

Supplementary document of the paper

Enforcement of Time-Constrained GMECs on Time-Colored Petri Nets

Yifan Dong, Dimitri Lefebvre, and Zhiwu Li

UniversiteLe Havre Normandie, GREAH, Normandy University

1 Nomenclature

N	Colored Petri net, $N = (P, T, C, Pre, Post)$
P	Set of places of a colored Petri net
T	Set of transitions of a colored Petri net
C	Set of colors of a colored Petri net
\mathbb{N}	Set of non-negative integers
Pre	Pre-incidence function of a Petri net
$Post$	Post-incidence function of a Petri net
M	Marking of a colored Petri net
(N, M_0)	Colored Petri net system
\mathcal{C}	Incidence matrix of a colored Petri net
$M[t]M'$	Transition t fires at M generating M'
$\mathcal{EN}(M)$	Set of transitions logically enabled at M
$M[\sigma]$	Transition sequence σ is sequentially enabled at M
$M[\sigma]M'$	Transition sequence σ fires at M generating M'
$R(N, M_0)$	Reachable marking set of a colored Petri net system (N, M_0)
G	Time-colored Petri net, $G = (N, M_0, I)$
I	Time function
\mathbb{Q}_0^+	Set of non-negative rational numbers
$c(t)$	Time value associated to t
(M, c)	State of a time-colored Petri net
$(M, c) \xrightarrow{\delta} (M, c + \delta)$	Continuous transition step
$(M, c) \xrightarrow{t} (M', c')$	Discrete transition step
$New(M, t)$	Newly enabled transition set at M
ϕ	Admissible timed trajectory
φ	Transition-time sequence
$\bar{\varphi}$	logical transition sequence
ρ	Logical trajectory
$\mathcal{TL}(G)$	Set of all admissible timed trajectory in G
$RI(G)$	Set of all reachable markings in G
SC	State class
Θ	Inequality set representing the remaining firing time interval
\bar{M}	$m \times o$ matrix derived from M
vt	C-GMEC
$\mathcal{R}(G, vt)$	Marking set defined by vt
VT	C-GMEC set
$\mathcal{R}(G, VT)$	Marking set defined by VT

V	Time-constrained C-GMEC
st	Time constraint function
\mathcal{V}	Time-constrained C-GMEC set
\mathcal{V}_{ma}	Mandatory time-constrained C-GMEC set
\mathcal{V}_{il}	Illegal time-constrained C-GMEC set
T_{so}	Soft transition set
T_{st}	Strict transition set
\mathcal{P}	Parameterized modified state class graph, $\mathcal{P} = (Q, E, f, q_0)$
Q	Vertex set in \mathcal{P}
E	Edge set in \mathcal{P}
χ	Path in \mathcal{P}
Q^{te}	Set of all terminal vertices in \mathcal{P}
$Q^{te}(vt_{ma,s})$	Set of all mandatory vertices with respect to $vt_{ma,s}$
$Q^{te}(vt_{il,s})$	Set of all illegal vertices with respect to $vt_{il,s}$
χ_{te}	Initial-to-terminal path in \mathcal{P}
X_{te}	Set of initial-to-terminal paths in \mathcal{P}
$\chi_{te}(vt_{ma,s})$	Initial-mandatory-terminal path with respect to $vt_{ma,s}$
$\chi_{te}(vt_{il,s})$	Initial-illegal-terminal path with respect to $vt_{il,s}$
$X_{te}(vt_{ma,s})$	Set of initial-mandatory-terminal paths with respect to $vt_{ma,s}$
$X_{te}(vt_{il,s})$	Set of initial-illegal-terminal paths with respect to $vt_{il,s}$
$\kappa(\mathcal{P})$	Edge density of \mathcal{P}

2 Time-Constrained C-GMEC Enforcement in a Port System

In this section, we present the detailed data of the constructed partial modified state class graph (PAMSCG) used in the case study of the paper “Enforcement of Time-Constrained GMECs on Time-Colored Petri Nets”. Particularly, in Section VI of this paper, we discuss the application of the proposed time-constrained C-GMEC enforcement in a port system within the maritime domain, where the layout (shown in Fig. 3), the operational behavior, and the TCoPN model (illustrated in Fig. 4) of the port system are presented.

For the enforcement of time-constrained C-GMECs, constructing the PAMSCG of the TCoPN is a key step. However, due to the excessive size of the PAMSCG in this case (including 75 state classes and 35 initial-to-terminal paths) and the space limitations of the paper, we present the PAMSCG in tabular form (Tables I–III). In addition, we also record all the initial-to-terminal paths of the PAMSCG in these tables, enabling a clearer understanding of how transition sequences unfold along the paths.

Table 1: Marking of the TCoPN G in Fig. 4 of the paper

$M_0 = (1, 2, 0)p_1 + (0, 0, 1)p_6$	$M_{10} = (0, 1, 0)p_1 + (0, 1, 1)p_2 + (1, 0, 0)p_4$
$M_1 = (0, 2, 0)p_1 + (1, 0, 0)p_2 + (0, 0, 1)p_6$	$M_{11} = (0, 1, 0)p_1 + (1, 0, 0)p_3 + (0, 1, 0)p_5 + (0, 0, 1)p_6$
$M_2 = (1, 1, 0)p_1 + (0, 1, 1)p_2$	$M_{12} = (1, 1, 1)p_2 + (0, 1, 0)p_5$
$M_3 = (0, 2, 0)p_1 + (1, 0, 0)p_3 + (0, 0, 1)p_6$	$M_{13} = (1, 0, 0)p_1 + (0, 2, 0)p_5 + (0, 0, 1)p_6$
$M_4 = (0, 1, 0)p_1 + (1, 1, 1)p_2$	$M_{14} = (0, 1, 0)p_1 + (1, 0, 0)p_4 + (0, 1, 0)p_5 + (0, 0, 1)p_6$
$M_5 = (1, 1, 0)p_1 + (0, 1, 0)p_5 + (0, 0, 1)p_6$	$M_{15} = (0, 1, 1)p_2 + (1, 0, 0)p_3 + (0, 1, 0)p_5$
$M_6 = (0, 2, 0)p_1 + (1, 0, 0)p_4 + (0, 0, 1)p_6$	$M_{16} = (1, 0, 0)p_2 + (0, 2, 0)p_5 + (0, 0, 1)p_6$
$M_7 = (0, 1, 0)p_1 + (0, 1, 1)p_2 + (1, 0, 0)p_3$	$M_{17} = (0, 1, 1)p_2 + (1, 0, 0)p_4 + (0, 1, 0)p_5$
$M_8 = (0, 1, 0)p_1 + (1, 0, 0)p_2 + (0, 1, 0)p_5 + (0, 0, 1)p_6$	$M_{18} = (1, 0, 0)p_3 + (0, 2, 0)p_5 + (0, 0, 1)p_6$
$M_9 = (1, 0, 0)p_1 + (0, 1, 1)p_2 + (0, 1, 0)p_5$	$M_{19} = (1, 0, 0)p_4 + (0, 2, 0)p_5 + (0, 0, 1)p_6$

