# CSE 512 Final Projection Distribution Document

Spring 2022

Dongyang Wang

**Normal Distribution:** $X \sim N(\mu, \sigma^2)$

PDF: $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$ for $\mu \in \mathbb{R}$ and $\sigma^2 \in \mathbb{R}_+$
Support: $x \in \mathbb{R}$

CDF: $F_X(x) = \frac{1}{2} + \frac{1}{2} erf(\frac{x-\mu}{\sigma\sqrt{2}})$ where $erf(\frac{x-\mu}{\sigma\sqrt{2}}) = \frac{2}{\sqrt{\pi}} \int\limits_0^{\frac{x-\mu}{\sigma\sqrt{2}}} e^{-t^2} dt$

MGF: $e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Mean: $\mu$
Variance: $\sigma^2$

Transformation to $\chi^2(n)$: $\sum_{i=1}^{n} (\frac{X_i - \mu}{\sigma})^2$

**Exponential Distribution:** $X \sim Exp(\lambda)$

PDF: $f_X(x) = \lambda e^{-\lambda x}$ for $\lambda > 0$
Support: $x \in [0, \infty)$
CDF: $F_X(x) = 1 - e^{-\lambda x}$

MGF: $\frac{\lambda}{\lambda - t}$ for $t < \lambda$
Mean: $\frac{1}{\lambda}$
Variance: $\frac{1}{\lambda^2}$

Transformation to $\chi^2(n)$: $2\lambda \sum_{i=1}^{n} X_i$
Transformation to $F(n_1, n_2)$: $\lambda = 1$, $\frac{X_1}{X_2}$
Transformation to $Gamma(1, \beta)$: $Exp(\beta) \equiv Gamma(1, \beta)$

Memoryless Property: $P(X > x + a | X > a) = P(X > x)$

**Gamma Distribution:** $X \sim Gamma(\alpha, \beta)$

PDF: $f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, where $\Gamma(\alpha) = \int\limits_0^\infty s^{\alpha-1} e^{-s} ds$, for $\alpha, \beta > 0$
Support: $x \in (0, \infty)$
CDF: $F_X(x) = \frac{\gamma(\alpha, \beta x)}{\Gamma(\alpha)}$ where $\gamma(\alpha, \beta x) = \int\limits_0^{\beta x} t^{\alpha-1} e^{-t} dt$

MGF: $(\frac{\beta}{\beta - t})^\alpha$
Mean: $\frac{\alpha}{\beta}$
Variance: $\frac{\alpha}{\beta^2}$

Transformation to $Exp(\beta)$: $Gamma(1, \beta) \equiv Exp(\beta)$
Transformation to $Beta(\alpha, \beta)$: $\frac{X_1}{X_1 + X_2}$
Transformation to $N(\mu, \sigma^2)$: $\mu = \frac{\beta}{\alpha}$, $\sigma^2 = \frac{\beta}{\alpha^2}$, $\beta \to \infty$
Transformation to $\chi^2(n)$: $\alpha = \frac{1}{2}$, $\beta = \frac{n}{2}$

**Beta Distribution:** $X \sim Beta(\alpha, \beta)$

PDF: $f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$ where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ for $\alpha, \beta > 0$
Support: $x \in [0, 1]$
CDF: $F_X(x) = I_x(\alpha, \beta) = \frac{\int\limits_0^x t^{\alpha-1}(1-t)^{\beta-1} dt}{B(\alpha, \beta)}$

MGF: $1 + \sum_{k=1}^\infty (\prod_{r=0}^{k-1} \frac{\alpha + r}{\alpha + \beta + r}) \frac{t^k}{k!}$
Mean: $\frac{\alpha}{\alpha + \beta}$
Variance: $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Transformation to $Unif(0, 1)$: $Beta(0, 1) \equiv Unif(0, 1)$

**Uniform Distribution:** $X \sim Unif(a,b)$

PDF: $f_X(x) = \frac{1}{b-a}$ for $-\infty < a < b < \infty$
Support: $x \in [a,b]$
CDF: $F_X(x) = \frac{x-a}{b-a}$

