Dynamical Tides in Compact Binaries

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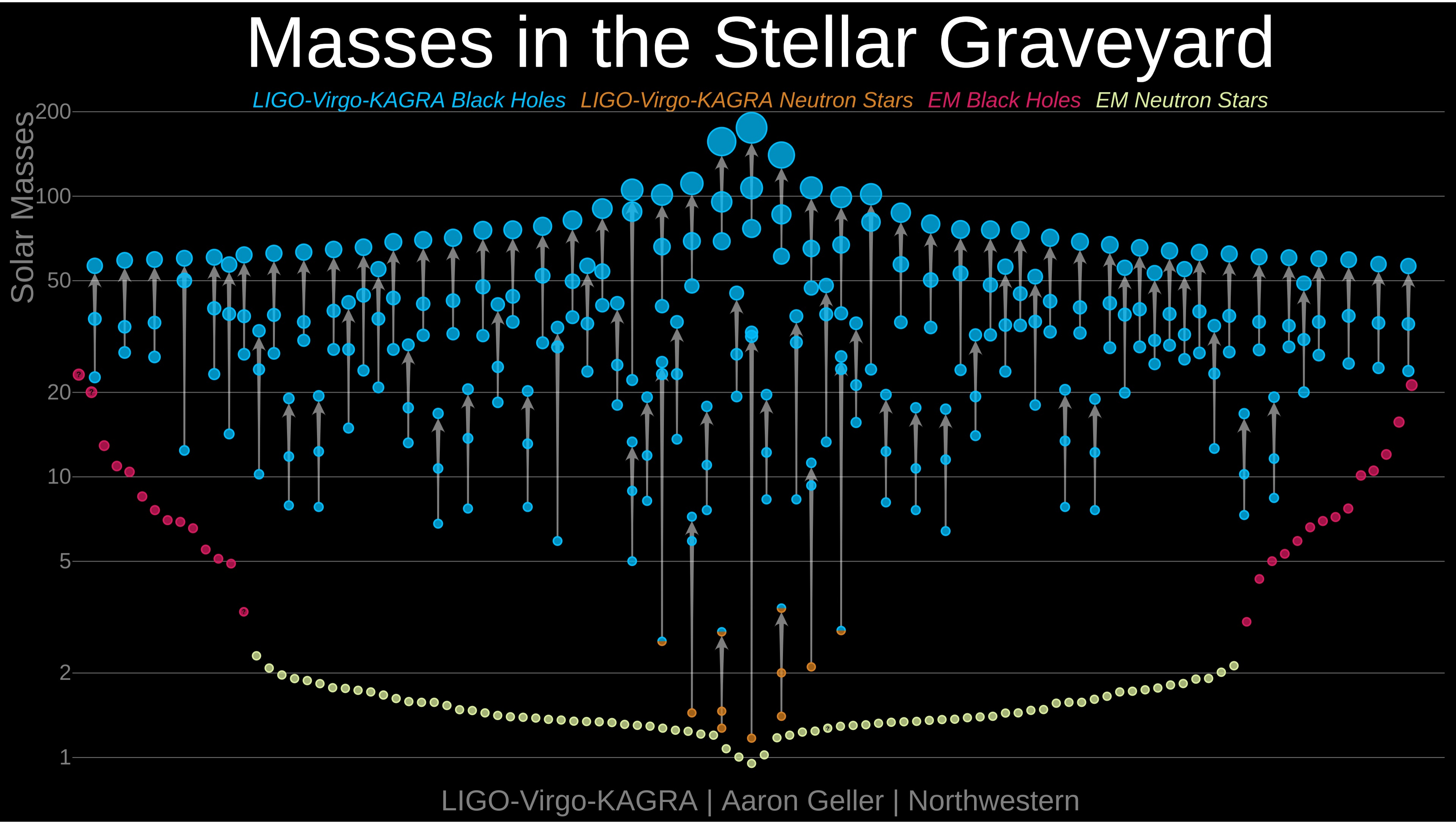
Cornell University

"Weather and Climate on Neutron Stars", PCTS (Princeton), April 5, 2022

Merging neutron star binaries

White dwarf binaries

Eccentric mergers



GWTC-3 (11/2021): 90 merger events, with 2 NS/NS mergers, 3 NS/BH mergers

Probe Nuclear EOS from NS Binary Mergers

Fate of the remnant of NS/NS

maximum TOV mass

Final merger GW of NS/NS or NS/BH

characteristic frequency $\sim(GM/R^3)^{1/2}$

Tidal effects on GW during Inspiral

Tidal Seismology in Merging NS Binaries

-- Quasi-Equilibrium tides

f-mode, p-modes

-- Resonant (Dynamical) tides

g-modes, r-modes, mixed modes

Nonradial Oscillation Modes of NS

Acoustic modes (p-modes):

Stationary sound waves, depends on sound speed $c_s = (\partial P/\partial \rho)^{1/2}$

f-mode: global acoustic mode (no radial node), $\sigma \sim \sqrt{G\bar{\rho}} \sim 2 \text{ kHz}$

G-modes (gravity modes = buoyancy modes):

Arise from stable stratification:

Density jumps in crust: interface modes

Core composition gradient: $x=n_p/n$ increases with density

$$N = g\left[\left(\frac{\partial \rho}{\partial x}\right)_P \left(\frac{dx}{dP}\right)\right]^{1/2}$$

~100 Hz or less

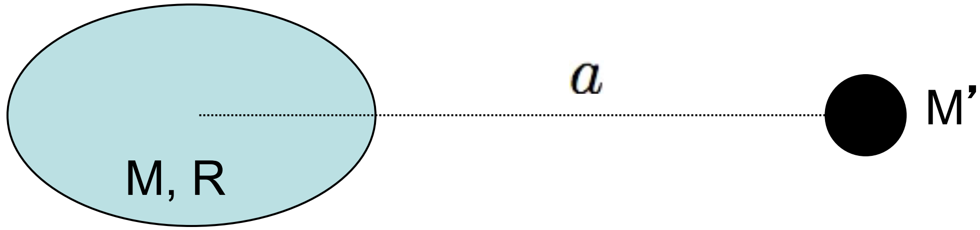
depends on symmetry energy

depends on superfluidity…

R-modes (inertial modes, Rossby modes):

arise from Coriolis force $\sigma \sim \Omega_s$

“Quasi-Equilibrium” Tide (F-mode Distortion)



Tidally induced quadrupole $Q \sim k_2 \frac{M' R^5}{a^3}$ $\quad k_2 = \text{Love number}$

$$V_0 = -\frac{MM'}{a}, \quad \Delta V \sim -\frac{M'Q}{a^3} \Longrightarrow \frac{\Delta V}{V_0} \sim \frac{\Delta\Omega}{\Omega_0} \sim \frac{Q}{Ma^2} \sim k_2 \frac{M'}{M}\left(\frac{R}{a}\right)^5$$

Additional GW from distorted NS $\Longrightarrow \frac{\Delta \dot{E}_{\rm GW}}{\dot{E}_{\rm GW}} \sim \frac{Q}{\mu a^2} \sim k_2 \frac{M_t}{M}\left(\frac{R}{a}\right)^5$

$$\Longrightarrow \frac{d\Phi}{df} = \left(\frac{d\Phi}{df}\right)_0 \left[1 - 2k_2\left(\frac{R}{a}\right)^5\left(\frac{11M'}{M} + \frac{M_t}{M}\right)\right]$$ (GW Phase Shift)

➔ Important only at small separation (just prior to merger)
(Kochenek 92; Bildsten & Cutler 92; Lai+93,94, etc)

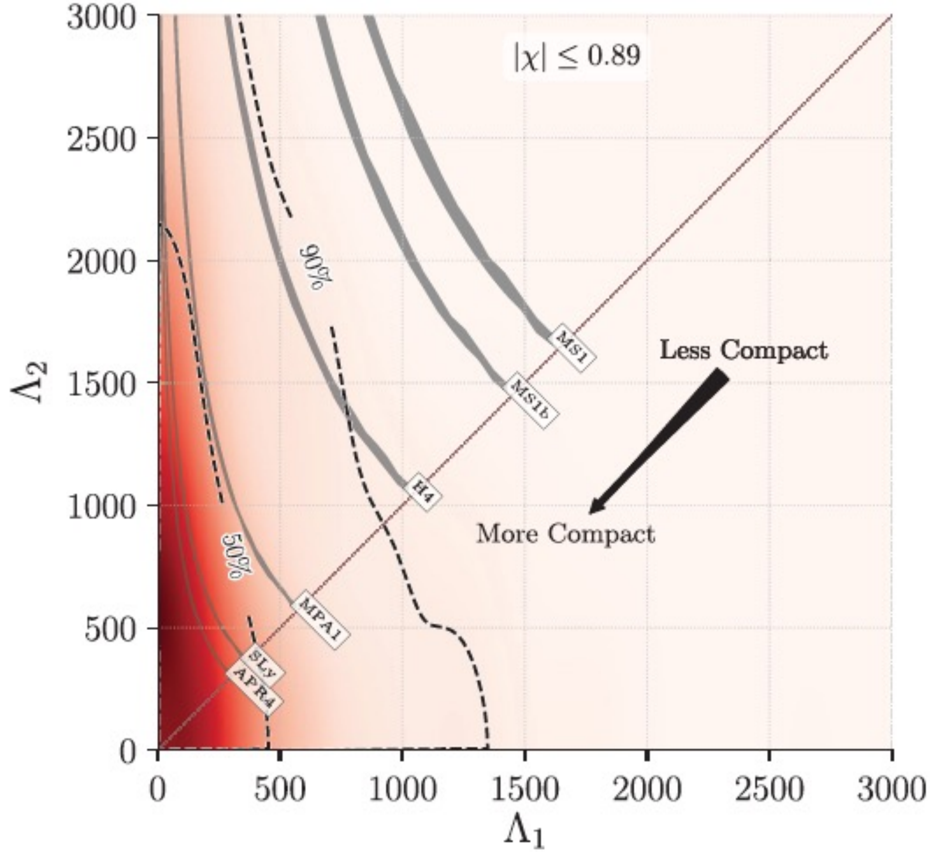
Numerical GR Quasi-equilibrium NS binary sequence

