# Flow Synthetic Assets Working Paper

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## 1 Introduction

A new type of fungible Cryptocurrency - stablecoin of Euro (***fEUR***) is created by our ***Flow*** protocol, to meet the DeFi market demand of stablecoins for non-USD currencies. Stablecoins of other currencies such as JPY will be created in the future.

## 2 Liquidity Pool

A liquidity provider can run his/her own liquidity pool after depositing USD stablecoins to be used as collateral to open positions when new fEUR tokens are minted. Flow protocol support co-existence of multiple liquidity pools.

## 3 Model Setup

The following model allows any trader to purchase (***fEUR***) with USD stablecoins through the protocol.

- $t$ - Time
- $m$ - Midpoint of Market Exchange Rate
  $m_0$ - Midpoint of Market Exchange Rate at opening of a position
  $m_t$ - Midpoint of Market Exchange Rate at time $t$
- $\theta$ - Market Fluctuation
  $m_t = m_0(1 + \theta_t)$ for any particular open position

- $b$ and $a$ - Bid or Ask spread between the Flow bid price $m_t(1-b)$ or ask price $m_t(1+a)$ to the market midpoint $m_t$ set by the liquidity provider of a pool that

$$b_t = \frac{m_t - m_t(1-b)}{m_t}$$

$$a_t = \frac{m_t(1+a) - m_t}{m_t}$$

  Bid and ask spread $b_t$ and $a_t$ are chosen by the liquidity provider that can be changed anytime.

- $v$ - Number of fEUR token issued at opening of a position

- $\alpha$ - Ratio of extra collateral required to open an position for minting any fEUR token, e.g. 10%. To mint $v$ fEUR tokens when market exchange rate is $m_0$, $\alpha m_0 v$ unit of USD stablecoins are required to be locked on top of $m_0 v$ in a liquidity pool as collateral against market fluctuations.

- $c$ - Collateral of an open position that belong to the liquidity provider
  For $v$ fEUR tokens issued, $c_0 = \alpha m_0 v$, $c_t = (1+\alpha)m_0 v - m_t v$.

- $\beta$ - Liquidation Threshold, e.g. 5%

- $\gamma$ - Extreme threshold, e.g. 1%

- $\mu$ - Ratio of Liquidity Provider's collateral to the current value of fEUR at an open position, $\mu_0 = \alpha$ and

$$\mu_t = \frac{\alpha - \theta_t}{1 + \theta_t}$$

- $\lambda$ - Represent how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold,

$$\lambda_t = \beta - \mu_t$$

- $\delta$ - Proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated,

$$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - \mu_t}{\beta - \gamma}$$

# 4 Mint Token & Open Position

Suppose a trader is interested in purchasing $v$ unit of fEUR tokens when market exchange rate midpoint is $m_0$, he/she deposits unit of $m_0(1+a_0)v$ USD stablecoins into a liquidity pool, $v$ unit of fEUR tokens are minted by Flow protocol after the liquidity provider of the pool opens a position by locking the fund as collateral

$$\{m_0 v \quad + \quad \alpha * m_0 v\}$$

- $m_0 v$ represents the current value of $v$ unit of fEUR tokens in USD
- $\alpha * m_0 v$ is the collateral against market fluctuation that belong to the liquidity provider, made of the spread part deposited by the trader $a_0 m_0 v$, and the rest deposited by the liquidity provider $(\alpha - a_0)m_0 v$
- total value of the position equals to $m_0 v + \alpha * m_0 v = (1+\alpha)m_0 v$

# 5 Burn Token & Close Position

Suppose a trader is interested in selling $v$ unit of fEUR tokens back to the liquidity pool when market exchange rate midpoint is $m_t$, he/she deposits the fEUR tokens into a liquidity pool, $v$ unit of fEUR tokens are burnt by Flow protocol after the liquidity provider of the pool closes the open position for the $v$ unit of fEUR tokens and transfers $m_t(1-b_t)v$ unit of USD stablecoins to the trader, and all remaining collateral of the position now belong to the liquidity provider and are returned into the pool.

