Labor Commitment and Expected Returns

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Motivation: Fact

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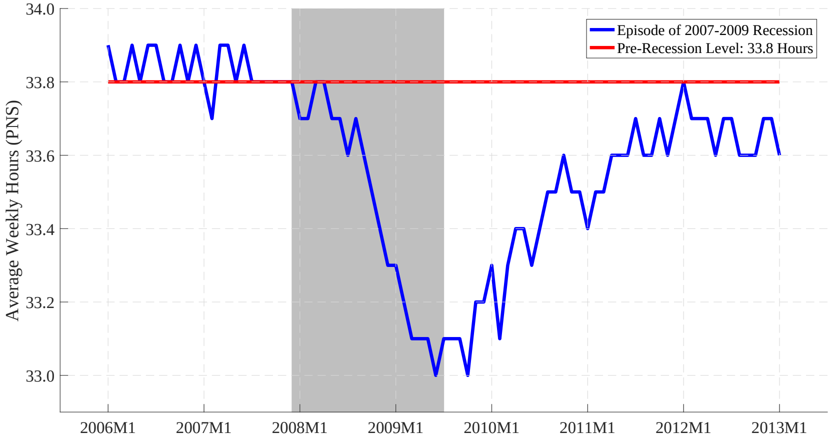

Figure: Average Weekly Hours around 2007-2009 Recession. This figure depicts the average weekly hours of production and nonsupervisory employees in all private sectors. NBER recessions are indicated by shaded areas. The plotted sample period is 2006:Q1 - 2013:Q1.

Motivation: Thought

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A **coherent** story

- that relates expected returns to hours
- (and to other real economy variables).

A **different** story

- because of asset pricing implications from hours
- (not from labor income).

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What I Did

$$\text{hours} + \cdots \to \text{expected returns}$$

- Model (Equilibrium)

$\Downarrow$

- Empirics: Model Validation (Cointegration & GMM)

$\Downarrow$

- Empirics: Asset Pricing Implication

What I Found

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Risk in intensive margin of labor supply is

- significantly higher priced than risk in consumption;
- overlooked in various models.
  - Marginal utility increases when labor commitment-consumption ratio increases.
  - Intertemporal marginal rate of substitution increases when labor commitment-consumption ratio growth rate increases.

Literature

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Most Closely Related

- Yogo (2006): Similar Framework
  - durable vs nondurable
    (labor (intensive) vs consumption)
  - consumption $\rightarrow$ expected returns
    (labor (intensive) $\rightarrow$ expected returns)
- Epstein and Zin (1991): Solution Algorithm

Asset Pricing Implications from Labor Market

- Campbell (1996); Jagannathan and Wang (1996); Santos and Veronesi (2006); Lustig and Van Nieuwerburgh (2006); Lustig et al. (2013); Campbell et al. (2016); Kilic and Wachter (2017)
- risk associated with labor (intensive)

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Model

- Labor Commitment (central to mechanism)
- Setup
- Equilibrium

Empirics

- Validation
  - Empirical evidence about labor commitment
  - Cointegration estimation of one FOC
  - GMM estimation of all FOCs (equilibrium)

- Implication
  - Transparency
  - Disentangle labor commitment vs consumption

Labor Commitment: Law of Motion

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The intensive margin of labor supply

- Model explicitly "Flow" vs "Stock"

A representative agent (household), in the period of t,

- provides H_t units of hours,
- receives N_t units of earnings (income per hour), and
- has L_t units of labor commitment in hours to fulfill.

Law of motion for labor

$$L_t = (1+\delta)L_{t-1} - H_t.$$

where $(1+\delta)$ is the labor development rate.

Labor Commitment: Intuition

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What is L_t labor commitment

intensive margin measure of labor supply

growing, **income-generating** stock of labor

What is δ

- simplification
- **extra** commitment to fulfill due to postponement

Writing a book in 12 months vs in 1 month

L_t = writing this book

δ = opening computers more times

+ looking up notation defined 3-month ago

+ etc.

Appendix: Labor Commitment

Setup: Budget Constrains

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Denote W_t the **Wealth** at the beginning of period t.

- C_t units of wealth to consume
- H_t units of hours
- A_t^n units of wealth to invest in asset n $(n = 1, 2, \cdots, N)$, which earns a gross return of R_{t+1}^n during period t

- The intratemporal budget constrain is

$$C_t + \sum_{n=1}^{N} A_t^n = W_t + H_t N_t.$$

- The intertemporal budget constrain is

$$W_{t+1} = \sum_{n=1}^{N} A_t^n R_{t+1}^n.$$

Setup: Agent Problem

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The agent problem

$$U_t = \max[u(C_t, L_t)^{1-1/\sigma} + \beta(\mathbb{E}_t[U_{t+1}]^{1-\gamma})^{\frac{1-1/\sigma}{1-\gamma}}]^{\frac{1}{1-1/\sigma}}$$

$$\text{st} \quad u(C_t, L_t) = [(1-\alpha){C_t}^{1-1/\rho} + \alpha {L_t}^{1-1/\rho}]^{\frac{1}{1-1/\rho}}$$

$$\Sigma_n A_t^n = W_t + H_t N_t - C_t$$

$$\Sigma_n A_t^n R_{t+1}^n = W_{t+1}$$

$$L_t = (1+\delta)L_{t-1} - H_t$$

- α the relative "importance"
- ρ the intratemporal elasticity of substitution
- β the subjective discount rate
- σ the intertemporal elasticity of substitution
- γ the relative risk aversion

