# Sequential Pattern Mining



## Outline

- What is sequence database and sequential pattern mining
- Methods for sequential pattern mining
- Constraint-based sequential pattern mining
- Periodicity analysis for sequence data



## Sequence Databases

- A sequence database consists of ordered elements or events
- Transaction databases vs. sequence databases

A *transaction database*

| TID | itemsets |
|---|---|
| 10 | a, b, d |
| 20 | a, c, d |
| 30 | a, d, e |
| 40 | b, e, f |

A *sequence database*

| SID | sequences |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |



## Applications

- Applications of sequential pattern mining
  - Customer shopping sequences:
    - First buy computer, then CD-ROM, and then digital camera, within 3 months.
  - Medical treatments, natural disasters (e.g., earthquakes), science & eng. processes, stocks and markets, etc.
  - Telephone calling patterns, Weblog click streams
  - DNA sequences and gene structures



## Subsequence vs. super sequence

- A sequence is an ordered list of events, denoted $< e_1\ e_2 \ldots e_l >$
- Given two sequences $\alpha =< a_1\ a_2 \ldots a_n >$ and $\beta =< b_1\ b_2 \ldots b_m >$
- $\alpha$ is called a subsequence of $\beta$, denoted as $\alpha \subseteq \beta$, if there exist integers $1 \le j_1 < j_2 < \ldots < j_n \le m$ such that $a_1 \subseteq b_{j1}$, $a_2 \subseteq b_{j2}, \ldots, a_n \subseteq b_{jn}$
- $\beta$ is a super sequence of $\alpha$
  - E.g. $\alpha$=< (ab), d> and $\beta$=< (abc), (de)>



## What Is Sequential Pattern Mining?

- Given a set of sequences and support threshold, find the complete set of *frequent* subsequences

A *sequence* : < (ef) (ab) (df) c b >

A *sequence database*

| SID | sequence |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |

An element may contain a set of items. Items within an element are unordered and we list them alphabetically.

<a(bc)dc> is a *subsequence* of <a(abc)(ac)d(cf)>

Given *support threshold* *min_sup* =2, <(ab)c> is a *sequential pattern*

## Challenges on Sequential Pattern Mining

- A huge number of possible sequential patterns are hidden in databases
- A mining algorithm should
  - find the complete set of patterns, when possible, satisfying the minimum support (frequency) threshold
  - be highly efficient, scalable, involving only a small number of database scans
  - be able to incorporate various kinds of user-specific constraints



## Studies on Sequential Pattern Mining

- Concept introduction and an initial Apriori-like algorithm
  - Agrawal & Srikant. Mining sequential patterns, [ICDE'95]
- Apriori-based method: GSP (Generalized Sequential Patterns: Srikant & Agrawal [EDBT'96])
- Pattern-growth methods: FreeSpan & PrefixSpan (Han et al.KDD'00; Pei, et al. [ICDE'01])
- Vertical format-based mining: SPADE (Zaki [Machine Leanining'00])
- Constraint-based sequential pattern mining (SPIRIT: Garofalakis, Rastogi, Shim [VLDB'99]; Pei, Han, Wang [CIKM'02])
- Mining closed sequential patterns: CloSpan (Yan, Han & Afshar [SDM'03])



## Methods for sequential pattern mining

- **Apriori-based Approaches**
  - **GSP**
  - **SPADE**
- **Pattern-Growth-based Approaches**
  - **FreeSpan**
  - **PrefixSpan**



## The Apriori Property of Sequential Patterns

- A basic property: Apriori (Agrawal & Sirkant'94)
  - If a sequence *S* is not frequent, then none of the super-sequences of *S* is frequent
  - E.g, <hb> is infrequent → so do <hab> and <(ah)b>

| Seq. ID | Sequence |
|---|---|
| 10 | <(bd)cb(ac)> |
| 20 | <(bf)(ce)b(fg)> |
| 30 | <(ah)(bf)abf> |
| 40 | <(be)(ce)d> |
| 50 | <a(bd)bcb(ade)> |

Given *support threshold*
*min_sup* =2



## GSP—Generalized Sequential Pattern Mining

- GSP (Generalized Sequential Pattern) mining algorithm
- Outline of the method
  - Initially, every item in DB is a candidate of length-1
  - for each level (i.e., sequences of length-k) do
    - scan database to collect support count for each candidate sequence
    - generate candidate length-(k+1) sequences from length-k frequent sequences using Apriori
  - repeat until no frequent sequence or no candidate can be found
- Major strength: Candidate pruning by Apriori



