CORE
Control + Optimization Research Lab

Trust Region Policy Optimization (TRPO)

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Outline

- Trust Region Policy Optimization (TRPO)
  - Learning process
  - Hyperparameter
  - Main loop
  - Train model
  - Train & TensorboardX
  - Learning curve & Test

Trust Region Policy Optimization (TRPO)

- TRPO Algorithm

Algorithm 1 Trust Region Policy Optimization

Input: initial policy parameters θ_0, initial value function parameters ϕ_0

Hyperparameters: KL-divergence limit δ, backtracking coefficient α, maximum number of backtracking steps K

for $k = 0, 1, 2, \ldots$ **do**

Collect set of trajectories $\mathcal{D}_k = \{\tau_i\}$ by running policy $\pi_k = \pi(\theta_k)$ in the environment.

Compute rewards-to-go $\hat{R}_t$.

Compute advantage estimates, $\hat{A}_t$ (using any method of advantage estimation) based on the current value function V_{ϕ_k}.

Estimate policy gradient as

$$\hat{g}_k = \frac{1}{|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)|_{\theta_k} \hat{A}_t.$$

Use the conjugate gradient algorithm to compute

$$\hat{x}_k \approx \hat{H}_k^{-1} \hat{g}_k,$$

where $\hat{H}_k$ is the Hessian of the sample average KL-divergence.

Update the policy by backtracking line search with

$$\theta_{k+1} = \theta_k + \alpha^j \sqrt{\frac{2\delta}{\hat{x}_k^T \hat{H}_k \hat{x}_k}} \hat{x}_k,$$

where $j \in \{0, 1, 2, \ldots K\}$ is the smallest value which improves the sample loss and satisfies the sample KL-divergence constraint.

Fit value function by regression on mean-squared error:

$$\phi_{k+1} = \arg\min_\phi \frac{1}{|\mathcal{D}_k| T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(V_\phi(s_t) - \hat{R}_t \right)^2,$$

typically via some gradient descent algorithm.

end for

source : https://spinningup.openai.com/en/latest/algorithms/trpo.html

Trust Region Policy Optimization (TRPO)

- Learning process
  1. 상태에 따른 행동 선택
  2. 환경에서 선택한 행동으로 한 time step을 진행한 후, 다음 상태와 보상을 받음
  3. Sample (s, a, r)을 trajectories set에 저장
  4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
     - Step 1 : Return 구하기
     - Step 2 : Critic network 업데이트
     - Step 3 : Surrogate loss를 통해 actor loss의 gradient 구하기
     - Step 4 : Conjugate gradient method를 통해 search direction 구하기
     - Step 5 : Step size와 maximal step 구하기
     - Step 6 : Backtracking line search를 통해 trust region안에서 actor network 업데이트

Trust Region Policy Optimization (TRPO)

- Actor network

```
4   class Actor(nn.Module):
5       def __init__(self, state_size, action_size, args):
6           super(Actor, self).__init__()
7           self.fc1 = nn.Linear(state_size, args.hidden_size)
8           self.fc2 = nn.Linear(args.hidden_size, args.hidden_size)
9           self.fc3 = nn.Linear(args.hidden_size, action_size)
10
11      def forward(self, x):
12          x = torch.tanh(self.fc1(x))
13          x = torch.tanh(self.fc2(x))
14
15          mu = self.fc3(x)
16          log_std = torch.zeros_like(mu)
17          std = torch.exp(log_std)
18
19          return mu, std
```

Trust Region Policy Optimization (TRPO)

- Actor network

```python
class Actor(nn.Module):
    def __init__(self, state_size, action_size, args):
        super(Actor, self).__init__()
        self.fc1 = nn.Linear(state_size, args.hidden_size)
        self.fc2 = nn.Linear(args.hidden_size, args.hidden_size)
        self.fc3 = nn.Linear(args.hidden_size, action_size)

    def forward(self, x):
        x = torch.tanh(self.fc1(x))
        x = torch.tanh(self.fc2(x))

        mu = self.fc3(x)
        log_std = torch.zeros_like(mu)
        std = torch.exp(log_std)

        return mu, std
```

The number of parameters

Layer	Weight	Bias
1	$3 \times 64 = 192$	64
2	$64 \times 64 = 4096$	64
3	$64 \times 1 = 64$	1
Sum	4352	129
Total	4481	

Trust Region Policy Optimization (TRPO)

- Critic network

```
21  class Critic(nn.Module):
22      def __init__(self, state_size, args):
23          super(Critic, self).__init__()
24          self.fc1 = nn.Linear(state_size, args.hidden_size)
25          self.fc2 = nn.Linear(args.hidden_size, args.hidden_size)
26          self.fc3 = nn.Linear(args.hidden_size, 1)
27
28      def forward(self, x):
29          x = torch.tanh(self.fc1(x))
30          x = torch.tanh(self.fc2(x))
31          value = self.fc3(x)
32
33          return value
```

