Labor Commitment

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Abstract

This paper proposes a economic mechanism that matches both the procyclicality of hours and countercyclicality of risk premium. Central to the mechanism is labor commitment, a measure of representative agent's growing, income-generating stock of labor. When utility is nonseperable between consumption and labor and intratemporal elasticity of substitution is high, intertemporal marginal rate of substitution (IMRS) in equilibrium increases when labor-consumption ratio growth rate increases. I find empirical evidences supportive of predictions from IMRS, and obtain reasonable estimation of equilibrium coefficients. I further test the model in asset pricing context. Firstly, I demonstrate that the equilibrium stochastic discount factor (SDF) implied linear factor model performs fat better than various models and about as good as Fama-French three-factor model in explaining cross-sectional portfolio excess returns. Secondly, in conditional linear factor setting, I show that using labor-consumption ratio growth rate as conditional variable improves more than using cay or durable consumption growth, in all models considered.

1 Introduction

From 1964 to 2013, the average weekly hours in U.S. drops from 38.5 to 33.7, a total of 4.8 hours decrease. The seven NBER defined recessions together contribute 4.3 hours, or 89% (see Figure 1, Panel A). The most recent episode of 2007-2009 recession sees a total of 0.8 hours decrease; while this recession spans 19 months, the average weekly hours in U.S. takes 31 months to recover to its pre-recession level (see Figure 1, Panel B).

[Insert Figure 1 Here]

I in this paper propose a novel economic mechanism that embeds the decrease of hours during recessions. The less flow of hours provided in the current period, the more commitment of hours to fulfill in the following periods, where such decision of intertemporal substitution could be made voluntarily or compulsorily. Central to this mechanism is the concept of labor commitment, a measure of forward-looking representative agent's life-time hours. Therefore, the labor commitment links the intensive margin of labor market across time. I formulate the labor commitment in a law of motion, where both the hours and the labor commitment enter, and the decision of one pins downs the decision of the other.

I model the economy with a representative agent abstract from the production side. A representative agent faces three choices, the consumption, the labor commitment (or equivalently, the hours), and the portfolio. In equilibrium, the first-order conditions of the model indicate a countercyclical intertemporal marginal rate of substitution (or equivalently, a stochastic discount factor), that decreases in consumption growth and portfolio return, and increases in labor commitment growth, a desired business-cycle characteristic. The equilibrium analysis shows that, the model is able to generate an extra channel of labor commitment, through which information regarding the economic downturns is captured.

Such stochastic discount factor also implies a macro-oriented linear factor model; therefore, I test the model, the labor commitment model, in the context of asset pricing. I start with the unconditional formulation and compare the model with both market-oriented (the CAPM and the three factors model) and macro-oriented (the consumption-based CAPM and the human capital CAPM) asset pricing models. Using 6 Fama-French portfolios formed on size and book-to-market, the labor commitment model is able to give a mean absolute pricing error of 0.3% and a R^2 of 0.74. Tested using various portfolio sets, the labor commitment model in general is favored most for good overall fit (R^2) and high prediction precision (α-test).

I furthermore examine the concept of labor commitment by first constructing the procyclical labor commitment-consumption ratio, and secondly testing it in the context of conditional linear factor model. With the three factors model underlying, I compare the conditional formulation using, as conditional variable, the labor commitment-consumption ratio, the wealth-consumption proxy (cay) and the durable consumption growth. The labor commitment-consumption ratio is favored most again for improvement in overall fit (R^2) and in prediction precision (α-test).

The idea of labor commitment provides a new perspective to look at labor market intensive margin. In the canonical neoclassical framework, there is no intertemporal substitution decision regarding the labor itself for labor is defined and provided from period to period. The total hours, the empirical measure of labor, fluctuates as much as the output, where the most of fluctuation come from the extensive margin (King and Rebelo [1999]). I in this paper distinguish the the flow and the stock of labor, the hours and the labor commitment, and investigate the intertemporal substitution mechanism along intensive margin. The novel labor commitment abstraction and the proposed intertemporal substitution mechanism are conceptually desirable and empirically powerful. They together give a new answer to the question of why and how the shifts of hours across time happen. Furthermore, in a broad sense, the labor commitment also matches the basic intuition of human capital theory, yet with a simpler construction. The equilibrium approach to identify the human capital involves usually the total labor income, a bundle of the extensive effectiveness, the intensive quantity and price, while equilibrium in the labor commitment model departures from that by highlighting the intensive margin in mimic of the human capital accumulation.

Connection from labor market to asset market has always been viewed through the lenses of aggregate labor income (Campbell [1996]; Jagannathan and Wang [1996]; Santos and Veronesi [2006]). The contribution to the stand of empirical asset pricing literature are twofold. Firstly, the labor commitment model has a relative simple description of labor market, yet the implied stochastic discount factor predicts equity excess returns with good overall fit and high prediction precision. The labor commitment, a new labor-market-relevant state variable drives variation in risk premium. In other words, the extra, novel channel of labor commitment passes the information in labor market to the equilibrium intertemporal marginal rate of substitution, and hence the stochastic discount factor recovered admits an extra term regarding the labor commitment. Secondly, by examining the labor commitment model in the context of conditional factor pricing model, I also identify that the labor commitment-consumption ratio, which is the conditional variable derived from the labor commitment model, improves three factors model.

My paper relates firstly to a large body of equilibrium asset pricing works in macro-finance literature. Starting from Mehra and Prescott [1985], many researches have addressed the equity premium puzzle using equilibrium approaches. One of the most influential work is done by Epstein and Zin [1989]. The recursive utility structure marks a conceptual transformation in viewing the determinants of risks intertemporally (Merton [1973]). It also gives an elegant way of separating the relative risk aversion and intertemporal elasticity of substitution, which technique is adopted in the labor commitment model. Bansal and Yaron [2004] and Bansal et al. [2010] extend the uncertainty risk in recursive utility by proposing the consumption long-run risk model. In the long-run risk model, the future uncertainty is captured by unexpected changes in consumption, which has a stochastic shock term covariates with dividends. Also looking at the covariance, Barro [2006], taking up the idea from Rietz [1988], investigates the effect of fear for rare disaster onto the covariance, between marginal utility and equity returns. Such rare disaster fear proves to be really powerful in explaining asset pricing puzzles (Gabaix [2012]). Instead of focusing on future consumption, Campbell and Cochrane [1999] look into the history of consumption itself and propose the concept of consumption habit. The slowly adjusting external habit level and highly nonlinear utility responses together suggest a time-varying, countercyclical equity excess returns. Taking a detour from consumption, Cochrane [1991], Belo [2010], and Jermann [1998] approaches the asset pricing from equilibrium implied in the production side of the economy. In a very broad sense, these equilibrium asset pricing models and the labor commitment model all focus on the intertemporal marginal rate of substitution from a representative agent point of view, and identify extra time-variate driving forces to the equilibrium implied stochastic discount factor. The differences are my focus on the labor market intensive margin and the labor commitment variable.

The implied linear factor model places my paper into a growing stand of empirical works that focus on factors and/or conditional variables. Perhaps the most widely used linear factor model is the CAPM (Sharpe [1964]; Lintner [1965]; Black [1972]); Fama and French [1993, 1995] extends the CAPM to include two additional factors that capture the size premium and value premium in data, hence the three factors model. Following the same spirit, Carhart [1997] locate the momentum factor and Fama and French [2015] contribute by locating two more regarding the operational profitability and the investment strategy. These linear factor models are market-oriented, i.e., contain factors that origins from the market participants' characteristics. The labor commitment model on the other hand is a macro-oriented linear factor model, due to the featuring factors are regarding the aggregate macroeconomic variables. On the other dimension, due to the failure of empirical test of consumption-based CAPM (Rubinstein [1976]; Breeden and Litzenberger [1978]; Lucas [1978]), several solutions using conditional variables applied into the linear factor pricing model are proposed. Lettau and Ludvigson [2001b] cointegrate the consumption, labor income and financial wealth, and find the consumption-wealth ratio proxy of *cay* being a powerful conditional variable. Yogo

[2006] suggests the consumption on durable goods, particularly its growth, also being one. Finally, focusing also on the labor market, Santos and Veronesi [2006] find that, the share of consumption that is financed by labor income also generates predictability of equity returns. I in this paper also formulate a linear factor pricing model using the implied stochastic discount factor recovered from the equilibrium of the labor commitment model; the new factor I identify is the hence the labor commitment. With the conditional formulation, I find that the labor commitment-consumption ratio outperforms the aforementioned three popular conditional variables.

The paper proceeds as follows. Section 2 presents the model with elaboration on the labor commitment concept; this section also derives the first-order conditions that lead to empirical part of the paper. I discuss the source and construction of data in Section 3; also in there I show the procyclicality of labor commitment-consumption ratio. For the empirical part of the paper, I in Section 4 provide, of the equilibrium in labor commitment model, the economic analysis and the generalized method of moments estimation. Section 5 continues the empirical analysis in the context of linear factor asset pricing model. Finally, Section 6 concludes and proposes two plausible extensions.

2 Model

I present the model in this section. It is a representative agent economy abstract from the production side. I construct a novel concept of labor commitment, which has two realistic features. It is a representative agent's rational expectation of the long-run labor units to provide; it captures both the intratemporal and the intertemporal substitutions in a representative agent's optimization of lifetime utility.

I start this section by discussing the intuition of labor commitment, and then present the representative agent's maximization problem and its solution. I end this section with a discussion of first-order conditions from the model and their implications.

2.1 Labor Commitment

The construction of the labor measurement in my model, the labor commitment, comes from basic observation and intuition. Firstly, in canonical neoclassical macroeconomic model, the representative agent provides labor from period to period, with no intertemporal consideration regarding labor itself. However, labor supply at a macroeconomic level moves across business cycles and at a microeconomics level varies over ages/time (see, for example, Altonji [1986], Card [1991], Imai and Keane [2004]). Therefore, I find it useful in the model to distinguish between the per-period labor hours, H_t, and the long-term labor commitment, L_t. Secondly, labor commitment develops by itself; that is, the labor commitment appreciates between today and tomorrow because of the postponement of fulfillment to the labor commitment by today's labor hours. Writing a 365-page book in one year needs more hours of work than writing the same 365-page book in one month, possibly due to reopening the computer several times or due to looking for the notation for a parameter defined three months ago, even if the hours spent per page are exactly the same.

To be concrete, a representative agent in the period of t provides H_t units of labor ($h$ours), and receives N_t units of income per unit of labor (ear$n$ings). The labor commitment is denoted L_t; in particular, it follows the law of motion

$$L_t = (1+\delta)L_{t-1} - H_t, \tag{1}$$

where $\delta > 0$ is the appreciation rate of labor commitment. Implicit assumption in this specification is that, the fulfillment flow from period hours is linear in labor commitment; therefore, I refer to "labor commitment" as "labor", and to "labor flow" as "hours" when no confusion.

The law of motion for labor commitment in Eq. (1) is plausible in two ways. Firstly, it models explicitly the fact that the workload grows. Such incrementality feature is not only realistic approximation of the real-world experience, but also matches the human capital accumulation theory in the literature (see, for example, Acemoglu [1996], Wang and Yao [2003], Imai and Keane [2004]), if L_t measures the human capital, and δ the accumulation rate. Secondly, Eq. (1) suggest the intertemporal substitution within labor itself. Entering the period of t, a representative agent chooses the hours H_t of this period, which automatically pins down the labor commitment to face in the next period of $t+1$. Such mechanism is simple yet powerful. It matches the fact of decrease in hours during recessions, as is shown in Figure 2; it embeds the optimization consideration of agent via labor commitment, since the decrease of hours in this period means the increase of labor commitment in the next period (assuming constant appreciation); it simplifies the system, without modeling the production side and hence the wage schedule.

[Insert Figure 2 Here]

2.2 Choices and Preference

In this economy, a representative agent makes three choices in each period t. The first two choices are consumption and labor. C_t units of consumption goods is purchased and entirely consumed in each period; H_t unit of labor flow is provided and earn labor income. The third choice is the portfolio choice.

There are N tradeable assets in the economy, indexed by $n = 1, 2, \cdots, N$. In each period t, the agent choses A_t^n units of wealth to invested in asset n, which yields a gross return of R_{t+1}^n during period t. Denote W_t the total wealth of a representative agent at the beginning of period t, then her intratemporal budget constrain is

$$C_t + \sum_{n=1}^{N} A_t^n = W_t + H_t N_t, \tag{2}$$

and her intertemporal budget constrain is

$$W_{t+1} = \sum_{n=1}^{N} A_t^n R_{t+1}^n. \tag{3}$$

The intratemporal utility index is assumed taking constant elasticity of substitution (CES) function form

$$u(C_t, L_t) = \left[(1-\alpha){C_t}^{1-1/\rho} + \alpha {L_t}^{1-1/\rho} \right]^{\frac{1}{1-1/\rho}}, \tag{4}$$

where parameter α controls the relative "importance" between utility from consumption and from labor commitment. The parameter ρ is the elasticity of substitution (ES) between intratemporal consumption and labor. Special case of $\rho = 1$ degenerates the utility index to Cobb-Douglas function form. It worth noticing that, while non-homothetic utility index forms, King et al. [1988] (KPR) and Greenwood et al. [1988] (GHH) preferences, are of great interest in such scenario, the implementation would pose difficulties in getting closed-form solution to the model.

The intertemporal utility function of a representative agent is in Epstein and Zin [1989, 1991] recursive form; that is,

$$U_t = \left[(1-\beta) u(C_t, L_t)^{1-1/\sigma} + \beta \left(\mathbb{E}_t [U_{t+1}]^{1-\gamma} \right)^{\frac{1-1/\sigma}{1-\gamma}} \right]^{\frac{1}{1-1/\sigma}}, \tag{5}$$

where the parameters β is the subjective discount rate, σ the intertemporal elasticity of substitution (IES), and γ the relative risk aversion (RRA).

As a result, the representative agent problem is as follows: she makes consumption, labor and portfolio choices, $\{C_t; H_t; A_t^1, \cdots, A_t^N\}$, to maximize Eq. (5) subject to Eq. (1), (2), (3), and (4).

2.3 First-Order Conditions

As is shown in Appendix B.1, the recursive structure of preference, the homotheticity of function forms, and the linearity of budget constrains together imply a closed-form solution. For convenience, denote R^W_{t+1} the gross return on the total wealth. The intertemporal marginal rate of substitution (IMRS) (or, equivalently, the stochastic discount factor (SDF), which would be more asset pricing relevant, and hence is interchangeably used in Section 5) is

$$M_{t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \left(\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)} \right)^{\frac{1}{\rho}-\frac{1}{\sigma}} \left(R^W_{t+1} \right)^{\frac{1/\sigma-\gamma}{1-\gamma}} \right]^{\frac{1-\gamma}{1-1/\sigma}} \tag{6}$$

where

$$v(L/C) = [(1-\alpha) + \alpha(L/C)^{1-1/\rho}]^{\frac{1}{1-1/\rho}}, \tag{7}$$

and $u(C, L) = C \cdot v(L/C)$. Before deriving the first-order conditions of the optimization problem, it is necessary to examine the stochastic discount factor, particularly the non-standard terms of L/C. Towards this end, assume the relative risk aversion and intertemporal elasticity of substitution $\gamma, \sigma > 1$ (Hansen and Singleton [1983], Colacito and Croce [2011, 2013]), so $\frac{1-\gamma}{1-1/\sigma} < 0$. Discuss only scenarios where $1/\rho \neq 1/\sigma$. One can readily seen that, the effect of the growth rate of L/C, i.e., $\Delta L/C$, onto M depends on the relative magnitudes of ρ and σ. If $\rho > \sigma$, the representative agent is more elastic to substitute intratemporally than intertemporally; thus $1/\rho - 1/\sigma < 0$, and Eq.(6) increases in the growth rate of L/C. Therefore, if L/C experiences higher growth during recessions, i.e., $\Delta L/C > 0$, the intertemporal marginal rate of substitution M shall increase correspondingly, resulting higher equity premium and lower risk-free rate. Similarly, Eq.(6) decreases in the growth rate of C, and hence, if C experiences lower growth during the recessions, the intertemporal marginal rate of substitution M shall increase as well. The key takeaway from the discussion of Eq.(6) therefore is the countercyclicality of intertemporal marginal rate of substitution, due to the countercyclicality of labor commitment-consumption ratio, as well as the procyclicality of consumption growth and wealth returns.

