# Indirect Learning Hybrid Memory Predistorter Based On Polynomial and Look-Up-Table

Zheren Long , Hua Wang, Ning Guan, Nan Wu, Dongxuan He
Fundamental Science on Multiple Information Systems Laboratory
School of Information and Electronics
Beijing Institute of Technology
Beijing, China

***Abstract*—Baseband predistortion is a popular and efficient method to linearize high power amplifier (HPA) in wireless communication systems. Polynomial (POLY) and look-up-table (LUT) are two methods to design baseband predistorter (PD). However, on the one hand, POLY-based method is complex to implement. On the other hand, LUT-based predistorter suffers convergence time and quantization error problem. In this paper, we propose a hybrid POLY and LUT predistorter for memory nonlinear system in wideband scenarios, it is also suitable for memoryless channel. Simulations show that the proposed hybrid structure outperforms the traditional one with lower complexity.**

***Keywords*—memory predistorter, look-up-table, polynomial, indirect, LMS.**

## I. Introduction

In wireless communication systems, the nonlinearity is usually caused by high power amplifier (HPA). When HPA works near the saturation point, higher power efficiency is achieved, however the nonlinearity becomes more serious. When the non-constant envelope higher-order modulation methods (such as APSK, QAM) are used to improve spectrum efficiency, amplitude modulation-amplitude modulation (AM-AM) and amplitude modulation-phase modulation (AM-PM) distortions caused by HPA become significant. Baseband predistortion is an efficient technology to counter the above effects.

Although many models are used to characterize HPA, look-up-table (LUT) and polynomial (POLY) are widely used for their advantages in terms of performance and complexity [1]. These two models are also used to design predistorter [2]. However, LUT and POLY have their own disadvantages. Due to the finite size, the quantization effect is serious and look-up-table predistorter (LUT PD) cannot be smooth enough. Better performance will be gotten when LUT with larger size is used, but more training samples are required. As to polynomial predistorter (POLY PD), the highest order is usually lower than the upsample rate of shaping filter, which limits its application. Moreover, the complexity of estimating the coefficients rises rapidly when the order grows, especially for memory polynomial structure [3].

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Corresponding author: Hua Wang (e-mail: wanghua@bit.edu.cn)

LUT PD can be categorized into two types: polar coordinate PD and rectangular coordinate PD. The latter one is more popular because it doesn't need any coordinate conversion. Nagata uses double two-dimensional LUT to map PD [4]. However, the quantization noise variance is too large. Cavers proposes a complex gain PD using one-dimensional LUT [5]. Multiplied by the original signal, the system obtains interpolation gain. In that case, the LUT is indexed by the power of signal, which is found not the best in practice [6]. The authors of [6] investigate different quantization methods, LUTs indexed by amplitude, power, $\mu$-law, and optimum unequal space are compared. Finally the equal space based on amplitude is found the most practical. Also, memory effects are taken into consideration, Hoh uses the short-term average power as the second dimension index to get a memory LUT for OFDM system [7]. On the other side, a POLY PD is described in [8]. Memory POLY PD is proposed by Kim [9]. In fact, memory POLY is a special case of Volttera series which is the best to describe nonlinear and memory system, and it is widely used. However, the POLY PD may be numerical instable when the input data is strong correlative. Some methods are proposed to alleviate this problem, such as orthogonal polynomial [10] and QRD-RLS [11]. According to the different characters between LUT PD and POLY PD, the authors of [3] propose a joint PD, which outperforms the single one. However, it is based on direct learning structure that cannot work offline, and the memory efforts are not taken into consideration.

This paper proposes a hybrid indirect learning predistorter based on polynomial and look-up-table for memory nonlinear system. According to the order of combination and training, four structures are compared. Through analysis and simulation, the predistorter under combination method of POLY followed by LUT, while POLY is trained first, is found the best. With lower polynomial order and smaller LUT entry size, the hybrid structure outperforms the single one. As a special case of memory scenarios, the new predistorter is suitable for memoryless nonlinear system too.