Table 2: Vertex components of the PAMSCG associated to G in Fig. 4 of the paper

q_0	$(M_0, \{\hat{l}_1 \leq \theta_1 < \hat{u}_1, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{38}	$(M_{16}, \{\max\{0, 35 - \Delta_4^1 - \Delta_5^4 - \Delta_4^{10} - \Delta_5^{21}\} \leq \theta_2 < 46 - \Delta_4^1 - \Delta_5^4 - \Delta_4^{10} - \Delta_5^{21}\})$
q_1	$(M_1, \{35 \leq \theta_2 < 46, \max\{0, \hat{l}_4 - \Delta_1^0\} \leq \theta_4 < \hat{u}_4 - \Delta_1^0\})$	q_{39}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_3^{23}\} \leq \theta_4 < \hat{u}_4 - \Delta_3^{23}\})$
q_2	$(M_2, \{\max\{0, \hat{l}_1 - \Delta_4^0\} \leq \theta_1 < \hat{u}_1 - \Delta_4^0, 40 \leq \theta_5 < 55\})$	q_{40}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_5^{11} - \Delta_4^{23}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{11} - \Delta_4^{23}, 40 \leq \theta_5 < 55\})$
q_3	$(M_3, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, \hat{l}_4 - \Delta_1^0 - \Delta_2^1\} \leq \theta_4 < \hat{u}_4 - \Delta_1^0 - \Delta_2^1\})$	q_{41}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_3^{12} - \Delta_4^{24}\} \leq \theta_4 < \hat{u}_4 - \Delta_3^{12} - \Delta_4^{24}\})$
q_4	$(M_4, \{\max\{0, 35 - \Delta_4^1\} \leq \theta_2 < 46 - \Delta_4^1, 40 \leq \theta_5 < 55\})$	q_{42}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_4^{24}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^{24}, 40 \leq \theta_5 < 55\})$
q_5	$(M_4, \{35 \leq \theta_2 < 46, \max\{0, 40 - \Delta_1^2\} \leq \theta_5 < 55 - \Delta_1^2\})$	q_{43}	$(M_{15}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_2^{25}\} \leq \theta_5 < 55 - \Delta_2^{25}\})$
q_6	$(M_5, \{\max\{0, \hat{l}_1 - \Delta_4^0 - \Delta_5^2\} \leq \theta_1 < \hat{u}_1 - \Delta_4^0 - \Delta_5^2, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{44}	$(M_{16}, \{\max\{0, 35 - \Delta_5^5 - \Delta_4^{12} - \Delta_5^{25}\} \leq \theta_2 < 46 - \Delta_5^5 - \Delta_4^{12} - \Delta_5^{25}\})$
q_7	$(M_6, \{\max\{0, \hat{l}_4 - \Delta_1^0 - \Delta_2^1 - \Delta_3^2\} \leq \theta_4 < \hat{u}_4 - \Delta_1^0 - \Delta_2^1 - \Delta_3^2\})$	q_{45}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_1^6 - \Delta_2^{13} - \Delta_3^{26}\} \leq \theta_4 < \hat{u}_4 - \Delta_1^6 - \Delta_2^{13} - \Delta_3^{26}\})$
q_8	$(M_7, \{\max\{0, \hat{l}_3 - \Delta_4^3\} \leq \theta_3 < \hat{u}_3 - \Delta_4^3, 40 \leq \theta_5 < 55\})$	q_{46}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_4^{26}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^{26}, 40 \leq \theta_5 < 55\})$
q_9	$(M_7, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_2^4\} \leq \theta_5 < 55 - \Delta_2^4\})$	q_{47}	$(M_{15}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_2^{27}\} \leq \theta_5 < 55 - \Delta_2^{27}\})$
q_{10}	$(M_8, \{\max\{0, 35 - \Delta_4^1 - \Delta_5^4\} \leq \theta_2 < 46 - \Delta_4^1 - \Delta_5^4, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{48}	$(M_{16}, \{\max\{0, 35 - \Delta_4^{13} - \Delta_5^{27}\} \leq \theta_2 < 46 - \Delta_4^{13} - \Delta_5^{27}\})$
q_{11}	$(M_7, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_1^2 - \Delta_5^3\} \leq \theta_5 < 55 - \Delta_1^2 - \Delta_5^3\})$	q_{49}	$(M_{15}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_1^{14} - \Delta_2^{28}\} \leq \theta_5 < 55 - \Delta_1^{14} - \Delta_2^{28}\})$
q_{12}	$(M_8, \{\max\{0, 35 - \Delta_5^2\} \leq \theta_2 < 46 - \Delta_5^2, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{50}	$(M_{16}, \{\max\{0, 35 - \Delta_5^{28}\} \leq \theta_2 < 46 - \Delta_5^{28}\})$
q_{13}	$(M_8, \{35 \leq \theta_2 < 46, \max\{0, \hat{l}_4 - \Delta_1^6\} \leq \theta_4 < \hat{u}_4 - \Delta_1^6\})$	q_{51}	$(M_{16}, \{35 \leq \theta_2 < 46\})$
q_{14}	$(M_9, \{\max\{0, \hat{l}_1 - \Delta_4^0 - \Delta_5^2 - \Delta_6^3\} \leq \theta_1 < \hat{u}_1 - \Delta_4^0 - \Delta_5^2 - \Delta_6^3, 40 \leq \theta_5 < 55\})$	q_{52}	$(M_{17}, \{40 \leq \theta_5 < 55\})$
q_{15}	$(M_{10}, \{40 \leq \theta_5 < 55\})$	q_{53}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{32}\} \leq \theta_5 < 55 - \Delta_3^{32}\})$
q_{16}	$(M_{10}, \{\max\{0, 40 - \Delta_3^8\} \leq \theta_5 < 55 - \Delta_3^8\})$	q_{54}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_4^3 - \Delta_5^8 - \Delta_4^{17} - \Delta_5^{32}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^3 - \Delta_5^8 - \Delta_4^{17} - \Delta_5^{32}\})$
q_{17}	$(M_{11}, \{\max\{0, \hat{l}_3 - \Delta_4^3 - \Delta_5^8\} \leq \theta_3 < \hat{u}_3 - \Delta_4^3 - \Delta_5^8, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{55}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{34}\} \leq \theta_5 < 55 - \Delta_3^{34}\})$
q_{18}	$(M_{10}, \{\max\{0, 40 - \Delta_4^4 - \Delta_3^9\} \leq \theta_5 < 55 - \Delta_4^4 - \Delta_3^9\})$	q_{56}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^9 - \Delta_4^{19} - \Delta_5^{34}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^9 - \Delta_4^{19} - \Delta_5^{34}\})$
