homework #9

Write up 12.1-12.3 (p. 336) and 13.20-13.22 (pp. 367-368).

**12.1**

$\forall x\, [(\mathsf{Brillig}(x) \lor \mathsf{Tove}(x)) \rightarrow (\mathsf{Mimsy}(x) \land \mathsf{Gyre}(x))]$
$\forall y\, [(\mathsf{Slithy}(y) \lor \mathsf{Mimsy}(y)) \rightarrow \mathsf{Tove}(y)]$
$\exists x\, \mathsf{Slithy}(x)$
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$\exists x\, [\mathsf{Slithy}(x) \land \mathsf{Mimsy}(x)]$

**Purported proof:** By the third premise, we know that something in the domain of discourse is slithy. Let $b$ be one of these slithy things. By the second premise, we know that $b$ is a tove. By the first premise, we see that $b$ is mimsy. Thus, $b$ is both slithy and mimsy. Hence, something is both slithy and mimsy.

12.1 Answer:

The argument is valid, and this proof is fine.

The proof would be better if it includes the rule used to prove. For example, as saying that b is a tove the proof can claim this conclusion is by using the ∀ Elim, ∨ Intro and → Elim.

13.20：

1. $\forall x[(\text{Brilling}(x) \lor \text{Tove}(x)) \rightarrow (\text{Mimsy}(x) \land \text{Gyre}(x))]$
2. $\forall y[(\text{Slithy}(x) \lor \text{Mimsy}(y)) \longrightarrow \text{Tove}(y)]$
3. $\exists x\ \text{Slithy}(x)$
4. $\boxed{e}$ $\text{Slithy}(e)$
5. $\text{Slithy}(e) \longrightarrow \text{Tove}(e)$ ∀ Elim: 2
6. $\text{Tove}(e)$ → Elim: 5,4
7. $\text{Tove}(e) \longrightarrow \text{Mimsy}(e)$ ∀ Elim: 1
8. $\text{Mimsy}(e)$ → Elim: 7,6
9. $\text{Slithy}(e) \land \text{Mimsy}(e)$ ∧ Intro: 4,8
10. $\exists x\ [\text{Slithy}(x) \land \text{Mimsy}(x)]$ ∃ Intro: 9
11. $\exists x\ [\text{Slithy}(x) \land \text{Mimsy}(x)]$ ∃ Elim: 3,4-10

**12.2**

$\forall x\,[\mathsf{Brillig}(x) \to (\mathsf{Mimsy}(x) \land \mathsf{Slithy}(x))]$
$\forall y\,[(\mathsf{Slithy}(y) \lor \mathsf{Mimsy}(y)) \to \mathsf{Tove}(y)]$
$\forall x\,[\mathsf{Tove}(x) \to (\mathsf{Outgrabe}(x, b) \land \mathsf{Brillig}(x))]$

$\forall z\,[\mathsf{Brillig}(z) \leftrightarrow \mathsf{Mimsy}(z)]$

**Purported proof:** In order to prove the conclusion, it suffices to prove the logically equivalent sentence obtained by conjoining the following two sentences:

(1) $\forall x\,[\mathsf{Brillig}(x) \to \mathsf{Mimsy}(x)]$

(2) $\forall x\,[\mathsf{Mimsy}(x) \to \mathsf{Brillig}(x)]$

We prove these by the method of general conditional proof, in turn. To prove (1), let $b$ be anything that is brillig. Then by the first premise it is both mimsy and slithy. Hence it is mimsy, as desired. Thus we have established (1).