## II. System Overview

The block diagram of the proposed PD is illustrated in Fig. 1, with indirect learning architecture [12] that can work offline. For a Volterra system, [13] has provided that the p-order postinverse is the same as p-order preinverse. Therefore, the response of an amplifier followed by postdistortion is the

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same as the response to the same model applied before it. The proposed PD is a kind of waveform PD which processes signal after shaping, and it can be adapted to different modulation methods.

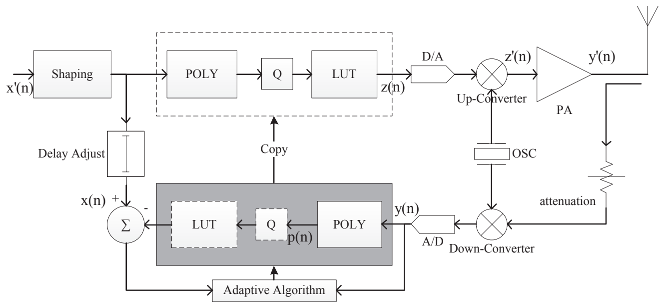


Fig. 1: block diagram of transmitter with PD

At the beginning, the PD does not work, and the shaping signal will be sent directly. After the original and feedback baseband signals are aligned (we assume the data has been aligned perfectly in this paper), the output is normalized to the input, then they are used to build the postinverse PD as the shaded part in Fig. 1. Finally, the postinverse PD is coped to preinverse PD and PD will work in the transmitter. The input and output of RF PA is $z'(n)$ and $y'(n)$. For a baseband PD, we are only interested in baseband signal $z(n)$ and $y(n)$. In the part below, we will discuss on baseband signal only and they are presented in complex form.

Saleh model is used to characterize memoryless power amplifier (PA), the AM/AM and the AM/PM characteristic are expressed as $f(\rho_{in})$ and $g(\rho_{in})$

$$f(\rho_{in}) = \frac{\alpha_\rho \rho_{in}}{1+\beta_\rho \rho_{in}^2}, g(\rho_{in}) = \frac{\alpha_\theta \rho_{in}^2}{1+\beta_\theta \rho_{in}^2} \tag{1}$$

where $\rho_{in}$ is the amplitude of input signal and $\alpha_\rho$, $\beta_\rho$, $\alpha_\theta$, $\beta_\theta$ are parameters.

Winner and memory polynomial models are used to characterize memory PA. Winner model is a memory system followed by a memoryless nonlinear system. Usually, the memory part is a FIR filter.

Memory polynomial model is expressed as

$$y(n) = \sum_{k=1}^{K} \sum_{i=0}^{I} b_{ki} x(n-i) |x(n-i)|^{k-1} \tag{2}$$

where $x(n)$ and $y(n)$ are the equivalent baseband input and output, $b_{ki}$ is the coefficient, $K$ and $I$ are the highest order and memory depth.

## III. Predistorter Structure and Adaptive Algorithm

LS, RLS and LMS are three popular algorithms in adaptive filter [14] that can be used for PD. However, matrix inversion is needed in LS and RLS which may have ill-conditioned matrix problem. LMS is used in this paper.

### A. Polynomial Predistorter

The diagram of memory polynomial predistorter (MP PD) with 2-memory depth is shown in Fig. 2, the shaded part is a POLY PD. Since even-order powers in the polynomial representing the PA does not reflect into the first harmonic zone of output, only odd powers are needed when implementing the POLY PD [15].

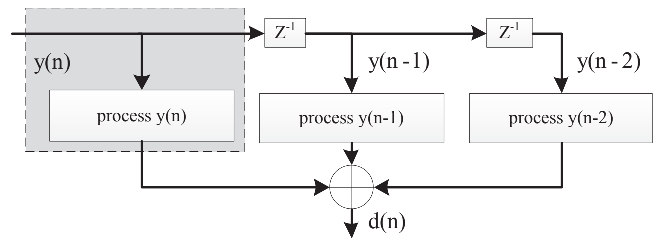


Fig. 2: block diagram of MP PD (memory depth is 2)

The output of a 2K+1 order MP PD with Q-memory depth at time $n$ can be described as

$$d(n) = y(n) \cdot P_0(|y(n)|) + y(n-1) \cdot P_1(|y(n-1)|) + \cdots + y(n-Q) \cdot P_Q(|y(n-Q)|) \tag{3}$$

where $P_q(|y(n-q)|) = a_{1,q} + a_{3,q} \cdot |y(n-q)|^2 + \cdots + a_{2k+1,q} \cdot |y(n-q)|^{2k} + \cdots + a_{2K+1,q} \cdot |y(n-q)|^{2K}$, $a_{2k+1,q}$ is the complex coefficient needed to be resolved.

