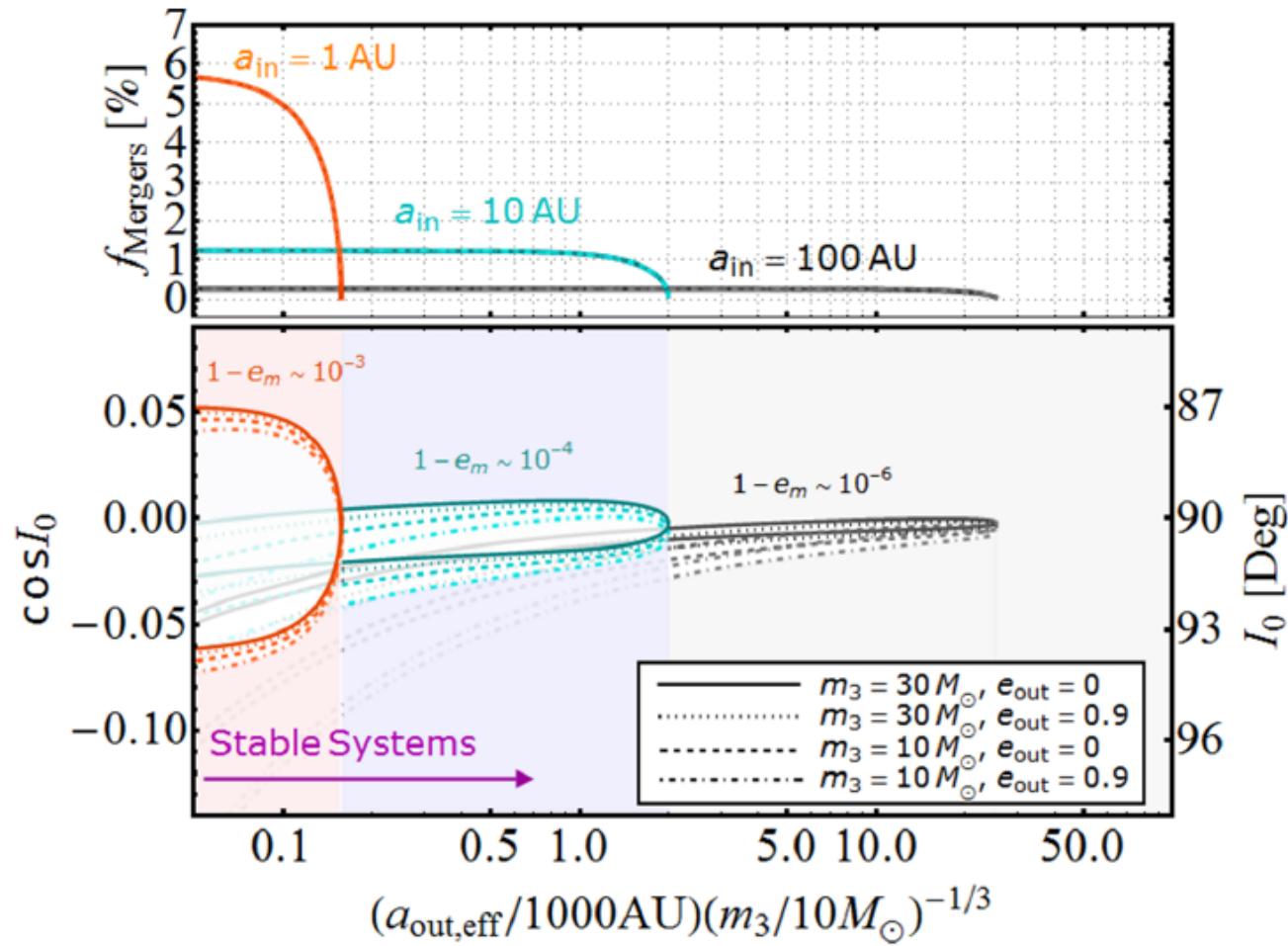
Xu & DL 2017

Not as important...



# Merger Window and Merger Fraction

## NS-NS binary with a tertiary



$$(a_{\text{out,eff}}/1000 \text{ AU})(m_3/10 M_\odot)^{-1/3}$$

$$a_{\text{out,eff}} = a_{\text{out}} \sqrt{1 - e_{\text{out}}^2}$$