Flow Synthetic Asset Whitepaper

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1 Introduction

A new type of fungible Cryptocurrency - stablecoin of Euro (**fEUR**) is created by our **Flow** protocol, to meet the DeFi market demand of stablecoins for non-USD currencies. Stablecoins of other currencies such as JPY will be created in the future.

2 Liquidity Pool

A liquidity provider can run his/her own liquidity pool after depositing USD stablecoins to be used as collateral to open positions when new fEUR tokens are minted. Flow protocol support co-existence of multiple liquidity pools.

3 Model Setup

The following model allows any trader to purchase (**fEUR**) with USD stablecoins through the protocol.

- t - Time

- m - Midpoint of Market Exchange Rate
  m_0 - Midpoint of Market Exchange Rate at opening of a position
  m_t - Midpoint of Market Exchange Rate at time t

- θ - Market Fluctuation
  $m_t = m_0(1 + \theta_t)$ for any particular open position

- s - Bid or Ask spread between the Flow bid price $m_t(1-s)$ or ask price $m_t(1+s)$ to the market midpoint m_t set by the liquidity provider of a pool that

$$s = \frac{m_t(1+s) - m_t}{m_t} = \frac{m_t - m_t(1-s)}{m_t}$$

- v - Number of fEUR token issued at opening of a position

- α - Ratio of extra collateral required to open an position for minting any fEUR token, e.g. 10%. To mint v fEUR tokens when market exchange rate is m_0, $\alpha m_0 v$ unit of USD stablecoins are required to be locked on top of $m_0 v$ in a liquidity pool as collateral against market fluctuations.

- c - Collateral of an open position that belong to the liquidity provider
  For v fEUR tokens issued, $c_0 = \alpha m_0 v$, $c_t = (1+\alpha)m_0 v - m_t v$.

- β - Liquidation Threshold, e.g. 5%

- γ - Extreme threshold, e.g. 1%

- μ - Ratio of Liquidity Provider's collateral to the current value of fEUR at an open position, $\mu_0 = \alpha$ and

$$\mu_t = \frac{\alpha - \theta_t}{1 + \theta_t}$$

- λ - Represent how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold,

$$\lambda_t = \beta - \mu_t$$

- δ - Proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated,

$$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - \mu_t}{\beta - \gamma}$$

4   Mint Token & Open Position

Suppose a trader is interested in purchasing v unit of fEUR tokens when market exchange rate midpoint is m_0, he/she deposits unit of $m_0(1+s)v$ USD stablecoins into a liquidity pool, v unit of fEUR tokens are minted by Flow protocol after the liquidity provider of the pool opens a position by locking the fund as collateral

$$\{m_0 v \quad + \quad \alpha * m_0 v\}$$

- $m_0 v$ represents the current value of v unit of fEUR tokens in USD

- $\alpha * m_0 v$ is the collateral against market fluctuation that belong to the liquidity provider, made of the spread part deposited by the trader $s m_0 v$, and the rest deposited by the liquidity provider $(\alpha - s) m_0 v$

- total value of the position equals to $m_0 v + \alpha * m_0 v = (1 + \alpha) m_0 v$

5 Burn Token & Close Position

Suppose a trader is interested in selling v unit of fEUR tokens back to the liquidity pool when market exchange rate midpoint is m_t, he/she deposits the fEUR tokens into a liquidity pool, v unit of fEUR tokens are burnt by Flow protocol after the liquidity provider of the pool closes the open position for the v unit of fEUR tokens and transfers $m_t(1 - s)v$ unit of USD stablecoins to the trader, and all remaining collateral of the position now belong to the liquidity provider and are returned into the pool.

5.1 Trader sells immediately after opening position

Suppose trader wants to sell the token back to the liquidity pool immediately after buying v unit of fEUR tokens when market exchange rate midpoint is m_0, he receives $(1 - s)m_0 v$ USD stablecoins back and makes loss of $-2 s m_0 v$ USD stablecoins.