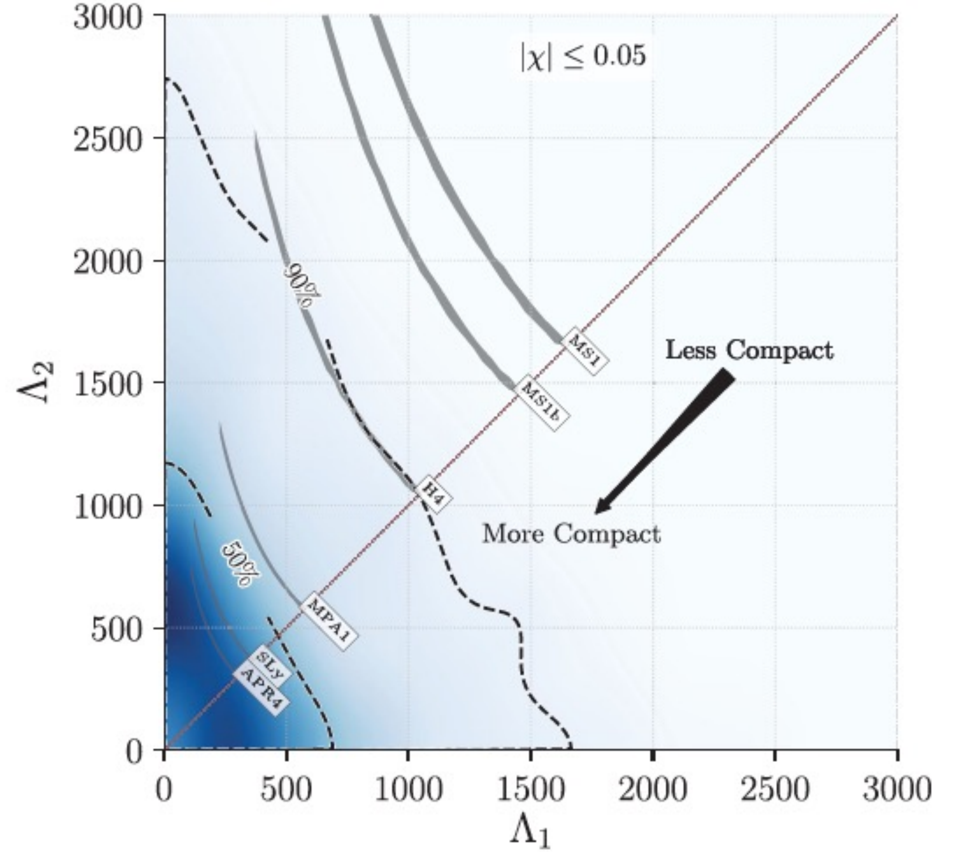
(Baumgarte, Shapiro, Teukolsky, Shibata, Meudon/Paris group, etc. late1990s--200x)

More recent GR calculations of tidal effect

(Hinderer, Flanagan, Poisson, Damour, Penner, Andersson, Jones, etc., 2008+)

GW170817:





$$\Lambda_2^{(i)} = \frac{2}{3} k_2^{(i)} \left[\left(\frac{c^2}{G} \right) \left(\frac{R_i}{M_i} \right) \right]^5$$

Quasi-Equilibrium Tide = F-mode $(l = 2)$

$$\ddot{\xi} + \sigma^2 \xi \sim e^{i2\Omega t}$$

Ω = orbital frequency

$$\Rightarrow \quad \xi \sim \frac{e^{i2\Omega t}}{\sigma^2 - (2\Omega)^2}$$

The usual tidal distortion calculation assumes $\sigma_{\mathrm{f}} \gg 2\Omega$

Dynamical correction to "equilibrium tide":

$$d\Phi = d\Phi^{(0)} \left[1 - \mathcal{O}\left(\frac{k_2 R^5}{a^5} \right) \frac{1}{1 - 4\Omega^2/\sigma_{\mathrm{f}}^2} \right]$$

Resonant Tides:
Excitations of Internal Modes

NS has low-frequency oscillation modes:

g-modes (~100 Hz)
r-modes

$$\ddot{\xi} + \sigma_\alpha^2 \xi \sim e^{im\Omega t}$$

Resonance: $\sigma_\alpha = m\Omega_{\rm orb}, \quad m = 2, 3, \cdots$

Rosonant tidal excitations of NS modes during inspiral

➔ transfer orbital energy to NS
➔ GW phase shift

Resonant Excitations of NS Oscillations During Inspiral

$$\Delta\Phi \propto \frac{(Q_{\alpha,2m})^2}{\epsilon_\alpha \sigma_\alpha}, \quad Q_{\alpha,2m} = \int d^3x \, (\delta\rho_\alpha)^\star r^2 Y_{2m}$$ (Tidal overlap coefficient)

σ_α = mode frequency in inertial frame
ϵ_α = mode frequency at zero-rotation

Non-rotating NS:

G-mode (DL 94; Reisenegger & Goldreich 94; Shibata 94)

Rotating NS:

G-mode, F-mode, R-mode (Ho & DL 99)
Inertial modes (DL & Wu 06)
R-mode (excited by gravitomagnetic force; Racine & Flanagan 06)

Resonant Excitations of NS Oscillations During Inspiral

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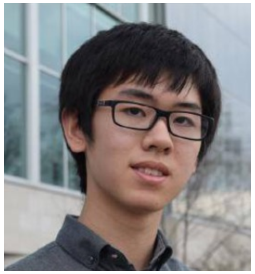
More recent works:

- Superfluid NSs: Yu & Weinberg 2017
- Gravity-inertial modes, scalings (parameterized EOS): Xu & DL 2017
- Nils Andersson and collaborators



Wenrui Xu
(Cornell '17
→ Princeton Ph.D.)

G-modes

crustal density discontinuities
stable composition stratification of core: symmetry energy, superfluidity

Parameterize the uncertainties:

$P \propto \rho^\gamma, \quad \Gamma = (\partial \ln P / \partial \ln \rho)_{\rm ad}$

$$\omega_\alpha \propto (\Gamma - \gamma)^{1/2} M^{1/2} R^{-3/2}$$
$$Q_{\alpha,2m} \propto \Gamma - \gamma,$$

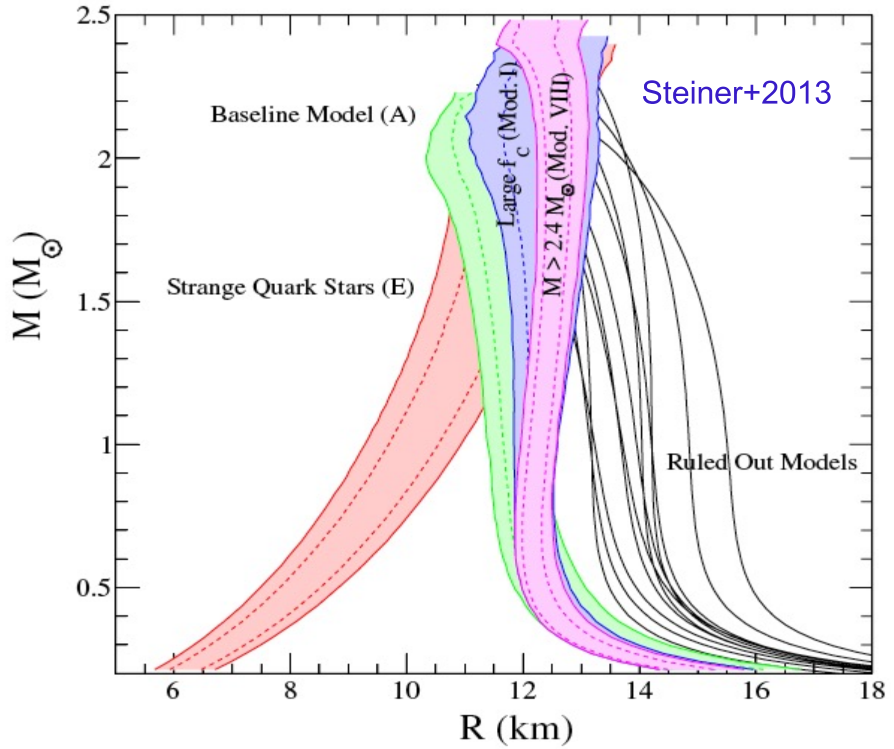
$$\Delta\Phi = -0.060 \left(\frac{R_{10}^8}{M_{1.4}^6}\right) \frac{2}{q(1+q)} \left(\frac{f_\alpha}{100\,\mathrm{Hz}}\right)^2 \times \left(\frac{\bar{f}_\alpha}{100\,\mathrm{Hz}}\right)^{-4} \left(\frac{\bar{Q}_{\alpha,22}}{10^{-3}}\right)^2,$$

Xu & DL 17

Note: $M = 1.2 M_\odot, \; R = 13 \text{ km NS}$

➔ x 21 Important !

NS Mass-Radius



Miller+2021: $R = 12.45 \pm 0.65$ km for $M = 1.4 M_\odot$

Raaijmakers+2021: $R = 12.33^{+0.76}_{-0.81}$ km for $M = 1.4 M_\odot$

G-mode excitation during binary inspiral

$$\Delta\Phi = -0.060\left(\frac{R_{10}^8}{M_{1.4}^6}\right)\frac{2}{q(1+q)}\left(\frac{f_\alpha}{100\,\mathrm{Hz}}\right)^2 \times\left(\frac{\bar{f}_\alpha}{100\,\mathrm{Hz}}\right)^{-4}\left(\frac{\bar{Q}_{\alpha,22}}{10^{-3}}\right)^2,$$

It is a coincidence that $|\Delta\Phi| \lesssim 1$...

Nuclear physics challenge:

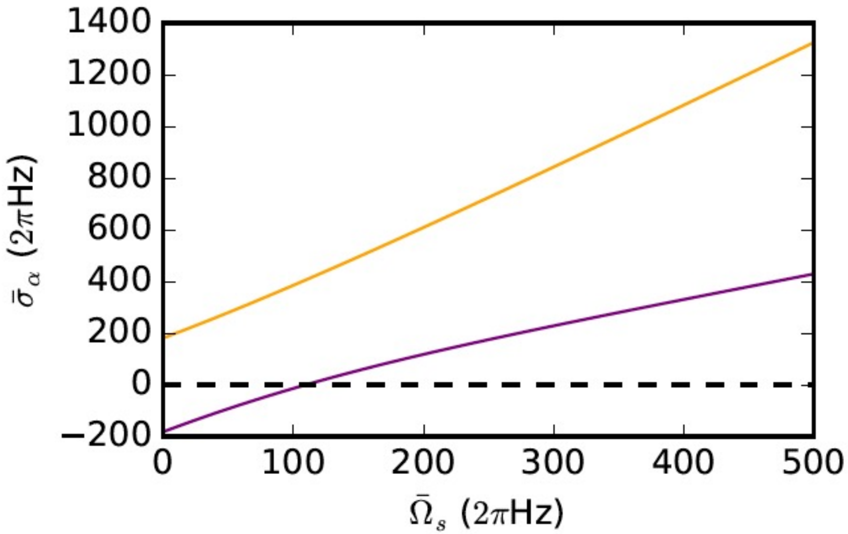
Survey $E_{\mathrm{sym}}(n)$ (constrained by exps/obs), calculate g-modes and $\Delta\Phi$

With rotation: pure r-modes (inertial modes)

