Improved Impulsive Noise Suppression Method: Joint Myriad Detection and Gaussian Fitting Robust Local Weighted Smoothing

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***Abstract*—Impulsive noise is a common impediment in many wireless communication systems, which prevents the system from error-free transmission. To this end, this paper aims to investigate the clipping and robust locally weighted regression (RLOESS) to mitigate the adverse effects of impulsive noise. To improve the impulsive noise suppression performance, a Myriad detection - Gaussian fitting robust local weighted regression smoothing (M-GLOESS) algorithm is designed. The proposed M-GLOESS finds the outliers with the help of the Myriad filter and realizes a better impulsive noise suppression performance by introducing a Gaussian fitting robust correction coefficient diagonal matrix. Simulation results are presented to verify the effectiveness of the proposed M-GLOESS, which is robust to the impulsive noise and has better performance than the traditional algorithms.**

***Index Terms*—Impulsive noise, MSK, Myriad filter, RLOESS, Clipping**

I. Introduction

Long wave propagation in the atmosphere has the characteristics of minimal attenuation, stable propagation and strong reliability, which has been widely used in military communication, such as shore-to-submarine and shore-to-ship communication. However, long-wave propagation is sensitive to impulsive noise, which is mainly caused by the atmospheric noise induced by lightning, artificial noise induced by ignition system, heavy duty power cord and current switch [1]. Different from Gaussian noise, such impulsive noise shows high amplitude, wide band and rapid appearance [2]. The long duration and high incidence of impulsive noise can significantly destroy the signal, resulting in a significant performance loss.

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In order to mitigate the adverse effects of impulsive noise, several techniques have been investigated, which mainly include blanking, clipping and their combinations [3]. For example, median filtering [4] and myriad filtering [5] were used to mitigate the impulsive noise with simple hardware and software implementation. However, their performance is typically limited, which cannot realize real-time adaption according to the time-variant channel and noise.

Another common impulsive noise suppression technique is sparse reconstruction algorithm, where compressed sensing (CS) and sparse Bayesian learning approach (SBL) are typically applied [6], [7]. Exploring the sparsity nature of impulsive noise, abundant works utilize the CS-aided and SBL-based techniques to reconstruct and mitigate the impulsive noise, which shows outstanding performance [8]–[11]. However, such techniques always require additional bandwidth for null subcarriers and strict sparsity of the impulsive noise, which is hard to implement in practice.

Locally weighted regression (LOESS) [12] can be used for Gaussian white noise mitigation. However, LOESS may lead to smooth distortion in the face of impulsive noise. To tackle this problem, robust locally weighted regression (RLOESS) was proposed, which is based on outlier data detection and iterative weighted modification [13]. By adjusting the weighting matrix during iterations, RLOESS can mitigate the impulsive noise that deviates from the normal data significantly. However, the calculation of the fitting parameters is computationally complicated due to the matrix inversion operation during the fitting weight matrix update, which leads to the low noise rejection efficiency of RLOESS.

In light of the above, a Myriad detection - Gaussian fitting robust local weighted regression smoothing (M-GLOESS) algorithm is proposed, which has reduced-complexity and better inhibitory effect for impulsive noise. More specifically, the proposed algorithm exploits the sensitivity of the Myriad

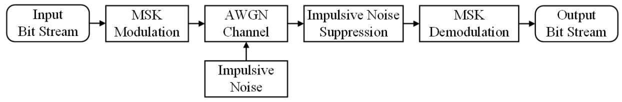

Fig. 1. Basic block diagram of the system

filter to impulsive noise, which is helpful to find the outliers. Meanwhile, the Gaussian fitting robust correction coefficient diagonal matrix is proposed, resulting in better performance for impulsive noise suppression. In addition, impulsive noise detection and local restoration algorithm are used to replace the iterations of weighting matrix adjustment in RLOESS algorithm, which reduces the computational complexity obviously.

II. System Model

The block diagram of the considered system is shown in Fig. 1, where MSK modulation signal transmission over an additive white Gaussian noise (AWGN) channel in the presence of Bernoulli-Gaussian impulsive noise is considered.