q_{19}	$(M_{11}, \{\max\{0, \hat{l}_3 - \Delta_5^9\} \leq \theta_3 < \hat{u}_3 - \Delta_5^9, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{57}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{36}\} \leq \theta_5 < 55 - \Delta_3^{36}\})$
q_{20}	$(M_{11}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, \hat{l}_4 - \Delta_2^{10}\} \leq \theta_4 < \hat{u}_4 - \Delta_2^{10}\})$	q_{58}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_4^{20} - \Delta_5^{36}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^{20} - \Delta_5^{36}\})$
q_{21}	$(M_{12}, \{\max\{0, 35 - \Delta_4^1 - \Delta_5^4 - \Delta_6^{10}\} \leq \theta_2 < 46 - \Delta_4^1 - \Delta_5^4 - \Delta_6^{10}, 40 \leq \theta_5 < 55\})$	q_{59}	$(M_{17}, \{\max\{0, 40 - \Delta_2^{21} - \Delta_3^{37}\} \leq \theta_5 < 55 - \Delta_2^{21} - \Delta_3^{37}\})$
q_{22}	$(M_{10}, \{\max\{0, 40 - \Delta_1^2 - \Delta_5^3 - \Delta_6^{11}\} \leq \theta_5 < 55 - \Delta_1^2 - \Delta_5^3 - \Delta_6^{11}\})$	q_{60}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^{37}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{37}\})$
q_{23}	$(M_{11}, \{\max\{0, \hat{l}_3 - \Delta_5^{11}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{11}, \hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{61}	$(M_{18}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3\})$
q_{24}	$(M_{11}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, \hat{l}_4 - \Delta_2^{12}\} \leq \theta_4 < \hat{u}_4 - \Delta_2^{12}\})$	q_{62}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{40}\} \leq \theta_5 < 55 - \Delta_3^{40}\})$
q_{25}	$(M_{12}, \{\max\{0, 35 - \Delta_5^5 - \Delta_1^{12}\} \leq \theta_2 < 46 - \Delta_5^5 - \Delta_1^{12}, 40 \leq \theta_5 < 55\})$	q_{63}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^{11} - \Delta_4^{23} - \Delta_5^{40}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{11} - \Delta_4^{23} - \Delta_5^{40}\})$
q_{26}	$(M_{11}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, \hat{l}_4 - \Delta_1^6 - \Delta_2^{13}\} \leq \theta_4 < \hat{u}_4 - \Delta_1^6 - \Delta_2^{13}\})$	q_{64}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{42}\} \leq \theta_5 < 55 - \Delta_3^{42}\})$
q_{27}	$(M_{12}, \{\max\{0, 35 - \Delta_4^{13}\} \leq \theta_2 < 46 - \Delta_4^{13}, 40 \leq \theta_5 < 55\})$	q_{65}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_4^{24} - \Delta_5^{42}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^{24} - \Delta_5^{42}\})$
q_{28}	$(M_{12}, \{35 \leq \theta_2 < 46, \max\{0, 40 - \Delta_1^{14}\} \leq \theta_5 < 55 - \Delta_1^{14}\})$	q_{66}	$(M_{17}, \{\max\{0, 40 - \Delta_2^{25} - \Delta_3^{43}\} \leq \theta_5 < 55 - \Delta_2^{25} - \Delta_3^{43}\})$
q_{29}	$(M_{13}, \{\max\{0, \hat{l}_1 - \Delta_4^0 - \Delta_5^2 - \Delta_6^4 - \Delta_5^{14}\} \leq \theta_1 < \hat{u}_1 - \Delta_4^0 - \Delta_5^2 - \Delta_6^4 - \Delta_5^{14}\})$	q_{67}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^{43}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{43}\})$
q_{30}	$(M_{14}, \{\hat{l}_4 \leq \theta_4 < \hat{u}_4\})$	q_{68}	$(M_{17}, \{\max\{0, 40 - \Delta_3^{46}\} \leq \theta_5 < 55 - \Delta_3^{46}\})$
q_{31}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_3^{17}\} \leq \theta_4 < \hat{u}_4 - \Delta_3^{17}\})$	q_{69}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_4^{26} - \Delta_5^{46}\} \leq \theta_3 < \hat{u}_3 - \Delta_4^{26} - \Delta_5^{46}\})$
q_{32}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_3^8 - \Delta_4^{17}\} \leq \theta_3 < \hat{u}_3 - \Delta_3^8 - \Delta_4^{17}, 40 \leq \theta_5 < 55\})$	q_{70}	$(M_{17}, \{\max\{0, 40 - \Delta_2^{27} - \Delta_3^{47}\} \leq \theta_5 < 55 - \Delta_2^{27} - \Delta_3^{47}\})$
q_{33}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_3^{19}\} \leq \theta_4 < \hat{u}_4 - \Delta_3^{19}\})$	q_{71}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^{47}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{47}\})$
q_{34}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_5^9 - \Delta_4^{19}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^9 - \Delta_4^{19}, 40 \leq \theta_5 < 55\})$	q_{72}	$(M_{17}, \{\max\{0, 40 - \Delta_1^{14} - \Delta_2^{28} - \Delta_3^{49}\} \leq \theta_5 < 55 - \Delta_1^{14} - \Delta_2^{28} - \Delta_3^{49}\})$
q_{35}	$(M_{14}, \{\max\{0, \hat{l}_4 - \Delta_2^{20} - \Delta_3^{20}\} \leq \theta_4 < \hat{u}_4 - \Delta_2^{20} - \Delta_3^{20}\})$	q_{73}	$(M_{18}, \{\max\{0, \hat{l}_3 - \Delta_5^{49}\} \leq \theta_3 < \hat{u}_3 - \Delta_5^{49}\})$
q_{36}	$(M_{15}, \{\max\{0, \hat{l}_3 - \Delta_2^{20}\} \leq \theta_3 < \hat{u}_3 - \Delta_2^{20}, 40 \leq \theta_5 < 55\})$	q_{74}	(M_{19}, \emptyset)
q_{37}	$(M_{15}, \{\hat{l}_3 \leq \theta_3 < \hat{u}_3, \max\{0, 40 - \Delta_2^{21}\} \leq \theta_5 < 55 - \Delta_2^{21}\})$	-	-