Equilibrium: 3 FOCs

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FOC wrt Consumption Choice C_t

$$M_{t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left[\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)}\right]^{\frac{1}{\rho}-\frac{1}{\sigma}}\left(R^W_{t+1}\right)^{\frac{1/\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-1/\sigma}}.$$

where

$$v(L/C) = [(1-\alpha)+\alpha(L/C)^{1-1/\rho}]^{\frac{1}{1-1/\rho}}.$$

Three Predictions

- (L/C) increases in Recessions.
- $\Delta\log(L/C)$ increases in Recessions.
- Intra-ES $\rho > \sigma$ and > 1.

Appendix: Equilibrium

Equilibrium: 3 FOCs

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FOC wrt Labor Commitment Choice L_t

$$\frac{u_{L_t}}{u_{C_t}} = N_t - (1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}],$$

where

$$\frac{u_L}{u_C} = \frac{\alpha}{1-\alpha}\left(\frac{L}{C}\right)^{-1/\rho}.$$

FOC wrt Portfolio Choices A_t^n

$$\mathbb{E}[M_{t+1}(R_{t+1}^n - R_{t+1}^1)] = 0, (n = 2, \cdots, N).$$

Appendix: Equilibrium

Summary: Model

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What I Defined

Labor commitment

Intensive: **growing**, **income-generating** stock of labor supply

What I Found

If Intra-ES $\rho >$ IES σ (and > 1),
MU increases in (L/C), and IMRS increases in $\Delta \log(L/C)$;
that is, (L/C) and $\Delta \log(L/C)$ increase in Recessions.

Why I Cared

IMRS response to $\Delta \log(L/C)$ pins down expected returns.

Now What

(1) validate the model first (2) asset pricing implications

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Data

Source

- C: Real Personal Consumption Expenditure p.c. (BEA)
- H: Average Weekly Hours in Total Private Sectors (BLS)
- N: Average Hourly Earnings in Total Private Sectors (BLS)
- R: Various Portfolio Returns (K. French Website + CRSP)

Matching

- "REAL-ize" returns using PCE p.c. implied deflater.

Frequency & Span

- From 1964:Q1 to 2013:Q4.

Appendix: Data

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Model

Empirics

- Validation
  - Empirical evidence about labor commitment

  $\Rightarrow$ (L/C) and $\Delta \log(L/C)$ increase in Recessions.
  - Cointegration estimation of one FOC

  $\Rightarrow$ Intra-ES $\rho >$ IES σ (and > 1)
  - GMM estimation of all FOCs (equilibrium)

  $\Rightarrow$ validation
- Implication (linear factor asset pricing model)
  - Within-model

  $\Rightarrow$ which has higher risk price
  - Across-model

  $\Rightarrow$ is risk associated with labor (intensive) an important, but often overlooked one.

Labor Commitment-Consumption Ratio

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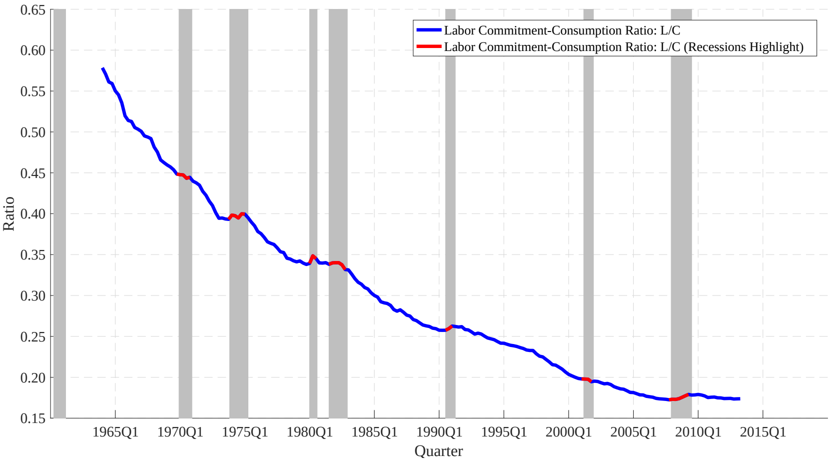

Figure: The Business-Cycle Characteristic of Labor Commitment-Consumption Ratio. This figure depicts the ratio of labor commitment to consumption (L/C). The labor commitment is constructed from average weekly hours of Production and Nonsupervisory (PNS) employees in all private sectors using law of motion for labor; the consumption is measured by real personal consumption expenditure per capita. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Labor Commitment-Consumption Ratio