Liquidity provider closes the open position $\{m_0 v \quad + \quad \alpha * m_0 v\}$, $(\alpha + s)m_0 v$ collateral is released and returned to the pool and he makes profit of $2 s m_0 v$ USD stablecoins.

And v unit of fEUR tokens are then burnt by Flow protocol.

5.2 Trader sells fEUR when EUR/USD ↓

Suppose EUR/USD goes down after opening of a position for v fEUR tokens minted, i.e. $\theta_t < 0$ and $m_t < m_0$, the position is now

$$\{m_t v \quad + \quad [(1 + \alpha)m_0 - m_t]v\}$$

$$\{(1 + \theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$$

where $(\alpha - \theta_t)m_0 v$ belong to the liquidity provider.

If trader chooses to sell v fEUR tokens back to the Liquidity pool, he receives $(1 + \theta_t - s)m_0 v$ USD stablecoins back and makes loss of $(\theta_t - 2s)m_0 v$ USD stablecoins.

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Liquidity provider closes the open position $\{(1 + \theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$, $(\alpha - \theta_t)m_0 v$ collateral is released and returned to the pool and he makes profit of $(2s - \theta_t)m_0 v$ USD stablecoins.

And v unit of fEUR tokens are then burnt by Flow protocol.

5.3 Trader sells fEUR when EUR/USD ↑

Suppose EUR/USD goes up after opening of a position for v fEUR tokens minted, i.e. $\theta_t > 0$ and $m_t > m_0$, the position is now

$$\{m_t v \quad + \quad [(1 + \alpha)m_0 - m_t]v\}$$

$$\{(1 + \theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$$

where $(\alpha - \theta_t)m_0 v$ belong to the liquidity provider.

The ratio of collateral that belongs to the liquidity provider to the current value of v fEUR tokens

$$\mu_t = \frac{(\alpha - \theta_t)m_0 v}{(1 + \theta_t)m_0 v} = \frac{\alpha - \theta_t}{1 + \theta_t}$$

When EUR/USD goes high enough (θ_t large enough), there is a risk that the current value of the fEUR tokens in USD stablecoins may exceed total collateral locked in the position, i.e. $(1 + \theta_t)m_0 v > (1 + \alpha)m_0 v$, in which case the liquidity provider will not be able to provide enough USD stablecoins if a trader tries to sell fEUR tokens back. A liquidity provider may choose to add more USD stablecoins into the open position to top up his collateral to increase μ. While if he fails to top up in time, the position is at risk.

The Liquidation Threshold β and the Extreme Threshold γ are utilized to avoid this problem.

- When Liquidity provider collateral ratio μ_t goes down and reaches the Liquidation Threshold β, any liquidator is allowed to close the position for the liquidity provider and receives **some proportional part** of the liquidity provider collateral as reward, depending on how small the ratio is.

- Once Liquidity provider collateral ratio μ_t becomes too small and reaches the Extreme Threshold γ, **all** liquidity provider collateral will be given to liquidators as reward to encourage them to close the position for the liquidity provider.

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5.3.1 Liquidity provider collateral ratio above Liquidation Threshold

When $\mu_t > \beta$, we have

$$\theta_t < \frac{\alpha - \beta}{1 + \beta}$$

For $\alpha = 10\%$ and $\beta = 5\%$, when $\mu_t > \beta$, we have $\theta_t < 4.76\%$.

If trader chooses to sell v fEUR tokens back to the Liquidity pool to close the open position, he receives $(1+\theta_t-s)m_0 v$ USD stablecoins back and makes profit of $(\theta_t - 2s)m_0 v$ USD stablecoins.

Liquidity provider closes the open position

$$\{(1 + \theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}, (\alpha - \theta_t)m_0 v$$

, collateral is released and returned to the pool and he makes loss of $(2s-\theta_t)m_0 v$ USD stablecoins.

And v unit of fEUR tokens are then burnt by Flow protocol.