Learning process

1. 상태에 따른 행동 선택

```
127        mu, std = actor(torch.Tensor(state))
128        action = get_action(mu, std)
```

train.py

```
5    def get_action(mu, std):
6        normal = Normal(mu, std)
7        action = normal.sample()
8
9        return action.data.numpy()
```

utils.py

- Normal(Gaussian) distribution example

```
mu = torch.Tensor([1,  0, -1])
std = torch.Tensor([1., 1., 1.])

from torch.distributions import Normal
normal = Normal(mu, std)
```

```
x = normal.sample()
print(x)
'''
tensor([-0.2713,  0.3903, -0.1373])
'''
```

Learning process

1. 상태에 따른 행동 선택

```
127        mu, std = actor(torch.Tensor(state))
128        action = get_action(mu, std)
```

train.py

```
5    def get_action(mu, std):
6        normal = Normal(mu, std)
7        action = normal.sample()
8
9        return action.data.numpy()
```

utils.py

- Normal(mu, std)
  - Normal(Gaussian) distribution에서 sampling을 할 경우, 분산을 일정하게 유지하면서 지속적인 exploration이 가능
  - std를 1로 고정함으로써 일정한 폭을 가지는 normal distribution에서 sampling

Learning process

2. 환경에서 선택한 행동으로 한 time step을 진행한 후, 다음 상태와 보상을 받음

```
130                 next_state, reward, done, _ = env.step(action)
```

3. Sample (s, a, r)을 trajectories set에 저장

```
110             trajectories = deque()
```

```
132                 mask = 0 if done else 1
133
134                 trajectories.append((state, action, reward, mask))
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 1 : Return 구하기
     - time step 6까지 진행하고 episode가 끝났을 경우를 가정

$$G_1 = R_2 + \gamma R_3 + \gamma^2 R_4 + \gamma^3 R_5 + \gamma^4 R_6$$
$$G_2 = R_3 + \gamma R_4 + \gamma^2 R_5 + \gamma^3 R_6$$
$$G_3 = R_4 + \gamma R_5 + \gamma^2 R_6$$
$$G_4 = R_5 + \gamma R_6$$
$$G_5 = R_6$$

     - 거꾸로 계산하며 계산해놓은 return값을 이용

$$G_5 = R_6$$
$$G_4 = R_5 + \gamma G_5$$
$$G_3 = R_4 + \gamma G_4$$
$$G_2 = R_3 + \gamma G_3$$
$$G_1 = R_2 + \gamma G_2$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 1 : Return 구하기
     - 거꾸로 계산하며 계산해놓은 return값을 이용

$$G_5 = R_6$$

$$G_4 = R_5 + \gamma G_5$$

$$G_3 = R_4 + \gamma G_4$$

$$G_2 = R_3 + \gamma G_3$$

$$G_1 = R_2 + \gamma G_2$$

utils.py

```
11  def get_returns(rewards, masks, gamma):
12      returns = torch.zeros_like(rewards)
13      running_returns = 0
14
15      for t in reversed(range(0, len(rewards))):
16          running_returns = rewards[t] + masks[t] * gamma * running_returns
17          returns[t] = running_returns
18
19      returns = (returns - returns.mean()) / returns.std()
20
21      return returns
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 2 : Critic network 업데이트
     - Critic Loss

$$J_V(\phi) = (V_\phi(s) - R)^2$$

Prediction Target

train.py

```
47      # ----------------------------
48      # step 2: update ciritic
49      criterion = torch.nn.MSELoss()
50
51      values = critic(torch.Tensor(states))
52      targets = returns.unsqueeze(1)
53
54      critic_loss = criterion(values, targets)
55      critic_optimizer.zero_grad()
56      critic_loss.backward()
57      critic_optimizer.step()
```

Learning process

- Learning process
  1. 상태에 따른 행동 선택
  2. 환경에서 선택한 행동으로 한 time step을 진행한 후, 다음 상태와 보상을 받음
  3. Sample (s, a, r)을 trajectories set에 저장
  4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
     - Step 1 : Return 구하기 Clear!
     - Step 2 : Critic network 업데이트 Clear!
     - Step 3 : Surrogate loss를 통해 actor loss의 gradient 구하기
     - Step 4 : Conjugate gradient method를 통해 search direction 구하기
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     - Step 6 : Backtracking line search를 통해 trust region안에서 actor network 업데이트

Learning process

- Optimization problem of theoretical TRPO → Surrogate objective function

$$maximize_\theta \ \mathcal{L}_{\theta_{old}}(\theta) = \mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)} A_{\pi_{\theta_{old}}}(s,a)\right]$$

$$s.t. \ \bar{D}_{KL}^{\rho_{\theta_{old}}}(\theta_{old},\theta) \le \delta$$

- But how do we solve it? → Approximation!