## Finding Length-1 Sequential Patterns

- Initial candidates:
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan database once, count support for candidates

*min_sup* =2

| Seq. ID | Sequence |
|---|---|
| 10 | <(bd)cb(ac)> |
| 20 | <(bf)(ce)b(fg)> |
| 30 | <(ah)(bf)abf> |
| 40 | <(be)(ce)d> |
| 50 | <a(bd)bcb(ade)> |

| Cand | Sup |
|---|---|
| **<a>** | **3** |
| **<b>** | **5** |
| **<c>** | **4** |
| **<d>** | **3** |
| **<e>** | **3** |
| **<f>** | **2** |
| ~~<g>~~ | 1 |
| ~~<h>~~ | 1 |

## Generating Length-2 Candidates

51 length-2 Candidates

| | <a> | <b> | <c> | <d> | <e> | <f> |
|---|---|---|---|---|---|---|
| <a> | <aa> | <ab> | <ac> | <ad> | <ae> | <af> |
| <b> | <ba> | <bb> | <bc> | <bd> | <be> | <bf> |
| <c> | <ca> | <cb> | <cc> | <cd> | <ce> | <cf> |
| <d> | <da> | <db> | <dc> | <dd> | <de> | <df> |
| <e> | <ea> | <eb> | <ec> | <ed> | <ee> | <ef> |
| <f> | <fa> | <fb> | <fc> | <fd> | <fe> | <ff> |

| | <a> | <b> | <c> | <d> | <e> | <f> |
|---|---|---|---|---|---|---|
| <a> | | <(ab)> | <(ac)> | <(ad)> | <(ae)> | <(af)> |
| <b> | | | <(bc)> | <(bd)> | <(be)> | <(bf)> |
| <c> | | | | <(cd)> | <(ce)> | <(cf)> |
| <d> | | | | | <(de)> | <(df)> |
| <e> | | | | | | <(ef)> |
| <f> | | | | | | |

Without Apriori property, 8*8+8*7/2=92 candidates

Apriori prunes 44.57% candidates

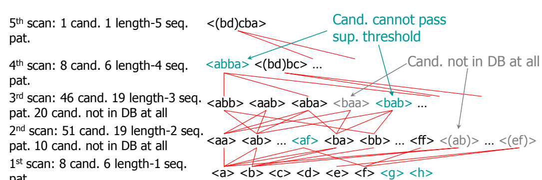


## Finding Lenth-2 Sequential Patterns

- Scan database one more time, collect support count for each length-2 candidate
- There are 19 length-2 candidates which pass the minimum support threshold
  - They are length-2 sequential patterns



## The GSP Mining Process




*min_sup* =2

| Seq. ID | Sequence |
|---|---|
| 10 | <(bd)cb(ac)> |
| 20 | <(bf)(ce)b(fg)> |
| 30 | <(ah)(bf)abf> |
| 40 | <(be)(ce)d> |
| 50 | <a(bd)bcb(ade)> |



## The GSP Algorithm

- Take sequences in form of <x> as length-1 candidates
- Scan database once, find $F_1$, the set of length-1 sequential patterns
- Let k=1; while $F_k$ is not empty do
  - Form $C_{k+1}$, the set of length-(k+1) candidates from $F_k$;
  - If $C_{k+1}$ is not empty, scan database once, find $F_{k+1}$, the set of length-(k+1) sequential patterns
  - Let k=k+1;



## The GSP Algorithm

- Benefits from the Apriori pruning
  - Reduces search space
- Bottlenecks
  - Scans the database multiple times
  - Generates a huge set of candidate sequences

There is a need for more efficient mining methods



## The SPADE Algorithm

- SPADE (Sequential PAttern Discovery using Equivalent Class) developed by Zaki 2001
- A vertical format sequential pattern mining method
- A sequence database is mapped to a large set of Item: <SID, EID>
- Sequential pattern mining is performed by
  - growing the subsequences (patterns) one item at a time by Apriori candidate generation