5.3.2 Liquidity provider collateral ratio below Extreme Threshold

When $\mu_t \in [0, \gamma)$, we have

$$\frac{\alpha - \gamma}{1 + \gamma} < \theta_t \leq \alpha$$

For $\alpha = 10\%$ and $\gamma = 1\%$, when $\mu_t \in [0, \gamma)$, we have $\theta_t \in (8.91\%, 10\%]$.

When μ_t goes below the Extreme Threshold, the position is at high risk, that **all** liquidity provider collateral will be given to liquidators as reward to encourage them to close the position for the liquidity provider.

If a liquidator would like to close an open position of v fEUR tokens when the market exchange rate midpoint is m_t, he could

- Buy v fEUR tokens from any liquidity provider at cost $(1 + s)m_t v$ or at any exchange at a similar cost.

- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1 - s)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus **everything** belongs to the liquidity provider in this position (including the income from bid spread at close of the position):

$$(1 - s)m_t v + [(1 + \alpha)m_0 - m_t]v + sm_t v$$

- Make profit $\Pi_l = (\alpha - \theta_t - s)m_0 v$ if his tokens are purchased from a liquidity provider.

5.3.3   Liquidity provider collateral ratio in between LT & ET

When $\mu_t \in [\gamma, \beta]$, we have

$$\mu_t = \frac{c_t}{m_t v} = \frac{(\alpha - \theta_t)m_0 v}{(1 + \theta_t)m_0 v}$$

where c_t is the collateral of an open position that belong to the liquidity provider at time t that $c_t = \mu_t m_t v$.

If a liquidator would like to close an open position of v fEUR tokens when the market exchange rate midpoint is m_t and $\mu_t \in [\gamma, \beta]$, he could

- Buy v fEUR tokens from any liquidity provider at cost $(1 + s)m_t v$ or at any exchange at a similar cost.

- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1 - s)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus **some proportion** of the collateral belongs to the liquidity provider in the position and the income from bid spread at close of the position:
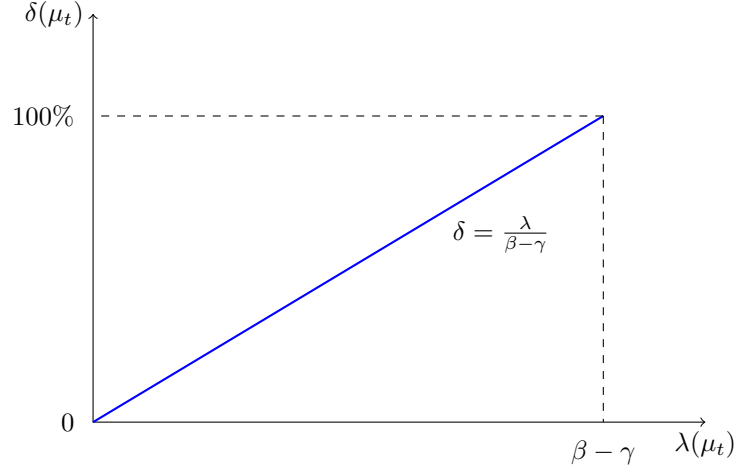
$$(1 - s)m_t v + \delta(c_t + s m_t v)$$

  where δ is the proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated that

$$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - \mu_t}{\beta - \gamma}$$

  and λ represents how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold that

$$\lambda_t = \beta - \mu_t$$

- Make profit

$$\Pi_l = \delta(c_t + sm_t v) - 2sm_t v$$

if his tokens are purchased from a liquidity provider.