$$\mathcal{L}_{\theta_{old}}(\theta) \approx g^T(\theta-\theta_{old}) \qquad g \doteq \nabla_\theta \mathcal{L}_{\theta_{old}}(\theta)\,|_{\theta_{old}}$$

$$\bar{D}_{KL}^{\rho_{\theta_{old}}}(\theta_{old},\theta) \approx \frac{1}{2}(\theta-\theta_{old})^T H(\theta-\theta_{old}) \qquad H \doteq \nabla_\theta^2 \bar{D}_{KL}^{\rho_{\theta_{old}}}(\theta_{old},\theta)\,|_{\theta_{old}}$$

Learning process

- Approximate optimization problem of practical TRPO

$$maximize_{\theta} \ \ g^T(\theta - \theta_{old})$$

$$s.t. \ \frac{1}{2}(\theta - \theta_{old})^T H(\theta - \theta_{old}) \leq \delta$$

- Approximate optimization problem of practical TRPO (argmax form)

$$\theta_{k+1} = \underset{\theta}{\operatorname{argmax}}\, g^T(\theta - \theta_k)$$

$$s.t. \ \frac{1}{2}(\theta - \theta_k)^T H(\theta - \theta_k) \leq \delta$$

- Solution to approximate problem

$$\theta_{k+1} = \theta_k + \underbrace{\sqrt{\frac{2\delta}{g^T H^{-1} g}}}_{\text{Step size}} \underbrace{H^{-1} g}_{\text{Search direction}}$$

Learning process

- Solution to approximate problem

$$\theta_{k+1} = \theta_k + \underbrace{\sqrt{\frac{2\delta}{g^T H^{-1} g}} H^{-1} g}_{\text{Maximal step}}$$

- TRPO adds a modification to this update rule → Backtracking line search

$$\theta_{k+1} = \theta_k + \underbrace{\alpha^j}_{\text{Backtracking coefficient}} \sqrt{\frac{2\delta}{g^T H^{-1} g}} H^{-1} g$$

Learning process

❖ 정리

1) Get surrogate objective function

$$\mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)}A_{\pi_{\theta_{old}}}(s,a)\right]$$

2) Find search direction, step size and maximal step through CGM and Hessian of KL

- Search direction : $H^{-1}g$
- Step size : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}$
- Maximal step : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$

3) Do line search on that direction inside trust region through Backtracking line search

$$\theta_{k+1}=\theta_k+\alpha^j\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 3 : Surrogate loss를 통해 actor loss의 gradient 구하기

train.py

```
59    # -----------------------------
60    # step 3: get gradient of actor loss through surrogate loss
61    mu, std = actor(torch.Tensor(states))
62    old_policy = get_log_prob(actions, mu, std)
```

utils.py

```
23    def get_log_prob(actions, mu, std):
24        normal = Normal(mu, std)
25        log_prob = normal.log_prob(actions)
26
27        return log_prob
```

- The probability density of the normal distribution

$$\pi_\theta(a|s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(a-\mu_\theta(s))^2}{2\sigma^2}\right)$$

- Multiply the log on both sides

$$\log \pi_\theta(a|s) = -\log\sqrt{2\pi} - \log\sigma - \frac{\left(a-\mu_\theta(s)\right)^2}{2\sigma^2}$$

Learning process

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   - Step 3 : Surrogate loss를 통해 actor loss의 gradient 구하기

train.py

```
59      # ----------------------------
60      # step 3: get gradient of actor loss through surrogate loss
61      mu, std = actor(torch.Tensor(states))
62      old_policy = get_log_prob(actions, mu, std)
63      actor_loss = surrogate_loss(actor, values, targets, states, old_policy.detach(), actions)
```

utils.py

```
29  def surrogate_loss(actor, values, targets, states, old_policy, actions):
30      mu, std = actor(torch.Tensor(states))
31      new_policy = get_log_prob(actions, mu, std)
32
33      advantages = targets - values
34
35      surrogate_loss = torch.exp(new_policy - old_policy) * advantages
36      surrogate_loss = surrogate_loss.mean()
37
38      return surrogate_loss
```

   - surrogate_loss

$$\mathcal{L}_{\theta_{old}}(\theta) = \mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)} A_{\pi_{\theta_{old}}}(s,a)\right]$$

Learning process

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```
59      # ----------------------------
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61      mu, std = actor(torch.Tensor(states))
62      old_policy = get_log_prob(actions, mu, std)
63      actor_loss = surrogate_loss(actor, values, targets, states, old_policy.detach(), actions)
64
65      actor_loss_grad = torch.autograd.grad(actor_loss, actor.parameters())
66      actor_loss_grad = flat_grad(actor_loss_grad)
```

   - torch.autograd.grad(outputs, inputs) - actor.parameter : 4481(weight : 4352, bias : 129)

> `torch.autograd.grad(`*`outputs, inputs, grad_outputs=None, retain_graph=None, create_graph=False, only_inputs=True, allow_unused=False`*`)` [SOURCE]
>
> Computes and returns the sum of gradients of outputs w.r.t. the inputs.

   - example

$$y = \sum_{i=1}^{4} 3 \times x_i$$

$$\nabla_x y = \left(\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \frac{\partial y}{\partial x_3}, \frac{\partial y}{\partial x_4}\right) = (3, 3, 3, 3)$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 3 : Surrogate loss를 통해 actor loss의 gradient 구하기

train.py

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62      old_policy = get_log_prob(actions, mu, std)
63      actor_loss = surrogate_loss(actor, values, targets, states, old_policy.detach(), actions)
64
65      actor_loss_grad = torch.autograd.grad(actor_loss, actor.parameters())
66      actor_loss_grad = flat_grad(actor_loss_grad)
```

