High-Order in FLASH

- Current release of FLASH is limited to 2nd order accuracy
  - Use of linear characteristics for predictor
    - Normal direction characteristic tracing & CTU
  - 1D reconstruction
  - Single point flux quadrature

ADER-CG

- Single step update for FVM based on multidimensional reconstruction in a modal basis

$$u(x,y,t) = \sum_{lmn} \hat{u}_{lmn} \phi_l(x) \phi_m(y) \phi_n(t)$$

$\mathbf{1; t; t^2; t^3}$

$\mathbf{1; x; x^2 - \frac{1}{12}; x^3 - \frac{3}{20}x}$

- Use WENO reconstruction to get the spatial modes: ϕ_l, ϕ_m
  - Use governing PDE to get “in the small” time evolution

[1]
D. S. Balsara, T. Rumpf, M. Dumbser, and C. D. Munz, “Efficient, high accuracy ADER-WENO schemes for hydrodynamics and divergence-free magnetohydrodynamics,” *Journal of Computational Physics*, vol. 228, no. 7, pp. 2480–2516, 2009.

ADER-CG

- Couple spatially reconstructed modes to time dependent modes through Galerkin projection

$$\int_{t^n}^{t^{n+1}} \int_{I_{ij}} \left[\frac{\partial \mathbf{U}(x,y,t)}{\partial t} + \frac{\partial \mathbf{F}(x,y,t)}{\partial x} + \frac{\partial \mathbf{G}(x,y,t)}{\partial y} \right] \Phi(x,y,t) dx dy dt = 0$$

Some moments only couple to spatial variation in fluxes

$$\hat{u}_{xxt} = -3\hat{f}_{xxx} - \hat{g}_{yxx}$$

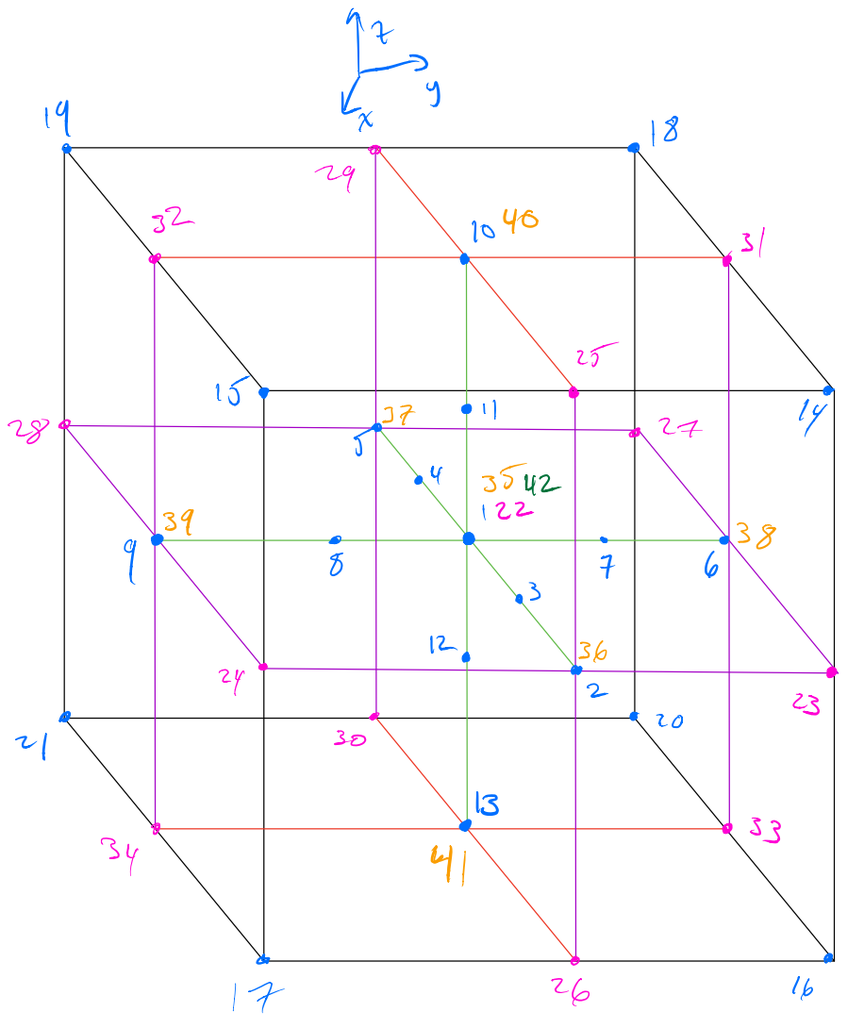
$$\hat{u}_{xtt} = -\hat{f}_{xxt} - \frac{1}{2}\hat{g}_{xyt}$$