## 5.1 Trader sells immediately after opening position

Suppose trader wants to sell the token back to the liquidity pool immediately after buying $v$ unit of fEUR tokens when market exchange rate midpoint is $m_0$, he receives $(1-b_0)m_0 v$ USD stablecoins back and makes loss of $-(a_0+b_0)m_0 v$ USD stablecoins.

Liquidity provider closes the open position $\{m_0 v \quad + \quad \alpha * m_0 v\}$, $\alpha m_0 v$ collateral is released and returned to the pool, together with profit made from bid spread, he makes profit of $(a_0+b_0)m_0 v$ USD stablecoins.

And $v$ unit of fEUR tokens are then burnt by Flow protocol.

## 5.2 Trader sells fEUR when EUR/USD ↓

Suppose EUR/USD goes down after opening of a position for $v$ fEUR tokens minted, i.e. $\theta_t < 0$ and $m_t < m_0$, the position is now

$$\{m_t v \quad + \quad [(1+\alpha)m_0 - m_t]v\}$$

$$\{(1+\theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$$

where $(\alpha - \theta_t)m_0 v$ belong to the liquidity provider.

If trader chooses to sell $v$ fEUR tokens back to the Liquidity pool, he receives $(1 + \theta_t - b_t)m_0 v$ USD stablecoins back and makes loss of $(\theta_t - (a_0 + b_t))m_0 v$ USD stablecoins.

Liquidity provider closes the open position $\{(1+\theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$, $(\alpha - \theta_t)m_0 v$ collateral is released and returned to the pool, together with profit made from bid spread, he makes profit of $((a_0 + b_t) - \theta_t)m_0 v$ USD stablecoins.

And $v$ unit of fEUR tokens are then burnt by Flow protocol.

## 5.3 Trader sells fEUR when EUR/USD ↑

Suppose EUR/USD goes up after opening of a position for $v$ fEUR tokens minted, i.e. $\theta_t > 0$ and $m_t > m_0$, the position is now

$$\{m_t v \quad + \quad [(1+\alpha)m_0 - m_t]v\}$$

$$\{(1+\theta_t)m_0 v \quad + \quad (\alpha - \theta_t)m_0 v\}$$

where $(\alpha - \theta_t)m_0 v$ belong to the liquidity provider.

The ratio of collateral that belongs to the liquidity provider to the current value of $v$ fEUR tokens

$$\mu_t = \frac{(\alpha - \theta_t)m_0 v}{(1+\theta_t)m_0 v} = \frac{\alpha - \theta_t}{1 + \theta_t}$$

When EUR/USD goes high enough ($\theta_t$ large enough), there is a risk that the current value of the fEUR tokens in USD stablecoins may exceed total collateral locked in the position, i.e. $(1+\theta_t)m_0 v > (1+\alpha)m_0 v$, in which case the liquidity provider will not be able to provide enough USD stablecoins if a trader tries to sell fEUR tokens back. A liquidity provider may choose to add more USD stablecoins into the open position to top up his collateral to increase $\mu$. While if he fails to top up in time, the position is at risk.

The Liquidation Threshold $\beta$ and the Extreme Threshold $\gamma$ are utilized to avoid this problem.

- When Liquidity provider collateral ratio $\mu_t$ goes down and reaches the Liquidation Threshold $\beta$, any liquidator is allowed to close the position for the liquidity provider and receives ***some proportional part*** of the liquidity provider collateral as reward, depending on how small the ratio is.
- Once Liquidity provider collateral ratio $\mu_t$ becomes too small and reaches the Extreme Threshold $\gamma$, ***all*** liquidity provider collateral will be given to liquidators as reward to encourage them to close the position for the liquidity provider.

### 5.3.1 Liquidity provider collateral ratio above Liquidation Threshold

When $\mu_t > \beta$, we have

$$\theta_t < \frac{\alpha - \beta}{1 + \beta}$$

For $\alpha = 10\%$ and $\beta = 5\%$, when $\mu_t > \beta$, we have $\theta_t < 4.76\%$ .

If trader chooses to sell $v$ fEUR tokens back to the Liquidity pool to close the open position, he receives $(1 + \theta_t - b_t)m_0v$ USD stablecoins back and makes profit of $(\theta_t - (a_0 + b_t))m_0v$ USD stablecoins.