   - ○ flat_grad(actor_loss_grad)

utils.py

```
94      def flat_grad(grads):
95          grad_flatten = []
96          for grad in grads:
97              grad_flatten.append(grad.view(-1))
98          grad_flatten = torch.cat(grad_flatten)
99          return grad_flatten
```

Weight	Bias
3×64	64
64×64	64
64×1	1

Learning process

❖ 정리

1) Get surrogate objective function Clear!

$$\mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)}A_{\pi_{\theta_{old}}}(s,a)\right]$$

2) Find search direction, step size and maximal step through CGM and Hessian of KL

- Search direction : $H^{-1}g$
- Step size : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}$
- Maximal step : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$

3) Do line search on that direction inside trust region through Backtracking line search

$$\theta_{k+1}=\theta_k+\alpha^j\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기

train.py

```
68      # ----------------------------
69      # step 4: get search direction through conjugate gradient method
70      search_dir = conjugate_gradient(actor, states, actor_loss_grad.data, nsteps=10)
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기
     - Use CGM to solve $Hx = g$ for $x = H^{-1}g$ ($Hx = \nabla_\theta\left((\nabla_\theta \bar{D}_{KL}(\theta_{old} \parallel \theta))^T \cdot x\right)$)

Iterative method

$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$
if $\mathbf{r}_0$ is sufficiently small, then return $\mathbf{x}_0$ as the result
$\mathbf{p}_0 := \mathbf{r}_0$
$k := 0$
repeat
  $\alpha_k := \frac{\mathbf{r}_k^\mathsf{T}\mathbf{r}_k}{\mathbf{p}_k^\mathsf{T}\mathbf{A}\mathbf{p}_k}$
  $\mathbf{x}_{k+1} := \mathbf{x}_k + \alpha_k\mathbf{p}_k$
  $\mathbf{r}_{k+1} := \mathbf{r}_k - \alpha_k\mathbf{A}\mathbf{p}_k$
  if $\mathbf{r}_{k+1}$ is sufficiently small, then exit loop
  $\beta_k := \frac{\mathbf{r}_{k+1}^\mathsf{T}\mathbf{r}_{k+1}}{\mathbf{r}_k^\mathsf{T}\mathbf{r}_k}$
  $\mathbf{p}_{k+1} := \mathbf{r}_{k+1} + \beta_k\mathbf{p}_k$
  $k := k + 1$
end repeat
return $\mathbf{x}_{k+1}$ as the result

https://en.wikipedia.org/wiki/Conjugate_gradient_method#As_an_iterative_method

utils.py

```
def conjugate_gradient(actor, states, b, nsteps, residual_tol=1e-10):
    x = torch.zeros(b.size())
    r = b.clone()
    p = b.clone()
    rdotr = torch.dot(r, r)

    for i in range(nsteps): # nsteps = 10
        Ap = hessian_vector_product(actor, states, p, cg_damping=1e-1)
        alpha = rdotr / torch.dot(p, Ap)

        x += alpha * p
        r -= alpha * Ap

        new_rdotr = torch.dot(r, r)
        betta = new_rdotr / rdotr

        p = r + betta * p
        rdotr = new_rdotr

        if rdotr < residual_tol: # residual_tol = 0.0000000001
            break
    return x
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기

```python
def hessian_vector_product(actor, states, p, cg_damping=1e-1):
    p.detach()
    kl = kl_divergence(new_actor=actor, old_actor=actor, states=states)
    kl = kl.mean()

    kl_grad = torch.autograd.grad(kl, actor.parameters(), create_graph=True)
    kl_grad = flat_grad(kl_grad)

    kl_grad_p = (kl_grad * p).sum()
    kl_hessian = torch.autograd.grad(kl_grad_p, actor.parameters())
    kl_hessian = flat_hessian(kl_hessian)

    return kl_hessian + p * cg_damping # cg_damping = 0.1
```

utils.py

   - $Hx = \nabla_\theta\left((\nabla_\theta \bar{D}_{KL}(\theta_{old} \parallel \theta))^T \cdot x\right)$
   - CGM의 Iteration을 돌 때마다 매번 Ap의 값을 업데이트해주기 위해서 kl_grad를 구한 후에 p를 곱해서 kl_hessian을 구함