Some moments only couple to spatial variation in fluxes

ADER-CG

- Need to iteratively build time-dependent flux modes through a conversion between modal and nodal representations

$$\hat{f}_{xxt} = 3\left[\left(1\tfrac{1}{2}00\right)^{2/3} + \left|-\tfrac{1}{2}00\right\rangle^{2/3}\right) - 2\left|000\right\rangle^{2/3}\Big] - \frac{3}{2}\hat{f}_{xx}$$



$t=0$ $t=1/3$ $t=2/3$ $t=1$

ADER-CG

- Achieve high-order (HO) in time by transforming HO spatial reconstructions into HO time dependence

- Fully nonlinear time advancement in predictor

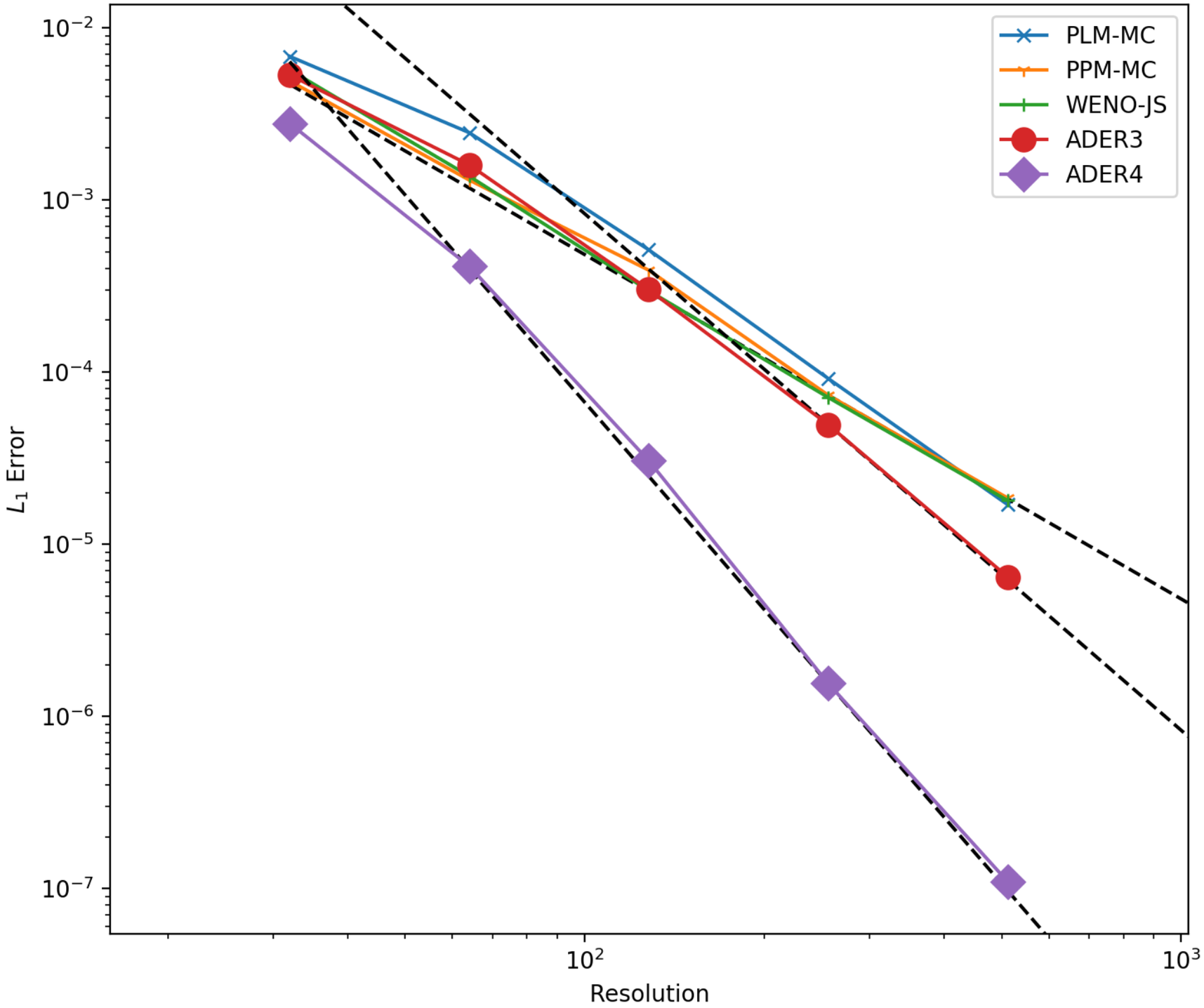
- Multidimensional reconstruction allows genuinely high-order reconstruction and high-order flux quadrature