Liquidity provider closes the open position

$$\{(1 + \theta_t)m_0v \quad + \quad (\alpha - \theta_t)m_0v\}, (\alpha - \theta_t)m_0v$$

, collateral is released and returned to the pool and he makes loss of $((a_0 + b_t) - \theta_t)m_0v$ USD stablecoins.

And $v$ unit of fEUR tokens are then burnt by Flow protocol.

### 5.3.2 Liquidity provider collateral ratio below Extreme Threshold

When $\mu_t \in [0, \gamma)$, we have

$$\frac{\alpha - \gamma}{1 + \gamma} < \theta_t \leq \alpha$$

For $\alpha = 10\%$ and $\gamma = 1\%$, when $\mu_t \in [0, \gamma)$, we have $\theta_t \in (8.91\%, 10\%]$ .

When $\mu_t$ goes below the Extreme Threshold, the position is at high risk, that ***all*** liquidity provider collateral will be given to liquidators as reward to encourage them to close the position for the liquidity provider.

If a liquidator would like to close an open position of $v$ fEUR tokens when the market exchange rate midpoint is $m_t$, he could

- Buy $v$ fEUR tokens from any liquidity provider at cost $(1 + a_t)m_t v$ or at any exchange at a similar cost.
- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1 - b_t)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus ***everything*** belongs to the liquidity provider in this position (including the income from bid spread at close of the position):

$$(1 - b_t)m_t v + [(1 + \alpha)m_0 - m_t]v + b_t m_t v$$

- Make profit $\Pi_l = [\alpha - a_t - (1 + a_t)\theta_t]m_0 v$ if his tokens are purchased from a liquidity provider.

#### 5.3.3 Liquidity provider collateral ratio in between LT & ET

When $\mu_t \in [\gamma, \beta]$, we have

$$\mu_t = \frac{c_t}{m_t v} = \frac{(\alpha - \theta_t)m_0 v}{(1 + \theta_t)m_0 v}$$

where $c_t$ is the collateral of an open position that belong to the liquidity provider at time $t$ that $c_t = \mu_t m_t v$.

If a liquidator would like to close an open position of $v$ fEUR tokens when the market exchange rate midpoint is $m_t$ and $\mu_t \in [\gamma, \beta]$, he could

- Buy $v$ fEUR tokens from any liquidity provider at cost $(1 + a_t)m_t v$ or at any exchange at a similar cost.
- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1 - b_t)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus ***some proportion*** of the collateral belongs to the liquidity provider in the position and the income from bid spread at close of the position:

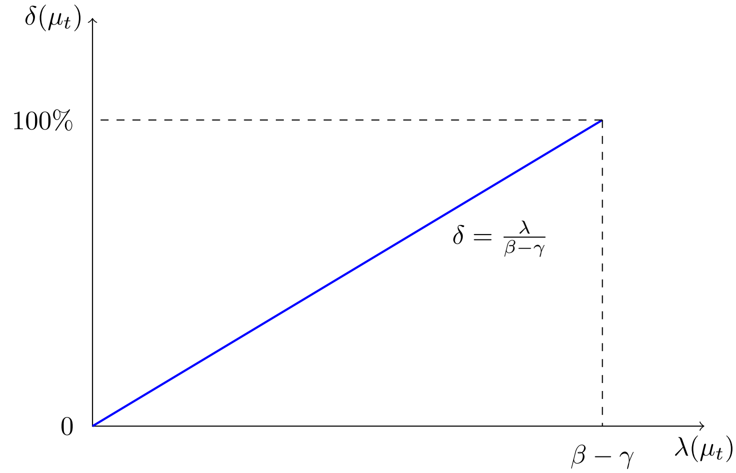
$$(1 - b_t)m_t v + \delta(c_t + b_t m_t v)$$

  where $\delta$ is the proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated that

$$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - \mu_t}{\beta - \gamma}$$

and $\lambda$ represents how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold that

$$\lambda_t = \beta - \mu_t$$




- Make profit

$$\Pi_l = \delta(c_t + b_t m_t v) - (a_t + b_t) m_t v$$

if his tokens are purchased from a liquidity provider.