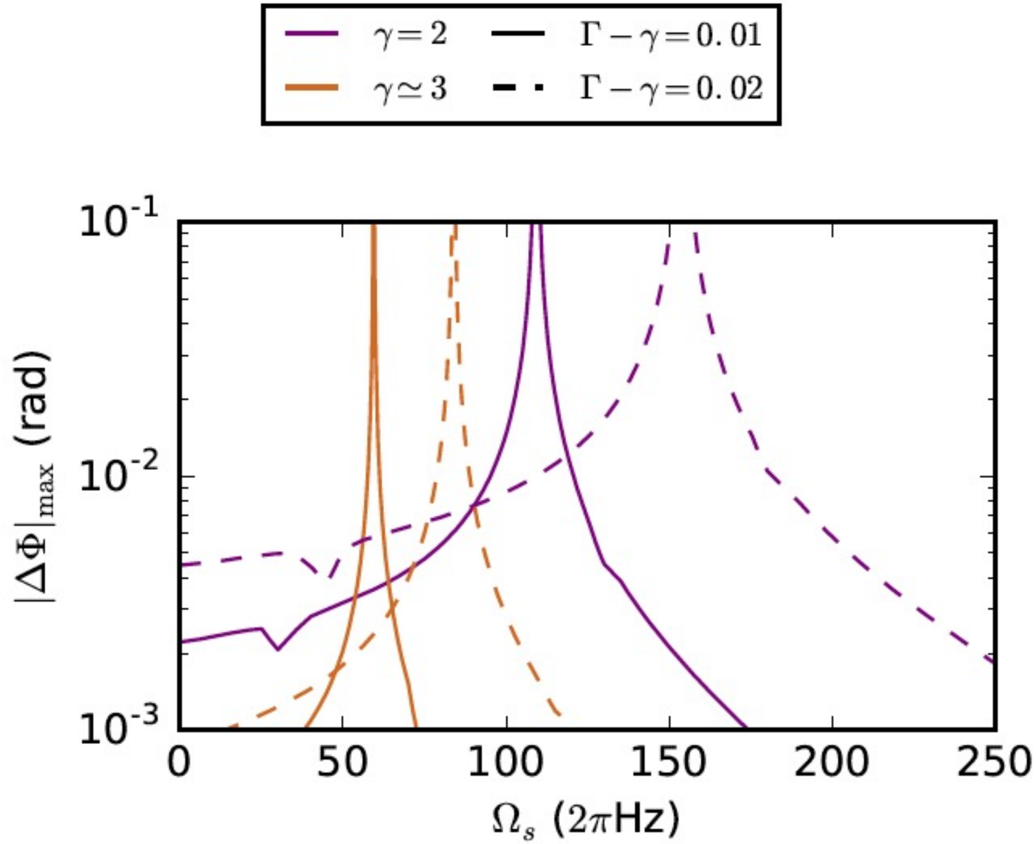
$$\Delta\Phi = \mp 0.0027 \left(\frac{R_{10}^8}{M_{1.4}^6}\right) \frac{2}{q(1+q)} \left(\frac{\epsilon_\alpha |\sigma_\alpha|}{\Omega_s^2}\right)^{-1} \times \left(\frac{Q_{\alpha,2m}}{0.02\hat{\Omega}_s^2}\right)^2 \left(\frac{f_s}{500\ \mathrm{Hz}}\right)^2 \left|\mathcal{D}_{m\pm 2}^{(2)}\right|^2,$$

Usually negligible (unless R^8/M^6 is large)…

With rotation: Mixed modes (Inertial-Gravity Modes)



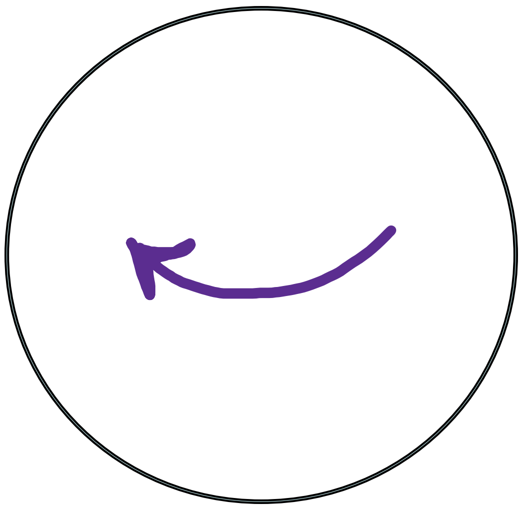
Rotation can reduce (retrograde) g-mode $|\sigma_\alpha|$, thus increase $|\Delta\Phi|$

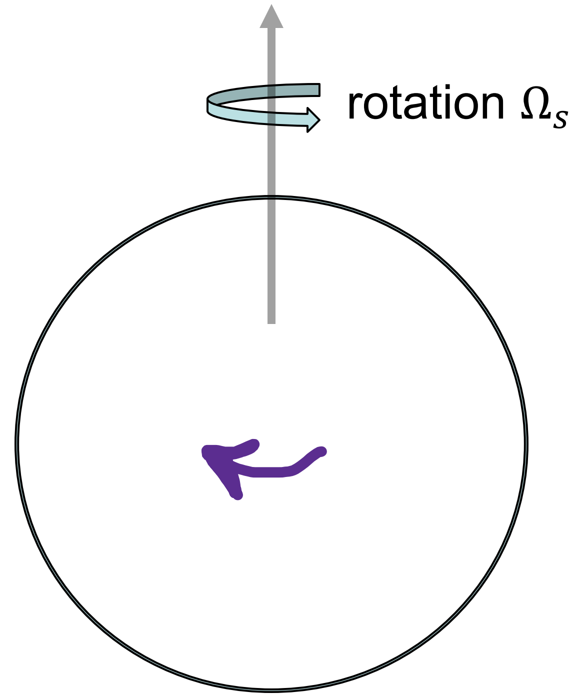


Xu & DL 2017

co-rotating frame

Inertial frame





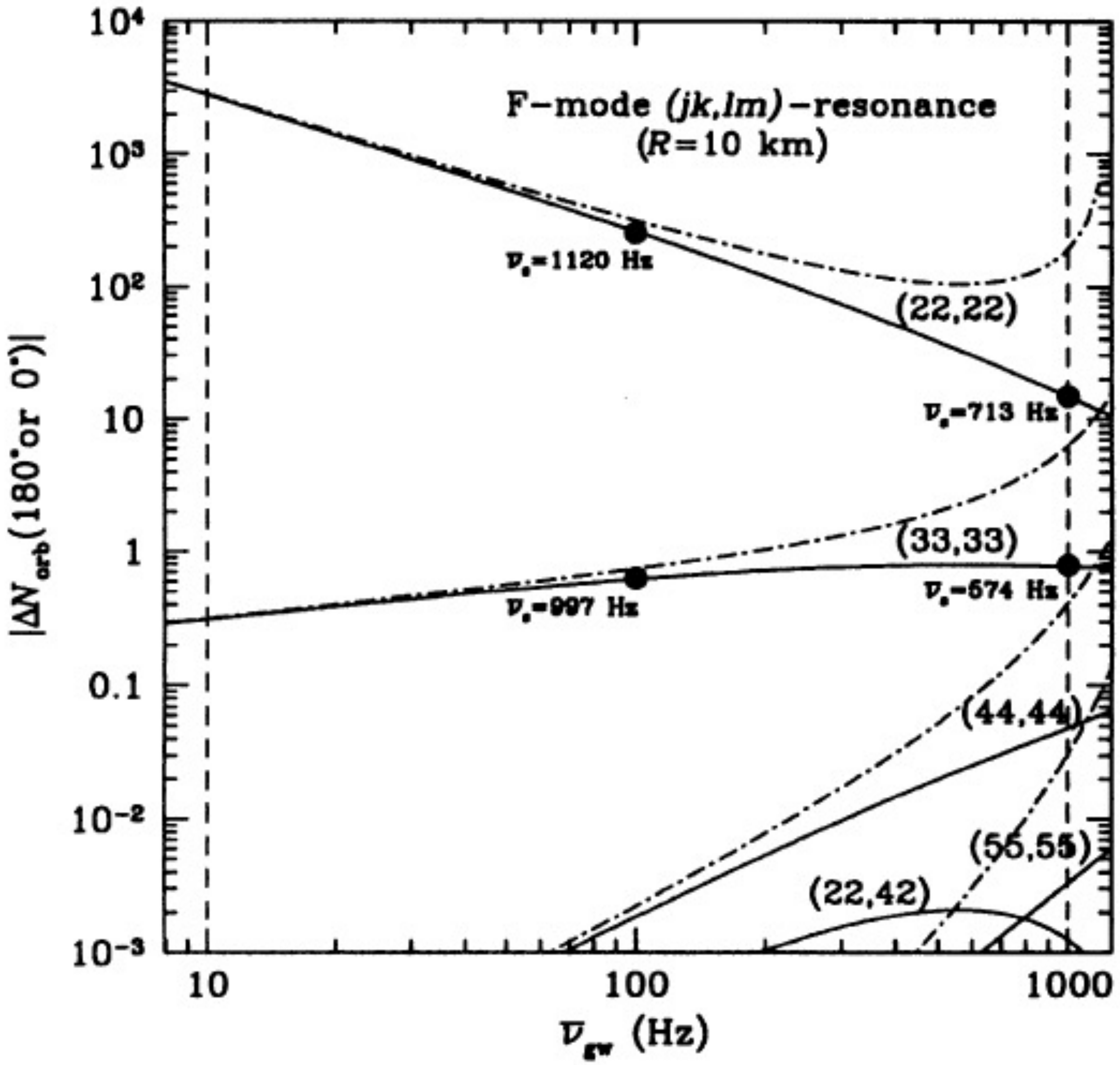
Retrograde mode $\omega_\alpha < 0$

$\sigma_\alpha = \omega_\alpha + m\Omega_s$ (with $m\Omega_s > 0$)

Resonant excitation of F-modes?

Very strong … possible with **very rapid (kHz) rotations**

Wynn Ho & DL 1999



Physics allows it.
Does Nature provide it?

Summary – Part I:

- Tidal distortion (non-resonant f-mode) constrains R-M

- Resonant tidal excitation of modes occurs at low frequency (< 100 Hz).
  If strong, would provide a unique probe of EOS (e.g. symmetry energy)

  -- g-modes: Important for low-M, large-R NSs:

  Possibly within reach for detection?