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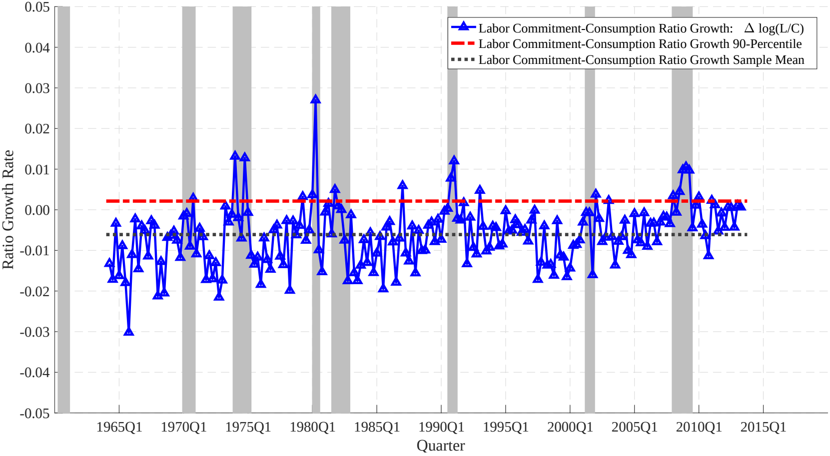

Figure: The Business-Cycle Characteristic of Labor Commitment-Consumption Ratio Growth Rate. This figure depicts the growth rate of ratio of labor commitment to consumption $\Delta \log(L/C)$. The red dashed horizontal line indicates the sample 90-percentile value; the black dotted horizontal line indicates the sample mean value. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Intratemporal Elasticity of Substitution

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From FOC wrt to L_t,

$$(l_t - c_t) - (-\rho)n_t + \text{constant} \sim I(0)$$

Test			ρ Est.		
Reject	p-value	t-Stat	$\hat{\rho}$	p-value	t-Stat
1	1.4e-3	-4.63	1.94	7.4e-5	-4.09

Table: The Cointegration Estimation of Intratemporal Elasticity of Substitution. This table tabulates the cointegration estimation result of intratemporal elasticity of substitution coefficient ρ using $(l_t - c_t) - (-\rho)n_t + \text{constant} \sim I(0)$. The cointegration test has NULL hypotheses of no cointegration; the cointegration test uses Engle-Granger algorithm with Augmented Dickey-Fuller unit root test. The sample period is 1979:Q1 - 2013:Q4.

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GMM Estimation

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Z_t	(1) α	(2) β	(3) γ	(4) ρ	(5) σ	(6) J-test
1	0.90***	2.68***	3.21	1.77	1.02***	6.07
	(0.14)	(0.24)	(4.67)	(3.60)	(0.05)	0.11
1	0.89***	2.41***	2.80	1.76	1.03***	22.2
	(0.12)	(0.19)	(9.58)	(2.85)	(0.17)	0.45
(dp,ep)	0.89***	2.51***	2.86***	1.77***	1.02***	92.16
	(0.01)	(0.04)	(0.20)	(0.18)	(0.00)	0.00

Table: The Generalized Method of Moments Estimation Results. This table tabulates the GMM estimation results of the labor commitment model. Column (1)-(5) list the point estimations (standard deviation) of five parameters $(\alpha, \beta, \gamma, \rho, \sigma)$; column (6) lists the J-statistics and the p-values. The first row is unconditional estimation (i.e., $Z_t = 1$) using Fama-French 6 portfolios bivariate sorted by size and book-to-market; the second row is unconditional estimation using Fama-French 25 portfolios; the third row is conditional estimation with instruments of dividend-price ratio and earnings-price ratio (i.e., $Z_t = (dp_t, ep_t)$) using Fama-French 6 portfolios. The *** denotes significance level 0.01. The sample period is 1964:Q1 - 2013:Q4.

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Stochastic Discount Factor: Linear Factor Model

Equilibrium Implied IMRS

$$m_{t+1} = \kappa \log(\beta) + b^1 \Delta c_{t+1} + b^2 \Delta l_{t+1} + b^3 r^W_{t+1}.$$

Stochastic Discount Factor

$$M_{t+1} = a + \boldsymbol{b}' \cdot \boldsymbol{f}_{t+1}$$

$$\boldsymbol{f} := (f^k)_{K=3} = \left[\Delta c, \Delta l, r^W\right]$$

Linear Factors Model (Fama-MacBeth)

$$\mathbb{E}[R^{en}] = \mathbb{E}[R^n - R^1], n = 2, \cdots N$$

$$\mathbb{E}[R^{en}] = \psi + \phi^{n\prime} \boldsymbol{f}$$

$$\mathbb{E}[R^{en}] = \boldsymbol{\lambda}' \phi^n$$

$$\boldsymbol{\lambda} := (\lambda^k)_{K=3} = \left[\lambda_{\Delta c}, \lambda_{\Delta l}, \lambda_{r^W}\right]$$

Pricing Errors

$$\varepsilon^n = R^{en} - \hat{\boldsymbol{\lambda}}' \hat{\phi}^n$$

Within Model: Labor Commitment vs Consumption

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$$M = a + \boldsymbol{b}' \cdot \boldsymbol{f}, \qquad \boldsymbol{f} = \left[\Delta c, \quad \Delta l, \quad r^W\right]'$$

$$\mathbb{E}[R^{en}] = \boldsymbol{\lambda}'\phi^n, \qquad \boldsymbol{\lambda} = \left[\lambda_{\Delta c}, \quad \lambda_{\Delta l}, \quad \lambda_{r^W}\right]'$$