Since

$$c_t = \mu_t m_t v \qquad m_t = (1+\theta_t) m_0$$

$$\delta_t = \frac{\beta - \mu_t}{\beta - \gamma} \qquad \mu_t = \frac{\alpha - \theta_t}{1+\theta_t}$$

$$\Pi_l(\theta_t) = \left[ \frac{\beta - \alpha + (\beta+1)\theta_t}{(1+\theta_t)(\beta-\gamma)} \left( \frac{\alpha - \theta_t}{1+\theta_t} + b_t \right) - (a_t + b_t) \right] (1+\theta_t) m_0 v$$

For $\mu_t \in [\gamma, \beta]$, i.e. when

$$\theta_t \in [\frac{\alpha-\beta}{1+\beta}, \frac{\alpha-\gamma}{1+\gamma}]$$

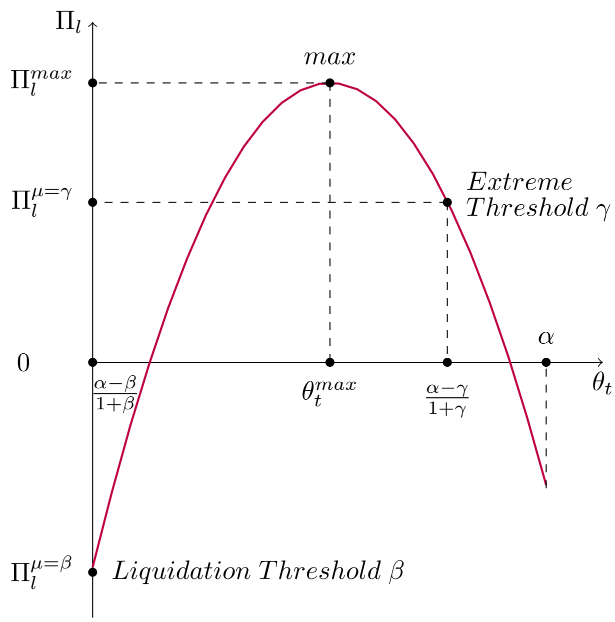
, there exists a local maxima solution of

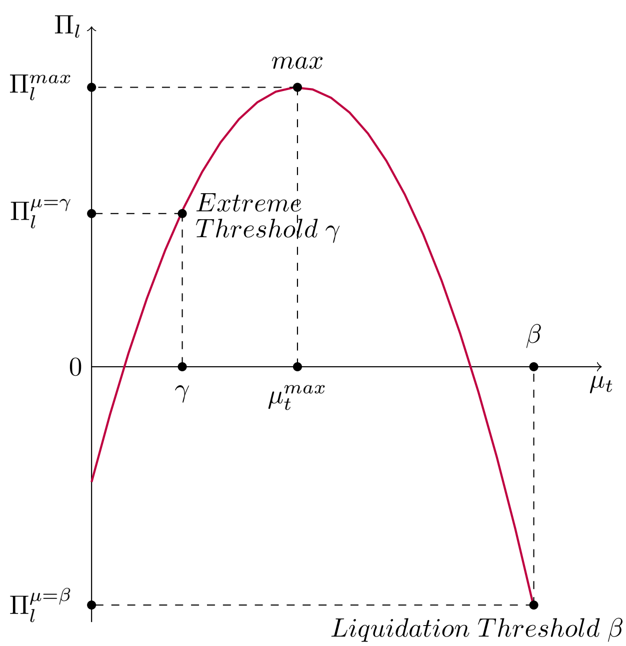
$$\theta_t^{max}(m_0, v, \alpha, \beta, \gamma, a_t, b_t) \quad and \quad \mu_t^{max}(m_0, v, \alpha, \beta, \gamma, a_t, b_t)$$