A. Signal Model

MSK signal is an FSK modulation with constant envelope and continuous phase. The baseband MSK modulation signal can be expressed as

$$s(t)=\cos\left(\omega_c t+\frac{a_k\pi}{2T_s}t+\varphi_k\right),\ kT_s<t<(k+1)T_s,\quad (1)$$

where w_c is carrier frequency, T_s is symbol period, a_k is input data, $a_k=\pm1$. The modulation frequency difference Δf is $\frac{1}{2T_s}$, the modulation index is $h=\Delta f/f_s=1/2$. φ_k is initial phase, given by

$$\varphi_k=\begin{cases}\varphi_{k-1} & a_k=a_{k-1}\\ \varphi_{k-1}+k\pi & a_k\neq a_{k-1},\end{cases}\quad (2)$$

where $k\in\mathbf{Z}$. Besides, MSK modulated signal can be obtained as

$$\begin{aligned}&s_{MSK}(t)=\cos\varphi_k\cos\tfrac{\pi t}{2T_s}\cos(\omega_c t)\\&-a_k\cos\varphi_k\sin\tfrac{\pi t}{2T_s}\sin(\omega_c t),\ (k-1)T_s\le t\le kT_s.\end{aligned}\quad (3)$$

From the above formula, we can see that the MSK modulation can be implemented with quadrature modulation, where the in-phase and quadrature components can be expressed as

$$\begin{aligned}I(t)&=I_k\cos\tfrac{\pi t}{2T_s}\cos(\omega_c t),\\Q(t)&=Q_k\sin\tfrac{\pi t}{2T_s}\sin(\omega_c t).\end{aligned}\quad (4)$$

B. Bernoulli-Gaussian Impulsive Noise

The Bernoulli-Gaussian model is the simplest form of impulsive noise. Here, the occurrence of the impulses is modeled by a binary Bernoulli distribution and the amplitude of the impulses are modeled by Gaussian distributions [14], [15]. Therefore, the impulsive noise is represented as a product of Bernoulli distribution and Gaussian distribution, given by

$$i_k=b_k g_k,\quad (5)$$

where b_k is the Bernoulli process, that is, an independent and identically distributed (i.i.d.) sequence that takes a value of 1 with a probability $p(b_k=1)=\lambda$ and a value of 0 with a probability $p(b_k=0)=1-\lambda$, and g_k is the Gaussian process. Here, $b_k=1$ indicates the presence of an impulse and $b_k=0$ means the absence of an impulse. Therefore, the combined noise seen at the receiver is given as [14]

$$n_k=\omega_k+b_k g_k,\quad (6)$$

where w_k is the background AWGN noise. The probability density function (PDF) of n_k is given by [15]

$$f(n_k)=\frac{1-\lambda}{\sqrt{2\pi\sigma_w^2}}\exp\left(-\frac{n_k^2}{2\sigma_w^2}\right)+\frac{\lambda}{\sqrt{2\pi\sigma_g^2}}\exp\left(-\frac{n_k^2}{2\sigma_g^2}\right),\quad (7)$$

where σ_w^2 is the variance of the background Gaussian noise and σ_g^2 is the variance of the impulsive noise. The whole model can be represented by σ_w^2, σ_g^2, and λ, and the total noise variance is $\sigma_n^2=(1-\lambda)\sigma_w^2+\lambda\sigma_g^2$.

III. Impulsive Noise Suppression

A common and simple approach for impulsive noise suppression is to detect high peak amplitudes in the time domain and reduce them. This method may reduce the effect of large received signal amplitudes which can be regarded as impulsive noise. In general, different nonlinear methods such as clipping and RLOESS have been widely investigated in case of MSK modulation transmission.

A. Conventional Impulsive Noise Suppression Schemes

In this section, we first present several traditional impulsive noise suppression techniques.

For clipping, the received signal samples are compared to a clipping threshold T_c. If the absolute value of any signal sample exceeds T_c, it is clipped as follows [16]

$$r_k=\begin{cases}y_k & if\ |y_k|\le \mathrm{T}_c\\ T_c e^{jarg(y_k)} & otherwise\end{cases},\quad (8)$$

where r_k is the clipped output of the received signal y_k.