(1) Δc	(2) Δl	(3) r^W	(4) MAE(%)	(5) R^2(%)	(6) ε-Test
-0.0032	0.0049****	0.0139**	0.20	89.89	3.59
(0.0037)	(0.0013)	(0.0064)		74.72	0.31
-0.0042*	0.0040****	0.0155**	0.39	45.45	27.33
(0.0025)	(0.0010)	(0.0065)		37.66	0.20

Table: Fama-Macbeth Regression Implied Risk Premia of Labor Commitment Model. This table tabulates the FMB regression results of the labor commitment model under unconditional specification. Column (1)-(3) list the point estimations (standard deviation) of risk premia λ for the three factors (Δc, Δl, r^W); column (4) lists mean absolute pricing errors (MAE) in percentage; column (5) lists the R^2 and adjusted R^2 in percentage; column (6) lists the ε-statistics and the p-values of the intercepts test. The first row uses Fama-French 6 portfolios bivariate sorted by size and book-to-market; the second row uses Fama-French 25 portfolios. The **** denotes significance level of 0.001, *** of 0.01, ** of 0.05, and * of 0.1. The sample period is 1964:Q1 - 2013:Q4.

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Across Model: Labor Commitment Important

λ	(1) CAPM	(2) Cons.	(3) FF-3	(4) Labor
Market	.0179***		.0135**	.0139**
	(.0066)		(.0064)	(.0064)
SMB			.0096**	
			(.0042)	
HML			.0114***	
			(.0043)	
Consumption		.0079***		-.0032
		(.0028)		(.0037)
Labor				.0049****
				(.0013)
MAE(%)	0.60	0.36	0.27	0.20
R^2(%)	-10.16	56.68	78.19	89.89
	-37.70	45.86	45.48	74.72
ε-Test	40.15	14.52	25.72	3.59
	0.00	0.01	0.00	0.31

Table: Unconditional Model Comparisons of Fama-Macbeth Regression Implied Risk Premia. This table tabulates the FMB regression results of various models under unconditional specification using Fama-French 6 portfolios bivariate sorted by size and book-to-market. Column (1)-(4) list estimation of ,respectively, the CAPM, the consumption-based CAPM, the Fama-French three factors model and labor commitment model. Row one to five list the point estimates (standard deviation) of risk premia associated with model-specific factors; row six lists mean absolute pricing errors (MAE) in percentage; row seven lists the R^2 and adjusted R^2 in percentage; row eight lists the ε-statistics and the p-values of the intercepts test. The **** denotes significance level of 0.001, *** of 0.01, ** of 0.05, and * of 0.1. The sample period is 1964:Q1 - 2013:Q4.

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Across Model: Labor Commitment Important

λ	(1) CAPM	(2) Cons.	(3) FF-3	(4) Labor
Market	.0185***		.0136**	.0155**
	(.0067)		(.0064)	(.0065)
SMB			.0093**	
			(.0043)	
HML			.0131***	
			(.0044)	
Consumption		.0077***		-.0042*
		(.0027)		(.0025)
Labor				.0040****
				(.0010)
MAE(%)	0.61	0.55	0.30	0.39
R^2(%)	-33.38	-16.67	65.51	45.45
	-39.18	-21.75	60.59	37.66
ε-Test	96.05	47.76	74.78	27.33
	0.00	0.00	0.00	0.20

Table: Unconditional Model Comparisons of Fama-Macbeth Regression Implied Risk Premia. This table tabulates the FMB regression results of various models under unconditional specification using Fama-French 25 portfolios bivariate sorted by size and book-to-market. Column (1)-(4) list estimation of, respectively, the CAPM, the consumption-based CAPM, the Fama-French three factors model and labor commitment model. Row one to five list the point estimates (standard deviation) of risk premia associated with model-specific factors; row six lists mean absolute pricing errors (MAE) in percentage; row seven lists the R^2 and adjusted R^2 in percentage; row eight lists the ε-statistics and the p-values of the intercepts test. The **** denotes significance level of 0.001, *** of 0.01, ** of 0.05, and * of 0.1. The sample period is 1964:Q1 - 2013:Q4.

Conclusion

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Key Theoretical Message

- Labor commitment is useful in thinking of substitution
    - intratemporally (vs. consumption) and
    - intertemporally (vs. hours).

Key Economic Message

- Marginal Utility increases in (L/C), and
- IMRS increases in $\Delta \log(L/C)$.

Key Empirical Message

- Risk in intensive margin of labor supply is
    - significantly higher priced than risk in consumption;
    - important to macroeconomy.

Appendix: Labor Commitment (Math)

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Human Capital A (Campbell, 1996)

$$\frac{P_t + D_t}{P_{t-1}} = R_t^P \Longrightarrow \quad \frac{A_t + Y_t}{A_{t-1}} = R_t^A.$$

where t Period Income is Y_t.

But what is Period Income?

$$Y_t = H_t N_t$$

Then

$$\frac{L_t N_t + H_t N_t}{L_{t-1} N_{t-1}} = R_t^A \Longrightarrow \quad \frac{L_t + H_t}{L_{t-1}} = \frac{R_t^A}{R_t^N} = R_t^L$$

L is Labor Commitment.