### B. LUT Predistorter

The diagram of memory look-up-table predistorter (MLUT PD) with 2-memory depth is shown in Fig. 3. Input signal is quantized in equal amplitude space, and Q module denotes a quantizer. $Q(|y(n)|)$ is the index of LUT corresponding to the amplitude of $y(n)$, and $L(Q(|y(n)|))$ is the mapping output which is updated during training process. Memoryless LUT PD is a special case of MLUT PD, as the shaded part in Fig. 3.

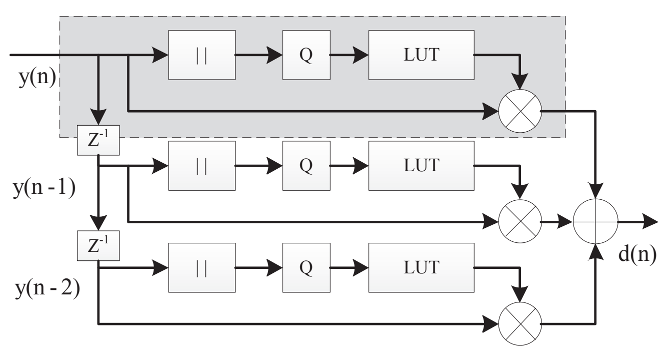


Fig. 3: block diagram of MLUT PD (memory depth is 2)

where $\boldsymbol{y}(n) = [y(n), y(n-1), \cdots, y(n-M)]$, $\boldsymbol{L}(n) = [L_0(Q(|y(n)|)), \ldots, L_M(Q(|y(n-M)|))]$, $N$ is the size of LUT entry, $Q(\cdot) \in (1, \cdots, N)$.

The output of a MLUT PD with M-memory depth at time $n$ can be given as:

$$d(n) = \boldsymbol{L}(n) \cdot \boldsymbol{y}^T(n) \tag{4}$$

In fact, LUT PD is a POLY PD with infinite order, so its performance is better when the system is convergent perfectly with enough LUT entry. However, during the training process, only one LUT will be updated, it is limited due to convergence time.

### C. Proposed Predistorter

Sharing the infinite order of LUT PD and quick convergence speed of POLY PD, hybrid PD combing low order POLY and small size LUT is proposed. As memoryless PD is a memory PD with 0-memory depth, we will only discuss on memory PD in the below part.

*1) Processing of Hybrid PD*

The hybrid PD has two configurable parameters: the cascade sequence and training order, and discussion will be given later. Here, the processing of hybrid PD with structure POLY followed by LUT while POLY part is identified first (we use '*POLY*+LUT' to denote this structure. The expression indicates the structure, and the italic part means the component trained first) is as follows:

*1.Train the POLY PD until it has been converged.*

This processing is the same as traditional POLY. At the beginning, the 1-order, 0-memory depth coefficient is set to one, others are all zeros. As shown in Fig. 1.

$$p(n) = \sum_{k=0}^{K}\sum_{q=0}^{Q} a_{2k+1,q}\, y(n-q)|y(n-q)|^{2k} = \sum_{q=0}^{Q} FP_q(y(n-q)) \tag{5}$$

where $2K+1$ is the highest order, $Q$ is the memory depth, and $FP_q(y(n-q))$ is the q-memory depth characteristic function of MP PD.

LMS algorithm is chosen for adaptive filter, and the coefficient $a_{2k+1,q}$ is iterated as follows:

$$a_{2k+1,q}^{i+1} = a_{2k+1,q}^{i} + \mu\cdot(y(n-q)\cdot|y(n-q)|^{2k})^{*} \cdot(x(n)-p(n)) \tag{6}$$

where $p(n)$, $x(n)$ are shown in Fig. 1. $i$ is the iteration number, and $\mu$ is the step size to control the balance between convergence speed and stability.