I summarize the Euler equations implied by the first-order conditions into two groups. The first group are kernel pricing formula applying to arbitrary asset n in this economy,

$$\mathbb{E}_t[M_{t+1} R^n_{t+1}] = 1, \tag{8}$$

where $n = 1, 2, \cdots, N$. The second are cross-sectional restrictions regarding excess return of asset n over the first asset

$$\mathbb{E}_t[M_{t+1}(R^n_{t+1} - R^1_{t+1})] = 0, \tag{9}$$

where $n = 2, 3, \cdots, N$. Implicitly from the specifications of Eq. (8) and (9), the first asset can be arbitrary regarded as the risk-free asset (treasury bills), while the remaining as the equities (or equity portfolios); the empirical sections (Section 4 and 5) in the later part follows such instinct.

Moreover, let u_c and u_L be the marginal utility from intratemporal utility index u with respect to C and L, respectively. The marginal rate of substitution between intratemporal consumption and labor is

$$\frac{u_L}{u_C} = \frac{\alpha}{1-\alpha}\left(\frac{L}{C}\right)^{-1/\rho}. \tag{10}$$

The FOC with respect to labor further requires

$$\frac{u_{L_t}}{u_{C_t}} = N_t - (1+\delta)\mathbb{E}_t[M_{t+1}N_{t+1}]. \tag{11}$$

The right-hand-side of Eq. (11) has two elements: the first is the benefit from providing 1 unit of labor in current period t, and the second is the benefit from providing $(1+\delta)$ units in the following period $t+1$. Consequently, the right-hand-side is the marginal benefit of providing unit of labor in current period t. Looking at the left-hand-side, it is the marginal utility from suppressing such unit of labor, or marginal cost of providing unit of labor, in period t, denominated by the marginal utility of consumption. As a result, Eq. (11) reveals the standard optimality condition that, the marginal cost of fulfillment to the labor commitment must equal to marginal benefit of such fulfillment, both measured in the unit of utility at period of time t.

To summarize these first-order conditions and end this section, Eq. (6) and (7) solidify the ground for macro-oriented asset pricing part of this paper (Section 5). On the other hand, from a macroeconomic point of view, Eq. (8), (9), (10) and (11) together give testable moment conditions to estimate equilibrium of the aforedefined economy (Section 4). To avoid possible confusion, I refer such economy (model) as labor commitment economy (model).

3 Data

I in this section discuss the data used, including its source and construction. The novel concept, the labor commitment-consumption ratio is also elaborated.

3.1 Source and Construction

While a more detailed description of data is attached in Appendix A, I place here the summary of two types, economic and financial, data.

Economic Data Consumption data of total personal consumption expenditure and of personal consumption expenditure on nondurable goods and services are from the National Income and Product Accounts (NIPA) tables of the U.S. Bureau of Economic Analysis. The flow data of per period hours and per hour earnings are measured using the average weekly hours and the average hourly earnings, respectively, of production and nonsupervisory (PNS) employees from the U.S. Bureau of Labor Statistics. Following Eq. (1), the stock data of labor commitment is constructed. To ensure the comparability to the literature, I use consumer price index (CPI) for all urban consumers from U.S. Bureau of Labor Statistics, and total population including all ages from U.S. Bureau of the Census. Throughout all series, the frequency of one quarter is used.

Financial Data The financial data are from various sources and of two types. The first type is market data. The Fama-French three factors data is from the data library (French [2017]), as well as some other market-oriented factors, namely the investment strategy, the operation profitability, the momentum, and the long-term and short-term reversal factors. The market return is approximated by the Center for Research in Security Prices (CRSP) value-weighted index return including NYSE, AMEX and NASDAQ, and risk-free rate is by the one-month treasury bill rate. I also collect and construct dividend-price ratio, earnings-price ratio and the yield spread.

For the portfolio type financial data, I use both univariate and bivariate portfolios formed on size and/or book-to-market; besides, based on industrial type, I also collect industrial portfolios. For comparison with existing literature, I use (i) dividend-price ratio (Campbell and Shiller [1988]), (ii) consumption-wealth ratio proxy (cay, Lettau and Ludvigson [2001b]), (iii) labor income share (Santos and Veronesi [2005]), and (iv) durable consumption growth (Yogo [2006]). The resulting sample period for the whole dataset is from 1964:Q1 to 2013:Q4.

3.2 Labor Commitment-Consumption Ratio

The key ingredient of the aforedefined labor commitment economy is L/C. To study how the labor commitment-consumption ratio affects the equilibrium quantities and prices, this section focuses firstly on its business-cycle characteristic. Figure 3 depicts the labor commitment-consumption ratio in the Panel (A), and its growth rate over the whole sample in the Panel (B).

[Insert Figure 3 Here]

From Panel (A), the labor commitment-consumption ratio L/C has a downward sloping trend, due to the growth of consumption and the boundedness of labor commitment; in particular, it falls during booms, and rises in recessions, as indicated by shaded regions. I plot the labor commitment-consumption ratio using both total personal consumption expenditure and nondurable consumption & service measured consumption; the lines suggest two series have no obvious departures. More interestingly, as is shown in Panel (B), the growth rate of labor commitment-consumption ratio, $\Delta \log L/C$, spikes in recessions, indicated by elevating over the sample 90-percentile reference line. It suggest that, during severe cases of recessions, L/C reverses its trend. From both panels, the year of 1986 and of 1997 give false recession pattern and the early 2000s recession does not reveal its appearance until the year of 2002.

Clearly, the labor commitment-consumption ratio does not produce a one-to-one mapping to the NBER designated recessions. However, I emphasize that perhaps it should not. The overall condition of the economy, including equity market, shall not be characterized only by recessions. Without a one-to-one mapping, the labor commitment-consumption ratio tells more. Section 4.1 further discusses the extra information extracted from labor commitment.

The business-cycle characteristics of labor commitment-consumption ratio, mainly the procyclicality is the key findings to start the empirical analysis. Such key empirical observation combined with Eq. (6) and (7) suggests that, the procyclicality of labor commitment-consumption ratio ensures the countercyclicality of intertemporal marginal rate of substitution, and hence the higher equity premium and lower risk-free rate during recessions.

Descriptive Summary In Table 1, I tabulate the standard deviation-to-mean ratios and first-order auto-correlation coefficients of some key variables, as well as their correlation coefficients with labor commitment-consumption ratio, L/C.

[Insert Table 1 Here]

From the labor market, the labor commitment (L) and the earnings (N) are both much more volatile than consumption (C), measured as both total of and nondurable goods and services of personal consumption expenditure (column (1)). This is intuitive, since consumption shall be much smoother, than the (intensive margin of) labor market. The labor commitment is highly, negatively correlated, and the consumption is highly, positively correlated with labor commitment-consumption ratio (L/C) by definition (column (3)). The earnings are only moderately, positively correlated, suggesting a combination of both wealth and substitution effects. Overall, all the macroeconomic variables are highly persistent (column (2)).

The Fama-French three factors, namely, excess returns on the market portfolio, returns on the SMB (Small Minus Big) portfolio, and returns on the HML (High Minus Low) portfolio, have rather small correlation with labor commitment-consumption ratio. Finally, the dividend-price ratio, earnings-price ratio and the yield spread, all of which are documented to predict equity returns, have relative moderate correlation with labor commitment-consumption ratio.

4 Economic Equilibrium

I provide analysis of economic equilibrium in the labor commitment model in this section. I start with a discussion of the extra channel of the labor commitment in providing information regarding the overall economic environment. I illustrate my point in a counting exercise. I then estimate the labor commitment model using generalized method of moments. I begin the estimation procedure with some intuition from the intertemporal marginal rate of substitution; I then find the estimation results in some parameters are relatively well while the those in others are not.

4.1 Decomposition

A natural question with respect to the business-cycle characteristic of the labor commitment-consumption ratio is that, whether it is completely driven by consumption alone. A quick decomposition of (the logarithm of) labor commitment-consumption ratio growth rate goes as follows. Denote the lower cases the logarithm of the upper cases,

$$\Delta \log L/C \equiv \Delta l - \Delta c. \tag{12}$$

Figure 4 depicts the two elements on the right-hand-side of Eq. (12). In Panel (A) is the growth rate of labor commitment Δl, and its 90 and 10-percentile reference line; in Panel (B) is the growth rate of consumption Δc (measured as total of personal consumption expenditure), and its 90 and 10-percentile reference line. The first observation is that, vertical axises are of different scales; that is, the labor commitment throughout the sample is more volatile than consumption (Table 1, Column (1)). Graphically, the growth rate of labor commitment in the upper panel is too active, in the sense that, the surges over 90-percentile reference line are less concentrated on episodes of NBER recessions. On the other hand, anticipated to behave better, the growth rate of consumption in the lower panel is too inactive, in the sense that, the drops under 10-percentile reference line are too concentrated on certain episodes, especially the recent one, of NBER recessions. To answer the question more carefully, I conduct a counting exercise in the following.

A Counting Exercise I shall mention that, before the counting exercise, the recessions by definition cannot fully describe the overall economic environment; they are rather, at best, an extremely simplified indicator of the economic downturns. The mere purpose of this counting exercise is to demonstrate, how extra information can be extracted from the novel channel, the labor commitment, aside from the consumption.

Eq. (6) and (7) and the accompany discussion suggest a reduced form of intertemporal marginal rate of substitution as follow,

$$M_{t+1} = M(-C_{t+1}/C_t, +L_{t+1}/L_t, -R^W_{t+1}), \tag{13}$$

the exact function form of which is fully derived in Eq. (17). I add the minus sign $(-)$ before C_{t+1}/C_t, R^W_{t+1} and plus sign $(+)$ before L_{t+1}/L_t to emphasize that, the intertemporal marginal rate of substitution is anticipated decreasing in consumption growth and increasing in labor commitment growth (i.e., in this function form, intertemporal marginal rate of substitution increases in all its arguments).

Construct three unconditional binary variables; in particular,

$$\begin{aligned} I^{\text{rec}} &= \mathbf{1}_{\text{recession}}, \\ I^{\Delta c} &= \mathbf{1}_{\Delta c \leq \mathbb{P}(\Delta c, 10)}, \\ I^{\Delta l} &= \mathbf{1}_{\Delta l \geq \mathbb{P}(\Delta l, 90)}, \end{aligned}$$

where $\mathbf{1}$ is the indicator function and $\mathbb{P}$ is the percentile function. Since I focus on the extra information provided by labor commitment besides consumption, the indicator for total wealth return R^W_{t+1} is not constructed nor to be discussed. From the three indicators, I calculate four conditional probabilities; specifically,

$$\begin{aligned} \Pr(I^{\Delta c} \mid I^{\text{rec}}) &= 0.41, \\ \Pr(I^{\Delta l} \mid I^{\text{rec}}) &= 0.12, \\ \Pr(I^{\Delta l \vee \Delta c} \mid I^{\text{rec}}) &= 0.47, \\ \Pr(I^{\Delta l \wedge \Delta c} \mid \neg I^{\text{rec}}) &= 0.01. \end{aligned}$$

The magnitude of improvement from adding labor commitment shall be appreciated. Firstly, all the variables involved in the calculation are simple binary ones, i.e., a sequence of zeros and ones. This binary comparison fails in capturing, for example, $\mathbb{P}(\Delta l, 88) < \Delta_t l < \mathbb{P}(\Delta l, 90)$, whereas such scenario contains obviously a lot of information about the economic downturn but is not counted in the indicators. Secondly, as is depicted in Figure 5, the recent episode spans seven quarters in the sample, and the indicators capture five $(5/7 = 0.71)$ of it; even the message is quite clear, the conditional probability is only moderate $(5/7 = 0.71)$.

[Insert Figure 5 Here]

Figure 5 shows the overall precision of such simply indicators in capturing recessions. The blue bars plot $(I^{\Delta c} \vee I^{\Delta l})$ given recessions; that is, the two indicators give the "true message" about the economic downturns. The red bars plot $(I^{\Delta c} \wedge I^{\Delta l})$ given non-recessions; that is, the two indicators give the "wrong message". The indicators captures overall with high precision; they mis-captures only the early 2000s recession and mis-signals only in the narrow window between early 80s recessions.

4.2 Generalized Method of Moments

I in this section estimate the equilibrium implied parameters using generalized method of moments. To start, I rewrite Eq. (8), (9), (10) and (11) into moment conditions. In here and what follows, I use the boldface letter to denote a column vector.

Suppose $\mathbf{Z}_t$ is a vector of size I, consist of I instrumental variables at time t. The resulting moment conditions are with regard to (i) the risk-free rate, (ii) the asset portfolios excess returns (equity premia), and, (iii) the labor; specifically,

$$\begin{aligned} \mathbb{E}_t\left[\left(M_{t+1}(R^n_{t+1}-1)\right)\otimes\mathbf{Z}_t\right] &= 0,\ n=1 \\ \mathbb{E}_t\left[\left(M_{t+1}(R^n_{t+1}-R^1_{t+1})\right)\otimes\mathbf{Z}_t\right] &= 0,\ n=2,\cdots,N \\ \mathbb{E}_t\left[\left(\frac{u_{L_t}/u_{C_t}}{N_t}+\frac{M_{t+1}(1+\delta)N_{t+1}}{N_t}-1\right)\otimes\mathbf{Z}_t\right] &= 0. \end{aligned} \tag{14}$$

The system represented by Eq. (14) has a total of $(N+1)I$ moment conditions and 5 parameters (γ, ρ, σ, α, and β). Therefore, there are $(N+1)I-5$ overidentification restrictions of the labor commitment model to be test through the J-test (Hansen [1982]). For reasons to be clear later, define two vectors: the portfolios vector

$$\mathbf{R}_{t+1} = (R^1_{t+1}; R^2_{t+1}, \cdots, R^N_{t+1}), \tag{15}$$

and the instrumental vector

$$\mathbf{Z}_{t+1} = (1; Z^2_{t+1}, \cdots, Z^I_{t+1}). \tag{16}$$

Parameters Before estimating the parameters, I provide some intuition on magnitudes of and relations across parameters. The key parameters in labor commitment model are (i) the relative risk aversion γ, (ii) the elasticity of substitution between intratemporal consumption and labor commitment ρ, and (iii) the intertemporal elasticity of substitution σ.

