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Adversarial Examples

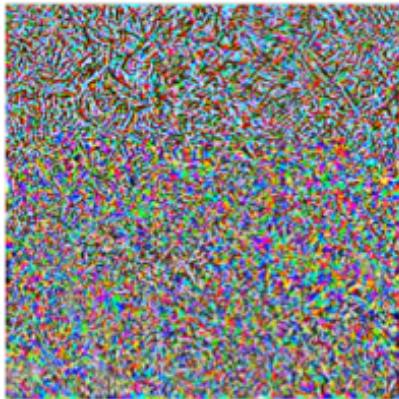


Clean images



Alps: 94.39%

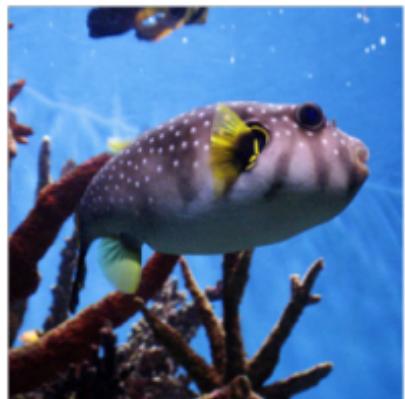
Adversarial noise



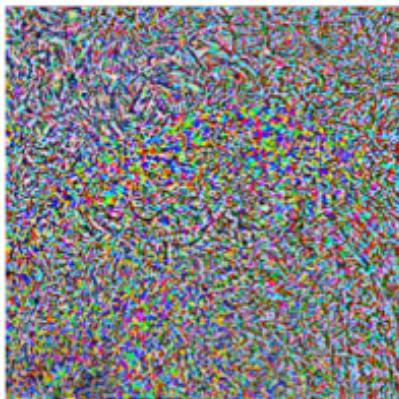
Adversarial examples



Dog: 99.99%



Puffer: 97.99%



Crab: 100.00%

(Figure is from Dong et al. 2018)

Adversarial Training

- From the optimization view, adversarial training (AT) can be formulated as a minimax optimization problem (Madry et al., 2018)

Outer minimization: train a robust classifier

$$\min_{\theta} \frac{1}{n} \sum_{i=1}^n \max_{\delta_i \in S} L(f_{\theta}(x_i + \delta_i), y_i)$$

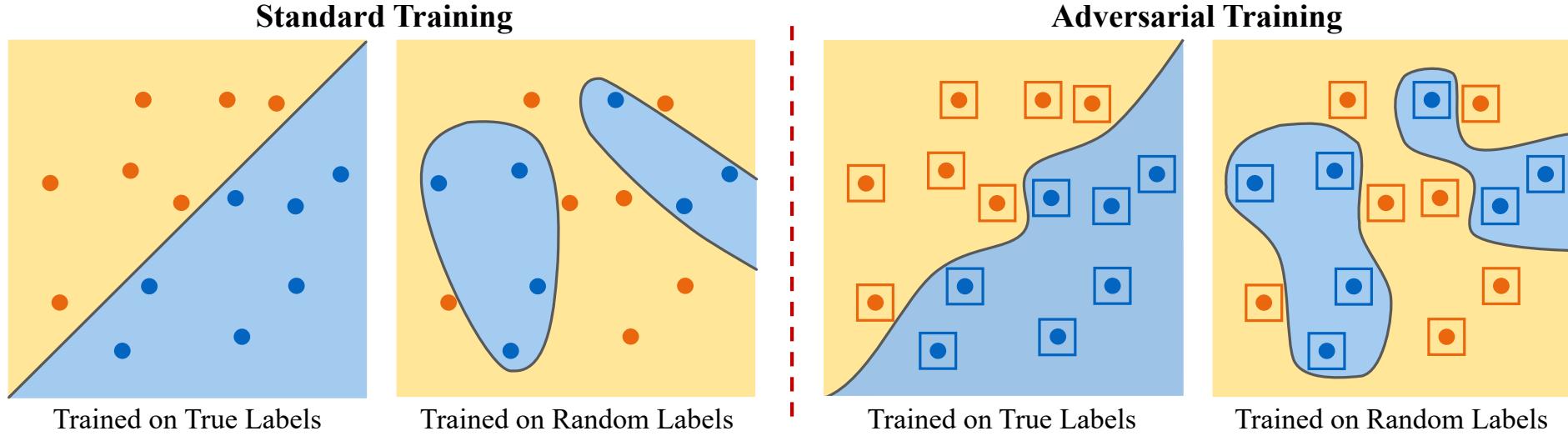
$\leftarrow S = \{\delta: \|\delta\|_{\infty} \leq \epsilon\}$