  -- rotation can enhance resonance

  -- f-mode resonance: not possible, unless very rapid rotation (kHz)

G-mode excitation during binary inspiral

Recall $\Delta\Phi \propto -\mathrm{R}^8$

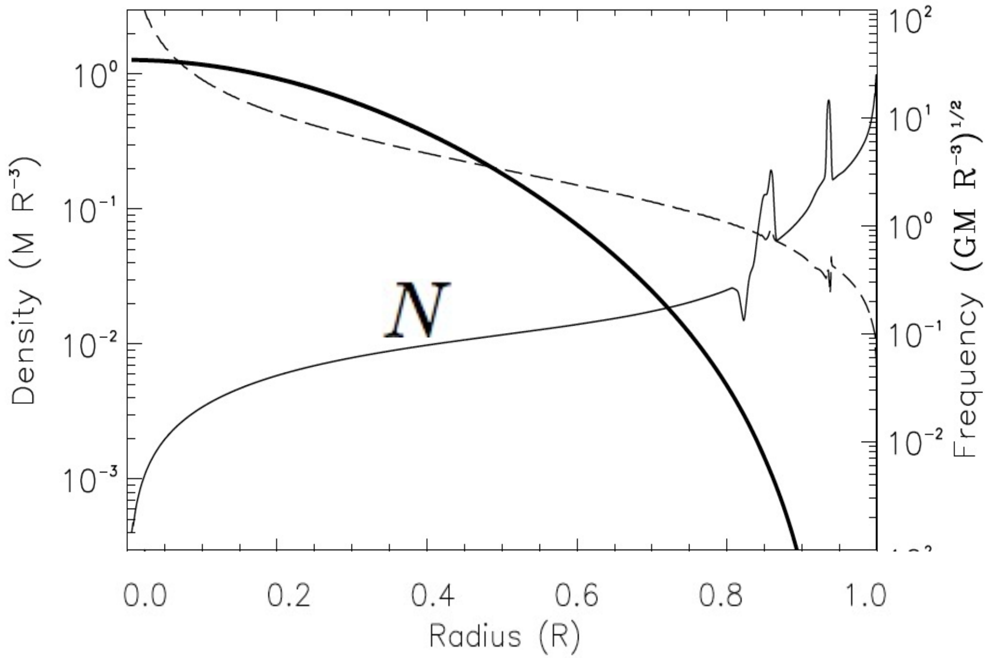
Very important for merging white dwarf binaries…

In fact, tidally excited g-modes (gravity waves) become nonlinear…

“Continuous” Excitation of Gravity Waves

Waves are tidally excited in the interior/envelope, propagate outwards and dissipate near surface

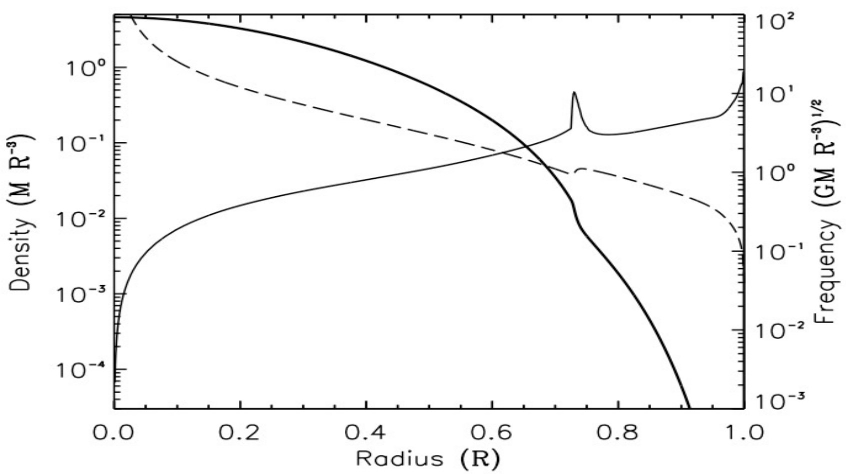
Wave Propagation inside White Dwarf



N = Local Buoyancy Freq

CO WD

$0.6M_\odot,\ 8720\,\mathrm{K}$

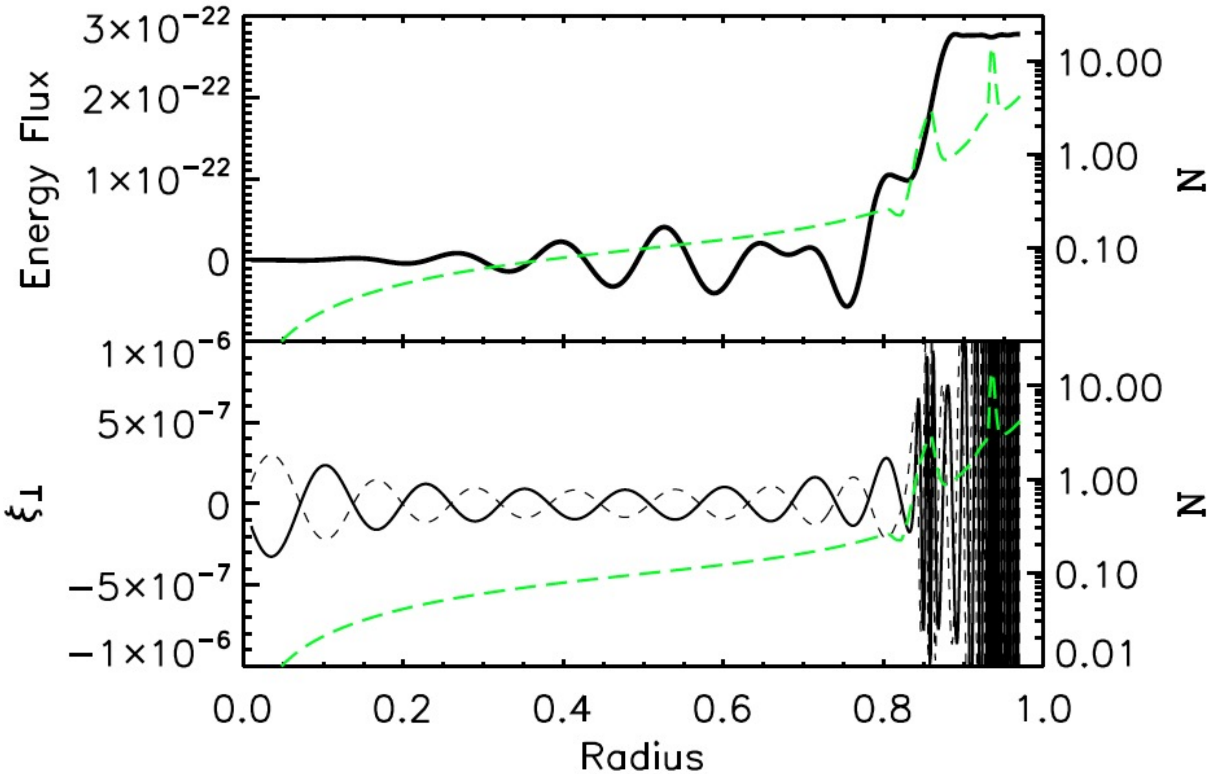


He-core WD

$0.3M_\odot,\ 12000\,\mathrm{K}$

“Continuous” Excitation of Gravity Waves

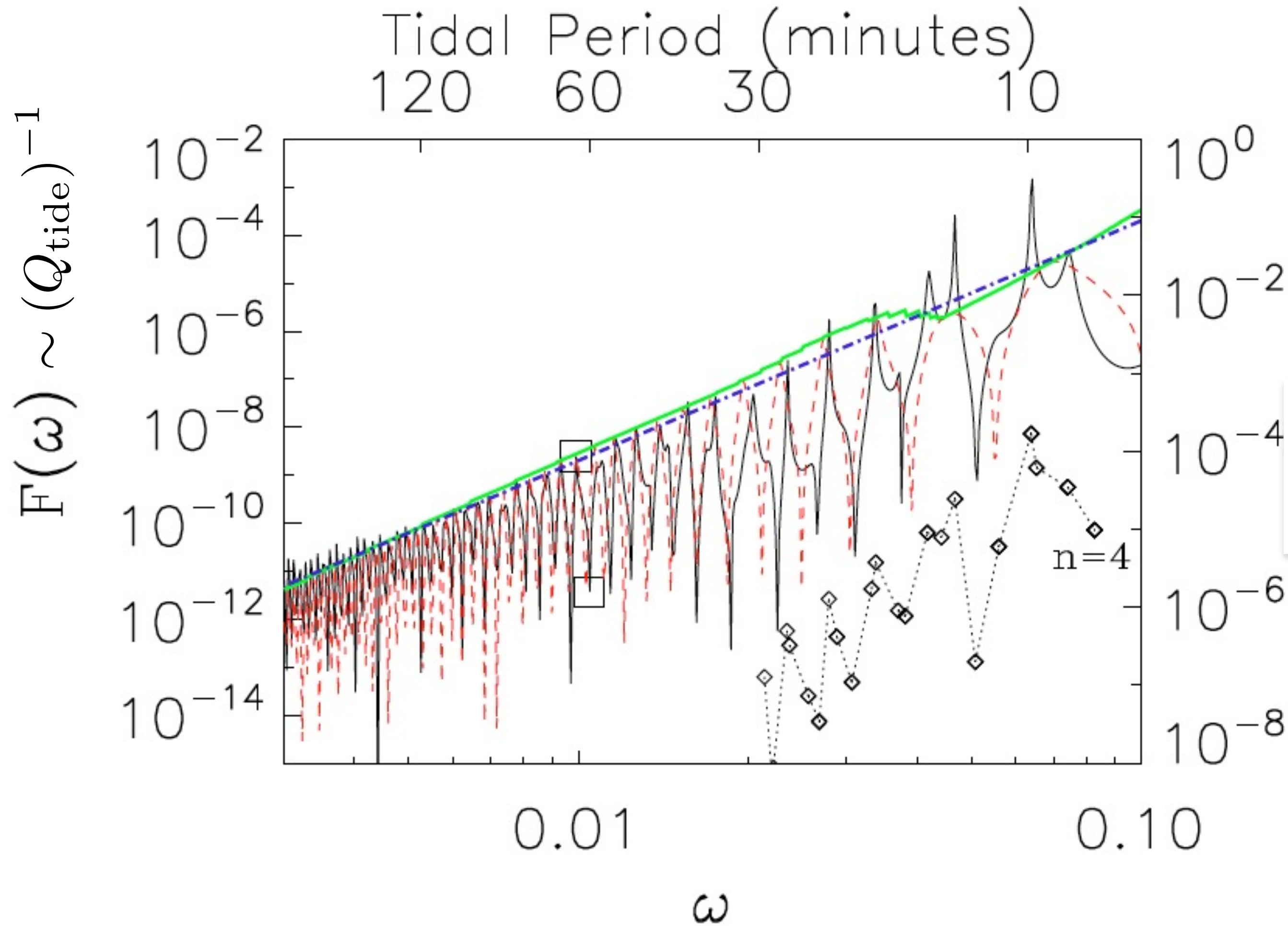
Waves are tidally excited in the interior/envelope, propagate outwards and dissipate near surface