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기

```python
def hessian_vector_product(actor, states, p, cg_damping=1e-1):
    p.detach()
    kl = kl_divergence(new_actor=actor, old_actor=actor, states=states)
    kl = kl.mean()

    kl_grad = torch.autograd.grad(kl, actor.parameters(), create_graph=True)
    kl_grad = flat_grad(kl_grad)

    kl_grad_p = (kl_grad * p).sum()
    kl_hessian = torch.autograd.grad(kl_grad_p, actor.parameters())
    kl_hessian = flat_hessian(kl_hessian)

    return kl_hessian + p * cg_damping # cg_damping = 0.1
```

utils.py

```python
def flat_hessian(hessians):
    hessians_flatten = []
    for hessian in hessians:
        hessians_flatten.append(hessian.contiguous().view(-1))
    hessians_flatten = torch.cat(hessians_flatten).data
    return hessians_flatten
```

utils.py

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기

utils.py

```python
def kl_divergence(new_actor, old_actor, states):
    mu, std = new_actor(torch.Tensor(states))

    mu_old, std_old = old_actor(torch.Tensor(states))
    mu_old = mu_old.detach()
    std_old = std_old.detach()

    # kl divergence between old policy and new policy : D( pi_old || pi_new )
    # pi_old -> mu_old, std_old / pi_new -> mu, std
    # be careful of calculating KL-divergence. It is not symmetric metric.
    kl = torch.log(std / std_old) + (std_old.pow(2) + (mu_old - mu).pow(2)) / (2.0 * std.pow(2)) - 0.5
    return kl.sum(1, keepdim=True)
```

   - KL-Divergence between two univariate Gaussians

$$KL(p,q) = -\int p(x)\log q(x)dx + \int p(x)\log p(x)dx$$

$$= \frac{1}{2}\log(2\pi\sigma_2^2) + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2}(1 + \log 2\pi\sigma_1^2)$$

$$= \log\frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2}$$

https://stats.stackexchange.com/questions/7440/kl-divergence-between-two-univariate-gaussians

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 4 : Conjugate gradient method를 통해 search direction 구하기

train.py

```
68      # ----------------------------
69      # step 4: get search direction through conjugate gradient method
70      search_dir = conjugate_gradient(actor, states, actor_loss_grad.data, nsteps=10)
```

Learning process

❖ 정리

1) Get surrogate objective function Clear!

$$\mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)}A_{\pi_{\theta_{old}}}(s,a)\right]$$

2) Find search direction, step size and maximal step through CGM and Hessian of KL

- Search direction : $H^{-1}g$ Clear!
- Step size : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}$
- Maximal step : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$

3) Do line search on that direction inside trust region through Backtracking line search

$$\theta_{k+1}=\theta_k+\alpha^j\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 5 : Step size와 maximal step 구하기

train.py

```
72      # ----------------------------
73      # step 5: get step size and maximal step
74      gHg = (hessian_vector_product(actor, states, search_dir) * search_dir).sum(0, keepdim=True)
75      step_size = torch.sqrt(2 * args.max_kl / gHg)[0]
76      maximal_step = step_size * search_dir
```

   - Step size : $\sqrt{\frac{2\delta}{g^T H^{-1} g}}$ $(Hx = g)$, (max_kl δ : 0.01)
   - Maximal step : $\sqrt{\frac{2\delta}{g^T H^{-1} g}} H^{-1} g$

Learning process

❖ 정리

1) Get surrogate objective function Clear!

$$\mathbb{E}_{s,a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a|s)}{\pi_{\theta_{old}}(a|s)}A_{\pi_{\theta_{old}}}(s,a)\right]$$

2) Find search direction, step size and maximal step through CGM and Hessian of KL

- Search direction : $H^{-1}g$ Clear!
- Step size : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}$ Clear!
- Maximal step : $\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$ Clear!

3) Do line search on that direction inside trust region through Backtracking line search

$$\theta_{k+1}=\theta_k+\alpha^j\sqrt{\frac{2\delta}{g^TH^{-1}g}}H^{-1}g$$

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 6 : Backtracking line search를 통해 trust region안에서 actor network 업데이트

train.py

```
78      # ----------------------------
79      # step 6: perform backtracking line search and update actor in trust region
80      params = flat_params(actor)
```

utils.py

```
109   def flat_params(model):
110       params = []
111       for param in model.parameters():
112           params.append(param.data.view(-1))
113       params_flatten = torch.cat(params)
114       return params_flatten
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 6 : Backtracking line search를 통해 trust region안에서 actor network 업데이트

train.py

```
78      # ----------------------------
79      # step 6: perform backtracking line search and update actor in trust region
80      params = flat_params(actor)
81
82      old_actor = Actor(state_size, action_size, args)
83      update_model(old_actor, params)
```

utils.py

```
def update_model(model, new_params):
    index = 0
    for params in model.parameters():
        params_length = len(params.view(-1))
        new_param = new_params[index: index + params_length]
        new_param = new_param.view(params.size())
        params.data.copy_(new_param)
        index += params_length
```