## The SPADE Algorithm

| SID | EID | Items |
|---|---|---|
| 1 | 1 | a |
| 1 | 2 | abc |
| 1 | 3 | ac |
| 1 | 4 | d |
| 1 | 5 | cf |
| 2 | 1 | ad |
| 2 | 2 | c |
| 2 | 3 | bc |
| 2 | 4 | ae |
| 3 | 1 | ef |
| 3 | 2 | ab |
| 3 | 3 | df |
| 3 | 4 | c |
| 3 | 5 | b |
| 4 | 1 | e |
| 4 | 2 | g |
| 4 | 3 | af |
| 4 | 4 | c |
| 4 | 5 | b |
| 4 | 6 | c |

| a | | b | | ··· |
|---|---|---|---|---|
| SID | EID | SID | EID | ··· |
| 1 | 1 | 1 | 2 | |
| 1 | 2 | 2 | 3 | |
| 1 | 3 | 3 | 2 | |
| 2 | 1 | 3 | 5 | |
| 2 | 4 | 4 | 5 | |
| 3 | 2 | | | |
| 4 | 3 | | | |

| ab | | | ba | | | ··· |
|---|---|---|---|---|---|---|
| SID | EID (a) | EID(b) | SID | EID (b) | EID(a) | ··· |
| 1 | 1 | 2 | 1 | 2 | 3 | |
| 2 | 1 | 3 | 2 | 3 | 4 | |
| 3 | 2 | 5 | | | | |
| 4 | 3 | 5 | | | | |

| aba | | | | ··· |
|---|---|---|---|---|
| SID | EID (a) | EID(b) | EID(a) | ··· |
| 1 | 1 | 2 | 3 | |
| 2 | 1 | 3 | 4 | |

## Bottlenecks of Candidate Generate-and-test

- A huge set of candidates generated.
  - Especially 2-item candidate sequence.
- Multiple Scans of database in mining.
  - The length of each candidate grows by one at each database scan.
- Inefficient for mining long sequential patterns.
  - A long pattern grow up from short patterns
  - An exponential number of short candidates



## PrefixSpan (Prefix-Projected Sequential Pattern Growth)

- PrefixSpan
  - Projection-based
  - But only prefix-based projection: less projections and quickly shrinking sequences
- J.Pei, J.Han,… PrefixSpan : Mining sequential patterns efficiently by prefix-projected pattern growth. ICDE'01.



## Prefix and Suffix (Projection)

- <a>, <aa>, <a(ab)> and <a(abc)> are *prefixes* of sequence <a(abc)(ac)d(cf)>
- Given sequence <a(abc)(ac)d(cf)>

| Prefix | *Suffix* (Prefix-Based *Projection)* |
|---|---|
| <a> | <(abc)(ac)d(cf)> |
| <aa> | <(_bc)(ac)d(cf)> |
| <ab> | <(_c)(ac)d(cf)> |



## Mining Sequential Patterns by Prefix Projections

- Step 1: find length-1 sequential patterns
  - <a>, <b>, <c>, <d>, <e>, <f>
- Step 2: divide search space. The complete set of seq. pat. can be partitioned into 6 subsets:
  - The ones having prefix <a>;
  - The ones having prefix <b>;
  - …
  - The ones having prefix <f>

| SID | sequence |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |

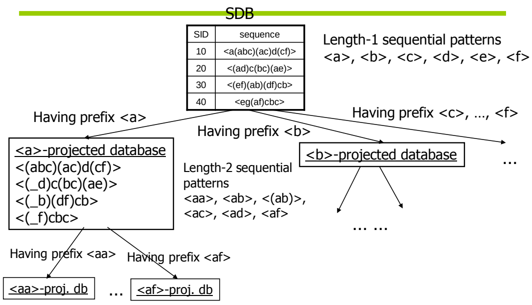


## Finding Seq. Patterns with Prefix <a>

- Only need to consider projections w.r.t. <a>
  - <a>-projected database: <(abc)(ac)d(cf)>, <(_d)c(bc)(ae)>, <(_b)(df)cb>, <(_f)cbc>
- Find all the length-2 seq. pat. Having prefix <a>: <aa>, <ab>, <(ab)>, <ac>, <ad>, <af>
  - Further partition into 6 subsets
    - Having prefix <aa>;
    - …
    - Having prefix <af>

| SID | sequence |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |

## Completeness of PrefixSpan






## The Algorithm of PrefixSpan

- **Input**: A sequence database S, and the minimum support threshold min_sup
- **Output**: The complete set of sequential patterns
- **Method**: Call PrefixSpan(<>,0,S)
- **Subroutine** PrefixSpan(α, l, S|α)
- **Parameters**:
  - α: sequential pattern,
  - l: the length of α;
  - S|α: the α-projected database, if α ≠<>; otherwise; the sequence database S