Table 3: Edges of the PAMSCG associated to G in Fig. 4 of the paper

$e_1 = (q_0, q_1)$	$f(e_1) = (t_1, \Delta_1^0 \in [\tilde{l}_1, \min\{\tilde{u}_1, \tilde{u}_4\}))$	$e_{55} = (q_{30}, q_{52})$	$f(e_{55}) = (t_4, \Delta_4^{30} \in [\tilde{l}_4, \tilde{u}_4])$
$e_2 = (q_0, q_2)$	$f(e_2) = (t_4, \Delta_4^0 \in [\tilde{l}_4, \min\{\tilde{u}_1, \tilde{u}_4\}))$	$e_{56} = (q_{31}, q_{52})$	$f(e_{56}) = (t_4, \Delta_4^{31} \in [\max\{0, \tilde{l}_4 - \Delta_4^{17}\}, \tilde{u}_4 - \Delta_4^{17}))$
$e_3 = (q_1, q_3)$	$f(e_3) = (t_2, \Delta_2^1 \in [35, \min\{46, \tilde{u}_4 - \Delta_2^0\}))$	$e_{57} = (q_{32}, q_{53})$	$f(e_{57}) = (t_3, \Delta_3^{32} \in [\max\{0, \tilde{l}_3 - \Delta_3^8 - \Delta_3^{17}\}, \min\{\tilde{u}_3 - \Delta_3^8 - \Delta_3^{17}, 55\}))$
$e_4 = (q_1, q_4)$	$f(e_4) = (t_4, \Delta_4^1 \in [\max\{0, \tilde{l}_4 - \Delta_4^0\}, \min\{46, \tilde{u}_4 - \Delta_4^0\}))$	$e_{58} = (q_{32}, q_{54})$	$f(e_{58}) = (t_5, \Delta_5^{32} \in [40, \min\{\tilde{u}_3 - \Delta_3^8 - \Delta_5^8 - \Delta_5^{17}, 55\}))$
$e_5 = (q_2, q_5)$	$f(e_5) = (t_1, \Delta_1^1 \in [\max\{0, \tilde{l}_1 - \Delta_1^0\}, \min\{\tilde{u}_1 + \Delta_1^0, 55\}))$	$e_{59} = (q_{33}, q_{52})$	$f(e_{59}) = (t_4, \Delta_4^{33} \in [\max\{0, \tilde{l}_4 - \Delta_4^{19}\}, \tilde{u}_4 - \Delta_4^{19}))$
$e_6 = (q_2, q_6)$	$f(e_6) = (t_5, \Delta_5^2 \in [40, \min\{\tilde{u}_1 - \Delta_1^0, 55\}))$	$e_{60} = (q_{34}, q_{55})$	$f(e_{60}) = (t_3, \Delta_3^{34} \in [\max\{0, \tilde{l}_3 - \Delta_3^9 - \Delta_3^{19}\}, \min\{\tilde{u}_3 - \Delta_3^9 - \Delta_3^{19}, 55\}))$
$e_7 = (q_3, q_7)$	$f(e_7) = (t_3, \Delta_3^2 \in [\tilde{l}_3, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_1^0 - \Delta_2^1\}))$	$e_{61} = (q_{34}, q_{56})$	$f(e_{61}) = (t_5, \Delta_5^{34} \in [40, \min\{\tilde{u}_3 - \Delta_3^9 - \Delta_5^{19}, 55\}))$
$e_8 = (q_3, q_8)$	$f(e_8) = (t_4, \Delta_4^3 \in [\max\{0, \tilde{l}_4 - \Delta_4^0 - \Delta_4^1\}, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_1^0 - \Delta_2^1\}))$	$e_{62} = (q_{35}, q_{52})$	$f(e_{62}) = (t_4, \Delta_4^{35} \in [\max\{0, \tilde{l}_4 - \Delta_4^{10} - \Delta_4^{20}\}, \tilde{u}_4 - \Delta_4^{10} - \Delta_4^{20}))$
$e_9 = (q_4, q_9)$	$f(e_9) = (t_2, \Delta_2^1 \in [\max\{0, 35 - \Delta_1^1\}, \min\{46 - \Delta_1^1, 55\}))$	$e_{63} = (q_{36}, q_{57})$	$f(e_{63}) = (t_3, \Delta_3^{36} \in [\max\{0, \tilde{l}_3 - \Delta_3^{20}\}, \min\{\tilde{u}_3 - \Delta_3^{20}, 55\}))$
$e_{10} = (q_4, q_{10})$	$f(e_{10}) = (t_5, \Delta_5^2 \in [40, \min\{46 - \Delta_1^1, 55\}))$	$e_{64} = (q_{36}, q_{58})$	$f(e_{64}) = (t_5, \Delta_5^{36} \in [40, \min\{\tilde{u}_3 - \Delta_4^{20}, 55\}))$
$e_{11} = (q_5, q_{11})$	$f(e_{11}) = (t_2, \Delta_2^2 \in [35, \min\{46 - \Delta_1^1, 55\}))$	$e_{65} = (q_{37}, q_{59})$	$f(e_{65}) = (t_3, \Delta_3^{37} \in [\tilde{l}_3, \min\{\tilde{u}_3, 55 - \Delta_1^{21}\}))$
$e_{12} = (q_5, q_{12})$	$f(e_{12}) = (t_5, \Delta_5^2 \in [\max\{0, 40 - \Delta_1^1\}, \min\{46, 55 - \Delta_1^1\}))$	$e_{66} = (q_{37}, q_{60})$	$f(e_{66}) = (t_5, \Delta_5^{37} \in [\max\{0, 40 - \Delta_1^1\}, \min\{\tilde{u}_3, 55 - \Delta_1^{21}\}))$
$e_{13} = (q_6, q_{13})$	$f(e_{13}) = (t_1, \Delta_1^0 \in [\max\{0, \tilde{l}_1 - \Delta_1^0 - \Delta_2^0\}, \min\{\tilde{u}_1 - \Delta_1^0 - \Delta_2^0, \tilde{u}_4\}))$	$e_{67} = (q_{38}, q_{61})$	$f(e_{67}) = (t_2, \Delta_2^{38} \in [\max\{0, 35 - \Delta_1^1 - \Delta_4^4 - \Delta_5^{10} - \Delta_5^{21}\}, 46 - \Delta_1^1 - \Delta_5^4 - \Delta_4^{10} - \Delta_5^{21}))$
$e_{14} = (q_6, q_{14})$	$f(e_{14}) = (t_4, \Delta_4^0 \in [\tilde{l}_4, \min\{\tilde{u}_1 - \Delta_1^0 - \Delta_2^0, \tilde{u}_4\}))$	$e_{68} = (q_{39}, q_{52})$	$f(e_{68}) = (t_4, \Delta_4^{39} \in [\max\{0, \tilde{l}_4 - \Delta_4^{23}\}, \tilde{u}_4 - \Delta_4^{23}))$
$e_{15} = (q_7, q_{15})$	$f(e_{15}) = (t_4, \Delta_4^1 \in [\max\{0, \tilde{l}_4 - \Delta_4^0 - \Delta_4^1\}, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_1^0 - \Delta_2^1 - \Delta_3^1\}))$	$e_{69} = (q_{40}, q_{62})$	$f(e_{69}) = (t_3, \Delta_3^{40} \in [\max\{0, \tilde{l}_3 - \Delta_3^{23}\}, \min\{\tilde{u}_3 - \Delta_3^{23} - \Delta_4^{21}, 55\}))$
$e_{16} = (q_8, q_{16})$	$f(e_{16}) = (t_3, \Delta_3^2 \in [\max\{0, \tilde{l}_3 - \Delta_3^1\}, \min\{\tilde{u}_3 - \Delta_3^1, 55\}))$	$e_{70} = (q_{40}, q_{63})$	$f(e_{70}) = (t_5, \Delta_5^{40} \in [40, \min\{\tilde{u}_3 - \Delta_1^{11} - \Delta_3^{23}, 55\}))$
$e_{17} = (q_8, q_{17})$	$f(e_{17}) = (t_5, \Delta_5^2 \in [40, \min\{\tilde{u}_3 - \Delta_3^1, 55\}))$	$e_{71} = (q_{41}, q_{52})$	$f(e_{71}) = (t_3, \Delta_3^{41} \in [\max\{0, \tilde{l}_3 - \Delta_3^{12} - \Delta_3^{24}\}, \tilde{u}_4 - \Delta_1^{12} - \Delta_3^{24}))$
$e_{18} = (q_9, q_{18})$	$f(e_{18}) = (t_3, \Delta_3^0 \in [\tilde{l}_3, \min\{\tilde{u}_3, 55\} - \Delta_4^1])$	$e_{72} = (q_{42}, q_{64})$	$f(e_{72}) = (t_3, \Delta_3^{42} \in [\max\{0, \tilde{l}_3 - \Delta_3^{24}\}, \min\{\tilde{u}_3 - \Delta_3^{24}, 55\}))$