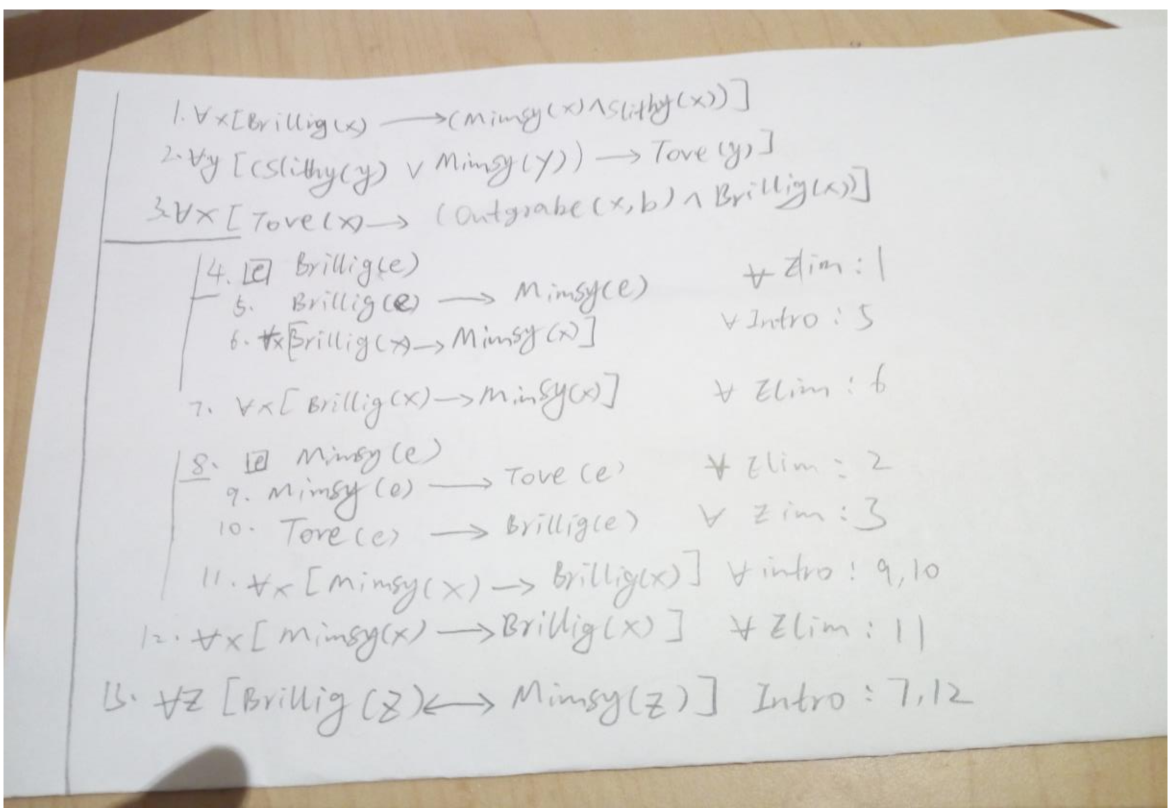
To prove (2), let $b$ be anything that is mimsy. By the second premise, $b$ is also tove. But then by the final premise, $b$ is brillig, as desired. This concludes the proof.

12.2 Answer:

The argument is valid, and this proof is logically correct.

We can optimize the proof by adding a statement says since b is an arbitrary object in the domain, we can conclude (for both (1) and (2)) the conclusion by universal generalization.

13.21：



1. $\forall x[Brillig(x) \to (Mimsy(x) \land Slithy(x))]$
2. $\forall y\,[((Slithy(y) \lor Mimsy(y)) \to Tove(y)]$
3. $\forall x\,[Tove(x) \to (Outgrabe(x,b) \land Brillig(x)]$
4. $\boxed{e}$ Brillig(e) ∀ Elim : 1
5. Brillig(e) → Mimsy(e) ∀ Intro : 5
6. $\forall x[Brillig(x) \to Mimsy(x)]$
7. $\forall x[Brillig(x) \to Mimsy(x)]$ ∀ Elim : 6
8. $\boxed{e}$ Mimsy(e)
9. Mimsy(e) → Tove(e) ∀ Elim : 2
10. Tove(e) → Brillig(e) ∀ Elim : 3
11. $\forall x[Mimsy(x) \to Brillig(x)]$ ∀ intro : 9,10
12. $\forall x[Mimsy(x) \to Brillig(x)]$ ∀ Elim : 11
13. $\forall z\,[Brillig(z) \leftrightarrow Mimsy(z)]$ Intro : 7,12

But then by the final premise, $b$ is brillig, as desired. This concludes the proof.

**12.3**

$\forall x\,[(\mathsf{Brillig}(x) \land \mathsf{Tove}(x)) \to \mathsf{Mimsy}(x)]$
$\forall y\,[(\mathsf{Tove}(y) \lor \mathsf{Mimsy}(y)) \to \mathsf{Slithy}(y)]$
$\exists x\,\mathsf{Brillig}(x) \land \exists x\,\mathsf{Tove}(x)$

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$\exists z\,\mathsf{Slithy}(z)$

**Purported proof:** By the third premise, we know that there are brillig toves. Let $b$ be one of them. By the first premise, we know that $b$ is mimsy. By the second premise, we know that $b$ is slithy. Hence, there is something that is slithy.

12.3 Answer:

The statement is valid and so the proof is fine.

13.22：

1. $\forall x\,[\mathrm{Brillig}(x) \land \mathrm{Tove}(x)) \to \mathrm{Mimsy}(x)]$
2. $\forall y\,[(\mathrm{Tove}(y) \lor \mathrm{Mimsy}(y)) \to \mathrm{Slithy}(y)]$
3. $\exists x\,\mathrm{Brillig}(x) \land \exists x\,\mathrm{Tove}(x)$

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4. $\boxed{e}$ $\mathrm{Brillig}(e) \land \mathrm{Tove}(e)$

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5. $[\mathrm{Brillig}(e) \land \mathrm{Tove}(e)] \to \mathrm{Mimsy}(e)$ — $\forall$ Elim: 1
6. $\mathrm{Mimsy}(e)$ — $\to$ Elim: 5,4
7. $\mathrm{Mimsy}(e) \to \mathrm{Slithy}(e)$ — $\forall$ Elim: 2
8. $\mathrm{Slithy}(e)$ — $\to$ Elim: 7,6
9. $\exists z\,\mathrm{Slithy}(z)$ — $\exists$ Intro: 8

10. $\exists z\,\mathrm{Slithy}(z)$ — $\exists$ Elim: 3, 4-9