Quick Review:

- Useful trig identities:

$$\sin^2\theta + \cos^2\theta = 1,$$
$$1 + \tan^2\theta = \sec^2\theta,$$
$$\sin 2\theta = 2\sin\theta\cos\theta,$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta.$$

- Trig substitution:

  When you see $\sqrt{a^2 - x^2}$, try $x = a\sin\theta$;

  when you see $\sqrt{a^2 + x^2}$, try $x = a\tan\theta$;

  when you see $\sqrt{x^2 - a^2}$, try $x = a\sec\theta$.

  The goal is: once these substitutions are applied, the square root can be taken care of!

Practice problems:

1.

$$\int \sin^3(x)dx$$

2.

$$\int \frac{dx}{x\sqrt{a^2 - x^2}}$$

3.
$$\int_1^{\sqrt{2}} \frac{dx}{x^3\sqrt{x^2-1}}$$

4.
$$\int_{-\sqrt{2}}^{-1} \frac{dx}{x^3\sqrt{x^2-1}}$$

5.
$$\int \sqrt{a^2+x^2}\,dx$$

(Hint: $\int \frac{1}{\cos^3\theta}d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$.)

6. (If time permits.) If the circle $(x-b)^2 + y^2 = a^2 (0 < a < b)$ is revolved about the y-axis, the resulting solid of revolution is called a *torus*. Find the volume of this torus.