  - We're using a "quadrature-free" Riemann solver that freezes the Riemann fan at each face

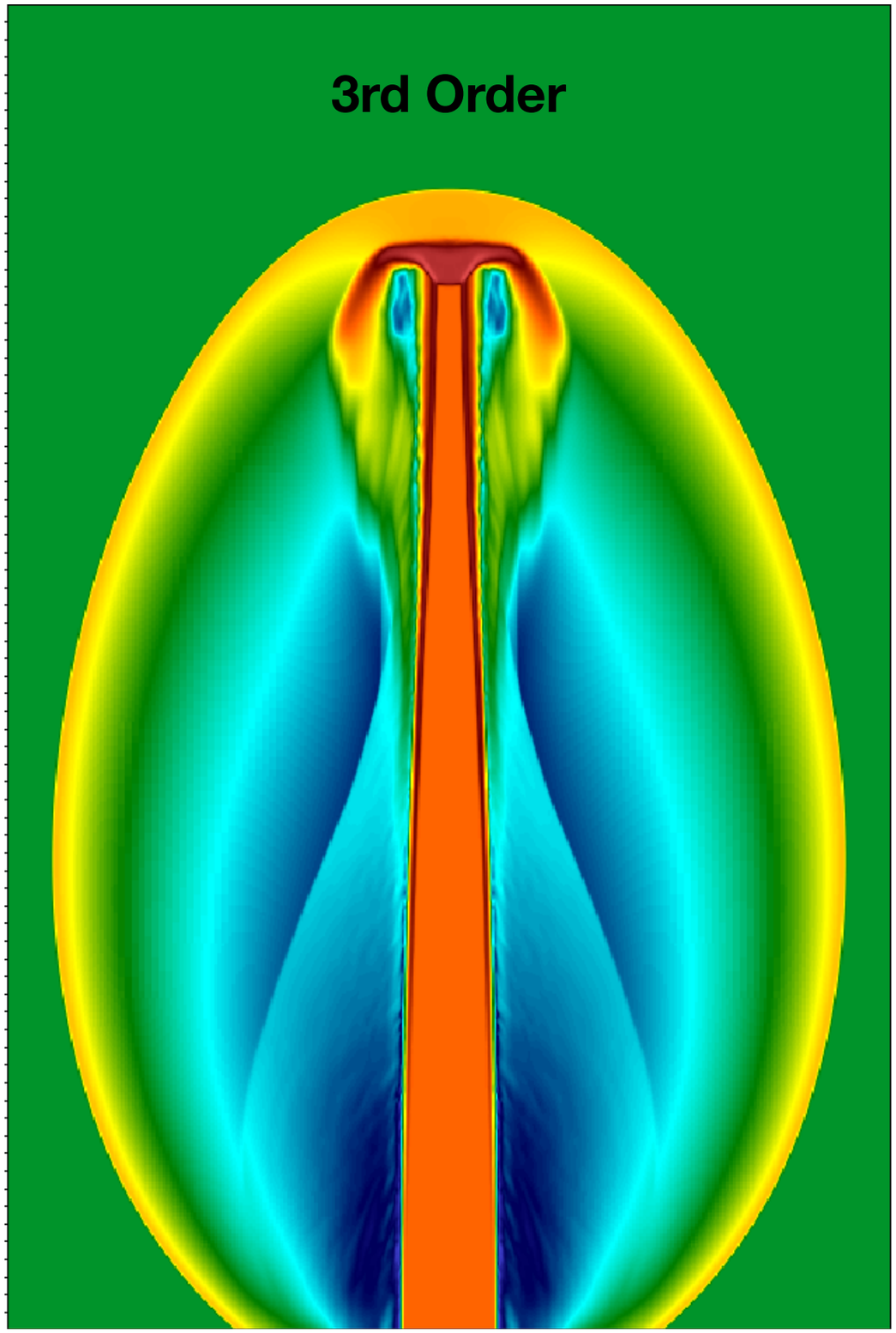
$$\mathbf{F}^{\mathrm{HLL}}(x_{i+1/2},y,z) = \frac{1}{S_R - S_L}\left[S_R\mathbf{F}^L(y,z) - S_L\mathbf{F}^R(y,z) + S_R S_L\left(\mathbf{U}^R(y,z) - \mathbf{U}^L(y,z)\right)\right]$$

