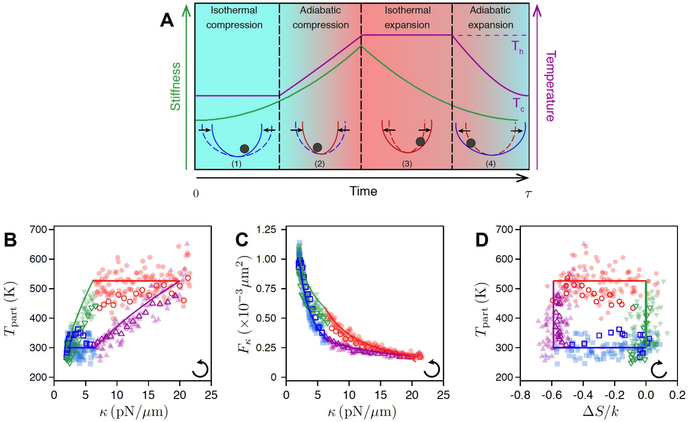
Self-propelled Brownian Particles in Carnot Cycle: As an Engine

Yulong Dong
Supervisor: Prof. Zhonghuai Hou

Department of Chemical Physics
University of Science and Technology of China

September 14, 2017

Background



1. dilute Brownian particles → analogous Carnot cyle → work
2. stochatic fluctuation of entropy → promising to violate Carnot limit

1 I. A. Mart nez, E.Roldan, et al. Brownian Carnot engine. Nature Physics. 2015 October 26; 12: 67-70.

Motivation

1. Fluctuation theorem, laws of thermodymanics are redefined in active Brownian particles.
2. active Brownian engine $\Rightarrow$ higher efficiency or higher power ?

Model

Rayleigh-Helmholtz self-propelled Model

$$m\ddot{x} + (\gamma - a + b\dot{x}^2)\dot{x} = -\partial_x U + \xi$$

where

$$\langle \xi(t)\xi(t') \rangle = 2\gamma kT\delta(t - t')$$

1. a: pump energy into system $\Rightarrow$ negative friction
2. b: increasement of friction caused by motion
3. like motor proteins NK11
4. Brownian particle is trapped by harmonic laser, where $U(x) = \frac{\lambda}{2}x^2$

2 Ganguly C, Chaudhuri D. Stochastic thermodynamics of active Brownian particles. Physical Review E. 2013 Sep;88(3):032102.

Theoretical Analysis

1. Langevin equation →

$$\begin{aligned} \mathrm{d}E &= \mathrm{d}(\frac{p^2}{2m} + U(x;\lambda)) \\ &= [\frac{\partial U}{\partial \lambda}\mathrm{d}\lambda] + [(-\gamma\frac{p}{m} + \xi)\mathrm{d}x] \\ &= \mathrm{d}W + \mathrm{d}Q \end{aligned}$$

2. Assumption: adiabatic process happens instantaneously without changing P.D.F.

$$W_a = \Delta_a E = \int \mathrm{d}x p(x,\tau)(U(x,\tau+0) - U(x,\tau-0))$$

3. $\eta \stackrel{d}{=} -\frac{\langle W\rangle}{\langle Q_h\rangle}$

3 Ken Sekimoto, Fumiko Takagi, Tsuyoshi Hondou. Carnot s cycle for small systems: Irreversibility and cost of operations. Physical Review E. 2014, 62(6): 7759-7767.

4 Tim Schmiedl and Udo Seifert. Efficiency at maximum power: An analytically solvable model for stochastic heat engines. arXiv:0710.4097v1.

Controling Protocols

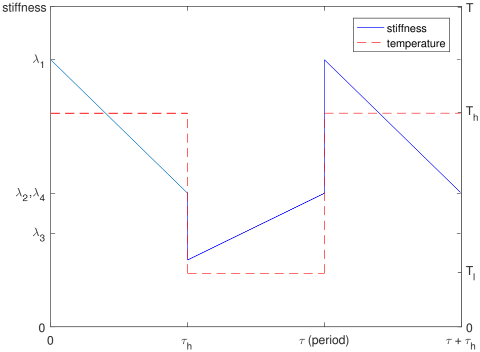


Figure: controling protocols as a function of time

5 S Rana, PS Pal, A Saha, AM Jayannavar. Single-particle stochastic heat engine. Physical Review E 90 (4), 042146.

Phase Diagram

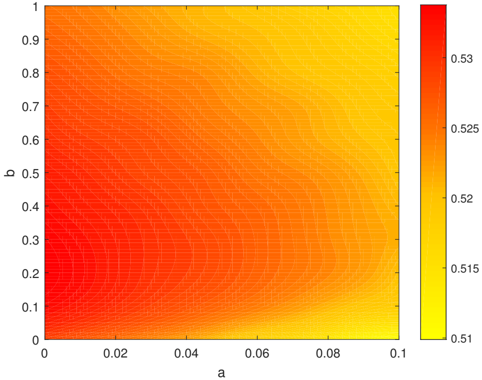


Figure: Phase diagram of efficiency (duration = 10).