```
params_length 192
params_length 64
params_length 4096
params_length 64
params_length 64
params_length 1
```

Weight	Bias
$3 \times 64 = 192$	64
$64 \times 64 = 4096$	64
$64 \times 1 = 64$	1
4352	129

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 6 : Backtracking line search를 통해 trust region안에서 actor network 업데이트

train.py

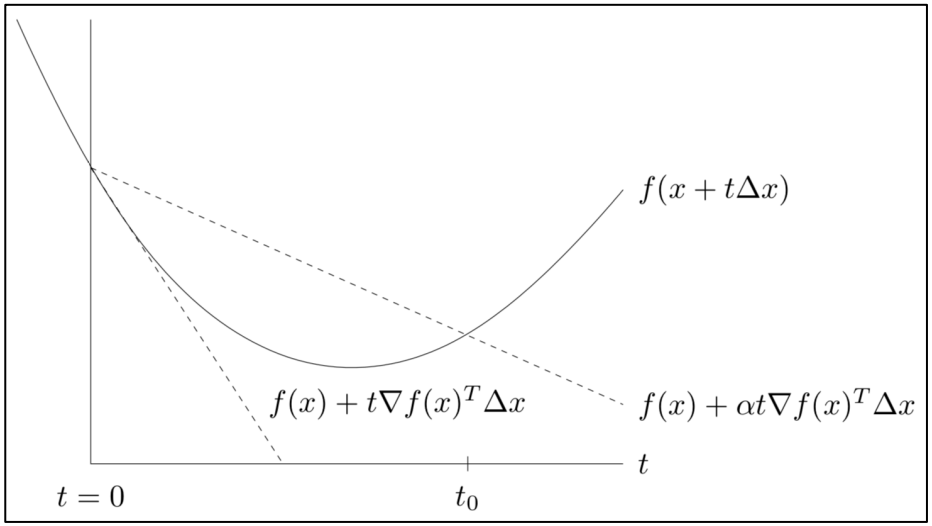
```
78      # ----------------------------
79      # step 6: perform backtracking line search and update actor in trust region
80      params = flat_params(actor)
81
82      old_actor = Actor(state_size, action_size, args)
83      update_model(old_actor, params)
84
85      backtracking_line_search(old_actor, actor, actor_loss, actor_loss_grad,
86                               old_policy, params, maximal_step, args.max_kl,
87                               values, targets, states, actions)
```

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 6 : Actor network를 업데이트하고, backtracking line search를 통해 trust region안에서 업데이트

Algorithm 9.2 *Backtracking line search.*

given a descent direction Δx for f at $x \in \mathbf{dom}\, f$, $\alpha \in (0, 0.5)$, $\beta \in (0, 1)$.
$t := 1$.
while $f(x + t\Delta x) > f(x) + \alpha t \nabla f(x)^T \Delta x$, $\quad t := \beta t$.



source : https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf

Learning process

4. 일정 sample들이 모이면 trajectories set으로 Actor & Critic network 업데이트
   - Step 6 : Actor network를 업데이트하고, backtracking line search를 통해 trust region안에서 업데이트

utils.py

```python
def backtracking_line_search(old_actor, actor, actor_loss, actor_loss_grad,
                             old_policy, params, maximal_step, max_kl,
                             values, targets, states, actions):
    backtrac_coef = 1.0
    alpha = 0.5
    beta = 0.5
    flag = False

    expected_improve = (actor_loss_grad * maximal_step).sum(0, keepdim=True)

    for i in range(10):
        new_params = params + backtrac_coef * maximal_step
        update_model(actor, new_params)

        new_actor_loss = surrogate_loss(actor, values, targets, states, old_policy.detach(), actions)

        loss_improve = new_actor_loss - actor_loss
        expected_improve *= backtrac_coef
        improve_condition = loss_improve / expected_improve

        kl = kl_divergence(new_actor=actor, old_actor=old_actor, states=states)
        kl = kl.mean()

        if kl < max_kl and improve_condition > alpha:
            flag = True
            break

        backtrac_coef *= beta

    if not flag:
        params = flat_params(old_actor)
        update_model(actor, params)
        print('policy update does not impove the surrogate')
```

Hyperparameter

```
14    parser = argparse.ArgumentParser()
15    parser.add_argument('--env_name', type=str, default="Pendulum-v0")
16    parser.add_argument('--load_model', type=str, default=None)
17    parser.add_argument('--save_path', default='./save_model/', help='')
18    parser.add_argument('--render', action="store_true", default=False)
19    parser.add_argument('--gamma', type=float, default=0.99)
20    parser.add_argument('--hidden_size', type=int, default=64)
21    parser.add_argument('--critic_lr', type=float, default=1e-3)
22    parser.add_argument('--max_kl', type=float, default=1e-2)
23    parser.add_argument('--max_iter_num', type=int, default=500)
24    parser.add_argument('--total_sample_size', type=int, default=2048)
25    parser.add_argument('--log_interval', type=int, default=5)
26    parser.add_argument('--goal_score', type=int, default=-300)
27    parser.add_argument('--logdir', type=str, default='./logs',
28                        help='tensorboardx logs directory')
29    args = parser.parse_args()
```

Main loop

- Initialization
  - Seed - random number 고정
  - Actor & Critic network
  - Critic optimizer
  - TensorboardX
  - Recent rewards