From Eq. (6) and (7), particular from Eq. (13), the intertemporal marginal rate of substitution is countercyclical; it decreases in consumption growth and in wealth return, and increase in labor growth. Therefore, in face with a economic downturn, where agents reduce consumption, pushes up labor commitment through voluntary or compulsory hours cut, and/or suffer from total wealth shrinkage, the intertemporal marginal rate of substitution elevates. Therefore, the countercyclicality of intertemporal marginal rate of substitution puts intuition on γ, ρ, and σ. The combined power of consumption growth being negative suggests $(1-\gamma)/(1-1/\sigma) < 0$, from which a economic sensible answer would be $\gamma > 1$ and $\sigma > 1$. This intuition also ensures the negative first-order derivative with respect to total wealth return. The critical observation comes from the labor commitment part. The intertemporal marginal rate of substitution increases in labor commitment requires $1/\rho - 1/\sigma > 0$; that is $\sigma < \rho$, i.e., the intertemporal elasticity of substitution is smaller than intratemporal elasticity of substitution. Such intuition means that the agents, with labor commitment, is more willing to substitutes intratemporally between consumption and labor commitment, than intertemporally.

The economic interpretation of these parameters are rather standard and hence omitted. In the following sections, I focus on cross-sectional and time-series estimation of labor commitment model using unconditional and conditional versions of moment conditions in Eq. (14), respectively. Details of specifications are in Appendix C.1.

Estimation The design of estimation is as follows. In cross-sectional estimation, I use unconditional moments, i.e., instrumental variable of constant, and set the portfolios number large. By design, I am able to estimation the labor commitment model across a large panel of portfolios. In time-series estimation, I use conditional moments, i.e., instrumental variables of lagged dividend-price ratio and earnings-price ratio. I set the portfolios number limited so as to check the consistency along time horizon. Estimation results are in Table 2.

[Insert Table 2 Here]

In cross-section estimation, I use (i) 10 portfolios of industrial type, (ii) 30 portfolios of industrial type, a finer version of (i), (iii) 10 portfolios formed on size, and (iv) 10 portfolios formed on book-to-market. The panel (A) column (3) to (6) shows the estimation results of them, respectively, and the column (7) shows the estimation results of pooling all portfolios. I use the instrumental variable of only a constant ($I = 1$), and the portfolios of $N = 70$. In panel (A), the relative "importance" between consumption and labor commitment is controlled by α and the estimation of it is around 0.8. This number suggests that the labor commitment affects relatively more. Intuition of the magnitude and role of α in explaining portfolios returns become more transparent in Section 5, where the context of linear factor pricing model formulation is discussed.

The estimation of the relative risk aversion γ is around 1.5, well within the range in line with the literature (Chetty [2003], for example). Such estimate also indicates the capability to solve the equity premium puzzle (Mehra and Prescott [1985]) with the labor commitment model framework. The estimation of the elasticity of substitution between intratemporal consumption and labor commitment ρ is around 2.2, and the estimation of the intertemporal elasticity of substitution σ is around 1.1. Both estimates are broadly consistent with the previous studies. Such result shall not be surprising, as the utility function already separates the relative risk aversion and intertemporal elasticity substitution Epstein and Zin [1989, 1991]. A final note is that the estimated magnitude of both elasticities are consistent with intuition mentioned earlier, $1 < \sigma < \rho$.

The estimate of the subjective discount factor β is around 7 with relative large variation using different portfolios set. The unrealistic estimate of the subjective discount factor marks the inability to explain the risk-free rate puzzle (Weil [1989]) with the labor commitment model framework. Table 1 shows that the standard deviation-to-mean ratio of labor commitment is higher than both that of consumption and that of market return (which is the proxy of total wealth return). To match the volatility of risk-free rate, a small effective discount factor has to be employed $\beta^{\frac{1-\gamma}{1-1/\sigma}}$, pushing up the estimate of the underlying subjective discount factor.

In time-series estimation, I use (i) 6 Fama-French portfolios, and (ii) 25 Fama-French portfolios, formed on size and book-to-market. The column (1) and (2) in Panel (B) shows the estimation results of them, respectively, and the column (7) shows the estimation results of pooling both portfolios. I use the instrumental variables of one-period lagged dividend-price ratio and earnings-price ratio ($I = 3$), and the portfolios of $N = 31$. The estimate of the relative "importance" between consumption and labor commitment α remains stable at around 0.8 across two panels. The estimate of the subjective discount factor β is around 1.3, an noticeable improvement from the unconditional moments. The estimates of the three key parameters fall short in matching the previous studies in literature. The estimate of the parameters across two panels points to a more transparent way in identifying the magnitudes of these parameters. In the following section, I answer to such demand under the linear factor pricing model framework.

5 Asset Pricing

In this section, I turn to the asset pricing implications of the labor commitment model. The key concept here is the (linear) factor asset pricing methodology, which links fundamental macroeconomic (or financial) factors to asset prices. Such methodology has the advantage of transparency, highlighting the central finding of business-cycle characteristic of the labor commitment. In addition, a large body of empirical works has contributed this line of literature in many directions, providing a strong background to examine performance of my model.

It makes things clearer for me to mention now that, I follow the classical tour by firstly linear-approximating the stochastic discount factor to get the linear factor representation $m_{t+1} = a + \boldsymbol{b}'\boldsymbol{f}_{t+1}$, and secondly establishing the multiple-beta representation of linear factor pricing model $\mathbb{E}[R_{t+1} - R^f_{t+1}] = \boldsymbol{\lambda}'\boldsymbol{\beta}$ to give estimated expected returns on an arbitrary asset. In here and what follows, I use the boldface letter to denote a column vector.

The performance of a specification of any model is discussed in two dimensions of interest. The first one is the model's overall fit, measured by R^2; the second one is the model's prediction precision, measured by intercepts joint test, i.e., α-test, by a bit abuse of notation.

5.1 Linear Factor Pricing Model

From the household's optimization, perhaps the most important insight is the intertemporal marginal rate of substitution in Eq. (6). Taking logarithm of both sides, the stochastic discount factor is then approximately

$$-m_{t+1} \approx -\kappa \log(\beta) + b^1 \Delta c_{t+1} + b^2 \Delta l_{t+1} + b^3 r^W_{t+1}, \tag{17}$$

where

$$\begin{aligned} b^1 &= \kappa[\alpha/\rho + (1-\alpha)/\sigma] \\ b^2 &= \alpha\kappa(1/\sigma - 1/\rho) \\ b^3 &= 1-\kappa \end{aligned}$$

and $\kappa = (1-\gamma)/(1-1/\sigma)$. Therefore, applying logarithm linear approximation to the stochastic discount factor gives

$$-M_{t+1} = a + \boldsymbol{b}' \cdot \boldsymbol{f}_{t+1}, \tag{18}$$

where $\boldsymbol{b}$ is the factor coefficient vector as defined above

$$\boldsymbol{b} := (b^k)_K = (b^1, b^2, b^3),$$

$\boldsymbol{f}$ is the fundamental factor vector including $K = 3$ factors

$$\boldsymbol{f} := (f^k)_K = (\Delta c, \Delta l, r^W),$$

and $a = -(1 + \kappa \log(\beta))$.

Before heading further to derive the actual multiple-beta equation of linear factor pricing model, I first dig a little deeper into Eq. (17) and (18), the linear factor representation of stochastic discount factor. It features three explanatory factors, namely, the growth rate of consumption, the growth rate of labor commitment, and the return on total wealth, which is usually empirically approximated by the return on market portfolio. Therefore, the labor commitment model, by this linear factor representation, nests the CAPM and consumption-based CAPM; furthermore, it also captures some portion of labor income variation at the intensive margin in human capital CAPM.

More generally, from first order conditions of Eq. (8) and (9), it can be shown that, in a more empirical-relevant presentation, Eq. (18) suggests the following linear factor pricing model

$$\mathbb{E}[R^n_{t+1} - R^1_{t+1}] = \boldsymbol{\lambda}' \cdot \boldsymbol{\beta}^n, \tag{19}$$

where $\boldsymbol{\lambda}$ is the factor premium vector,

$$\boldsymbol{\lambda} := (\lambda^k)_K = (b^k \mathbb{E}[M_{t+1}]^{-1} \mathbb{V}[f^k_{t+1}])_K,$$

and $\boldsymbol{\beta}^n$ is the factor exposure vector of arbitrary asset n,

$$\boldsymbol{\beta}^n := (\beta^{nk})_K = (\mathbb{V}[f^k_{t+1}]^{-1} \mathbb{V}[f^k_{t+1}, R^n_{t+1} - R^1_{t+1}])_K.$$

Details of derivation and approximation are in Appendix B.2. Note that the factor exposure vector $\boldsymbol{\beta}^n$ consists of the multiple regression coefficients of R^n_{t+1} on $\{f^k\}_K$ with a constant, i.e., asset-dependent. More importantly, such derivation above establishes that, any fundamental factor $\boldsymbol{f}$ presentation of stochastic discount factor M_{t+1} in Eq. (18), is equivalent to a beta $\boldsymbol{\beta^n}$ pricing presentation of linear factor model $\mathbb{E}[R^n_{t+1}]$ in Eq. (19). Additionally, this equivalence suggests the regression methodology of Fama-MacBeth procedure (Fama and MacBeth [1973]), the implementation details of which is in Appendix C.2.

5.2 Unconditional Formulation

In Eq. (19), I make no assumption of the information set, i.e., the expectation $\mathbb{E}[\cdot]$ is taken as the mathematical expectation operator. Under this circumstance, the linear factor pricing model of Eq. (19) is indeed a unconditional formulation.

I present in this section the estimation and analysis of factor premia ($\boldsymbol{\lambda}$) from labor commitment model, as well as two other macro-oriented models (consumption-based and human capital CAPM) and two market-oriented asset pricing models (CAPM and Fama-French three factors model). I conclude that, based on two criteria, the overall model fit (R^2) and the prediction precision (α-test), the labor commitment model is favored.

Estimation of Bivariate Portfolios In this part, I use the Fama-French portfolios bivariately sorted by size and book-to-market values. Table 3 tabulates the estimation results. In Panel (A) 6 portfolios (2×3) are used while in Panel (B) 25 (5×5) are. I examine the factor premium $\boldsymbol{\lambda}$ from specifications of the CAPM, the consumption-based CAPM, the human capital CAPM, the labor commitment model and the Fama-French three factors model from column (1) to (5), respectively; the standard deviations corrected for Shanken's critique (Shanken [1992]) are reported in parentheses below every point estimate.. I also report results overall fit (R^2) and prediction precision (α-Test).

[Insert Table 3 Here]

In Panel (A), the CAPM (column (1)) has a positive, significant factor premium estimation on the market return. The literature has mixed ways of defining and reporting R^2 based on econometric specification; in here, I define R^2 one minus the ratio of mean squared pricing errors to the variance of average portfolios returns (Campbell and Vuolteenaho [2004] and Yogo [2006]). The mean absolute pricing error from CAPM is 0.0077 and the R^2 from CAPM is -0.40. Such results imply the model has less explanatory power than simply predicting constant realized average returns of portfolios, a documented failure of CAPM in cross-sectional data (e.g., Basu [1977], Banz [1981], Shanken [1985a,b], and Fama and French [1992]).

The other market-oriented asset pricing model, the Fama-French three factors model (column (5)) sees a large improvement in terms of R^2 of 0.86. Given the estimation that, both factor premia of SMB and HML factors are positive and highly significant, such improvement shall be viewed as (partial) elimination of omitted variable bias. However, one major drawback of such market-oriented asset pricing models comes from failure of α-test; both CAPM and Fama-French three factors model fail in the α-test, a test of overall model fit by looking at the joint magnitude of intercepts from cross-sectional regression. CAPM specification has a α-test statistics of $33.3 (> 11.1)$ and Fama-French three factors model of $19.3 (> 7.8)$, both of which strongly reject the α-test.

On the other hand, both consumption-based CAPM (column (2)) and human capital CAPM (column (3)) both pass the α-test, which I view as an empirically improvement of conceptually switching from market-oriented to macro-oriented asset pricing model. The consumption-based CAPM has positive, highly significant coefficient estimation. The massive achievements of consumption-based CAPM and of its many second-generation derivatives is revealed in here as well; that is, the precision and power from one single factor. The consumption-based CAPM has mean absolute pricing error of 0.0033 and R^2 of 0.69. Even though both of the coefficients estimation are highly significant from human capital CAPM, its specification falls short compared to consumption-based CAPM in terms of mean absolute pricing error and R^2, of 0.0045 and 0.43, respectively.

The labor commitment model (column (4)) has a mean absolute pricing error of 0.0030, only marginal higher than that of Fama-French three factors model; the R^2 from labor commitment model is 0.74, highest in macro-oriented asset pricing models and again slightly less than that of Fama-French three factors model. It also passes the α-test like the other two macro-oriented asset pricing models. The coefficients estimation are unanimously significant at conventional level.

The estimation results in Panel (B) generally is consistent with results in Panel (A). However, I bring to attention two observations of particular interest. Firstly, both market-oriented asset pricing models, the CAPM and the Fama-French three factors model, fail the α-test consistently. Secondly, labor commitment model is the only macro-oriented asset pricing model that (i) gives positive R^2, and (ii) passes α-test. Figure 6 provides a visual summary of its outperformance over other macro-oriented asset pricing models according to Panel (B). The realized average returns are on the vertical y-axis. On the horizontal x-axis is the fitted expected returns from predictions of models, which are the CAPM, the consumption-based CAPM, the human capital CAPM and the labor commitment model in panel (A) to (D), respectively.

[Insert Figure 6 Here]

The points in any of the panel are the 25 Fama-French portfolios; thus the horizontal distance from any point to the 45-degree ray represents the pricing error of that corresponding portfolio. From the CAPM (panel (A)) to the consumption-based CAPM (panel (B)), the improvements are not only the overall movement towards the diagonal line, but also the enhancement of precision over the small-value, big-growth and big-value portfolios. However, the point of small-growth portfolio stays almost the same position. The human capital CAPM (panel (C)) prices the small-growth portfolio fairly well, while not so well the small-value portfolio. The pricing errors from the labor commitment model (panel (D)) are much smaller compared to the consumption-based CAPM and the human capital CAPM, as the points are less horizontally spread-out.

Analysis and Intuition The Fama-French bivariate portfolios are extensively tested in applied macroeconomic and finance literature. Nature of such portfolios is market characteristic, e.g., size, book-to-market ratio, operating profitability and/or investment strategy. As a result, market-oriented asset pricing models and their further generations (Carhart [1997],Fama and French [2012, 2015]) are expected to give high prediction precision (small mean squared error and passing α-test) and good overall fit (large R^2 value). Estimation results in Table 3 confirm such expectation.

Nevertheless, the CAPM and the Fama-French three factors model fails in α-test consistently. In Table 4, the Panel (A) tabulates the intercepts (i.e., α) estimation and standard deviation from the second stage of Fama-MacBeth procedure (i.e., cross-sectional regression). The Panel (B) tabulates the deviations in absolute t-statistics from the CAPM, i.e., the absolute value of t-statistics from the CAPM minus that from the corresponding models. The Panel (C) tabulates the deviations in absolute t-statistics from the Fama-French three factors model. The design of absolute t-statistics differences is to show how the market-oriented asset pricing models fail in prediction precision. The Panel (B) and (C) give a clear message, that the intercepts estimation from market-oriented asset pricing models are generally more significantly different from zero, than those from macro-oriented asset pricing models, even though the mean absolute pricing errors might be relatively smaller as the case of Fama-French three factors model.

[Insert Table 4 Here]

While the human capital CAPM might be of an exception if compared to Fama-French three factors model in Panel (C), the consumption-based CAPM and, most importantly, the labor commitment model provide supportive evidence of inadequacy in market-oriented asset pricing models; furthermore, failures in α-test pose demands in both theoretical and empirical sides, as is shown in the latter part of this paper (Section 5.3).

