

HW 2 SOLUTION

Problem 1

Formulate the linear programming model for this problem:

(1) Decision variables:

A = number of pounds of brand A

B = number of pounds of brand B

(2) Objective: Minimize

$$\text{Cost} = \$0.8 * A + \$0.8 * B (\$)$$

(3) Constraints:

Protein) $6A+4B \geq 24$ ounces

Carbohydrates) $3A+10B \geq 30$ ounces

Vitamins) $50A+100B \geq 400$ units

In LINDO:

```
min 0.8a+0.8b
st
6a+4b >= 24
3a+10b >= 30
50a+100b >= 400
```

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 4.000000

VARIABLE	VALUE	REDUCED COST
A	2.000000	0.000000
B	3.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.100000
3)	6.000000	0.000000
4)	0.000000	-0.004000

NO. ITERATIONS= 2

Optimal solution is 2 pounds of brand A and 3 pounds of brand B. The minimum cost is \$4.

The first and third bounds (protein and vitamin) are binding.

Problem 2

Million \$ Money in:

First mortgages: x_1

Second mortgages: x_2

Personal loans: x_3

Commercial loans: x_4

Government securities: x_5

Objective:

$$\text{Max } 9x_1 + 12x_2 + 15x_3 + 8x_4 + 6x_5$$

Constraints:

Budget) $x_1 + x_2 + x_3 + x_4 + x_5 = 100$ million \$

Avg Risk) $(3x_1 + 6x_2 + 8x_3 + 2x_4 + x_5) / (x_1 + x_2 + x_3 + x_4 + x_5) \leq 5$
 or $(3x_1 + 6x_2 + 8x_3 + 2x_4 + x_5) \leq 5(x_1 + x_2 + x_3 + x_4 + x_5)$
 or $3x_1 + 6x_2 + 8x_3 + 2x_4 + x_5 \leq 500$
 commercial loans) $x_4 \geq 20$ million \$
 two mortgages) $x_1 + x_2 \leq 50$ million \$
 second mortgage and personal loans) $x_2 + x_3 - x_1 \leq 0$

Problem 3

Formulate the linear programming model for this problem:

(1) Decision variables:

x_{1b} = number of units of product buying in Month 1
 x_{2b} = number of units of product buying in Month 2
 x_{3b} = number of units of product buying in Month 3
 x_{4b} = number of units of product buying in Month 4
 x_{1s} = number of units of product selling in Month 1
 x_{2s} = number of units of product selling in Month 2
 x_{3s} = number of units of product selling in Month 3
 x_{4s} = number of units of product selling in Month 4

(2) Objective: Maximize profits (\$)

Profits = income from sales – payment of purchase – storage fees for four months
 $= 55x_{1s} + 44x_{2s} + 66x_{3s} + 55x_{4s} - 50x_{1b} - 40x_{2b} - 60x_{3b} - 50x_{4b} - (300 + x_{1b} - x_{1s}) -$
 $(300 + x_{1b} - x_{1s} + x_{2b} - x_{2s}) - (300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} + x_{3b} - x_{3s}) - (300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} + x_{3b} -$
 $x_{3s} + x_{4b} - x_{4s})$
 $= 59x_{1s} + 47x_{2s} + 68x_{3s} + 56x_{4s} - 54x_{1b} - 43x_{2b} - 62x_{3b} - 51x_{4b} - 1200$ \$

(3) Constraints:

Storage1) $300 + x_{1b} - x_{1s} \leq 500$ %warehouse limitation for the first month
 Storage2) $300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} \leq 500$
 Storage3) $300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} + x_{3b} - x_{3s} \leq 500$
 Storage4) $300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} + x_{3b} - x_{3s} + x_{4b} - x_{4s} = 300$ %end of month 4 requirement
 No exceed1) $x_{1s} \leq 300$ % sales in Month 1 should not exceed initial inventories
 No exceed1) $x_{2s} \leq 300 + x_{1b} - x_{1s}$ % sales in Month 2 should not exceed Month 1 inventories
 No exceed1) $x_{3s} \leq 300 + x_{1b} - x_{1s} + x_{2b} - x_{2s}$
 No exceed1) $x_{4s} \leq 300 + x_{1b} - x_{1s} + x_{2b} - x_{2s} + x_{3b} - x_{3s}$

In LINDO:

```

max 59x11+47x21+68x31+56x41-54x12-43x22-62x32-51x42-c
st
c = 1200
x12-x11 <= 200
x12-x11+x22-x21 <= 200
x12-x11+x22-x21+x32-x31 <= 200
x12-x11+x22-x21+x32-x31+x42-x41 = 0
x11 <= 300
x21-x12+x11 <= 300
x31-x12+x11-x22+x21 <= 300
x41-x12+x11-x22+x21-x32+x31 <= 300

```

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 13700.00

VARIABLE	VALUE	REDUCED COST
X11	300.000000	0.000000
X21	0.000000	7.000000
X31	500.000000	0.000000
X41	0.000000	6.000000
X12	0.000000	0.000000
X22	500.000000	0.000000
X32	0.000000	0.000000
X42	300.000000	0.000000
C	1200.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-1.000000
3)	500.000000	0.000000
4)	0.000000	25.000000
5)	500.000000	0.000000
6)	0.000000	-51.000000
7)	0.000000	5.000000
8)	0.000000	11.000000
9)	0.000000	6.000000
10)	0.000000	11.000000

NO. ITERATIONS= 6

So the optimal buying and selling choice should be:

Month	Units of selling	Units of buying	Inventories
1	300	0	0
2	0	500	500
3	500	0	0
4	0	300	300

The optimal profits thereby should be \$13700.