Model: Labor

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Appendix: Labor Commitment (Design)

L is Labor Commitment.

$$L_t = R_t^L L_{t-1} - H_t, \quad R_t^L = R_t^A / R_t^N$$

Model Design

- R_t^A is Not Directly Observed.
- R_t^N is Not Explicitly Modeled (No Production Side).

Let

$$R_t^L \equiv R^L = (1 + \delta)$$

Model: Labor

Appendix: Equilibrium (IMRS)

FOC wrt Consumption Choice C_t

$$M_{t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left[\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)}\right]^{\frac{1}{\rho}-\frac{1}{\sigma}}\left(R^W_{t+1}\right)^{\frac{1/\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-1/\sigma}}$$

where

$$v(L/C) = [(1-\alpha)+\alpha(L/C)^{1-1/\rho}]^{\frac{1}{1-1/\rho}}$$

How to Interpret the New Element?

- Marginal Utility increases in Labor-Consumption Ratio.
- IMRS increases in Labor-Consumption Ratio Growth Rate.

3 Empirical Checkpoints

- (L/C) increases in Recessions.
- $\Delta \log(L/C)$ increases in Recessions.
- Intratemporal ES $\rho > \sigma$ and > 1.

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Appendix: Equilibrium (Intra)

FOC wrt Labor Choice L_t

$$\frac{u_{L_t}}{u_{C_t}} = N_t - (1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}],$$

where

$$\frac{u_L}{u_C} = \frac{\alpha}{1-\alpha}\left(\frac{L}{C}\right)^{-1/\rho}.$$

How to Interpret the New Element?

- One Unit of Labor in H_t: N_t
- One Unit of Labor in L_t: $(1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}]$
- Intratemporal Substitution Between L_t and $H_t(C_t)$

Denote RHS the Implicit Earnings of Labor

$$P_t := N_t - (1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}]$$

Model: Equilibrium

Appendix: Equilibrium (Kernel)

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FOC wrt Portfolio Choices A_t^n, $(n = 1, \cdots, N)$

$$\mathbb{E}[M_{t+1} R_{t+1}^n] = 1.$$

Rewrite into

$$\mathbb{E}_t[M_{t+1}(R_{t+1}^n - R_{t+1}^1)] = 0 \quad n = 2, \cdots, N$$

Model: Equilibrium

Appendix: Data

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Labor commitment

$$L_t = \sum_{s=1}^{\infty} \frac{1}{(1+\delta)^s} H_{t+s}$$

$$\hat{L}_t = \sum_{s=1}^{\bar{s}=16} \frac{1}{(1+0.3)^s} H_{t+s}$$

Why

- The median years of tenure with current employer for employed wage and salary workers is 4.2 year (BEA, 2016).
- δ is chosen such that $\forall t : H_{t+17}/(1+\delta)^{17} <$ ten minutes.

Additional

- Varying $\bar{s}$ and δ does not change behavior of L/C nor L'/L.
- Errors from $\hat{L}_{t-1} - H_t - \hat{L}_{t+1} < 2.5$ minutes.

Data

Appendix: Validation (Cointegration)

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Denote RHS the Implicit Earnings of Labor

$$P_t := N_t - (1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}]$$

Transformation of Intra-MRS

$$\frac{u_{L_t}}{u_{C_t}} = \frac{\alpha}{1-\alpha}\left(\frac{L_t}{C_t}\right)^{-1/\rho}, \quad \frac{u_{L_t}}{u_{C_t}} = P_t \quad \Rightarrow \quad \frac{u_{L_t}/u_{C_t}}{N_t} = \frac{P_t}{N_t}.$$

Suppose Spot Earnings N_t and Implicit Earnings P_t are Cointegrated,

$$(l_t - c_t) - (-\rho)n_t + \text{constant} = p_t - n_t \sim I(0).$$

- Cointegration Estimation: (Unconventional) ρ.
- Also a First-Pass Test of Model.

Validation

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Appendix: Validation (GMM)

Combine All FOCs

$$\mathbb{E}_t\left[\left(M_{t+1}(R^n_{t+1} - R^1_{t+1})\right) \otimes Z_t\right] = 0,\ n = 2, \cdots, N$$

$$\mathbb{E}_t\left[\left(\frac{u_{L_t}/u_{C_t}}{N_t} + \frac{M_{t+1}(1+\delta)N_{t+1}}{N_t} - 1\right) \otimes Z_t\right] = 0.$$

A System of NI Moments with 5 Parameters

- I instrumental variables in Z_t
- γ, ρ, σ, α, and β to estimate

Empirics

- R^1_{t+1} = Three-Month T-bill Rate
- R^n_{t+1} = Fama-French (6/25) Portfolio Returns ($n = 2, \cdots, N$)

Validation

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Stochastic Discount Factor: Linear Factor Model

Equilibrium Implied IMRS

$$M_{t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left[\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)}\right]^{\frac{1}{\rho}-\frac{1}{\sigma}}\left(R^W_{t+1}\right)^{\frac{1/\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-1/\sigma}},$$

$$m_{t+1} = \kappa\log(\beta) + b^1\Delta c_{t+1} + b^2\Delta l_{t+1} + b^3 r^W_{t+1}.$$