RLOESS is a non-parametric smoother with linear regression. The general idea of RLOESS is to take a point x as the center, intercept a data sequence of length N_d both forward and backward and make a weighted regression on the data with a weight function w. For all L data points, L weighted regression lines can be made, and the line of the center value of each regression line is the RLOESS curve of this data.

Let the local data sequence be $D=[(x_n,y_n)], n=1,2,...,N$, where x_n is the position parameter of the n-th data, y_n is the value of the n-th data, N is the length of the sequence and usually odd.

RLOESS is essentially a polynomial fitting smoothing process, which can be expressed as $\mathbf{Y}=\mathbf{X}\boldsymbol{\alpha}$, where $\mathbf{X}\in\mathbf{R}^{N\times(M+1)}$ is a polynomial location parameter matrix, whose element in the n-th row and m-th column is x_n^{m-1}, M is the polynomial order, $\mathbf{Y}\in\mathbf{R}^{N\times1}$ is a local data matrix, whose element in the n-th row is y_n, $\boldsymbol{\alpha}\in\mathbf{R}^{(M+1)\times1}$ is a polynomial fitting parameter matrix of $\mathbf{X}$, whose element in the m-th row is the $m-1$ order fitting parameter α_{m-1}.

Based on the polynomial fitting smoothing process, RLOESS is a robust smoothing process of outliers by adaptively adjusting the fitting weights. RLOESS algorithm uses weighted least squares method to solve for $\boldsymbol{\alpha}$ and obtain $\boldsymbol{\alpha}_{RLOESS}$, given by

$$\boldsymbol{\alpha}_{RLOESS} = (\mathbf{X}^T\mathbf{W}\boldsymbol{\Delta}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}\boldsymbol{\Delta}\mathbf{Y}, \tag{9}$$

where $\boldsymbol{\Delta} \in \mathbf{R}^{N\times N}$ is a diagonal matrix of robust correction coefficients, whose element in the n-th row and n-th column is the robust correction coefficient δ_n of the fitting weight for the n-th point, and its initial value is 1, $\mathbf{W} \in \mathbf{R}^{N\times N}$ is a diagonal matrix of fitting weights, whose element in the n-th row and n-th column is the fitting weight w_n for the n-th point valued according to a cubic kernel function as follows [12]

$$w_n = \left(1 - \left|\frac{x_s - x_n}{d(x_s)}\right|^3\right)^3, \tag{10}$$

where x_s is the filtering position, $d(x_s)$ is the farthest distance from x_s to other points in the sequence window,

$$d(x_s) = \max\{|x_s - x_n|\} \quad n = 1, 2, ..., N. \tag{11}$$

After $\boldsymbol{\alpha}_{RLOESS}$ is obtained, the smoothed data series $\mathbf{Y}_{RLOESS}$ can be obtained as follows

$$\mathbf{Y}_{RLOESS} = \mathbf{X}\boldsymbol{\alpha}_{RLOESS}, \tag{12}$$

and the smoothing residuals $\mathbf{r} \in \mathbf{R}^{N\times 1}$ are calculated as follows, where the n-th row element is r_n,

$$\mathbf{r} = |\mathbf{Y} - \mathbf{Y}_{RLOESS}|. \tag{13}$$

Take the median of $\mathbf{r}$ as ρ, use 6ρ as the outlier detection threshold, calculate δ_n as follows

$$\delta_n = \begin{cases} (1 - |\frac{r_n}{6\rho}|^2)^2 & |r_n| < 6\rho \\ 0 & |r_n| \geq 6\rho \end{cases}. \tag{14}$$

Maximum number of iterations is denoted as T_R. Besides, stop the iteration and output $\mathbf{Y}_{RLOESS}$ if the smoothing residuals are all less than 6ρ, otherwise recalculate $\boldsymbol{\alpha}_{RLOESS}$ and $\mathbf{Y}_{RLOESS}$ by Eq(9) and Eq(12).

B. Proposed M-GLOESS

In this section, an improved RLOESS is proposed to obtain better suppression performance of impulsive noise.