Inner maximization: generate an adversarial example

- Solve the inner maximization by projected gradient descent

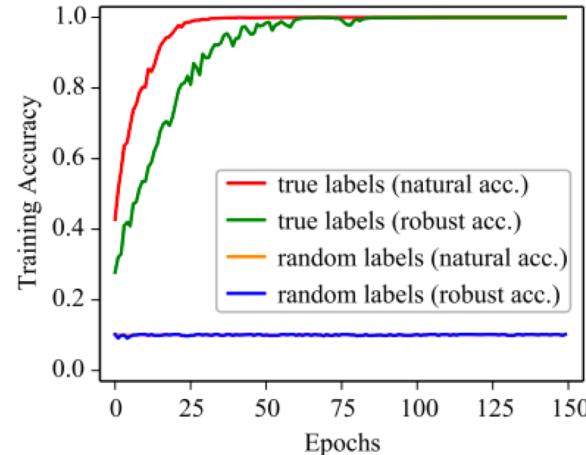
$$\delta_i^{t+1} = \Pi_S \left(\delta_i^t + \alpha \cdot \text{sign} \left(\nabla_x L(f_{\theta}(x_i + \delta_i^t), y_i) \right) \right)$$

Memorization

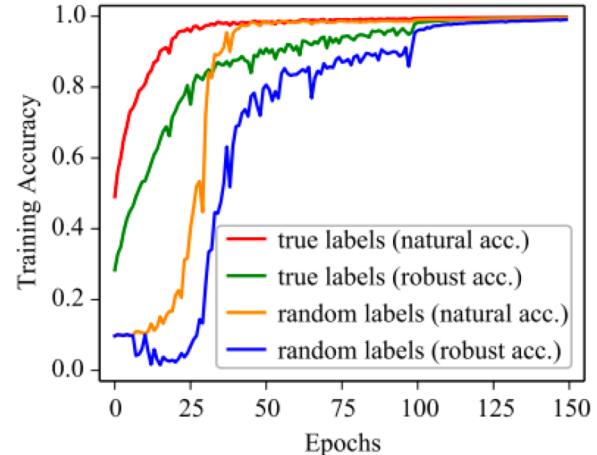


Goal: To facilitate a deeper understanding of model capacity, training convergence, robust generalization, and robust overfitting of the adversarially trained models.