Estimation of Univariate Portfolios I present and discuss the estimation result using Fama-French univariate portfolios. I shall mention that due to the limit size of panel (portfolios number), the point estimate would be heavily affected by the outliers. Table 5 tabulates the estimation results of factor premia ($\boldsymbol{\lambda}$) using 5 portfolios sorted by size (panel (A)), 5 portfolios sorted by book-to-market (panel (B)), 10 portfolios sorted by size (panel (C)), and 10 portfolios sorted by book-to-market (panel (D)), under the unconditional formulation of the CAPM (column (1)), the consumption-based CAPM (column (2)), the human capital CAPM (column (3)), and the labor commitment model (column (4)).

[Insert Table 5 Here]

One can immediately notice that, all model specifications across four portfolio sets pass the α-test, which should not be a surprise due to the panel size (portfolio numbers within each portfolio set, i.e., each panel). A immediate follow-up observation is that, due to the small panel size, the point estimates of the same model vary quite a lot across different portfolio sets.

Interestingly enough, reading across panels, the labor commitment model consistently has (i) the lowest mean absolute pricing error, and (ii) the highest R^2 in all panels. Reading within panel, roughly speaking, consumption growth predicts value premium better while fluctuation in labor market predicts size premium better.

5.3 Conditional Formulation

From the theory side, one particular concern regarding Eq. (18) is parameters' time-dependence. From the empirical side, many have contributed in resolving such time-dependence via the use of conditional variable (vector) $\boldsymbol{z}_t$ (for example, Jagannathan and Wang [1996], Lettau and Ludvigson [2001a,b, 2004], Santos and Veronesi [2006], Yogo [2006]).

In this section, I again focus on the model's overall fit (R^2) and prediction precision (α-test). I use the labor commitment-consumption ratio as the conditional variable. For reasons to be clear later, I consider only the cases with single conditional variable, and hence case where the conditional variable vector consisting of dividend-price ratio and term premium (Jagannathan and Wang [1996]) is not discussed.

The main dimension analyzed here is conditional variable and the underlying model is set as Fama-French three factors model across all estimation, for reasons to be discussed in context. I find that, labor commitment-consumption ratio enables the conditional Fama-French three factors model to (i) improve overall fit estimating industrial portfolios, and (ii) to gain prediction precision estimating size and book-to-market bivariate portfolios.

Scaled Factors Suppose the stochastic discount factor is assumed as in Eq. (18), whereas its true formula is

$$-M_{t+1} = a_t + \boldsymbol{b}_t' \cdot \boldsymbol{f}_{t+1}, \tag{20}$$

where a_t and $\boldsymbol{b}_t$ are rather time-variate parameters. Following Cochrane [2009], I model such time-dependence explicitly via some conditional variable vector $\boldsymbol{z}_t$, i.e., time-variate parameters are functions of conditional variable vector $\boldsymbol{z}_t$. Let

$$a_t = a(\boldsymbol{z}_t) \text{ and } \boldsymbol{b}_t = (b^k(\boldsymbol{z}_t))_K.$$

Empirically such first-step simplification yields no returns. Therefore, I further make two second-step simplification in line with the literature. Firstly, time-variate parameters are linear functions of conditional variable vector $\boldsymbol{z}_t$. The linearity is not restrictive at all but rather just a formulative re-written, since, obviously, any non-linear element can be grouped into conditional variable vector $\boldsymbol{z}_t$ and re-enters the functions in a linear manner. The other simplification is that, conditional variable vector $\boldsymbol{z}_t$ contains only one element z_t. Such simplification enables me to focus not on discovering but on comparing conditional variables. As a result, parameters in Eq. (20) are then

$$a_t = a(\boldsymbol{z}_t) := \boldsymbol{\zeta}' \cdot (1, z_t) = \zeta_0 + \zeta_1 z_t,$$
$$\boldsymbol{b}_t = (b^k(\boldsymbol{z}_t))_K := (\boldsymbol{\eta}^{k\prime} \cdot (1, z_t))_K = (\eta_0^k + \eta_1^k z_t)_K.$$

Therefore, the stochastic discount factor is

$$-M_{t+1} = \xi + \boldsymbol{\theta}' \cdot \boldsymbol{F}_{t+1}, \tag{21}$$

where $\boldsymbol{F}_{t+1}$ is the fundamental factor vector $\boldsymbol{f}_{t+1}$ scaled by the conditional variable z_t, or simply, the scaled factors,

$$\boldsymbol{F}_{t+1} = (F_{t+1}^k)_{2K+1} := (z_t, \boldsymbol{f}_{t+1}, z_t \otimes \boldsymbol{f}'_{t+1}),$$

and ξ is a constant and $\boldsymbol{\theta}$ is a constant vector. Detailed derivation of general scaled fundamental factor representation of stochastic discount factor is in Appendix B.3. I shall point out that, by the above argument, the time-dependent parameters in stochastic discount factor in (20) have been completely replace by aforedefined, time-independent parameters as in (21).

Obviously, I can modify the unconditional formulation of linear factor pricing model and write the conditional formulation; however, I omit doing so to avoid any possible confusion, as, by the argument at the end of last paragraph, all changes to the multi-beta representation of unconditional linear factor pricing model in Eq. (19) are not material but rather conceptual.

Estimation of Industrial Portfolios As the intuition I discussed in previous section, the Fama-French three factors model should and does predict portfolios formed on market characteristic. In here, I ask two further questions. The first one is, how the unconditional Fama-French three factors model performs with portfolios formed not on market characteristic but rather some exogenous characteristic, such as industrial type. Particularly, how is the overall fit and how is the prediction precision.

If the unconditional Fama-French three factors model indeed performs relatively poorly, then the second question arises that, what conditional variable improves its performance and by how much. Table 6 tabulates the estimation results using both unconditional and conditional formulation of Fama-French three factors model with industrial portfolios.

[Insert Table 6 Here]

I would like to bring to immediate attention the lowest mean absolute pricing error and highest R^2 using the labor commitment-consumption ratio (column (4)) as conditional variable.

Even though the unconditional formulation of Fama-French three factors model has relatively low mean absolute pricing error and passes the α-test, its overall fit (R^2) is anticipatively bad at -0.28, worse than simply predicting using average realized return of each portfolio. This observation confirms the intuition in the previous section, and answers my first question. The conditional formulation using conditional variables of the consumption-wealth ratio proxy (or, cay, Lettau and Ludvigson [2004]) and of the durable consumption growth (Yogo [2006]) sees noticeable improvements. However, these two conditional variables fall short compared to the labor commitment-consumption ratio in overall fit (R^2). Therefore, my second question is answered. That is, among the conditional variables test in this context, the labor commitment-consumption ratio is favored the most.

One final observation turns out quite interesting that, all specifications, including the unconditional formulation, of Fama-French three factors model tested here pass the α-test, i.e., give relative high precision, in spite of the quite variant overall fits (R^2). A quick speculation of the reason is the panel size (portfolios number). Recalling that the unconditional formulation of Fama-French three factors model fails the α-test using bivariate portfolios, I address such speculation in the following.

Estimation of Bivariate Portfolios To resolve the speculation and, most importantly, to analyze along the dimension of prediction precision (α-test), Table 7 tabulates the estimation results using both unconditional and conditional formulation of Fama-French three factors model with bivariate portfolios formed on size and book-to-market.

[Insert Table 7 Here]

As is already seen, the unconditional formulation fails the α-test (column (1)), with a α-statistic of 65.36 (> 33.92). Adding conditional variables of the consumption-wealth ratio proxy, i.e., cay (column (2)) and the durable consumption growth (column (3)) improves the mean absolute pricing error and the overall fit (R^2), but not the model prediction precision (α-test). In fact, the α-statistics remain relative high, 31.22 (> 28.87) and 50.18 (> 28.87).

In column (4), estimation of conditional formulation using conditional variable the labor commitment-consumption ratio gives, again, the smallest mean absolute pricing error and the highest overall fit R^2. Most importantly, such specification passes the α-test with a large margin, 16.58 (< 28.87). This resolves my speculation that, as the increase of panel size (portfolios number), the failure of α-test remains problematic in market-oriented asset pricing model, Fama-French three factors model. It also highlights the plausibility of labor commitment-consumption ratio as conditional variable under conditional formulation.

6 Conclusion

The central finding of this paper suggest that, there is much empirical content in the theoretical paradigm of consumption-based asset pricing. The insight of any consumption-based asset pricing framework is that, the marginal utility of consumption relates to agents' view of risk. This paper shows that, additionally, marginal utility of labor commitment is critically relevant as well.

The key ingredient is the labor commitment, which follows a law of motion regarding the intensive margin of labor market, and the its nonseparability from consumption in intratemporal utility index, where the intratemporal elasticity of substitution between the two is higher than the intertemporal elasticity of substitution from intertemporal utility function. The key mechanism replies on the procyclicality of labor commitment-consumption ratio, which is one of the driving forces to the countercyclicality of intertemporal marginal rate of substitution, and thus the high equity premium and low risk-free rate during recessions.

The linear factor model from implied stochastic discount factor transparentizes the role of labor commitment in explaining the portfolio excess returns. In dimensions of overall fit measured by R^2 and prediction precision measured by α-test, the labor commitment model outperforms its competitors. With industrial portfolios where the Fama-French three factors model is incompetent, the labor commitment-consumption ratio as conditional variable is favored most.

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Tables

Table 1. Descriptive Statistics of the Data Variables.

This table tabulates two summary statistics of some key variables in the data set. The column (1) and (2) are the standard deviation-to-mean ratios and first-order auto-correlation coeffficients. The column (3) are the correponding variable's correlation coefficient with respect to labor commitment-consumption ratio L/C. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

Variable	Std/Mean	First-Order Auto-correlation	Correlation w/ L/C
	(1)	(2)	(3)
Labor-Consumption Ratio	2.7062	0.9985	
C (PCE)	3.2296	0.9998	-0.9457
C (NDS)	3.6303	0.9999	-0.9613
L	22.9534	0.9172	0.9513
N	18.6765	0.9925	0.3826
Excess Return	11.8208	0.0630	-0.0543
Size Factor (SMB)	17.6941	-0.0114	0.0828
Value Factor (HML)	17.1660	0.1249	0.0374
Dividend-Price Ratio	2.9783	0.9634	0.4417
Earnings-Price Ratio	2.8248	0.9652	0.3614
Yield Spread	1.5394	0.8175	-0.4627

Table 2. Estimation of Labor Commitment Model Under Generalized Method of Moments with Various Portfolios.

This table tabulates (A) cross-sectional and (B) time-series estimation of labor commitment model using unconditional and conditional versions of moment conditions, respectively. I use (1) 6 Fama-French portfolios, and (2) 25 Fama-French portfolios, formed on size and book-to-market, (3) 10 portfolios of industrial type, (4) 30 portfolios of industrial type, a finer version of (3), (5) 10 portfolios formed on size, and (6) 10 portfolios formed on book-to-market. Column (7) uses the pooling portfolios in each panel. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

Portfolio	6 Fama-French	25 Fama-French	Industrial Type: 10	Industrial Type: 30	Size	Book-to-Market	All
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel (A): Unconditional Estimation: Instruments of Constant							
γ			1.3167	1.5117	1.2929	1.5631	1.4716
ρ			2.3402	2.2602	2.1079	2.1739	2.1916
σ			1.0244	1.0399	1.0244	1.0163	1.0942
α			0.8302	0.8275	0.8215	0.8242	0.8249
β			6.5292	6.6674	7.8183	7.1522	7.4408
J-Stat			0.0000	0.0000	0.0000	0.0000	0.0000
Pass J-Stat			Yes	Yes	Yes	Yes	Yes
Panel (B): Conditional Estimation: Instruments of Dividend- and Earnings- Price Ratio							
γ	4.3783	4.5451					4.5973
ρ	2.6697	4.1228					2.6665
σ	1.0000	1.0000					1.0000
α	0.8395	0.8608					0.8394
β	1.3328	1.3347					1.3375
J-Stat	0.0000	0.0000					0.0000
Pass J-Test	Yes	Yes					Yes

Table 3. Estimation of Linear Factor Pricing Model Under Unconditional Formulation with the Bivariate Portfolios.

This table tabulates the estimation results of factor premia $\boldsymbol{\lambda}$ under unconditional formulation of the CAPM, the consumption-based CAPM, the human capital CAPM, the labor commitment model and the Fama-French three factors model from column (1) to (5), respectively. The standard deviations corrected for Shanken's critique (Shanken [1992]) are reported in parentheses below every point estimate. Results of overall fit (mean absolute pricing error and R^2) and prediction precision (α-test result) are also reported. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

Factor Premium ($\boldsymbol{\lambda}$)	CAPM	Consumption -based CAPM	Human Capital CAPM	Labor Commitment Model	Fama-French Three Factor Model
	(1)	(2)	(3)	(4)	(5)
Panel (A): 6 Fama-French Portfolios Formed on Size and Book-to-Market (2×3)					
Market	0.0214		0.0213	0.0193	0.0191
	(0.0057)		(0.0063)	(0.0061)	(0.0062)
SMB					0.0115
					(0.0040)
HML					0.0102
					(0.0041)
Consumption		0.0112		0.0054	
		(0.0029)		(0.0030)	
Labor			-0.0237	-0.0115	
			(0.0085)	(0.0063)	
MAE	0.0077	0.0033	0.0045	0.0030	0.0027
R^2	-0.40	0.69	0.54	0.74	0.86
Pass α-Test	No	Yes	Yes	Yes	No
Panel(B): 25 Fama-French Portfolios Formed on Size and Book-to-Market (5×5)					
Market	0.0208		0.0240	0.0205	0.0195
	(0.0054)		(0.0064)	(0.0061)	(0.0062)
SMB					0.0112
					(0.0041)
HML					0.0115
					(0.0042)
Consumption		0.0108		0.0010	
		(0.0028)		(0.0017)	
Labor			-0.0074	-0.0214	
			(0.0031)	(0.0048)	
MAE	0.0081	0.0063	0.0058	0.0045	0.0027
R^2	-0.78	-0.32	0.11	0.41	0.74
Pass α-Test	No	Yes	No	Yes	No

Table 4. Intercept Estimation from Cross-Sectional Regression of Linear Factor Pricing Model Under Unconditional Formulation with the Fama-French Portfolios.