Stochastic Discount Factor

$$M_{t+1} = a + \boldsymbol{b}'\cdot\boldsymbol{f}_{t+1},$$

$$\boldsymbol{b} := (b^k)_{K=3} = \begin{bmatrix} b^1 \\ b^2 \\ b^3 \end{bmatrix}, \boldsymbol{f} := (f^k)_{K=3} = \begin{bmatrix} \Delta c \\ \Delta l \\ r^W \end{bmatrix},$$

Linear Factors Model

$$\mathbb{E}_t[M_{t+1}R^{en}_{t+1}] = 0, \quad (n = 2, \cdots, N)$$

$$R^{en} = R^n - R^1.$$

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Stochastic Discount Factor: Fama-MacBeth

Stochastic Discount Factor

$$M = a + \boldsymbol{b}' \cdot \boldsymbol{f}.$$

Fama-MacBeth Procedure

- Time-Series Regression

$$\mathbb{E}[R^{en}] = \alpha + \boldsymbol{\beta}^{n\prime} \cdot \boldsymbol{f}.$$

- Cross-Sectional Regression

$$\mathbb{E}[R^{en}] = \boldsymbol{\lambda}' \boldsymbol{\beta}^{n},$$

where

$$\begin{array}{rl} \boldsymbol{\lambda} & \text{factor risk premia,} \\ \boldsymbol{\beta}^{n} & \text{factor risk exposures.} \end{array}$$

- Pricing Errors

$$\varepsilon^{n} = R^{en} - \hat{\boldsymbol{\lambda}}' \hat{\boldsymbol{\beta}}^{n}$$

Stochastic Discount Factor: Fama-MacBeth

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Derivation

$$\mathbb{E}[R^n_{t+1} - R^1_{t+1}] = \boldsymbol{\lambda}' \cdot \boldsymbol{\beta}^n,$$

where $\boldsymbol{\lambda}$ is the factor premium vector,

$$\boldsymbol{\lambda} := (\lambda^k)_K = (b^k \mathbb{E}[M_{t+1}]^{-1} \mathbb{V}[f^k_{t+1}])_K,$$

and $\boldsymbol{\beta}^n$ is the factor exposure vector of arbitrary asset n,

$$\boldsymbol{\beta}^n := (\beta^{nk})_K = (\mathbb{V}[f^k_{t+1}]^{-1} \mathbb{V}[f^k_{t+1}, R^n_{t+1} - R^1_{t+1}])_K.$$

Conditional (Scaled) Factors Model

Unconditional

$$M_{t+1} = a + \boldsymbol{b}' \cdot \boldsymbol{f}_{t+1}.$$

Time-Varying Dependence (on Conditional Variable z_t)

$$\begin{aligned} M_{t+1} &= a_t + \boldsymbol{b}_t' \cdot \boldsymbol{f}_{t+1} \\ a_t &= a_0 + a_1 z_t \\ b_t^k &= b_0^k + b_1^k z_t \quad (k = 1, \cdots, K) \end{aligned}$$

Define Conditional Factors

$$\boldsymbol{F}_{t+1} = z_t \otimes \boldsymbol{f}_{t+1}$$

Conditional Linear Factors Model

$$\begin{aligned} M_{t+1} &= a + \boldsymbol{b}' \cdot \boldsymbol{F}_{t+1}. \\ \mathbb{E}[R^{en}] &= \boldsymbol{\lambda}' \boldsymbol{\beta}^n. \end{aligned}$$

Propose the New Conditional Variable

$$z_t = \Delta \log(L_t / C_t)$$

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Empirical Checkpoint: Compare Variables (1/2)

FF 25 Portfolios + 3 Factors

	(1)	(2)	(3)	(4)
Model	Unc'l	Con'l		
z_t	1	cay_t	$\Delta \log(C_t^D)$	$\Delta \log(L_t/C_t)$
MAE(%)	0.30	0.27	0.28	0.18
R^2(%)	65.51	74.96	71.03	90.31
	60.59	64.25	59.11	86.32
ε-Test	74.78	32.36	48.29	14.12
	0.00	0.00	0.00	0.72

Table: Conditional Variable Comparisons of Fama-Macbeth Regression MAE, R-Squared and Intercept-Test. This table tabulates the FMB regression results of mean absolute pricing error (MAE), R^2 and intercepts ε-Test under both unconditional and conditional specifications of Fama-French three factors model using Fama-French 25 portfolios bivariate sorted by size and book-to-market. Column (1) lists results under unconditional specification (i.e., $z_t = 1$); column (2)-(4) lists results under conditional specification, where, respectively, the conditional variables are cay_t (Lettau and Ludvigson, 2004), durable consumption growth ΔC_t^D (Yogo, 2006), and labor-consumption ratio growth $\Delta \log(L_t/C_t)$. Row one lists mean absolute pricing errors (MAE) in percentage; row two lists the R^2 and adjusted R^2 in percentage; row three lists the ε-statistics and the p-values of the intercepts test. The sample period is 1964:Q1 - 2013:Q4.