$e_{19} = (q_9, q_{19})$	$f(e_{19}) = (t_5, \Delta_5^0 \in [\max\{0, 40 - \Delta_1^1\}, \min\{\tilde{u}_3, 55 - \Delta_1^1\}))$	$e_{73} = (q_{42}, q_{65})$	$f(e_{73}) = (t_5, \Delta_5^{42} \in [40, \min\{\tilde{u}_3 - \Delta_4^{24}, 55\}))$
$e_{20} = (q_{10}, q_{20})$	$f(e_{20}) = (t_2, \Delta_2^1 \in [\max\{0, 35 - \Delta_1^1\}, \min\{46 - \Delta_1^1 - \Delta_5^4, \tilde{u}_4\}))$	$e_{74} = (q_{43}, q_{66})$	$f(e_{74}) = (t_3, \Delta_3^{43} \in [\tilde{l}_3, \min\{\tilde{u}_3, 55 - \Delta_5^{25}\}))$
$e_{21} = (q_{10}, q_{21})$	$f(e_{21}) = (t_4, \Delta_4^0 \in [\tilde{l}_4, \min\{46 - \Delta_1^1 - \Delta_5^4, \tilde{u}_4\}))$	$e_{75} = (q_{43}, q_{67})$	$f(e_{75}) = (t_5, \Delta_5^{43} \in [\max\{0, 40 - \Delta_5^{25}\}, \min\{\tilde{u}_3, 55 - \Delta_5^{25}\}))$
$e_{22} = (q_{11}, q_{22})$	$f(e_{22}) = (t_3, \Delta_3^1 \in [\tilde{l}_3, \min\{\tilde{u}_3, 55 - \Delta_1^1 - \Delta_2^1\}))$	$e_{76} = (q_{44}, q_{61})$	$f(e_{76}) = (t_2, \Delta_2^{44} \in [\max\{0, 35 - \Delta_1^1 - \Delta_4^{12} - \Delta_5^{25}\}, 46 - \Delta_5^5 - \Delta_4^{12} - \Delta_5^{25}))$
$e_{23} = (q_{11}, q_{23})$	$f(e_{23}) = (t_5, \Delta_5^1 \in [\max\{0, 40 - \Delta_1^1 - \Delta_5^1\}, \min\{\tilde{u}_3, 55 - \Delta_1^1 - \Delta_5^1\}))$	$e_{77} = (q_{45}, q_{52})$	$f(e_{77}) = (t_4, \Delta_4^{45} \in [\max\{0, \tilde{l}_4 - \Delta_4^6 - \Delta_4^{13} - \Delta_4^{26}\}, \tilde{u}_4 - \Delta_4^6 - \Delta_4^{13} - \Delta_4^{26}))$
$e_{24} = (q_{12}, q_{24})$	$f(e_{24}) = (t_2, \Delta_2^2 \in [\max\{0, 35 - \Delta_1^1\}, \min\{46 - \Delta_5^5, \tilde{u}_4\}))$	$e_{78} = (q_{46}, q_{68})$	$f(e_{78}) = (t_3, \Delta_3^{46} \in [\max\{0, \tilde{l}_3 - \Delta_3^{26}\}, \min\{\tilde{u}_3 - \Delta_4^{26}, 55\}))$
$e_{25} = (q_{12}, q_{25})$	$f(e_{25}) = (t_4, \Delta_4^2 \in [\tilde{l}_4, \min\{46 - \Delta_5^5, \tilde{u}_4\}))$	$e_{79} = (q_{46}, q_{69})$	$f(e_{79}) = (t_5, \Delta_5^{46} \in [40, \min\{\tilde{u}_3 - \Delta_4^{26}, 55\}))$
$e_{26} = (q_{13}, q_{26})$	$f(e_{26}) = (t_2, \Delta_2^3 \in [35, \min\{46, \tilde{u}_4 - \Delta_4^6\}))$	$e_{80} = (q_{47}, q_{70})$	$f(e_{80}) = (t_3, \Delta_3^{47} \in [\tilde{l}_3, \min\{\tilde{u}_3, 55 - \Delta_5^{27}\}))$
$e_{27} = (q_{13}, q_{27})$	$f(e_{27}) = (t_1, \Delta_1^3 \in [\max\{0, \tilde{l}_1 - \Delta_1^0\}, \min\{46, \tilde{u}_4 - \Delta_1^6\}))$	$e_{81} = (q_{47}, q_{71})$	$f(e_{81}) = (t_5, \Delta_5^{47} \in [\max\{0, 40 - \Delta_5^{27}\}, \min\{\tilde{u}_3, 55 - \Delta_5^{27}\}))$
$e_{28} = (q_{14}, q_{28})$	$f(e_{28}) = (t_1, \Delta_1^4 \in [\max\{0, \tilde{l}_1 - \Delta_1^0 - \Delta_1^1\}, \min\{\tilde{u}_1 - \Delta_1^0 - \Delta_2^0 - \Delta_1^1, 55\}))$	$e_{82} = (q_{48}, q_{61})$	$f(e_{82}) = (t_2, \Delta_2^{48} \in [\max\{0, 35 - \Delta_1^1 - \Delta_5^{27}\}, 46 - \Delta_4^{13} - \Delta_5^{27}))$
$e_{29} = (q_{14}, q_{29})$	$f(e_{29}) = (t_5, \Delta_5^4 \in [40, \min\{\tilde{u}_1 - \Delta_1^0 - \Delta_2^0 - \Delta_1^1, 55\}))$	$e_{83} = (q_{49}, q_{72})$	$f(e_{83}) = (t_3, \Delta_3^{49} \in [\tilde{l}_3, \min\{\tilde{u}_3, 55 - \Delta_4^{14} - \Delta_5^{28}\}))$
$e_{30} = (q_{15}, q_{30})$	$f(e_{30}) = (t_5, \Delta_5^5 \in [40, 55])$	$e_{84} = (q_{49}, q_{73})$	$f(e_{84}) = (t_5, \Delta_5^{49} \in [\max\{0, 40 - \Delta_4^{14} - \Delta_5^{28}\}, \min\{\tilde{u}_3, 55 - \Delta_4^{14} - \Delta_5^{28}\}))$
$e_{31} = (q_{16}, q_{30})$	$f(e_{31}) = (t_5, \Delta_5^{46} \in [\max\{0, 40 - \Delta_3^3\}, 55 - \Delta_5^8])$	$e_{85} = (q_{50}, q_{61})$	$f(e_{85}) = (t_2, \Delta_2^{50} \in [\max\{0, 35 - \Delta_5^8\}, 46 - \Delta_5^{28}])$
$e_{32} = (q_{17}, q_{31})$	$f(e_{32}) = (t_3, \Delta_3^7 \in [\max\{0, \tilde{l}_3 - \Delta_3^4 - \Delta_3^8\}, \min\{\tilde{u}_3 - \Delta_3^4 - \Delta_5^8, \tilde{u}_4\}))$	$e_{86} = (q_{51}, q_{61})$	$f(e_{86}) = (t_2, \Delta_2^{51} \in [35, 46])$
$e_{33} = (q_{17}, q_{32})$	$f(e_{33}) = (t_4, \Delta_4^7 \in [\tilde{l}_4, \min\{\tilde{u}_3 - \Delta_3^4 - \Delta_5^8, \tilde{u}_4\}))$	$e_{87} = (q_{52}, q_{74})$	$f(e_{87}) = (t_5, \Delta_5^{52} \in [40, 55])$
$e_{34} = (q_{18}, q_{30})$	$f(e_{34}) = (t_5, \Delta_5^{38} \in [\max\{0, 40 - \Delta_3^4 - \Delta_3^8\}, 55 - \Delta_2^4 - \Delta_3^8])$	$e_{88} = (q_{53}, q_{74})$	$f(e_{88}) = (t_5, \Delta_5^{53} \in [\max\{0, 40 - \Delta_3^{32}\}, 55 - \Delta_3^{32}])$
$e_{35} = (q_{19}, q_{33})$	$f(e_{35}) = (t_3, \Delta_3^9 \in [\max\{0, \tilde{l}_3 - \Delta_3^8\}, \min\{\tilde{u}_3 - \Delta_3^8, \tilde{u}_4\}))$	$e_{89} = (q_{54}, q_{74})$	$f(e_{89}) = (t_3, \Delta_3^{54} \in [\max\{0, \tilde{l}_3 - \Delta_3^8 - \Delta_3^{17} - \Delta_3^{32}\}, \tilde{u}_3 - \Delta_3^8 - \Delta_3^8 - \Delta_4^{17} - \Delta_3^{32}])$
$e_{36} = (q_{19}, q_{34})$	$f(e_{36}) = (t_4, \Delta_4^9 \in [\tilde{l}_4, \min\{\tilde{u}_3 - \Delta_3^8, \tilde{u}_4\}))$	$e_{90} = (q_{55}, q_{74})$	$f(e_{90}) = (t_5, \Delta_5^{55} \in [\max\{0, 40 - \Delta_3^{34}\}, 55 - \Delta_3^{34}])$
$e_{37} = (q_{20}, q_{35})$	$f(e_{37}) = (t_3, \Delta_3^{20} \in [\tilde{l}_3, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_2^{10}\}))$	$e_{91} = (q_{56}, q_{74})$	$f(e_{91}) = (t_3, \Delta_3^{56} \in [\max\{0, \tilde{l}_3 - \Delta_3^9 - \Delta_3^{19} - \Delta_5^{34}\}, \tilde{u}_3 - \Delta_3^9 - \Delta_4^{19} - \Delta_5^{34}])$