This table tabulates in Panel (A) the intercepts (i.e., α) estimation and standard deviation from the second stage of Fama-MacBeth procedure (i.e., cross-sectional regression), in Panel (B) the deviations in absolute t-statistics from the CAPM, and in Panel (C) the deviations in absolute t-statistics from the Fama-French three factors model. The deviations in absolute t-statistics is calculated as the absolute value of t-statistics from the CAPM/Fama-French three factors model minus that from the corresponding models. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

Size	Small			Big		
Book-to-Market	Low	Med	High	Low	Med	High
	(1)	(2)	(3)	(4)	(5)	(6)
Panel (A): Intercept α Estimation and Standard Deviation						
CAPM	-0.0144	0.0018	0.0057	0.0058	0.0080	0.0107
	(0.0028)	(0.0012)	(0.0025)	(0.0039)	(0.0033)	(0.0035)
3-Factor	-0.0035	0.0027	0.0021	0.0034	-0.0014	-0.0029
	(0.0008)	(0.0009)	(0.0009)	(0.0009)	(0.0013)	(0.0012)
Consumption	-0.0070	0.0018	0.0008	0.0057	0.0039	0.0003
	(0.0033)	(0.0012)	(0.0028)	(0.0040)	(0.0027)	(0.0027)
Human	-0.0045	0.0073	0.0058	-0.0003	-0.0050	-0.0041
	(0.0020)	(0.0017)	(0.0019)	(0.0024)	(0.0023)	(0.0018)
Commitment	-0.0075	0.0016	0.0013	0.0011	0.0027	0.0037
	(0.0031)	(0.0009)	(0.0012)	(0.0009)	(0.0021)	(0.0017)
Panel (B): Deviations in Absolute t-Statistics from CAPM						
Consumption	+3.0196	-0.0355	+1.9947	+0.0362	+0.9925	+2.9411
Human	+2.8581	-2.7319	-0.8237	+1.3500	+0.2294	+0.7604
Commitment	+2.7297	-0.3792	+1.1625	+0.2238	+1.1169	+0.8543
Panel (C): Deviations in Absolute t-Statistics from Fama-French 3 Factors Model						
Consumption	+2.2445	+1.5945	+1.9958	+2.2890	-0.3827	+2.2096
Human	+2.0830	-1.1019	-0.8226	+3.6028	-1.1458	+0.0289
Commitment	+1.9546	+1.2508	+1.1636	+2.4766	-0.2582	+0.1228

Table 5. Estimation of Linear Factor Pricing Model Under Unconditional Formulation with the Univariate Portfolios.

This table tabulates the estimation results of factor premia ($\boldsymbol{\lambda}$) using 5 portfolios sorted by size (panel (A)), 5 portfolios sorted by book-to-market (panel (B)), 10 portfolios sorted by size (panel (C)), and 10 portfolios sorted by book-to-market (panel (D)), under the unconditional formulation of the CAPM (column (1)), the consumption-based CAPM (column (2)), the human capital CAPM (column (3)), and the labor commitment model (column (4). The standard deviations corrected for Shanken's critique (Shanken [1992]) are reported in parentheses below every point estimate. Results of overall fit (mean absolute pricing error and R^2) and prediction precision (α-test result) are also reported. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

Factor Premium ($\boldsymbol{\lambda}$)	CAPM (1)	Consumption -based CAPM (2)	Human Capital CAPM (3)	Labor Commitment Model (4)
Panel (A): 5 Fama-French Portfolios Formed on Size				
Market	0.0196		0.0229	0.0207
	(0.0055)		(0.0064)	(0.0061)
Consumption		0.0102		-0.0010
		(0.0028)		(0.0043)
Labor			0.0004	-0.0140
			(0.0041)	(0.0152)
MAE	0.0044	0.0058	0.0020	0.0015
R^2	-0.07	-0.38	0.78	0.90
Pass α-Test	Yes	Yes	Yes	Yes
Panel (B): 5 Fama-French Portfolios Formed on Book-to-Market				
Market	0.0318		0.0224	0.0217
	(0.0082)		(0.0060)	(0.0060)
Consumption		0.0124		0.0067
		(0.0031)		(0.0027)
Labor			-0.0036	-0.0055
			(0.0040)	(0.0082)
MAE	0.0018	0.0030	0.0034	0.0010
R^2	0.85	0.45	0.15	0.94
Pass α-Test	Yes	Yes	Yes	Yes
Panel (C): 10 Fama-French Portfolios Formed on Size				
Market	0.0198		0.0232	0.0218
	(0.0055)		(0.0064)	(0.0062)
Consumption		0.0103		0.0019
		(0.0028)		(0.0026)
Labor			0.0022	-0.0021
			(0.0034)	(0.0062)
MAE	0.0043	0.0059	0.0018	0.0012
R^2	-0.00	-0.84	0.78	0.91
Pass α-Test	Yes	Yes	Yes	Yes
Panel (D): 10 Fama-French Portfolios Formed on Book-to-Market				
Market	0.0315		0.0232	0.0222
	(0.0080)		(0.0061)	(0.0060)
Consumption		0.0118		0.0057
		(0.0029)		(0.0024)
Labor			-0.0007	0.0001
			(0.0025)	(0.0066)
MAE	0.0020	0.0043	0.0040	0.0018
R^2	0.80	-0.05	0.21	0.81
Pass α-Test	Yes	Yes	Yes	Yes

Table 6. Estimation of Fama-French Three Factor Model Under Unconditional and Conditional Formulations with the Industrial Portfolios.

This table tabulates the estimation results of factor premia ($\boldsymbol{\lambda}$) using the Fama-French three facctos model and 10 industial portfolios. The specification are unconditional formulation (column (1)), conditional on the consumption-wealth ratio proxy (column (2)), on the durable consumption growth (column (3)), and on the labor commitment-consumption ratio (column (4). The standard deviations corrected for Shanken's critique (Shanken [1992]) are reported in parentheses below every point estimate. Results of overall fit (mean absolute pricing error and R^2) and prediction precision (α-test result) are also reported. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

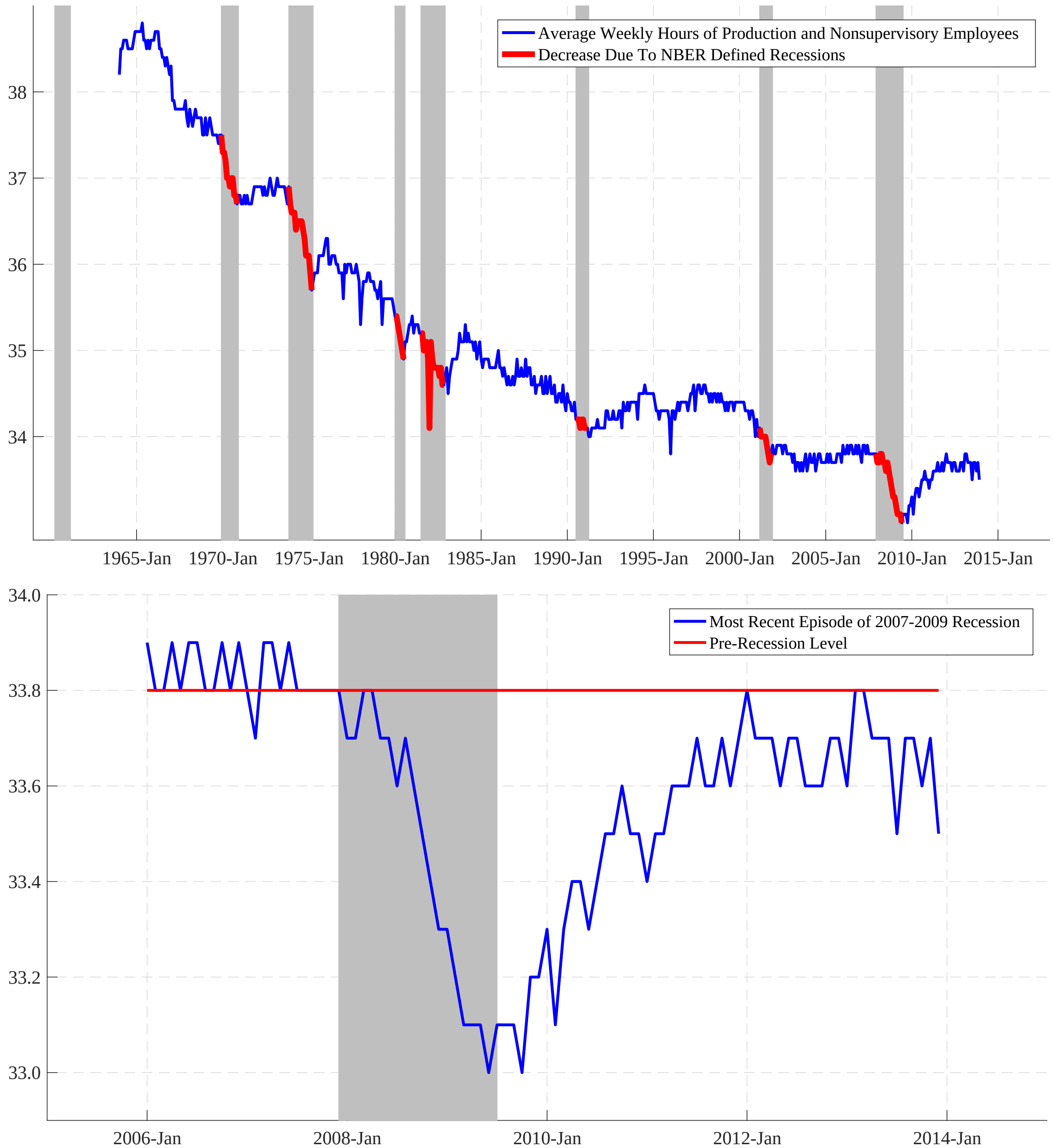
Factor Premium ($\boldsymbol{\lambda}$)	Unconditional	Consumption-Wealth Ratio (*cay*)	Durable Consumption Growth	Labor Cmmt-Consumption Ratio
	(1)	(2)	(3)	(4)
Market	0.0237	0.0235	0.0243	0.0234
	(0.0062)	(0.0062)	(0.0062)	(0.0062)
SML	-0.0074	-0.0041	0.0023	-0.0054
	(0.0075)	(0.0070)	(0.0111)	(0.0079)
HML	-0.0063	-0.0144	-0.0124	-0.0119
	(0.0058)	(0.0066)	(0.0082)	(0.0077)
Conditional		0.0073	-0.0357	-0.0127
		(0.0093)	(0.0353)	(0.0643)
Conditional $\times$				
Market		-0.0010	-0.0045	0.0092
		(0.0006)	(0.0033)	(0.0027)
SML		-0.0008	-0.0011	-0.0042
		(0.0006)	(0.0037)	(0.0035)
HML		0.0003	0.0003	-0.0046
		(0.0003)	(0.0031)	(0.0040)
MAE	0.0029	0.0016	0.0018	0.0011
R^2	-0.28	0.65	0.53	0.77
Pass α-Test	Yes	Yes	Yes	Yes

Table 7. Estimation of Fama-French Three Factor Model Under Unconditional and Conditional Formulations with the Bivariate Portfolios Formed on Size and Book-to-Market (5×5).

This table tabulates the estimation results of factor premia ($\boldsymbol{\lambda}$) using the Fama-French three facctos model and 25 bivariate portfolios formed on size and book-to-market. The specification are unconditional formulation (column (1)), conditional on the consumption-wealth ratio proxy (column (2)), on the durable consumption growth (column (3)), and on the labor commitment-consumption ratio (column (4). The standard deviations corrected for Shanken's critique (Shanken [1992]) are reported in parentheses below every point estimate. Results of overall fit (mean absolute pricing error and R^2) and prediction precision (α-statistic and α-test result) are also reported. The data are quarterly and sample period is 1964:Q1 - 2013:Q4.

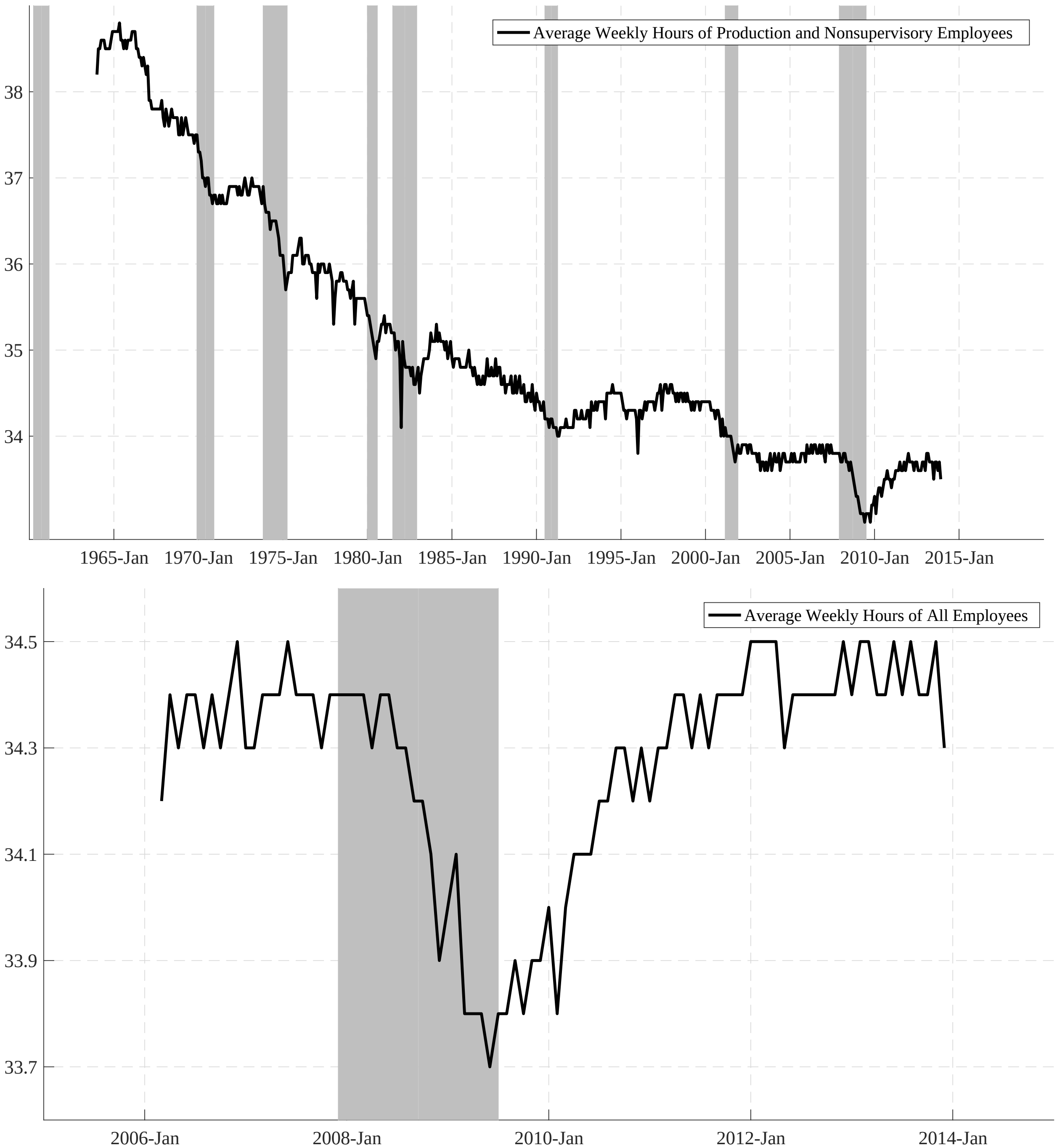
Factor Premium ($\boldsymbol{\lambda}$)	Unconditional	Consumption-Wealth Ratio (*cay*)	Durable Consumption Growth	Labor Cmmt-Consumption Ratio
	(1)	(2)	(3)	(4)
Market	0.0195	0.0198	0.0188	0.0207
	(0.0062)	(0.0062)	(0.0062)	(0.0062)
SML	0.0112	0.0107	0.0113	0.0086
	(0.0041)	(0.0041)	(0.0041)	(0.0041)
HML	0.0115	0.0102	0.0104	0.0086
	(0.0042)	(0.0042)	(0.0042)	(0.0042)
Conditional		0.0138	-0.0250	-0.0655
		(0.0052)	(0.0156)	(0.0280)
Conditional $\times$				
Market		-0.0010	0.0001	0.0123
		(0.0004)	(0.0012)	(0.0030)
SML		-0.0008	0.0003	0.0026
		(0.0002)	(0.0010)	(0.0018)
HML		0.0003	0.0004	0.0047
		(0.0003)	(0.0009)	(0.0016)
MAE	0.0027	0.0024	0.0025	0.0018
R^2	0.74	0.81	0.78	0.91
α-Statistic	65.36	31.22	50.18	16.98
Pass α-Test	No	No	No	Yes