Jim Fuller & Lai
2012-2013

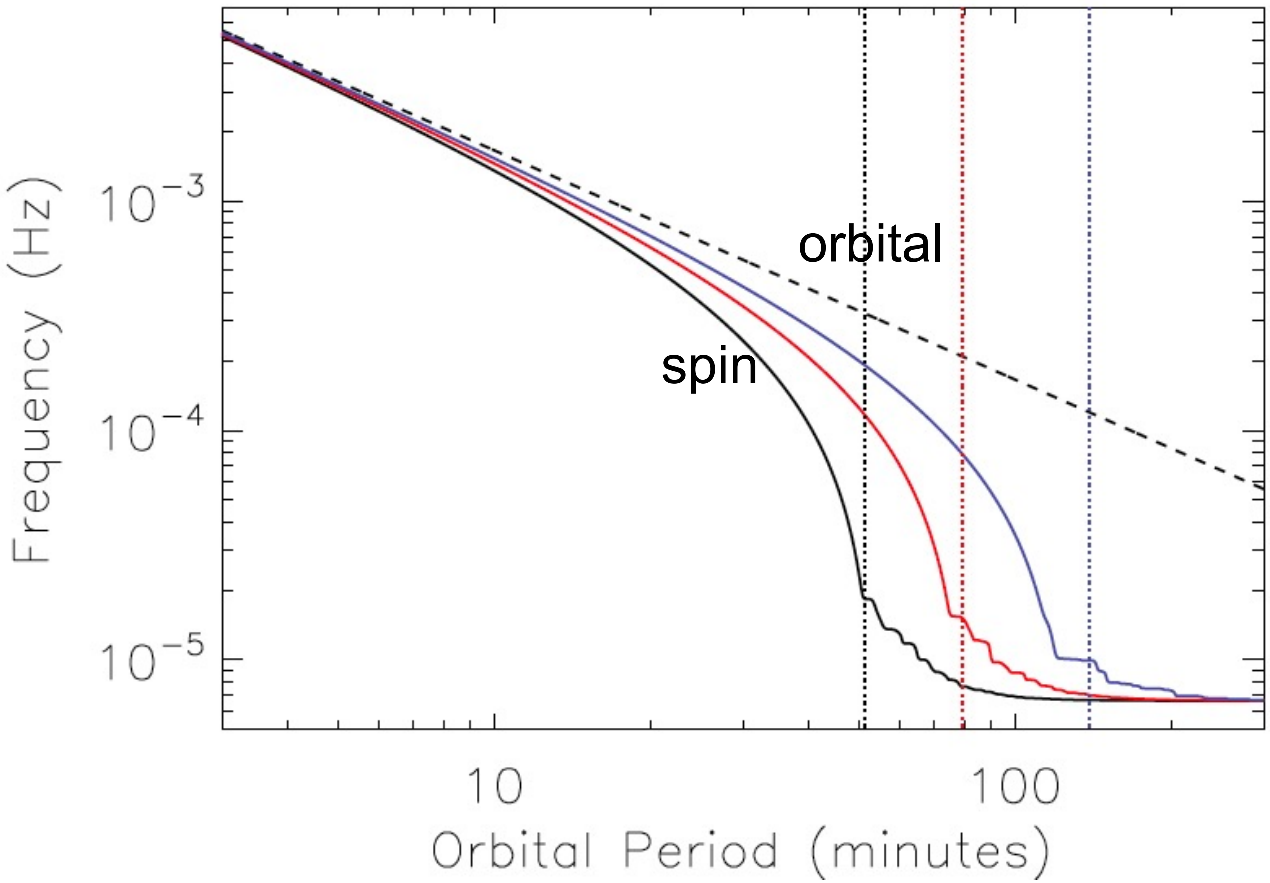
$$M = 0.6 M_{\odot}, \quad \omega = 0.01$$

Tidal torque on WD = $G\left(\frac{M'}{a^3}\right)^2 R^5\, F(\omega)$



Tidal forcing frequency

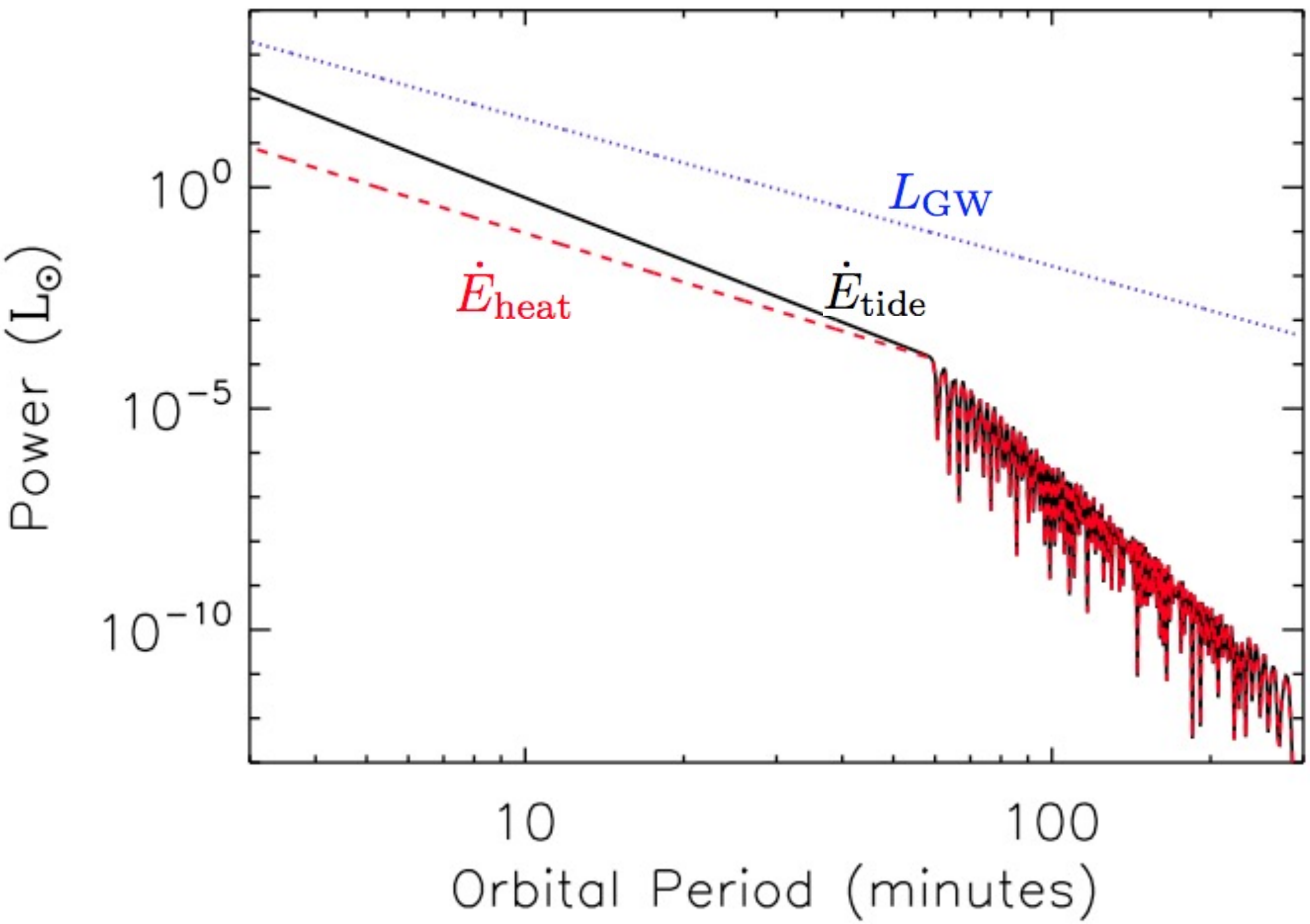
Tidal torque drives spin-orbit towards near (but not complete) synchronization for $P \lesssim 20\,\mathrm{min}$



Fuller & DL 2012

See also Yu, Weinberg, et al. 2020

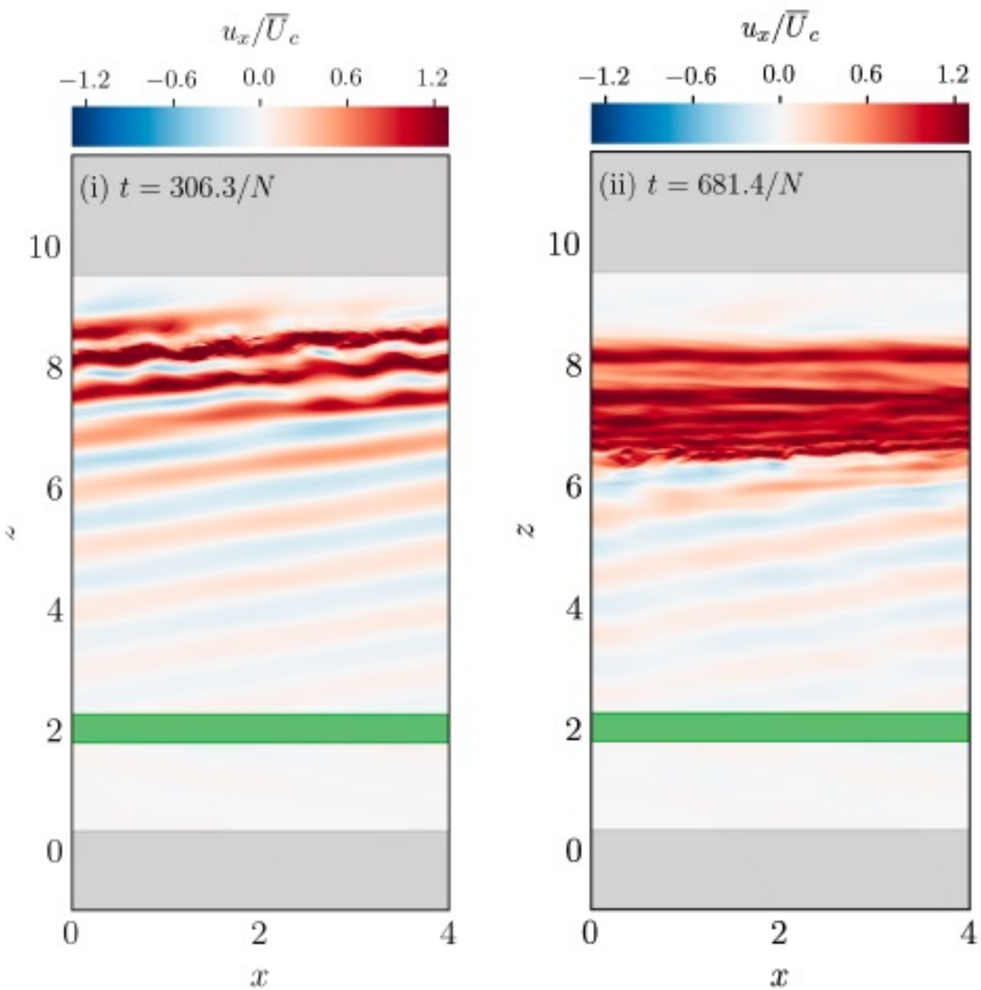
Tidal Heating Rate



$$\dot{E}_{\rm heat} = \dot{E}_{\rm tide}\left(1 - \frac{\Omega_s}{\Omega_{\rm orb}}\right)$$

Physics of gravity wave breaking/absorption in WD envelope

Simulations of gravity waves in stratified plane-parallel atmosphere



Su, Lecoanet & Lai 2020

Yubo Su
(Cornell Ph.D.22
→Princeton)

Nonlinear breaking of waves

➔ Formation of critical layer
(above which horizontal flow velocity
= phase velocity of the wave);
waves are absorbed at the critical layer;