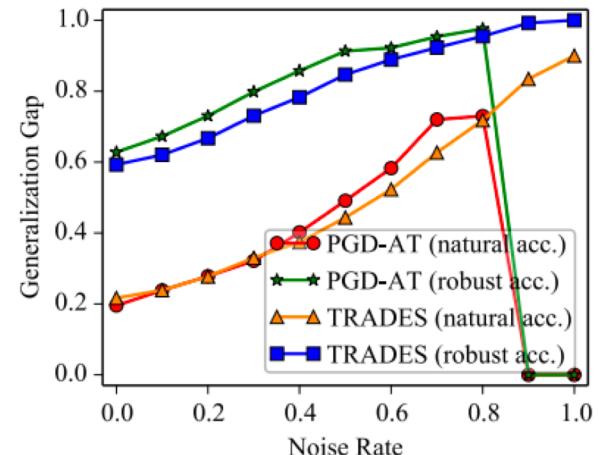
AT with random labels



(a) PGD-AT



(b) TRADES



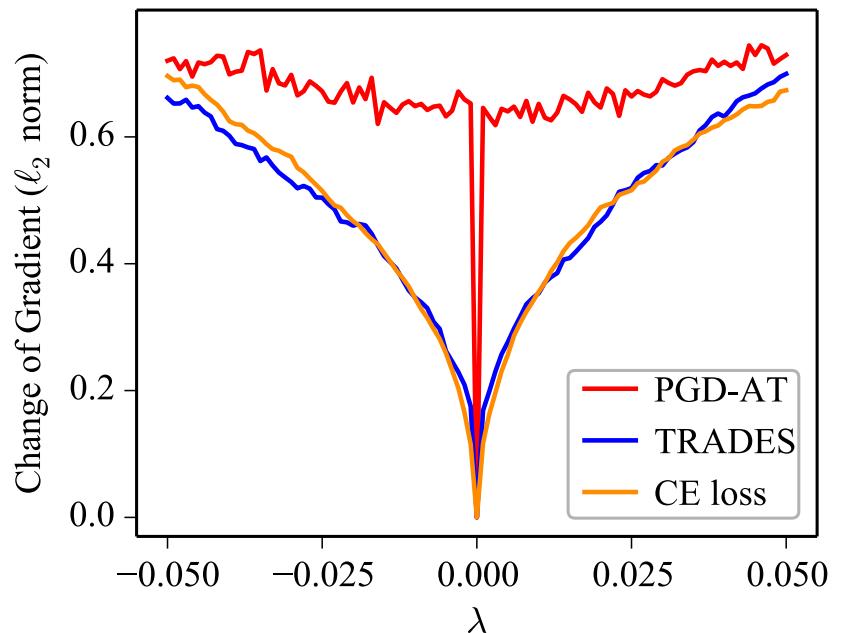
(c) Generalization gap growth

Finding: PGD-AT cannot converge with random labels, but TRADES can.

- This finding holds with different training settings, including network architecture, attack steps, optimizer, perturbation budget, regularizations

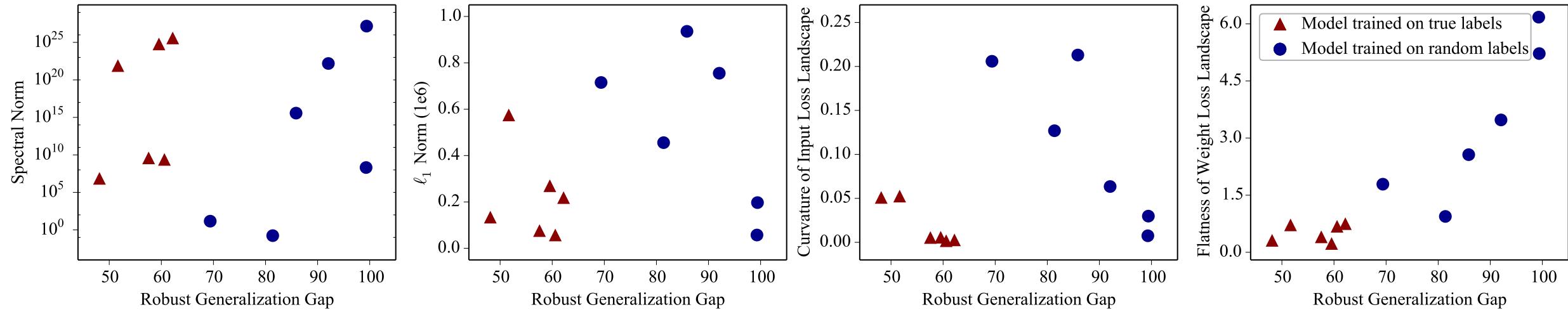
Convergence Analysis

- **Gradient instability issue:** the gradient of the adversarial loss in PGD-AT changes more abruptly than TRADES.