Figures



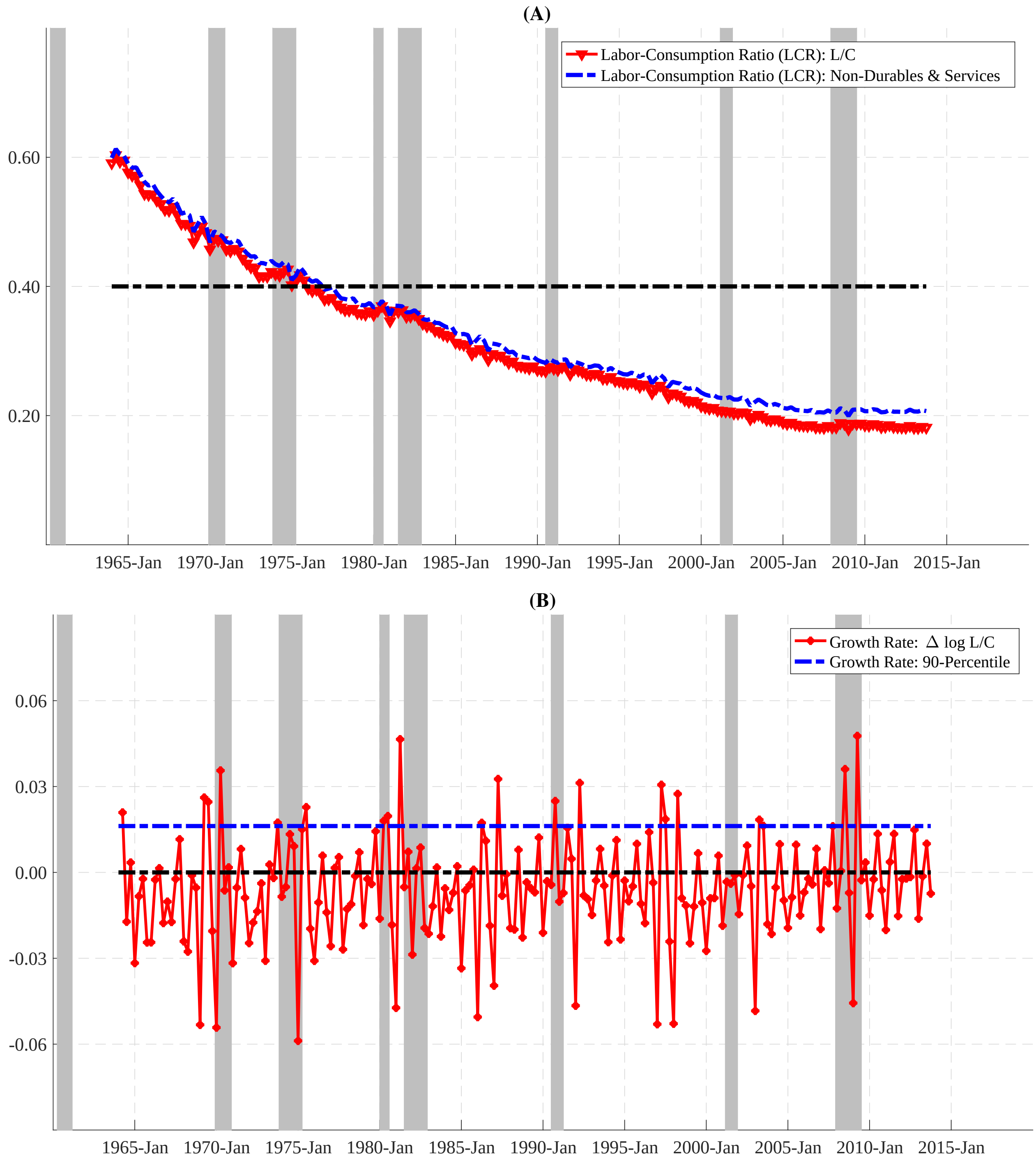
This figure depicts the average weekly hours of Production and Nonsupervisory (PNS) employees in all private sectors. The Panel (A) plots the whole sample period from 1964:Q1 to 2013:Q4; the Panel (B) plot the most recent episode from 2006:Q1 to 2013:Q4.

Figure 1. The Decrease of Hours in U.S. Due to Recessions.



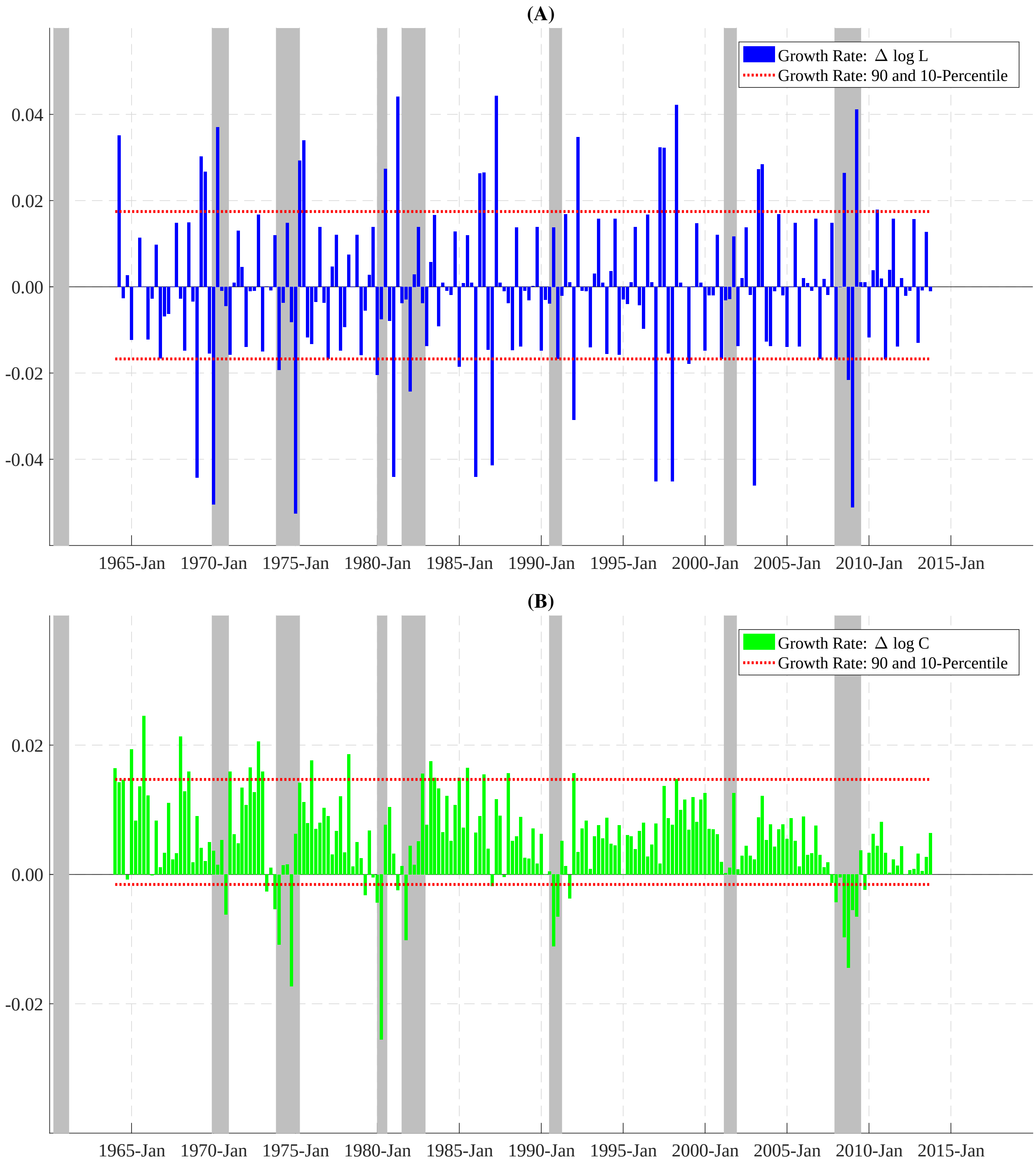
This figure depicts the average weekly hours across all private sectors. The Panel (A) plots the average weekly hours of only Production and Nonsupervisory (PNS) employees; the Panel (B) plots that of all employers, with a shorter horizon. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Figure 2. The Business-Cycle Characteristic of Average Weekly Hours In All Private Sectors.



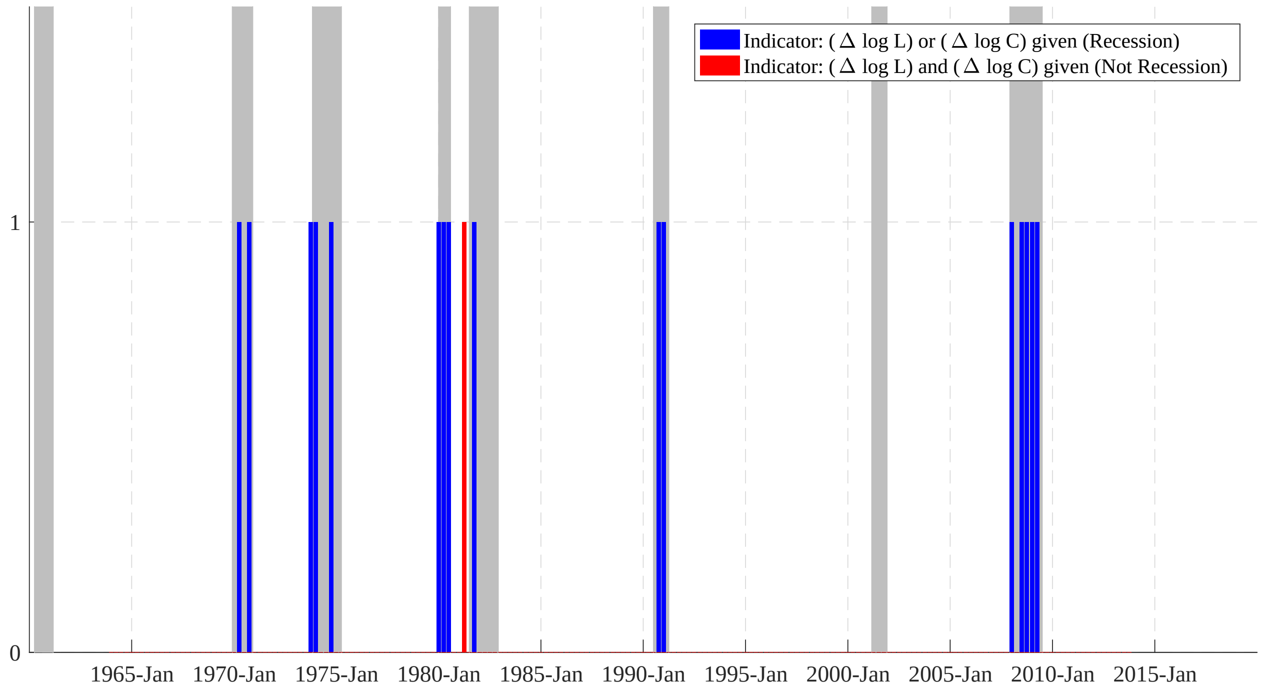
This figure depicts the labor commitment-consumption ratio L/C in the Panel (A), and its growth rate $\Delta\log(L/C)$ over the whole sample in the Panel (B); both panels demonstrate the business-cycle characteristic. In Panel (A), a compnay plot is L/C with consumption measured by nondurable goods & services consumption. In Panel (B), the reference horizontal line is the 90-percentile of growth rate $\Delta\log(L/C)$ in the sample. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Figure 3. The Business-Cycle Characteristic of L/C and $\Delta\log(L/C)$: Labor Commitment-Consumption Ratio and Growth Rate.



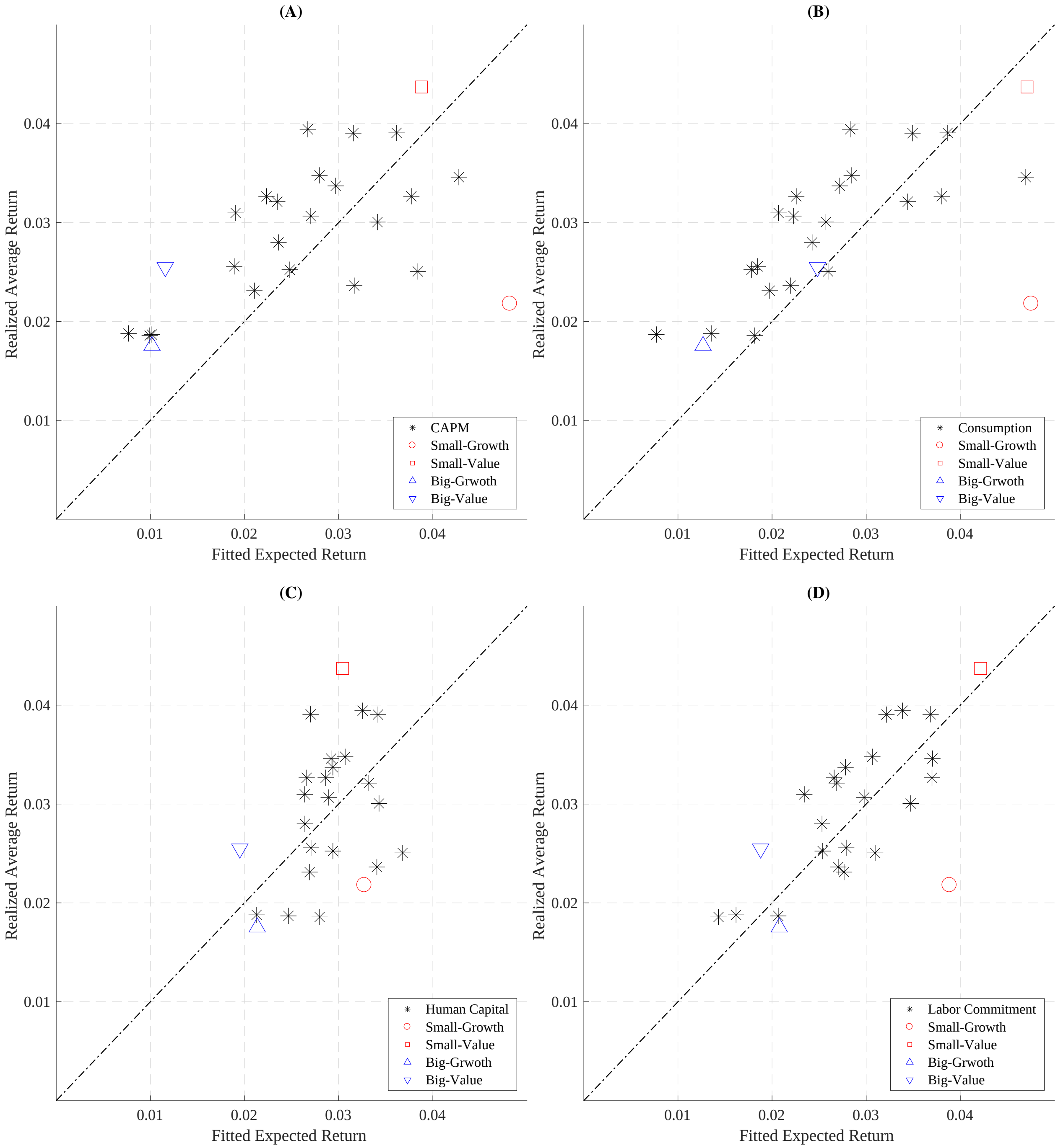
This figure depicts in Panel (A) the growth rate of labor commitment $\Delta \log L$ and its 90-percentile reference line, and in Panel (B) the growth rate of consumption $\Delta \log C$ (measured as total of personal consumption expenditure) and its 10-percentile reference line. Both panels combined together indicates the decomposition of labor commitment-consumption ratio growth rate $\Delta \log L/C \equiv \Delta \log L - \Delta \log C$. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Figure 4. The Decomposition of Labor Commitment-Consumption Ratio Growth Rate $\Delta \log(L/C)$.