$e_{38} = (q_{20}, q_{36})$	$f(e_{38}) = (t_4, \Delta_4^{20} \in [\max\{0, \tilde{l}_4 - \Delta_4^{10}\}, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_2^{10}\}))$	$e_{92} = (q_{57}, q_{74})$	$f(e_{92}) = (t_5, \Delta_5^{57} \in [\max\{0, 40 - \Delta_3^{36}\}, 55 - \Delta_3^{36}])$
$e_{39} = (q_{21}, q_{37})$	$f(e_{39}) = (t_2, \Delta_2^{21} \in [\max\{0, 35 - \Delta_1^1 - \Delta_4^4 - \Delta_5^{10}\}, \min\{46 - \Delta_1^1 - \Delta_4^4 - \Delta_5^{10}, 55\}))$	$e_{93} = (q_{58}, q_{74})$	$f(e_{93}) = (t_3, \Delta_3^{58} \in [\max\{0, \tilde{l}_3 - \Delta_3^{20} - \Delta_3^{36}\}, \tilde{u}_3 - \Delta_3^{20} - \Delta_3^{36}])$
$e_{40} = (q_{21}, q_{38})$	$f(e_{40}) = (t_5, \Delta_5^{21} \in [40, \min\{46 - \Delta_1^1 - \Delta_4^4 - \Delta_5^{10}, 55\}))$	$e_{94} = (q_{59}, q_{74})$	$f(e_{94}) = (t_5, \Delta_5^{59} \in [\max\{0, 40 - \Delta_3^{31} - \Delta_3^{37}\}, 55 - \Delta_3^{31} - \Delta_3^{37}])$
$e_{41} = (q_{22}, q_{30})$	$f(e_{41}) = (t_5, \Delta_5^{22} \in [\max\{0, 40 - \Delta_1^1 - \Delta_2^1 - \Delta_3^1\}, 55 - \Delta_1^1 - \Delta_2^1 - \Delta_3^1])$	$e_{95} = (q_{60}, q_{74})$	$f(e_{95}) = (t_3, \Delta_3^{60} \in [\max\{0, \tilde{l}_3 - \Delta_3^{37}\}, \tilde{u}_3 - \Delta_3^{37}])$
$e_{42} = (q_{23}, q_{39})$	$f(e_{42}) = (t_3, \Delta_3^{23} \in [\max\{0, \tilde{l}_3 - \Delta_5^{11}\}, \min\{\tilde{u}_3 - \Delta_5^{11}, \tilde{u}_4\}))$	$e_{96} = (q_{61}, q_{74})$	$f(e_{96}) = (t_3, \Delta_3^{61} \in [\tilde{l}_3, \tilde{u}_3])$
$e_{43} = (q_{23}, q_{40})$	$f(e_{43}) = (t_4, \Delta_4^{23} \in [\tilde{l}_4, \min\{\tilde{u}_3 - \Delta_5^{11}, \tilde{u}_4\}))$	$e_{97} = (q_{62}, q_{74})$	$f(e_{97}) = (t_5, \Delta_5^{62} \in [\max\{0, 40 - \Delta_3^{40}\}, 55 - \Delta_3^{40}])$
$e_{44} = (q_{24}, q_{41})$	$f(e_{44}) = (t_3, \Delta_3^{24} \in [\tilde{l}_3, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_2^{12}\}))$	$e_{98} = (q_{63}, q_{74})$	$f(e_{98}) = (t_3, \Delta_3^{63} \in [\max\{0, \tilde{l}_3 - \Delta_3^{11} - \Delta_3^{23} - \Delta_3^{40}\}, \tilde{u}_3 - \Delta_3^{11} - \Delta_3^{23} - \Delta_3^{40}])$
$e_{45} = (q_{24}, q_{42})$	$f(e_{45}) = (t_4, \Delta_4^{24} \in [\max\{0, \tilde{l}_4 - \Delta_2^{12}, \min\{\tilde{u}_3\}, \tilde{u}_4 - \Delta_2^{12}\}))$	$e_{99} = (q_{64}, q_{74})$	$f(e_{99}) = (t_5, \Delta_5^{64} \in [\max\{0, 40 - \Delta_3^{42}\}, 55 - \Delta_3^{42}])$
$e_{46} = (q_{25}, q_{43})$	$f(e_{46}) = (t_2, \Delta_2^{25} \in [\max\{0, 35 - \Delta_5^5 - \Delta_4^{12}\}, \min\{46 - \Delta_5^5 - \Delta_4^{12}, 55\}))$	$e_{100} = (q_{65}, q_{74})$	$f(e_{100}) = (t_3, \Delta_3^{65} \in [\max\{0, \tilde{l}_3 - \Delta_3^{24} - \Delta_4^{12}\}, \tilde{u}_3 - \Delta_3^{24} - \Delta_5^{42}])$
$e_{47} = (q_{25}, q_{44})$	$f(e_{47}) = (t_5, \Delta_5^{25} \in [40, \min\{46 - \Delta_5^5 - \Delta_4^{12}, 55\}))$	$e_{101} = (q_{66}, q_{74})$	$f(e_{101}) = (t_5, \Delta_5^{66} \in [\max\{0, 40 - \Delta_5^{25} - \Delta_3^{43}\}, 55 - \Delta_5^{25} - \Delta_3^{43}])$
$e_{48} = (q_{26}, q_{45})$	$f(e_{48}) = (t_3, \Delta_3^{26} \in [\tilde{l}_3, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_1^0 - \Delta_2^1\}))$	$e_{102} = (q_{67}, q_{74})$	$f(e_{102}) = (t_3, \Delta_3^{67} \in [\max\{0, \tilde{l}_3 - \Delta_3^{43}\}, \tilde{u}_3 - \Delta_3^{43}])$
$e_{49} = (q_{26}, q_{46})$	$f(e_{49}) = (t_4, \Delta_4^{26} \in [\max\{0, \tilde{l}_4 - \Delta_1^0 - \Delta_2^1\}, \min\{\tilde{u}_3, \tilde{u}_4 - \Delta_1^0 - \Delta_2^1\}))$	$e_{103} = (q_{68}, q_{74})$	$f(e_{103}) = (t_5, \Delta_5^{68} \in [\max\{0, 40 - \Delta_3^{46}\}, 55 - \Delta_3^{46}])$
$e_{50} = (q_{27}, q_{47})$	$f(e_{50}) = (t_2, \Delta_2^{27} \in [\max\{0, 35 - \Delta_1^{13}\}, \min\{46 - \Delta_1^{13}, 55\}))$	$e_{104} = (q_{69}, q_{74})$	$f(e_{104}) = (t_3, \Delta_3^{69} \in [\max\{0, \tilde{l}_3 - \Delta_3^{26} - \Delta_4^{46}\}, \tilde{u}_3 - \Delta_3^{26} - \Delta_4^{46}])$
$e_{51} = (q_{27}, q_{48})$	$f(e_{51}) = (t_5, \Delta_5^{27} \in [40, \min\{46 - \Delta_1^{13}, 55\}))$	$e_{105} = (q_{70}, q_{74})$	$f(e_{105}) = (t_5, \Delta_5^{70} \in [\max\{0, 40 - \Delta_3^{27} - \Delta_4^{47}\}, 55 - \Delta_3^{27} - \Delta_4^{47}])$
$e_{52} = (q_{28}, q_{49})$	$f(e_{52}) = (t_2, \Delta_2^{28} \in [35, \min\{46, 55 - \Delta_1^{14}\}))$	$e_{106} = (q_{71}, q_{74})$	$f(e_{106}) = (t_3, \Delta_3^{71} \in [\max\{0, \tilde{l}_3 - \Delta_3^8\}, \tilde{u}_3 - \Delta_3^8])$
$e_{53} = (q_{28}, q_{50})$	$f(e_{53}) = (t_5, \Delta_5^{28} \in [\max\{0, 40 - \Delta_1^{14}\}, \min\{46, 55 - \Delta_1^{14}\}))$	$e_{107} = (q_{72}, q_{74})$	$f(e_{107}) = (t_5, \Delta_5^{72} \in [\max\{0, 40 - \Delta_1^{14} - \Delta_3^{28} - \Delta_3^{49}\}, 55 - \Delta_1^{14} - \Delta_3^{28} - \Delta_3^{49}])$
$e_{54} = (q_{29}, q_{51})$	$f(e_{54}) = (t_1, \Delta_1^{29} \in [\max\{0, \tilde{l}_1 - \Delta_1^0 - \Delta_2^0 - \Delta_4^4 - \Delta_5^4\}, \tilde{u}_1 - \Delta_1^0 - \Delta_2^0 - \Delta_4^4 - \Delta_5^{14}])$	$e_{108} = (q_{73}, q_{74})$	$f(e_{108}) = (t_3, \Delta_3^{73} \in [\max\{0, \tilde{l}_3 - \Delta_3^{49}\}, \tilde{u}_3 - \Delta_3^{49}])$