```
90  def main():
91      env = gym.make(args.env_name)
92      env.seed(500)
93      torch.manual_seed(500)
94
95      state_size = env.observation_space.shape[0]
96      action_size = env.action_space.shape[0]
97      print('state size:', state_size)
98      print('action size:', action_size)
99
100     actor = Actor(state_size, action_size, args)
101     critic = Critic(state_size, args)
102     critic_optimizer = optim.Adam(critic.parameters(), lr=args.critic_lr)
103
104     writer = SummaryWriter(args.logdir)
105
106     recent_rewards = deque(maxlen=100)
107     episodes = 0
```

Main loop

- Episode 진행
  - Initialize trajectories set
  - 상태에 따른 행동 선택
  - 다음 상태와 보상을 받음
  - Trajectories set에 저장

```
109     for iter in range(args.max_iter_num):
110         trajectories = deque()
111         steps = 0
112 
113         while steps < args.total_sample_size:
114             done = False
115             score = 0
116             episodes += 1
117 
118             state = env.reset()
119             state = np.reshape(state, [1, state_size])
120 
121             while not done:
122                 if args.render:
123                     env.render()
124 
125                 steps += 1
126 
127                 mu, std = actor(torch.Tensor(state))
128                 action = get_action(mu, std)
129 
130                 next_state, reward, done, _ = env.step(action)
131 
132                 mask = 0 if done else 1
133 
134                 trajectories.append((state, action, reward, mask))
135 
136                 next_state = np.reshape(next_state, [1, state_size])
137                 state = next_state
138                 score += reward
139 
140                 if done:
141                     recent_rewards.append(score)
```

Main loop

- Train model
- Print & Visualize log
- Termination : 최근 100개의 episode의 평균 score가 -300보다 크다면
  - Save model
  - 학습 종료

```
143         actor.train(), critic.train()
144         train_model(actor, critic, critic_optimizer,
145                     trajectories, state_size, action_size)
146
147         writer.add_scalar('log/score', float(score), episodes)
148
149         if iter % args.log_interval == 0:
150             print('{} iter | {} episode | score_avg: {:.2f}'.format(iter, episodes, np.mean(recent_rewards)))
151
152         if np.mean(recent_rewards) > args.goal_score:
153             if not os.path.isdir(args.save_path):
154                 os.makedirs(args.save_path)
155
156             ckpt_path = args.save_path + 'model.pth.tar'
157             torch.save(actor.state_dict(), ckpt_path)
158             print('Recent rewards exceed -300. So end')
159             break
```

Train model

- Trajectories → Numpy array
- Trajectories에 있는 2200개의 sample들을 각각 나눔
  - state - (2200, 3)
  - action - (2200, 1)
  - reward - (2200)
  - mask - (2200)

```
31  def train_model(actor, critic, critic_optimizer,
32                  trajectories, state_size, action_size):
33      trajectories = np.array(trajectories)
34      states = np.vstack(trajectories[:, 0])
35      actions = list(trajectories[:, 1])
36      rewards = list(trajectories[:, 2])
37      masks = list(trajectories[:, 3])
38
39      actions = torch.Tensor(actions).squeeze(1)
40      rewards = torch.Tensor(rewards).squeeze(1)
41      masks = torch.Tensor(masks)
```

Train model

- **returns** - (2200)
- **values** - (2200, 1)
- **targets** - (2200, 1)

```
43      # ----------------------------
44      # step 1: get returns
45      returns = get_returns(rewards, masks, args.gamma)
46
47      # ----------------------------
48      # step 2: update ciritic
49      criterion = torch.nn.MSELoss()
50
51      values = critic(torch.Tensor(states))
52      targets = returns.unsqueeze(1)
53
54      critic_loss = criterion(values, targets)
55      critic_optimizer.zero_grad()
56      critic_loss.backward()
57      critic_optimizer.step()
```

Train model

- old_policy - (2200, 1)
- actor_loss_grad - (4481)
- search_dir - (4481)