*2.Train the LUT PD with POLY PD.*

After POLY part has been trained, LUT part will be trained next. All mapping relationships in 0-memory depth LUT are ones, and others are all zeros. The mapping relationship $FL(Q(|y(n)|))$is updated as follows:

$$\boldsymbol{FL}^{i+1}(n) = \boldsymbol{FL}^{i}(n) + \mu\cdot\boldsymbol{p}(n)^{*}\cdot(x(n)-d(n))$$
$$d(n) = \boldsymbol{FL}^{i}(n)\cdot\boldsymbol{p}^{T}(n) \tag{7}$$

where $\boldsymbol{p}(n) = [p(n), p(n-1), \cdots, p(n-M)]$, $\boldsymbol{FL}^{i}(n) = [FL_0^i(Q(|p(n)|)), \ldots, FL_M^i(Q(|p(n-M)|))]$, and $M$ is the memory depth of LUT entry. $Q(\cdot)\in(1,\cdots,N)$ , $N$ is the size of LUT entry. $i$ is the iteration number, $\mu$ is the step size.

*3.Copy the postinverse PD to the preinverse PD.*

When both parts in hybrid PD have been trained, the parameters of postivese PD are copied to the preinverse one as illustrated in Fig.1. After that, the hybrid PD is ready to work.

*2) Cascade sequence and training order*

As mentioned above, according to the cascade sequence and training order, four hybrid PDs: *POLY*+LUT, POLY+*LUT*, *LUT*+POLY, LUT+*POLY* exist. If the structure is POLY followed by LUT and LUT is identified first(POLY+*LUT*), the output of POLY may exceed the range of LUT when POLY is being trained. POLY+*LUT* is impractical, and will not be included in the results. In order to reduce hardware complexity, LUT with small size will be used. When LUT is identified first, the quantization effect is serious, leading to sawtooth transmission characteristics that cannot be compensated by POLY. Hence, the POLY should be trained first in hybrid PDs. Through simulations, it is found that better performance will be achieved when POLY is in front of LUT. It could be due to the unequal quantization of soure signal. In that circumstances, POLY and PA are separated by LUT.

*3) Complexity comparison*

The complexity of single and hybrid PD consists of two parts: the hardware complexity of predistorter and the computational complexity of adaptive filter updating. The comparison between 2K+1-order MP PD with Q-memory depth, M-memory depth MLUT PD with N-entry and hybrid PD will be given below.

As an expansion of memoryless PD, the memory PD is (memory depth+1) times the hardware complex of of memoryless PD. According to [3], the hybrid PD combining 3-order MP PD with 2-memory depth and 2-memory depth MLUT PD with 16-entry requires 60% transistors of 7-order MP PD with 2-memory depth, 40% transistors of 2-memory depth MLUT PD with 64-entry.

As for computational complexity, because subtraction is used just for calculating the difference between expectative and real output, only addition and multiplication are discussed. The computational complexity is listed in Table. I. Due to the independence between the complexity and the size of LUT entry, only MP PD and hybrid PD are given.

TABLE I: Computational Complexity comparison

| | MP PD | Hybrid PD |
|---|---|---|
| addition | $(2K+1)(Q+1)$ | $(2K+1)(Q+1)+M+1$ |
| multiplication | $(\frac{K^2+7K-2}{2})(Q+1)$ | $(\frac{K^2+7K-2}{2})(Q+1)+2(M+1)$ |

As shown in Table. I, the total number of addition and multiplication of 7-order MP PD with 2-memory depth, are 21 and 42, while 12 additions and 15 multiplications for hybrid PD combining 3-order MP PD with 2-memory depth and 2-memory depth MLUT PD with 16-entry. The computational complexity of hybrid PD is lower than MP PD. Also the size of hybrid PD's LUT entry is much smaller than MLUT PD with 128-entry, much smaller iterations are needed for convergence and a better performance will be gotten as shown in simulation results.

## IV. Simulation Results

### A. Performance estimation

In order to test the hybrid PD, power spectrum density (PS-D) is used for estimating the out-of-band performance, error vector magnitude (EVM) is used for in-band performance. And normalized mean square error (NMSE) is observed for convergence speed and total performance.

Welch method is used for PSD estimating, and 'Reference Signal' is chosen as normalization method for EVM.

