NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2016/2017

MA1102R Calculus **Homework Assignment 3**

IMPORTANT: Please write your ***name***, ***matriculation card number*** and ***tutorial group number*** on the answer script, and submit by 11th November 2016 (Friday).†

The assignment carries a total of 70 *marks. The marks for each individual question or part are as indicated.*

1. Find the following limits.

   (a) $\lim\limits_{x\to 0^+} (\ln \cot x)^{\tan x}$. [4]

   (b) $\lim\limits_{x\to\infty} \left(x - x^2 \ln(1+\frac{1}{x})\right)$. [4]

2. Evaluate the following integrals.

   (a) $\int \ln\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$. [4]

   (b) $\int \dfrac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\dfrac{x^2+1}{x}\right)} dx$. [4]

   (c) $\int \dfrac{x^3+3}{(x+1)(x^2+1)} dx$. [4]

3. Suppose $n \in \mathbb{Z}^+$. Derive reduction formulas for the following integrals.

   (a) $I_n = \int (\ln x)^n \, dx$. [4]

   (b) $I_n = \int \dfrac{x^n}{\sqrt{x^2+a^2}} dx$. [6]

4. Using **both the disk/washer and cylindrical shell methods**, find the volume of the solid generated by revolving the region bounded by

   (a) the curve $y = 9 - x^2$ and the line $y = 5$ about $y = 2$ [4]

   (b) the curves $y = \sin^2 x$ and the lines $y = \cos^2 x$ $(0 \le x \le \pi/4)$ about $x = 2$ [6]

†The homework scripts can also be submitted on 10th and 11th November during the lecture time.

(c) the curve $x^2 - y^2 = 9$ and the lines $y = \pm 4$ about [14]
(i) The x-axis. (ii) The y-axis.

5. Find the lengths of the following curves.

(a) $x = \frac{1}{12}y^3 + y^{-1}, \quad 1 \leq y \leq 3.$ [4]

(b) $y = \ln \frac{e^x - 1}{e^x + 1}, \quad 1 \leq x \leq 2.$ [4]

6. Find the areas of the surfaces formed by revolving the given regions about the axes.

(a) $y = x^2, \quad 0 \leq x \leq 1, \quad$ about the y-axis. [4]

(b) $\frac{x^2}{4} + y^2 = 1, \quad$ about the x-axis. [4]