Since both c_t and δ_t depend on μ_t that Π_l is maximized when

$$\frac{\partial \Pi_l}{\partial \mu_t} = \frac{(\beta - s - 2\mu_t)m_t v}{\beta - \gamma} = 0$$
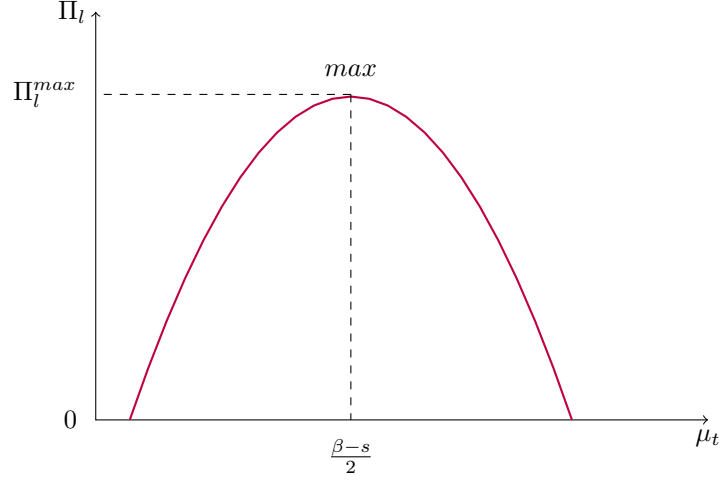
Thus, we have

$$\mu_t^{max} = \frac{\beta - s}{2}$$

and

$$\Pi_l^{max}(\mu_t) = \frac{(\beta + s)^2 m_t v}{4(\beta - \gamma)} - 2sm_t v$$

For $\beta = 5\%$, $\gamma = 1\%$, $s = 0.5\%$, $v = 10,000$ and $m_t = 1.2$, when $\mu_t \in [\gamma, \beta]$, we have $\mu_t^{max} = 2.25\%$ and $\Pi_l^{max} = 106.875$ USD stablecoins.

After the position is liquidated by a liquidator, the liquidity provider receives the remaining proportion of the collateral,

$$(1 - \delta)(c_t + sm_tv) > 0$$

and make a loss of

$$(1 - \delta)(c_t + sm_tv) - (\alpha - s)m_0v < 0$$

6   One Position in One Liquidity Pool

As traders place the first, second, third, i_{th} to n_{th} trade orders to buy fEUR tokens, open positions are created at each order as collateral. While in reality, liquidity provider of a liquidity pool combines all positions together and holds only one position that for total open unit of fEUR tokens

$$V = \sum_{i=1}^{n} v^i = v^1 + v^2 + v^3 + ... + v^n$$

Total collateral that belong to the liquidity provider

$$C_t = \sum_{i=1}^{n} (\alpha - \theta_t^i) m_t^i v^i = \mu_t m_t V$$

as ratio of liquidity provider's collateral to the current value of all minted fEUR tokens backed up by the pool

$$\mu_t = \frac{C_t}{m_t V}$$

Now when $\mu_t \in [\gamma, \beta]$, i.e. the liquidity provider collateral ratio falls in between the Extreme Threshold and Liquidation Threshold, any liquidator is allowed to liquidate a fraction or the entire open position in the pool, by

- Buy ρV fEUR tokens from any liquidity provider at cost $(1+s)m_t\rho V$ or at any exchange at a similar cost, where $\rho \in (0,1]$ represent the fraction of the open position that liquidator would like to close.

- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1-s)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus **some proportion** of the collateral belongs to the liquidity provider in the fraction of the position and the income from bid spread at close of the fraction of the position:
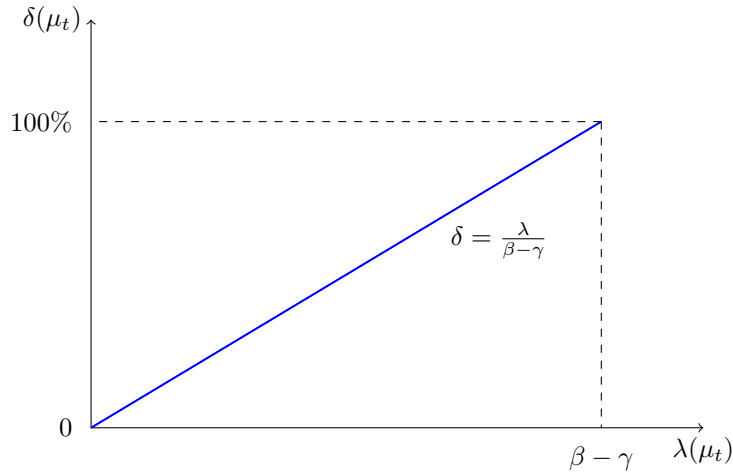
$$(1-s)m_t\rho V + \delta(\rho C_t + sm_t\rho V)$$