Width of critical layer limited by KH instability

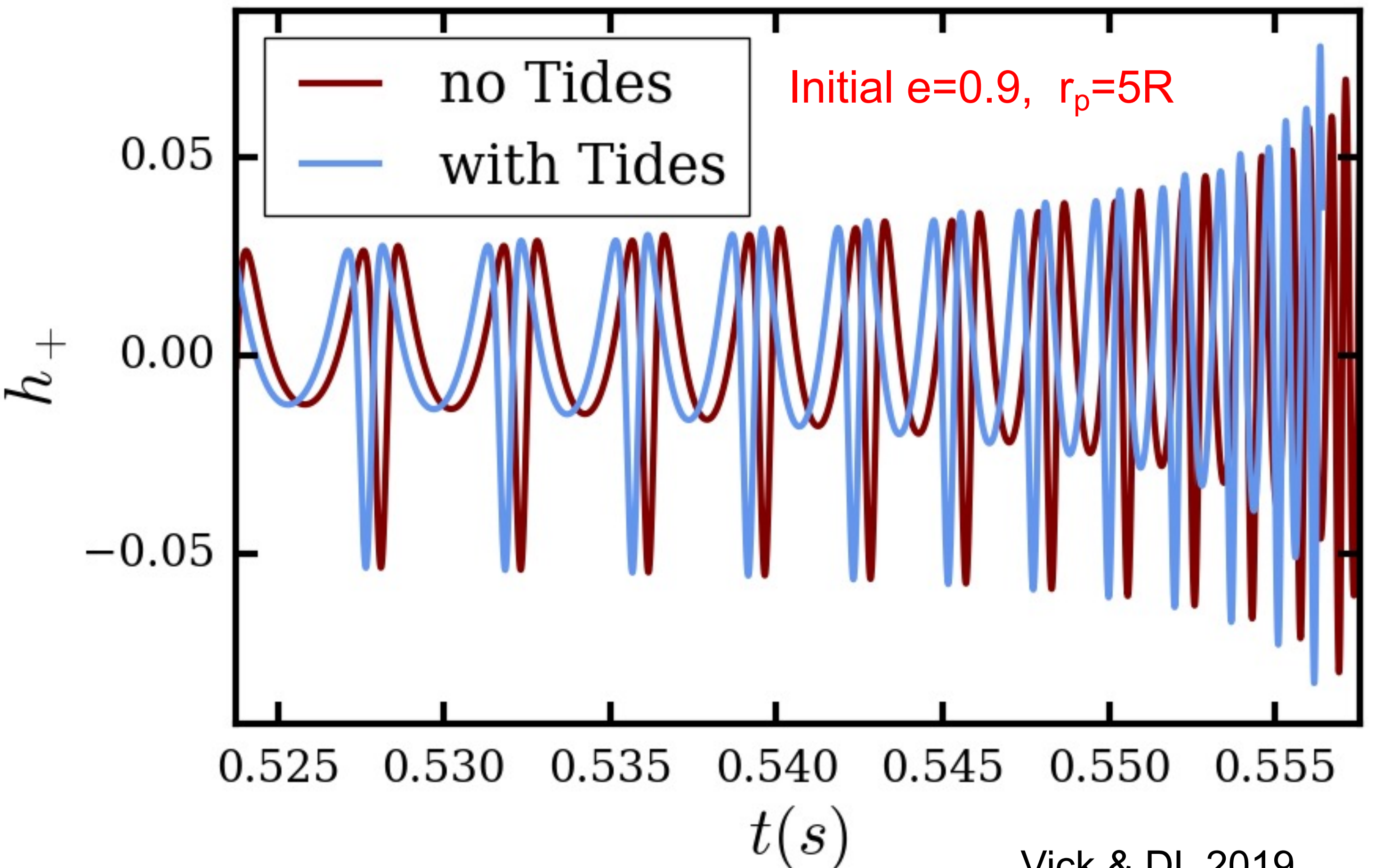
$$R_i = \left.\frac{N^2}{(\partial \bar{u}_x/\partial z)^2}\right|_{z_c} = \frac{1}{4}$$

So far: Circular mergers

What about eccentric NS (BH) mergers?

Tidal effect in eccentric NS mergers

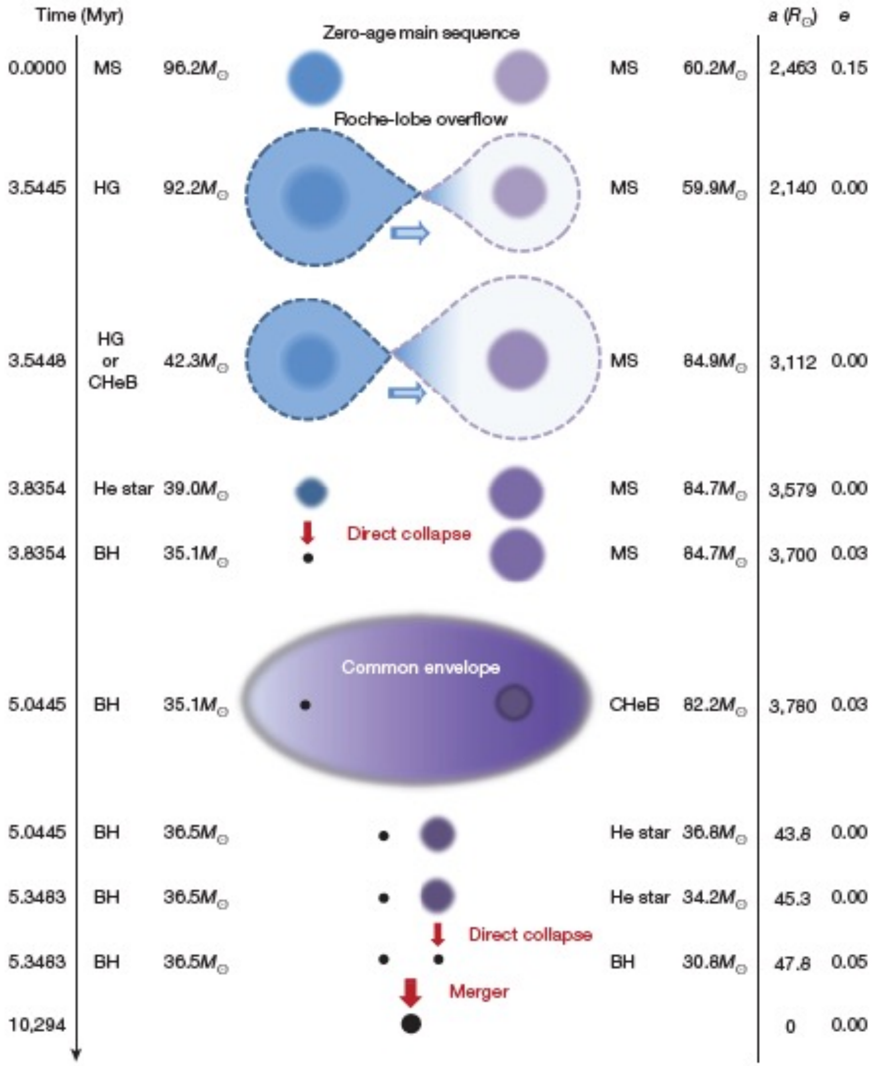
Phase shift (due to f-mode) is much enhanced compared to circular systems



Vick & DL 2019

Michelle Vick
(Cornell Ph.D.20 → Northwestern)

Standard Binary Evolution Channel ➔ Circular mergers



Belczynski +16

Possibility of eccentric mergers:
Dynamical formation merging compact binaries

-- Binary + single scatterings dense clusters

e.g. Portegies Zwart & McMillan 2000; Rodriguez et al.2015; Chatterjee et al.2017; Samsing et al. 2018; …

-- Binary mergers induced by external companion (SMBH, stellar triple)

e.g. Miller & Hamilton 2002; Wen 2003; Antonini+2017; Silsbee & Tremaine 2017; Bin Liu & Lai 2018,19

➔ Produce mostly circular mergers; a small fraction have $e \neq 0$ in LIGO band

Long-Term Evolution of Tightly-Packed Stellar BHs in AGN Disks:

Highly Eccentric BH Binary Mergers

Li, Lai & Rodet

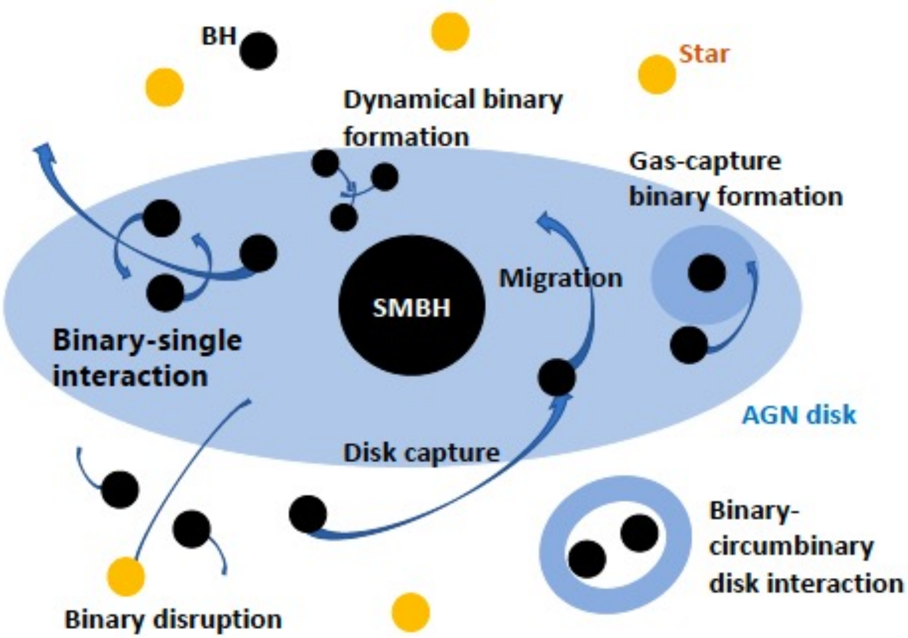
arXiv:2203.05584

Jiaru Li

(Cornell Ph.D. 2023)

BH (NS) Mergers in AGN Disks…

Where do BH binaries in AGN disks come from?



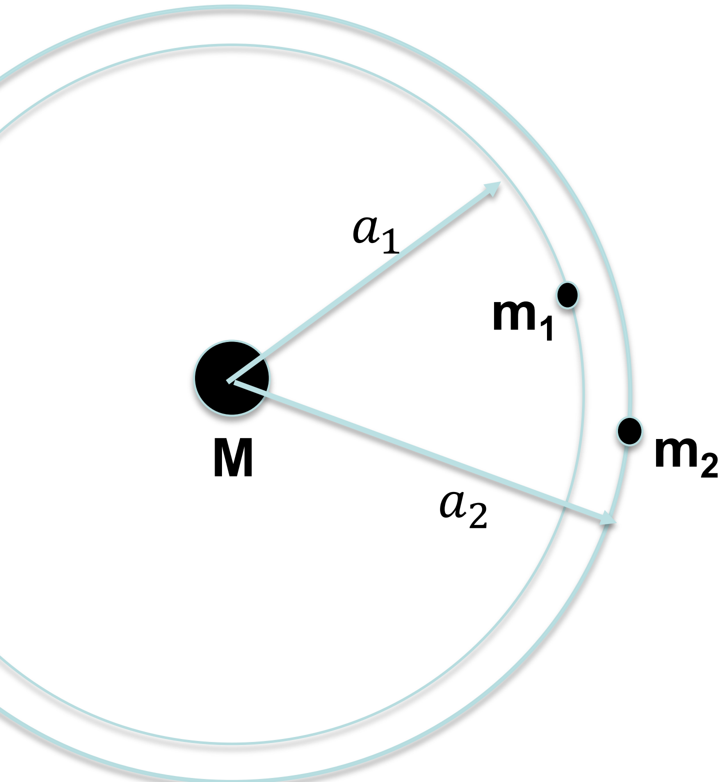
1. Binaries form in disks via GI (~pc)
2. Binaries in nuclear clusters get captured in disks
3. Single BHs in AGN disks get captured in binaries

Tagawa, Haiman, Kocsis 2020

See also Bartos+17; Stone+17; Secunda+18;….

