1.

To prove: (A^B) → (C V D)

Premise: ~D → ~B

Prove by conditions:

Already known that if ~D is true then ~B is true by premise. This is equal to if B is true then D is true by contraposition.

Need to show that when (A^B) is true then (C V D) is true. So, let's assume that (A^B) is true, then we know B is true. Now, since B is true then D is true by what we got above.

Since D is true so (D V C) or (C V D) is true. Proof is done.

2. To prove: ~C v ~D

premises:

1) C --> ~(A v B)

2) ~B --> ~D

Prove by conditions:

Assume C is true, then ~(A v B) is true by premise 1. And ~(A v B) is just ~A ^ ~B by De Morgan's laws. Based on this, we get ~B.

And by the premise 2 we will get ~D.

So, this means C → ~D which is just (~C v ~D)

3.

To prove: ~B v ~C

premises:

1) D

2) (B ^ C) --> A

3) A --> ~D

Proof:

By premise 1, we know that D is true.

Premise 3 says when A is true then D is not true. However, we already know that D is true, then by the modus tollens we get ~A.

Since we have ~A and premise 2, and still with modus tollens, we get ~(B ^ C).

And by De Morgan's laws, we get ~B v ~C. The proof is done then.

4. To prove: (B v C) --> (~B --> C)

(B v C) is equivalent to (~B → C)

So, (B v C) → (~B →C).