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Empirical Checkpoint: Compare Variables (2/2)

Industrial Type (30) Portfolios: MAE (Mean Absolute Pricing Errors)

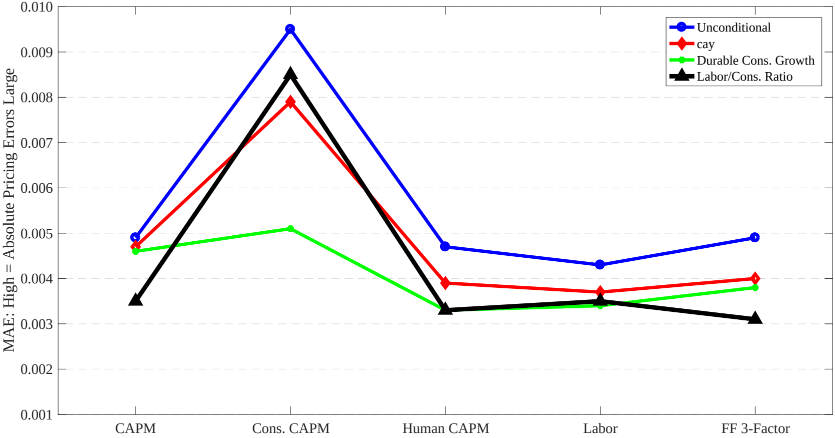

Figure: Conditional Variable Comparisons of Fama-Macbeth Regression MAE. This figure depicts mean absolute pricing errors (MAE) using 30 portfolios defined by industrial type (details on K. French website). The model specifications compared (horizontal axis) are, from left to right, the CAPM, the consumption-based CAPM, the human capital CAPM (Campbell, 1996; Jagannathan and Wang, 1996), the labor commitment model and the Fama-French 3 factors model; the conditional variables compared (plot legends) are, from top to bottom, the unconditional (i.e., $z_t = 1$), the cay_t (Lettau and Ludvigson, 2004), the durable consumption growth ΔC_t^D (Yogo, 2006), and the labor-consumption ratio growth $\Delta \log(L_t / C_t)$. The sample period is 1964:Q1 - 2013:Q4.

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Industrial Type (30) Portfolios: R-Squared Values (R^2)

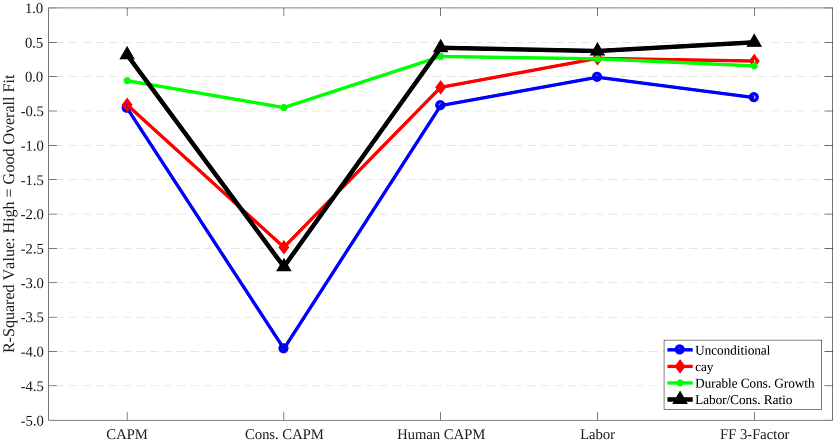

Figure: Conditional Variable Comparisons of Fama-Macbeth Regression R-Squared Values. This figure depicts R-squared values (R^2) using 30 portfolios defined by industrial type (details on K. French website). The model specifications compared (horizontal axis) are, from left to right, the CAPM, the consumption-based CAPM, the human capital CAPM (Campbell, 1996; Jagannathan and Wang, 1996), the labor commitment model and the Fama-French 3 factors model; the conditional variables compared (plot legends) are, from top to bottom, the unconditional (i.e., $z_t = 1$), the cay_t (Lettau and Ludvigson, 2004), the durable consumption growth ΔC_t^D (Yogo, 2006), and the labor-consumption ratio growth $\Delta \log(L_t / C_t)$. The sample period is 1964:Q1 - 2013:Q4.

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Empirical Checkpoint: Compare Variables (2/2)

Industrial Type (30) Portfolios: Adjusted R-Squared Values ($\bar{R}^2$)