In that way,

$$EVM_{RMS} = \sqrt{\frac{\sum_{n=1}^{N} |d(n) - x(n)|^2}{\sum_{n=1}^{N} |x(n)|^2}} * 100\% \quad (8)$$

The NMSE can be defined as

$$NMSE = \frac{\sum_{n=1}^{N} |d(n) - x(n)|^2}{\sum_{n=1}^{N} |d(n)|^2} \quad (9)$$

where $N$ is the length of data, $d(n)$ is the real signal and $x(n)$ is the ideal signal.

### B. Simulation Parameters and Results

For all scenes, the baseband input is 16-quadrature amplitude modulation (16-QAM). The upsample rate is 8. Root raised-cosine shaping filter with roll-off factor of 0.35 is used. The step size of LMS algorithm for POLY is 0.1 and 0.5 for LUT. For memory system, traditional MP PD is 7-order with 2-memory depth, MLUT PD is 2-memory depth with 128-entry, and Hybrid PD consists 3-order MP PD with 2-memory depth and 2-memory depth MLUT PD with 16-entry. For memoryless system, parameters are the same excepting that memory depth is 0 and traditional LUT is 64-entry.

*1) Memory system*

Wiener model PA is tested, the coefficients of FIR filter is [0.7692 0.1538 0.0769] and the nonlinear part is Saleh model.

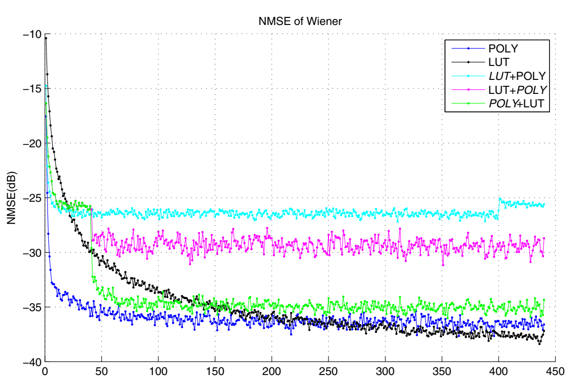


Fig. 4: NMSE learning curve for Wiener PA

Learning trajectories based on NMSE are given in Fig. 4. In order to make sure every part is well convergent, more iterations are used. The top three NMSEs are obtained by LUT, POLY and *POLY*+LUT. The performance of LUT exceeds *POLY*+LUT after more than 25000 iterations. However, due to the low order of POLY and small size of LUT, only less than 15000 iterations are needed for *POLY*+LUT to become convergent which is more practical. The slow convergence speed of LUT with large entry is shown clearly. 5-order MP PD with 2-memory depth is worse than *POLY*+LUT which is not plotted. As analysed before, *LUT*+POLY is the worst, nearly 10dB worse than *POLY*+LUT. The quantization effects caused by small LUT entry cannot be compensated by POLY, even worse as shown in the 80000 iterations. So, *LUT*+POLY will not be included in other models for simplicity. Also it is interesting to find that *POLY*+LUT is nearly 5 dB better than LUT+*POLY* when POLY is not connected with PA directly.

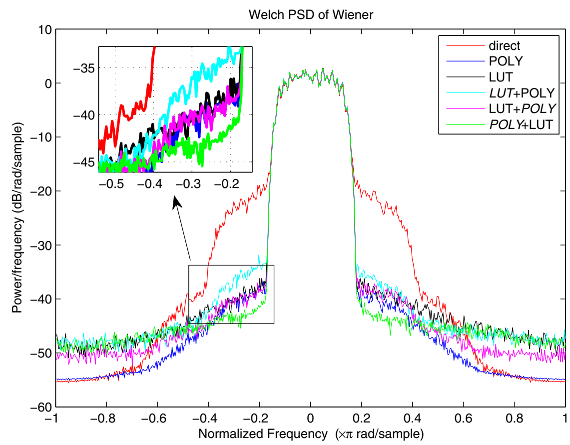


Fig. 5: PSD of Wiener PA output

PSDs with and without PD are shown in Fig. 5. The side lobe of *POLY*+LUT is suppressed best at $\pm 0.2\pi$, nearly 21 dB better than original signal, 6 dB better than *LUT*+POLY, and 3 dB better than LUT+*POLY*. The PDs consisting of LUT obtain worse performance than POLY in farther sideband, as POLY is the best from $\pm 0.4\pi$ to $\pm \pi$. In that zone, signal can be filtered easily and the PSD of *POLY*+LUT is less than -45dB which is good enough.