The Problem:

Li, Lai & Rodet 2022



Two BHs (m_1, m_2) on closely-packed, nearly circular, nearly-coplanar orbits around a SMBH (M)
(e.g. brought together by migration in AGN disks)

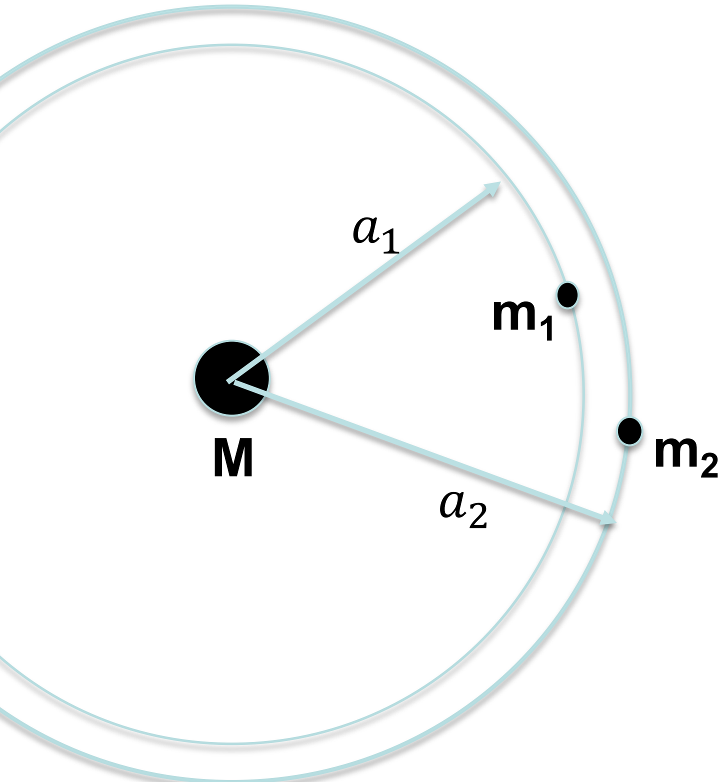
When $a_2 - a_1 \lesssim 2\sqrt{3}\,R_{\rm H}$

$$R_{\rm H} = a_1 \left(\frac{m_{12}}{3M}\right)^{1/3}, \quad m_{12} = m_1 + m_2$$

orbits are dynamically unstable.

What happens to the two BHs?

Neglect gas effect for now…

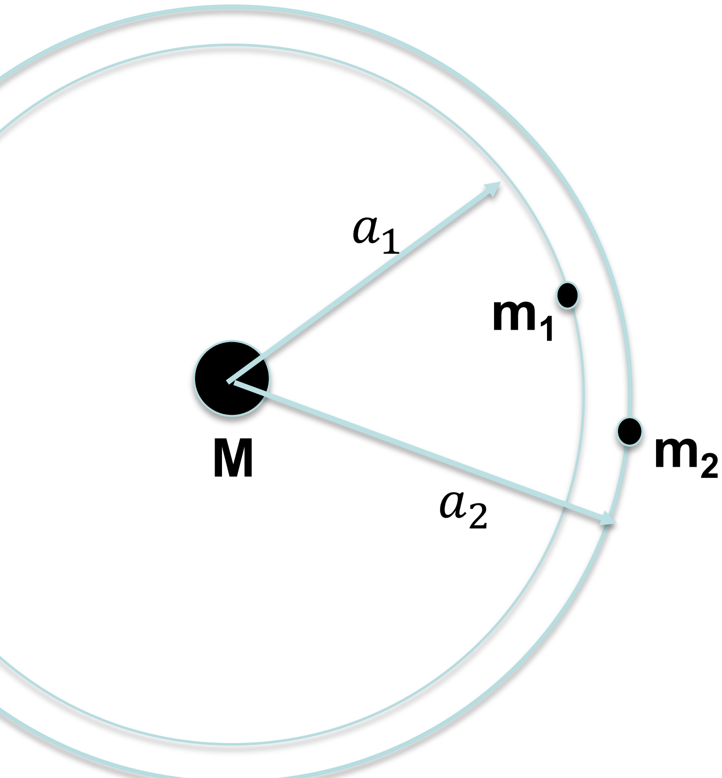


Two planets in unstable orbits around a star:

Mainly two outcomes:

1. Ejection of lower-mass planet
2. Planet-planet collision

See Li, Lai, Anderson & Pu 2021 and refs therein



Two planets in unstable orbits around a star:

Mainly two outcomes:

1. Ejection of lower-mass planet
2. Planet-planet collision

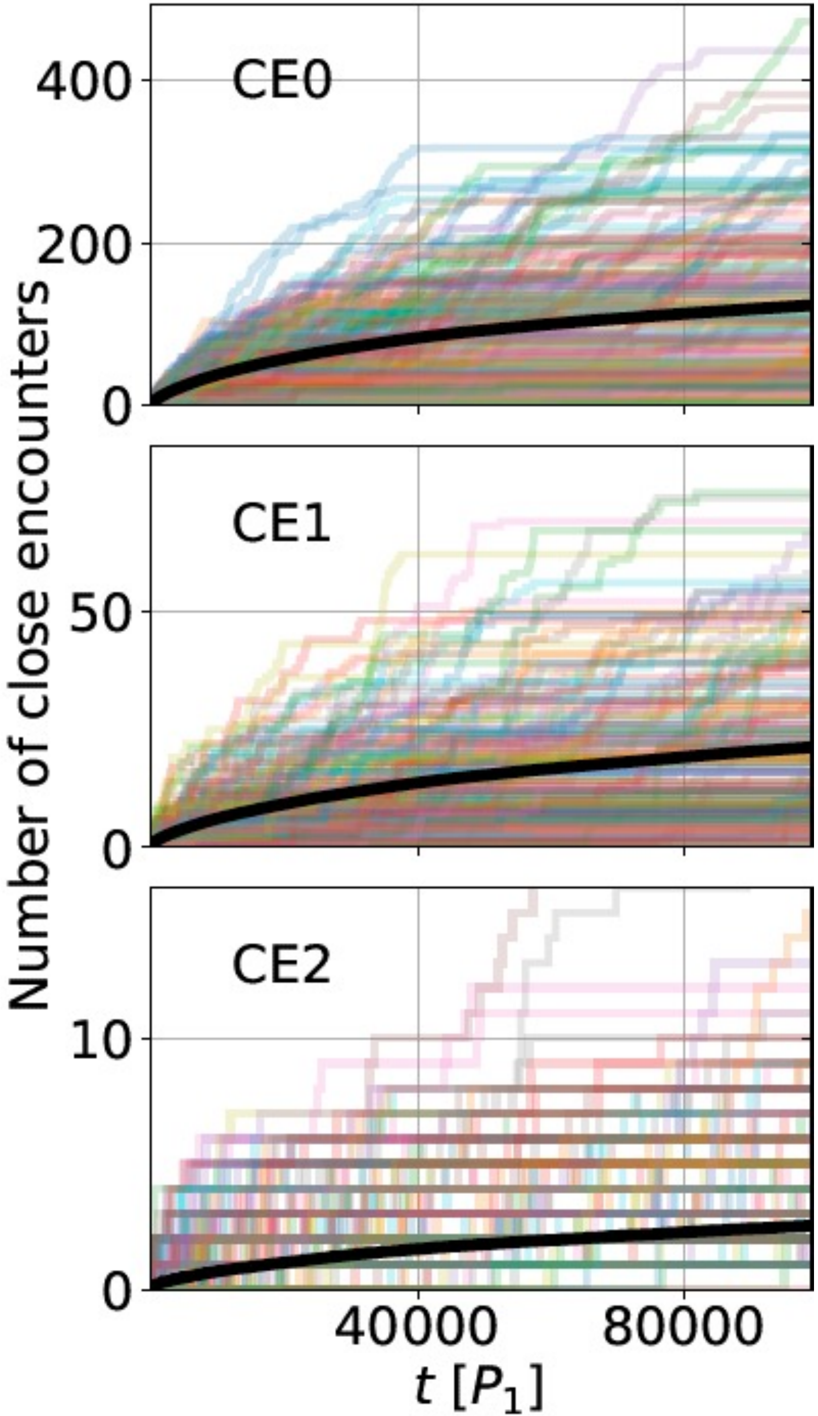
See Li, Lai, Anderson & Pu 2021 and refs therein

Two BHs in unstable orbits around a SMBH:

Since $M/m_{12} \sim 10^6 \gg 1$
ejection is not possible (takes many orbits > Hubble time)

Since $\frac{GMm_{1,2}}{a} \gg \frac{Gm_1 m_2}{a}, \frac{Gm_1 m_2}{R_{\rm H}}$

➔ The two BHs undergo "chaotic" motion, experience recurring closer encounters (separation < R_H)

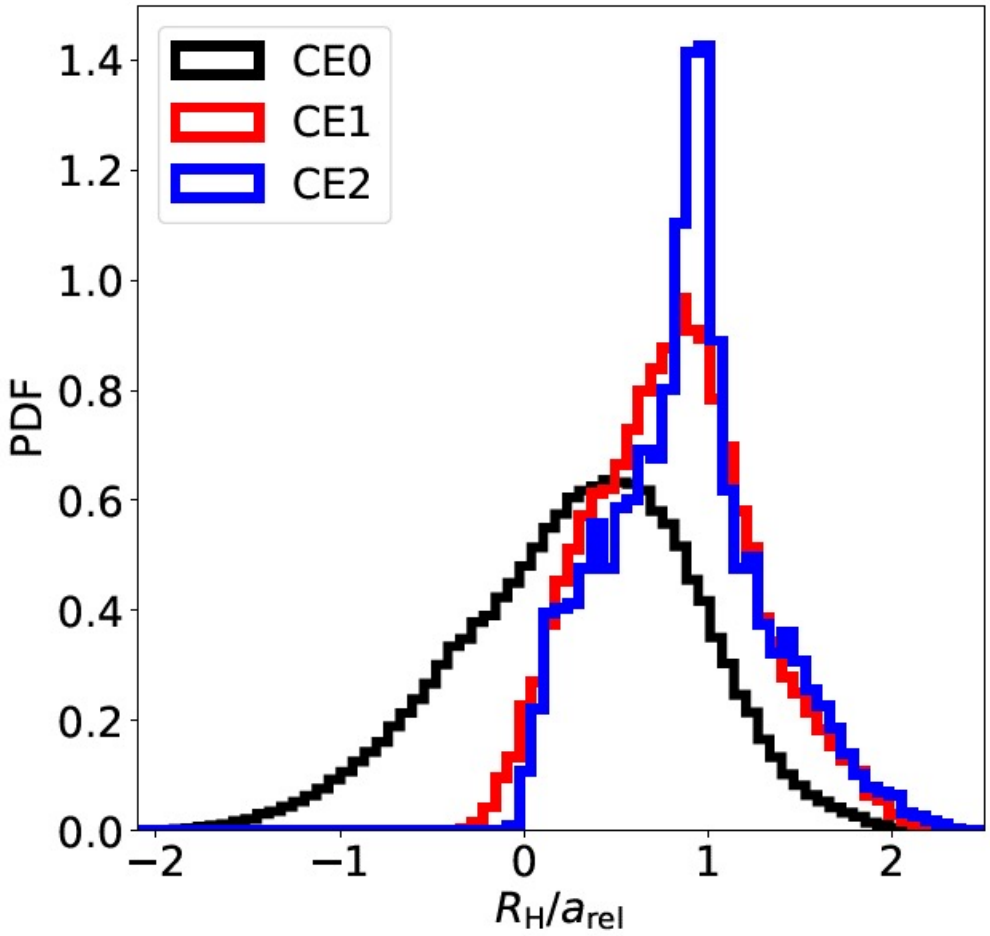


Close encounters with $r_{\rm rel} < R_{\rm H}$

Close encounters with $r_{\rm rel} < 0.1 R_{\rm H}$

Close encounters with $r_{\rm rel} < 10^{-2} R_{\rm H}$

During close encounters, the BH pairs are temporarily bound with $a_{\rm rel} \sim R_{\rm H}$



