

# 1 Introduction

Fair division of goods among competing agents is a fundamental problem in Economics and Computer Science. There is a set  $M$  of  $m$  goods, and the goal is to allocate goods among  $n$  agents in a fair way. An allocation is a partition of  $M$  into disjoint subsets  $X_1, \dots, X_n$  where  $X_i$  is the set of goods given to agent  $i$ . When can an allocation be considered “fair”? One of the most well-studied notions of fairness is Envy-freeness. Every agent has a value associated with each subset of  $M$ , and agent  $i$  envies agent  $j$  if  $i$  values  $X_j$  more than  $X_i$ . An allocation is envy-free if no agent envies another. An envy-free allocation can be regarded as a fair and desirable partition of  $M$  among the  $n$  agents since no agent envies another; as mentioned in [?], such a mechanism of partitioning land dates back to the Bible.

Unlike land, which is divisible, goods in our setting are indivisible, and an envy-free allocation of the given set of goods need not exist. Consider the following simple example with two agents and a single good that both agents desire: one of the agents has to receive this good, and the other agent envies her. Since envy-free allocations need not exist, several relaxations have been considered.