This figure depicts the two kinds of messages. The first are true messages in blue bars; that is, indicators of $I^{\Delta c} = \mathbf{1}_{\Delta c \leq \mathbb{P}(\Delta c, 10)}$ || $I^{\Delta l} = \mathbf{1}_{\Delta l \geq \mathbb{P}(\Delta l, 90)}$ given recessions. The second are false messages in red bar; that is, indicators of $I^{\Delta c} = \mathbf{1}_{\Delta c \leq \mathbb{P}(\Delta c, 10)}$&&$I^{\Delta l} = \mathbf{1}_{\Delta l \geq \mathbb{P}(\Delta l, 90)}$ given not recessions. $\mathbf{1}$ is the indicator function and $\mathbb{P}$ is the percentile function; || stands for logical or and && stands for logical and. NBER recessions are indicated by shaded areas. The sample period is 1964:Q1 - 2013:Q4.

Figure 5. The Counting Exercise: Indicators Using Both Consumption $\Delta \log C$ and Labor Commitment $\Delta \log L$.



This figure depicts the realized average returns are on the vertical y-axis, and the fitted expected returns on the horizontal x-axis. Every point in any of the panel represents one of the 25 Fama-French portfolios formed on size and book-to-market. From Panel (A) to (D) are the CAPM, the consumption-based CAPM, the human capital CAPM and the labor commitment model, respectively, all of which are under unconditional formulation

Figure 6. Realized Average Versus Fitted Expected Returns Under Unconditional Formulation with the Fama-French Portfolios.

Appendix A Data Description

The consumption data is measured in two ways, with the first the main measure and the second the robust measure. The Real Personal Consumption Expenditures Per Capita in Chained Year-2009 Dollars at frequency of Quarter with Seasonally Adjusted Annual Rate is the first measure; the Real Personal Consumption Expenditures Per Capita on Nondurable Goods and Services in Chained Year-2009 Dollars at frequency of Quarter with Seasonally Adjusted Annual Rate is the second measure. Both measure of consumption measures are in the National Income and Product Accounts (NIPA) tables from the U.S. Bureau of Economic Analysis.

The labor flow is measured as the Average Weekly Hours of Production and Nonsupervisory (PNS) Employees in Total Private Sectors; it is in Hours at frequency of Month with Seasonal Adjustment. The period income is measured as the Average Hourly Earnings of Production and Nonsupervisory (PNS) Employees in Total Private; it is in Dollar-per-Hour at frequency of Month with Seasonal Adjustment. Both series are in Current Employment Statistics (CES) from the U.S. Bureau of Labor Statistics. Following Eq. (1), the labor stock, i.e., the labor commitment is constructed. Additionally, I use Consumer Price Index of All Items for All Urban Consumers from U.S. Bureau of Labor Statistics to deflate the period income; it is a Monthly index with Seasonal Adjustment targeting at Year-1982 to Year-1984 being 1.

The market data of factors are all from French website (French [2017]). Specifically, the market factor is the excess returns on the market portfolio relative to the the one-month Treasury-Bill rate; the size factor is the return on the SMB portfolio; the value factor is the return on the HML portfolio. The market portfolio is defined as a value-weighted portfolio of all NYSE, AMEX, and Nasdaq stocks; the SMB and HML portfolios are based on the six Fama–French benchmark portfolios sorted by size (breakpoint at the median) and book-to-market equity (breakpoints at the 30th and 70th percentiles). The SMB return is the difference in average returns between three small and three big stock portfolios. The HML return is the difference in average returns between two high and two low book-to-market portfolios.

I obtain the wealth return rate, the risk-free rate, the dividend-to-price ratio and the earnings-to-price ratio from the Center for Research in Security Prices (CRSP), where the wealth return rate is calculated from value-weighted index level of NYSE/AMEX/Nasdaq stocks, and the risk-free rate is measured as the 30-day Treasury-Bill rate. The dividend-to-price ratio is calculated from the value-weighted index level including and excluding dividend; the earnings-to-price ratio is obtained using SP500 index data including contributions from

CRSP and earnings data from Shiller website (Shiller [2017]).

The portfolio sets used in this paper are mainly of two kinds. The first kind is Fama-French portfolios formed on size and/or book-to-market; the second kind is Industrial portfolios formed on industrial types. Specifically, Fama–French portfolios are constructed from an frequency-independent sort of all NYSE, AMEX, and Nasdaq stocks into quintiles based on size and book-to-market. Industrial portfolios are constructed by assigning each NYSE, AMEX, and NASDAQ stock to an industry portfolio based on the four-digit SIC code.

Finally, the dataset sample finalized is from 1964:Q1 to 2013:Q4, at frequency of one quarter. The data used are either obtained directly from the source (for example, French [2017]; Shiller [2017]; authors' websites), or exacted using FRED (Federal Reserve Bank of St. Louis) and WRDS (University of Pennsylvania).

Appendix B Analytical Derivations

This section derives the analytical results with details.

B.1 The Representative Agent Problem

In Section 2.2, a representative agent makes three choices, consumption, labor and portfolio, $\{C_t; H_t; A_t^1, \cdots, A_t^N\}$, to maximize life-time utility function of intertemporal recursive structure Eq. (5), subject to (i) the law of motion for labor Eq. (1), (ii) the intratemporal budget constrain Eq. (2), (iii) the intertemporal budget constrain Eq. (3), and (iv) the intratemporal utility index specification Eq. (4). In this section, I provide, possibly among many, one solution to this problem. The main reference is Epstein and Zin [1989, 1991].

Restatement To solve, I start with simplification of the problem via changes of variables. Define for the $(N+1)$th asset

$$A_t^{N+1} = L_t N_t,$$
$$R_{t+1}^{N+1} = (1+\delta) N_{t+1}/N_t,$$

and the general wealth, the total wealth taking into consideration the labor, is then

$$\overline{W}_t := W_t + (1+\delta) L_{t-1} N_t. \tag{B.1}$$

Therefore, the intratemporal budget constrain in Eq. (2) and the intertemporal budget constrain in Eq. (3) become, respectively,

$$C_t + \sum_{n=1}^{N+1} A_t^n = \overline{W}_t, \tag{B.2}$$

and

$$\overline{W}_{t+1} = \sum_{n=1}^{N+1} A_t^n R_{t+1}^n. \tag{B.3}$$

where $\overline{W}_t$ is defined in Eq. (B.1). Furthermore, denote the share of asset holding A_t^n in general wealth $\overline{W}_t$ less consumption C_t

$$s_t^n := \frac{A_t^n}{\overline{W}_t - C_t}. \tag{B.4}$$

As a result, the intratemporal budget constrain in Eq. (B.2) and the intertemporal budget constrain in Eq. (B.3) further become, respectively,

$$\sum_{n=1}^{N+1} s_t^n = 1, \tag{B.5}$$

and

$$\overline{W}_{t+1} = (\overline{W}_t - C_t) \sum_{n=1}^{N+1} s_t^n R_{t+1}^n. \tag{B.6}$$

where $\{s_t^n\}_{n=1,\cdots,N,(N+1)}$ is defined in Eq. (B.4).

The second step goes as the transformation to the intratemporal utility index of constant elasticity of substitution form. Note that

$$L_t = \frac{A_t^{N+1}}{N_t} = s_t^{N+1} \frac{(\overline{W}_t - C_t)}{N_t}.$$

Thus the intratemporal utility index in Eq. (4) becomes

$$u(C_t, L_t) = C_t \cdot \overline{v}(C_t/\overline{W}_t, s_t^{N+1}), \tag{B.7}$$

where $\overline{v}$ is function of two arguments given price

$$\overline{v}(C/\overline{W}, s) := \left[(1-\alpha) + \alpha \left(s \frac{(\overline{W}/C - 1)}{N} \right)^{1-1/\rho} \right]^{\frac{1}{1-1/\rho}}. \tag{B.8}$$

After the above two steps, I restate the representative agent problem as follows. Given the general wealth level $\overline{W}_t$, a representative agent makes consumption and general portfolio choices, $\{C_t; s_t^1, \cdots, s_t^N; s_t^{N+1}\}$, to maximize life-time utility function of intertemporal recursive structure Eq. (5), subject to (i) the intratemporal budget constrain Eq. (B.5), (ii) the intertemporal budget constrain Eq. (B.6), and (iii) the intratemporal utility index specification of Eq. (B.7) and (B.8).

Solution The Bellman equation (Merton [1973]) for the representative agent problem is

$$J_t(\overline{W}_t) = \max_{C_t;s_t^1,\cdots,s_t^N;s_t^{N+1}} \left\{ \left[(1-\beta)\left(C_t \cdot \overline{v}(C_t/\overline{W}_t, s_t^{N+1}) \right)^{1-1/\sigma} \right.\right. \\ \left.\left. +\beta\left(\mathbb{E}_t[J_{t+1}(\overline{W}_{t+1})]^{1-\gamma} \right)^{\frac{1-1/\sigma}{1-\gamma}} \right]^{\frac{1}{1-1/\sigma}} \right\} \tag{B.9}$$

The characteristics of the representative agent problem, in particular, the recursive structure of itertemporal preference, the homotheticity of intratemporal index forms, and the linearity of budget constrains, imply that the optimal value is proportional to general wealth, that is,

$$J_t(\overline{W}_t) = \phi_t \overline{W}_t \tag{B.10}$$

Denote the solution to the representative agent problem and hence the above Bellman equation is $\{C_t^*; s_t^{1*}, \cdots, s_t^{N*}; s_t^{N+1*}\}$, and follow Epstein and Zin [1989, 1991], it can be shown that

$$\phi_t = \left(\frac{C_t^*}{\overline{W}_t} \right)^{\frac{1}{1-\sigma}} \{(1-\alpha-\beta+\alpha\beta)[\overline{v}(C_t^*/\overline{W}_t, s_t^{N+1*})]^{\frac{1}{\rho}-\frac{1}{\sigma}}\}^{\frac{1}{1-1/\sigma}}. \tag{B.11}$$

Therefore, taking Eq. (B.10) and (B.11) back to Bellman equation Eq. (B.9) gives the value function, the derivative of which yields first-order conditions that are solutions of optimality.

Specifically, denote the gross return on general wealth $R_{t+1}^{\overline{W}}$ and hence with the portfolio choices $\{s_t^{n*}\}_{n=1,\cdots,N,(N+1)}$ at optimality

$$R_{t+1}^{\overline{W}*} = \sum_{n=1}^{N+1} s_t^{n*} R_{t+1}^n.$$

Let, for convenience,

$$\chi = \left(1 - s_t^{N+1*} \frac{u_{L_t}/u_{C_t}}{N_t} \right). \tag{B.12}$$

Then the derivatives of value function (Eq. (B.9), (B.10) and (B.11)), with respect to C_t is

$$\mathbb{E}[M_{t+1}^* R_{t+1}^{\overline{W}*}] = \chi^{\frac{1-\gamma}{1-1/\sigma}}, \tag{B.13}$$

with respect to $s_t^n, (n = 1, 2, \cdots, N)$ is

$$\mathbb{E}[M_{t+1}^* R_{t+1}^n] = \chi^{\frac{1/\sigma-\gamma}{1-1/\sigma}}, \tag{B.14}$$

and with respect to s_t^{N+1} is

$$\mathbb{E}[M_{t+1}^* R_{t+1}^{N+1}] = \chi^{\frac{1/\sigma-\gamma}{1-1/\sigma}} \left(1 - \frac{u_{L_t}/u_{C_t}}{N_t} \right), \tag{B.15}$$

where the general intertemporal marginal rate of substitution M^*_{t+1} is

$$M^*_{t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left(\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)}\right)^{\frac{1}{\rho}-\frac{1}{\sigma}}\left(R^{\overline{W}*}_{t+1}\right)^{\frac{1/\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-1/\sigma}}. \tag{B.16}$$

Finally, to recover the gross return of total wealth and the intertemporal marginal rate of substitution in contexts, normalize the general versions as follows

$$\begin{aligned} R^W_{t+1}\cdot\chi^{-1} &= R^{\overline{W}*}_{t+1} \\ M_{t+1}\cdot\chi^{\frac{\gamma-1/\sigma}{1-1/\sigma}} &= M^*_{t+1} \end{aligned} \tag{B.17}$$

Substitute the normalization Eq. (B.17) in Eq. (B.13) - (B.16) yields Eq. (6) - (11) in the main contexts.

B.2 Linear Factors Model

The stochastic discount factor of the labor commitment model implies a linear factors pricing model. To recapitulate the starting point, the stochastic discount factor of the labor commitment model is as defined as intertemporal marginal rate of substitution in Eq. (6)

$$M_{t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left(\frac{v(L_{t+1}/C_{t+1})}{v(L_t/C_t)}\right)^{\frac{1}{\rho}-\frac{1}{\sigma}}\left(R^{W}_{t+1}\right)^{\frac{1/\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-1/\sigma}},$$

where $v(\cdot)$ is as defined in Eq. (7)

$$v(L/C) = [(1-\alpha)+\alpha(L/C)^{1-1/\rho}]^{\frac{1}{1-1/\rho}}.$$

Logarithm-Linearization of Stochastic Discount Factor To obtain the linear factors pricing model, I firstly logarithm-linearize the stochastic discount factor

$$m_{t+1} = \kappa\left[\log(\beta) + \left(-\frac{1}{\sigma}\right)\Delta c_{t+1} + \left(\frac{1}{\rho}-\frac{1}{\sigma}\right)\log\left(\frac{v_{t+1}}{v_t}\right) + \left(\frac{1/\sigma-\gamma}{1-\gamma}\right)r^W_{t+1}\right], \tag{B.18}$$

where

$$\kappa = \frac{1-\gamma}{1-1/\sigma}$$

and

$$v_t := v\left(\frac{L_t}{C_t}\right) = \left[(1-\alpha)+\alpha\left(\frac{L_t}{C_t}\right)^{1-1/\rho}\right]^{\frac{1}{1-1/\rho}}. \tag{B.19}$$

The second step is to get a linear approximation form for term $\log(v_{t+1}/v_t)$ in Eq. (B.18); toward this end, approximate Eq. (B.19) around the special case of Cobb-Douglas function form, i.e., $\rho = 1$. Through change of notation, Eq. (B.19) is then

$$v\left(\frac{L_t}{C_t};\rho\right) = v\left(\frac{L_t}{C_t};\rho=1\right) + (\rho-1)\cdot \left.\frac{\partial v(L_t/C_t;\rho)}{\partial \rho}\right|_{\rho=1}, \tag{B.20}$$

where the derivative of v with respect to ρ at $\rho = 1$ is exactly 0. Therefore, the logarithm-linearization in Eq. (B.18) becomes

$$-m_{t+1} \approx -\kappa \log(\beta) + b^1 \Delta c_{t+1} + b^2 \Delta l_{t+1} + b^3 r^W_{t+1}, \tag{B.21}$$

where

$$\begin{aligned} b^1 &= \kappa[\alpha/\rho + (1-\alpha)/\sigma] \\ b^2 &= \alpha\kappa(1/\sigma - 1/\rho) \\ b^3 &= 1-\kappa \end{aligned} \tag{B.22}$$

which are exactly as in Eq. (17) in the main context.