List of all initial-to-terminal paths in the PAMSCG of the TCoPN G in Fig. 4 of the paper.

- $\chi_{te,1} = q_{0,1} \xrightarrow{e_{1,1}} q_{1,1} \xrightarrow{e_{3,1}} q_{3,1} \xrightarrow{e_{7,1}} q_{7,1} \xrightarrow{e_{15,1}} q_{15,1} \xrightarrow{e_{30,1}} q_{30,1} \xrightarrow{e_{55,1}} q_{52,1} \xrightarrow{e_{87,1}} q_{74,1}$
- $\chi_{te,2} = q_{0,2} \xrightarrow{e_{1,2}} q_{1,2} \xrightarrow{e_{3,2}} q_{3,2} \xrightarrow{e_{8,2}} q_{8,2} \xrightarrow{e_{16,2}} q_{16,2} \xrightarrow{e_{31,2}} q_{30,2} \xrightarrow{e_{55,2}} q_{52,2} \xrightarrow{e_{87,2}} q_{74,2}$
- $\chi_{te,3} = q_{0,3} \xrightarrow{e_{1,3}} q_{1,3} \xrightarrow{e_{3,3}} q_{3,3} \xrightarrow{e_{8,3}} q_{8,3} \xrightarrow{e_{17,3}} q_{17,3} \xrightarrow{e_{32,3}} q_{31,3} \xrightarrow{e_{56,3}} q_{52,3} \xrightarrow{e_{87,3}} q_{74,3}$
- $\chi_{te,4} = q_{0,4} \xrightarrow{e_{1,4}} q_{1,4} \xrightarrow{e_{3,4}} q_{3,4} \xrightarrow{e_{8,4}} q_{8,4} \xrightarrow{e_{17,4}} q_{17,4} \xrightarrow{e_{33,4}} q_{32,4} \xrightarrow{e_{57,4}} q_{53,4} \xrightarrow{e_{88,4}} q_{74,4}$
- $\chi_{te,5} = q_{0,5} \xrightarrow{e_{1,5}} q_{1,5} \xrightarrow{e_{3,5}} q_{3,5} \xrightarrow{e_{8,5}} q_{8,5} \xrightarrow{e_{17,5}} q_{17,5} \xrightarrow{e_{33,5}} q_{32,5} \xrightarrow{e_{58,5}} q_{54,5} \xrightarrow{e_{89,5}} q_{74,5}$
- $\chi_{te,6} = q_{0,6} \xrightarrow{e_{1,6}} q_{1,6} \xrightarrow{e_{4,6}} q_{4,6} \xrightarrow{e_{9,6}} q_{9,6} \xrightarrow{e_{18,6}} q_{18,6} \xrightarrow{e_{34,6}} q_{30,6} \xrightarrow{e_{55,6}} q_{52,6} \xrightarrow{e_{87,6}} q_{74,6}$
- $\chi_{te,7} = q_{0,7} \xrightarrow{e_{1,7}} q_{1,7} \xrightarrow{e_{4,7}} q_{4,7} \xrightarrow{e_{9,7}} q_{9,7} \xrightarrow{e_{19,7}} q_{19,7} \xrightarrow{e_{35,7}} q_{33,7} \xrightarrow{e_{59,7}} q_{52,7} \xrightarrow{e_{87,7}} q_{74,7}$
- $\chi_{te,8} = q_{0,8} \xrightarrow{e_{1,8}} q_{1,8} \xrightarrow{e_{4,8}} q_{4,8} \xrightarrow{e_{9,8}} q_{9,8} \xrightarrow{e_{19,8}} q_{19,8} \xrightarrow{e_{36,8}} q_{34,8} \xrightarrow{e_{60,8}} q_{55,8} \xrightarrow{e_{90,8}} q_{74,8}$
- $\chi_{te,9} = q_{0,9} \xrightarrow{e_{1,9}} q_{1,9} \xrightarrow{e_{4,9}} q_{4,9} \xrightarrow{e_{9,9}} q_{9,9} \xrightarrow{e_{19,9}} q_{19,9} \xrightarrow{e_{36,9}} q_{34,9} \xrightarrow{e_{61,9}} q_{56,9} \xrightarrow{e_{91,9}} q_{74,9}$
- $\chi_{te,10} = q_{0,10} \xrightarrow{e_{1,10}} q_{1,10} \xrightarrow{e_{4,10}} q_{4,10} \xrightarrow{e_{10,10}} q_{10,10} \xrightarrow{e_{20,10}} q_{20,10} \xrightarrow{e_{37,10}} q_{35,10} \xrightarrow{e_{62,10}} q_{52,10} \xrightarrow{e_{87,10}} q_{74,10}$
- $\chi_{te,11} = q_{0,11} \xrightarrow{e_{1,11}} q_{1,11} \xrightarrow{e_{4,11}} q_{4,11} \xrightarrow{e_{10,11}} q_{10,11} \xrightarrow{e_{20,11}} q_{20,11} \xrightarrow{e_{38,11}} q_{36,11} \xrightarrow{e_{63,11}} q_{57,11} \xrightarrow{e_{92,11}} q_{74,11}$
- $\chi_{te,12} = q_{0,12} \xrightarrow{e_{1,12}} q_{1,12} \xrightarrow{e_{4,12}} q_{4,12} \xrightarrow{e_{10,12}} q_{10,12} \xrightarrow{e_{20,12}} q_{20,12} \xrightarrow{e_{38,12}} q_{36,12} \xrightarrow{e_{64,12}} q_{58,12} \xrightarrow{e_{93,12}} q_{74,12}$
- $\chi_{te,13} = q_{0,13} \xrightarrow{e_{1,13}} q_{1,13} \xrightarrow{e_{4,13}} q_{4,13} \xrightarrow{e_{10,13}} q_{10,13} \xrightarrow{e_{21,13}} q_{21,13} \xrightarrow{e_{39,13}} q_{37,13} \xrightarrow{e_{65,13}} q_{59,13} \xrightarrow{e_{94,13}} q_{74,13}$
- $\chi_{te,14} = q_{0,14} \xrightarrow{e_{1,14}} q_{1,14} \xrightarrow{e_{4,14}} q_{4,14} \xrightarrow{e_{10,14}} q_{10,14} \xrightarrow{e_{21,14}} q_{21,14} \xrightarrow{e_{39,14}} q_{37,14} \xrightarrow{e_{66,14}} q_{60,14} \xrightarrow{e_{95,14}} q_{74,14}$
- $\chi_{te,15} = q_{0,15} \xrightarrow{e_{1,15}} q_{1,15} \xrightarrow{e_{4,15}} q_{4,15} \xrightarrow{e_{10,15}} q_{10,15} \xrightarrow{e_{21,15}} q_{21,15} \xrightarrow{e_{40,15}} q_{38,15} \xrightarrow{e_{67,15}} q_{63,15} \xrightarrow{e_{96,15}} q_{74,15}$
- $\chi_{te,16} = q_{0,16} \xrightarrow{e_{2,16}} q_{2,16} \xrightarrow{e_{5,16}} q_{5,16} \xrightarrow{e_{11,16}} q_{11,16} \xrightarrow{e_{22,16}} q_{22,16} \xrightarrow{e_{41,16}} q_{30,16} \xrightarrow{e_{55,16}} q_{52,16} \xrightarrow{e_{87,16}} q_{74,16}$
- $\chi_{te,17} = q_{0,17} \xrightarrow{e_{2,17}} q_{2,17} \xrightarrow{e_{5,17}} q_{5,17} \xrightarrow{e_{11,17}} q_{11,17} \xrightarrow{e_{23,17}} q_{23,17} \xrightarrow{e_{42,17}} q_{39,17} \xrightarrow{e_{68,17}} q_{52,17} \xrightarrow{e_{87,17}} q_{74,17}$
- $\chi_{te,18} = q_{0,18} \xrightarrow{e_{2,18}} q_{2,18} \xrightarrow{e_{5,18}} q_{5,18} \xrightarrow{e_{11,18}} q_{11,18} \xrightarrow{e_{23,18}} q_{23,18} \xrightarrow{e_{43,18}} q_{40,18} \xrightarrow{e_{69,18}} q_{62,18} \xrightarrow{e_{97,18}} q_{74,18}$
- $\chi_{te,19} = q_{0,19} \xrightarrow{e_{2,19}} q_{2,19} \xrightarrow{e_{5,19}} q_{5,19} \xrightarrow{e_{11,19}} q_{11,19} \xrightarrow{e_{23,19}} q_{23,19} \xrightarrow{e_{43,19}} q_{40,19} \xrightarrow{e_{70,19}} q_{63,19} \xrightarrow{e_{98,19}} q_{74,19}$
- $\chi_{te,20} = q_{0,20} \xrightarrow{e_{2,20}} q_{2,20} \xrightarrow{e_{5,20}} q_{5,20} \xrightarrow{e_{12,20}} q_{12,20} \xrightarrow{e_{24,20}} q_{24,20} \xrightarrow{e_{44,20}} q_{41,20} \xrightarrow{e_{71,20}} q_{52,20} \xrightarrow{e_{87,20}} q_{74,20}$
- $\chi_{te,21} = q_{0,21} \xrightarrow{e_{2,21}} q_{2,21} \xrightarrow{e_{5,21}} q_{5,21} \xrightarrow{e_{12,21}} q_{12,21} \xrightarrow{e_{24,21}} q_{24,21} \xrightarrow{e_{45,21}} q_{42,21} \xrightarrow{e_{72,21}} q_{64,21} \xrightarrow{e_{99,21}} q_{74,21}$
- $\chi_{te,22} = q_{0,22} \xrightarrow{e_{2,22}} q_{2,22} \xrightarrow{e_{5,22}} q_{5,22} \xrightarrow{e_{12,22}} q_{12,22} \xrightarrow{e_{24,22}} q_{24,22} \xrightarrow{e_{45,22}} q_{42,22} \xrightarrow{e_{73,22}} q_{65,22} \xrightarrow{e_{100,22}} q_{74,22}$
- $\chi_{te,23} = q_{0,23} \xrightarrow{e_{2,23}} q_{2,23} \xrightarrow{e_{5,23}} q_{5,23} \xrightarrow{e_{12,23}} q_{12,23} \xrightarrow{e_{25,23}} q_{25,23} \xrightarrow{e_{46,23}} q_{43,23} \xrightarrow{e_{74,23}} q_{66,23} \xrightarrow{e_{101,23}} q_{74,23}$
- $\chi_{te,24} = q_{0,24} \xrightarrow{e_{2,24}} q_{2,24} \xrightarrow{e_{5,24}} q_{5,24} \xrightarrow{e_{12,24}} q_{12,24} \xrightarrow{e_{25,24}} q_{25,24} \xrightarrow{e_{46,24}} q_{43,24} \xrightarrow{e_{75,24}} q_{67,24} \xrightarrow{e_{102,24}} q_{74,24}$
- $\chi_{te,25} = q_{0,25} \xrightarrow{e_{2,25}} q_{2,25} \xrightarrow{e_{5,25}} q_{5,25} \xrightarrow{e_{12,25}} q_{12,25} \xrightarrow{e_{25,25}} q_{25,25} \xrightarrow{e_{47,25}} q_{44,25} \xrightarrow{e_{76,25}} q_{61,25} \xrightarrow{e_{96,25}} q_{74,25}$
- $\chi_{te,26} = q_{0,26} \xrightarrow{e_{2,26}} q_{2,26} \xrightarrow{e_{6,26}} q_{6,26} \xrightarrow{e_{13,26}} q_{13,26} \xrightarrow{e_{26,26}} q_{26,26} \xrightarrow{e_{48,26}} q_{45,26} \xrightarrow{e_{77,26}} q_{52,26} \xrightarrow{e_{87,26}} q_{74,26}$