## The Algorithm of PrefixSpan(2)

- **Method**

1. Scan S|α once, find the set of frequent items b such that:
   a) b can be assembled to the last element of α to form a sequential pattern; or
   b) <b> can be appended to α to form a sequential pattern.
2. For each frequent item b, append it to α to form a sequential pattern α', and output α';
3. For each α', construct α'-projected database S|α', and call PrefixSpan(α', l+1, S|α').



## Efficiency of PrefixSpan

- No candidate sequence needs to be generated
- Projected databases keep shrinking
- Major cost of PrefixSpan: constructing projected databases
  - Can be improved by bi-level projections



## Optimization in PrefixSpan

- Single level vs. bi-level projection
  - Bi-level projection with 3-way checking may reduce the number and size of projected databases
- Physical projection vs. pseudo-projection
  - Pseudo-projection may reduce the effort of projection when the projected database fits in main memory
- Parallel projection vs. partition projection
  - Partition projection may avoid the blowup of disk space



## Scaling Up by Bi-Level Projection

- Partition search space based on length-2 sequential patterns
- Only form projected databases and pursue recursive mining over bi-level projected databases

## Speed-up by Pseudo-projection

- Major cost of PrefixSpan: projection
  - Postfixes of sequences often appear repeatedly in recursive projected databases
- When (projected) database can be held in main memory, use pointers to form projections
  - Pointer to the sequence
  - Offset of the postfix

s=<a(abc)(ac)d(cf)>

<a>

s|<a>: ( , 2) <(abc)(ac)d(cf)>

<ab>

s|<ab>: ( , 4) <(_c)(ac)d(cf)>

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## Pseudo-Projection vs. Physical Projection

- Pseudo-projection avoids physically copying postfixes
  - Efficient in running time and space when database can be held in main memory
- However, it is not efficient when database cannot fit in main memory
  - Disk-based random accessing is very costly
- Suggested Approach:
  - Integration of physical and pseudo-projection
  - Swapping to pseudo-projection when the data set fits in memory

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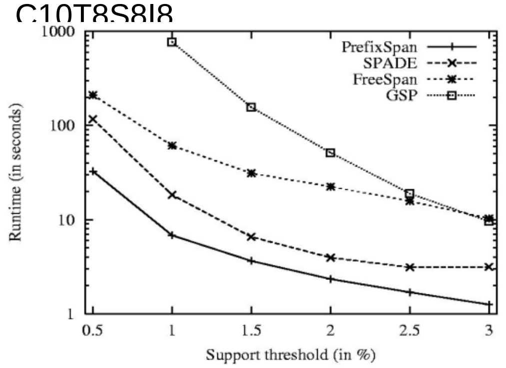
## Performance on Data Set C10T8S8I8

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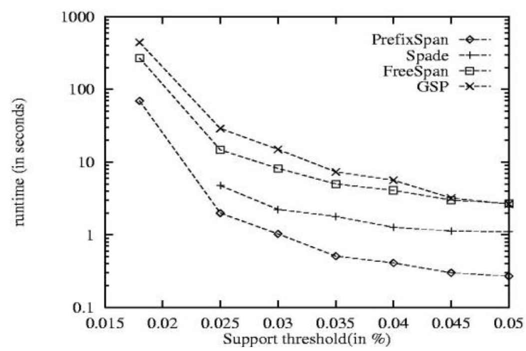
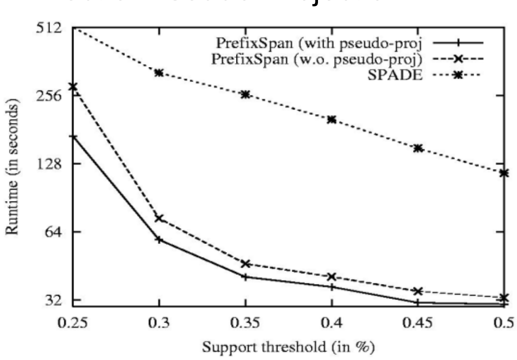
## Performance on Data Set Gazelle