Since $\boldsymbol{\alpha}_{RLOESS}$ has to be recalculated after each iteration of updating the fitting weight robust correction coefficient, the matrix multiplication and inversion operations have high computational complexity, which leads to low denoising efficiency. The local smoothing residual median of the finite length window is greatly affected by the local geometric feature randomness, which is easy to have a large median value and leads to low reliability of the outlier detection threshold. Moreover, it is difficult to characterize the normal level of background noise intensity, which may cause small amplitude random impulsive noise to be misjudged as background noise. Since the fitting weight robust correction coefficient is related to the smoothing residual and median, it makes the frequency domain distortion random, which makes it difficult to compensate for the oscillation sequence analysis results.

Based on the above analysis, we propose the Myriad detection - Gaussian fitting robust local weighted regression smoothing (M-GLOESS) method, which reduces the computational complexity while maintaining good denoising ability.

The proposed M-GLOESS method consists of two main parts: outlier detection based on the Myriad filtering scheme and local regression smoothing with fitted Gaussian distributions for the outliers.

1) Myriad Detection:

Myriad filter is a robust nonlinear filter developed on the basis of mean filter and median filter, which is very sensitive to outliers and suitable for impulsive noise environment.

In our proposed scheme, we use the Myriad filter to exploit its sensitivity to impulsive noise for outlier detection. The received signal is processed using the Myraid algorithm, given by

$$\tau = \arg\left\{\min_{\beta} \prod_{i=1}^{L} [K^2 + (|y_i| - \beta)^2]\right\}, \tag{15}$$

where $\mathbf{Y} = [y_1, y_2, ..., y_L]^T$ denotes the noisy signal sequence, and $|y_i|$ is the amplitude of $y_i, i = 1, 2, ..., L$. L is the signal length, K is the linearization parameter, which can be adjusted according to the strength of the impulsive noise. It should be noticed that the result of the Myriad algorithm is to obtain the value that minimizes the loss function. When K is fixed, choose the optimal τ to minimize the sum of logarithms of squared distances from each sample point to the ideal point [17].

For convenience, the above formula can be re-expressed as

$$\tau = \arg\left\{\min_{\beta} \sum_{i=1}^{L} \log[K^2 + (|y_i| - \beta)^2]\right\}. \tag{16}$$

The output result is used as a threshold to detect the impulsive noise, and the position of the impulsive noise is recorded as $\mathbf{U} = [u_1, u_2, \ldots, u_H]^T$, where $u_h, h = 1, 2, ..., H$ is the position of detected outliers, and H is the number of detected outliers.

2) Gaussian Fitting Robust Local Weighted Smoothing:

The RLOESS algorithm uses a robust correction coefficient calculated by the amplitude of the residuals to suppress impulsive noise. It requires global smoothing of all data and does not make full use of channel information. Considering the above problems, a robust correction coefficient based on Gaussian distribution is introduced.

Assuming that the signal sequence $\hat{\mathbf{Y}} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_{L-H}]^T$ after removing the outliers does not contain impulsive noise, the envelope of $\hat{\mathbf{Y}}$ can be approximated by a Gaussian distribution. The PDF of the corresponding Gaussian distribution is calculated as follows

$$f_G(a) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(a-\mu)^2}{2\sigma^2}\right), \tag{17}$$

where μ and σ^2 are the mean and variance of the amplitude of the signal sequence $\hat{\mathbf{Y}}$ respectively, a represents the input variable of the PDF, which is the amplitude of signal.

The impulsive noise influences the original data, so local smoothing and repair are needed at the impulsive noise positions. A sliding window with a length of N is used. The impulsive noise point u_h is taken as the midpoint of the window and Gaussian fitting robust local weighted smoothing is performed. The local data sequence in the sliding window is denoted as $\mathbf{Y}_N = [y_{u_k-N_d}, \ldots, y_{u_k}, \ldots, y_{u_k+N_d}]^T$, where N_d denotes half the window length, $N_d = (N-1)/2$. The corresponding polynomial location parameter matrix is denoted as $\mathbf{X}$.