TABLE II: EVM after Wiener PA

| | Original | POLY | LUT |
|---|---|---|---|
| EVM | 29.36% | 0.67% | 1.17% |
| | *LUT*+POLY | LUT+*POLY* | *POLY*+LUT |
| EVM | 2.21% | 2.22% | **0.62%** |

EVMs under different conditions are listed in Table. II. *POLY*+LUT outperform others obviously, the EVM falls from 29.36% to 0.62%. Taking NMSE and PSD into consideration, we get the conclusion that *POLY*+LUT is the best choice for hybrid PD.

Also, memory polynomial model PA is tested. Similar results are gotten but not shown here.

*2) Memoryless system*

As a special case of memory system with 0-memory depth, the hybrid PD is suitable for memoryless system too. Saleh model under parameters $\alpha_\rho = 2$, $\beta_\rho = 2$, $\alpha_\theta = \pi/3$, $\beta_\theta = 1$ is tested.

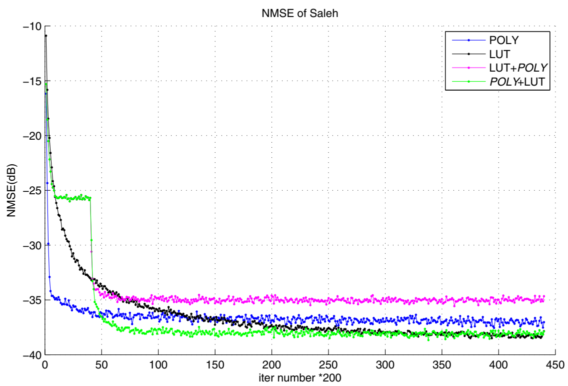


Fig. 6: NMSE learning curve for Saleh PA

Learning trajectories based on NMSE is shown in Fig. 6. *POLY*+LUT achieves the best performance, its NMSE is 1dB better than POLY, 2.5dB better than LUT+*POLY*. The performance of LUT approaches *POLY*+LUT after nearly 60000 iterations that is too large for practice.

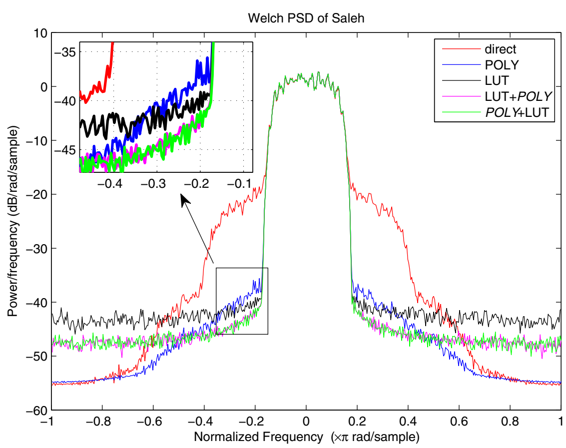


Fig. 7: PSD of Saleh PA output

PSDs with and without PD are shown in Fig. 7, The side lobes of *POLY*+LUT and LUT+*POLY* are are difficult to be distinguished and suppressed best at $\pm 0.2\pi$, nearly 22dB better than original signal, 5dB better than POLY, and 3dB better than LUT. The PDs consisting of LUT obtain worse performance than POLY at farther sideband, as POLY is best from $\pm 0.4\pi$ to $\pm\pi$. In that zone, signal can be filtered easily and the PSD of *POLY*+LUT is less than -45dB which is good enough.

TABLE III: EVM after Saleh PA

| | Orignal | POLY | LUT | LUT+*POLY* | *POLY*+LUT |
|---|---|---|---|---|---|
| EVM | 29.21% | 1.22% | 0.74% | **0.52%** | 0.54% |

EVMs under different conditions are listed in Table. III. *POLY*+LUT and LUT+*POLY* are the best and nearly the same with 3% difference. The EVM falls from 29.2% to nearly 0.5%. Taking NMSE and PSD into consideration, *POLY*+LUT is found the best hybrid PD.