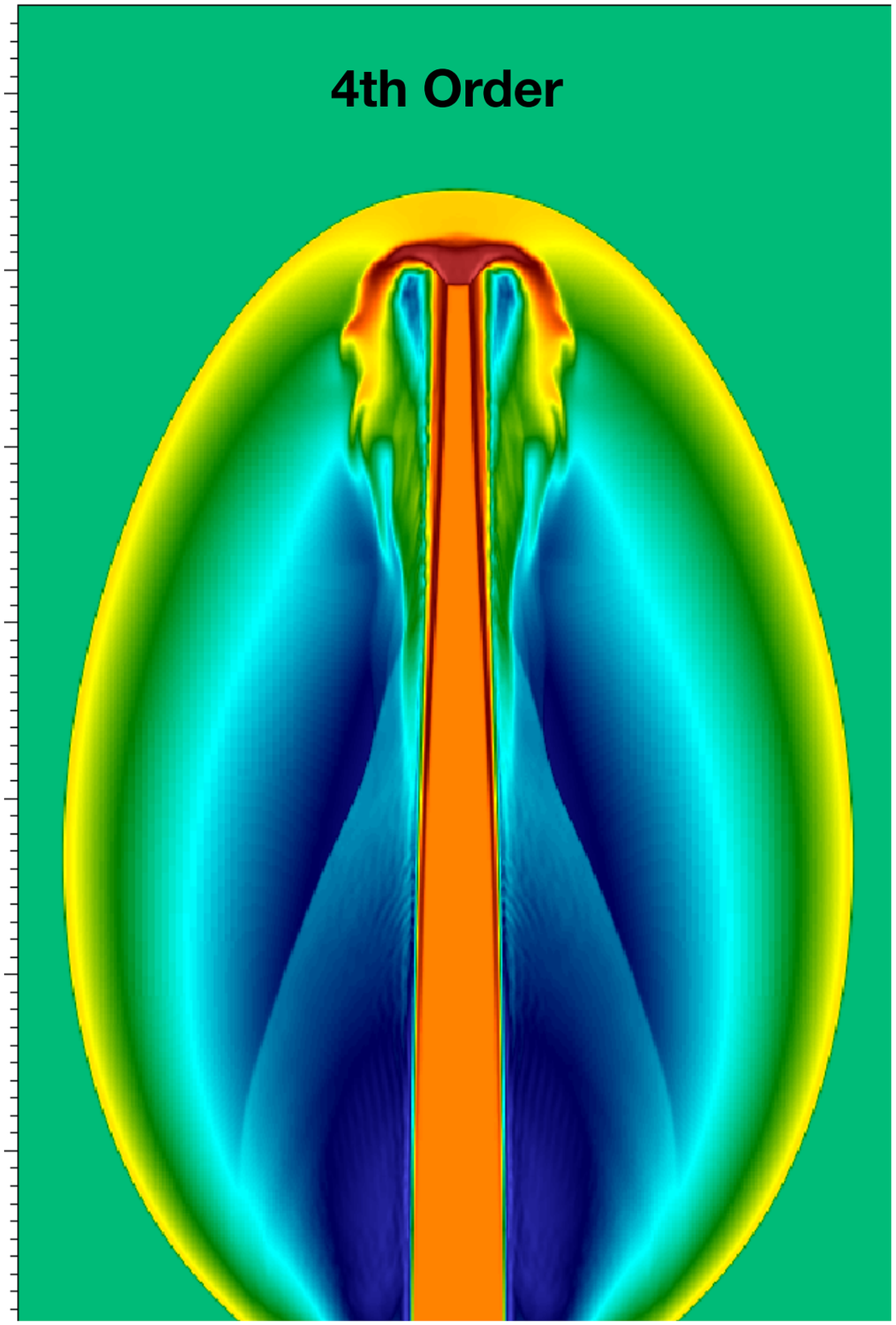
Typically these vary w/ y&z

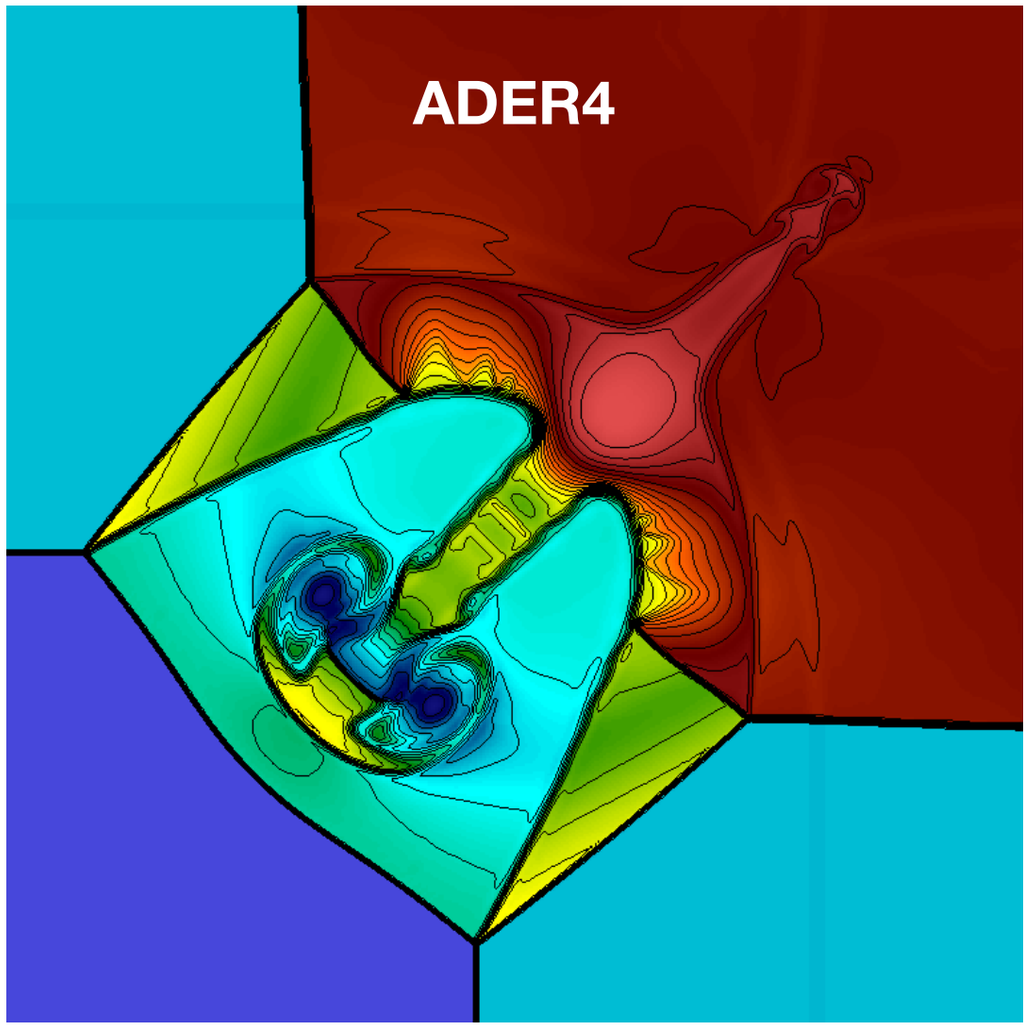
Order of Accuracy