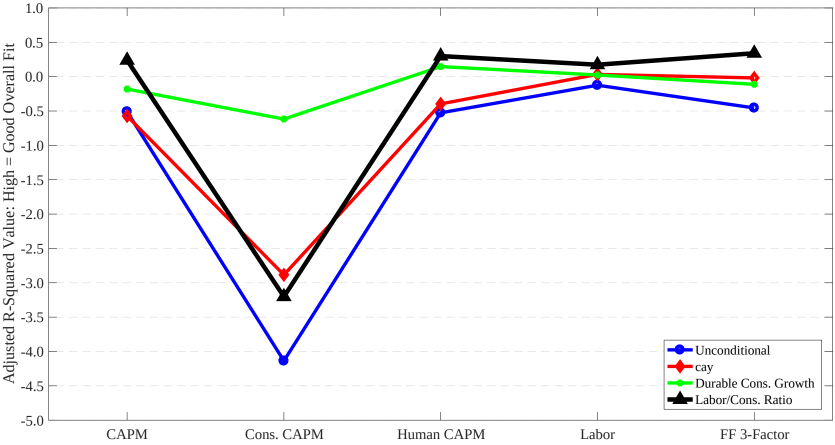

Figure: Conditional Variable Comparisons of Fama-Macbeth Regression Adjusted R-Squared Values. This figure depicts adjusted R-squared values ($\bar{R}^2$) using 30 portfolios defined by industrial type (details on K. French website). The model specifications compared (horizontal axis) are, from left to right, the CAPM, the consumption-based CAPM, the human capital CAPM (Campbell, 1996; Jagannathan and Wang, 1996), the labor commitment model and the Fama-French 3 factors model; the conditional variables compared (plot legends) are, from top to bottom, the unconditional (i.e., $z_t = 1$), the cay_t (Lettau and Ludvigson, 2004), the durable consumption growth ΔC_t^D (Yogo, 2006), and the labor-consumption ratio growth $\Delta \log(L_t / C_t)$. The sample period is 1964:Q1 - 2013:Q4.

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Industrial Type (30) Portfolios: Intercept-Test Statistics (ε-Test)

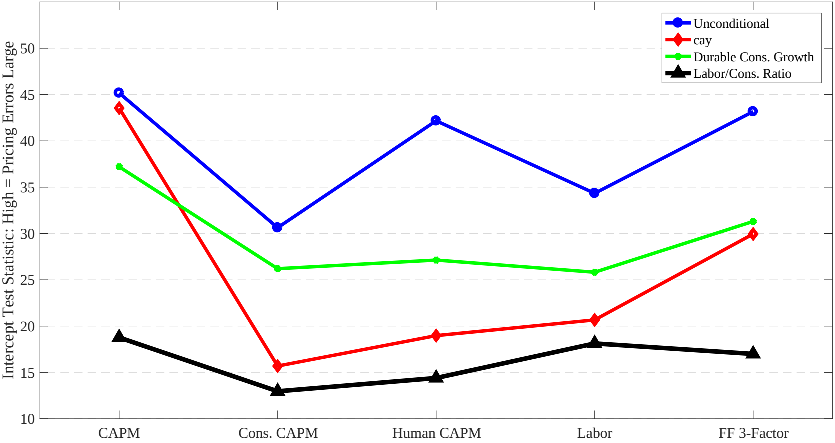

Figure: Conditional Variable Comparisons of Fama-Macbeth Regression Intercept-Test Statistics. This figure depicts intercept-test statistics (statistics of ε-test) using 30 portfolios defined by industrial type (details on K. French website). The model specifications compared (horizontal axis) are, from left to right, the CAPM, the consumption-based CAPM, the human capital CAPM (Campbell, 1996; Jagannathan and Wang, 1996), the labor commitment model and the Fama-French 3 factors model; the conditional variables compared (plot legends) are, from top to bottom, the unconditional (i.e., $z_t = 1$), the cay_t (Lettau and Ludvigson, 2004), the durable consumption growth ΔC_t^D (Yogo, 2006), and the labor-consumption ratio growth $\Delta \log(L_t/C_t)$. The sample period is 1964:Q1 - 2013:Q4.

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Industrial Type (30) Portfolios: Intercept-Test p-Value

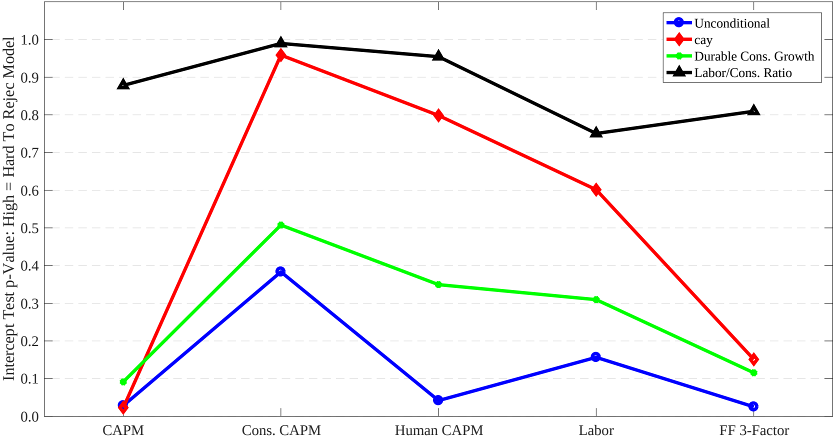

Figure: Conditional Variable Comparisons of Fama-Macbeth Regression Intercept-Test p-Values. This figure depicts intercept-test p-values using 30 portfolios defined by industrial type (details on K. French website). The model specifications compared (horizontal axis) are, from left to right, the CAPM, the consumption-based CAPM, the human capital CAPM (Campbell, 1996; Jagannathan and Wang, 1996), the labor commitment model and the Fama-French 3 factors model; the conditional variables compared (plot legends) are, from top to bottom, the unconditional (i.e., $z_t = 1$), the cay_t (Lettau and Ludvigson, 2004), the durable consumption growth ΔC_t^D (Yogo, 2006), and the labor-consumption ratio growth $\Delta \log(L_t / C_t)$. The sample period is 1964:Q1 - 2013:Q4.