where δ is the proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated that

$$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - \mu_t}{\beta - \gamma}$$

and λ represents how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold that

$$\lambda_t = \beta - \mu_t$$

- Make profit

$$\Pi_l = \delta(\rho C_t + sm_t\rho V) - 2sm_t\rho V$$

if his tokens are purchased from a liquidity provider.

Since both C_t and δ_t depend on μ_t that Π_l is maximized when

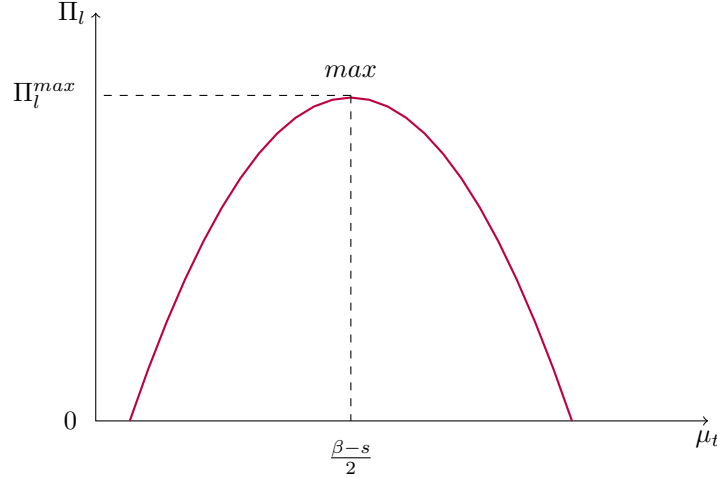$$\frac{\partial \Pi_l}{\partial \mu_t} = \frac{(\beta - s - 2\mu_t)\rho m_t V}{\beta - \gamma} = 0$$

Thus, we have

$$\mu_t^{max} = \frac{\beta - s}{2}$$

and

$$\Pi_l^{max}(\mu_t) = \frac{(\beta + s)^2 m_t \rho V}{4(\beta - \gamma)} - 2sm_t\rho V$$

For $\beta = 5\%$, $\gamma = 1\%$, $s = 0.5\%$, $v = 100,000$, $m_t = 1.2$ and $\rho = 20\%$, when $\mu_t \in [\gamma, \beta]$, we have $\mu_t^{max} = 2.25\%$ and $\Pi_l^{max} = 213.75$ USD stablecoins.

After the ρ fraction of the position is liquidated by a liquidator, the liquidity provider receives the remaining fraction of spread income from buying back fEUR tokens $(1 - \delta)sm_t\rho V$, and keeps the remaining fraction of collateral of the closed position $(1 - \delta)\rho C$. And the new position becomes

$$\{(1 - \rho)m_t V \qquad + \qquad C^{old}(1 - \delta\rho) + (1 - \delta)sm_t\rho V\}$$

and the new liquidity provider collateral ratio

$$\mu_t^{new} = \frac{C^{new}}{(1 - \rho)m_t V} = \frac{C^{old}(1 - \delta\rho) + (1 - \delta)sm_t\rho V}{(1 - \rho)m_t V} > \mu^{old}$$

If $\mu_t^{new} > \beta$, there will be no immediate arbitrage opportunity. While if we still have $\mu_t^{new} \leq \beta$, any liquidator can keep closing more fraction of the open position, until the latest $\mu_t > \beta$, i.e. when the current liquidity provider collateral ratio exceeds the Liquidation Threshold.