The Gaussian fitting robust local weighted smoothing form can be written according to Eq(9), (12). $\mathbf{Y}_{GLOESS}$ can be obtained as

$$\mathbf{Y}_{GLOESS} = \mathbf{X}\boldsymbol{\alpha}_{GLOESS}, \tag{18}$$

where $\boldsymbol{\alpha}_{GLOESS}$ can be calculated as

$$\boldsymbol{\alpha}_{GLOESS} = (\mathbf{X}^T\mathbf{W}\boldsymbol{\Delta}_G\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}\boldsymbol{\Delta}_G\mathbf{Y}_N, \tag{19}$$

where $\boldsymbol{\Delta}_G \in \mathbf{R}^{N\times N}$ is the proposed improved Gaussian fitting robust correction coefficient diagonal matrix, whose element in the n-th row and n-th column is the robust repair weight δ_{Gn} of the n-th point. δ_{Gn} is the probability density value of the fitted Gaussian distribution, which can be calculated as

$$\delta_{Gn} = f_G(|y_n|), \tag{20}$$

where y_n is the n-th sample point of $\mathbf{Y}_N$, $n = 1, 2, ..., N$, and $|y_n|$ is the amplitude of y_n.

To obtain the smoothed signal sequence, at the detected outliers position, the central value of $\mathbf{Y}_{GLOESS}$ is used. By sliding the window, Gaussian fitting robust local weighted smoothing is performed at each impulsive noise point $u_h, h = 1, 2, ..., H$. The details of M-GLOESS is summarized in Algorithm 1.

Algorithm 1 Proposed M-GLOESS algorithm

Input: $\mathbf{Y} = [y_1, y_2, \ldots, y_L]^T$.
Solve (16) to obtain τ.
Initialization: $k = 1$.
for $i = 1, 2, \ldots, L$ **do**
  if $y_i > \tau$ **then**
    $u_k = i,\ k = k + 1$.
  end if
end for
Obtain the outliers position $\mathbf{U} = [u_1, u_2, \ldots, u_H]^T$.
Remove the outliers from $\mathbf{Y}$ to obtain $\hat{\mathbf{Y}}$.
Solve (17) to obtain the PDF $f_G(a)$.
for $k = 1, 2, \ldots, H$ **do**
  $\mathbf{Y}_N = [y_{u_k-N_d}, \ldots, y_{u_k}, \ldots, y_{u_k+N_d}]^T$.
  $\boldsymbol{\delta}_n = f_G(|\mathbf{Y}_N|)$.
  $\boldsymbol{\Delta}_G = diag(\boldsymbol{\delta}_n)$.
  Solve (19) to obtain $\boldsymbol{\alpha}_{GLOESS}$.
  Solve (18) to obtain $\mathbf{Y}_{GLOESS}$.
  Replace y_{u_k} with the central value of $\mathbf{Y}_{GLOESS}$.
end for
Output: $\mathbf{Y}_{deal} = [y'_1, y'_2, \ldots, y'_L]^T$.

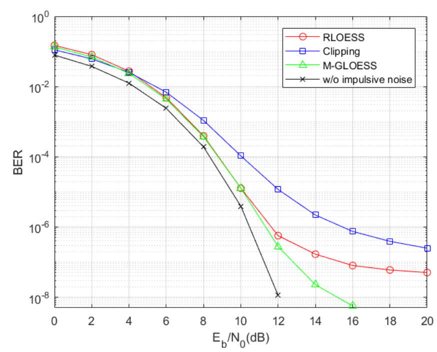

Fig. 2. BER comparison of various algorithms versus E_b/N_0 with impulsive noise $\lambda = 0.05$, SINR $= -30$dB

IV. Numerical Results

In this section, numerical simulations are presented to evaluate the performance of the proposed algorithm. The simulation modulates data using MSK modulation. Besides, Bernoulli Gaussian model is used to model the impulsive noise, and an AWGN channel is considered. The ratio of bit energy to noise power spectral density E_b/N_0 is set to $E_b/N_0 = [0:2:20]$dB. The impulsive noise is generated with $\lambda = [0.01, 0.05]$ and SINR $= [-30, -20]$dB, which means 1% or 5% samples in the time domain are affected by an impulsive noise with SINR $= -30$dB or -20dB. The signal to impulsive noise ratio (SINR) is defined as the ratio of signal power to impulsive noise power.