- $\chi_{te,27} = q_{0,27} \xrightarrow{e_{2,27}} q_{2,27} \xrightarrow{e_{6,27}} q_{6,27} \xrightarrow{e_{13,27}} q_{13,27} \xrightarrow{e_{26,27}} q_{26,27} \xrightarrow{e_{49,27}} q_{46,27} \xrightarrow{e_{78,27}} q_{68,27} \xrightarrow{e_{103,27}} q_{74,27}$
- $\chi_{te,28} = q_{0,28} \xrightarrow{e_{2,28}} q_{2,28} \xrightarrow{e_{6,28}} q_{6,28} \xrightarrow{e_{13,28}} q_{13,28} \xrightarrow{e_{26,28}} q_{26,28} \xrightarrow{e_{49,28}} q_{46,28} \xrightarrow{e_{79,28}} q_{69,28} \xrightarrow{e_{104,28}} q_{74,28}$
- $\chi_{te,29} = q_{0,29} \xrightarrow{e_{2,29}} q_{2,29} \xrightarrow{e_{6,29}} q_{6,29} \xrightarrow{e_{13,29}} q_{13,29} \xrightarrow{e_{27,29}} q_{27,29} \xrightarrow{e_{50,29}} q_{47,29} \xrightarrow{e_{80,29}} q_{70,29} \xrightarrow{e_{105,29}} q_{74,29}$
- $\chi_{te,30} = q_{0,30} \xrightarrow{e_{2,30}} q_{2,30} \xrightarrow{e_{6,30}} q_{6,30} \xrightarrow{e_{13,30}} q_{13,30} \xrightarrow{e_{27,30}} q_{27,30} \xrightarrow{e_{50,30}} q_{47,30} \xrightarrow{e_{81,30}} q_{71,30} \xrightarrow{e_{106,30}} q_{74,30}$
- $\chi_{te,31} = q_{0,31} \xrightarrow{e_{2,31}} q_{2,31} \xrightarrow{e_{6,31}} q_{6,31} \xrightarrow{e_{13,31}} q_{13,31} \xrightarrow{e_{27,31}} q_{27,31} \xrightarrow{e_{51,31}} q_{48,31} \xrightarrow{e_{82,31}} q_{61,31} \xrightarrow{e_{96,31}} q_{74,31}$
- $\chi_{te,32} = q_{0,32} \xrightarrow{e_{2,32}} q_{2,32} \xrightarrow{e_{6,32}} q_{6,32} \xrightarrow{e_{14,32}} q_{14,32} \xrightarrow{e_{28,32}} q_{28,32} \xrightarrow{e_{52,32}} q_{49,32} \xrightarrow{e_{83,32}} q_{72,32} \xrightarrow{e_{107,32}} q_{74,32}$
- $\chi_{te,33} = q_{0,33} \xrightarrow{e_{2,33}} q_{2,33} \xrightarrow{e_{6,33}} q_{6,33} \xrightarrow{e_{14,33}} q_{14,33} \xrightarrow{e_{28,33}} q_{28,33} \xrightarrow{e_{52,33}} q_{49,33} \xrightarrow{e_{84,33}} q_{73,33} \xrightarrow{e_{108,33}} q_{74,33}$
- $\chi_{te,34} = q_{0,34} \xrightarrow{e_{2,34}} q_{2,34} \xrightarrow{e_{6,34}} q_{6,34} \xrightarrow{e_{14,34}} q_{14,34} \xrightarrow{e_{28,34}} q_{28,34} \xrightarrow{e_{53,34}} q_{50,34} \xrightarrow{e_{85,34}} q_{61,34} \xrightarrow{e_{96,34}} q_{74,34}$
- $\chi_{te,35} = q_{0,35} \xrightarrow{e_{2,35}} q_{2,35} \xrightarrow{e_{6,35}} q_{6,35} \xrightarrow{e_{14,35}} q_{14,35} \xrightarrow{e_{29,35}} q_{29,35} \xrightarrow{e_{54,35}} q_{51,35} \xrightarrow{e_{86,35}} q_{61,35} \xrightarrow{e_{96,35}} q_{74,35}$