Functional Intrinsic Volumes

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A functional Z defined on a space of real-valued functions $\mathcal{F}$ is called a *valuation* if

$$\mathrm{Z}(f \vee g) + \mathrm{Z}(f \wedge g) = \mathrm{Z}(f) + \mathrm{Z}(g)$$

for all $f, g \in \mathcal{F}$ such that $f, g, f \vee g, f \wedge g \in \mathcal{F}$. Here $f \vee g$ is the pointwise maximum of f and g, while $f \wedge g$ is their pointwise minimum. The important, classical notion of valuations on convex bodies is a special case of the rather recent notion of valuations on function spaces.

We present a complete classification of all continuous, epi-translation and rotation invariant valuations on the space of super-coercive convex functions on $\mathbb{R}^n$. This result corresponds to Hadwiger's celebrated theorem on the classification of continuous, translation and rotation invariant valuations on the space of convex bodies. The valuations obtained in our theorem are functional versions of the classical intrinsic volumes. Representations and important properties will be described.

(Based on joint work with Andrea Colesanti and Fabian Mussnig)