# STAT 502 - Homework 7

Due date: Thursday, December 9. Total points: 18 + 2 Bonus points.

1. (**6 Points**) What is the underlying experimental design of the following figures? Identify blocks and factors, if applicable.

   (a) (**2Pts**) Vending success by targeted special offers (A to E).

	Store				
Weekday	1	2	3	4	5
Monday	B	C	A	D	E
Tuesday	E	D	C	A	B
Wednesday	C	A	B	E	D
Thursday	D	B	E	C	A
Friday	A	E	D	B	C

   (b) (**2Pts**) Field test with two types of ground covers (b_1, b_2) and three fertilizer levels (d_0, d_1, d_2) (replicated analogously at four more locations).

$b_2 d_0$	$b_2 d_1$	$b_1 d_2$
$b_2 d_2$	$b_1 d_0$	$b_1 d_1$
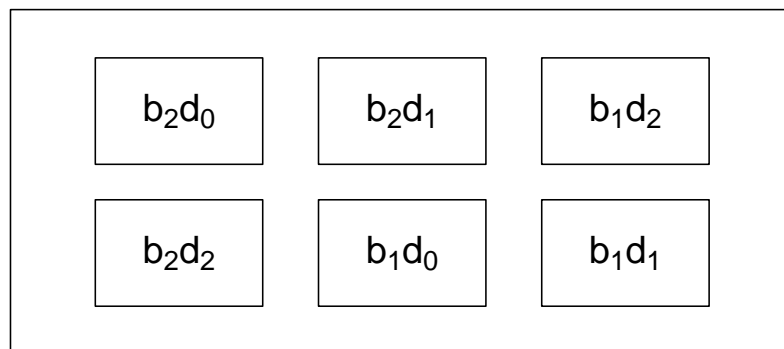
   (c) (**2Pts**) Comparison of the efficacy of five different antirheumatic ointments (A to E).

	Patient									
Application region	1	2	3	4	5	6	7	8	9	10
Left arm	A	A	D	E	B	B	E	C	C	D
Right arm	B	C	A	A	C	D	B	D	E	E

2. (**4 Points**) Questions about balanced incomplete block design:

   (a) (**2Pts**) An experimenter wishes to compare four treatments in blocks of size two. Find a BIBD with six blocks.

   (b) (**2Pts**) Consider the following incomplete block experiment with nine treatements (A-I) in nine blocks of size three.

	Block								
	1	2	3	4	5	6	7	8	9
	C 54	B 35	A 48	G 46	D 61	C 52	A 54	B 45	A 31
	H 56	G 36	G 42	H 56	E 61	I 53	H 59	I 46	B 28
	D 53	D 40	E 43	I 59	F 54	E 48	F 62	F 47	C 25

Read in the data (`ibd.RDS`) from the course Canvas page. Is this a balanced incomplete block design? Explain your reasoning.

3. **(8 Points)** For parts (a) and (c) of this last question, you may simply print and fill out the given tables.

   (a) **(2Pts)** Fill in the columns for the following table of contrasts of a full 2^4 design.

levels	2^4 Design				2^4 factorial design interactions										
	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ACD	ABD	BCD	ABCD
(1)	−	−	−	−											
a	+	−	−	−											
b	−	+	−	−											
ab	+	+	−	−											
c	−	−	+	−											
ac	+	−	+	−											
bc	−	+	+	−											
abc	+	+	+	−											
d	−	−	−	+											
ad	+	−	−	+											
bd	−	+	−	+											
abd	+	+	−	+											
cd	−	−	+	+											
acd	+	−	+	+											
bcd	−	+	+	+											
$abcd$	+	+	+	+											

   (b) **(2Pts)** Assume that you would like to test five two-level factors A, B, C, D, E, and you only can afford 16 experimental units. You would like to do a half-fraction factorial design 2^{5-1} constructed from a full 2^4 factorial design with factors A, B, C and D as in part (a). Which column would you use for the levels of the new factor E? Why?

   (c) **(2Pts)** In the following table, fill in your five factor factorial design, including the levels in the first column, based on your choice in part (b). There are many interaction terms in a full 2^5 design, but you can only list 10 of them in the top row for interaction terms of your (wise) choice. List in the bottom row their aliases (as well as the aliases of the main effects). Make sure to align each alias with the column of their alias pair.

levels	A	B	C	D	E	2^{5-1} fractional factorial design interactions											
	−	−	−	−													
	+	−	−	−													
	−	+	−	−													
	+	+	−	−													
	−	−	+	−													
	+	−	+	−													
	−	+	+	−													
	+	+	+	−													
	−	−	−	+													
	+	−	−	+													
	−	+	−	+													
	+	+	−	+													
	−	−	+	+													
	+	−	+	+													
	−	+	+	+													
	+	+	+	+													

The table header spans: columns A–E are under "2^{5-1} design"; the remaining columns are under "2^{5-1} fractional factorial design interactions". The bottom row spanning the interaction columns is labelled **Aliases**.

(d) **(2Pts)** Are their any interaction term(s) of full 2^5 design not listed in the above table? If there are any, what are the levels of the interaction term(s)?

4. **(Bonus: 2 Points)** Suppose you are given two runs (replicates) of the following 2^{3-1} factorial design:

A	B	C	AB	AC	BC	ABC	response
+	−	−	−	−	+	+	y_{2111}, y_{2112}
−	−	+	+	−	−	+	y_{1121}, y_{1122}
−	+	−	−	+	−	+	y_{1211}, y_{1212}
+	+	+	+	+	+	+	y_{2221}, y_{2222}

In this case, the linear model is:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijkl},$$

with $\epsilon_{ijkl} \sim \mathcal{N}(0,4)$ i.i.d. $i, j, k, l \in \{1, 2\}$ and $\sum_i \alpha_i = \sum_j \beta_j = \sum_k \gamma_k = 0$. What test statistic would you use to test the total main effect of factor A, that means, $H_0 : \mu_{2jk} = \mu_{1jk}$ for all j, k. What distribution does this test statistic follow under H_0?