We measure the L2 distance between the gradient at θ and $\theta + \lambda d$

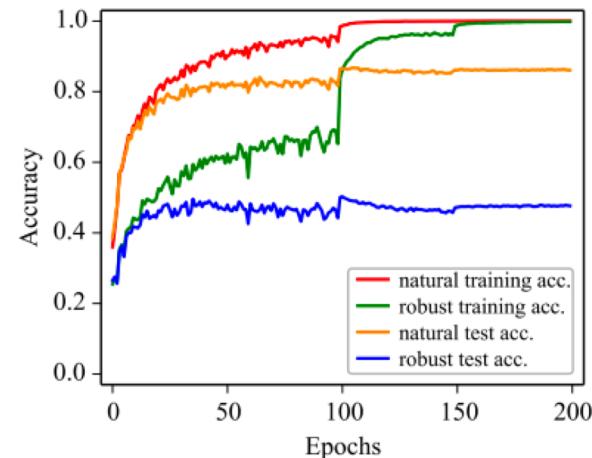
Generalization Analysis



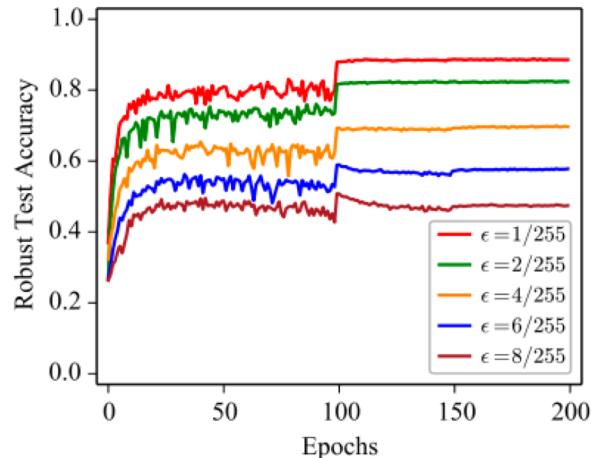
- We consider two norm-based measures and two sharpness/flatness-based measures.
- **Finding:** None of them can adequately explain and ensure robust generalization.

Robust Overfitting Analysis

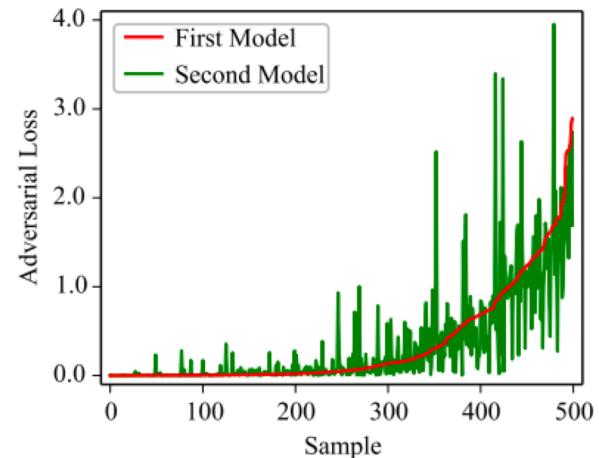
- **Argument:** robust overfitting is caused by excessive memorization of (noisy) one-hot labels.



(a)



(b)



(c)

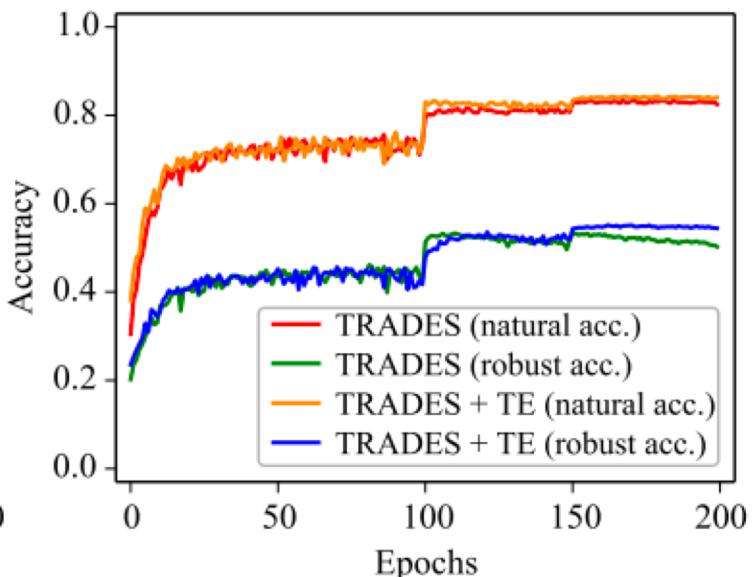
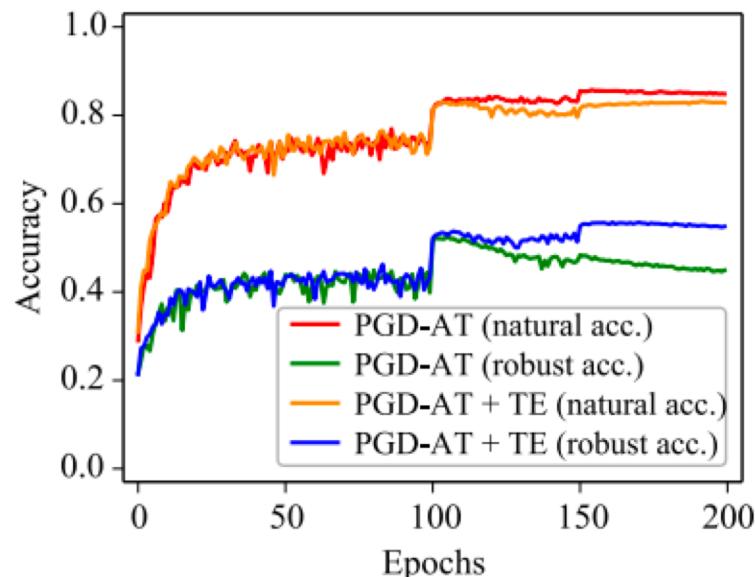
1. Robust overfitting does not occur with a smaller perturbation budget (e.g., $\epsilon = 1/255$).
2. The hard examples are consistent across different networks.