MGF: $\begin{cases} \frac{e^{tb}-e^{ta}}{t(b-a)}, & \text{for } t = 0 \\ 1, & \text{for } t \neq 0 \end{cases}$
Mean: $\frac{a+b}{2}$
Variance: $\frac{(b-a)^2}{12}$

Transformation to $Exp(\lambda)$: $\frac{-log(Y)}{\lambda}$ where $Y = \frac{X-a}{b-a}$

**Chi-Squared Distribution:** $X \sim \chi^2(n)$

PDF: $f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$
Support: $n \in \mathbb{N}_+$
CDF: $F_X(x) = \frac{1}{\Gamma(n/2)} \gamma(\frac{n}{2}, \frac{x}{2})$

MGF: $(1-2t)^{-\frac{n}{2}}$ for $t < \frac{1}{2}$
Mean: $n$
Variance: $2n$

Transformation to $F(n_1, n_2)$: $\frac{X_1/n_1}{X_2/n_2}$
Transformation to $Exp(\lambda)$: $\chi^2(2) \equiv Exp(\frac{1}{2})$

**F Distribution:** $X \sim F(x; d_1, d_2)$

PDF: $f_X(x) = \frac{\sqrt{\frac{(d_1 x)^{d_1} {d_2}^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x B(\frac{d_1}{2}, \frac{d_2}{2})}$, $d_1, d_2 > 0$ are degrees of freedom
Support: $x \in (0, \infty)$ if $d_1 = 1$, otherwise $x \in [0, \infty)$
CDF: $F_X(x) = I_{\frac{d_1 x}{d_1 x + d_2}}(\frac{d_1}{2}, \frac{d_2}{2})$

MGF: Does Not Exist
Mean: $\frac{d_2}{d_2 - 2}$ for $d_2 > 2$
Variance: $\frac{d_2}{d_2 - 2}$ for $d_2 > 4$

Transformation to $\chi^2(n)$: $n_1 X_1$, $n_2 \to \infty$

**Binomial Distribution:** $X \sim Binomial(n, p)$

PMF: $f_X(x) = \binom{n}{k} p^k (1-p)^{n-k}$ for $p \in [0, 1]$
Support: $k \in \{0, 1, 2...n\}$
CDF: $F_X(x) = \sum_{i=0}^{k} \binom{n}{k} p^k (1-p)^{n-k}$

MGF: $(1 - p + pe^t)^n$
Mean: $np$
Variance: $np(1-p)$

Transformation to $N(\mu, \sigma^2)$: $\mu = np$, $\sigma^2 = np(1-p)$, $n \to \infty$
Transformation to $Poisson(\lambda)$: $\lambda = np$, $n \to \infty$

**Poisson Distribution:** $X \sim Pois(\lambda)$

PMF: $f_X(x) = \frac{\lambda^k e^{-\lambda}}{k!}$ for $\lambda \in (0, \infty)$
Support: $k \in \mathbb{N}_0$
CDF: $F_X(x) = \sum_{i=0}^{k} \frac{\lambda^k e^{-\lambda}}{k!}$

MGF: $e^{\lambda(e^t - 1)}$
Mean: $\lambda$
Variance: $\lambda$

Transformation to $N(\mu, \sigma^2)$: $\mu = \lambda$, $\sigma^2 = \lambda$, $\lambda \to \infty$

**Geometric Distribution:** $X \sim Geom(p)$

PMF: $f_X(x) = p(1-p)^{k-1}$ for $0 < p \leq 1$
Support: number of trials $k \in \{1, 2, 3...\}$
CDF: $F_X(x) = 1 - (1-p)^k$

MGF: $\frac{pe^t}{1-(1-p)e^t}$
Mean: $\frac{1}{p}$
Variance: $\frac{1-p}{p^2}$

Transformation to $Poisson(\lambda)$: $\sum_{i=1}^{r} X_i$, $\lambda = \frac{r}{p}$, $r \to \infty$

Memoryless Property: $P(X > x + a | X > a) = P(X > x)$