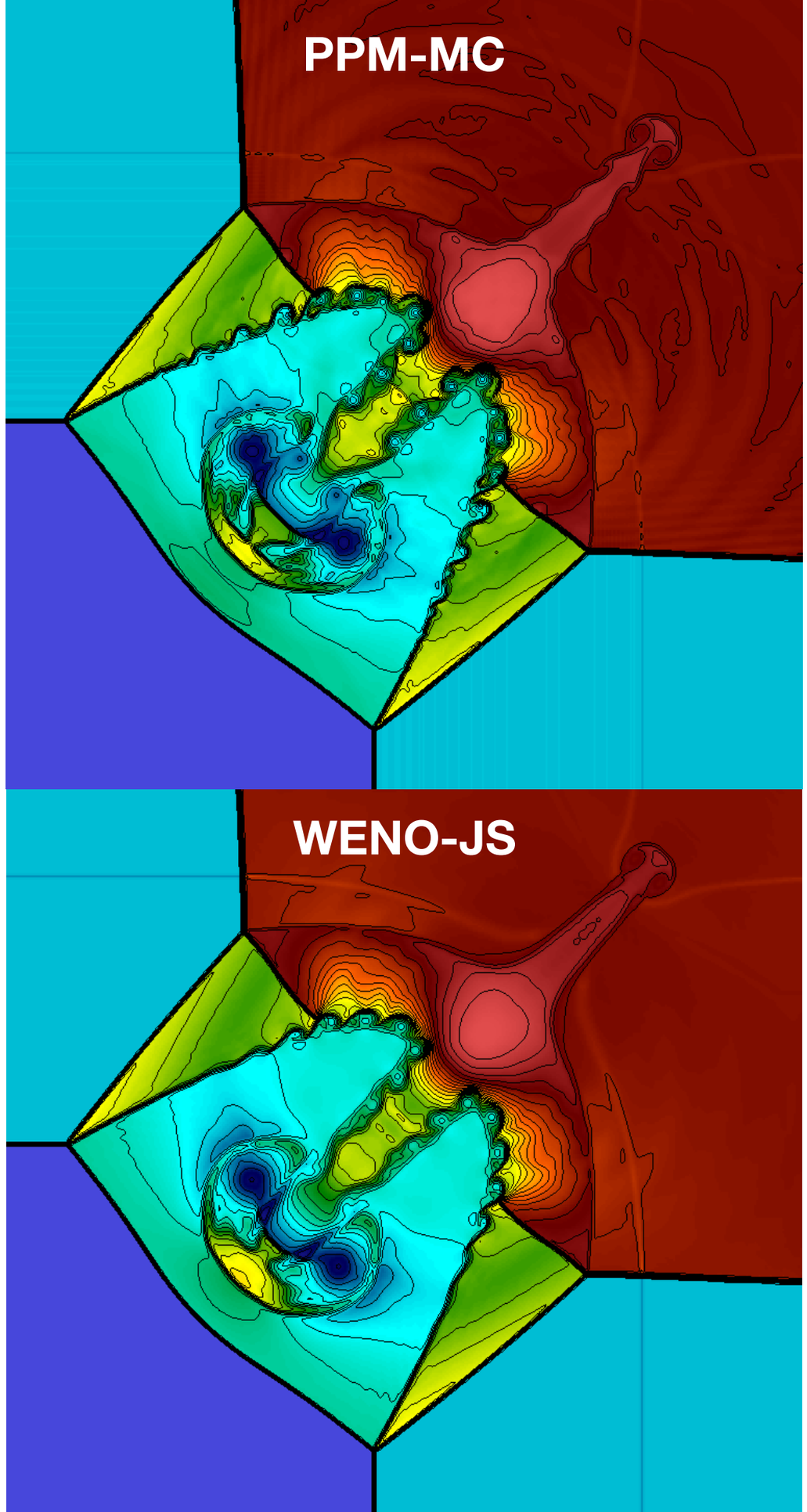


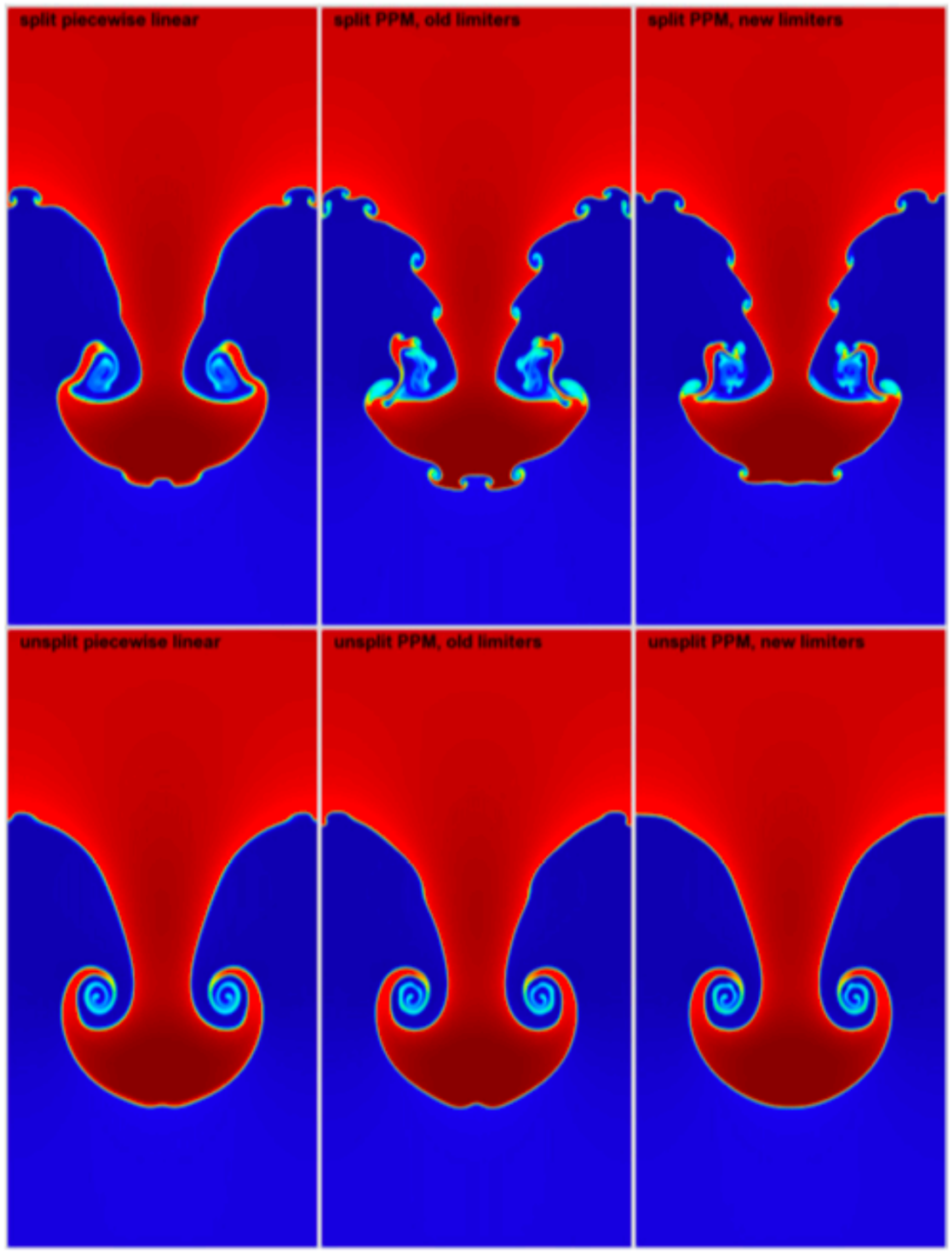
Mach 800 Jet





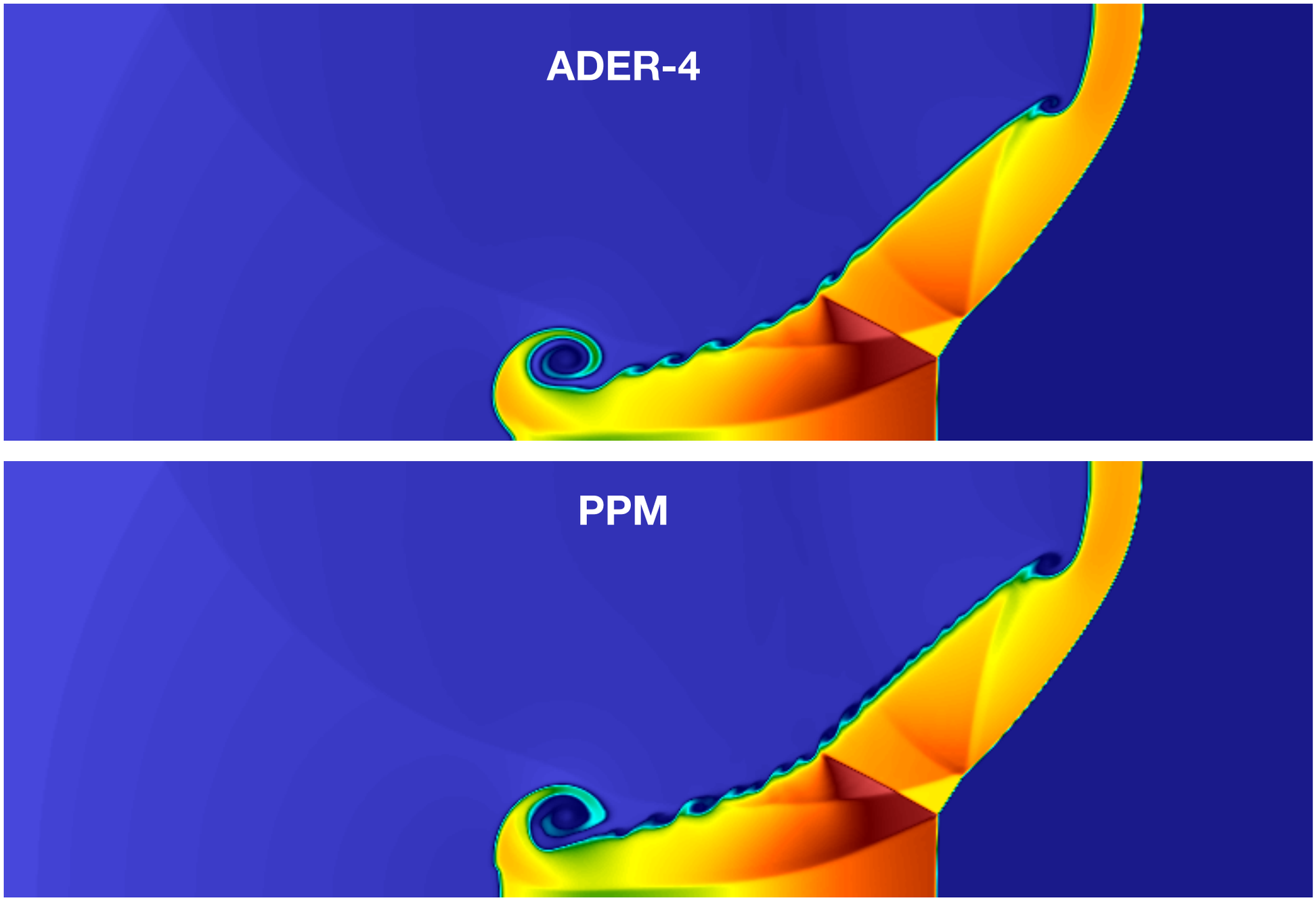

ADER4


PPM-MC
WENO-JS



Almgren, A. S., V. E. Beckner, J. B. Bell, M. S. Day, L. H. Howell, C. C. Joggerst, M. J. Lijewski, A. Nonaka, M. Singer, and M. Zingale. “CASTRO: A NEW COMPRESSIBLE ASTROPHYSICAL SOLVER. I. HYDRODYNAMICS AND SELF-GRAVITY.” *The Astrophysical Journal* 715, no. 2 (May 2010): 1221–1238. https://doi.org/10.1088/0004-637X/715/2/1221.

Shock Refraction



Turbulence in a Box