Mitigation Algorithm

- Incorporate the **Temporal Ensembling (TE)** approach into AT

$$\min_{\theta} \frac{1}{n} \sum_{i=1}^n \max_{\delta_i \in S} \{L(f_{\theta}(x_i + \delta_i), y_i) + w \cdot \|f_{\theta}(x_i + \delta_i) - \hat{p}_i\|_2^2\}$$

- $\hat{p}_i = \frac{p_i}{\|p_i\|_2}$, $p_i = \eta p_i + (1 - \eta)f_{\theta}(x_i)$



Learning curves
on CIFAR-10

Empirical Results



Method	Natural Accuracy			PGD-10			PGD-1000			C&W-1000			AutoAttack		
	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff
PGD-AT	83.75	84.82	-1.07	52.64	44.92	7.72	51.22	42.74	8.48	50.11	43.63	7.48	47.74	41.84	5.90
PGD-AT+TE	82.35	82.79	-0.44	55.79	54.83	0.96	54.65	53.30	1.35	52.30	51.73	0.57	50.59	49.62	0.97
TRADES	81.19	82.48	-1.29	53.32	50.25	3.07	52.44	48.67	3.77	49.88	48.14	1.74	49.03	46.80	2.23
TRADES+TE	83.86	83.97	-0.11	55.15	54.42	0.73	53.74	53.03	0.71	50.77	50.63	0.14	49.77	49.20	0.57

(a) The evaluation results on **CIFAR-10**.

Method	Natural Accuracy			PGD-10			PGD-1000			C&W-1000			AutoAttack		
	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff
PGD-AT	57.54	57.51	0.03	29.40	21.75	7.65	28.54	20.63	7.91	27.06	21.17	5.89	24.72	19.34	5.38
PGD-AT+TE	56.45	57.12	-0.67	31.74	30.24	1.50	31.27	29.80	1.47	28.27	27.36	0.91	26.30	25.34	0.96
TRADES	57.98	56.32	1.66	29.93	27.70	2.23	29.51	26.93	2.58	25.46	24.42	1.04	24.61	23.40	1.21
TRADES+TE	59.35	58.72	0.63	31.09	30.12	0.97	30.54	29.45	1.09	26.61	25.94	0.67	25.27	24.55	0.72

(b) The evaluation results on **CIFAR-100**.

Method	Natural Accuracy			PGD-10			PGD-1000			C&W-1000			AutoAttack		
	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff	Best	Final	Diff
PGD-AT	89.00	90.55	-1.55	54.51	46.97	7.54	52.22	42.85	9.37	48.66	44.13	4.53	46.61	38.24	8.37
PGD-AT+TE	90.09	90.91	-0.82	59.74	59.05	0.69	57.71	56.46	1.25	54.55	53.94	0.61	51.44	50.61	0.83
TRADES	90.88	91.30	-0.42	59.50	57.04	2.46	52.78	50.17	2.61	52.76	50.53	2.23	40.36	38.88	1.48
TRADES+TE	89.01	88.52	0.49	59.81	58.49	1.32	58.24	56.66	1.58	54.00	53.24	0.76	51.45	50.16	1.29

(c) The evaluation results on **SVHN**.



Thanks for listening

Code is available at:
<https://github.com/dongyp13/memorization-AT>