that maximizes $\Pi_l(\theta_t)$ when

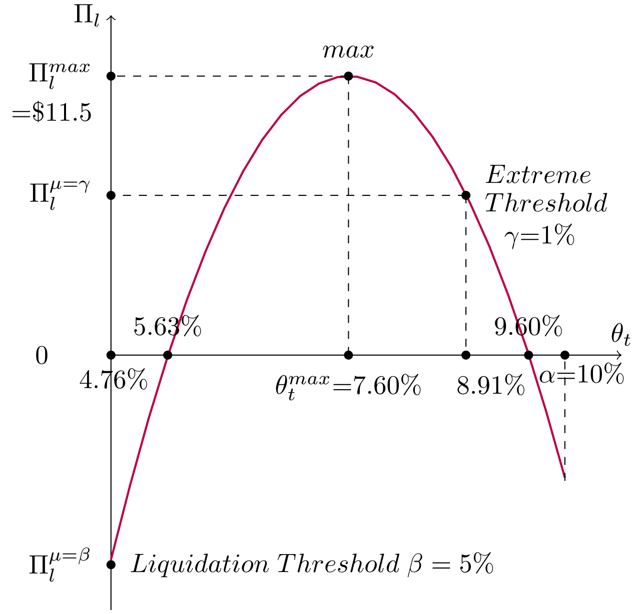
$$\frac{\partial \Pi_l}{\partial \theta_t} = 0$$

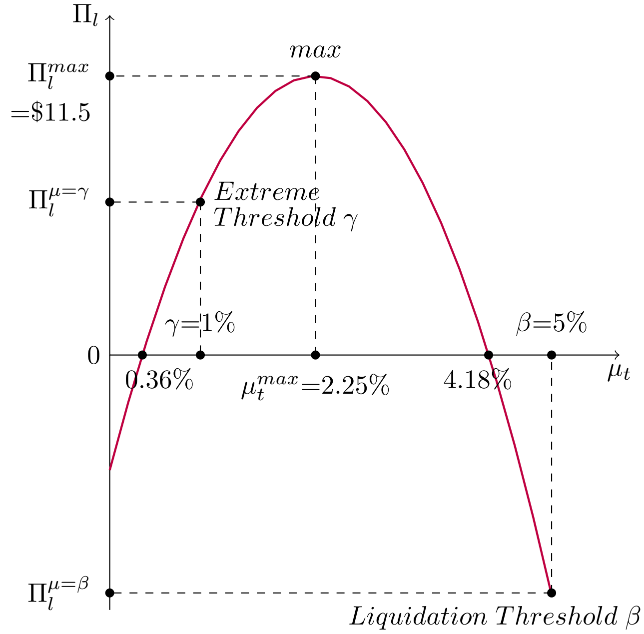
, generating maximized profit $\Pi_l^{max}(\theta_t^{max})$, i.e. $\Pi_l^{max}(\mu_t^{max})$



For $\alpha = 10\%$, $\beta = 5\%$, $\gamma = 1\%$, $a_t = 0.5\%$, $b_t = 0.5\%$, $v = 1,000$ and $m_0 = 1.2$, when $\mu_t \in [\gamma, \beta]$, we have $\mu_t^{max} = 2.25\%$, $\theta_t^{max} = 7.6\%$ and $\Pi_l^{max} = 11.5$ USD stablecoins.



After the position is liquidated by a liquidator, the liquidity provider receives the remaining proportion of the collateral,

$$(1-\delta)(c_t + b_t m_t v) > 0$$

and make a loss of

$$(1-\delta)(c_t + b_t m_t v) - (\alpha - a_0) m_0 v < 0$$

# 6 One Position in One Liquidity Pool

As traders place the first, second, third, $i_{th}$ to $n_{th}$ trade orders to buy fEUR tokens, open positions are created at each order as collateral. While in reality, liquidity provider of a liquidity pool combines all positions together and holds only one position that for total open unit of fEUR tokens

$$V = \sum_{i=1}^{n} v^i = v^1 + v^2 + v^3 + ... + v^n$$

Total collateral that belong to the liquidity provider

$$C_t = \sum_{i=1}^{n} c_t^i = \sum_{i=1}^{n} \mu_t^i m_t v_t^i = \sum_{i=1}^{n} (\alpha - \theta_t^i) m_0^i v^i$$

as ratio of liquidity provider's collateral to the current value of all minted fEUR tokens backed up by the pool

$$M_t = \frac{C_t}{m_t V}$$

Note that $M_t$ is a weighted average of $\mu_t^i$ as

$$M_t = \sum_{i=1}^{n} \frac{v_t^i}{V} \mu_t^i$$

Now when $M_t \in [\gamma, \beta]$, i.e. the liquidity provider collateral ratio falls in between the Extreme Threshold and Liquidation Threshold, any liquidator is allowed to liquidate a fraction or the entire open position in the pool, by