Fig. 2 compares the bit error rate (BER) performance of the proposed M-GLOESS algorithm, clipping, and RLOESS algorithm, where the impulsive noise is generated with $\lambda = 0.05$, $SINR = -30$dB. It can be observed that the BER performance of the proposed M-GLOESS algorithm is better than other methods. Especially, the gap between the proposed algorithm and the comparison methods increases with the E_b/N_0. It shows that the proposed Myriad-GLOESS algorithm delays the error platform in the presence of impulsive noise.

Fig. 3 shows the BER performance of the proposed algorithm under different impulsive noise conditions. It can be observed that the proposed algorithm has better BER performance with smaller λ, which means fewer signal samples are disturbed. When SINR is smaller, the difference between the impulsive noise and the transmitted signal in amplitude is more obvious, and it is easier to detect the impulsive noise in the first step of Myriad detection. When $\lambda = 0.01$ and SINR $= -30$dB, the BER curve of the proposed algorithm is progressively coincident with the BER curve without impulsive noise, indicating that the proposed algorithm can effectively suppress impulsive noise.

The above simulation results show that the BER performance of the proposed algorithm is significantly improved compared with clipping and RLOESS, which is more robust

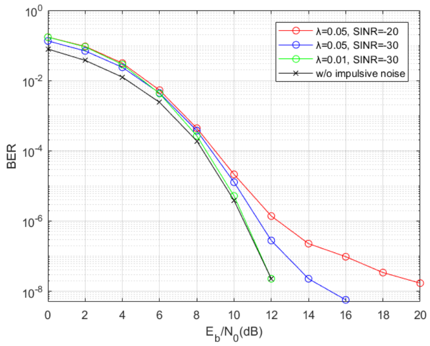

Fig. 3. BER performance of proposed algorithm with different impulsive noise conditions

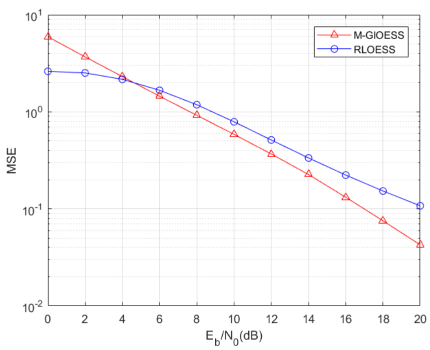

Fig. 4. MSE performance comparison of proposed algorithm and RLOESS

to impulsive noise.

Fig. 4 compares the mean-square error (MSE) performance of the proposed algorithm and RLOESS algorithm for impulsive noise suppression. The impulsive noise is generated with parameters $\lambda = 0.05$ and SINR $= -20$dB. It can be observed that after impulsive noise suppression by the proposed algorithm, the MSE of the processed signal and the original signal has a significantly decrease compared with RLOESS algorithm. It is proved that the proposed algorithm can effectively suppress impulsive noise.

Finally, we compare the computational complexity of the proposed algorithm and RLOESS. RLOESS needs to perform regression smoothing at each signal sample, and improve the robustness to impulsive noise through iterative operation, which requires a lot computational resource. The proposed algorithm firstly performs Myriad detection and then performs regression smoothing only at the outliers, which greatly reduces the amount of computation. The two algorithms both are based on the least square method, the number of least squares of RLOESS is $L * I$, where L is the length of signal samples, I is the number of iterations. While the number of least squares solutions of the proposed algorithm is H, where H is the number of outlier samples detected. Considering time domain sparsity of the impulsive noise, $H \ll L$. The proposed algorithm effectively reduces the time complexity and improves the impulsive noise suppression efficiency.

V. Conclusion

In this paper, we design a Myriad detection - Gaussian fitting robust local weighted regression smoothing (M-GLOESS) algorithm to suppress the impulsive noise in the MSK modulated systems. The algorithm utilizes the sensitivity of the Myriad filter to impulsive noise to find the affected outliers, and introduces a Gaussian fitting robust correction coefficient diagonal matrix to obtain better impulsive noise suppression performance. Simulation results show that the proposed algorithm is robust to impulsive noise and has better impulsive noise suppression performance than tradition clipping and RLOESS method.

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