1. Slight improvement
2. how about long-time limit

Efficiency

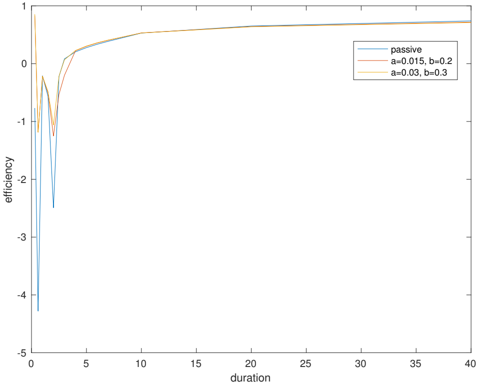


Figure: Efficiency as a function of duration.

1. non-engine region: separated
2. engine region(efficiency>0): without significantly difference

Future Work

1. theoretically prove passive engine possesses higher efficiency than R-H one?
2. nonlinear effect of external field

Appendix A: Validation of Experimental Realization

1. In experiements, adiadbatic process $\Rightarrow \Delta S = S_t - S_0 = 0$
   With slow change of protocols, utilizing statistical mechanics

$$\Delta S = k\Delta \ln(\mathcal{Q}\exp(\frac{T\partial_T \mathcal{Q}}{\mathcal{Q}})) = 0$$

$$\Rightarrow \mathcal{Q}\exp(\frac{T\partial_T \mathcal{Q}}{\mathcal{Q}}) = constant$$

2. Corresponding F-P equation $\Rightarrow$ P.D.F.

$$\mathcal{Q} = \iint_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}v \exp(-\frac{b}{4\gamma kT}(\frac{m}{2}v^2 + \frac{\lambda}{2}x^2 - H_0)^2)$$

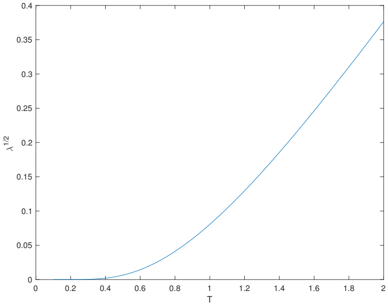
$$= 2\pi^{3/2}\sqrt{\frac{\gamma kT}{m\lambda b}}(1 + erf(\frac{\mathcal{P}}{\sqrt{T}}))$$

with $\mathcal{P} \stackrel{d}{=} \frac{H_0}{2}\sqrt{\frac{b}{\gamma k}}$

Validation

$$\sqrt{\lambda} = \mathcal{C}\frac{\sqrt{T}}{\mathcal{P}}(1 + erf(\frac{\mathcal{P}}{\sqrt{T}}))\exp(\frac{1}{2} - \frac{\mathcal{P}}{2\sqrt{T}}\frac{\exp(-\frac{\mathcal{P}^2}{T})}{1 + erf(\frac{\mathcal{P}}{\sqrt{T}})})$$

with $\mathcal{C} \in \mathbb{R}$



Appendix B: Data Analysis by Large Deviation Technique

1. central limit theorem $\Rightarrow$ for X_i i.i.d. $\mu = 0, \sigma = 1 \quad \forall i = 1, 2, ..., n$

$$p_n(l) \overset{d}{=} \mathbb{P}[\sum_{i=1}^{n} X_i/\sqrt{n} \geqslant \sqrt{n}l\,] \overset{n\to\infty}{\longrightarrow} \frac{1}{\sqrt{\pi}} \int_{\sqrt{n}l}^{\infty} e^{-x^2} \mathrm{d}x$$
$$\sim e^{-nl^2+o(n)} \sim e^{-nI(l)+o(n)}$$

$\Rightarrow$ define rate function (estimator) as $I(l) \overset{d}{=} \lim_{n\to\infty} -\frac{1}{n} \ln p_n(l)$

2. define empirical P.D.F. as
$p_{em}^{(n)}(x)\mathrm{d}x = \sum_{i=1}^{N} \mathbb{I}(X_i^{(n)} \in [x, x+\mathrm{d}x)])/N$

$$I(\eta) \overset{d}{=} \lim_{n\to\infty} -\frac{1}{n\tau} \ln p_{em}^{(n)}(\eta)$$
$$\Rightarrow \frac{1}{n\tau} \ln p_{em}^{(n)}(\eta) = -I(\eta) + \frac{\mathcal{A}}{n\tau}$$

plot $\frac{1}{n\tau} \ln p_{em}^{(n)}(\eta)\ vs\ \frac{1}{n\tau}$ to get estimator
$\mathbb{P}[\{X_i\}_{i=1}^{n} = x] \sim \exp(-n\tau I(x))$