Taking the results of Wiener, memory polynomial and Saleh model PAs into consideration, it is obvious that *POLY*+LUT is the most effective hybrid PD.

## V. Conclusions

In this paper, according to different cascade sequence and training order, four hybrid predistorters consisting of polynomial and look-up-table under indirect structure, were proposed for nonlinear system. Through analysis and simulations, *POLY*+LUT hybrid predistorter, the structure of which is memory polynomial followed by memory look-up-table and polynomial is trained first outperforms others. Compared with traditional predistorter, the proposed hybrid predistorter with low order polynomial and small size look-up-table achieves better performance with lower complexity for both memory and memoryless nonlinear system.

## References

[1] F. M. Ghannouchi and O. Hammi, “Behavioral modeling and predistortion,” *Microwave Magazine, IEEE*, vol. 10, no. 7, pp. 52–64, 2009.

[2] R. Piazza, S. Mysore, R. Bhavani, and B. Ottersten, “Multicarrier lut-based data predistortion for non-linear satellite channels,” in *Communications (ICC), 2014 IEEE International Conference on*, June 2014, pp. 4319–4324.

[3] H. H. Chen, C. H. Lin, P. C. Huang, and J. T. Chen, “Joint polynomial and look-up-table predistortion power amplifier linearization,” *Circuits and Systems II: Express Briefs, IEEE Transactions on*, vol. 53, no. 8, pp. 612–616, 2006.

[4] Y. Nagata, “Linear amplification technique for digital mobile communications,” in *Vehicular Technology Conference, 1989, IEEE 39th*. IEEE, 1989, pp. 159–164.

[5] J. Cavers, “Amplifier linearization using a digital predistorter with fast adaptation and low memory requirements,” *Vehicular Technology, IEEE Transactions on*, vol. 39, no. 4, pp. 374–382, 1990.

[6] J. K. Cavers, “Optimum table spacing in predistorting amplifier linearizers,” *Vehicular Technology, IEEE Transactions on*, vol. 48, no. 5, pp. 1699–1705, 1999.

[7] Z. Y. He, G. J. Hua, S. J. Geng, and G. Wang, “An improved look-up table predistortion technique for hpa with memory effects in ofdm systems,” *Broadcasting, IEEE Transactions on*, vol. 52, no. 1, pp. 87–91, 2006.

[8] S. P. Stapleton, G. S. Kandola, and J. K. Cavers, “Simulation and analysis of an adaptive predistorter utilizing a complex spectral convolution,” *Vehicular Technology, IEEE Transactions on*, vol. 41, no. 4, pp. 387–394, 1992.

[9] J. Kim and K. Konstantinou, “Digital predistortion of wideband signals based on power amplifier model with memory,” *Electronics Letters*, vol. 37, no. 23, pp. 1417–1418, 2001.

[10] R. Raich, H. Qian, and G. T. Zhou, “Orthogonal polynomials for power amplifier modeling and predistorter design,” *Vehicular Technology, IEEE Transactions on*, vol. 53, no. 5, pp. 1468–1479, 2004.

[11] S. D. Muruganathan and A. B. Sesay, “A qrd-rls-based predistortion scheme for high-power amplifier linearization,” *Circuits and Systems II: Express Briefs, IEEE Transactions on*, vol. 53, no. 10, pp. 1108–1112, 2006.

[12] M. Rawat, K. Rawat, F. M. Ghannouchi, S. Bhattacharjee, and H. Leung, “Generalized rational functions for reduced-complexity behavioral modeling and digital predistortion of broadband wireless transmitters,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 63, no. 2, pp. 485–498, Feb 2014.

[13] M. Schetzen, *The Volterra and Wiener theories of nonlinear systems*. {John Wiley & Sons}, 1980.

[14] S. S. Haykin, *Adaptive filter theory*. Pearson Education India, 2008.

[15] S. Stapleton and J. Cavers, “A new technique for adaptation of linearizing predistorters,” in *Vehicular Technology Conference, 1991. Gateway to the Future Technology in Motion., 41st IEEE*. IEEE, 1991, pp. 753–758.