```
59      # ----------------------------
60      # step 3: get gradient of actor loss through surrogate loss
61      mu, std = actor(torch.Tensor(states))
62      old_policy = get_log_prob(actions, mu, std)
63      actor_loss = surrogate_loss(actor, values, targets, states, old_policy.detach(), actions)
64
65      actor_loss_grad = torch.autograd.grad(actor_loss, actor.parameters())
66      actor_loss_grad = flat_grad(actor_loss_grad)
67
68      # ----------------------------
69      # step 4: get search direction through conjugate gradient method
70      search_dir = conjugate_gradient(actor, states, actor_loss_grad.data, nsteps=10)
```

Train model

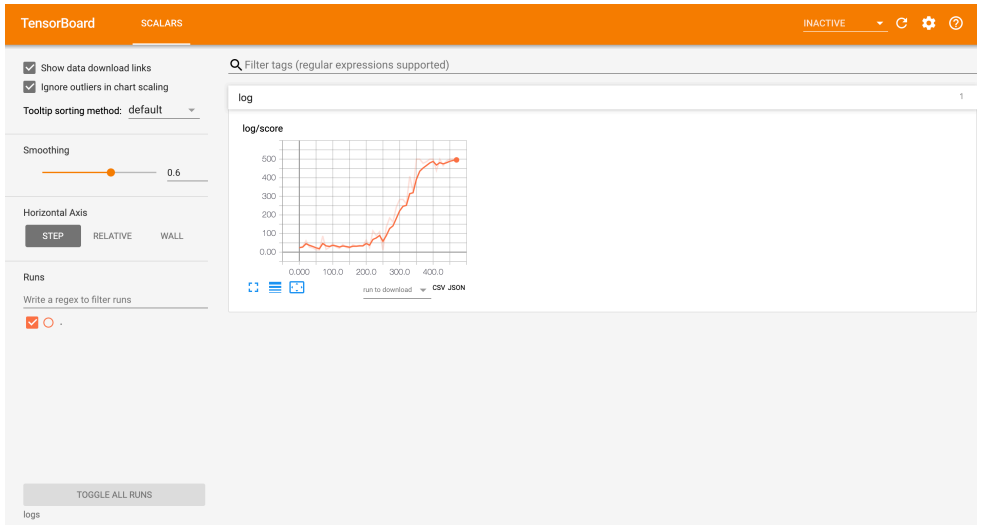
- **params** - (4481)

```
72      # ----------------------------
73      # step 5: get step size and maximal step
74      gHg = (hessian_vector_product(actor, states, search_dir) * search_dir).sum(0, keepdim=True)
75      step_size = torch.sqrt(2 * args.max_kl / gHg)[0]
76      maximal_step = step_size * search_dir
77
78      # ----------------------------
79      # step 6: perform backtracking line search and update actor in trust region
80      params = flat_params(actor)
81
82      old_actor = Actor(state_size, action_size, args)
83      update_model(old_actor, params)
84
85      backtracking_line_search(old_actor, actor, actor_loss, actor_loss_grad,
86                               old_policy, params, maximal_step, args.max_kl,
87                               values, targets, states, actions)
```

Train & TensorboardX

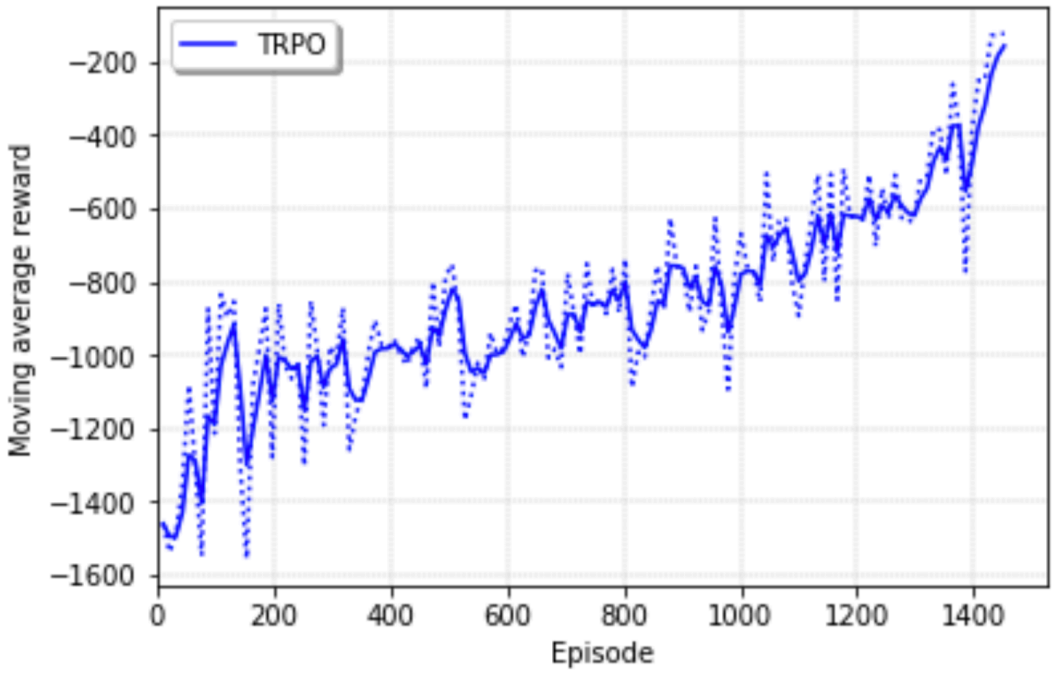
- Terminal A - train 실행
  - conda activate env_name
  - python train.py

- Terminal B - tensorboardX 실행
  - conda activate env_name
  - tensorboard --logdir logs
  - (웹에서) localhost:6006



Learning curve & Test

- Learning curve



- Test
  - ➢ python test.py

Thank you