- Buy $\rho V$ fEUR tokens from any liquidity provider at cost $(1 + a_t) m_t \rho V$ or at any exchange at a similar cost, where $\rho \in (0, 1]$ represent the fraction of the open position that liquidator would like to close.

- Deposit the tokens into the liquidity pool to close the position (by selling the exact amount of fEUR tokens at bid price $(1 - b_t)m_t$ back to close the original open position) and receive the sale return of the fEUR tokens plus ***some proportion*** of the collateral belongs to the liquidity provider in the fraction of the position and the income from bid spread at close of the fraction of the position:

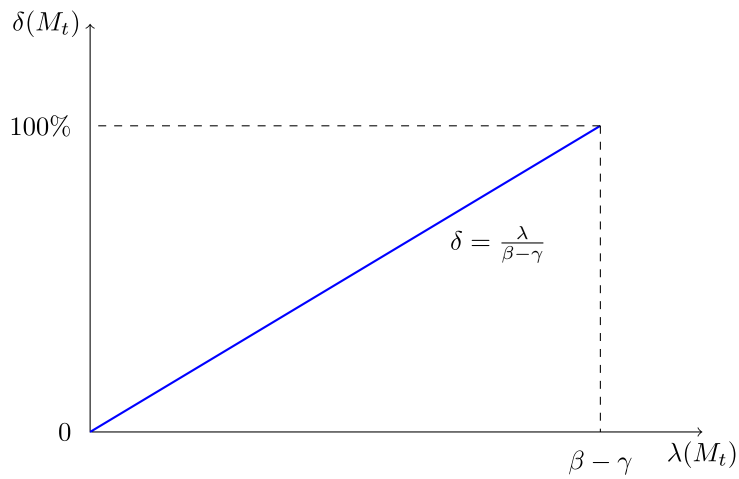
  $$(1 - b_t)m_t\rho V + \delta(\rho C_t + b_t m_t \rho V)$$

  where $\delta$ is the proportion of Liquidity Provider's collateral to be given to the liquidator if a position is liquidated that

  $$\delta_t = \frac{\lambda_t}{\beta - \gamma} = \frac{\beta - M_t}{\beta - \gamma}$$

  and $\lambda$ represents how far away the current Liquidity Provider's collateral ratio is from the Liquidation Threshold that

  $$\lambda_t = \beta - M_t$$




- Make profit

  $$\Pi_l = [b_t(\delta - 1) - a_t]m_t\rho V + \delta\rho C_t$$

  if his tokens are purchased from a liquidity provider.

  Since

  $$\delta_t = \frac{\beta - M_t}{\beta - \gamma}$$

  $$C_t = m_t V M_t$$

  $$m_t = \frac{(\alpha + 1)\sum_{i=1}^{n} m_0^i v^i}{(M_t + 1)V}$$

$$\Pi_l(M_t) = \left[\frac{(b_t + M_t)(\beta - M_t)}{\beta - \gamma} - (a_t + b_t)\right] \frac{(\alpha+1)\rho \sum_{i=1}^{n} m_0^i v^i}{M_t + 1}$$