Fundamental Factor Presentation The fundamental factor vector $\boldsymbol{f}$ connects the observable factors $\{f^k\}_{k\in K}$ to the unobservable stochastic discount factor M. I in this section derive the fundamental factor presentation of stochastic discount factor, as in Eq. (18), firstly in the general case and secondly in the labor commitment model.

Suppose, as in Eq. (B.21), an approximation form of the logarithm of stochastic discount factor goes as

$$m_t = \overline{a} + \boldsymbol{b}' \cdot \boldsymbol{f}_t \tag{B.23}$$

where $\boldsymbol{b}$ and $\boldsymbol{f}$ are vectors of length K, representing the coefficients of factors and factors themselves; that is, for $k = 1, 2, \cdots, K$,

$$\boldsymbol{b} = \left[b^k\right]_K, \tag{B.24}$$

and

$$\boldsymbol{f}_t = \left[f^k_t\right]_K. \tag{B.25}$$

By the first-order Taylor expansion, the approximated stochastic discount factor in terms of its logarithm is

$$M_t = e^{m_t} \approx 1 + m_t. \tag{B.26}$$

Therefore, the fundamental factor presentation of stochastic discount factor is

$$M_t \approx a + \boldsymbol{b}' \cdot \boldsymbol{f}_t \tag{B.27}$$

where $a = 1 + \overline{a}$.

In the labor commitment model, I set

$$-m_{t+1} \approx -\kappa \log(\beta) + \boldsymbol{b}' \cdot \boldsymbol{f}_t, \tag{B.28}$$

where $\boldsymbol{b}$ and $\boldsymbol{f}$ are vectors of length $K = 3$,

$$\boldsymbol{b} = (b^1, b^2, b^3), \tag{B.29}$$

as defined in Eq. (B.22), and

$$\boldsymbol{f}_t = (\Delta c_t, \Delta l_t, r_t^W). \tag{B.30}$$

Therefore, the fundamental factor presentation of stochastic discount factor is

$$-M_t \approx a + \boldsymbol{b}' \cdot \boldsymbol{f}_t \tag{B.31}$$

where $a = 1 + \kappa \log(\beta)$.

Multi-Beta Representation In the main context, an equivalence between the fundamental factor $\boldsymbol{f}$ presentation of stochastic discount factor M, and the multi-beta $\boldsymbol{\beta^n}$ presentation of linear factor model $\mathbb{E}[R^n]$. In this section, I establish the equivalence, again, firstly in the general case and secondly in the labor commitment model. For what is following, I omit the time t subscripts.

Start with the kernel pricing formula, which states that, any arbitrary asset n with value x in the next period shall be price at p in current period; in excess return form, it is

$$\mathbb{E}[M(R^n - R^1)] = 0, \tag{B.32}$$

where $(R^n - R^1)$ is the excess return of asset n relative asset 1 (say, risk-free asset), and M is the stochastic discount factor. A few steps of statistical decomposition to Eq. (B.32) shows that

$$\mathbb{E}[R^n - R^1] = \mathbb{V}\left[\frac{-M}{\mathbb{E}[M]}, (R^n - R^1)\right]. \tag{B.33}$$

Suppose, the fundamental factor presentation of stochastic discount factor is

$$-M = a + \boldsymbol{b}' \cdot \boldsymbol{f} \tag{B.34}$$

Combining Eq. (B.33) and (B.34) together yields

$$\mathbb{E}[R^n - R^1] = \boldsymbol{\lambda}' \cdot \boldsymbol{\beta}^n, \tag{B.35}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}^n$ are vectors of length K, representing the factor premia and factor exposures; that is, for $k = 1, 2, \cdots, K$,

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda^k \end{bmatrix}_K = \begin{bmatrix} b^k \dfrac{\mathbb{V}[f^k]}{\mathbb{E}[M]} \end{bmatrix}_K,$$

and

$$\boldsymbol{\beta}^n = \begin{bmatrix} \beta^{nk} \end{bmatrix}_K = \begin{bmatrix} \dfrac{\mathbb{V}[f^k, (R^n - R^1)]}{\mathbb{V}[f^k]} \end{bmatrix}_K.$$

In the labor commitment model, the factor premium vector and the factor exposure vector are calculated exactly following Eq. (B.2) and (B.2), where fundamental factors and factor coefficients are as in Eq. (B.30) and (B.29).

B.3 Conditional Linear Factor Model

The conditional linear factor pricing model aims at resolving the time dependence of coefficient in fundamental factor presentation of stochastic discount factor. I in here derive the conditional linear factor pricing model up to the scaled fundamental factor presentation of stochastic discount factor; I show result firstly in the general case and secondly in the labor commitment model. It is useful to keep in mind that vector is in boldface font and in column format.

Denote the fundamental factor presentation of stochastic discount factor, with time-variate factor coefficients, as

$$M_{t+1} = (a_t, \boldsymbol{b}_t)' \cdot (1, \boldsymbol{f}_{t+1}), \tag{B.36}$$

where $\boldsymbol{b}_t$ and $\boldsymbol{f}_{t+1}$ are vectors of length K, the number of fundamental factors. Suppose the time-dependence of factor coefficients a_t and $\boldsymbol{b}_t$ is (i) fully captured by conditional variable vector $\boldsymbol{z}_t$ of length J, and (ii) in linear form with time-invariate coefficients of $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$; in particular,

$$a_t = \boldsymbol{\zeta}' \cdot (1, \boldsymbol{z}_t) \tag{B.37}$$

and

$$\boldsymbol{b}_t = \boldsymbol{\eta}' \cdot (1, \boldsymbol{z}_t) \tag{B.38}$$

where $\boldsymbol{\zeta}$ is vector of length $J+1$ and $\boldsymbol{\eta}$ is matrix of size $(J+1) \times K$. Therefore, the stochastic discount factor in Eq. (B.36) can be re-written as

$$\begin{aligned} M_{t+1} &= (1, \boldsymbol{z}_t)' \cdot [\boldsymbol{\xi}, \boldsymbol{\eta}] \cdot (1, \boldsymbol{f}_{t+1}) \\ &= (1, \boldsymbol{f}_{t+1})' \otimes (1, \boldsymbol{z}_t)' \cdot vec([\boldsymbol{\xi}, \boldsymbol{\eta}]) \end{aligned} \tag{B.39}$$

where $(1, \boldsymbol{f}_{t+1})$ is vector of length $K+1$, $(1, \boldsymbol{z}_t)$ is vector of length $J+1$, and the Kronecker product term $(1, \boldsymbol{f}_{t+1})' \otimes (1, \boldsymbol{z}_t)'$ is matrix of size $1 \times [(K+1)(J+1)]$; $[\boldsymbol{\xi}, \boldsymbol{\eta}]$ is matrix of size $(K+1) \times (J+1)$ and hence the vectorization term $vec([\boldsymbol{\xi}, \boldsymbol{\eta}])$ is a matrix of size $[(K+1)(J+1)] \times 1$. Let the scaled fundamental factor $\boldsymbol{F}_{t+1}$ be defined via

$$(1, \boldsymbol{F}_{t+1})' = (1, \boldsymbol{f}_{t+1})' \otimes (1, \boldsymbol{z}_t)', \tag{B.40}$$

and denote arbitrary

$$(\xi, \boldsymbol{\theta}) = vec([\boldsymbol{\xi}, \boldsymbol{\eta}]). \tag{B.41}$$

The above Eq. (B.40) and (B.41) implies

$$M_{t+1} = (\xi, \boldsymbol{\theta})' \cdot (1, \boldsymbol{F}_{t+1}), \tag{B.42}$$

which is equivalent to Eq. (21) in the main context.

Appendix C Empirical Specification

This section specifies the empirical procedures with details.

C.1 Generalized Method of Moments Procedure

From Eq. (14) in the main context, I have a total of $(N+1)I$ moments from the labor commitment model, where N is number of equity portfolios and I is the number of instrumental variables. In here, I discuss the general receipt of the estimation procedure, and the unconditional and conditional moments using different portfolio sets. The main references are Hansen [1982] and Cochrane [2009].

General Receipt For simplicity, consider the case with only one moment condition,

$$\mathbb{E}[u_{t+1}(b)] = 0, \tag{C.1}$$

where $u_{t+1}(b)$ is the moment function, of the parameter b, or equivalently in this case the moment errors. Define the sample mean of the moment errors

$$g_T(b) = \frac{1}{T} \sum_{t=1}^{T} u_t(b) = \mathbb{E}_T[u_{t+1}(b)], \tag{C.2}$$

where the subscript T denotes the mean across the whole sample of length T.

Therefore, the first-stage estimation of parameter b minimize the quadratic form of the sample moment errors,

$$b_1 = \text{argmin}_b g_T(b)' \cdot W_1 \cdot g_T(b), \tag{C.3}$$

where W_1 is the first-stage moments weight matrix, set as identity matrix. Furthermore, define from the first-stage estimation,

$$S = \frac{1}{T^2} \sum_{t=1}^{T} u_t^2(b). \tag{C.4}$$

The second-stage estimation of parameter b still minimize the quadratic form similarly

$$b_2 = \text{argmin}_b g_T(b)' \cdot W_2 \cdot g_T(b), \tag{C.5}$$

where W_2 is the second-stage moments weight matrix, set as $W_2 = S^{-1}$. Additionally, the standard deviation of estimation b_2 is from the variance-covariance matrix

$$var(b_2) = \frac{1}{T}(d'S^{-1}d)^{-1}, \tag{C.6}$$

where the gradient d is

$$d = \mathbb{E}_T \left(\frac{\partial u_{t+1}}{\partial b} \right) \Bigg|_{b=b_2}. \tag{C.7}$$

Jumping outside the above simply example of one moment condition, the labor commitment model represented by Eq. (14) has a total of $(N+1)I$ moment conditions and five parameters (γ, ρ, σ, α, and β). Therefore, there are $(N+1)I - 5$ overidentification restrictions of the labor commitment model to be test through the J-test (Hansen [1982]). The J-test tests the null hypothesis that the pricing errors are jointly close to zero across the N test assets. The J-test is conceptually similar to the GRS-test (Gibbons et al. [1989]), in the sense that the core statistic of both is a quadratic form measuring the joint magnitude of all pricing errors. Specifically, J-test states that, T times the J-statistic, the minimized value of the second-stage quadratic form objective, is distributed χ^2 with degrees of freedom equal to the number of moments less the number of estimated parameters,

$$TJ_T = T \cdot g_T(b_2)' \cdot W_2 \cdot g_T(b_2) \sim \chi^2[(N+1)I - 5]. \tag{C.8}$$

Unconditional and Conditional Moments The argument of implementing both unconditional and conditional moments is as follows. In cross-sectional estimation, I use unconditional moments, i.e., instrumental variable of constant, and set the portfolios number large. By design, I am able to estimation the labor commitment model across a large panel of portfolios. In time-series estimation, I use conditional moments, i.e., instrumental variables of lagged dividend-price ratio and earnings-price ratio. I set the portfolios number limited so as to check the consistency along time horizon.

Specifically, in cross-sectional estimation, I use portfolio sets of (i) 10 industrial portfolios, (ii) 30 industrial portfolios, (iii) 10 market portfolios sorted by size, and (iv) 10 market portfolios sorted by book-to-market; I use one instrumental variable of constant. Therefore, I in total use 60 unconditional moments. In time-series estimation, I use portfolio set of 25 bivariate portfolios sorted by size and book-to-market; I use three instrumental variables of (i) constant, (ii) one-period lagged dividend-to-price ratio, and (iii) one-period lagged earnings-to-price ratio. Therefore, I in total use 25 moments conditional on 3 instruments, a total of 75 conditional moments.

C.2 Fama-MacBeth Procedure

From Eq. (19) in the main context, I have the multi-beta representation of linear factor pricing model. In here, I discuss the general receipt of the estimation procedure only, given the unconditional and conditional formulation are almost identical, conceptually and empirically. The main references are Fama and MacBeth [1973] and Cochrane [2009].

Consider generally the case with N assets and K factors, where such K factors can either be fundamental factors $\boldsymbol{f}$ or scaled factors $\boldsymbol{F}$. The first-stage of Fama-MacBeth procedure is time-series regression of any asset n across all time periods

$$R_t^{en} = a^n + \boldsymbol{\beta}^{n\prime} \boldsymbol{f}_t + \epsilon_t^n, t = 1, 2, \cdots, T \tag{C.9}$$

where, for simplicity, denote superscript e the excess return relative to risk-free asset. The second-stage of Fama-MacBeth procedure is cross-sectional regression at each time period across all assets

$$R_t^{en} = \boldsymbol{\lambda}_t' \boldsymbol{\beta}^n + \alpha_t^n, n = 1, 2, \cdots, N \tag{C.10}$$

where $\boldsymbol{\beta}^n$ for any asset n is obtained from first-stage, and

$$\boldsymbol{\alpha}_t = \left[\alpha_t^n\right]_{n=1,2,\cdots,N}$$

is vector of pricing errors from time period t.

Fama-MacBeth Procedure is an intuitively appealing. Pricing error is, after all, about how a statistic would vary from one sample to the other. Following this sense, it is natural to cut the sample in half, and see how the pricing errors varies from the upper half to the lower half. The Fama-MacBeth procedure carries such intuition by cut the sample of size T into its theoretical limit T subsamples.

The estimates of interest are thus factor premia vector

$$\boldsymbol{\lambda} = \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{\lambda}_t \tag{C.11}$$

of length K, and pricing error vector

$$\boldsymbol{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{\alpha}_t \tag{C.12}$$

of length N. Due to Shanken [1992], a correction term, taking into account that the $\boldsymbol{\beta}^n$ for any asset n in second-stage of cross-sectional regression is not observed but estimated, is

$$S = (\boldsymbol{\lambda}' \Sigma_f^{-1} \boldsymbol{\lambda} + 1), \tag{C.13}$$

where $\Sigma_{\boldsymbol{f}} = \mathbb{V}[\boldsymbol{f}, \boldsymbol{f}']$ is variance-covariance matrix of factors.

The α-test tests the null hypothesis that the pricing errors are jointly close to zero across the N test assets. The α-test is conceptually similar to the GRS-test (Gibbons et al. [1989]), in the sense that the core statistic of both is a quadratic form measuring the joint magnitude of all pricing errors. Specifically, J-test states that, with correction for sample errors in $\boldsymbol{\beta}^n$, the quadratic form of estimates of pricing errors normalized by its variance-covariance matrix, is distributed χ^2 with degrees of freedom equal to the number of assets less the number of factors,

$$\boldsymbol{\alpha}' (\Sigma_{\boldsymbol{\alpha}} \cdot S)^{-1} \boldsymbol{\alpha} \sim \chi^2 (N - K). \tag{C.14}$$

where $\Sigma_{\boldsymbol{\alpha}} = \mathbb{V}[\boldsymbol{\alpha}]$ is variance of pricing errors.