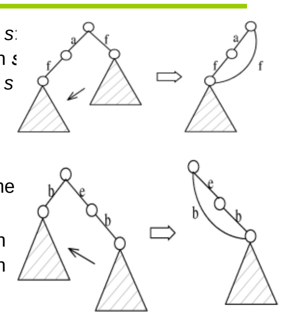



## Effect of Pseudo-Projection

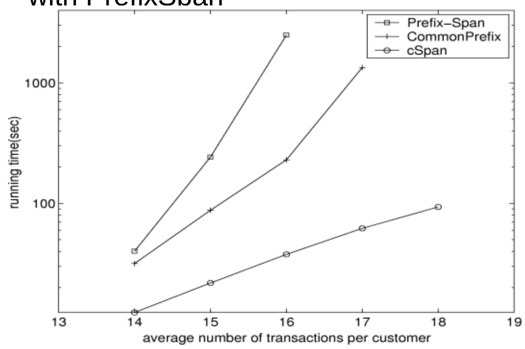



## CloSpan: Mining Closed Sequential Patterns

- A closed sequential pattern *s*: there exists no superpattern *s*' such that *s*' ⊃ *s*, and *s*' and *s* have the same support
- Motivation: reduces the number of (redundant) patterns but attains the same expressive power
- Using Backward Subpattern and Backward Superpattern pruning to prune redundant search space



## CloSpan: Performance Comparison with PrefixSpan




## Constraints for Seq.-Pattern Mining

- Item constraint
  - Find web log patterns only about online-bookstores
- Length constraint
  - Find patterns having at least 20 items
- Super pattern constraint
  - Find super patterns of “PC → digital camera”
- Aggregate constraint
  - Find patterns that the average price of items is over $100

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## More Constraints

- Regular expression constraint
  - Find patterns “starting from Yahoo homepage, search for hotels in Washington DC area”
  - Yahootravel(WashingtonDC|DC)(hotel|motel|lodging)
- Duration constraint
  - Find patterns about $\pm$24 hours of a shooting
- Gap constraint
  - Find purchasing patterns such that “the gap between each consecutive purchases is less than 1 month”

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## From Sequential Patterns to Structured Patterns

- Sets, sequences, trees, graphs, and other structures
  - Transaction DB: Sets of items
    - $\{\{i_1, i_2, \ldots, i_m\}, \ldots\}$
  - Seq. DB: Sequences of sets:
    - $\{<\{i_1, i_2\}, \ldots, \{i_m, i_n, i_k\}>, \ldots\}$
  - Sets of Sequences:
    - $\{\{<i_1, i_2>, \ldots, <i_m, i_n, i_k>\}, \ldots\}$
  - Sets of trees: $\{t_1, t_2, \ldots, t_n\}$
  - Sets of graphs (mining for frequent subgraphs):
    - $\{g_1, g_2, \ldots, g_n\}$
- Mining structured patterns in XML documents, bio-chemical structures, etc.

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## Episodes and Episode Pattern Mining

- Other methods for specifying the kinds of patterns
  - Serial episodes: A → B
  - Parallel episodes: A & B
  - Regular expressions: (A | B)C*(D → E)
- Methods for episode pattern mining
  - Variations of Apriori-like algorithms, e.g., GSP
  - Database projection-based pattern growth
    - Similar to the frequent pattern growth without candidate generation

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## Periodicity Analysis

- Periodicity is everywhere: tides, seasons, daily power consumption, etc.
- Full periodicity
  - Every point in time contributes (precisely or approximately) to the periodicity
- Partial periodicit: A more general notion
  - Only some segments contribute to the periodicity
    - Jim reads NY Times 7:00-7:30 am every week day
- Cyclic association rules
  - Associations which form cycles
- Methods
  - Full periodicity: FFT, other statistical analysis methods
  - Partial and cyclic periodicity: Variations of Apriori-like mining methods

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## Summary

- Sequential Pattern Mining is useful in many application, e.g. weblog analysis, financial market prediction, BioInformatics, etc.
- It is similar to the frequent itemsets mining, *but* with consideration of ordering.
- We have looked at different approaches that are descendants from two popular algorithms in mining frequent itemsets
  - Candidates Generation: AprioriAll and GSP
  - Pattern Growth: FreeSpan and PrefixSpan