For $M_t \in [\gamma, \beta]$, there exists a local maxima solution

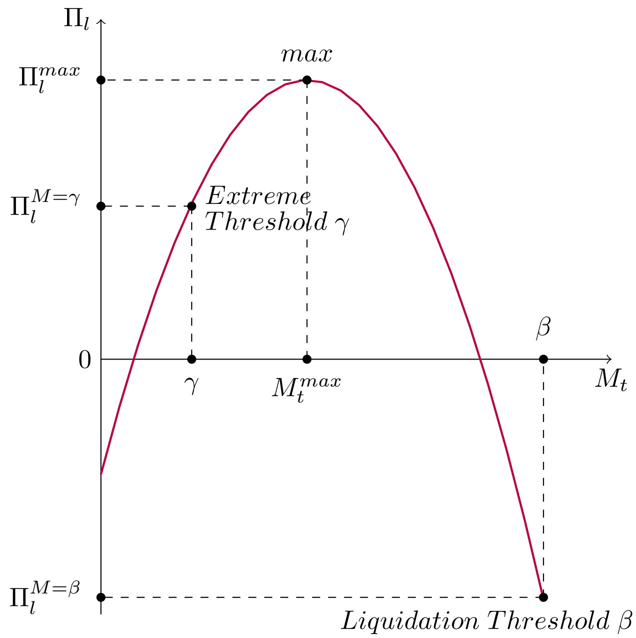
$$M_t^{max}(m_0^i, v^i, \alpha, \beta, \gamma, a_t, b_t)$$

that generate maximized profit

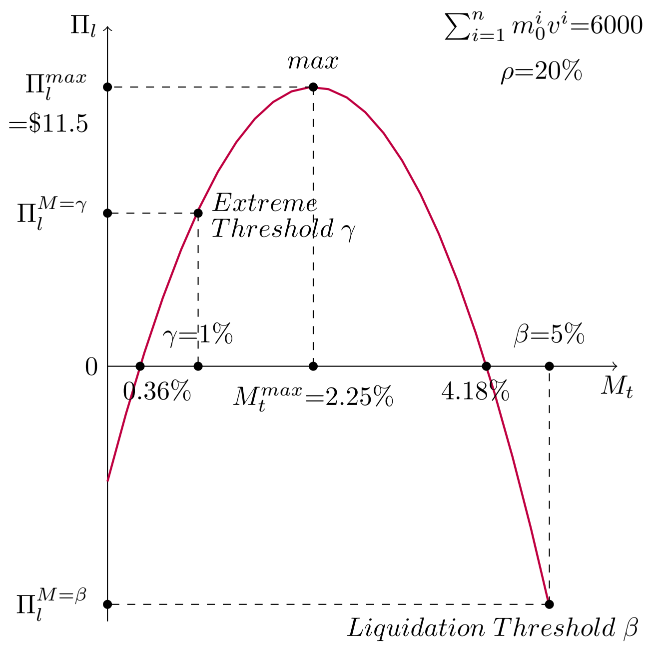
$$\Pi_l^{max}(\rho, M_t^{max})$$

when

$$\frac{\partial \Pi_l}{\partial M_t} = 0$$



For $\alpha = 10\%$, $\beta = 5\%$, $\gamma = 1\%$, $a_t = 0.5\%$, $b_t = 0.5\%$, $\sum_{i=1}^{n} m_0^i v^i = 6,000$ and $\rho = 20\%$, when $M_t \in [\gamma, \beta]$, we have $M_t^{max} = 2.25\%$ and $\Pi_l^{max} = 11.5$ USD stablecoins.




After the $\rho$ fraction of the position is liquidated by a liquidator, the liquidity provider receives the remaining fraction of spread income from buying back fEUR tokens $(1-\delta)b_t m_t \rho V$, and keeps the remaining fraction of collateral of the closed position $(1-\delta)\rho C_t$. And the new position becomes

$$\{(1-\rho)m_t V \qquad + \qquad C_t(1-\delta\rho) + (1-\delta)b_t m_t \rho V\}$$

and the new liquidity provider collateral ratio

$$M_t^{new} = \frac{C_t^{new}}{V^{new}} = \frac{C_t^{old}(1-\delta\rho) + (1-\delta)b_t m_t \rho V}{(1-\rho)m_t V} > M_t^{old}$$

- If $M_t^{new} > \beta$, there will be no immediate arbitrage opportunity.

- While if we still have $M_t^{new} \leq \beta$, any liquidator can keep closing more fraction of the open position, until reaching $M_t > \beta$, i.e. when the current liquidity provider collateral ratio exceeds the Liquidation Threshold.