But they are all short-lived (destroyed by tide from SMBH in ~ one orbit)

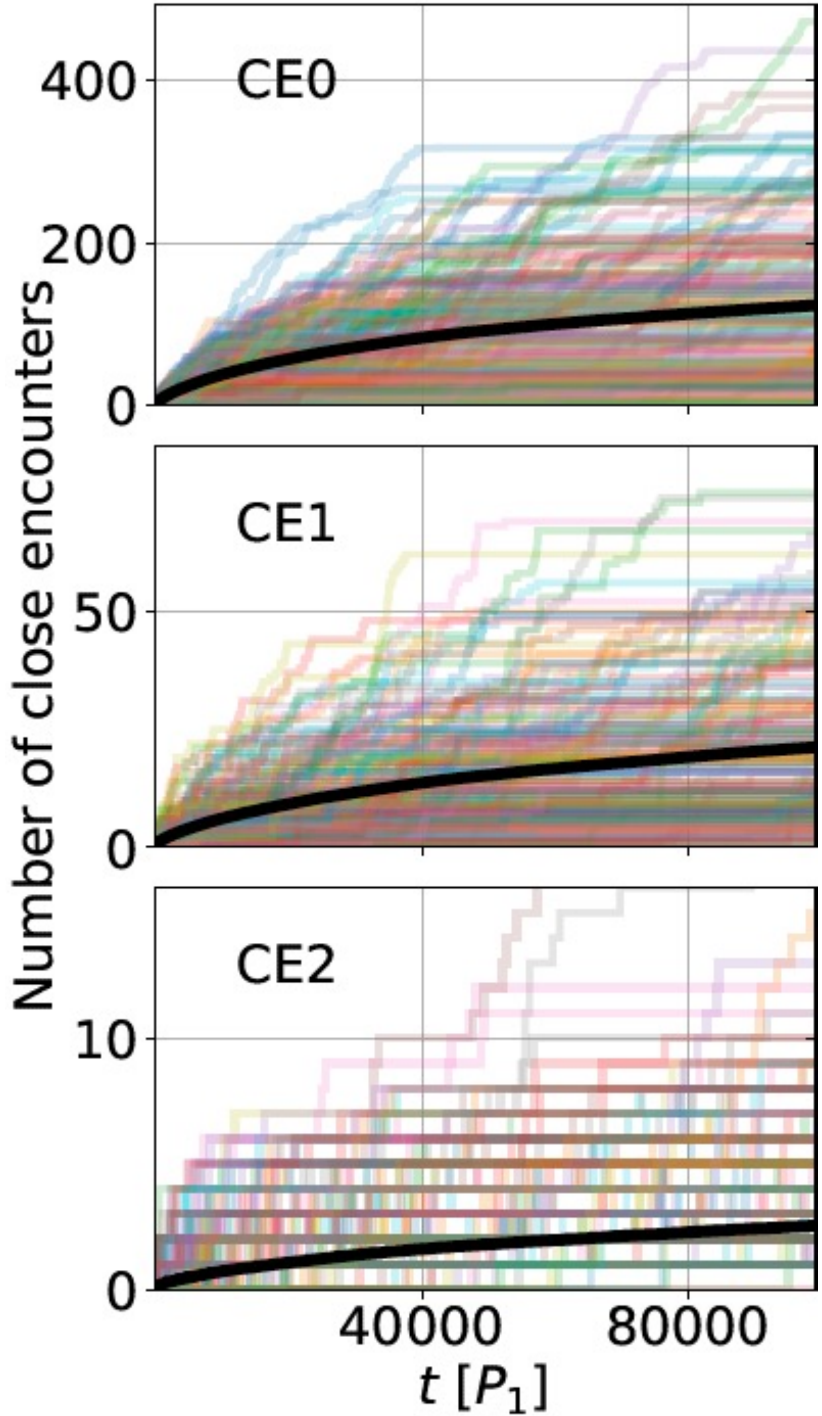
For VERY close encounter:

GW emission $\Delta E_{\rm GW} \sim \frac{\mu^2 m_{12}^{5/2}}{r_{\rm rel}^{7/2}} \gtrsim \frac{G m_1 m_2}{R_{\rm H}}$

$$\Rightarrow \quad \frac{r_{\rm rel}}{R_{\rm H}} \lesssim 10^{-4} \left(\frac{4\mu}{m_{12}}\right)^{2/7} \left(\frac{10^6 m_{12}}{M}\right)^{10/21} \left(\frac{a_1}{100M}\right)^{-5/7}$$

Capture radius for forming “permanent” binary due to GW bremsstrahlung

What is the cumulative capture rate (i.e. CE4 rate)?



Close encounters with $r_{\mathrm{rel}} < R_{\mathrm{H}}$

Close encounters with $r_{\mathrm{rel}} < 0.1R_{\mathrm{H}}$

Close encounters with $r_{\mathrm{rel}} < 10^{-2}R_{\mathrm{H}}$

For a typical "SMBH + 2 BHs" system (in unstable orbits), what is the cumulative capture rate to form real bound binary?

$$l_{\rm rel} \simeq \sqrt{2 m_{12} r_{\rm rel}}$$

$$\frac{{\rm d}P}{{\rm d}\,l_{\rm rel}} \propto l_{\rm rel} \quad \Rightarrow \quad P(< r_{\rm rel}) \propto r_{\rm rel}$$

$$\Longrightarrow \quad \langle N_{\rm cap}(t) \rangle \simeq 6 \times 10^{-5} \left(\frac{t}{P_1}\right)^{0.52} \left(\frac{r_{\rm cap}}{10^{-4} R_{\rm H}}\right)$$

It takes $10^8 P_1$ (on average) for two BHs to capture into bound merging binary

Captured BH binary as GW source

$$f_{\rm cap} \simeq (1.4\,{\rm Hz}) \left(\frac{4\mu}{m_{12}}\right)^{-3/7} \left(\frac{M}{10^8 M_\odot}\right)^{-2/7} \left(\frac{m_{12}}{100 M_\odot}\right)^{-5/7} \left(\frac{a_1}{100 M}\right)^{-3/7}$$

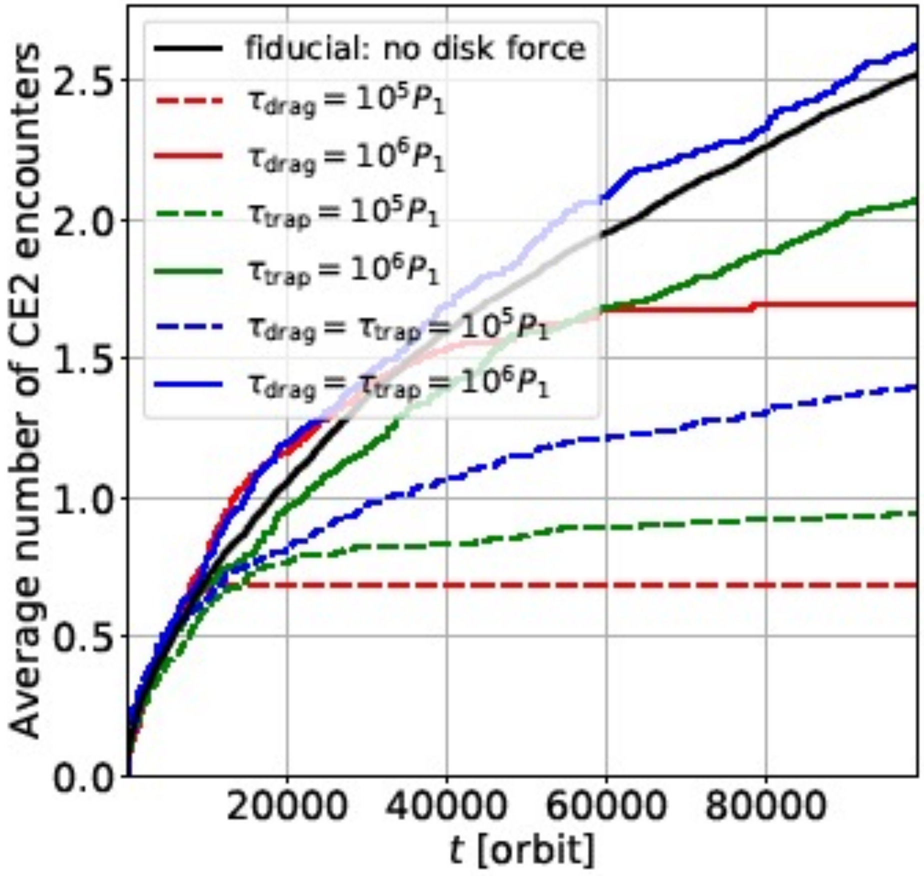
Once capture, it will take a few orbits to merge

it enters LIGO band with $e \gtrsim 0.5$

What about the gas effect? (uncertain)

In N-body simulations, add

$$\boldsymbol{F}_{\mathrm{drag}} = -\frac{\boldsymbol{v} - \boldsymbol{v}_{\mathrm{K}}}{\tau_{\mathrm{drag}}} \quad \text{and/or} \quad \boldsymbol{F}_{\mathrm{trap}} = -\frac{\Omega_{\mathrm{K},0}(r - r_0)}{\tau_{\mathrm{trap}}}\hat{\boldsymbol{\theta}}$$



Gas does not increase the capture rate

Summary

- Tidal distortion (non-resonant f-mode) constrains NS R-M
- Resonant tidal excitation of modes occurs at low frequency (< 100 Hz).
  If strong, would provide a unique probe of EOS (e.g. symmetry energy)
  - -- g-modes: Important for low-M, large-R NSs: maybe within reach?
  - -- rotation may enhance resonance.
  - -- f-mode resonance: not possible unless very rapid rotation

- Resonant tides are important for WD binaries
  physics: Gravity wave excitation and dissipation by nonlinear breaking
  and absorption in critical layers
  affects gravitational waveform in LISA band, tidal heating of WDs

- Highly eccentric mergers